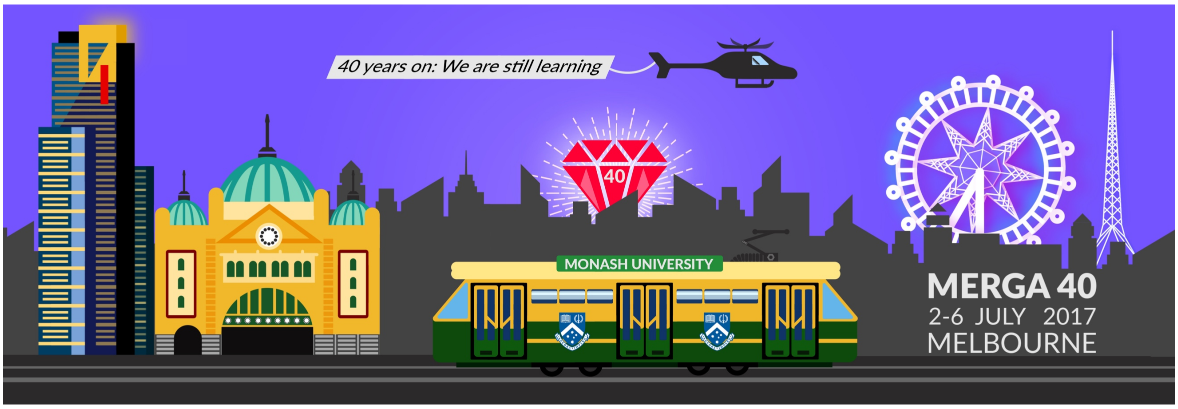

40 years on: We are still learning!

Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia

Edited by Ann Downton, Sharyn Livy, & Jennifer Hall

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Preface

This publication is a record of the proceedings of the celebratory 40th conference of the Mathematics Education Research Group of Australasia (MERGA), which, like the inaugural MERGA conference, was held at Monash University in Clayton, Melbourne. The proceedings are made available to conference delegates on a USB and are also published on the MERGA website at www.merga.edu.au.

The theme of this 40th anniversary conference was *40 years on: We are still learning!* This theme was chosen to acknowledge the significant contributions of Australasian researchers over the past 40 years, was inspired by a group of currently active researchers who attended both MERGA1 and MERGA40, and is linked to the Monash University motto, *Ancora Imparo* (We are still learning). The theme also highlights the impact and importance of our collective research for enabling new learning, innovation, and critique of mathematics education for those in our region and beyond.

MERGA40 conference participants presented research papers, symposia, round table discussions, and short communications that covered a broad range of topics relevant to mathematics education across all countries, with a particular focus on the Australasian region. The MERGA40 conference also included a series of nine workshops focused on research-related issues and 15 Research Interest Area (RIA) discussion groups aligned with chapter themes in the most recent four-yearly review of mathematics education research in Australasia (Makar et al., 2016). All workshops and RIA discussion groups were led by MERGA members who are acknowledged in the proceedings and conference program. We thank these members for their important contribution, leadership, and generosity.

In accordance with established MERGA procedures, all research papers were blind peer-reviewed by panels of mathematics education researchers with appropriate expertise in the field. Papers were accepted for presentation only, or for both presentation and publication in the conference proceedings. Only those research papers accepted for presentation and publication are published in full in these proceedings. Symposia papers and the abstracts of all short communications and round tables were also peer-reviewed. The published proceedings include the keynote papers; the Beth Southwell Practical Implications Award paper; symposia papers; abstracts for round tables, short communications, and research papers accepted for presentation; and the titles of all workshops and Research Interest Area discussion groups.

We acknowledge, with gratitude, the efforts of the MERGA40 review panel chairs, reviewers, and the Monash editorial team, in reading and providing constructive feedback to presenters in a short timeframe. Ensuring that the published papers are of a high academic quality is an important responsibility of the MERGA community. We thank the proceedings editors, Ann Downton, Sharyn Livy, and Jennifer Hall, for their hard work and care in preparing these proceedings for publication.

Ann Gervasoni and Helen Forgasz
(Co-Conveners of the MERGA40 conference on behalf of the MERGA40 Monash organising committee)

Reference

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KEYNOTES

The “M” in STEM: National Perspectives

Dr Alan Finkel
Chief Scientist of Australia

From splitting the bill at a restaurant to wrangling over real estate, there’s no getting away from the need for mathematics. English is the language of communication; maths is the language of science. In this data-hungry world of digital innovation, it’s a language that is only gaining power – and Australia can’t afford to be left behind. We need everyone to contribute to overcome stagnation in our national numeracy results, and switch our students on to maths. From teachers in school through to the critical role of our universities, Chief Scientist Dr Alan Finkel will highlight the need for Australia to embrace the “M” in STEM.

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We are Still Learning to Integrate Affect (and Mathematics) into our Research

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Mathematics enables us to investigate, explain, and make sense of the world in which we live (Ministry of Education, 2007). Many people, however, are unable to do this fully because of their affective views and responses to mathematics. In this paper, a review of affective research in mathematics education is presented to provide a context for exploring research that includes affective aspects at MERGA conferences over the last 25 years, and to position my own research. Researchers of affect are challenged to maintain a focus on mathematics, and researchers working in the broader field of mathematics education are challenged to incorporate affective aspects into their research.

Introduction

We conduct research in mathematics education to improve students' learning of mathematics. This goal is embedded in the policies of the Mathematics Education Research Group of Australasia (MERGA) and is a worthwhile aim on which to spend a lifetime's work. Indeed, in the 40-year life of MERGA our community has learnt a lot about research, students, learning and mathematics. We are still learning, and the process of challenging ourselves helps to ensure this continues. In this paper, I seek to challenge myself as an affective researcher, but also to challenge others, whether they consider themselves to be an affective researcher or not.

During my membership of MERGA, several challenges from MERGA members have particularly resonated with me. Tom Lowrie (2015) discussed the implications of the trend towards non-mathematicians in MERGA, which led me to query my own background as a researcher. This is linked to Robyn Jorgensen's (2014) caution that the focus in mathematics education research seems to have shifted from the core learning of mathematics to the social conditions within which learning occurs. In turn, this resonated with me because of its connections with what Bill Barton (2003) said when he underlined the importance of connecting affective research with students' learning: "We should be careful about doing research that is easy, rather than research that contributes to our understanding of mathematics learning" (p. 85). Conversely, Andy Begg (2000) has regularly reminded me that the role of a teacher is to teach students, rather than mathematics, and I need to extrapolate this wisdom about teaching to my role of mathematics education researcher.

In this paper, I first outline affective research in mathematics education. This is a broad-brush approach only; recent exhaustive reviews of affective research can be found elsewhere (see Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016; Goldin et al., 2016; Grootenboer & Marshman, 2016). I will map affective research presented at MERGA conferences over the last 25 years, and use these understandings to provide a context and critique for my own research and my own developing identity as both an affective researcher and a mathematics education researcher.

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Researching Mathematical Affect

Affect is an umbrella term used to describe a range of aspects of the human mind that go beyond cognition, such as beliefs or the experience of feelings and emotions (Hannula, 2012; McLeod, 1992). Affect is well established as an integral part of students' processes of learning mathematics (Op 't Eynde, De Corte, & Verschaffel, 2006). When an individual engages in mathematics, there is more going on than the application of logic and reasoning; engagement is mediated by the individual's affective views and responses.

Researchers working in the field of mathematics education have a variety of reasons for the focus on or inclusion of affective factors in their research. There is compelling evidence that a person's confidence and proficiency in applying their mathematical knowledge impacts not only their mathematical learning, but also their career opportunities and participation in our technologically-rich society (Anthony & Walshaw, 2009). Declining engagement within school (Sullivan, McDonough, & Harrison, 2004), and declining numbers of students participating in mathematics in the senior school when it is no longer compulsory, and as a major at university, are factors that often preface research (e.g., Brown, Brown, & Bibby, 2008). Assisting primary preservice teachers (e.g., Ingram & Linsell, 2014), or those undertaking mathematics minors at university (e.g., Nardi, 2016), to have robust mathematical content knowledge are also cited as catalysts for affective research in mathematics education. Perhaps most importantly, many students do not enjoy mathematics (Evans, 2000) and some researchers assume that a person's positive affect is a worthwhile outcome in itself for research (e.g., Falsetti & Rogríguez, 2005).

Grootenboer and Marshman (2016) describe the researching of affect in mathematics education as a "contested space" (p. 13). Certainly, it is a busy one. Affective researchers in mathematics education come from a variety of backgrounds. These include psychology, sociology, neurophysiology, and education. There is resulting variety, and at times, muddiness in researchers' focus and theoretical perspectives. Affective researchers in mathematics education have variously explored participants' beliefs, attitudes, values, identity, self-efficacy, moods, norms, goals, confidence, emotions, anxiety, and motivation, and they have explored these factors in relation to each other as well as gender, ethnicity, socio-economic status, engagement, achievement, and participation. The affective views and responses of a range of participants have been studied, although studies of school students and preservice teachers dominate (Attard et al., 2016).

Early affective research was initiated by social psychologists in the 1970s (Hannula, 2014) and centred on studies of mathematical anxiety and attitude, and these types of studies continue (e.g., Jennison & Beswick, 2010; Wilson, 2009). *Mathematics anxiety* is often described (e.g., Ashcraft & Moore, 2009) as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). *Mathematics attitude* is usually defined as a general liking or disliking of mathematics and a predisposition to respond in a favourable or unfavourable way (Hart, 1989). Early researchers generally had an atheoretical approach with varying and overlapping definitions for these constructs, making it somewhat difficult to interpret and compare results. Affective variables were studied to prove causal correlations between the constructs and other measures such as achievement. Usually measured by self-report questionnaires, these became more sophisticated over time, moving from small scale studies designed to measure a single affective dimension (e.g., Aiken & Dreger, 1961) to large-scale multivariate investigations (e.g., Chiu & Henry, 1990). There is robust critique in the affective literature of quantitatively analysed, large-scale studies because of their perceived inability to fully describe student experience (see Grootenboer & Marshman, 2016). Open-ended surveys and interviews as well as mixed-methods have, in latter years, been widely employed (e.g., Boyd, Foster, Smith, & Boyd, 2014). Mathematics anxiety and attitude has variously been related to general and test anxiety, gender and achievement (Hembree, 1990; Ma & Xu, 2004). However, whether the direction of influence between affect and achievement is negative or positive is often unclear or unexplored, and there has been a shift from a causal-relationship paradigm to an interpretative one (Goldin et al., 2016).

McLeod's (1992) conception of an affective domain dominated an era of affective research related to problem solving. He defined *affect* as a "wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). There were three elements in this conception: beliefs, attitudes, and emotion. This conceptualisation was significant because it captured some of the complexity of a person's affect and allowed for relationships between elements to be explored; important in order to understand students' mathematical learning (Grootenboer & Marshman, 2016). Beliefs, attitudes, and emotions were conceptualised as varying in stability, intensity, and cognitive involvement; a person's emotions change rapidly but a person's beliefs are cool, stable constructs that develop over a relatively long period of time. *Beliefs* are regarded as an individual's subjective conceptions, or anything that they regard as true (Beswick, 2007). Beliefs, although somewhat cognitive in nature (Hannula, 2012) can, arguably, be distinguished from *knowledge*, which is shared, external and accepted by the mathematics community (Furinghetti & Pehkonen, 2002). The beliefs students have about mathematics, largely shaped by students' experiences in the mathematics classroom, have implications for student behaviour and "extraordinarily powerful" consequences for affect and learning (Schoenfeld, 1992, p. 359). *Emotions* were generally described as hot reactions to mathematics, occurring when there is a discrepancy between the individual's expectations and the demands of an on-going activity (Mandler, 1989). Experiencing emotion is thought to lead to a reduction in the conscious capacity available because the process of emotional construction itself requires conscious capacity. Emotions can bias attention and memory, and may activate actions as students reflect on and try to control them (Mandler, 1989).

Debate continues about the inclusion of the elements in McLeod's conceptualisation. It is not clear where, for example, concepts such as motivation and identity fit. Similar to earlier work into anxiety and attitude, the research lacks sufficient theorising, the terms are often ill-defined, or definitions are only implicitly implied through research instruments. Furthermore, this conceptualisation emerged from relatively slow problem-solving tasks; there is little "fine-grained" (Ashcraft, 2001, p. 224) examination of mental representations and processes in this conceptualisation (see Hannula, 2012 for further critique).

Hannula (2012) more recently captured the complexity of the affective domain through proposing a meta-theory of mathematics-related affect (see Figure 1). He included three distinct dimensions of affect:

1. Cognitive (beliefs), motivational and emotional aspects. Beliefs deal with information about the self and the environment, and motivation directs behaviour. Success and failure are reflected by emotions, which provide feedback about cognitive and motivational processes;
2. Movement between state and trait, where state refers to rapidly changing affective states when faced with a mathematical situation and traits are relatively stable affective tendencies;

3. Physiological (embodied), psychological (individual) and social theories of affect (Goldin et al., 2016; Hannula, 2012).

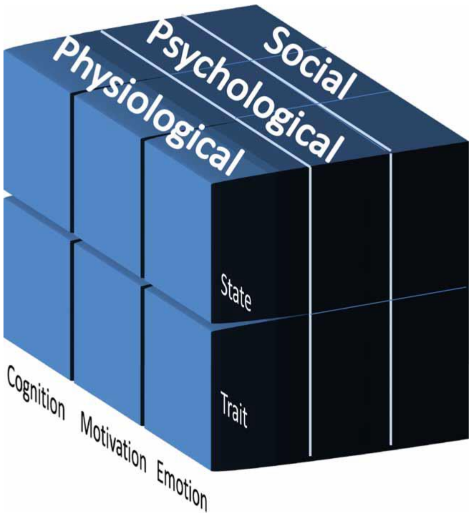

Figure 1. The three dimensions of mathematics-related affect (Hannula, 2007).

This useful meta-theory enables connections to be made across different theories of mathematical-related affect and across different eras of research in the affective domain. It also highlights aspects that have not received sufficient attention; Hannula (2012) identifies that traits have been focused on over states, and psychological studies have been focused on over others. Physiological theories of affect are rarely considered in mathematics education, although there is extensive neuropsychological research on mathematics anxiety (Goldin et al., 2016).

Affective Research Presented at MERGA Conferences 1992-2016

Given affect has been established as an integral part of mathematical learning for 25 years, I sought to find out how established it is as a research domain within the MERGA community. I chose to map research presented at MERGA conferences because participation in the conference is the enactment of our community. I did not review the findings; there are already extensive reviews on affective research in mathematics education within our region over this time period (e.g., Attard et al., 2016). Rather, my scope was to find out the extent to which our community has taken affect into account, what affective research has been done and by whom.

For a researcher/paper to be included in this mapping, there needed either to be emphasis in the title or abstract on aspects of affect, or affective aspects had to be noted in the findings or implied by the reference list. For each affective paper since 1992, I noted the researcher, the affective construct/s used, the research participants, and I noted any affective literature informing the research. I have not included short communications or round tables because abstracts often did not tell the whole story and references were needed to understand the depth of the affective focus. This means some affective research has not been included, such as the tantalising round table on emotional engagement by Higgins and Bobis in 2015.

Findings

In the last 25 years, 10.4% (202) of the full research papers included in the conference proceedings and presented at MERGA conferences have included an affective aspect. In other words, studies in mathematics education that relate to affect are relatively small in proportion to more cognitively-oriented studies. 42% of the affective research reports focused on beliefs (The next most prevalent aspect was attitudes, at 10%). A full range of participants was involved, with a notable exception of early childhood students and teachers. Research involving primary preservice teachers dominated. These findings are similar to reviews of affective research in mathematics education (e.g., Goldin et al, 2016).

Of note is the quality of affective research. It is pleasing that 79% of the affective research reports were grounded by affective literature. Those research reports that did not refer to this body of literature usually related to intervention studies (e.g., What is the effect of X on attitudes?) or the use of research tools to capture students' affect (e.g., capturing students' perceptions/beliefs/views of mathematics using X). These research papers often contained affective terms that were not adequately defined for research to be compared. Affective research needs to have constructs that are carefully defined and located within the wider body of affective literature.

The last 20 years of affective research in mathematics reflect, to varying extents, the growing interest and focus on the social and cultural context of the classroom (Sfard & Prusak, 2005), and focus on theories that see meaning, thinking, and reasoning as social products (Lerman, 2000). Again, this social turn in mathematics education is reflected by the research presented at MERGA. Jorgensen (2014) cautioned that these social perspectives have meant that focus is shifting away from the core learning of mathematics, and there is evidence the affective literature at MERGA has contributed to this. Too often, mathematics is dealt with very generally and there is a sense that the word "mathematics" could be removed and replaced with another subject such as "physics" or "economics" without much loss of understanding.

