# Singapore Enactment Project

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The *Enactment Project* is a Programmatic Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2016 and its aim is to study the enactment of the Singapore mathematics curriculum across the whole spectrum of secondary schools within the jurisdiction. There were two phases in the project: the first involved in-depth examination of 30 experienced and competent mathematics to draw out characteristics of their practices; in the second phase, we study the extent of these characteristics through a survey of 677 mathematics teachers. A symposium was organised in MERGA 42 in 2019 where the foundational elements of this project were presented; we would like to share more findings of this project in this year's conference.

**Paper 1:** Berinderjeet Kaur *Models of mathematics teaching practice in Singapore secondary schools*

This paper revisits the models of mathematics teaching practice that were proposed by earlier researchers of the Singapore mathematics classrooms: Traditional Instruction (TI), Direct Instruction (DI), and Teaching for Understanding (TfU). The data from the survey in this project point to hybridisation of these models.

**Paper 2:** Tin Lam Toh *An experienced and competent teacher's instructional practice for normal technical students: A case study*

This paper presents a case of how an experienced and competent teacher engaged mathematics "low-attainers" in the learning of mathematics in a way that was responsive to their learning needs while upholding the ambitious goal of helping them acquire relational understanding of mathematical concepts.

**Paper 3:** Joseph Boon Wooi Yeo *Imbuement of desired attitudes by experienced and competent Singapore secondary mathematics teachers*

One of the components of the Singapore Pentagonal curricular framework is "Attitude". This paper presents findings of a survey that point to specific strategies used by Singapore mathematics teacher to imbue positive attitude towards mathematics in their students.

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**Paper 4:** Yew Hoong Leong & Lu Pien Cheng *Singapore mathematics teachers' design of instructional materials*

Case studies based on the data in Phase 1 of the project revealed that the teachers crafted their own instructional materials based on modifications of reference materials. This paper summarises some of the moves teachers adopted when designing instructional materials for their lessons.

# Models of mathematics teaching practice in Singapore secondary schools

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A model of instruction is a set of strategies that guide teachers in their instructional practice. The purpose of this paper is to dispel the myth that mathematics teaching in Singapore schools is all about drill and practice, as perceived of many Asian systems. This paper draws on data of a large project that examined the enactment of school mathematics curriculum in Singapore secondary schools. Based on the teaching practices of 30 experienced and competent teachers, a survey was constructed and administered to 677 teachers. The data from the survey showed that teachers go well beyond traditional forms of instruction in their teaching practices in Singapore secondary schools.

Leung (2001) noted that in East Asian mathematics classrooms

> Instruction is very much teacher dominated and student involvement minimal. … [Teaching is] usually conducted in whole group settings, with relatively large class sizes. … [There is] virtually no group work or activities, and memorization of mathematics is stressed … [and] students are required to learn by rote. … [Students are] required to engage in ample practice of mathematical skills, mostly without thorough understanding. (Leung, 2001, pp. 35–36).

Hogan et al. (2013) examined the instructional practices of Grade 9 mathematics teachers and found that several models of instruction were prevalent in the practices. All of which had the goal of mastery and examination preparation. In a synthesis of past mathematics classroom studies done in Singapore, Kaur (2017) conjectured that instructional practices for mathematics in Singapore classrooms, based on the data of the study by Hogan et al. (2013) and the Learners Perspective Study carried out in Singapore (Kaur, 2009), cannot be considered either Eastern or Western but a coherent combination of both. Basis of the claim is that: i) Traditional Instruction (TI) provides the foundation of the instructional order, and ii) Direct Instruction (DI) builds on TI practices and extends and refines the instructional repertoire. While Teaching for Understanding/ Co-regulated Learning Strategies (TfU/CRLS) practices build on TI and DI practices and extend the instructional repertoire even further in ways that focus on developing student understanding and student-directed learning. The study reported in this paper further illuminates models of teaching practices of mathematics teachers in Singapore secondary schools.

## The Study

The study reported in this paper is part of a larger project, details of which are available elsewhere (Kaur et al., 2018; Toh et al., 2019). A study of mathematics lessons enacted by 30 experienced and competent mathematics teachers in Singapore secondary schools revealed that teacher and student actions from three main models of instruction were guiding teachers in their instructional practice. We elaborate the models and provide examples of teacher and student actions that were observed in the lessons of the experienced and competent teachers (which are marked *) as well as those that were not but were included in the survey. For actions that are marked * we also indicate the respective courses of study which are Integrated Programme (IP), Express Course (EX), Normal (Academic) Course

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(NA) and Normal (Technical) Course (NT) where the actions were observed. The IP is for the mathematically able students and the NT is for the least able ones.

