Exploring the Cognitive Demand and Features of Problem Solving Tasks in Primary Mathematics Classrooms

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Student learning is greatest in classrooms where students engage in problem solving tasks that are cognitively demanding. However, there are growing concerns that many Australian students are given limited opportunities to engage in these types of tasks. 108 upper primary school teachers were surveyed to examine task features and cognitive demand of problem solving tasks. Teachers are found to favour tasks with lower-level cognitive demand above those with higher-level cognitive demand when developing problem solving proficiency.

There is growing evidence that students learn best when they are presented with academically challenging tasks that focus on problem solving and reasoning (Kilpatrick, Swafford, & Findell, 2001). Guided by this evidence, the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2010) provides a new framework for teaching mathematics in an attempt to enhance student learning outcomes and address concerns regarding the "syndrome of shallow teaching" (Stacey, 2003, p. 119). Not only does the curriculum provide a new definition for problem solving as one of four *proficiency strands* (Understanding, Problem Solving, Reasoning, and Fluency) but it also provides a clear rationale for the teaching of problem solving. However despite the amount of policy advice and resources to support problem-solving practices, there are growing concerns that many Australian students are given limited opportunities to engage in problems other than those of low procedural complexity. Stacey (2003) contends that students are often asked to follow procedures without reason, solve low complexity problems with excessive repetition and given limited opportunities for reasoning or classroom discourse. The following report investigates this phenomenon through research undertaken as part of the *Encouraging Persistence, Maintaining Challenge* (EPMC) project. This study aims to explore how primary teachers describe their efforts to engage students in complex problem solving through their choice of task. In particular the features of problem solving tasks and the likely level of cognitive demand are examined.

Problem Solving Tasks

The type of tasks that teachers choose to use in the mathematics classroom not only determine what content students learn but also how they think about, use, develop, connect, and make sense of mathematics, in essence, their opportunities to learn (Stein, Grover, & Henningsen, 1996).

Students benefit from opportunities to engage in tasks that are problematic in nature, that may take some time to solve, as they mirror the unfamiliar, challenging, multifaceted problems in the real world for which we are preparing them (National Council of Teachers of Mathematics, 2014). Such tasks are characterised by features that not only influence students thinking processes and behaviour but ultimately their opportunity to learn (Stein,

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Grover, & Henningsen, 1996). The theoretical model by Carpenter and Fennema (1991) reflects this notion and emphasises that teaching does not *directly* influence student learning but instead influences student thinking, which, in turn, influences their learning. This model provides a framework for this research, which focuses on how teachers prioritise student thinking when choosing tasks with specific features and cognitive demand to support student learning.

Features of Problem Solving Tasks

There have been many attempts to research and classify problem tasks according to their recommended purposes and associated features. Some of the earlier classifications include application tasks but more recently the focus on problems that require high-order thinking, such as open-ended and unfamiliar problems have been promoted. Anderson (2003) delineated these features as follows:

- Application tasks are commonly found in textbooks and may be routine or non-routine in nature. Typically these tasks are preceded by direct instruction and examples are provided of the topic currently studied.
- Open-ended tasks have multiple solutions, varied pathways to solve and record the solution, and are non-routine in nature.
- Unfamiliar tasks are closed, not regularly encountered and involve non-routine problems.

Many scholars contend that "*real* problem solving" involves working on problems that are unfamiliar and out of context (Kilpatrick, Swafford, & Findell, 2001; Schoenfeld, 1992). Likewise, Anderson (2000) argues "open-ended and unfamiliar problems offer real challenges to students and provide rich tasks to be used in problem-solving contexts" (p. 44).

Other features include real world investigation tasks that incorporate real life problems (relating what is learned in mathematics to their daily lives) and are argued to enhance student motivation and competencies in mathematical problem solving (English & Sriraman, 2010). These types of tasks demand a higher degree of thinking and advocated by the Victorian Department of Education (2015) website. Students are required to identify the operations, clarify the question, identify the assumptions, reason and justify their answers, and work on problems over an extended period of time.

In an analysis of mathematical tasks Stein, Grover, and Henningsen (1996, p. 456) summarises the particular task features used in United States reform classrooms saying:

> One also finds consistent recommendations for the exposure of students to meaningful and worthwhile mathematical tasks, tasks that are truly problematic for students rather than simply a disguised way to have them practise an already-demonstrated algorithm. In such tasks, students need to impose meaning and structure, make decisions about what to do and how to do it, and interpret the reasonableness of their actions and solutions. Such tasks are characterized by features such as having more than one solution strategy, as being able to be represented in multiple ways, and as demanding that students communicate and justify their procedures and understandings in written and/or oral form.