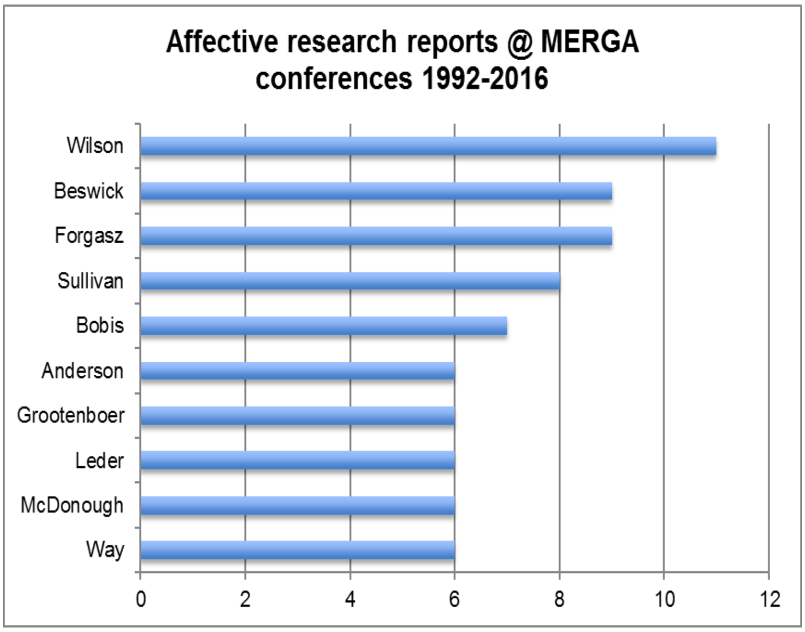

Figure 2. Affective research reports at MERGA conferences 1992-2016.

Since 1992, 184 researchers have presented papers that include affective aspects. The five most active (see Figure 2) study quite separate aspects of affect, but their research is informed by common bodies of literature. They are mostly informed by the research of their MERGA colleagues (Beswick, Forgasz, Leder, Sullivan, and Grootenboer), and the research of McLeod, Hannula, Ernest, Pajares, Boaler, Ma, and Thompson.

Another commonality of the most active affective researchers is that they began their careers mathematics teachers: Bobis as a primary teacher and the others in the secondary sector. As Lowrie (2015) highlighted, the discipline of mathematics education has now stronger foundations within education than it does within mathematics. Looking at the list of affective MERGA researchers, this seems true in this sub-field also. These affective researchers may or may not consider themselves to be mathematicians, psychologists, or neurophysiologists. However, they are experienced researchers and classroom teachers in the field of mathematics education and as such, they operate effectively at the nexus of these disciplines. They are well positioned to undertake research in this field.

My Own Research

My own background is also in mathematics teaching. I am an experienced secondary mathematics teacher, with a postgraduate mathematics degree and a doctorate in mathematics education. I am currently a researcher and lecturer in secondary and primary mathematics education. Lowrie (2015) lamented the lack of mathematicians in our research community, and although I do not teach mathematics at a tertiary level, I do consider myself to be a mathematician, but I have never formally studied psychology, physiology, or sociology. As with the more prolific affective researchers, I have experience that psychologists, physiologists, and sociologists may not have. I fulfill my role armed with my knowledge of mathematics, the New Zealand school mathematics curriculum, effective pedagogy of mathematics (Anthony & Walshaw, 2009), my experience in the classroom, my knowledge of New Zealand's unique context, and my research-based understanding that each student has a unique relationship with mathematics.

I am informed by the research of others. My main influences are a range of researchers working within and beyond the affective domain. These researchers are my touchstones, and I situate my own work within the context of this wider research (See Figure 3).

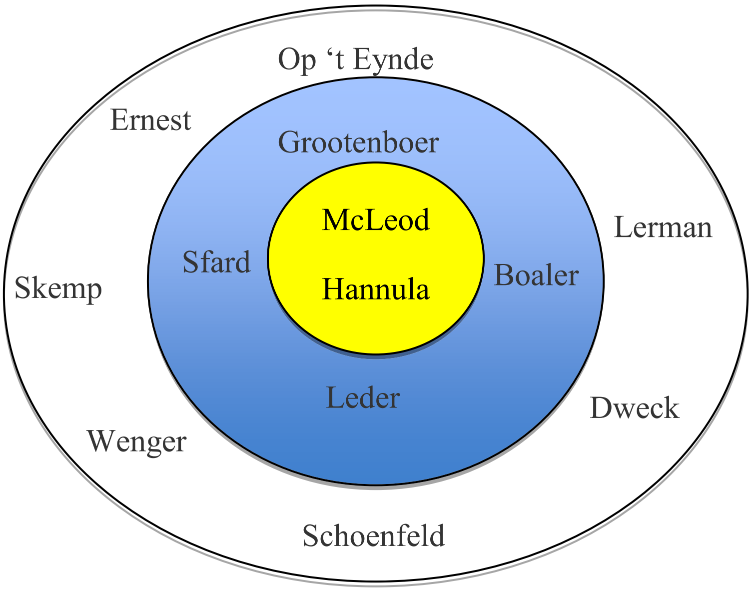

Figure 3. My research influences.

This background of affective literature provides a context for my own research. I either focus on the affective domain, or incorporate an affective aspect into my work. My main research thus far has been the study of mathematical journeys of a group of 31 adolescents. I collated data on these students from Years 7-12, but studied them intensely over two years, when they were aged 13-16 (Years 10 and 11 in New Zealand). During this period, I observed them in their mathematics and English classes, using video and audiotapes to capture their voices and facial expressions. I interviewed the students, their teachers, and surveyed their parents. I further captured students' views of mathematics through metaphor, drawings of mathematicians and personal journey graphs. This large collection of data continues to occupy me, and has resulted in several publications (Ingram, 2007, 2008, 2009, 2011, 2013).

In other affective research, I have completed several iterations of a project in which the use of Show and Tell tablet technology applications have been investigated in primary and secondary mathematics classroom. Particular attention was paid to the quality of students' engagement during problem solving (e.g., Ingram, Williamson-Leadley, & Pratt, 2015). In an iteration of the Encouraging Persistence, Maintaining Challenge Project, I was part of a team that included an exploration into how teachers encourage students to persist in challenging angles tasks (Ingram, Holmes, Linsell, Livy, & Sullivan, 2016). I am currently in the midst of a longitudinal study of preservice primary teachers called "Growing Mathematics Teachers" that aims to understand how preservice teachers' identities as teachers of mathematics develop.

It has been a privilege to engage in this research and to discover for myself the connections between affect and mathematical learning. Learning mathematics is, to me, change in both the cognitive and affective elements of an individual's relationship with mathematics. A summary of my understandings is described below. These understandings locate me among the affective researchers as having a broad and dynamic view of affect.

1. **Individuals have unique relationships with mathematics.** The school students in my longitudinal study described relationships with mathematics with five interacting elements: views related to mathematics, macro-feelings[1] about the subject of mathematics, mathematical knowledge, identities related to expectations of being able to do the mathematics, and habits of engagement, including engagement skills. These contributed to the context within which they engaged in mathematics. This construct of a relationship is similar to constructs in affective literature called mathematical self-systems (Malmivuori, 2006), identities or dispositions (Op 't Eynde et al., 2006), which variously include: mathematical content knowledge (Malmivuori, 2006); beliefs about mathematics (Op 't Eynde et al., 2006); related goals (Csikszentmihalyi, 1988) and needs of autonomy, competency, and social belonging (Hannula, 2006); global affects or attitudes; meta-knowledge, which involves knowledge about meta-cognitive functioning and affect (Malmivuori, 2006); and, habitual, re-current affective pathways and behaviours in mathematics (Goldin, 2002).

2. **When a student is exposed to a mathematical situation, each student interprets the situation according to his or her unique relationship with the subject**. The student may assess what knowledge may be needed for the task, if they have that knowledge and how accessible it is (Malmivuori, 2006). Students have varying interpretations of how well they will be able to complete the task, whether it adheres to their view of

[1] These students had just been introduced to macro and micro-economics in social studies.

what the nature of mathematics is or the type of task they might expect to get in that class. A student interprets the mathematical situation according to their *context of the moment;* the current context in which they find themselves. They also interpret the situation according to their macro-feelings about the subject of mathematics overall and they may experience a level of motivation because doing it will help them to fulfill their goals (Hannula, 2012). As a result of these complex interpretations, the individual experiences a wide range of affective responses. The students in my research called these micro-feelings, but in the literature, they are referred to as local affects (Goldin, 2002) or state affects (Hannula, 2012). These affective responses could be hot emotions with accompanying physiological arousal such as anxiety or joy; or less hot responses such as boredom or interest. The student's interpretation of the response may then disrupt or distract the learning process and affect the level of capability while engaging in the task (Mandler, 1989).

3. **The student engages in the mathematical task.** The context of the moment, and aspects of the student's relationship with mathematics, including their engagement skills, mediates the level and quality of engagement. The individual continues to experience micro-feelings during their engagement, and these affective responses are interpreted and perhaps acted on by the student. DeBellis and Goldin (2006) usefully described a positive affective pathway as one where the student begins by experiencing curiosity and puzzlement if the problem is unfamiliar and difficult. They are motivated to better understand the problem. As the problem solving continues, the person goes through a stage of bewilderment and frustration, which carries the meaning that the strategies employed so far have led to insufficient progress. One or more changes of strategy eventually yield pleasure and satisfaction. In a negative pathway, frustration does not lead to a change of strategy and ends in the student experiencing anxiety and despair, which evokes avoidance strategies and defense mechanisms.
4. **Students have unique performances and learning experiences.** These experiences are further interpreted in relation to his or her relationship with mathematics and these interpretations reinforce or, if sufficiently powerful or repeated often enough, alter their dynamic relationship. Students who have completed a task successfully may have expected to do so. Others may have given up quickly and do not attempt to understand it further. There will be little change in aspects of their relationship with mathematics as a result. Some students may have completed a task successfully, after several attempts, gaining new knowledge and gaining confidence in that particular type of problem. Others may have faced particular trouble with the task, when normally they find mathematics easy. Their experience may have been powerful because their difficulty was in front of the class. For these students, elements of their relationship with mathematics may alter. New, important, or personally significant mathematics learning experiences further build up or alter students' relationships with mathematics. These relationships with mathematics are therefore constantly changing and re-negotiated during every learning experience in the classroom.

An individual's doing of mathematics is rather like doing a long jump. Before approaching the pit, athletes have a view of long jump, which depends on their experience at previous events, whether it is compulsory to attend, and what the weather is like. When it is their turn for the long-jump, the athlete pauses at the mark; anticipating the jump, perhaps dreading it, perhaps thinking through the number of steps or their technique. They are aware of who is watching, where they are expected to land and where they probably will land. This whole plethora of complex factors remains with them as they jump, land and as they decide if their jump was successful. So it is with doing mathematics.

My research has had, thus far, a modest effect on the practice of teaching and learning mathematics. Through local and national mathematical associations, I encourage teachers to get to know their students, understand their relationships with mathematics and monitor these relationships over time. I ask them to make affective aspects of learning mathematics explicit in their teaching, and to build students' engagement skills by embracing and normalising the confusion of doing mathematics and to reflect on their doing.

As Lowrie (2015) said, "mathematics colleagues are concerned with the decrease in the number of students wanting to undertake degrees with a mathematics specialization" (p. 18). This is true for our university and, as an associate member of the mathematics department, I work to understand the decline of participation in mathematics as a major and to bridge the gap between school and university mathematics in terms of breadth and level of content knowledge of first year undergraduates.

Conclusion

It is timely to review the prevalence and influence of affective research at this point in MERGA's history. Just as learning mathematics is a social practice, socially and culturally constituted, so is learning in research. I may be isolated from the other researchers in this field by space and time, but by connecting with their research, I am participating in social practice.

> When our social practice of learning involves questioning and querying what [has already been done], then individuals will find new ways to do things … It is the social practice of learning through querying and questioning that enables the individual to produce new knowledge (Meaney, 2009).

Similar to the word "significant" in research, affective terms, such as "belief" and "attitude", need to be used with caution. When a researcher uses clear definitions of affect and their research is grounded in wider affective literature of the community and the field, this enables research to be compared to other research, utilised and built upon. Building on previous research is a way to ensure that that each generation of mathematics educators does not end up wrestling with many of the same problems the preceding generations thought they had "solved" (Kilpatrick, 1984; highlighted previously by Leder, 1994)

I challenge affective researchers to remain, become or return to being mathematics education researchers, and I challenge others in the mathematics education field to integrate affect into their research. Mathematics is not a "purely intellectual endeavour, where emotion has no place" (Hannula, Evans, Philippou, & Zan, 2004, p. 109). I fail to see how a researcher can understand why students struggle with algebra in a fine-grained way without factoring in affect. However, the idea of affective research in our field where mathematics has no place is equally fraught. Some affective studies at MERGA and, at times, my own research, would not hold up well to the question: 'Could this finding apply to any other subject or is this unique to mathematics?' Research in mathematics education needs to include cognitive *and* affective aspects rather than cognitive *or* affective aspects to understand students' learning of mathematics. Researchers of affect are challenged to maintain a focus on mathematics, and researchers working in the broader field of mathematics education are challenged to incorporate affective aspects into their research.

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CLEMENTS-FOYSTER LECTURE

The Clements-Foyster Lecture acknowledges an eminent mathematics education researcher from Australia, New Zealand or a South East Asian rim country, who is invited to present a keynote address at the annual MERGA conference. This annual keynote address is named in honour of Ken Clements and John Foyster who initiated and organised the first Mathematics Education Research Group of Australia Conference at Monash University in 1977. This led to the establishment of the organisation now known as MERGA.

In Search of Mathematical Structure: Looking Back, Beneath, and Beyond – 40 Years On

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This presentation reflects on over three decades of research focused on the development of mathematical structure. 'Looking back', it traces the key theoretical influences that informed a series of studies on children's imagery, patterns and relationships including multiplicative reasoning and spatial structuring. 'Looking beneath', the development of *Awareness of Mathematical Pattern and Structure* is highlighted through an interview-based assessment and pedagogical program. 'Looking beyond', it raises key questions about the importance of developing deep mathematical thinking leading to generalisation, and realising this possibility for young children.

This annual Clements/Foyster lecture provides a unique opportunity to be both "reflective and forward thinking" about our research and its impact. Under the theme*, 40 years on: We are still learning!*, we can celebrate our research strengths, our collegiality, and Australasia's place in mathematics education research internationally. While this presentation provides a reflection on my contribution to mathematics education research for more than three decades, it traces the significant impact of those colleagues whose ideas shaped my ever-developing theoretical perspectives and the many research questions that I sought to answer.

'Looking Back': Mathematical Thinking

While my interest in how children develop mathematical ideas stemmed from my initial study of educational psychology and years of primary teaching, it was not until I studied under the guidance of Professor Brian Low from 1984-1990 that my search for the origins of mathematical thinking in children took hold. As one of the foundational members, Brian emanated MERGA's collegial spirit and was pivotal in forming a strong group of mathematics education researchers at Macquarie University in the early 1980s. This was a time when scholars such as Richard Skemp, John Mason, and Alan Bishop influenced the research direction of many Australian mathematics education researchers. My attempts to narrow down a purposeful research investigation always led to a more fundamental question – What is mathematical thinking and how does it develop?

The ICME-5 conference in Adelaide 1984 was a critical opportunity to discuss firsthand the cutting-edge research and various theoretical perspectives of eminent scholars such as Alan Bell, Kath Hart, John Mason, Tom Romberg, Tom Carpenter, Jeremy Kilpatrick, Gerard Vergnaud, Efraim Fischbein, and Les Steffe, to name just a few. The Working Group on Primary Mathematics provided different perspectives on how to investigate such a broad and complex question. But a key message was the need to research how children's informal mathematics can develop prior to formal instruction – studies that describe and explain informal mathematical thinking and the strategies that children develop to solve mathematical problems.

An investigation of children's development of multiplication and division concepts seemed a focused and logical extension of the work by Steffe and other studies, for example, on additive word problems and counting strategies. My longitudinal study of

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children aged six to seven years that ensued would hopefully yield some new evidence of the informal mathematical strategies and children's representations of multiplicative situations.