### *Traditional Instruction (TI)*

A method of instruction that is teacher-centred, rather than learner-centred, in which the focus is on rote-learning and memorisation. In the context of Asian classrooms it is often associated with drill and practice (Biggs & Watkins, 2001; Hogan et al., 2013; Leung, 2006). There were altogether 13 TI teacher actions, and examples of two such actions are as follows:

Teacher –

- *asking students direct questions to stimulate students' recall of past knowledge / check for understanding of concepts being developed in the lesson (EX, NA)
- *providing students with sufficient questions from textbooks / workbooks / other sources to practise so as to develop procedural fluency (EX, NA, NT)

### *Direct Instruction (DI)*

A method of instruction that involves an explicit step-by-step strategy, often teacher-centred, with checks for mastery of procedural or conceptual knowledge (Hattie, 2003; Hogan et al., 2013; Good & Brophy, 2003). There were altogether nine teacher actions and two student actions and examples of two each are as follows:

Teacher –

- *using the "I do, We do, You do" strategy, i.e.
    - o Demonstrating how to apply a concept / carry out a skill on the board [I do]
    - o Demonstrating again the same using another similar example but with inputs from students [We do]
    - o Asking the students to do a similar question by themselves [You do] (EX, NA, NT)
- *explaining what exemplary solutions of mathematics problems must contain (logical steps and clear statements and / or how marks are given for such work during examinations) (IP, EX, NA)

Students –

- *asking questions when they do not understand (IP, EX, NA, NT)
- *practising a similar problem after the teacher has shown them how to do a similar one on the board (IP, EX, NA, NT)

### *Teaching for Understanding (TfU)*

A method of instruction that places student learning at the core. Teacher facilitates, monitors and regulates student learning through student-centred approaches (Hogan et al., 2013; Good & Brophy, 2003; Perkins, 1993). There were 13 teacher actions and 15 student actions, and examples of two each are as follows:

Teacher –

- *focusing on mathematical vocabulary (such as equations, expressions) to help students build mathematical concepts (IP, EX, NA, NT)
- *providing collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework (IP, EX, NA, NT)

Students –

- *explaining how their solutions or how their answers are obtained (IP, EX, NA, NT)
- *discussing and helping each other while doing individual seatwork (IP, EX, NA, NT)

### *The Survey*

The survey had three parts. The first part had 60 items (36 describing teacher actions and another 24 describing student actions). Amongst these items were the seven items on TI, 11 items on DI and 28 items on TfU. In the survey, teachers were asked to reflect on their lessons for a course (IP, EX, NA or NT) they were teaching, and respond to the items indicating the frequency of their actions on a Likert Scale of 1 (Never/Rarely) to 4 (Mostly/Always). 691 teachers completed the survey. In the preliminary screening of the data, some responses were removed as they did not meet the requirements of the survey. The data of 677 teachers were used for subsequent analyses. Forty percent of the teachers were male while 60 % were female and this was representative of the demographic of the teacher population in secondary schools which were 36 % males and 64 % females (MOE, 2018). In addition, the representation by course of study, almost 65% for the IP and EX, and 35% for the NA and NT courses was also coherent with the demographic of the student population in secondary schools which was 64% and 36% respectively for the IP and EX and NA and NT courses (MOE, 2018). Forty-five percent of the teachers had more than three but less than 10 years of mathematics teaching experience while the rest 55% had more than 10 years of the same experience.

### *What models of instruction guide mathematics teaching in the classrooms of mathematics teachers in Singapore secondary schools, in general?*

Table 1
*Means of the three models of instruction*

| Course of Study | Mean+ Model of Instruction | | |
|---|---|---|---|
| | TI | DI | TfU |
| All (n=677) | 2.78 | 3.11 | 2.86 |
| Integrated Programme (IP) (n=58) | 2.42 | 3.07 | 3.00 |
| Express (EX) (n=380) | 2.78 | 3.10 | 2.88 |
| Normal (Academic) (NA) (n=151) | 2.81 | 3.10 | 2.77 |
| Normal (Technical) (NT) (n=88) | 2.94 | 3.17 | 2.85 |

+maximum = 4; minimum = 1.

Table 1 shows that teachers appear to draw on teaching moves from all the three models of instruction, though with differing emphasis to enact their lessons. Direct Instruction appears to be the dominant model that teachers draw on in all the four courses of study. In the NA and NT classes, Direct Instruction and Traditional Instruction are apparently more prevalent whilst in the IP and EX classes Direct Instruction and Teaching for Understanding are apparently more prevalent. We next examined the survey items for each course of study that had a mean greater than 3 and a standard deviation of less than or equal to 0.7. The following teaching/learning actions were found to be common across all the four courses of study.

- Teacher providing students with sufficient questions from textbooks / workbooks / other sources to practise so as to develop procedural fluency
- Students asking questions when they do not understand
- Teacher walking around the class and providing students with between-desk instruction (i.e. help them with their difficulties) when they are doing their work at their desks
- Teacher walking around the class noting student work that teacher would draw on to provide the class feedback during whole class review
- Teacher only progressing to the next objective of the lesson when he/she is confident that students have grasped the one before
- Teacher providing feedback to individuals for in-class work and homework to serve as information and diagnosis so that students can correct their errors and improve
- Teacher providing collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework
- Teacher focusing on mathematical vocabulary (such as factorise, solve) to help students adopt the correct skills needed to work on mathematical tasks
- Students explaining how their solutions or their answers are obtained

We conclude that the model of instruction that mathematics teachers in Singapore secondary schools adopt is a hybrid one comprising TI, DI and TfU. This finding lends to strengthen our earlier conjecture that mathematics instruction in Singapore secondary schools is neither Eastern nor Western but a coherent combination of both, i.e. a hybridisation of TI, DI and TfU.