Connected to this, Silver and Stein (1996) state that the above mentioned task features, described by Stein, Grover, and Henningsen (1996), have long been associated with conceptions of mathematical understanding and proficiencies, and similarly associated with cognitive flexibility and the ability to view problems from multiple perspectives – both characterised as high order thinking and reasoning.

While some features are more likely to provide opportunities to engage students in high order thinking all of the features mentioned above can vary in complexity. Therefore analysis of these tasks "must go beyond superficial features to focus on the kinds of thinking in which students must engage to complete the task" (Smith & Stein, 1998, p. 345).

Cognitive Demand

A number of studies have found that the highest learning gains are related to the extent to which tasks are set up and implemented to engage students in high levels of cognitive thinking and reasoning (Smith & Stein, 1998). Therefore, choosing tasks that engage students in higher-levels of thinking through problem solving and reasoning is an important aspect of teachers' decision-making. Smith and Stein (1998) have attempted to differentiate task features and developed a taxonomy of mathematical tasks based on the level of *cognitive demand* of the task. They identify four levels of cognitive demand:

Lower-level cognitive demands (memorisation). Routine exercises that involve the memorisation of formulas, algorithms, or procedures, and without connection to the underlying concepts or meaning.

Lower-level cognitive demands (procedures without connections). Tasks that are algorithmic and focus solely on describing the procedure that was used. Little ambiguity exists about what needs to be done and how; and there is no connection to the concepts or meaning that underlie the procedure being used.

Higher-level cognitive demands (procedures with connections). Tasks that focus on the use of broad general procedures for developing deeper understating of concepts and ideas. These tasks are usually represented multiple ways and require a degree of cognitive effort to complete the task successfully.

Higher-level cognitive demands (doing mathematics). Tasks that require complex and non-algorithmic thinking to explore and understand the nature of mathematical concepts processes and relationships. These tasks may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

When exploring the implementation of higher-level cognitively demanding tasks, Smith and Stein (1998) found that many teachers experienced difficulty implementing these tasks, and while the task features remained the same, the level of cognitive demand tended to decline after implementation. It appears that the challenge for teachers appears to not only lie in the *choice* of task, but *implementing* these tasks to ensure the challenge is maintained. Pedagogical approaches for maintaining challenge are beyond the scope of this paper but may be of relevance to the future direction of the study.

Reflected in this taxonomy and convincingly argued by many mathematics educators and philosophers, is the notion that in order to have a complete understanding of mathematics, students need more than just knowledge of mathematical concepts, principles and structure. It is not enough to simply memorise formulas and apply procedures. Complete understanding is achieved when students develop deep and interconnected understanding through: "framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging and so on" (Stein, Grover, & Henningsen, 1996, p. 456). Therefore, challenging tasks that focus on problem solving and reasoning, have multiple entry points, and varied solution strategies, potentially provide the best opportunities for students to learn mathematics and prepare them for their future lives. This premise was explored through the overarching research question that guided the research: *How do primary teachers describe their efforts to integrate problem solving through their choice of task?* The above mentioned task features (Anderson, 2003; Victorian Department of Education, 2015) and taxonomy of mathematical tasks (Smith & Stein, 1998) are used as a framework for analysis.

Survey Design

The survey was part of a larger project titled *Encouraging Persistence Maintaining Challenge* (Sullivan et al., 2014), an Australian initiative that explores the use of challenging tasks as the basis of classroom lessons. The EPMC project utilises a design-based methodological approach that entails both engineering specific forms of learning and systematically studying them within the assigned context. Anderson and Shattuck (2012) describe design-based research as an approach that uses mixed methods, comprises of multiple iterations, and involves a collaborative partnership between researchers and practitioners. The EPMC intervention involved the presentation of mathematical tasks and supporting pedagogies for teachers to pose to their students. These tasks would match what the teachers intended to teach and data was gathered from the teachers and students on their experience.