At a deeper level, I was also searching for some clues about how recognising patterns and relationships, and the processes of modeling, representing, visualising, symbolising, abstracting, and generalising were central to mathematical thinking (Mason, Burton, & Stacey, 1982; Skemp, 1976). From the outset, I considered that any investigation of domain specific concepts, skills, or strategies would need to look more deeply at these processes. While a Piagetian view of developmental stages still prevailed, I questioned whether the processes of abstraction and generalisation could be developed in young children, even prior to formal instruction.

I will show later in his paper how I have returned many times to these origins, and how my present research investigations still emulate the convergence of these ideas.

My initial work on multiplication and division problems provided an opportunity to look more deeply into underlying mathematical processes. The research focused on children's multiplicative structures and their representations which could be traced to a number of theoretical perspectives: the Structure of Observed Learning Outcomes (SOLO) model (Biggs & Collis, 1982), intuitive models theory (Fischbein, 1977), conceptual fields (Mulligan & Vergnaud, 2006), and multiplicative reasoning (Steffe, 1994).

Taking on multiple perspectives encouraged the integration of different but seemingly complementary ideas about mathematical structure. Ideally the longitudinal study of children's multiplication and division concepts was one conceptual domain that could allow further exploration into the application of mathematical structure. On one hand, the analysis of semantic structure of word problems enabled an investigation of mathematical structure in terms of 'theorems in action'. From another perspective, the initial analysis of children's solutions to word problems applied the SOLO taxonomy, represented as response maps to multiplicative word problems. Adapting the SOLO model allowed 'structure' to be described, and so this informed the direction of subsequent analyses.

Fishbein's notion of 'implicit primitive models' directed my attention to exploring the underlying influences of these on children's solution strategies. Children's intuitive models for multiplication and division were analysed through their solutions to a variety of semantic structures of word problems (Mulligan & Mitchelmore, 1997). It was found that instructional approaches were not necessarily the basis for children's implicit models – Multiplicative concepts were found already well developed prior to formal instruction. Robust formation of these concepts was essentially based on an equal-groups structure and strategies that reflected this structure, such as multiple and double counting, grouping, partitioning, and patterning processes. These were represented by children's inscriptions, and articulated through verbal and written explanations. However, children often chose to impose their own, often inappropriate, structures such as additive rather than equal groups, based on their imagistic representations of the problem situation.

While the findings of this study advanced our understanding of children's developing strategies for solving multiplicative problems, it raised a much more fundamental question of how children's imagistic representations influenced the structural development of mathematical concepts.

In collaboration with Jane Watson (Mulligan & Watson, 1998), we embarked on a secondary analysis of students' representations (drawn recordings, notations, and verbal explanations). Using a "more powerful lens" to look more closely at these data, we aimed to identify and describe structural characteristics using the Structure of Observed Learning

Outcomes (SOLO) model (Biggs & Collis, 1982). Modes of functioning such as ikonic or concrete-symbolic modeling were aligned with increasing levels of structural development (pre-structural, uni-structural, multi-structural, relational). Children's internal representations at pre-structural and uni-structural levels in the ikonic mode reflected the equal-grouping structure of multiplication. From longitudinal tracking of individual children's images, it was found that pre-structural images became more organised mathematically in the ikonic mode, that is, random inscriptions were developed into groupings. Children's pre-structural responses became less reliant on physical models, and idiosyncratic images were replaced by numerical and symbolic features.

However, we found that analysing students' responses according to SOLO did not provide sufficiently fine-grained categories to find relationships between mathematical structures across mathematical concepts. Working with young children proved to be more challenging because there had been no systematic in-depth studies applying the SOLO model to early concept development.

'Looking Within': Children's Internal Images of Mathematics

My attention was then turned to children's representations and how they used imagery in various ways to construct and interpret mathematical ideas. Internal, imagistic representation is essential to virtually all mathematical insight and understanding—interactions with external, imagistic representations are important to facilitating the construction of powerful internal imagistic systems in students (Goldin, 1996).

I questioned how these systems were fundamental to developing abstraction and generalisation in mathematics. Features of imagistic systems included visual, verbal and non-notational inscriptions, and kinaesthetic and tactile strategies for encoding mathematical meanings. Children's images that could be classified were viewed as either static or dynamic in nature. I began to take this perspective seriously as a different way of accessing children's underlying development and representation of concepts.

Several new studies on counting and estimation, subitising, the number line, the number system, fractions, and decimals were formulated.

Two new research questions were raised:

- If children's internal imagistic representations are closely linked to the structural development of mathematical concepts, how should these be integrated with assessment and instruction?
- What if there is a mismatch between the child's individual and informal mathematical structures and those imposed by instruction and curricula?

The study of mathematical structure was central to the work of Noel Thomas who explored the relationship between children's counting, grouping, and place-value knowledge, and their conceptual development of the base-ten numeration system. By analysing children's recordings for features of structural development, it was found that children's internal representations of numbers were highly imagistic and that their imagistic configurations embody structural features of the number system to widely varying extents, and often in unconventional ways. Close analysis of these structural features provided new evidence that counting and place value knowledge were influenced to a large extent by the way children imagined the counting sequence. We were able to describe several mathematical structural features in their representations: counting and symbols, number patterns and sequences, groupings by tens, use of ten as an iterable unit, recursive grouping, and multiplicative structure supporting place value knowledge. We found a wider use of structure than we had anticipated. What was more powerful was the evidence that children's representational systems were subject to change, and they could eventually become powerful autonomous systems (Thomas, Mulligan & Goldin, 2002). However, the structural development of the number system did not closely resemble the curriculum sequences that were typical of instructional programs.

Second Graders' Representations and Conceptual Development of Number: A Longitudinal Study

In a new investigation, a three-year longitudinal study investigated 120 second graders' representations of number involving counting, grouping, base ten structure, multiplicative and proportional reasoning (Mulligan, Mitchelmore, Outhred, & Russell, 1997). Although many studies were focused on early numeracy programs at that time, this study investigated the role of imagery in children's representations of a range of numerical situations. The study was considered by some colleagues at the time, as a departure from mainstream studies. Goldin's model was adapted to analyse representations across an alternative range of tasks, such as visualising the counting sequence, and imaging and drawing "what do you see between 0 and 1". Analysis of children's visualisations, drawings, ikons, symbols, and explanations of their representations identified how they imposed structure, or lack thereof, on numerical situations. Low achievers were more likely to produce poorly organised, pictorial, and ikonic representations that were lacking in structure. These children lacked flexibility in their thinking; they were only able to copy recordings produced by others. Essentially these children lacked a grasp of the number system, of an underlying equal-groups structure, and believed that unitary counting could be used to solve any mathematical problem. Difficulties faced with simple ratio tasks were also linked to children's inability to visualise unit fractions. High achievers, however, used abstract notational representations with well-developed structures from the outset.

A follow-up study of 24 of these children tracked to Grade 5 indicated that low achievers consistently lacked mathematical structure; pictorial and ikonic representations dominated responses with little evidence of meaningful notational systems being developed (Mulligan, 2002).

These studies were consistent with the literature on the differential effects of imagery use in the development of elementary arithmetic, and the finding that students who recognise the structure of mathematical processes and representations tend to acquire deep conceptual understanding (Gray, Pitta, & Tall, 2000). We formed the hypothesis that:

> the more a student's internal representational system has developed structurally, the more coherent, well organized, and stable in its structural aspects will be their external representations and the more mathematically competent the student will be.

The studies that followed this focused on the relationships between structural features and the formation of mathematical concepts.

There were other studies that influenced the direction of the larger suite of studies that followed. Students' representations were essentially spatial in nature and these features could not be separated from the process of structuring. The study of two- and three-dimensional structures (Battista, Clements, Arnoff, Battista, & Borrow, 1998), and measurement concepts (Outhred & Mitchelmore, 2000) focused on "spatial structuring" because it involved the process of constructing an organisational form to the mathematical ideas. The depictions of groups, arrays, grids, equal-sized units, and graphs all relied on some aspects of spatial structuring.

'Looking Beneath': Awareness of Mathematical Pattern and Structure

Building on the studies on imagery and multiplicative structures, a suite of related studies with four- to eight-year-olds was designed, with the aim to describe as explicitly as possible the structural characteristics in children's mathematical development. It was postulated that there was an underlying common feature that was critical to developing mathematical patterns and relationships, and ultimately the formation of simple generalisations. Awareness of Mathematical Pattern and Structure (AMPS) was thought to comprise two interdependent components: one cognitive—knowledge of structure, and one meta-cognitive—a tendency to seek and analyse patterns (Mulligan & Mitchelmore, 2009).

Another aim was to develop a reliable assessment that could give qualitative and possibly quantitative indicators of structural development. This assessment would inform the development of a pedagogical program that could potentially promote structural thinking with a broader goal of developing generalisation in early mathematics learning.

Early signs of the development of AMPS were gleaned from the investigation of children's early formation, for example, of subitising and other patterns, representations of shapes, and arrays and grids. Figures 1 and 2 depict a seven-year-old child's (Child A) drawn image of the numbers 10, 11, and 12. There is some indication that the child draws on some emerging features of spatial structuring, such as the outline of a square, rows, and columns but the structure of the numbers is somewhat random and does reflect equal grouping. In contrast Figure 3 depicts the image produced by another seven-year-old (Child B) showing a sequence of highly structured representations based on a 3 x 3 array, extended to 4 x 3 for 12. Spatial structuring is used in the construction of the array.

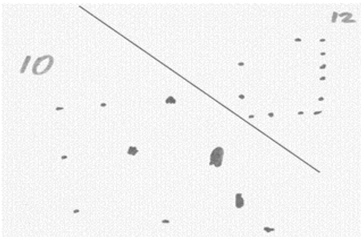

Figure 1. Child A's images of 10 and 12.

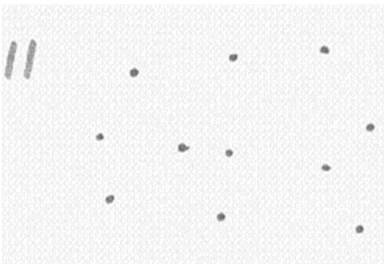

Figure 2. Child A's image of 11.

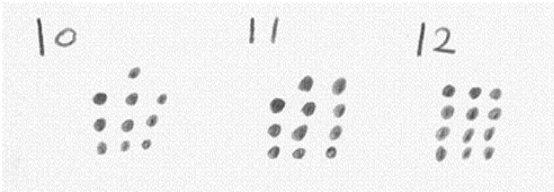

Figure 3. Child B's image of 10, 11, and 12 (Structural).

The focus on AMPS provided crucial information in the assessment of the children's mathematical concepts. While these examples (Figures 1 to 3) show that both children have learned to count, represent, and symbolise number correctly, the underlying lack of structure for child A (Figures 1 and 2) may not be visible using traditional forms of numeracy assessment. Similarly, the well-developed multiplicative structure for Child B could be overlooked.

Another key question was raised: Why do some children naturally develop and represent pattern and structure in their mathematical representations, and others do not?

Preschoolers' Representations of Patterning: An Intervention Study

Papic framed a new study focused on the early representational development of patterning with preschoolers (Papic, Mulligan, & Mitchelmore, 2011). The development of patterning strategies during the year prior to formal schooling was studied in 53 children from two similar preschools. One preschool implemented a six-month intervention focusing on repeating and spatial patterns. An interview-based Early Mathematical Patterning Assessment (EMPA) was developed and administered pre- and post-intervention, and again following the first year of formal schooling. Assessment tasks comprised identification, representation, extension, transformation, and justification of simple repetitions, and growing patterns. The intervention group outperformed the comparison group across a wide range of patterning tasks at the post and follow-up assessments. Intervention children demonstrated greater understanding of unit of repeat and spatial relationships, and most were also able to extend patterns. The notion of unit of repeat informed the subsequent studies on pattern and structure.

Studies on Pattern and Structure

An interview-based assessment, the Pattern and Structure Assessment (PASA), was developed and trialed with 109 Grade 1 students with follow up case studies in Grade 2 (Mulligan & Mitchelmore, 2009).

Three research questions were formulated:

1. Can the structure of young students' responses to a wide variety of mathematical tasks be reliably classified into categories that are consistent across the range of tasks?
2. Do individuals demonstrate consistency in the structural categories shown in their responses?
3. If so, is the individual student's general level of structural development related to their mathematical achievement?

Thirty-nine tasks covered many mathematical concepts and processes such as multiple counting, unitising, subitising, partitioning, simple repetition, spatial structuring, multiplication and division, and proportional reasoning and transformation. All the tasks required the child to identify, draw, and explain their visualisations, representations, and aspects of pattern and structure. Responses to these tasks were coded dichotomously (correct or incorrect), moreover each response was categorised for features of pattern and structure. These responses were later reliably assigned to one of five levels of structure as follows:

- Pre-structural: Students pick on particular features that appeal to them but are often irrelevant to the underlying mathematical concept.
- Emergent: Students recognise some relevant features, but are unable to organise them appropriately.
- Partial structural: Students recognise most relevant features of the structure, but their representations are inaccurate or incomplete.
- Structural: Students correctly represent the given structure.
- Advanced structural: Students provide accurate, efficient, and generalised use of underlying structure.

The findings showed that there was a high level of consistency in individuals' structural level across tasks. An extremely high correlation between students' structural level and the total number of correct PASA responses was also evident, considered a measure of their mathematical achievement level. Classroom teachers also identified the pre-structural students as low achievers and the structural students as high achievers.

A follow-up study investigated structural development among the eight lowest-achieving students and the eight highest-achieving students over the subsequent 18 months Consistent with earlier results, substantial differences were found between the two groups of students. The high achievers made significant progress over the 18 months, and many of their responses fell into the advanced structural level. Low achievers made little progress, and their representations became more disorganised and incoherent over time. They had not developed an initial awareness of patterns and structure, so their ongoing mathematical learning became meaningless and more 'crowded' over time.

Further development and trialling of the PASA continued, as well as the development of a pedagogical program to promote pattern and structure across mathematical concepts. This was supported by a year-long whole school intervention which resulted in significant advances for students on PASA and numeracy assessments, particularly in the first three years of schooling. The program was further developed through a study of Kindergarten students over a 15-week period. Students' showed rapid and sustained development of simple and complex repetitions, growing patterns, spatial structuring, base ten, and multiplicative reasoning was central to the program; measurement and geometry tasks were developed as a vehicle to develop number concepts.

The elements of the program were further refined and extended, and subject to an intensive longitudinal evaluation study from Kindergarten to Grade 1 (see Mulligan, English, Mitchelmore, & Crevensten, 2013). The work of English had a significant influence on the development of the study and a review of the PASA (English, 2004). This evaluation study of 316 Kindergartners employed a new form of the PASA and a standardised measure of mathematical achievement (I Can Do Maths). A PASMAP intervention program was trialled with an experimental group over the entire first year of schooling. Analysis indicated highly significant differences on the PASA between intervention students and the 'regular' group at the retention point ($p < 0.002$), and higher levels of structural development for the intervention students. The study validated the instrument (PASA) and constructed a Rasch scale indicating item fit.

Following the longitudinal evaluation study, a new validation study re-developed the PASA instrument, where it was administered to a reference sample of 618 five- to six-year-olds, and was subjected to a Rasch analysis (Mulligan, Mitchelmore, & Stephanou, 2015). Three forms of PASA provided a reliable and valid measure of AMPS, and it was found highly correlated with a test of mathematical achievement (PATMaths). The important outcome of this aspect of the research was that a measure of AMPS could be provided, as well as reliable indicators of structural features that were effective for teacher interpretation.

The analysis also enabled the PASA to be categorised into five structural groupings: sequences, shape and alignment, equal spacing, structured counting, and partitioning.

The Pattern and Structure Mathematics Awareness Program (PASMAP)

The PASMAP provides teachers with exemplars and explanations of core structural features gleaned from the research (Mulligan & Mitchelmore, 2016). An emphasis is placed on developing mathematical structures such as equal grouping, equivalence and commutativity, the relationship between metric units, transformations and pattern, and structuring data. The pedagogical approach takes what might seem to be a collection of inquiry-based tasks to a different level. What's critical is moving beyond the modelling and representing processes to visualising and generalising. The pedagogical approach focused on promoting and connecting concepts and relationships, and ultimately generating simple mathematical generalisation directs learning sequences to particular AMPS levels in particular structures, giving the teacher explicit descriptors and examples to inform pedagogical choices. The challenge for the teacher is to recognise and then capitalise on opportunities for developing pattern and structure, that is, can you show the same pattern (structure) in a different mode? A more critical question is how we develop teacher content knowledge and pedagogical content knowledge to support the type of thinking that leads to generalisation.