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# An experienced and competent teacher's instructional practice for normal technical students: A case study

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This paper presents a case study of an experienced and competent mathematics teacher's classroom instructional practice in a Normal Technical Mathematics course. The topic that was observed was Volume and Surface Area of a Pyramid, a subtopic within the mensuration topic in Secondary Two syllabus. The teacher used a video clip on the Egyptian Pyramids to integrate students' prior knowledge on pyramids, which raised their attention on the topic. This was followed by engaging the students in hands-on activity to understand the formulae.

The case study is part of the larger research project on enactment of the curriculum in the mathematics classroom as reported by this symposium.

## Low Attaining Students

Studies have shown that low attaining students are generally visual and kinaesthetic learners (e.g. Amir & Subramaniam, 2007; Rayneri & Gerber, 2003). The mainstream education programmes worldwide are usually more theory-based than skill-based with ample hands-on opportunity for individual learners (Glass, 2003). Therefore, it is not at all surprising that this dissonance puts the low attaining students, who usually learn best through visual and physical engagement, at a disadvantage in the education system.

Low attaining students generally have little interest in academic subjects. They lack focus during lessons, have short attention span and hence tend to be restless in classes (Lui et al., 2009). Thus, typical teacher-centric teaching approaches might not be most appropriate for them. Myron and Keith (2007) stressed that in order for teachers to be more successful in working with the low attaining students, they must be more cognizant of the various learning styles of their students and attempt different teaching approaches for different groups of students.

## Normal Technical Students in Mathematics

Singapore mathematics teachers are genuinely concerned about the performance in mathematics among the Normal Technical students (Toh & Lui, 2014). This concern is not unfounded as many of the Normal Technical mathematics students exhibit many of the characteristics of low attainers (Toh & Kaur, 2019).

Studies have also shown that Singapore teachers are not passively using traditional instructional materials and resource for teaching Normal Technical students. As the students' difficulties with mathematics and reasons for their lack of interest in the subject are various, teachers' effort to reach out to this group of students is also diverse. In addition to honing their pedagogical skills in the classrooms, teachers are also actively adapting less conventional instructional approaches and developing unconventional instructional material to address the learning needs of this group of students (Toh & Lui, 2014).

To have a first-hand glimpse into how mathematics lessons are conducted by a experienced and competent teacher in a typical Normal Technical class, the author (hereafter, first person pronoun) followed through one such identified teacher's lessons for two weeks on teaching a subtopic of mensuration in a Secondary Two Normal Technical

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mathematics class in a Singapore mainstream school. A few striking observations that were made will be reported in this paper.

## Method

All the lessons that were observed in this study, the teacher interview, and the student interviews were video-recorded and transcribed. The video-recording, adapting the Complementary Accounts Methodology of Clarke (1998, 2001), used three video cameras to focus on: (1) the classroom as seen from the teacher's perspective; (2) the activity of two particular students in each lesson; and (3) the classroom from the perspective of an observer at the back of the classroom.

The teacher, Lucy-Marianne (pseudonym), was identified as an experienced and competent mathematics teacher by the mathematics education community. She was a Senior Teacher in her school, in her mid-forties at the time of our study, had more than ten years of experience teaching in the school and had been teaching mathematics in Express, Normal Academic and Normal Technical stream for more than fifteen years at the time when this study was conducted. In a discussion with her during the teacher interview, she expressed her passion in teaching the group of low attaining students. According to Lucy-Marianne, this group of students "deserved our attention more". She was trained to teach both Mathematics and Computer Applications.

## Observation and Discussion

In unpacking teacher Lucy-Marianne's pedagogical practices from the entire set of video-recordings of her lessons, a very skilful scaffolding sequence to facilitate her students in understanding a complex concept was visible:

1. she first elicited her students' prior knowledge related to the concept;
2. she aroused her students' interest about the concept;
3. she built on their induced interest to further develop the mathematics concept;
4. she engaged her students in hands-on activities to "derive" the formula; and
5. she gave students ample opportunity to practise the application of the formulae.

During the teacher interview, she revealed that this was the constant sequence in teaching the other mathematical topics as well as to her Normal Technical students.

### *Eliciting students' prior knowledge*

Her teaching of the subtopic on surface area and volume of a pyramid is the focus here. She built on her students' prior knowledge selectively for her lesson development, as illustrated by a portion of the dialogue below. Letters T and S denote the teacher and student participant.