The present study is an informed exploration of teachers' work related to the problem solving proficiency, and findings will be used to guide and inform further research and the development of an intervention. Therefore, involvement in the EPMC project was limited to accessing the initial online survey over two separate EPMC iterations in 2015. Both surveys were administered using Qualtrics (2015) during the first face-to-face professional development day and before the EPMC intervention. I constructed one open-response survey items, with the intent of establishing some base line characteristics of teacher priorities. The item was designed to prompt self-reports to increase the likelihood that responses were honest and accurate portrayals of teachers planning, decision-making and practice. There were more items included in the survey but they are not reported in this paper. Additionally, there were other items that identified the level and school of each teacher to assist the analysis. The open-response survey item asked teachers to: "Describe a task you taught in 2015 that included a substantial expectation of student problem solving." This item was constructed to gain an insight into the types of tasks that teachers are using, and to further identify if the issue regarding the syndrome of shallow teaching continues to be evident. The task features described by Anderson (2003) and the Victorian Department of Education (2015) along with the taxonomy of cognitive demand described by Smith and Stein (1998) informed the questions posed and are used to analyse and categories the data collected.

Survey responses are first sorted by student year level, inspected for relevance to the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2015), then categorised with the above-mentioned task features and levels of cognitive demand. They are then sorted into school teams, and tasks were re-examined for similarities in task descriptions and re-categorised. Given my teaching background I was able to recognise and locate the described tasks in popular texts such as Macmillan, Nelson Maths, Maths300, and Nrich Maths to assist in data analysis. The categories of responses along with their interpretations are presented below.

Results

Data was collected from three groups of teachers in New South Wales, Victoria and Tasmania from a combination of Years 4, 5, and 6. Participants included 45 teachers in NSW, 8 teachers in Tasmania and 55 teachers in Victoria, who taught in a range of primary schools.

Table 1 presents the findings from the survey item. The left hand column presents the task features along with the correlating level of cognitive demand.

Table 1:
Features of problem solving tasks and associated level of cognitive demand (n=108)

	Lower-Level Memorisation	Lower-Level Procedures without connection	Higher-Level Procedures with connections	Higher-Level Doing Mathematics	Total
Application	0	44	1	0	45 (42%)
Real World Investigations	0	19	9	2	30 (28%)
Open-ended	0	1	6	15	22 (20%)
Unfamiliar	0	0	2	9	11 (10%)
Total		64 (59%)	18 (17%)	26 (24%)	108

The findings indicate that a higher proportion of teachers (59%) reported using problem-solving tasks that are of lower-level cognitive demand (*procedures without connections*); 44 of which are application tasks, 19 real world investigation tasks, and 1 open-ended task. While overall there are 45 teachers who described application type tasks, 44 involved *procedures without connections*. Application task 1 presents an example response with classification.

Application task 1: Example response and classification

> At xxxx we use the Macmillan problem solving tasks. At team planning each week we select a task to focus on and we dedicate a 30 min time slot every week to explicitly completing a problem-solving card and introducing a new strategy to assist with problem solving.

This task is classified as an application tasks as the procedures are explicitly taught before applying on routine problems. The task is of lower-level cognitive demand (*procedures without connections*); as it is algorithmic and presents no connection to the concepts or meaning that underlie the procedure. Little ambiguity exists to which students know what they need to do to solve the problem. The task is focussed on producing correct answers and does not require an explanation. This task requires limited cognitive demand for successful completion.

Tasks with lower-level cognitive demand, such as application and real world investigation tasks, are more commonly reported to develop problem solving proficiency. This exemplifies a traditional approach to teaching where knowledge and skills are transferred, rather than constructed. One interpretation is that teachers believe they must first 'teach' students the concepts and procedures, and provide opportunities to practise before students are able to solve cognitively demanding problems successfully.

In contrast, 24% of teachers reported using problem-solving tasks that are of higher-level cognitive demand (*doing mathematics*); 15 are open-ended, 9 unfamiliar, and 2 real world investigation tasks. It is imperative to note that 16 of these responses are exact replications of past EPMC project tasks and presents three implications. First, the EPMC project has had an influence on some teachers practice and understanding of tasks that promote problem solving. Second, teachers may have described EPMC tasks due to cognitive bias, which resulted in socially desirable responses. Third, few teachers who were independent of the EPMC gave responses that are of higher-cognitive demand. Of the 16 EPMC tasks reported, 7 are open-ended and of higher-level cognitive demand (see EPMC task 1), and 9 are unfamiliar and of higher-cognitive demand (see EPMC task 2).

EPMC tasks 1: Example response and classification:

> We did a fractions lesson that was looking at addition and subtraction of fractions. I gave the class 6 single digit numbers and they had to come up with as many problems as they could that equalled 2. The task involved students investigating a number proper and improper fractions and adding/subtracting them against another fraction to achieve the answer, 2.

