Perspective taking: Spatial reasoning and projective geometry in the early years

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Spatial reasoning is seen as increasingly important in STEM fields. Within mathematics, geometry is a potential site to study and support young children's spatial reasoning. In this paper we revisit Piaget and his colleagues' theoretical perspective on children's development of geometry concepts and take note of projective geometry in that theory. We outline critiques of Piaget and Inhelder's (1967) theory of topological primacy and then situate the criticisms within current spatial reasoning literature. We suggest a return to research on projective geometry holds promise for exploring and expanding opportunities that promote spatial reasoning in the early years.

For more than two decades, the push for STEM (Science, Technology, Engineering, Mathematics) skills worldwide has called attention to the importance of spatial skills, and specifically the role spatial reasoning plays in each of these domains as well as STEM-related fields. Spatial reasoning is integral to spatial skills and more generally, spatial ability, can be defined as "the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects" (Bruce et al., 2017, p. 147). Studies indicate that spatial ability is a critical attribute for entry into and success in STEM professions (Wai et al., 2009). Moreover, everyday activities such as assembling furniture, packing a suitcase, or using a web-based mapping system to get from one location to another not only require spatial know-how but also spatial know-why.

In fundamental ways, spatial reasoning shapes what we do, how we experience the world, and the ways we make sense of and think within it. Yet while spatial reasoning underlies all STEM domains, it is mathematics, in particular, that enables examination and communication of spatial concepts (Smith, 1964). Arguably then, spatial reasoning and spatial skills can be explored and developed in depth within the domain of mathematics.

In this paper, we report on key conceptual areas informing our inquiry regarding the spatial reasoning involved in projective geometry. Focusing on spatial reasoning as generally defined and recognized within STEM, we identify its relevance within mathematics education, and specifically, its relationships to geometry in the early years of mathematics education. Following this discussion, we make the case for spatial reasoning being taught through projective geometry—a largely forgotten area of research. We provide a brief summary regarding Piaget and his colleagues' theorizing of young children's conceptions of projective space, including key critiques. We then bring the three criticisms of Piagetian theory forward to 2021 and situate them within current spatial reasoning literature. What results is a change of theoretical perspectives and the emergence of new sightlines for early years research in mathematics education.

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 385-392. Singapore: MERGA.

Spatial Reasoning, Mathematics Education, and the Case for Geometry

There is a strong link between spatial skills and STEM professions (Mix & Cheng, 2012), and increasingly studies reveal connections between spatial skills and mathematics performance (Gilligan et al., 2017). Such skills observed with young children appear to be critical predictors of mathematics achievement—even beyond measures of verbal and quantitative scores, throughout schooling and into STEM-field careers (Cheng & Mix, 2014). Individual differences in spatial ability are apparent as early as the preschool years. For example, children who build with blocks, put together puzzles, and play with shapes tend to have stronger spatial reasoning skills than children who do not (Verdine et al., 2014).

Once viewed as a static and innate attribute of intelligence, spatial ability is now proving to be malleable (Uttal et al., 2013). These findings suggest early skill development could enhance mathematics performance for students. In fact, spatial training on tasks such as mentally rotating objects has led to improved performance in mathematics for 6- to 8-year-olds (Cheng & Mix, 2014). While findings differ, one conclusion is that some spatial skills are more likely than others to impact performance, such as constructing a mental number line (Lakoff & Núñez, 2000), solving missing term equations (Lourenco et al., 2018), and scaling related to proportional reasoning (Gilligan et al. 2018).

Spatial skills entail many constructs such as spatial thinking, spatial sense, visualisation, spatial cognition, and spatial orientation. There is a debate as to which skills are relevant. In this paper, we use Newcombe et al.'s (2013) categorisation of spatial skills, and specifically, tool making. *Tool making* refers to skills involved in manipulating, transforming, and creating objects, as well as using these objects as tools. Doing so requires dynamic spatial (reasoning) processes such as rotating, bending, and scaling. More than ever, research is connecting these process skills and tools with mathematics performance, especially in the early years (Mix et al., 2017).

However, despite evidence for spatial reasoning and spatial skills being essential to mathematics, especially in the early years, a clear absence of spatial skill development persists in K-12 mathematics classrooms (Woolcott et al., 2020). Given that it is mathematics "through which we communicate ideas that are essentially spatial" (Clements & Sarama, 2011, p. 134), it is concerning that mathematics curricula do not emphasise spatial concepts, processes, skills, and thinking (Davis et al., 2015).