| Dialogue | | Commentary |
|---|---|---|
| (after housekeeping matter) | | |
| T: | Now let's move on to volume of pyramid – uh no, surface of pyramid [first]. OK by the way, let me introduce the word "pyramid". **What is [a] pyramid**? | *Teacher elicited her students' prior knowledge on pyramid.* |
| S: | A 3D. | |
| T | OK It's a 3-dimensional object... A pyramid is no longer flat [tapped the table],it's no longer flat [tapped whiteboard], but it's a 3-dimensional object. But **what does it look like and how does it look like…**? | *Teacher responded to a student's use of the term 3D (3 dimensional) by distinguishing between 3D and 2D objects (prior to this lesson, the students learnt mensuration of circle – a 2D object).* |
| S: | Cone. | |

| | | |
|---|---|---|
| T: | It looks something like a cone. Oh, OK... "Looks something like" doesn't mean it's exactly the same. So later we are going to learn cone, today let's talk about pyramid. **Anyone can describe pyramid?** | *Teacher was careful to acknowledge the response that a pyramid looks something like a cone, but did not want to elaborate the concept of cone to avoid confusing the students (mensuration of a cone would be the next subtopic).* |
| | ::::::::::: | |
| S: | Huh? Oh triangle. | |
| T: | Thank you … they (two students) are right... There are triangles on pyramids. So this is a pyramid like what you see in Egypt. Now, **if I were to look from top down, what do you think is on the ground?** What shape?<br>S: I know, a square. | *Teacher elicited the responses of "triangle" and "square" from the students about their knowledge of pyramid. However, teacher did not further elaborate that the bases of pyramids can be made of other shapes at this juncture.* |

### *Arousing students' interest and curiosity in the concept.*

Teacher Lucy-Marianne skilfully related the geometrical figure of a pyramid to the Egyptian Pyramids at Giza. She discussed the historical function of the Egyptian Pyramids after showing a short video clip selected from YouTube about the Egyptian Pyramids. The content of the video clip raised students' awareness of mathematics in the real world; this is aligned to the Ministry of Education (MOE)'s desire to "prepare its citizens for a productive life in the 21st century" (MOE, 2012, p. 2). The selected video covered the students' responses: the sides of the pyramids (consisting of triangles), the plan view of the pyramids (squares), the dimensions and the historical functions of the pyramids. The use of videos in education is particularly useful for low attaining students, as it has the ability to reduce their cognitive load and facilitate their understanding of abstract concepts (Han & Toh, 2019)

### *Reinforcing the concept of the lateral side faces of a pyramid.*

Teacher Lucy-Marianne emphasized the sides and base of a pyramid from different angles and by decomposing a three-dimensional pyramid into two-dimensional parts. Teacher Lucy-Marianne next used a worksheet (Figure 1) to reinforce the identification of the sides. Here, she unravelled the next part of the "truth" that the base of a pyramid is not necessarily a square or rectangle. She introduced pyramids with various polygonal bases. This was also the first time she insisted on the precise mathematics terminologies (lateral sides and base of a pyramid) illustrated in the dialogue below Figure 1.

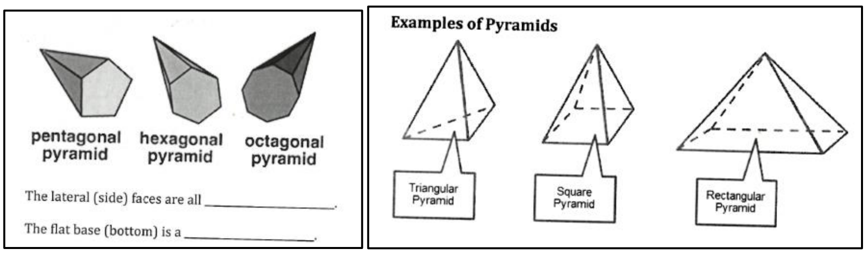


*Figure 1.* A portion of the worksheet used by Lucy-Marianne in introducing the faces of a pyramid

T: I want you to look at the word, the lateral side faces are? The word 'lateral' means side. Side means lateral. So the side faces are what kind of shape? … I will like to introduce a word, the flat base water missile, I call it 'polygon'. Polygon means it can be 3 sides, 4 sides, 5 sides, 6 sides, 7, 8, etc.

### *Deriving the procedure for calculating the total surface area of a pyramid.*

The video clip and the identification the various parts of the pyramid led to the calculation of the surface area of a pyramid by considering the nets of a pyramid. She engaged her students in deriving the formulae using a hands-on approach by engaging them to cut up a pyramid into its nets to identify the total surface area of a pyramid as the sum of the areas of the polygons in its corresponding net. This "experimental derivation" was observed in her lessons throughout this subtopic. In determining the volume of a pyramid in the succeeding subtopic, Lucy-Marianne conducted a "laboratory lesson" to demonstrate the relation between the volume of a pyramid and its related prism. The topic mensuration at the secondary level can be taught either in a very procedural manner, or one that engages the students with hands-on activities as proposed by Lim-Teo and Ng (2008). Teacher Lucy-Marianne had chosen the latter to better match the needs of her students.

Ample opportunity to practice. As in other observation of the Singapore classrooms, teacher Lucy-Marianne designed her worksheets to give sufficient structured and guided practice for her students. This will not be elaborated in this paper.

## Conclusion

This is an episode of teaching mathematics to Normal Technical students by an experienced and competent teacher. While the teacher was cognizant of the importance of maintaining the rigor of the mathematics curriculum even for the low attaining students, the teacher was also skilful in engaging her students in activating their prior knowledge, exciting them with the mathematics in the real-world, and chunking up big group of mathematical content into manageable bites for her students. The teacher strove to develop in her students a relational understanding of the mathematical concepts through appropriate student engagement, while using video clip and storytelling to excite her students in the mathematical concepts. The lesson was evidence of her attempt at striking a balance between developing her students' cognitive and affective aspects of learning.

