The development of predictive reasoning in Grades 3 through 4

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This longitudinal study aimed to determine changes in students' predictive reasoning across one year. Forty-four Australian students predicted future temperatures from a table of maximum monthly temperatures, explained their predictive strategies, and represented the data at two time points: Grade 3 and 4. Responses were analysed using a hierarchical framework of structural statistical features. Students were more likely in Grade 4 than Grade 3 to make reasonable predictions (87% vs 54%), to demonstrate data transnumeration in their representations (71% vs 19%), and to describe data prediction strategies based on extraction, clustering, aggregation of data, and observations of measures of central tendency.

Predictive reasoning is an everyday process where the decision-making process is informed by chance events placed in a context of underlying causal variation (Makar & Rubin, 2018). Research with primary school students on probability and prediction often focuses upon deterministic experiments using devices such as random draws; for example, balls or lollies (Reading & Shaughnessy, 2004). These tasks are helpful for investigating and promoting student reasoning because they reduce the number of sources of variation under consideration. However, at the heart of statistical analysis lies an understanding of the relationship between variables (Biehler et al., 2018). To answer many "real world" statistical questions of interest, students must also be capable of making distinctions between authentic correlations and genuine randomness (Bryant & Nunes, 2012). Designing predictive tasks, which include both causal and random variation, can therefore have exceptional potential for exploring some of the big ideas about probability and variation, such as distribution, expectation and randomness, and inference and sampling.

Appreciating distributional relationships and identifying authentic data patterns while predicting can be challenging for young students. Watson and Moritz (2001) described predictive strategies used by young students which included seeking missing or unused numbers, as also reported in Oslington et al., (2020). This may be linked to students' perception of "fairness" whereby variation is controlled by even allocation across groups (Reading & Shaughnessy, 2004). When describing the data predictions of Grade 2 students, for example, Ben-Zvi and Sharett-Amir (2005) noted that students in their sample appeared to conceive the data as flat distribution with all values equally likely. However, Grade 3 students with an aggregate view of data were generally able to make reasonable predictions (Oslington et al., 2020) suggesting a developmental progression towards an understanding of data as a distribution containing a central signal with random variability around the signal (Konold & Pollatsek, 2002).

Conceptual framework

Students' development of predictive reasoning competencies can be supported in multiple ways, including via opportunities to create and analyse data (English, 2012; Makar,

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2016), opportunities to represent data by drawing, describing and graphing (English, 2012; Mulligan, 2015), and via other low-stakes learning experiences in which predictive reasoning language, context, and content is scaffolded (Kazak et al., 2015; Makar, 2016). To measure these competencies, researchers have drawn upon frameworks that observe what students can *do*—such as the Awareness of Mathematical Pattern and Structure (AMPS) (Mulligan & Mitchelmore, 2009) or the Structure of Observed Learning Outcomes (SOLO) model (Watson et al., 2017)—as well as what students *notice and describe* (English, 2012; Konold et al., 2015; Mulligan, 2015).

Mulligan and Mitchelmore's (2009) five-level AMPS conceptual model demonstrated that some young students spontaneously sought patterns, structures, and relationships by noticing similarities and differences between mathematical quantities, objects or relationships. In this process, students showed an understanding of emerging generality when they identified and applied common structural features and noticed regularities of spatial structures. These students could also think relationally. Based on these findings, AMPS describes two interdependent components: one cognitive—a knowledge of structure, and one meta-cognitive—a tendency to seek and analyse patterns. When applied to the development of statistical reasoning, it is implied that pattern and structure refer to the general properties within the data set, which can be expressed through relationships between the elements or subsets of the data set (Mason et al., 2009).