'Looking Beyond': Pattern and Structure and Spatial Reasoning

Pattern and Structure 'Meet' Spatial Reasoning: Connecting Mathematics Learning with Spatial Reasoning (2017-2020)

Adopting a transdisciplinary perspective has raised new questions about how an Awareness of Mathematical Pattern and Structure is inextricably linked with spatial reasoning (Mulligan, Woolcott, Mitchelmore, & Davis, 2017). The Knowledge Synthesis of Spatial Reasoning (Bruce et al., 2016), and the studies on pattern and structure (Mulligan, 2015; Mulligan & Mitchelmore, 2013; Mulligan et al., 2013;), had gained impetus in creating transformative pedagogies that will promote spatial reasoning as integral to mathematics learning for the future. A Spatial Reasoning Mathematics Program will be created for Grades 3 to 5 engaging students in spatial problem solving, and encouraging them to generalise their solutions by looking for similarities, differences, and structural connections. The project aims to provide a more challenging and integrated view of mathematics learning by leveraging the recent progress made by the international Spatial Reasoning Study Group.

An integrated conceptual frame underpins this proposed study, which will allow analyses of the complex conceptual connectivity involved in learning mathematics, using visual maps created through network analytic tools. Network mapping will provide a representation of how students' learning of mathematical and spatial concepts is interconnected, rather than as a linear, compartmentalised view. This may demonstrate that there are many different pathways that individuals adopt through a complex system of mathematics learning.

The issues discussed at the recent Topic Group in Early Childhood Mathematics at the ICME-13 in 2016 supported greater consensus about the need for studies focused on the big ideas, or the study of underlying mathematical processes. It seemed that 30 years since ICME-5 we had come a long way – a more holistic and integrated perspective on mathematics learning. While studies on domain-specific concepts and traditional aspects of early numeracy were still represented, the common aim of the group was to explain and describe the wide variation in early mathematical competence. Some new questions were proposed about the importance of mathematical structural development and whether simple forms of mathematical generalisation could be promoted much earlier than traditionally expected. Participants questioned whether the long-term influence of the early development of mathematical structure could result in more effective, but very different, learning outcomes for older students. Another approach discussed the impact of technological toys and tools on mathematical structural development and how this would provide a more coherent picture of how children's mathematical development may be changing and adapting to dynamic learning environments.

'Looking forward' we can aim to explore further aspects of AMPS: the possibility that low AMPS in early childhood could predict poor performance in mathematics throughout schooling, particularly in relation to algebraic thinking. Extending the AMPS construct to the later years of schooling will involve studies of learning trajectories of students beyond the early years of schooling whose mathematical and scientific reasoning is enhanced by a structural approach. My interest also lies in the application of the PASMAP approach to assisting those students with special needs, students with low levels of AMPS who may be prone to difficulties in learning mathematics, and students with advanced AMPS who are gifted at mathematics. This presentation has raised many questions about the way that we might view early mathematics learning and the development of deep mathematical thinking. I will raise just one more critical question as my concluding remark —What are the consequences for those children who do not develop mathematical structures at an early age, and how can we as researchers ensure that we make positive impact on teaching and learning?

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INVITED PANEL:
MERGA1 TO MERGA40

Progressing Along a "Road Less Traveled": The History of School Mathematics

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I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.
Robert Frost, 1916

Much of my own University of Melbourne PhD thesis (Clements, 1979) was concerned with the history of school mathematics in Victoria. Since completing that work, I have gradually extended my knowledge of the history of school mathematics so that it now encompasses much wider international perspectives than ever before. I took "the road less traveled."

During my eight-year period at Monash University (1974-1982) I taught a graduate course on the history of school mathematics, and for over 40 years now, I have taken every opportunity to extend my knowledge on the history of mathematics education in all Australian states, in Europe, in Asia, and finally in North America.

In 1988, Nerida Ellerton and I had an article published on the history of school mathematics in Australia in a special issue of the *Australian Journal of Education* which commemorated 200 years of European settlement in Australia. A few years later, Deakin University published my history of school mathematics in Victoria (Clements, 1991). Then, during a seven-year stint in Brunei Darussalam, I researched the history of school mathematics in Southeast Asia (see, e.g., Horwood & Clements, 2000). Over the past five years, Nerida Ellerton and I have had two books published on the history of school mathematics in North America (Clements & Ellerton, 2015; Ellerton & Clements, 2012; 2014), and another on the history of school mathematics in England (Ellerton & Clements, 2017). Another Springer book, which deals with the history of school algebra from an international perspective (Kanbir, Clements, & Ellerton, in press), is about to appear. I think that this last-mentioned book opens up a whole new area of academic interest – specifically, the history of school mathematics curricula, written from international perspectives.

The first chapter (Clements, Keitel, Bishop, Kilpatrick, & Leung, 2013) of Springer's *Third International Handbook of Mathematics Education*, for which I had overriding editorial responsibility, provided an overview of the history of school mathematics. It was not an easy chapter to write, for nothing similar was available for reference. Each of the four sections in that *Third International Handbook* was structured so that the first chapter was specifically concerned with the history of the main theme addressed by that section.

The way I see things, my journey down the "road less traveled" was justified when in 2016, Nerida and I were honored to be appointed editors of Springer's special series on the history of mathematics education. I am very pleased that one of the early books in that series was written by Australian and Papua New Guinea authors (Owens, Lean, Paraide, & Muke, 2017). I think it would be fair to say that my major legacy to mathematics education

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 43–46). Melbourne: MERGA.

has been, and will continue to be, in the field of the history of mathematics education, and especially the history of school mathematics.

However, I am fully aware that my "road less traveled" is still one which has relatively few travelers. Towards the end of this paper, I argue that it has been my own experience that in mathematics education research, providing a strong historical base for a study is as important as providing a strong "theoretical framework".

The Need for Well-Researched Historical Bases for Mathematics Education Research

There is a tradition in mathematics education of requiring graduate students who are preparing a research thesis to provide a clear theoretical framework for their study. However, there is no firmly established tradition requiring researchers to provide a historical framework for their proposed research, and I am persuaded that there should be.

As mentioned above, in Springer's *Third International Handbook of Mathematics Education* (Clements, Bishop, Keitel, Kilpatrick, & Leung, 2013), there were four major sections, each dealing with a different theme, and each section was structured on the basis of past, present and future aspects of the theme. The first chapter in each section was concerned with analyses of antecedents ("How did we get to where we are now?"); the "middle" chapters provided analyses of present-day key issues for the theme ("Where are we now, and what recent events have been especially significant?"); and the final chapter in each section reflected on future policy ("What should we do to improve the quality of the teaching and learning in the future?"). One of the reasons the editorial team for the *Third International Handbook* adopted this past-present-future organizational structure was to suggest the kind of structure that ought normally to be present in high-quality mathematics education research.

In one of the papers at this 40th annual MERGA conference, Nerida Ellerton, Sinan Kanbir, and I offer a historical perspective on the purposes of school algebra. That paper is based on Chapter 2 of a forthcoming book (Kanbir, Clements, & Ellerton, in press). That chapter provides a substantial overview of the history of school algebra viewed from international vantage points. The last two chapters of the Kanbir et al. (in press) book look to the future, and in the intervening chapters we describe an intervention study in which Sinan Kanbir set out to improve present practices in relation to middle-school algebra. Hopefully, this past-present-future way of thinking about mathematics education research will become more commonly accepted and practised. Certainly, in the United States of America, we have not found many mathematics education studies in which more than peripheral reviews of historical antecedents are provided.

The lack of high-quality histories of school mathematics written from fully internationalized perspectives is a serious matter given that the seventeenth, eighteenth, and nineteenth centuries were marked by massive colonisation programs, whereby the colonisers (mainly European nations) tended to introduce school mathematics textbooks into their colonies, and the languages used in most of those textbooks were those of the colonising powers. The chief authors of the textbooks were, almost always, based in Europe, and textbooks were written which seemed to suggest that school mathematics should be a culture-free exercise. Even for students in the European homelands, the textbooks were designed to suit the perceived needs of children of elites. The first school mathematics textbooks used in the "colonies" were usually written from high mathematics vantage points and, I would argue, were entirely unsuited to meet the needs of indigenous children, or of children of convicts, or of the children of slaves, or of other children whose command of the spoken and written language used by mathematics teachers or authors of mathematics textbooks was not strong (Clements, Grimison, & Ellerton, 1989).

There were attempts to change the situation. In 1855, in Victoria, Australia, for example, two professors (Professor Hearn and Professor Wilson) at the recently-established University of Melbourne, wanted to create university-entrance regulations which were different from those of the old "home" universities in Europe. The professors argued:

> There are many parents who wish their sons to enter life at an early age but would gladly send them to the University if they could obtain there the amount and quality of education which they wish them to acquire. Such persons think that the study of the classics or the higher mathematics is a needless expenditure of time, and that these subjects, while they have no direct bearing upon their children's future occupations, tend to distract young men from, and give them a distaste for, more practical pursuits. The soundness of such views is not the question. If such an opinion exists, and it is prevalent at home, and probably still more so here, the making these studies a *sine qua non* for a degree would amount to a practical exclusion of the class to whom I have referred. (University of Melbourne, 1855)

However, the colonial conservatives who would administer the yet-to-be-opened University of Melbourne did not approve of such a radical point of view, and they decided that passes in Greek, Latin, Arithmetic, Algebra, and Euclid, at the university's matriculation examination, would be required of all persons wishing to take degrees. Although the course prescribed for matriculation, Algebra, for example, only went as far as "quadratic equations in one unknown" (Clements et al., 1989), any idea that algebra should be "for all children" was not part of the thinking of those who administered the university.

The above University of Melbourne episode draws attention to the need to recognize that, from its beginnings, school mathematics in Australia was, *by design*, not intended for everyone. Secondary school mathematics, in particular, was for "the chosen" (Sharp, Farr, Farr, & Farr, 1936). That was the intention, and any respectable history of school mathematics should make that clear. It took centuries before the idea that school mathematics might be for all would be put forward with any degree of conviction. And, even when that did occur, the challenge of unravelling the forms of mathematics education which, over the centuries, had taken root as "normal", was something which society had to face – usually against staunch opposition from those who wanted to maintain the status quo.

The subconsciously-held traditions of what was normal led to the prescription of forms of school mathematics which were not suited to the needs and backgrounds of many of the students who would be required to study it. In order to study the history of school mathematics adequately, one needs not only to take account of the *intended* curriculum (as summarized in textbooks, and in formal curriculum statements prepared by local, state, or national education authorities), but also the *implemented* curriculum (as represented in cyphering books, or workbooks, or what transpired in mathematics classes), and the *received* curriculum (as represented by student recollections, and data from tests and examinations) (Westbury, 1980). There is a need to regard bottom-up, school-based, perspectives on the history of school mathematics as being as important as top-down, purely mathematics-based perspectives.

I close by asking you to reflect on how you react to the following statement that Nerida Ellerton and I recently felt moved to make:

> We confess to feeling frustrated when, from time to time, we mix with mathematicians who want to colonise the history of school mathematics so that it becomes little more than a study of contributions which great mathematicians have made to mathematics education. We feel equally

> frustrated with mathematics teachers and educators who think that the study of the history of school mathematics beyond, say, the "New Math(s)," or the "national curriculum," or the "NCTM *Standards*," is an extravagant use of time when so much needs to be done to help to improve the existing state of school mathematics. (Ellerton & Clements, 2014, p. 337)

"The history of mathematics education" should be one of MERGA's research themes.

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Forty Years on: Mathematical Modelling in and for Education

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The topic from MERGA1 (structural modelling) was abandoned as a topic of study within months. This paper reflects on mathematical modelling, also a research interest since that time. Mathematical modelling has grown enormously in currency, and this is a mixed blessing, given the propensity for private interpretations to muddy its meaning and purpose. In this presentation, live issues from the field are discussed, including conceptions of modelling, modelling cycles, competencies, and developments in metacognitive understanding. Opportunities presented by recent international initiatives are indicated.

Background

The topic I spoke to at MERGA1 was abandoned within months as a research interest, and consequently, this present contribution is not a continuation of that theme. Instead it reflects on developments, opportunities, and challenges that attend the incorporation of mathematical modelling into educational theory and practice. This is a field with which I have been continually engaged over the same period of time.

It is perhaps significant that the first memorable contributions to the field of mathematical modelling in education were not put forward by professional educators, nor did they emerge from conventional educational settings. One of the earliest papers noting the lack of attention to real world problem solving in mathematics teaching (Pollak, 1969) made no mention of mathematical modelling as such. Simply titled "How can we teach applications of mathematics?", it focused on the unrealistic nature of word problems posing as "application problems" in school textbooks. In later contributions, he included descriptions of mentoring learners to become modellers – where the setting was not a classroom, but the research laboratory of the Bell Telephone Company. Similarly, the origins of the International Community for the Teaching of Mathematical Modelling and Applications (ICTMA) featured staff in British Polytechnics who had previously been engaged in applying mathematics to solve problems in industrial settings. They were concerned that their tertiary students did not know how to go about solving problems in real-world contexts, and sought to promote this ability. Around the same time, Treilibs (1979) demonstrated that secondary school students had little idea how to apply existing mathematical knowledge to solve problems located in the world outside the classroom. His identification of the formulation stage of modelling as a key gatekeeper to success has been reinforced and extended in many later studies, situated both at school and university level.

To engage with the subject of mathematical modelling in mathematics education we should first clarify the goal of the latter. Here is an attempt at a succinct statement

> Mathematics education aims to enable/enhance the teaching/learning of more mathematics, to more people, more effectively.

Secondly, we should agree on how to determine success. In their respective fields, chemists and engineers (for example) choose and apply mathematics in ways appropriate to their professional needs. Our field of application for mathematics is education, so in our case the interest is in promoting its effective teaching/learning, for which a key component is the scaffolding provided to facilitate this goal. Sadler (2008) argues that for some practitioners scaffolding becomes so elaborate, and the level of assistance so detailed, that

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the learner cannot help but ‘succeed’. They then fail when it is removed. Yet scaffolding is supposed to be a temporary arrangement that supports the building process, following which the building needs to stand on its own merits. So, a test for the effectiveness of learning/teaching in mathematical modelling would be:

> When scaffolding is withdrawn, what are students able to do independently in solving real-world problems that they could not do before?

Models of Modelling

Statements indicating the importance of students enabled to apply mathematics to real-world situations, including everyday life, society, and the workplace, are found increasingly in national curricula (e.g., ACARA, 2016; CCSSI, 2012).

Rarely do these seem to progress beyond lip service and, from the literature, readers new to the field could be forgiven for inferring that there are many modelling genres. The contention here is that there are but two – depending on where the ultimate authority is located (Julie & Mudaly, 2007). Either “modelling” is used to give a context for lesson material aimed at developing conceptual mathematics – the various approaches being controlled ultimately by curriculum priorities and perceived constraints on teaching. Alternatively, the purpose is to provide experiences requiring students to call on mathematical knowledge to address problems in the real world beyond the text book and the classroom. Authority here is vested in the requirements of the problem being addressed, as providing the evaluation standard for a modeller’s activity. Such modelling cannot live entirely in the classroom, as a captive of the education industry. This is also the approach necessary for participation in the recently inaugurated international Mathematical Modelling Challenge (IM2C, 2017). That word problem culture continues to be an issue for the treatment of real life problems in mathematics is illustrated specifically in recent studies on text-book content (e.g., Gatabi, Stacey, & Gooya, 2012), and more generally (e.g., Verschaffel & Greer, 2010).

Modelling Cycles

Arguments still engage around the structure and characteristics of the modelling process. A typical essential modelling cycle has the following components:

Understanding the task → formulating mathematical model → solving the mathematics → interpreting outcomes → validating/evaluating the outcomes in terms of the real context → documenting the solution process and outcomes

The components are necessary in the sense that every thoroughly conducted modelling project contains them; and analytical in the sense that they describe an objective problem solving process. In providing support consistent with the activities of professional modellers, they do not describe the activities that take place inside a modeller’s head. The pathway of these mental activities is anything but cyclic, as to and fro movement between stages occurs in response to how a solution is progressing. The essential cycle acts as a guide in terms of checking and progressing solution attempts. Novices typically refer to it constantly as an external referent, experienced modellers internalise it as a mental construct as noted in the *Stepping Stone* article (Galbraith & Clatworthy, 1990). The term *mathematising* is sometimes used to describe activities across the first three components of the cycle. This is legitimate, and an important emphasis, but creates problems when presented as if this is all that modelling entails.

The essential modelling cycle may be augmented for purposes of pedagogy or research. The cycle devised by Blum and Leiß (2007) contains additional structure of "real model" and "situation model", introduced to scaffold the formulation phase of the modelling process. These are not necessary components, as a variety of other successful modelling implementations do not employ them, but they have been used successfully in teaching modelling, especially in Germany. The approach using modelling eliciting activities also inserts pedagogical criteria into its carefully designed framework (Lesh & Doerr, 2003).