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# Imbuement of desired attitudes by experienced and competent Singapore secondary mathematics teachers

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This paper reports how 30 experienced and competent Singapore secondary mathematics teachers attempted to imbue desired attitudes in their students and some possible factors that might have influenced the teachers' choice of instructional approaches. It was found from the analysis of lesson observations of these teachers that most of those teaching lower-ability students tended to build their students' confidence and perseverance, while those teaching higher-ability students were more inclined to help their students appreciate the relevance of mathematics. Only a minority of the teachers tried to make lessons fun by using mathematics-related resources or telling non-mathematics-related jokes. It was also discovered from the teacher interviews that two factors appeared to influence the teachers' choice of the types of positive attitudes to develop in their students: the abilities of their students and the beliefs of the teachers on what mathematics is.

Most research on the affective domain in mathematics education focuses on finding out students' existing attitudes and their effect on other variables such as test performance (Aiken, 1970; Leder & Forgasz, 2006; McLeod, 1992), and students' and teachers' beliefs (Leder et al., 2002; Maaβ & Schlöglmann, 2009; Pepin & Roesken-Winter, 2015). In Singapore, research studies on affective variables also follow the international trend (e.g. Kay, 2003; Ng-Gan, 1987; Tan, 2011) and there are few intervention studies on changing students' attitudes (Yeo, 2018; Yeo et al., 2019).

This paper reports how some mathematics teachers attempted to imbue desired attitudes among their students as part of a programmatic research study on how 30 experienced and competent Singapore teachers enacted the secondary school mathematics curriculum. In the Mathematics Framework for the Singapore school curriculum (Ministry of Education, 1990; 2012), attitudes is one of the main components, consisting of beliefs, interest, appreciation, confidence and perseverance. It is beyond the scope of the research to study whether or how the teachers tried to affirm or change their students' beliefs about mathematics. Instead, this paper will report how most of these 30 teachers attempted to instil confidence in their students, encourage them to persevere, help them to appreciate mathematics and make lessons fun to interest them.

## Methodology

In the programmatic research, 30 experienced and competent teachers were videoed teaching a topic for two to three weeks to find out how they implemented the curriculum. For the purpose of this project, an experienced and competent teacher was one who had taught the same course of study for a minimum of five years, and was recognized by the school or school cluster as a competent teacher who had developed an effective approach of teaching mathematics. There are four courses of study in Singapore secondary schools: Integrated Programme (IP), Express, Normal (Academic) (NA) and Normal (Technical) (NT). In general, the abilities of the students decrease from IP to Express to NA and then to NT. For each lesson, two different focus students were also videoed to observe how they responded during the lesson and how they did the mathematics tasks.

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Each teacher was also interviewed four times: once before the first lesson, twice at appropriate junctures during the series of lessons and the last time after the last lesson. The purpose of the teacher interviews was to find out more about how and why the teachers had chosen to enact the curriculum in the ways observed during their lessons. At the end of each lesson, the two focus students were also interviewed separately to find out their reactions to the lesson and how much they had learnt. For more details on the data collection, the reader can refer to Toh et al. (2019).

This paper only reports on one aspect of the curriculum enactment: the imbuement of desired attitudes in the students. To analyse the data, the 211 lessons of the 30 teachers were examined to pick up episodes of the teachers trying to cultivate positive attitudes in the classroom. These episodes were then classified according to the sub-components of attitudes in the Mathematics Framework described earlier. The transcripts of the teacher and student interviews were also analysed to triangulate the data obtained from the lesson observations.

## Findings and Discussion

Table 1 on the following page shows the number (and percentage) of the 30 teachers in the four courses of study who attempted to imbue desired attitudes in their students using the respective instructional strategies. For each of the first three sub-categories of confidence, perseverance and appreciation, the teachers mainly utilised one instructional approach as shown in the table; while for the last sub-category of interest, the teachers generally employed two pedagogical strategies: using mathematics-related resources and/or telling non-mathematics-related stories or jokes. Some teachers also tried to develop more than one type of desired attitude.

From Table 1, we observed that most of the teachers (26 out of 30, or 86.7%) had tried to imbue desired attitudes in their students. Their foci were mainly in the areas of building students' confidence in doing mathematics by starting with tasks that students could do before progressing to more difficult tasks (20 out of 30, or 66.7%), followed by encouraging the class to persevere and to do well in mathematics (15 out of 30, or 50%). Of lower priorities were helping students appreciate the relevance of mathematics by showing real-life examples and/or applications (11 out of 30, or 36.7%) and making lessons fun to arouse the interest of their students (6 out of 30, or 20%). What was not shown in the table was that slightly more teachers (4 teachers) made lessons interesting by telling non-mathematics-related stories or jokes than those (3 teachers) who did this by using mathematics-related resources, including a teacher who did both.