Our recent study (Oslington et al., 2020) found individual differences in reasoning among Grade 3 students. For example, students who viewed the data represented in a table as a single dataset also tended to use that same table as a resource for making predictions; integrating data with personal experiences and general knowledge. However, other third graders used the table in an inconsistent and idiosyncratic way. The current study, which included the same cohort at the beginning of Grade 4, focused on the changes in structural features of student predictions, reasoning, and representations between Grades 3 and 4. The AMPS framework was extended to capture the structural features of statistical development, drawing on research which suggests that the development of complex cognitive processes underpinning predictive reasoning occurs over time and from an early age. These studies imply that students who engage in pattern-seeking behaviours such as seeking similarities and differences are likely to understand the mathematical and statistical structures behind these patterns, while those who do not notice the patterns are likely to focus upon idiosyncratic or non-mathematical features. Thus, the developmental process of predictive reasoning might be interpreted through observing the pattern-seeking behaviour of students while engaged in a predictive reasoning task.

The Design Study

An earlier report (Oslington et al., 2020) described the first of three iterations of a design study on predictive reasoning conducted with a single student cohort (Oslington, 2020). This report describes the second of the three iterations and two research questions were addressed:

1. *Which structural features of the data were identified by students when predicting from, reasoning about, and representing a data table?*
2. *How do the predictions, reasoning, and representations of Grade 3 students compare with those same students when in Grade 4?*

Participants

Students attended a K-12 independent school in metropolitan Sydney. The school was relatively advantaged with an ICSEA score of 1080 and 75% of students were in the top half of Australian students. The report here covers two data points: the first week of Term 1 for Grade 3 students and the first week of Grade 4 with the same students. Forty-four students were available at both time data points representing 96% of the year group. Ethical permission was provided for all students. Data collection on both occasions was by the first author as teacher-researcher.

Table 1
Coding for student predictions, reasoning and representations (AMPS level)

AMPS level	No. of reasonable[a] predictions	Explanations	Representations
Advanced structural	12	Identified and described relationships between variables across the table	Relevant data transnumeration demonstrating association between variables of time and temperature (e.g., dot and line graphs)
Structural	10-11	Patterns observed across the row and column structure	Relevant data transnumeration through reorganisation of variables of time and temperature (e.g., bar graphs, tables extracting highest values from years or months)
Partial	5-9	Patterns observed using column structure	Attempted transnumeration, such as changing the orientation of the table without relevant data extraction (e.g., reorienting the data table so time series is on the horizontal axis)
Emergent	3-4	Patterns described not related to data context or content	Reproduction of data set without transnumeration (i.e., copy of table)
Pre-structural	≥ 2	Systematic pattern seeking not employed	No interaction with the numbers in the data table observed (e.g., weather pictures, empty grids or invented data)

[a] Predictions were considered reasonable if falling within the 5th and 95th percentiles of temperatures historically recorded for the month at Observatory Hill meteorological station, Sydney.

Lesson design and data collection

As previously described (Oslington et al., 2020), students were withdrawn from the classroom in convenience groups of 9–12 students. Each student independently: (1) predicted future maximum monthly temperatures for Sydney using a table of past maximum temperatures, (2) constructed a representation of the temperature data, and (3) explained their predictive strategies via a videoed interview. The data collection process was identical in both Grades 3 and 4, with the exception that the temperature table provided to Grade 4 students (Figure 1) contained one extra year (2017) of temperatures. Data consisting of student predictions, video interviews, and representations for students at each time point were coded using a five level AMPS scaffold (Table 1).

Highest daily temperature in the month												
Max Temp	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
2010	41	38	31	31	27	21	21	25	27	29	30	31
2011	35	42	34	27	23	21	21	26	33	34	37	27
2012	30	29	27	26	23	20	20	21	24	26	27	29
2013	46	30	34	29	26	22	24	25	32	37	34	36
2014	37	32	30	30	27	23	25	23	34	34	37	32
2015	36	32	36	32	28	23	21	28	30	37	41	35
2016	39	33	31	34	28	22	26	25	25	34	33	38
2017	39	38	32	29	26	20	27	26	34	35	28	38
2018												

Figure 1. Maximum temperatures table for Sydney provided to Grade 4 students.