Geometry, which underlies most if not all mathematical thinking (e.g., Bronowski 1947), is the curricular area with the greatest potential for providing educational experiences in spatial reasoning (Lowrie & Logan, 2018). However, geometry receives the least attention in schools K-12 (e.g., Larkin et al., 2016). Geometry content is often limited to sorting and naming 2D shapes (Clements & Sarama, 2011), yet young children are motivated by, capable of, and need opportunities to apply, analyse, and investigate geometric transformations of 2D and 3D shapes through mental rotation, use of symmetry, multiple representations, and de/constructing parts (Frick et al., 2014).

The importance of spatial reasoning within mathematics and the supporting role that geometry plays, prompted us to turn to educational research in projective geometry initiated over a half century ago. While studies in this area were essentially abandoned in the 1990s, we assert projective geometry employs extensive spatial processes, many of which are underexplored. As such, re-examining and extending research focused on young children's projective thinking is vital to early years mathematics.

Projective Geometry

Projective geometry involves the relationship between objects and images and their mappings or projections onto other surfaces. For example, what geometric properties are maintained between an object and the shadow it casts. Spatial transformations, such as rotation, translation, scaling, and shearing are central to projective geometry. Historically, projective geometry grew out of attempts by artists and architects to use perspective to draw or paint (i.e., project) the 3D world onto a flat surface. Today, we take for granted the ability of artists to draw with perspective. Yet, compared with Euclid's geometry which is over 2000 years old, perspective drawings only appeared 600 years ago, during the Renaissance. A key difference between Euclidean geometry, on which most of school geometry is built, and projective geometry, is that while lines remain lines and points remain points, in the latter, lengths, angles, and areas are not all preserved under projection. Figure 1 illustrates a 2D presentation in which the 90-degree angles, edge lengths, and surface areas of a cube are not preserved. We know the image represents a cube because we have learned to read, interpret, and thus see it as possessing the geometric properties that can only be observed when actually holding the physical object itself. This common example illuminates how projective geometry lies at the very intersection between perceptual and representational space, and as such, holds tremendous potential for young children's spatial reasoning in mathematics. Current studies on how children come to make sense of projective concepts are virtually nonexistent in the literature. As such, our inquiry starts with the research of Piaget and his colleagues on young children's conception of space.

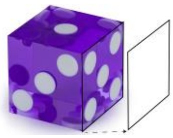

Figure 1. Projective image of a cube

Children's Spatial Reasoning within Projective Geometry

The spatial reasoning research by Piaget and his colleagues (e.g., Inhelder, Meyer) preceding and during the 1950s generated many further studies in the decades that followed up until and including the 1990s. Here we highlight key aspects of the theory concerning projective geometry, related studies, and critiques of the research.

The Child's Conception of Space

Piaget (1953) described young children's discovery of spatial relationships as spontaneous geometry. Central to Piaget and Inhelder's (1967) theory on how children come to make mathematical sense of space is that unlike a child's perceptual space which directly reflects their sensorimotor schema of a spatial environment, geometric representations of space result from their ongoing building up of motor and internalised actions into logical operational systems. Piaget (1953) contended that children's geometric conceptions follow a deductive or axiomatic progression, opposite in sequence to the historical development of the mathematics. That is, at 3 years old, the child first makes internal or topological distinctions about a particular figure (i.e., open and closed structures, interiority and exteriority, proximity and separation). By age 7, they begin to construct the projective concept of the straightness of a line. Then, at 9 or 10 years of age, the child understands problems involving perspective, such as angle of vision and point of view, and Euclidean concepts relating figures with other figures through angles, sides, parallelism, and distance. It is only when the child reaches this point that they have "complete conception of how to represent space" (Piaget, 1953, p. 79). Known as the topological primary thesis, Piaget (1953) asserted that young children's spatial reasoning was associated with their geometric representation of space as "another example of the kinship between psychological construction and the logical construction of science itself" (p. 75).

Piaget and Inhelder (1967) suggested that children begin to engage in projective geometry when they no longer view or represent objects in isolation but in relation to different viewpoints, including perspective in their drawings. For example, children at 7 to 8 years old can correctly infer by drawing what the doll's perspective or line of vision of the object is, when the doll is standing on the table and an object is placed in a certain direction in front of it. A similar experiment involved predicting the shape of an object's shadow when the object is placed in a certain position between a light source and a screen. Piaget (1953) concluded, based on the findings from this task, that the coordination of different viewpoints does not occur until the child is 9 or 10 years old. Further, Piaget and Inhelder (1967) theorised that differently from children's spatial constructs which are perceptual and experiential, or grounded in single viewpoint, their conception of basic projective relations requires their conscious constructing of reference systems through operationally connecting and coordinating all possible perspectives.