Stillman (2011) provides an example of a research augmented modelling cycle. The essential cycle is included, but is augmented by structure addressing mental activity, that enables examination and release of blockages that impede modelling performance. The additional structure is included for research purposes, and is not necessary for scaffolding the modelling process. Confusion in the field would be lessened by recognition that not all representations have the same purpose, nor contribute equally to enhancing performance.

Modelling Competencies

Sadler (2008) exposes the inadequacy of fragmented criteria to assess performance on holistic tasks, arguing that overall competence involves more than ticking off success on a set of individual sub-competencies. It requires also a demonstrated ability to integrate them is a complete solution. In terms of modelling, the following statement, deriving from ICMI Study 14, addresses competency as a synthesis of sub-competencies in the sense of Sadler:

> mathematical modelling competency means the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation. (Niss, Blum, & Galbraith, 2007, p. 12)

Contained in this statement are sub-competencies that contribute to the whole, but whose sufficiency can only be evaluated in terms of performance on the whole. In an extensive study Maaß (2006) addressed the matter of sub-competencies for modelling, identifying in excess of 20 components such as the following: to carry out the single steps of the modelling process; to solve mathematical questions within a mathematical model; to interpret mathematical results in a real situation; to validate solutions; to structure real world problems; and to work with a sense of direction for a solution.

The question remains as to whether modelling sub-competencies, when developed and tested separately, can be identified as contributing definitively to overall modelling competence. The alternative of enhancing sub-competencies through reflection and analysis within the context of complete modelling problems also has substantial support, and the arguable advantage that sub-competencies and overall modelling competence are then developed and assessed in a single setting.

Anticipatory Metacognition

The importance of reflection on actions undertaken in addressing real-world problems, whether checking mathematical accuracy, evaluating a solution against contextual implications, or examining interim decisions is well known, and such classical metacognition remains important. However, studies of metacognition are moving in additional directions. Anticipatory metacognition refers to modellers' metacognitive processes, as they attempt to anticipate necessary cognitive actions, and identify opportunities, not yet undertaken, but essential for the success of their modelling endeavour. Work to date suggests there are three distinct dimensions: (1) meta-

metacognition (2) implemented anticipation, and (3) modelling oriented noticing (Stillman et al., 2016).

Concluding Thoughts

Mathematical modelling as real world problem solving has two complementary goals: to enable the solution of specific real-world problems, but over time to nurture and enable modelling abilities for students to apply in their own life environments. What learning endures when scaffolding support is removed is an issue. Other more restricted purposes for modelling exist, and it is important to keep the different purposes conceptually distinct. In this short paper, some issues of continuing importance to the former purpose have been discussed. Of immediate significance is the initiation of the International Mathematical Modelling Challenge (IM2C, 2017). Australian participation is managed by a reference group containing mathematicians, mathematics educators, teachers, AAMT representation, and representatives from industry. A visit to the website will access the extraordinary modelling quality of which students are capable when properly guided.

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Mathematics Performance and Future Occupation: Are They (Still) Related?

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At the first MERGA conference, I presented a paper in which I explored possible links between students' mathematics performance at school and their intended occupations. Whether such connections differed for boys and girls was also examined. Diverse sets of evidence are provided to illustrate that, 40 years later, these issues are still relevant and continue to attract research and community attention.

Introduction

Attendance at the first MERGA conference served for many Australian mathematics educators as a powerful initiator into academia. For me, it provided an opportunity to present, and receive feedback, on one of the components explored in my doctoral thesis. A strong recognition that not only cognitive but also affective factors influence students' learning of mathematics permeated the content of the presentation.

My First MERGA Paper

Whether performance in mathematics seemed linked to students' career intentions was the core issue explored. The sample involved 133 boys and 114 girls in Grades 10 and 11. Performance data were gathered through the administration of the grade-level appropriate ACER Tests of Reasoning in Mathematics (TRIM; Australian Council for Educational Research, 1971). These results were checked for each class against the ranking of students by their mathematics teacher. Three questions were used to elicit students' occupational intentions: (1) Do you intend to continue with your studies after you leave school?, (2) If NO, what do you expect to do after school?, and (3) What sort of work do you expect to be doing 15 years from now? The occupational intentions offered by the students were classified in three different ways: by social status, in terms of the "sexiness" or masculinity ratio of the job, and by the occupation's mundaneness ratio. In brief, the "sexiness" of the intended occupation was calculated by finding, using the then most recent available Census data (1971), the proportion of the total workforce engaged in that occupation who were males. The mundaneness ratio was defined, separately for males and females, as the proportion of, respectively, the male or female workforce engaged in that particular occupation. Further explanations of the three measures are found in Leder (1977) and Leder and White (1980). The key findings can be summarised as follows:

- For the boys in both Grades 10 and 11, statistically significant correlations were found between the TRIM score and the intended occupation measures
- For the girls in Grade 10, but not for those in Grade 11, statistically significant correlations were found between the TRIM score and the intended occupation.

This grade related difference between the measure of mathematics achievement and intended occupation led to a further question: *Could the difference in findings for boys and girls in Grade 11 be attributed to differences in social pressures felt by the older students who had to make subject (and eventual career) choices at the end of Grade 10?*

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To test this eventuality, students were also asked to respond to three further items in which a character of the same sex as the respondent was portrayed as successful in three different settings, for example: "Anne/John came top of her/his mathematics/English class last term. Describe Anne/John". By analysing the responses to these cues each student was assigned an M_{-s} score. Reactions of particular interest included references to interpersonal engagement, to the presence or absence of instrumental activity, and to contingent and non-contingent negative consequences. The M_{-s} or "fear of the consequences of success" concept (often unhelpfully shortened to "fear of success", or simply FS) falls within the framework of the expectancy-value theory of achievement motivation. It is considered particularly relevant to high-ability, high-achievement oriented females who are capable of, and aspire to success, but are at the same time concerned about the negative consequences that may accompany this success. For further details of this measure, see Leder (1982).

The findings for this further component revealed that:

- For boys at both grade levels there was no statistically significant correlation between the M_{-s} and the social status, masculinity, or mundaneness job measures
- For the girls, and particularly for those in Grade 11, relatively high correlations were found (effect size around 0.5 for Grade 11 girls) between M_{-s} and the occupational measures.

Although keenly aware that correlational relationships do not necessarily imply causal relationships, I nevertheless hypothesized that:

> An increasing realization that attainment of an ambitious goal may be a mixed blessing and may have negative personal consequences may well lead to a lowering of personal goals. Alternately, this growing anxiety about the consequences of attaining an ambitious goal may act as an impediment on performance (Leder, 1977, pp. 186-187).

To what extent the findings of the study reported 40 years ago warrant contemporary attention is discussed in the next section.

The Current Context: Some Exhibits

Both within and beyond Australia there is widespread acceptance that mathematics often serves as a gate keeper to further studies and career choices. More recently such debate has also turned to participation and achievement in STEM (Science, Technology, Engineering, and Mathematics), with mathematics considered an integral part of the STEM cluster.

According to Rickard and Crowther's (2015) collation of the *2015 Survey of Women in the STEM Professions*, "respondents reported that the three greatest barriers to advancement in their working lives were balancing their work/life responsibilities, work place culture and the lack of access to senior roles for women" (p. 8). Many (40.2 %) "did not believe they received equal compensation for work of equal value compared to their male professional colleagues" (p. 4). The Office of the Chief Scientist (2016) furthermore reported: "Across all (occupational) fields a higher percentage of those with University qualifications had an income in the highest bracket compared to those with VET qualifications. The increase was larger for those with STEM qualifications than Non-STEM qualifications" (p. 30). Differences between males and females were also reported, with "almost three times the percentage of male STEM graduates in the highest income bracket ($104 000 or above) compared to female STEM graduates" (p. 36). This disparity was not a function of a higher proportion of females who work part-time.

Recently Helen Forgasz and I conducted a survey about schooling, careers, and STEM pathways, which attracted well over 1,000 responses (see Forgasz & Leder, in press; Leder & Forgasz, in press, for some preliminary findings). Since space constraints allow only a few, but certainly instructive, snippets to be reported here, the focus is on two of the younger respondents, both aged between 21 and 30.

Participant A, a mining engineer, wrote in response to the item "Who or what served as barriers to your career path(s)/goal(s)?":

> None of my friends went into STEM fields after school (most did law/arts including the males), and it took me a long time to find friends studying similar subjects as me. The general attitude towards me studying STEM was 'wow, that's unusual' or 'wow, you must be really smart' - this was meant as a compliment but for me it just highlights my non-conformance to society's expectations of me".

In response to the survey item, "To promote a boy's/girl's interest in STEM-related studies would you recommend a single-sex school/a co-educational school/could be either, depending on the child", participant B, a speech pathologist, explained that for a boy, the school setting was not important: "I've seen male relatives achieve well in both settings". However, for a girl, she would recommend a single-sex school:

> Social pressures of having males around are not as much of an issue in single sex schools for females. In my experience I've seen females being picked on by males if they are "too smart" or "nerdy" therefore they will dumb themselves down to avoid this and get male attention. However when you take the males out of the equation they can be themselves academically and don't have the social pressures of the male population. This is purely from my experience in a single sex school. I also found girls were less distracted in general without the boys around.

Participant B was not unique in the different recommendations she made for the optimum school setting for boys and girls. In our full sample (see Leder & Forgasz, in press), with respect to the recommendation for boys, 14% recommended a single-sex school, 10% a co-educational school, and 76% responded: "depends on the child". However, for the item that referred to the optimum school setting for girls to promote an interest in STEM-related studies, 43% recommended a single-sex school, 8% a co-educational school, and 49% checked the "depends on the child" option.

Collectively, the material included in this section reveals not only that there appears to be a continuing link between mathematical background and occupational outcomes, but also that, at least among relatively young and well-educated women, differences in societal expectations for males and females persist.

What about the Future?

Sufficient evidence has now been presented to indicate that the topics considered in that first MERGA paper are still "live" issues. Not surprisingly, research foregrounding the expectancy-value theory of motivation continues to appear. A quick Google scholar search conducted in mid-March of this year with the words expectancy-value theory of motivation yielded about 2,300 results since 2016 and 300 since 2017. Restricting the search by adding the words fear of success yielded about 650 results since 2016, and adding instead motive to avoid success generated 750 results since 2016. In my early discussion of the M_{-S} /FS concept (Leder, 1982), I noted the terminological confusion found in extant reviews of the literature, and the range of personal and situational variables invoked in research on this construct. Under related headings such as stereotype threat, motivational differences, attribution theory, self-efficacy, and autonomous learning, research on gender differences in a range of settings and endeavours is pursued seemingly unabated. Does such a plethora of apparently different perspectives stimulate or obstruct a systematic study of the field?

Those familiar with research on gender and mathematics education are well aware that the terms *sex differences* and *gender differences* are both readily found in the literature. Originally, the term sex differences was used uniquely and consistently. In more recent times, sex has more commonly been used to denote biologically-based differences. The use of *gender* evolved following debates on whether all differences could or should be attributed to biology alone. Gender, as a term, was consequently often used to describe differences between males and females that are not attributable to biology. Recently, the sufficiency of the male-female binary distinction has, however, been challenged:

> In the April [2016] issue of *AERA Highlights*, AERA announced that members would soon have the option to select from an expanded list of gender identity categories when renewing their membership or joining the association… (A) two-step approach to collecting data on gender: the first being the collecting of data on the biological sex assigned at birth, and the second asking members how they describe their gender…. (Levine, 2016)

How, or whether, these new categorizations will impinge on research on gender/sex differences in mathematics learning still remains to be seen.

Finally, to my predictions for the future. I draw on the views of a theoretical physicist from my country of birth (the Netherlands), Hendrik Anthony Kramers, who tantalizingly argued: “In the world of human thought generally, and in physical science particularly, the most important and fruitful concepts are those to which it is impossible to attach a well-defined meaning” (n.d.). Acceptance of this scenario offers a strong incentive to refine our terminology and embrace a common language as we strive to understand better the most complex aspects of human behaviour.

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“Does This Mean That Kindergarten Will Be a Remedial Year?”

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At MERGA1, I presented a paper entitled *Mathematics in the Pre-School,* which was a summary of tentative ventures into this emerging field. The role of the prior-to-school years in children’s mathematics learning was not yet under serious consideration but an understanding of the importance of play in children’s experiences was building. Forty years on, early childhood mathematics education is perceived as a critical frontier for mathematics education and other (e.g., psychology, sociology, childhood and family studies) research. Early intervention is seen as the Holy Grail. In this brief paper, I identify some of the achievements and some of the side effects of this changing context.

My first MERGA paper (Perry, 1977) was also my first paper at a “research” conference. It was a description of a project, Early Mathematical Experiences (EME Project, 1991), on which I had worked during my first sabbatical leave in 1976. This was also the year I graduated PhD in Pure Mathematics, with a thesis entitled Analytic Functions over Banach Algebras. Many people, including myself, have seen both the irony and the complementarity of these two fields of endeavour.

The key messages from my 1977 paper were:

1. there was a need for materials and activities to assist pre-school children and their educators to develop mathematical concepts;
2. these materials and activities should be drawn from children’s everyday activities, including their play and interests; and
3. pre-school children and educators should have a major role in the development of the materials and activities.

Even in this early paper, I sounded a warning that the introduction of mathematical activities and concepts into the pre-school might have some damaging side effects.

> These [materials] are meant to be used as reference material for the teacher – to make her aware of possibilities in the pre-school, so that her ability to introduce a mathematical slant to everyday activities becomes second nature. Of course, there is a danger that this ‘mathematisation’ will become an obsession. To counter this, the project materials stress the notion of ‘intervention without interference’. Hopefully then, the sensitive teacher, while planning some activities with a definite bias towards mathematics, will be careful not to spoil imaginative and creative play by intruding at an inappropriate time or trying to coerce children, who are not ready, into mathematical games. (Perry, 1977, p. 267)

Early childhood mathematics education research within MERGA has extended well beyond these tentative beginnings. MERGA members are influential international leaders in the field. Some of the most recent books (English & Mulligan, 2013; Perry, MacDonald, & Gervasoni, 2016; Phillipson, Gervasoni, & Sullivan, 2017) and papers and book chapters too numerous to mention show the impact Australasian early childhood mathematics education researchers have had on the field internationally. Within Australia, two key STEM projects funded by the Australian government – *Let’s Count* (The Smith Family, 2017) and *Early Learning STEM Australia* (ELSA; University of Canberra, 2017) - continue the emphasis on prior-to-school mathematics. The field has developed greatly since 1977, and MERGA is to be congratulated on its support and encouragement.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 55–58). Melbourne: MERGA.

The Role of MERGA in Early Mathematics Education Research

MERGA has and continues to provide support through its conferences and publications – journals, conference papers, and books, including the four-yearly reviews of research. There has been a special issue of MERJ on early childhood mathematics. Notable themes were highlighted in the Editorial (Perry & Diezmann, 2005).

> Australasian early childhood mathematics educators and researchers have adapted much of the recent general early childhood research involving early brain development, play, argumentation and investigation, reflection and recording to demonstrate the power of young children's mathematical thinking and how this thinking affects children's learning in the early years. (p. 1)
>
> ... what is common in all of the papers [in this special issue] is a celebration of the mathematical potential in young children. (p. 3)
>
> .. we hope that it [the special issue] helps to recognise and celebrate the strength of early childhood mathematics education research in Australasia and beyond and provides an ongoing stimulus to researchers to continue to develop the mathematical power of our young children and their educators and to share these experiences with others. (p. 4)

In preparing this paper, I chose to focus on the four-yearly MERGA reviews of mathematics education research with their themes of synthesis, critique, and celebration. There have been nine such reviews, beginning with one in 1984 written specifically to highlight Australian research to the International Congress on Mathematical Education (ICME-5). Although the first two of these reviews did not include chapters specifically directed to early childhood mathematics education research, each of the remaining seven reviews has featured such a chapter. Reading these chapters again has been not only a trip through history but also a recognition of how much impetus so many members of MERGA have provided in order to build the field of early childhood mathematics education research. Not surprisingly, in the 1980s and 1990s, the bulk of the research undertaken in the field in Australasia concentrated on challenges of learning and teaching about number in the post-Piagetian world (McIntosh & Dole, 2000; Perry, Mulligan, & Wright, 1992; Wright, Mulligan, Stewart, & Bobis, 1996). Other areas of early childhood mathematics learning were also covered, but they were much less well represented in the available Australasian research.