Results

As shown in Table 2, evidence of structural and advanced structural awareness underpinning temperature predictions was more widespread in the Grade 4 cohort relative to Grade 3. In Grade 4, 87% (n=38) of students predicted sequences of temperatures with at least 10 reasonable predictions. In contrast, just 54% (n=24) of the students had reached this same milestone in Grade 3. Almost half the Grade 4 cohort (48%) made reasonable predictions for all 12 monthly values, compared with 27% of Grade 3 students. The percentage of students making emergent or pre-structural predictions declined from 22% of Grade 3 students to only 9% of Grade 4 students.

Table 2
AMPS levels of student predictions, representations and explanations in Grades 3 and 4

	Predictions (%)		Explanations (%)		Representations (%)	
	Grade 3	Grade 4	Grade 3	Grade 4	Grade 3	Grade 4
Adv structural	27	48	2	7	5	12
Structural	27	39	15	36	7	45
Partial	23	5	34	36	7	14
Emergent	11	7	25	14	36	20
Pre-structural	11	2	23	7	45	9

Reasonable predictions relied on identification of patterns in the data, which some students could articulate at interview. Strategies unrelated to the data table (pre-structural), or that erroneously applied personal knowledge and experience in isolation from the data (emergent), were reported by 48% (n=21) of students in Grade 3 but just 20% (n=9) of the same students in Grade 4. By Grade 4,79 % (n=21) of the cohort described some relevant aspects of the data table when explaining their predictive strategies (i.e., a partial, structural, or advanced structural response) relative to 22 (50%) in Grade 3. Students' strategies that focused upon a single aspect of the data were categorised as partial. These explanations often reflected awareness of the similarities or patterns in the vertical column structure of the table only, but were not coordinated with the seasonality implicit in the horizontal row structure. Other strategies included matching the tens' digits or looking at the highest or lowest value in each column. Approximately one third of each of the Grade 3 and 4 cohorts' (34% and 36% respectively) elicited explanations that were coded at the partial level. Structural responses successfully integrated seasonal knowledge and changes in temperature with the yearly trends of the data table by coordination of vertical and horizontal data entries. Such responses were more common in Grade 4 (36%) than Grade 3 (15%). Grade 4 student Luca, for example, first described observing the range of values in columns, and noticed that some values were more frequent than others, implying an expectation of a clumped distribution. He then explained that the previous two July temperatures were the hottest July values, predicting a similarly hot value for 2018.

Finally, and in contrast to the temperature prediction data, verbal explanations at the advanced structural level were quite uncommon, observed in only one Grade 3 student (2%) and three Grade 4 students (7%). These students described the data table holistically, often observing multiple interrelated components (e.g., noticing a higher level of variability with increasing temperatures). They differed from those with structural explanations by describing how they had sought central tendencies in the data, or by selecting an average or representative figure. In Grade 3, for example, Joseph observed differences in monthly data range and benchmarked values by intentionally making adjacent months a few degrees higher or lower than adjacent ones. Grade 4 student, Rhys, explained he sought the "approximate average temperatures", describing these as middle values, closely aligned to the median. Grade 4 student, Stuart, started his predicting with winter temperatures where the variability was least, and explicitly linked the higher range of temperatures in the summer months to the amplified impact of climate change on warmer seasons relative to cooler ones.

Students' representations of predictions in Grade 3 were predominately pre-structural, or emergent (Table 2). Pre-structural representations did not include data from the original temperature table, instead consisting of weather pictures, empty grids, and tables with invented data (pre-structural, Figure 2). Students with emergent responses appeared to recognise the importance of the original temperature data by copying the table, but provided no interpretation or additional manipulation to illustrate understanding (emergent, Figure 2). In both Grade 3 (7%) and Grade 4 (14%), a relatively small number of students also constructed representations at the partial structural level, using aspects of the table data in a non-systematic way. For example, the transnumeration labelled partial in Figure 2 was constructed as a time sequence on the *x*-axis prior to stacking the temperature data values on the *y*-axis above.