On closer inspection, across the four courses of study, it is observed that all the teachers teaching the NT and NA courses (which are for lower-ability students) and 8 out of the 10 Express teachers (i.e. 80%) had attempted to develop desired attitudes in their students, but only two of the four IP teachers (i.e. 50%) had done the same. For the NT, NA and Express classes, most of the teachers focused on building students' confidence and encouraging the class to persevere, followed by helping students appreciate the relevance of mathematics and making lessons interesting. But for the IP course of study (which is for higher-ability students), the focus of the teachers was more on helping students appreciate the relevance of mathematics. In fact, only one of the four IP teachers had tried to encourage her class to persevere on only one occasion in all her seven one-hour lessons that were observed over more than two weeks, i.e. encouraging their students did not seem to be a high priority among IP teachers.

Table 1
*Instructional Strategies for Imbuing Desired Attitudes in Students*

| Instructional Approach | Number (and Percentage) of Teachers | | | | |
|---|---|---|---|---|---|
| | IP ($n$ = 4) | EX ($n$ = 10) | NA ($n$ = 8) | NT ($n$ = 8) | Total ($N$ = 30) |
| Building students' confidence in doing mathematics by starting with tasks that students can do before progressing to more difficult tasks | 0 (0%) | 6 (60%) | 8 (100%) | 6 (75%) | 20 (66.7%) |
| Encouraging the class to persevere and to do well in mathematics. | 1 (25%) | 5 (50%) | 5 (62.5%) | 4 (50%) | 15 (50%) |
| Helping students appreciate the relevance of mathematics by showing real-life examples and/or applications | 2 (50%) | 2 (20%) | 4 (50%) | 3 (37.5%) | 11 (36.7%) |
| Making lessons interesting by using mathematics-related resources and/or telling non-mathematics-related stories | 0 (0%) | 2 (20%) | 2 (25%) | 2 (25%) | 6 (20%) |
| Attempting to imbue any desired attitudes in students | 2 (50%) | 8 (80%) | 8 (100%) | 8 (100%) | 26 (86.7%) |

From the above analysis, it seems that one factor that might have influenced the teachers' instructional strategies in imbuing what sub-category of desired attitudes is the abilities of the students whom they were teaching in their respective course of study: for lower-ability students, their teachers focused on building their confidence and encouraging them to persevere, but for higher-ability students, their teachers were more inclined to help them appreciate the relevance of mathematics. This is further confirmed by interviews with the teachers. For example, a teacher said that her type of students needed motivation to solve more difficult mathematical problems and so she used an amusing video to provide the link to real life and to entice her class to solve the problems. The following shows part of a transcript of an interview with the teacher.

Interviewer: So what is your purpose for showing them this video?

Teacher: It's actually to entice them to be interested in doing mathematics because … when you keep on practising and they don't see how it can be linked, it is very difficult. So we want to see, eh, ancient times people are already using Pythagoras' theorem ... Because, my class, I think they need this kind of motivation, because some of them will fall into a world of their own very easily. So we wanted them to … entice them to this kind of thing … so after this, what they will do is, the king [from the video] has a series of problems, so they will try to solve the king's problems by Pythagoras' theorem.

Another factor that might have influenced the teachers' instructional approaches in cultivating which kind of positive attitudes is the beliefs of the teachers. For example, a teacher encouraged his students to try to score at least a few marks for a difficult exam-type question because he revealed during an interview that he believed that mathematics was about resilience and so he was attempting to convince his class not to give up on such examination questions.

## Conclusion

The study has shown how some experienced and competent teachers in Singapore attempted to imbue desired attitudes in their students. They focused mainly on building their lower-ability students' confidence and perseverance, while helping higher-ability students appreciate the relevance of mathematics. The least priority among the teachers was making lessons interesting. An implication for local teachers is maybe they should emulate the examples of the experienced and competent teachers in developing confidence, perseverance and appreciation in their students (if they are not already doing so), but at the same time, they could perhaps pay more attention to arousing in their students interest in mathematics. A possible area for future research is to study whether the students had developed the desired attitudes under the instructional strategies adopted by the teachers.

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# Singapore mathematics teachers' design of instructional materials

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This paper reports on one aspect of a bigger project: teachers' design of instructional materials. We found a number of design moves used by the teachers in our study. In this paper, we report three of them: Making things explicit, making connections, and re-sequencing practice examples.

This paper focuses on one major component of the project which examined the enactment of the Singapore mathematics curriculum in the Secondary Schools: the design and use of instructional materials by the teachers. We define instructional materials to be classroom-ready materials that teachers incorporate into their lessons for students' direct access for their learning. We make a distinction between *instructional materials (IM)* and *reference materials (RM)*. The latter are resources (including textbooks) which teachers refer to while planning for lessons; the former are the actual materials that are brought into their classrooms for use in their mathematics instruction. For most teachers which were the subjects of our study, their instructional materials differ substantially from their reference materials – it is this 'transformational space' that is an area of interest to us. For the rest of this paper, we will briefly describe a few such transformational moves as illustrated by some teachers in our study and their underlying intentions.

## Transform Move 1: Making things explicit

The fuller version in the examination of this move is in Leong et al. (2019). We provide a brief description here. This move is illustrated by Teacher Teck Kim. Repeatedly, in the interviews with him, he mentioned "making explicit" as a major goal in the design of instructional materials. That is, in selecting and modifying from RM (mainly the textbook subscribed by the school), he considered some of the contents as displayed in the textbook not sufficiently clear to the students; in crafting the IM, he was thus consciously governed by the principle of making the mathematical content more explicit to the students.