Later four-yearly reviews have shown a gradual diversification of the areas of early childhood mathematics education being researched, with strong representation in areas such as patterning and structure, data, measurement, dispositions, and social and cultural contexts.

> The focus on the social and cultural contexts of children highlights a growing awareness of the impact of these areas not only on what children learn, but also on how it is learned and how it is taught. (Perry & Dockett, 2004, p. 103)
>
> Another aspect of the broadening of impetus and focus has been the increased diversity of early childhood mathematics education fields in which Australasian researchers are working. While the influence of systemic numeracy programs in the early years of school can still be seen in the directions of this research, changes in emphasis towards, for example, early algebra development, assessment and mathematics learning in the years prior-to-school have been added to the collective repertoire of Australasian early childhood mathematics education researchers. (Perry, Young-Loveridge, Dockett, & Doig, 2008, pp.17-18)

While MERGA has nurtured early childhood mathematics education research and development throughout its history, it was also influential in Australia among other professional associations. In 2006, the Australian Association of Mathematics Teachers (AAMT), not until then known for its promotion of the early childhood field, and Early

Childhood Australia (ECA), not until then known for its promotion of mathematics education, combined to produce their *Position Statement on Early Childhood Mathematics* (AAMT & ECA, 2006). Many MERGA members were involved in the development of this statement. The position espoused clearly shows movement towards a dual purpose for mathematics in the early childhood years and the diversity of settings in which children learn this mathematics.

> The Australian Association of Mathematics Teachers and Early Childhood Australia believe that all children in their early childhood years are capable of accessing powerful mathematical ideas that are both relevant to their current lives and form a critical foundation for their future mathematical and other learning. Children should be given the opportunity to access these ideas through high quality child-centred activities in their homes, communities, prior-to-school settings and schools. (p. 1)

Another influence in Australia on the direction of early childhood mathematics education research and practice has been the introduction of national curricula documents in both the prior-to-school and school years (Australian Curriculum, Assessment and Reporting Authority, 2016; Department of Education, Employment and Workplace Relations, 2009). Many MERGA members were actively involved in the development of these documents. The title of the prior-to-school document, *Belonging, Being & Becoming,* clearly places prior-to-school education as needing to be relevant both to children's present as well as their future lives, just as the AAMT/ECA (2006) statement does for mathematics education.

The introduction to the latest MERGA review chapter (Macdonald, Goff, Dockett, & Perry, 2016) and Phillipson et al. (2017) continue the theme of diverse contexts and diverse people, particularly highlighting the contributions of children and families.

> The research presented in this chapter takes into consideration a range of early childhood contexts, including home, school, and early childhood education services. Similarly, the chapter considers research which has been undertaken with a range of stakeholders in early childhood mathematics education, including early childhood and school educators, families and the children themselves. Indeed, the views of children and families in early mathematics education are well-represented in the Australasian research, and this is to be celebrated. (MacDonald et al., 2016, p. 166)

Conclusion

So, early childhood mathematics education research needs to consider diversity in context, people, pedagogy, and purpose in order to bring its insights to bear on practice. All of this has to be achieved in an educational atmosphere of systemic, national and international accountability. There are expectations that children will leave prior-to-school settings 'ready' for the school mathematics mandated in the Foundation year of the Australian Curriculum. These expectations place demands on early childhood educators, families and children. Most states and territories in Australia have entry testing in numeracy as children start school. Nominally, these are to allow for the development of appropriate programs of learning, although one may question their validity as the first experience of a child in a new situation and with a new adult. One thing that is obvious is the impact these tests can have on prior-to-school programs. Another is that, while some starting school children may be deemed "not ready" for the mathematics they will meet in the Foundation year, many of their peers will far exceed these "readiness" expectations as they start school (Gould, 2012).

In 1977, during the discussion following the presentation of my first MERGA paper, I was asked by a prominent mathematics educator "Does this mean that Kindergarten will be a remedial year?" (In this question, "Kindergarten" means the first year of school.) I scoffed at the suggestion, believing that first year of school teachers did and should welcome children to active, creative, and play-based learning contexts in which they could build on the children's knowledge and dispositions established in the prior-to-school years. In spite of the laudatory research that has been conducted by many MERGA colleagues, I now think that this question was quite perspicacious, and that disappoints me a great deal.

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Forty Years of Teaching Problem Solving

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This paper presents a reflection on problem solving, stimulated by re-reading my paper on teaching problem solving, after 40 years. It describes how seeing problem solving as the ultimate goal of mathematics education reached its zenith in the early 1990s, and how subsequently this has been largely replaced by a less ambitious agenda where working on interesting problems is seen mostly as a teaching methodology to serve other curriculum goals. Equipping students to use whatever mathematics they know to solve problems that arise within and outside mathematics is an elusive goal, but it is the most important.

At the first MERGA conference, I presented a paper entitled "Teaching Problem Solving" (Stacey, 1977). I was in my third year of working in teacher education and I had a special responsibility for the mathematics discipline studies for pre-service primary and secondary teachers. During my doctoral studies in pure mathematics, I had regularly participated in research conferences, but MERGA 1 provided my first opportunity to engage with research in mathematics education. It was a very exciting occasion.

Too much has happened in the intervening 40 years to give a fair and comprehensive summary of progress in this field. There are multiple reasons. First, solving problems is the central activity of mathematics, and to help students do it well is for me the central goal of mathematics teaching. Consequently, this field encompasses all mathematical topics and all year levels, along with applications to other subjects and to life beyond school. As theoretical analysis and empirical studies have clearly demonstrated across the years, most aspects of the classroom environment and student learning have an impact on achieving the problem solving goal: conceptual understanding, procedural fluency, heuristic strategies, understanding the problem solving process, cognition and metacognition, student attitudes and emotions, productive teaching practices, equity, socio-cultural aspects and more.

A summary is also difficult because mathematics educators working in this field are engaged in three mutually supportive but different activities:

- advocacy (influencing teachers and curriculum authorities to address problem solving goals more thoroughly);
- curriculum development and evaluation (e.g., what to teach about problem solving, how to teach for it and assess it, how to design good problems for teaching); and
- systematic research into task and student variables, teacher behaviours, success of professional development, curriculum design, and many other questions.

(Niss, Blum, & Galbraith, 2007 link these with successive time periods.) As a consequence, this short article is a personal account touching only on a few aspects.

Work in problem solving has always been plagued by communicating definitions, especially for advocacy to teachers. In 1977, I saw problem solving as the most important goal of mathematics education (I still do), and I used the phrase to include all problems where mathematical thinking or knowledge contributes to a solution. However, the focus was to assist students to tackle non-routine problems, rather than those that good students "should" be able to solve routinely (e.g., easily recognising that proportional reasoning applies to a certain situation and being able to use it correctly, or solving an equation).

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 59–62). Melbourne: MERGA.

The 1977 paper and beyond

The paper described an experimental course in problem solving that Susie Groves and I designed at what was then Burwood State College, now Deakin University. We were team-teaching it in 1977 for the second time. The 60-hour problem solving course was nearly half of the first year optional mathematics major (or minor). The paper discussed issues arising in teaching the course, illustrated by examples. The sample problems included designing a car park (geometry of turning circles involved), predicting patterns that can be produced by the Spirograph drawing toy, problems involving mathematics in sport, and finding the best size for a 500 mL drink can (It is not the minimum surface area).

The paper reported a small evaluation of the 1976 course: Half of the students liked it more than their traditional mathematics subjects, one quarter liked it less, and one quarter were neutral. I remember that one "neutral" girl explained that she appreciated how much effort we had put into the course, but whatever we did it was still just work to her. In subsequent years, I have often found a similar 50:25:25 ratio when evaluating innovation.

It is clear from the set of problems described that the label 'problem solving' included both intra-mathematical and real world problem solving. Indeed, what is now called mathematical modelling was the central component. The problems were intended to be non-routine (i.e., not standard applications of learned mathematics) with many of them requiring substantial investigation and a solid written report for assessment. The problems were always open; some open at the start, some in the middle, and some at the end. Solving them successfully required some independence of thought, originality, and common sense, directed by strategic thinking and metacognitive control, and supported by deep mathematical knowledge and a productive disposition. Both Susie and I were strongly motivated to share the joy of mathematical discovery with our students, and to show how mathematics gives insight into real world situations. Our central goal was "teaching for problem solving" to help our students become better problem solvers, giving them confidence and strategies to use whatever mathematics they knew to understand the world.

The Context

In 1977, problem solving was still a fringe activity in teaching mathematics but it was attracting growing interest. For example, Georg Polya had written his famous books about heuristics and made an influential film (Polya, 1966) demonstrating how he worked on a challenging problem with a class. Henry Pollack (1968) was conducting an experimental course at Teachers' College (Columbia) that used mathematical modelling of accessible problems to motivate and illustrate mathematics.

At that time, interest in problem solving was especially strong in university teacher education. Prospective teachers have less need for specific content in tertiary mathematics, but especially need a broad view (Stacey 2008). Sadly, the opportunity to design tertiary mathematics programs specifically for prospective teachers is now rare in Australia.

At a similar time, to support the developing research into learning mathematics, work in psychology on human problem solving was maturing, developing information processing theories and using research methods such as protocol analysis for looking at cognitive processes. This research opened up beyond the cognitive in later decades.

Beyond 1977

Susie and I ran the course for several years, developing it in many ways. We built a wonderful collection of rich problems from many sources – far too many to use them all.

We learned that problem solving was better taught by looking at a fewer problems in depth, encouraging students to generalise and extend solutions, thereby achieving reflection on the solution (Polya's "looking back" phase) by "looking forward". Many teachers beginning to teach problem solving feel they do not have enough good problems, but in fact they "waste" good problems with narrow interpretations of the task until they come to appreciate the gems hiding below the surface, and see multiple solutions.

We also learned to engineer problems for classroom use: for example, to present them so that everyone can start, and to ensure that there was something interesting to find out (ideally a surprise). Many of the problems that we developed, and what I learned from our students' solutions, were included in *Thinking Mathematically* (Mason, Burton, & Stacey, 2010), written mostly in 1980 and now frequently described as "a classic".

Soon, we began to take our problem solving pre-service students to teach problem solving in primary schools. They (and we!) were able to observe children's thinking first hand, and to appreciate how a problem can be solved in multiple ways and at multiple levels. For example, upper primary students could often find a general solution to Arithmogon puzzles when the problem was presented as finding an unknown number of beans hidden in matchboxes arranged around a triangle or square. Students working cooperatively could often create convincing proofs of their method, expressed in concrete terms. At the same time, a general solution using algebra involves the independence/dependence of systems of linear equations that the pre-service teachers were learning in Linear Algebra. These experiences led us to work with teachers in schools to develop problem solving lessons for early secondary school students, and thence into mathematics education research. This work culminated in "Strategies for Problem Solving" (Stacey & Groves, 2006) first published in 1985. The word "Strategies" in the title refers to both teaching strategies and mathematical heuristic strategies.

The Zenith of Problem Solving as the Central Goal of School Education

Australia's attention to problem solving was greatly boosted when the NCTM's Agenda for Action (1980) declared that "Problem solving should be the focus of mathematics education in 1980s" (p. 1). Advocacy exploded and research and development blossomed. Within a few years, every educational jurisdiction in Australia proclaimed problem solving as central to all levels of the school curriculum. In MERGA's 1988 four-yearly review, we published an annotated bibliography (Groves & Stacey, 1988) of 238 recent Australian articles on problem solving. Although classification is somewhat arbitrary, 56 articles described innovative practices, 59 mainly discussed the importance of problem solving in the mathematics curriculum, 65 addressed aspects of teaching problem solving, 51 looked at cognitive processes and seven were concerned with assessment. Lester (1994), a speaker at MERGA1, provides a detailed summary of mainly American work from the 1970s until 1994, when a socio-cultural perspective was added, and Schoenfeld (2008) extends the timeframe.

Just a few years later, assessment of problem solving became a highly controversial issue. Ambitious curriculum and assessment change was implemented in Victoria to cement a place for problem solving and mathematical modelling in the mathematics subjects of the Victorian Certificate of Education. Students worked on substantial 20 hour mathematics projects (investigations) and solved challenging problems, writing up solutions for assessment that contributed directly to tertiary entrance scores. These changes to Year 12 assessment quickly rippled down throughout the secondary school years. Implementation problems caused a backlash, and the initiative has not yet been regained.

Where is Problem Solving now?

Until the 1990s, the strongest thrust in advocacy, research, and development was directed to teaching for and about problem solving, where the main goal is to make students better problem solvers. However, from about 1990, problem solving was split so that reasoning, communication, and connections became separate proficiencies (e.g., in the NCTM Standards of 1989), and the main goal became "teaching through problem solving" in order to teach the specified curriculum better. Whilst this has always been the main inclination of most teachers, it was also boosted by changing national accountability and assessment regimes for schools, as well as national and international thinking. The Australian Curriculum: Mathematics v8.3 (ACARA, n.d.) illustrates this very clearly. The proficiency strands, of which problem solving is one, are said to "describe how content is explored and developed [and] provide a meaningful basis for the development of concepts […]". Making students better problem solvers in any broad sense is not a prominent goal.

At best, the themes that originated in the early problem solving movement are now harnessed in a cluster of related new styles of teaching. Labels include inquiry teaching, sense-making, reform teaching, and standards-based. The themes include the importance of purpose developing autonomy as a learner and as a thinker, understanding the purpose of what you are learning, developing productive habits of mind including persistence, appreciating rigorous arguments, making conjectures. The evaluation of the performance of students who have been consistently taught in this way (mainly US studies comparing 'reform' and traditional curriculum programs) generally shows that 'reform' students perform about the same on skills, and better on concepts and applying their knowledge.

Problem solving in the 1977 sense is *the* reason for teaching mathematics. It cannot just be thought of as a teaching method or one of a number of goals. We need to reinvigorate efforts to value and work towards this most elusive, but most fundamental, benefit of learning mathematics.

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BETH SOUTHWELL
PRACTICAL IMPLICATIONS AWARD

The Beth Southwell Practical Implications Award was initiated and sponsored by the National Key Centre for Teaching and Research in School Science and Mathematics, Curtin University, Perth, Western Australia. Curtin sponsored the "Practical Implications Award" (PIA), as it was then known, for the first 10 years. The Australian Association of Mathematics Teachers (AAMT) now sponsors the Award. In 2008, MERGA was honoured to be able to rename the PIA as the Beth Southwell Practical Implications Award, in honour of MERGA's and AAMT's esteemed late member, Beth Southwell.

The award is designed to stimulate the writing of papers on research related to mathematics teaching or learning or mathematics curricula. Application for the award is open to all members of MERGA who are registered for the conference. Applications for the PIA are judged against specific criteria set by a four-member panel. The panel consists of two members from MERGA, two from AAMT, and is chaired by the MERGA Vice President (Development).

Framing, Assessing, and Developing Children's Understanding of Time

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An understanding of time which goes beyond the reading of clocks and calendars is crucial to full participation in society. This paper reports on classroom experiences and pedagogies that assisted Year 3/4 children's development when learning about time, drawing upon a Framework for the Learning and Teaching of Time, interview data, an eight-lesson intervention and student improved performance on the interview following the intervention.

Introduction and Theoretical Framework

Time is complex, but it plays a crucial role in our full participation in society. An understanding of divisions of time and temporal patterns allows adults to anticipate future events (Friedman, 1991, 2000; Hudson & Mayhew, 2011) and to have memories of times past. Fraisse (1984) considered time as an intricate subject, being associated with world time and personal time. Friedman (2011) described time as "many things: recurrent sequences of events, natural and conventional time patterns, invariant causal sequences, logical relations between succession and duration, the past-present-future distinction and many others" (p. 398).

The learning and teaching of time is listed in the *Australian Curriculum: Mathematics* under Measurement with the focus on students learning to operate with clocks and calendars (Australian Curriculum Assessment and Reporting Authority [ACARA], 2016). Personal classroom experience, other teachers' anecdotes and past NAPLAN test results accord with some research literature (e.g., Kamii & Long, 2003) in convincing us that for some children the concept of time is complex and complicated. The teaching of time should include a broad range of experiences (Casasanto, Fotakopoulou, & Boroditsky, 2010; Kamii & Long, 2003; Piaget, 1969) and include aspects of time such as duration and succession (Fraisse, 1984; Vakali, 1991) and psychological time (Friedman, 1978; Vakali, 1991). Dickson, Brown, and Gibson (1984) emphasised the distinction between telling the time and a concept of time as children may be trained to read the dials on a timepiece but have difficulty in understanding a concept of time. Other studies consider an understanding of time develops gradually from infancy to adolescence (Friedman, 2011; Piaget, 1969; Trosborg, 1982). While scholars have contributed to our understanding of concepts of time and its development, it would seem there is a paucity of research relative to other curriculum areas.