Finally, by Grade 4 more than half of the students created representations that were categorised at the structural (45%) or advanced structural levels (12%). These students were able to organise their data in a new way, distinct from the original data table: either by sorting (e.g., listing years as hottest to coldest), grouping the original data to create a new variable (e.g., determining hottest temperature for each month), or by focusing on a specific statistical exemplar, such as the median value. While some students listed these as abridged tables or lists, students in Grade 4 (39%) were often able to coordinate two sets of variables and create bar graphs (23%) or line graphs and scatterplots (16%). Only 7% of students in Grade 3 were able to do the same. The examples of structural and advanced structural representations in Figure 2 include structural elements such as approximate equal spacing, the range on the *y*-axis starting above zero, correct sequencing of months, and coordination of bivariate data.

Discussion

The aim of the study was to track the development of students' predictive reasoning capacities across one year. Students demonstrated important gains in development by Grade 4, evident through the reasonableness of their predictions, and examples of transnumeration and coordinate graphing in their representations. In order to accurately make predictions, students required an understanding of the variability in data, the capacity to reason logically about random events, and to appreciate associations between events (Bryant & Nunes, 2012). Consistent with Watson and Moritz (2001), who found increases over time in students' ability to represent, predict, and interpret pictograms, students' predictions in the current study progressed markedly between grade levels. By Grade 4, many students' explanations of their predictions reflected an understanding of multiple forms of variability (see Shaughnessy, 2007), for example, noticing extremes and outliers in temperatures, discussing changes over time, noticing variability in the table or monthly range; variability associated with seasonality, and awareness of distributions.

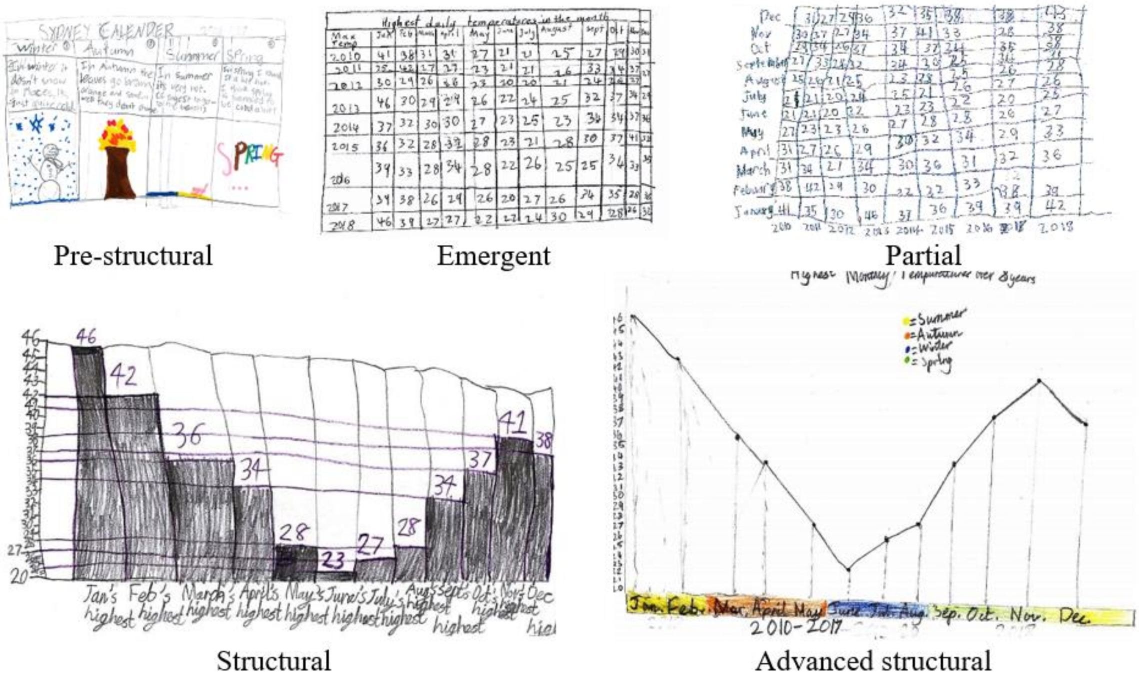

Figure 2. Student representations of Sydney maximum temperatures at five AMPS levels.