This task is classified as open-ended as there are a number of different solutions to this task, thus students can engage with the task at different levels. The task is classified to be of higher-level cognitive demand (doing mathematics), as the task requires students to explore and understand the underlying concepts, processes and relationships in order to solve the problem. Due to the varied solution pathways that are not explicitly suggested, and students need to access relevant knowledge and experiences to work on the task. The task requires a considerable amount of cognitive effort to solve the problem and involves some level of anxiety.

EPMC task 2: Example response and classification

> A fractions problem in a year 3/4 class. Students were given the following problem: 'Twins Ann and Josh had a birthday pizza party. The 4 girls shared 6 pizzas while the 6 shared 9 pizzas. Who got more pizza, the boys or the girls.' Students worked in pairs or triples using materials, drawing or paper folding to solve the problem. At various points in the lesson we stopped and shared students thinking.

This task is classified as unfamiliar as it involves a non-routine problem that students in Grade 3/4 are unlikely to have encountered before. In reference to the Australian Curriculum: Mathematics (ACARA, 2015) achievement standards, the concepts and procedures in this tasks are likely to be unfamiliar to Grade 3/4 due to their limited experience with fractions. The task is of higher-level cognitive demand (doing mathematics) as there are varied solution pathways that are not explicitly suggested, and students need to explore and understand the underlying concepts, processes and relationships in order to solve the problem. The task is not open-ended although requires a considerable amount of cognitive effort that may also involve some level of anxiety for the students.

Furthermore, 17% of teachers reported using tasks that are of higher-level cognitive demand (*procedures with connections*); 9 are real world investigation tasks, 6 are open-ended, 2 unfamiliar, and 1 application task. While real world investigation tasks have the potential to prioritise high order thinking, majority are of lower-level cognitive demand. An example of a real world investigation task with high-level cognitive demand is seen below with classification (see real world investigation task 1).

Real world investigation task 1: Example response and classification

> Students designed a venue/theatre/stadium that had a capacity of 'x' (depending on growth point). As the task developed over the week, students needed to prove that the venue had that capacity by recording it in a creative way, sell tickets at a designated cost (depending on seat location), experiment with various shapes...

This task is classified as a real world investigation tasks as it involves a real world problem and developed over a weeklong period. The task is of higher-level cognitive demand (procedures with connections), as students need to identify a possible solution pathway by engaging with the conceptual ideas that underlie the procedure. Although general procedures may be followed, they cannot be followed without careful consideration. This tasks can be represented in multiple ways and require a degree of cognitive effort.

While it is evident from Table 1 that the tasks overlap with the level of cognitive demand, there is a distinct pattern suggesting that both application and real world investigation tasks are commonly of lower-level cognitive demand, whereas open-ended and unfamiliar tasks are more likely to be of the higher-level cognitive demand. Despite the EPMC's influence on some teachers responses, very few teachers reported using tasks that are open-ended, unfamiliar and of higher-level cognitive demand.

Discussion and Conclusion

The present study explored how primary teachers describe their efforts to integrate problem solving through their choice of task, and in particular examined the features and cognitive demand of reported tasks. Overall, direct instruction followed by application tasks, as well as real world investigation tasks, are more favourable among the participating teachers when developing problem solving proficiency in the classroom. These tasks tended to be of lower level cognitive demand (*procedures without connections*) requiring previously taught facts and procedures to be applied on routine problems. While real world investigation tasks have the potential to promote higher order thinking, the complexity of the chosen tasks appear to be reduced as identified by Smith and Stein (1998) and previously discussed. Furthermore, participation in the EPMC project appears to have influenced some teachers' problem solving practices to the extent that teachers are choosing and implementing challenging and unfamiliar tasks.

From these findings it appears that students are given limited opportunities to solve challenging and unfamiliar problem solving tasks. As described by Stacey (1999), the notion of shallow teaching continues to be evident in the reported practice, where students are asked to follow procedures without connections and solve low complexity problems with excessive repetition. Not only will students be unable to develop complete mathematical understanding, but also their ability to solve challenging and unforeseen problems in the real world will be greatly compromised. Further research is required to explore this phenomenon. Furthermore, it appears that interventions such as the EPMC project have provided experiences that are impactful and long lasting for teachers. This particular finding could be of interest to task designer and those in leadership roles. This study contributes to and extends on Anderson (2003), Kilpatrick, Swafford, and Findell (2001) and Stein, Grover, and Henningsen (1996) to explore primary teachers problem-solving practices. The research is an initial effort to gain an insight into the types of problem solving tasks used in classrooms. It is acknowledged that for a more accurate portrayal of teachers' practices it is beneficial to augment survey data with observational data. This is the future direction of the research.

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