Figure 1 shows an example of such an explication deliberated by Teacher Teck Kim. He made the following adaptations (among others): (i) ln the RM, the textual explanation of column vectors was located at a section that was separate from the vector diagram. in Teck Kim's IM, he merged the textual mode into the visual representation of column vectors. Not only was the label of $\binom{-3}{4}$ placed beside the drawn vector, the explanation of translation of "-3" and "4" was also summarily fused into the diagram. This merging of representational modes was the way in which Teck Kim made explicit—in this case the links among the drawn vector, the column vector notation, and the translational significance. (ii) The two examples in the RM were $\binom{2}{3}$ and $\binom{-1}{-4}$. The two examples in Teck Kim's notes were $\binom{-3}{4}$ and $\binom{-3}{-4}$[the latter is not shown in Figure 1 due to space constraints]. Apart from the fact that the magnitudes of these vectors yielded an integer value, not a surd, and thus potentially reduce computational complexity so that the focus was on the definition and method of obtaining the magnitude, the choice of $\binom{-3}{4}$ and $\binom{-3}{-4}$ shows a one-component variation only in the translation in the y-direction, allowing the teacher to focus students' attention on the

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translational significance when "4" is replaced with "-4", thus highlighting the need to attend carefully to signs. In other words, Teck Kim re-worked the examples to make explicit critical ideas (perhaps, even potential student mistakes) which may have otherwise been unnoticed by the students. (iii) [Not shown in Figure 1] The task implicit in the RM required students' to "write" the given drawn vector in column vector notation; the task in Teck Kim's IM [not shown in Figure 1] required students to do the reverse: to "draw" vector given its column vector notation. He made explicit by filling a gap in the textbook. In this case, the gap was the skill of drawing vectors.

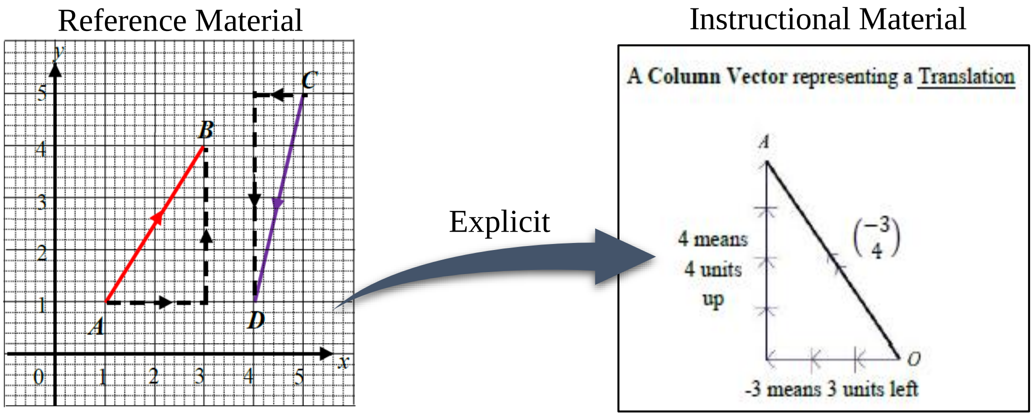


*Figure 1.* Making explicit from reference materials to instructional materials

## Transform Move 2: Making connections

We illustrate this move by drawing upon the IM of Teacher Siew Ong. The phrase "making connections" – and similar phrases – occur frequently in her talk during our interview sessions with her. This move is particularly significant as connection-making in instructional work is highlighted as desirable in Singapore's official documents: "connections refer to the ability to see and make linkages among mathematical ideas …" (Ministry of Education, 2012, p. 15, emphasis added).

The context was the method of "completing the square". The RM presents an "investigative task" consisting of a table with four columns entitled (from left to right): "Quadratic Expression", "Number that must be added to complete the square", "Half the coefficient of x", "Quadratic expression in the form $(x + a)^2 - b$". An example as a row entry was then given for "$x^2 + 2x$" in the first column, "$1^2 = 1$" in the second column, "$\frac{2}{2} = 1$" in the third column, and the algebraic working to obtain $(x + 1)^2 - 1$ in the last column. Other blank rows were given in the table below this first entry to provide working space for other samples of algebraic expressions of the form $x^2 + px$.

The IM designed by Teacher Siew Ong was an adaptation of the RM. She retained the four columns and kept largely to the titles of the first and the fourth columns (the 'beginning form' and the 'targeted complete square form'). She renamed the middle two columns as "Geometric representation" (second column) and "Term to be added" (third column). Figure 2 shows how the entry in the second column looks like for the same example of $x^2 + 2x$.