The perceived inadequacy of both the curriculum and the dearth of research literature led to the development of a more comprehensive Framework for the Learning and Teaching of Time ("the Framework", see Figure 1) that encompassed major underpinning components of time, the first version of which was reported in Thomas, Clarke, McDonough, and Clarkson (2016).

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 64–73). Melbourne: MERGA.

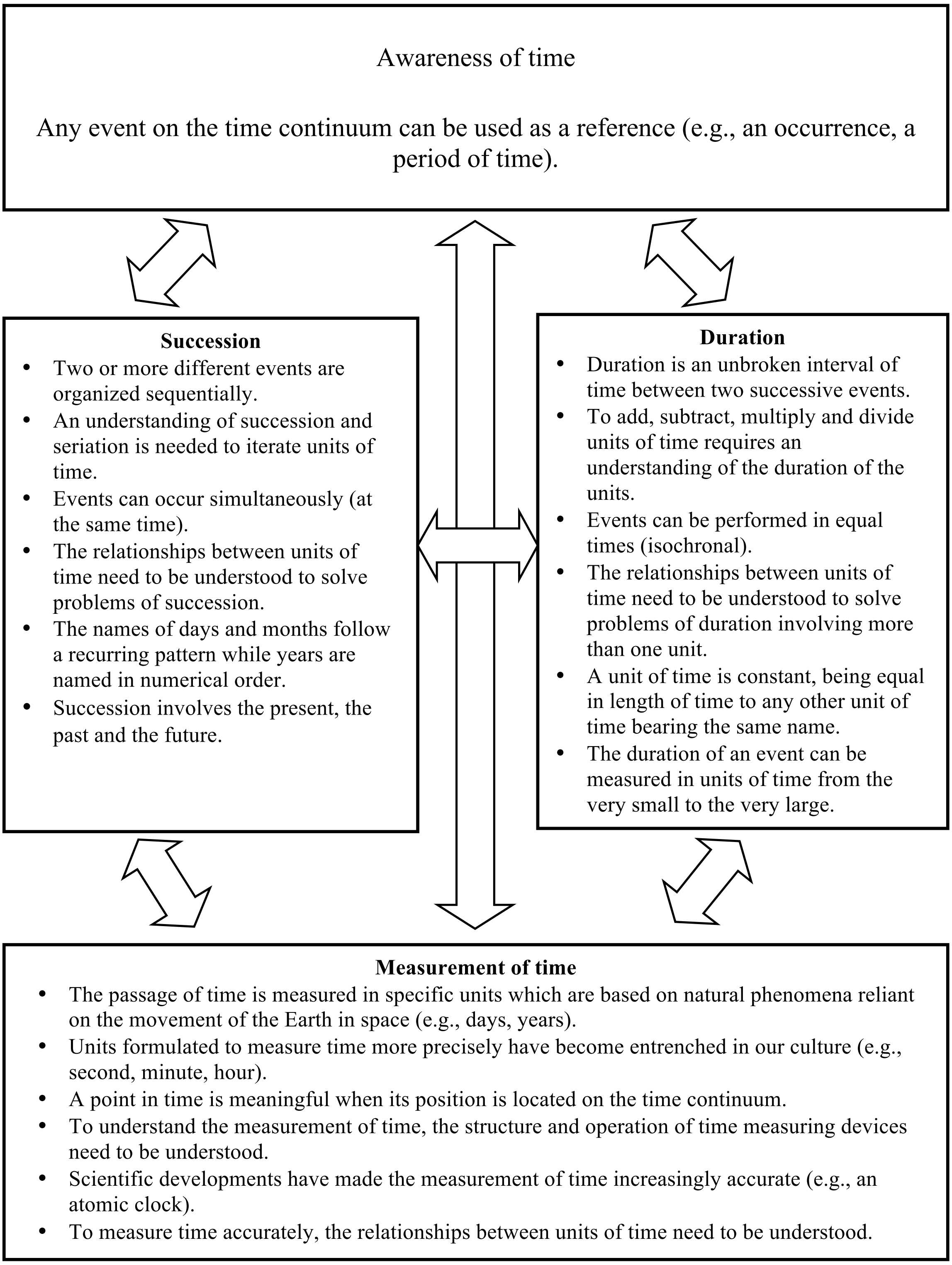

Figure 1. A framework for the learning and teaching of time.

It was argued that for students to understand, interpret, and be fully equipped to use time effectively, they need to have an understanding of these four components: an Awareness of time, an understanding of Succession of time and Duration of time, and be able to Measure time. The key ideas listed as dot points under each component further explain the Framework and add to its value and importance for teachers and researchers alike.

After analysing the data from an eight lesson intervention focussing on time, the Framework was reviewed with several refinements being made to improve its clarity. The final version of the Framework is presented in Figure 1. Apart from presenting the final version of the Framework, this paper also reports on an eight lesson teaching intervention and pre and post students' results from a one-to-one interview tool outlined previously (Thomas, McDonough, Clarkson, & Clarke, 2016).

Methodology

The one-to-one interview was selected as an assessment tool as it was considered to be an informative and reliable tool. Clements and Ellerton (1995) had raised questions about the reliability of pencil and paper tests to assess mathematical understanding, whereas by talking to students in a one-to-one interview, teachers are able to develop a deeper understanding of the students' thought processes (Webb, 1992), strategies and cognitive processes (Ginsberg, 2009). When interviewing their students, teachers are able to diagnose misconceptions and assess a student's ability to express mathematical knowledge verbally (Huinker, 1993), particularly during the early years of schooling when reading and writing skills may be limited (Clarke, 2001).

The Framework was the basis for the development of the one-to-one task based interview to assess a group of Year 3 and 4 students on their understanding of time (Thomas, McDonough, et al., 2016). Twenty-seven students from a class of 30 (five girls and nine boys from Year 3 and six girls and seven boys from Year 4) were interviewed on two occasions; the first interview was prior to the eight lessons, with the second interview three weeks after the intervention. The interview proved to be a comprehensive assessment tool as it was formulated around three of the four major components of the Framework (Succession, Duration, and Measurement). Awareness of time was deemed to be incorporated into each item and hence it too was assessed, though not reported specifically.

Each item had a range of anticipated responses. Responses to each of the 69 items which were assessed as demonstrating a full understanding gained two points, a partial understanding gained one point, and if the student demonstrated no understanding at all, they did not gain any points. This marking regime followed that of Clements and Ellerton (1995). To analyse each student's understanding of each component of the Framework, their points for each item listed under that component were tallied and a score given. A total score for all items was also calculated to give an overall summary for each student and to allow for comparisons to be made between students. Addressing each of the key ideas in the interview proved to be an effective way to assess each student's understanding of each component.

It was decided to make the focus of the intervention lessons the mathematics underpinning those interview items for which the total score from the class was less than 75% of the maximum possible score (a raw score of 40 or less from a maximum possible score of 54) on the pre-test. Although 75% was a somewhat arbitrary figure, it indicated those key ideas for which improvement was desirable and hopefully achievable. A sample of items for which performance indicated the need for attention in the eight lessons is shown in Table 1 alongside the score for the item and the components assessed: S (Succession), D (Duration), and M (Measurement).

Investigation of the individual student scores on the pre-test showed that fewer than half the students scored more than 75% in any of the components. For the Succession items, there were 11 students who gained more than 75%; for the Duration items, there were three students; and for Measurement, 11 students scored over 75%. Although the highest score achieved by a student was 90%, the results demonstrated the need for the eight lesson intervention, as it was anticipated that all students would benefit in some way.

Table 1
A Sample of Items for which Performance Indicated Needed Attention in the Eight Lessons

Interview item	Score and	Component
What can you tell me about the rotation and revolution of the Earth?	6	SDM
If you had a calculator, how would you work out your age in days?	10	DM
Tell me how we use clocks to measure time.	11	DM
Today is Wednesday. When will Wednesday finish?	19	SD
What units to measure time do you know?	27	M
What was the date exactly one month ago?	29	SM
Write this digital time as seen on an analogue clock. ¼ to 6.	30	M
How long does it take for the hour hand to move from the 8 to the 9?	32	DM
I have been given enough eggs for exactly one week (one per day). If I ate the first egg on Wednesday, which day would I eat the last egg?	32	S
How many minutes does it take for the minute hand to move from 4 to 5?	35	DM
What will the date be two years from now?	36	SM
What year were you in Prep?	40	SM

The eight lesson intervention focussed on student activities related to the measurement of time such as the rotation and revolution of the Earth, the observation of seconds and minutes on working clocks, and the measurement of hours from any given minute on the clock face. As a stimulus to their thinking, each lesson began with a text that related to the focus of the lesson. The texts, which included both fiction and non-fiction, were read to and discussed with the children. Data collected from the lessons included audio-recording of the children completing the tasks, anecdotal notes from classroom observations, classroom artefacts such as children's written and drawn task recordings, and letters and self-reflections written by the children about their learning. Three weeks after the intervention, the children were reassessed using the same one-to-one task based interview.

Results

The maximum number of possible points gained by responding to all items of the interview with full understanding was 138 (Succession 56, Duration 62, and Measurement 98). The maximum scores for components do not sum to 138 as a number of items were linked to more than one component. (Note the double arrows in Figure 1, which suggest overlap between the components.) The results from the pre-intervention interview show that from a possible maximum score of 138, the students' scores ranged from 48 to 124, with a mean score of 93.4, and a median score of 96 (see Table 2). All the students showed an improvement in their scores for the post-intervention interview, with scores ranging from 63 to 133, with a mean score of 108.0 and a median score of 112. The minimum score increased from the pre-intervention interview to the post-intervention interview by 15 points and the maximum score increased by 9 points.

Table 2
A Comparison of Scores: Pre-Intervention and Post-Intervention Interview

	Pre-Intervention Interview	Post-Intervention Interview
Mean	93.4	108
Median	96	112
Minimum	48	63
Maximum	124	133
Maximum Score possible	138	138

Although all students gained a higher *total* score for the second interview, not all of the students' scores on *individual* interview items increased. Of the 1,863 responses to the individual items, 432 scores (23%) increased, 1,263 scores (68%) remained the same, and 168 scores (9%) decreased. A more detailed analysis for the increase and decrease in scores by item can be seen in the crosstabulation in Table 3.

Table 3
Student Responses from the Pre-and Post- Intervention Interviews

		Post-intervention interview			
		0	**1**	**2**	**Total**
Pre-intervention Interview	**0**	152	96	184	**432**
	1	42	146	152	**340**
	2	52	74	965	**1091**
	Total	**246**	**316**	**1301**	**1863**

Clearly, the majority of responses (52%) are in the cell 2 > 2 implying that for many items, students scored maximum points in the first interview and did so again when interviewed the second time. In other words, these students had reached a ceiling in this scoring regime before any intervention and for these items. It was expected for many reasons that some students might fall from an initial two back to zero, and this did happen, but for relatively few responses (3%). More encouraging, 10 percent of responses moved from zero to two.

A further useful set of analyses is the changes that occurred for each of the three components of the Framework targeted by this assessment tool. The relative descriptive statistics for Succession, Duration, and Measurement are shown in Table 4.

Table 4
Descriptive Statistics for Succession, Duration and Measurement

	Succession		**Duration**		**Measurement**	
	Sept 2015	Nov 2015	Sept 2015	Nov 2015	Sept 2015	Nov 2015

Mean	38.7	43.7	36.1	44.8	65.7	76.7
Median	40	44	35	47	66	78
SD	6.8	6.3	9.4	8.0	16.0	13.0
Minimum	16	23	13	26	29	38
Maximum	50	54	54	59	88	96

In summary, the results in Table 4 suggest that this group of students not only improved their overall performance on interview items linked to the Framework, but there was improvement in their performance on each of three components: Duration, Succession and Measurement. For some insight as to why this improvement did occur it is instructive to review key elements of the intervention pedagogy.

Discussion

The significant improvement in children's understanding can reasonably be attributed to the eight lessons and in this section we outline the key pedagogies that we believe led to the students' improved scores from the first to the second interview.

1. Literature. Each lesson began with the reading of a book to the children. The text was directly related to the focus of the lesson and was selected to promote interest in the topic and engage the children in discussion. For example, *Clocks and More Clocks* (Hutchins, 1970) promoted discussion on duration. Reading and discussing children's books which relate to the mathematical focus of the lesson has been found to enhance the students' learning of mathematics (Elia, van den Heuvel-Panhuizen, & Georgiou, 2010; van den Heuvel-Panhuizen, van den Boogaard, & Doig, 2009).

2. Physical involvement. One of the most notable lessons involving the movement of the children was a lesson relating to the rotation and revolution of the Earth in space. As the children had limited knowledge of the Earth's movements, the teacher-researcher introduced them to rotation and revolution by firstly reading a story, and secondly, by having the children act out the movement and position of the Sun and the Earth whilst giving a narrative of their actions. Informal discussions during the days on the intervention, and an increase in scores for the interview item about rotation, led us to the conclusion that this lesson was one of the most memorable because of the physical involvement.

3. Equipment. The selection of equipment for each lesson played an important role in engaging the students in each activity. The most useful pieces of equipment were real clocks. In order for the children to be able to measure the passing of time, it was essential for them to use working clocks to observe the second and minute hands moving. The use of real clocks meant that the children could 'see' the duration of a second or a minute and appreciate that a minute is measured on the clock by the space that the hand had moved through with the minute lines showing the beginning or end of the duration. Seeing the movement of the hands as the clock ticked assisted the children to count elapsed minutes and seconds and to understand how a clock is read. Other important pieces of equipment were sand timers, which were checked for accuracy with a clock, large balls used as the Earth and the Sun, and items such as laminated numbers and walking sticks, to build a clock on the floor.

4. Correct terminology. The use of the correct terminology was instrumental in assisting the children to understand the focus of each lesson. The children who were accustomed to the terms ‘big hand’ and ‘small hand’ were intrigued to find that these hands could just as easily be named the ‘hour hand’ and the ‘minute hand’ thereby reducing any confusion. Terms such as duration and succession were used frequently after being introduced to the classroom.

5. Group work, discussion, and self-reflection. The classroom teacher’s mathematics groups comprised students with similar levels of understanding, based on her assessment. The activities undertaken during the intervention relied however on mixed ability groups working together and discussing their findings. At the conclusion of each lesson, children were encouraged to share their learning with the remainder of the class to reinforce their learning following which, all the children were required to write a self-reflection to consolidate their individual learning experiences. To assist their reflections, the children were given a different strategy each lesson. For example, a three, two, one reflection required children to list three things remembered from the lesson, to give two examples of something they had learned, and to write one question regarding something which was confusing to them. The self-reflections encouraged the students to consider the purpose of each lesson. At the commencement of the following lesson, the students were asked to recall what they had learned during the previous lesson.

6. Time to complete a variety of activities. Timetabling eight lessons for the intervention allowed many different activities to be experienced by the children. Lessons were planned to be sequential so that ideas from one lesson could be developed further in the next lesson. Past lessons were reviewed at the commencement of each new lesson, so that questions could be asked and ideas shared.

Summary

The 69-item one-to-one interview was shown to be very effective in identifying assumed strengths and weaknesses in the children’s understanding of time. Prior to the intervention, the children did not appear to understand the notion of time being measured and that units such as second, minute and hour were used to measure durations of time. Introducing the children to the rotation of the Earth gave many of the children an understanding of a 24-hour day, which includes periods of light and dark. Observing the revolution of the Earth around the Sun helped the children to understand why the calendar year has 365 days and why we intercalate a day every four years to make a Leap Year. By observing the second and minute hands of a working clock as they moved across the spaces between the minute marks, the children could ‘see’ the clock measuring the duration of a second and a minute. Giving the students real clocks, a variety of activities, interesting books and time to reflect on and discuss their learning were all found to be beneficial to the children’s improved understanding of time.

Practical Implications for Teaching

The *Australian Curriculum: Mathematics* lists the learning and teaching of time under Measurement with the focus on students learning to use the tools of time measurement: clocks and calendars (ACARA, 2016). The outcomes to be achieved by the end of Year 3 include knowing the months and the seasons, a knowledge of the calendar, and reading a clock to the minute. However, the concept of time has been found to be complex and challenging for many children and much more than just its measurement (Casasanto et al., 2010; Dickson et al., 1984; Kamii & Long, 2003; Piaget, 1969). Despite some research on the development of our understanding of concepts of time (Friedman, 1991, 2000; Friedman & Lyon, 2005; Hudson & Mayhew, 2011), there seems to be a dearth of research into the learning and teaching of time. In this study we have identified major components of time and incorporated them into a Framework for Learning and Teaching Time, developed a one-to-one task based interview to assess student understanding, and undertaken an eight-lesson intervention on time. The practical implications for each are detailed below.