Critiques of Topological Primacy Theory

Subsequent studies by researchers who replicated Piaget and Inhelder's experiments have, for the most part, confirmed their findings (Laurendeau & Pinard, 1970; Page, 1959). However, while not dismissed outright, Piaget and Inhelder's (1967) theory on topological primacy is not supported. We discuss three key criticisms of the theory which inform this initial stage of our research on children's spatial reasoning through projective concepts.

First, children's conceptions of space may not follow the logical order of topological ideas then projective and Euclidean concepts. Research findings which revealed varied results in drawing experiments (Dodwell, 1963; Lovell, 1959) open the possibility that all three types of geometric concepts might occur simultaneously and over time, children develop the ideas by further integrating and synthesising them. For example, drawings by 4-year-old children which were not predominantly topological suggest it might not be topological features that allow children to draw homeomorphic copies. Instead, it could be their coordinating of projective and Euclidean properties which enables topological properties to be maintained (Martin, 1976). Other research by Rosser et al. (1988) suggests an alternative developmental sequence wherein children progress in their spatial reasoning through three levels: from reproducing a set of geometric figures exclusively by encoding (i.e., given a set of shapes and then matching them to the original); to building the same configuration from memory; to then matching by building an identical configuration of geometric shapes from memory after a rotation or taking another's perspective. In their study, preschool children achieved the first two levels but not the third. Rosser et al's (1988) study also emphasise the need for research to focus on how children's projective processes relate to their thinking and activity in geometry.

This point leads to the second critique of topological primacy theory regarding young children's engagement as they develop projective ideas (Clements & Battista, 1992; Rosser et al., 1988). The contention concerns the overemphasis in Piagetian theory on identifying logical errors which then precludes examination of projective concepts that may not yet be fully fledged, articulated, and perhaps are altogether different. For example, Frick et al.

(2014) demonstrate how children, as young as 3 years old, engage in perspective-taking tasks when allowed to move the objects around or when provided with a model of the room. Here, the children encode the location of small objects through the use of landmarks. Additionally, these studies indicate that children's development of projective space may not only require operationally connecting and coordinating all possible perspectives, but also forming and integrating an external framework as part of their reference system for spatial organisation.

Third, while children's geometric conceptions of space may not be direct reproductions of their sensorimotor perceptions of the environment, at the same time, it is unlikely that their representations are purely logical operational systems. Clements and Battista (1992) identified this point as a key aspect that researchers had not yet examined in any depth. Drawing on the work of Fischbein (1987), Clements and Battista (1992) argued that space representations (e.g., a square shadow of a cube die projected on a screen) are more complex than exclusively abstract properties of space (e.g., four edge lengths and 90-degree vertices). Rather, children's conceptions of space entail "sensorimotor and intellectual skills organized into a system of beliefs and expectations that constitute an implicit theory of space. Most important, intuitions thus constructed are enactively meaningful... because they express the direct behavioral meaningfulness of an idea" (Clements & Battista, 1992, p. 426). Today, even more, it is necessary for researchers to examine how Piagetian theory and theories that emphasise the intuitive and complex nature of cognition can inform children's spatial reasoning through projective concepts.

Changing Perspectives

We now take the three critiques and consider them further by situating each of them within more current and general spatial reasoning literature. In doing so, we distinguish complementary theoretical perspectives which not only offer possibilities for how we might observe anew the ways young children reason spatially in projective geometry, but also prompt sightlines for reconceptualisation.

1. Children's conceptions of projective space may not follow the logical order proposed by Piaget and Inhelder (1967).

This point calls into question several aspects of Piaget and Inhelder's (1967) notion of topological primacy such as ages, stages, linear sequencing, mutual exclusion of the three types of geometries, and contexts. Moving deeper and changing tact, what might it mean if young children's conceptions of projective space were not characterised once and for all, as a predetermined sequence, or prescriptive stages?