Coding with the AMPS conceptual framework highlighted the degree to which students identified statistically-meaningful patterns within the data. While previous research has shown that students with a high awareness of patterns are also likely to develop coherent mathematical concepts and relationships (Mulligan et al., 2020) and representations (Mulligan, 2015; Oslington et al., 2020), this longitudinal study is the first to apply the AMPS framework to statistical reasoning (also see Cycle 1 findings in Oslington et al., 2020). Awareness of mathematical pattern and structure was fundamental to students' perception and representation of the data set, enabling an understanding of how the variables of time and temperature could be related and organised, how students coordinated bivariate lent data, and an understanding of collinearity. For example, students in the cohort were frequently able to identify the base-ten structure evident in the monthly columns, note the repeated data values, and identify variations and similarities in data range. Students who had structural or advanced structural AMPS levels demonstrated conceptual understanding of central tendency, variability and could abstract and generalise about relationships within the data, which are both crucial components of AMPS (Mulligan & Mitchelmore, 2009).

The data set chosen in this study reflected underlying causal variation due to seasonal change, which was apparent to students' reasoning at the structural and advanced structural levels. In each case, students were able to identify this causal pattern and use it in their own predictions and representations. However, the variables of time and temperature could also be organized in several ways, each giving a different grasp of the distribution (Gattuso & Ottaviani, 2011). If a student selected the temperature as the dependent variable, for example, the independent variable could be either month or year. Selection of month as the independent variable emphasised the relationship between months and temperatures, provided they were organised in calendar order. This made it easier for students to read beyond the data (Makar & Rubin, 2018) and make inferences about other years. For some students, drawing their own (structural) predictions in a figure or graph clarified this relationship in a way that simply viewing the historical data table could not. This became apparent at interview. Students who selected the year as the independent variable still demonstrated coordination of bivariate data, but their resulting graph lacked the relational component apparent when months were selected. This is because, at least on the timescale used, the data showed no clear trend across years and temperatures. This type of representation provided less opportunity for inference, as interpretation was limited to noticing (for example) the hottest or coldest year out of the limited range.

Limitations and directions for further study

This study illustrated development in the predictions, explanations, and representations of students at Grade 3 and 4. The study was nonetheless limited by focusing on a single aspect of predictive reasoning with one cohort. It also involved a repeated task and the impact of task familiarity upon the achievement of the students has not been explored. Caution is therefore required before such findings are generalised to other contexts. Notwithstanding these limitations, there are also recommendations for future research.

First, future research should investigate the interplay of students' prior knowledge, autobiographical memory, and data interpretation skills when making predictions. In the current study, students' explanations revealed gaps in their assumed semantic knowledge of relevant concepts such as the timing of winter, the importance of the values in a data table, and the repeated pattern of months. This was particularly true in Grade 3. For example, few students appreciated that the two dimensions of the data table actually represented one continuous data sequence, which formed a pattern of highs and lows repeated every 12 units. In contrast, references to students' prior experiences were common. The Grade 4 data collection occurred in February 2019, after an Australian summer widely promoted as having record-setting maximum temperatures. While predictive strategies described by the Grade 4 students were typically based upon either the data values, the seasons, or an integration of both season and data table contents, the discussion of temperature within the media and community may still have helped students to better understand what might count as a realistic temperature. Indeed, practice at making predictions in other contexts—such as literacy studies, problem-solving in mathematics, social science lessons on climate change and geography—may also have contributed to the growth in predictive reasoning capacity between the two grades.

Secondly, for more advanced and older students, further research should consider the optimal timing for introduction of formal statistical concepts. Several of the students already appear to be independently seeking central tendencies in the data. Research on the use of exploratory graphing as a tool for promoting a relational understanding of data sets would also provide guidance regarding appropriate statistical learning sequences.

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