Different from the RM, she intended to help students connect "square" in "completing the square" to a "geometric square". There is thus a deliberate design decision to draw students' attention to intermodal links – between the algebraic mode and the geometric mode of representation. The geometric square provided a more natural motivation and hint as to what value need "to be added" (language of Column 3) within the perforated small square to "complete the (geometric) square". This shift of focus rendered the step in Column 3 of the RM ("half the coefficient …") unnecessary as it would have become more intuitive from the geometric mode of representation within the context of forming a geometric square. [As an aside, the algebraic working in Column 4 now takes on a different function: it is not merely an algebraic procedure to complete; it is a static record (algebraically) of what happens dynamically over the entries in the last three columns. This further strengthens the algebraic-geometric connection].

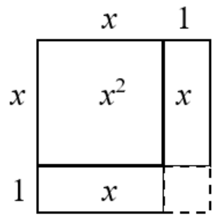


*Figure 2.* Geometric representation of $x^2 + 2x$ to set up for completing the square

In addition, to set up this way of thinking by students, that is, to view a quadratic expression as 'almost a square', she designed a prior page (not found in RM) where numbers (more accessible to students initially than algebraic expressions) were also represented geometrically as almost a square. As an example, 120 where written as $121 – 1 = 11^2 – 1$. This was also represented geometrically as a square of side 11 with a tiny square of $1^2$ at the corner snipped off. This additional preamble that she designed revealed her deliberate effort at connection in at least these ways: (i) intermodal connections not only between algebraic and geometric representations, but also numerical to algebraic and geometric; (ii) conceptual connections – she recognised that students had prior familiarity with numerical perfect squares such as $121 = 11^2$. She drew from this prior conception to connect it to their other prior familiar imagery of geometric squares. These were then linked and further developed into 'almost square' in anticipation of connecting to the method of completing the square. In other words, she connected concepts by developing tightly from earlier concepts.

## Transform Move 3: Re-sequencing practice examples

The details for this move can be found in Leong et al. (In press). As in the first move, we provide here a brief description. The teacher we studied for this move was Teacher Beng Choon. She designed the IM for the purpose of helping students gain proficiency with some 'rules' within the topic of differentiation. For the purpose of this paper, we restrict our consideration to the 'formula' of $\frac{d}{dx}(x^n) = nx^{n-1}$.

In her case, we were unsure as to the specific RM she relied upon most. Being an experienced teacher for many years, she could not specify a particular textbook she adapted from as her IM had evolved throughout the years over many rounds. For the purpose of this discussion, we referred to one common textbook to serve as a comparison to the examples she sequenced for this same section immediately after the introduction of the formula. The textbook provided three examples for application of this formula in this order: $\frac{1}{x^2}$, $\sqrt{x}$, and 1. The examples that appeared in Beng Choon's IM were: $x^3$, 5, $\frac{1}{x}$, and $\sqrt{x}$. Figure 3 provides a summary of what she wrote on the board for each item and how she explained the procedure to obtain the final answer. Her main goal was to help students recognise the form $x^n$ so that they can apply the formula correctly. As such, she needed to *vary the form* – so that they can 'see' how surface forms that do not initially look like $x^n$ can be re-written in such a form for correct use of the formula. At the same time, she was cognizant that students did not get discouraged by difficulties and so she *proceeded gradually from simpler cases* of the form. A brief chronology: She started with $x^3$ as it is most recognisable as $x^n$. The switch to "5" was deliberate as she wanted to draw students away from fixation of formula-application; rather, they can think graphically and connect to differentiation as "finding gradient". The third and the fourth items show progressive complexity in recognising and rewriting into the form.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (a) | $x^3$ | $\xrightarrow[\text{formula}]{\text{apply}}$ | $3x^2$ | | | | |
| (b) | 5 | $\xrightarrow{\text{gesture}}$ | hor. line | $\xrightarrow[\text{gradient}]{\text{find}}$ | 0 | | |
| (c) | $\frac{1}{x}$ | $\xrightarrow{\text{rewrite}}$ | $x^{-1}$ | $\xrightarrow[\text{formula}]{\text{apply}}$ | $-x^{-2}$ | $\xrightarrow{\text{rewrite}}$ | $-\frac{1}{x^2}$ |
| (d) | $\sqrt{x}$ | $\xrightarrow{\text{rewrite}}$ | $x^{\frac{1}{2}}$ | $\xrightarrow[\text{formula}]{\text{apply}}$ | $\frac{1}{2}x^{-\frac{1}{2}}$ | $\xrightarrow{\text{rewrite}}$ | $\frac{1}{2\sqrt{x}}$ |

*Figure 3.* Summary of the procedures explained for each item by Teacher Beng Choon

## Discussion

Clearly, these moves as described are not exhaustive nor are they unrelated. A cursory reflection would reveal that a teacher who wishes to adopt such moves may do so in an integrated way for the same activity – that is, making things explicit, making connections, and re-sequencing of practice examples can be applied concurrently. The purpose, however, of this article is to illustrate examples of each of these moves as they were adopted by the teachers in our study. This paper highlights that Singapore secondary mathematics teachers do not merely 'teach from the textbook'; rather, they make intentional moves to adapt the reference materials in ways that fit their instructional purposes which are largely 'sound' both from a theoretical perspective and in terms of concurrence to policy mandates. Often, these moves are elusive to a casual observer. The results of this study reminds us as researchers that we should avoid the simple route of pigeonholing pedagogical enactments based on cursory observations.

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