In *Spatial Reasoning in the Early Years: Principles, Assertions, and Speculations*, Davis et al. (2015) argue for theoretical perspectives and ways of interpreting spatial reasoning in mathematics which not only move beyond isolating observable and measurable aspects, but at the same time, more closely reflect learners' cognitive activities as they engage in-situ where mathematics teaching and learning happen. Here the authors characterise spatial reasoning as:

> a clearly discernible whole that cannot be fully comprehended by reducing it to its components. Such forms arise in the entangled interactions of many aspects, agents, or subsystems – and, within those interactions, new and unpredictable possibilities can arise. Those possibilities, in turn, can affect and occasion the entire system's current and future properties and behaviors. (Davis et al., 2015, p. 140)

Both the description and circular model proposed by Davis et al. (2015) (see Figure 9.1, p. 141) reflect spatial reasoning as neither linear nor hierarchical but instead dynamically emergent and ever-changing. This perspective which complements Piaget and Inhelder's psychological axiomatic order, facilitates more nuanced research, both theoretical and empirical, and enables alternate ways to understand young children's spatial reasoning in projective geometry. We illustrate and discuss these aspects next.

2. There is a need for research that examines children's projective concepts which may not yet be fully fledged, integrated, or articulated, and perhaps are altogether different.

Taking an emergent view of young children's spatial reasoning in projective geometry implies that while their cognitive activities may be unpredictable in foresight, for example, the forms they take or the ways they manifest moment to moment in everyday learning settings such as mathematics classrooms, they can potentially be understandable in hindsight. Here the value of both artifacts and acts of children's spatial reasoning is emphasised (Thom & McGarvey, 2015) as well as the prospect of gaining insight into the moments and contexts during which children bring projective ideas into being.

Within this theoretical perspective, we revisit Piaget's (1953) experiment with children predicting the shape of the shadow projected by an object (e.g., a six-sided die). Studying the emergence of their spatial reasoning demands paying even closer attention to children's cognitive activities as they engage with the object, its projected image, and different points of view. Using Davis et al.'s (2015) descriptive terms, several discernable and possibly co-emergent *transformings* include *movings* (e.g., *rotations*), *alterings* (e.g., *dialating/contracting, distorting/morphing*), situatings (e.g., *dimension-shifting, orienting, and locating*), and *(de)constructings* (e.g., *de/re/composing, sectioning*). Elements of *understanding* that could arise involve *interpreting* (e.g., *diagramming, comparing, relating*) and *sensating* (e.g., *perspective-taking, visualising, imagining, tactilising*).

Further, we see still other and potentially different opportunities to examine young children's conceptions of projective space within today's contexts. Projective geometry underlies many different designing and map-making activities associated with computer modelling, 3D printing, digital photography and editing, perspective drawing, engineering and architectural plans, as well as other imaging applications. These activities of *designing* and *map*-making, along with *projecting* are also identified by Davis et al. (2015) in their circular model as emergent competencies.

3. While children's geometric conceptions of space may not be direct reproductions of their sensorimotor perceptions of their environment, at the same, it is unlikely their representations are purely logical operational systems.

It is worth repeating that what makes projective geometry striking, complex, and unique is how the concepts are inextricably perceptual and representational. Relevant theories that allow for inquiry into young children's perceptual and representational conceptions of projective space include those which enable cognition to be viewed as dynamic, contextually contingent and body-centred, whereby logical forms of knowing are not separate from perceptually-guided activity (Varela et al., 1991).

Perspectives such as those rooted in enactive and/or embodiment theories, take cognitive structures and activities to be co-implicated by our biological bodies and our social-cultural ways of knowing. That is, what we come to know, how we think, and that to which we choose to attend is influenced by how our material bodies move through space and in relation to other bodies, as well as historical and cultural significances (Nemirovsky et al., 2020;

Varela et al., 1991). Cognitive scientists increasingly show the vital role our body plays in the conceptual development of both simple and seemingly abstract mathematical concepts (Marghetis & Núnez, 2013).

Past and current studies in mathematics education elucidate bodily aspects of young children's spatial thinking such as the spontaneous and deliberate ways they use their bodies to express concepts and develop meanings, though not necessarily related to projective geometry, for 2D and 3D figures and transformations (e.g., Bussi & Baccaglini-Frank, 2015; Thom, 2018). These include gestures, movement, sound, speech, rhythm, and drawing(s). Thus, it seems reasonable to assume that the body and perception are not simply the means by which children progress to more formal projective thinking, but rather, the means with which their conceptual thinking depends, emerges, and evolves. Here lies tremendous potential for research to expand understanding of spatial reasoning in projective geometry, in terms of critical spatial skills, processes, and tools that young children 'know' as well as how they demonstrate and develop these perceptually, logically, informally, and formally, mentally and physically.

Acknowledgments

This work was funded by a grant from the Social Sciences and Humanities Research Council of Canada.

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