The Framework for the Learning and Teaching of Time. The Framework incorporates four major components of time: Awareness of time, Succession, Duration, and Measurement of time. Rather than being a linear model, the Framework demonstrates how notions of time are not learnt in sequence, but over an extended period in interconnected ways, ensuring a deep understanding. Awareness of time includes knowing about a point in time, the language of time, temporal patterns and psychological time. The literature indicates that an Awareness of time seems to be the natural starting point before moving to untangle the deeper notions of the Framework (Ames, 1946; Friedman, 1977, 1990). Succession is the sequential ordering of time (Fraisse, 1984). Duration is the passage of time, with each duration requiring a starting and a finishing time (Fraisse, 1984). The Measurement of time is crucial to the understanding of time, requiring a knowledge of specific units of time and time measuring tools. The Framework has been designed as an important tool to inform both teachers and researchers, and to guide curriculum writers and teachers in the planning and implementation of lessons on the concept of time. By emphasizing the interconnectedness of the components, we try to counteract the notions of teaching each independently and serially.

The one-to-one task based interview. The interview has been designed to assess children in the middle primary school years on three of the four major components of the Framework: Succession, Duration, and Measurement of time. Children in the middle years of primary school were assumed to have an Awareness of time, and as such, it was not assessed separately but deemed to be incorporated into each item. An individual child's responses to the 69 items in the interview are calculated to inform the teacher of his/her apparent strengths and weaknesses in each of the four major components of time. The interview has been designed to be repeated over time as not all children are expected to demonstrate full understanding of each item. The interview proved to be easy to use but offered informative insights.

Pedagogies implemented during the eight lesson intervention. Given that all students improved in their understanding following the intervention lessons, as measured by the assessment interview, it is important to describe the underlying pedagogies of the lessons. Based on the experience of teaching the lessons, we would encourage the use of picture books and actual working clocks in any lesson dealing with time. Using correct terminology such as minute hand and hour hand aids in reducing the confusion some children experience when identifying the hands of the clock. Language such as rotation, revolution, duration and succession were readily learnt and used by students. Children need a variety of experiences when learning about time, and active involvement was important with students drawing, writing and discussing their ideas. Activities related to time need to be timetabled regularly and over a lengthy period to promote learning, as time is not just important in mathematics. We would recommend children be given opportunities to draw, write, discuss and share their learning throughout the lesson, particularly at the end.

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INVITED LECTURE

The "M" in STEM: AMSI's Perspective

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AMSI Director

The Australian mathematical sciences have a number of interesting internal interfaces, not least the one between educational researchers and mathematics researchers. This interface is currently looking less like a brick wall and more like an open door; this is important at a time when the mathematics education pipeline is so challenged. Why are there twice as many boys as girls taking Year 12 Advanced Maths? Why is the underachieving tail growing so fast according to the testing regimes? Is secondary school maths over-assessed and too competitive? How prevalent is maths anxiety in parents and teachers? How pervasive should real-world context be in the classroom? Why are so many tertiary students incompetent at the basics? Why is senior school maths so unsophisticated compared to some of the humanities subjects? How can we cope with the heightened expectations of employers and government? I will bring an AMSI perspective to some of these questions.

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RESEARCH REPORTS

The Prevalence of the Letter as Object Misconception in Junior Secondary Students

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In this study, we investigated students' thinking about the use of letters in algebra. Responses from over 1,400 Australian secondary school students to a set of three algebra items were analysed to determine the prevalence of the "letter as object" misconception. We estimate that 50% to 80% of Year 7 students bring this misconception to their initial learning of algebra. Over 50% of Year 8 students and over 40% of Year 9 students in the sample also selected responses consistent with this misconception.

If you speak to a group of adults about their learning of school mathematics, you are likely to find adults who comment that they found mathematics easy until they met algebra. There has been about three decades of research into difficulties/misconceptions that students experience when they learn algebra; some of this research focusses more on the manipulations required to solve equations while other research (such as this paper) focusses on students' thinking about the meaning of algebraic notation.

Steinle, Gvozdenko, Price, Stacey, and Pierce (2009) distinguish between two groups of misconceptions in algebra: *numerical* and *non-numerical*. The first group consists of student thinking regarding the numerical values which letters stand for; e.g. some students reject the solution $x = 8$ and $y = 8$ to $x + y = 16$, as they believe that different letters should be replaced by different numbers. Another numerical misconception is the *alphabetical value,* where $a = 1$, $b = 2$, etc. (see, for example, MacGregor & Stacey, 1997).

The second group of misconceptions that Steinle et al. (2009) refer to is the *non-numerical* misconceptions, which includes the *letter as object* misconception (using the terminology of Küchemann, 1981) where, for example, students think that *a* stands for *apples* rather than the *number of apples*. Clement (1982) noted this error when he used the Students and Professors problem (below) in a set of problems given to a sample of first year engineering students.

> Write an equation, using the variables *S* and *P* to represent the following statement: "At this university there are six times as many students as professors". Use *S* for the number of students and *P* for the number of professors.

Given that these students were enrolled in a mathematics-related degree, Clement was surprised by the considerable number of students who were unable to provide the correct equation (6P = S), instead writing 6S = P. Clement concluded that schools appeared to be "more successful in teaching students to manipulate equations than they have in teaching students to formulate equations in a meaningful way" (p. 28).

In this paper, we are focussing on this *letter as object* misconception, noting that there are several variations in the terminology used in the literature; for example, *letter as abbreviation*, and *letter as unit*. Akhtar and Steinle (2013) reported the prevalence of this misconception in a preliminary study of 850 students, and this paper builds on this earlier work with a larger sample of students.

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Literature Review

One of the foundational studies into students' understanding and skills in mathematics was the large-scale CSMS study in the U.K. Küchemann (1981) reported on the algebra items in this study and described six ways that students interpreted letters. One of the items (referred to here as Pencils) was as follows (p. 107):

> Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence. If *b* is the number of blue pencils bought and if *r* is the number of red pencils bought, what can you write down about *b* and *r*?

The correct answer ($5b + 6r = 90$) was provided by 10% of the 14 year olds in the study. Just under 20% answered $b + r = 90$, and another 6% answered $6b + 10r = 90$ or $12b + 5r = 90$. The last two equations are consistent with "6 blue pencils and 10 red pencils cost 90 pence" and "12 blue pencils and 5 red pencils cost 90 pence", respectively, which indicates that the letters are representing objects rather than numbers. Note that these two equations involve coefficients which are possible solutions to the problem, that is (6, 10) and (12, 5), rather than the information given in the worded problem.

The Students and Professors problem has been used by various researchers on samples of both secondary and tertiary students in various countries (e.g., Rosnick, 1981; Clement, 1982). The incorrect use of a letter in algebra to indicate an object or abbreviation is widespread. Rosnick (1981) indicated that this tendency is "deeply entrenched" (p. 419), and Warren (1998) noted that, "Even students who were considered by their teacher to be very capable of understanding algebraic concepts, included the letter standing for an object in a number of their responses" (p. 666). More recent research indicates that this incorrect use of letters still exists. Egodawatte (2011) used the following problem with Grade 11 students in Canada: Shirts cost *s* dollars each and pants cost *p* dollars a pair. If I buy 3 shirts and 2 pairs of pants, explain what $3s + 2p$ represents? One of the students interviewed (Colin) stated that "3s would equal to 3 shirts and 2p would equal to 2 pairs of pants. So, this would represent the total amount of items he bought… So, in total, there would be 5 items" (p. 119). Colin then proceeded to use the same logic on the next question to state that B stands for "Blue cars".

Textbooks in Australia (Chick, 2009; MacGregor & Stacey, 1997) have been found to contain explanations which use "fruit salad algebra", that is, *a* stands for apples and *b* for bananas. MacGregor and Stacey concluded that the students at one of the schools in their study (School C) were adversely affected by the use of a textbook that stated that letters can be used as abbreviated words and labels. Chick (2009) used a page from an Australian Year 8 mathematics textbook in her study of teachers' pedagogical content knowledge. The teachers in this project were asked to comment on two explanations of the distributive law, the first explanation used images of apples and bananas to show that $2(3a + 2b)$ was equal to $6a + 4b$. Of the 32 teachers who responded to the question about the fruit salad explanation, over 70% indicated that they would use this in the future. About one quarter of the group indicated that they had concerns with this explanation as it would reinforce the *letter as object* misconception.

Akhtar and Steinle (2013) analysed a sample of 850 students and reported that 50% to 70% of Year 7 students brought the *letter as object* misconception to their initial learning of algebra and this decreased to about half of Year 8 students and about one quarter of Year 9 students in the sample. The goal of this paper is to determine if this prevalence is confirmed for a larger sample.

Methodology

The instrument used in this study was a "SMART test" containing three algebra items, two of which were based on the work of Küchemann (1981). SMART tests (Specific Mathematics Assessments that Reveal Thinking) are designed to identify students' misconceptions in particular mathematics topics; in this case, about the use of letters in algebra. SMART tests are short, online, diagnostic tests which are automatically marked so that teachers have instant access to the results. Wherever possible, the tests are based on research findings. We intend that teachers pre-test their students before teaching a topic so that they can use this formative assessment to inform their teaching to better meet their students' learning needs.

Figure 1 contains the text (but not images) of the three items in this SMART test *Letters for numbers or objects?* (www.smartvic.com/smart/index.htm). Note that the multiple-choice options, listed here as dot points, appear in drop-down boxes in the test.

The data for this study came from 26 schools in Melbourne where teachers have chosen to use this SMART test with their students. Of the 1,449 Year 7, 8, and 9 students who attempted this test during 2015, 16 students did not complete the three items, and hence their data was removed. This left 1,433 students in total: 648 students from Year 7, 651 from Year 8, and 134 from Year 9. The Year 7 and 8 sample sizes are larger than our previous study (Akhtar & Steinle, 2013), but the Year 9 sample is of similar size. While this sample is not randomly chosen, we have no reason to believe that it is not representative.

Doughnuts (item 2377)	Garden (item 2387)	Wheels (item 2391)
Lucy bought 6 doughnuts for 12 dollars. She wanted to work out how much each doughnut cost. She wrote the equation $6d=12$. In Lucy's equation, *d* stands for: • one doughnut • dollars • the number of doughnuts • doughnuts • the cost of one doughnut	For my garden, I bought **r** red roses and **g** white gardenia bushes. The roses cost \$4 each. The gardenias cost \$5 each. Choose the equation that says that the total cost was \$70: • 4r + 5g = 70 • 10r + 6g = 70 • r + g = 70	At a bike shop there are **b** bikes (2 wheels each) and **t** trikes (3 wheels each). Choose the equation that says that there are a total of 100 wheels: • 2b + 3t = 100 • b + t = 100 • 35b + 10t = 100

Figure 1. Items from algebra SMART test: Letters for numbers or objects?

Results and Discussion

Figure 2 illustrates the distribution of students' responses across the five options in Doughnuts. The correct option is last; 15% of Year 7 and about 30% of Year 8 and Year 9 chose this option. The option which was chosen most often, however, was the fourth option which is a very clear indication of the *letter as object* misconception; students thinking that "*d* stands for doughnuts". Over 40% of Year 7 students chose this option, while just over 30% of the Year 8 and Year 9 students chose this.

Recall that SMART tests are intended to be used in advance of teaching and, if teachers are using them for this purpose, the data for the Year 7 students should be regarded as pre-test rather than a post-test data. The Year 8 and 9 students, however, have met pronumerals before and the large numbers of students choosing the *letter as object* (LO) option instead of the correct option is of concern.

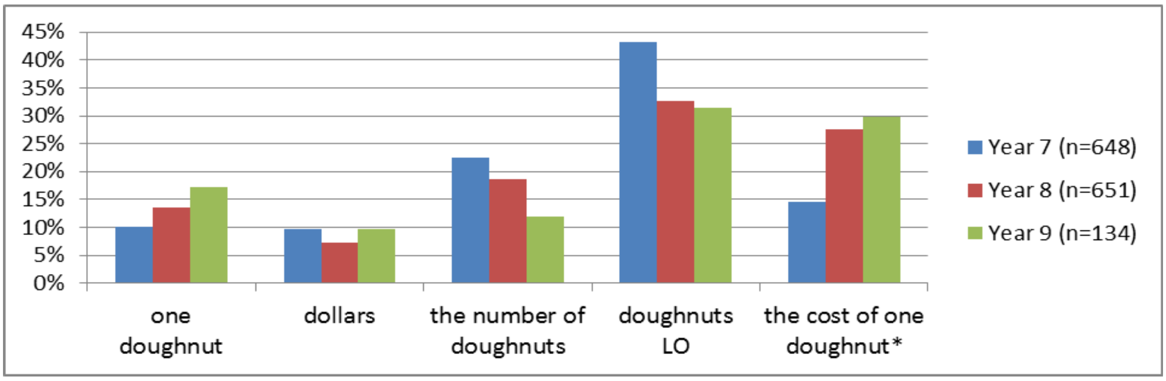

Figure 2. Distribution of responses on Doughnuts (*correct, LO: Letter as Object).

Figure 3 contains the distribution of students' responses across the three options in (a) Garden and (b) Wheels. These are designed to be parallel items and are discussed below. Note the rearrangement of the order of the multiple-choice options in (b) Wheels to match (a) Garden.

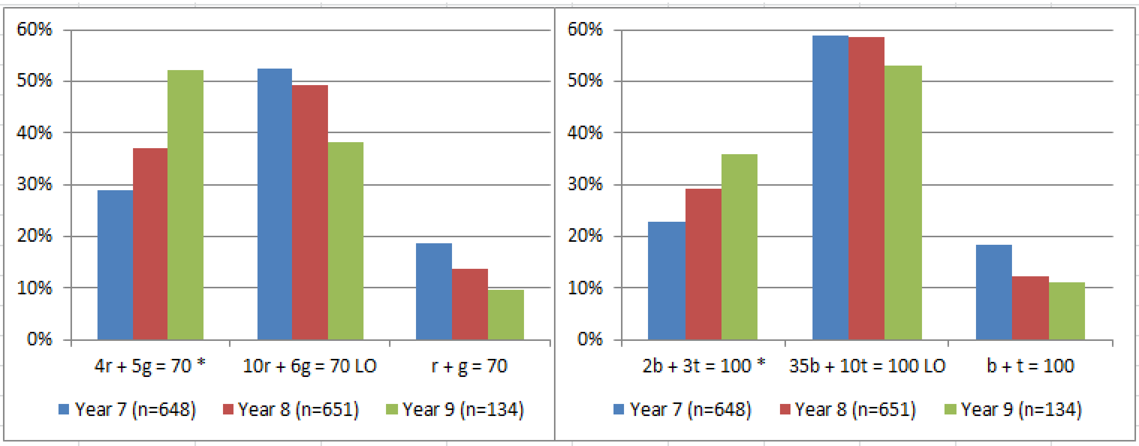

Figure 3. Distribution of responses on (a) Garden and (b) Wheels (*correct, LO: Letter as Object).

Figure 3 indicates that, within each item, there is an increasing trend in the facility from Year 7 to Year 9, but only 52% of the Year 9 students were correct on Garden and only 36% on Wheels. Küchemann (1981) reported that 10% of the 14-year-olds answered the Pencils item correctly; of the Year 8 students in this sample, 37% were correct on Garden and 29% on Wheels. The higher facility in this study is most likely to be due to the multiple-choice format of these items, compared to pen and paper tests reported in Küchemann.

Of the Year 8 students in this sample, 14% chose $r + g = 70$ in Garden and 12% chose $b + t = 100$ in Wheels, similar to the 17% Küchemann noted who wrote $b + r = 90$.

Küchemann reported 6% of the sample wrote $6b + 10r = 90$ or $12b + 5r = 90$ which are indicative of the *letter as object* misconception. In this study, considerably more Year 8 students chose $10r + 6g = 70$ in Garden (49%) and chose $35b + 10t = 100$ in Wheels (59%). The higher prevalence of a response indicative of the *letter as object* misconception in this study is (again) most likely due to the multiple-choice format. In a pen and paper test, students who believe that they need to *solve* the problem *before* they can write the equation are making the task much more difficult for themselves. It will take these *solvers* longer to complete such problems and they might even give up due to the difficulty. In a multiple-choice test, however, these *solvers* can consider each of the given options and choose the one that fits their interpretation (such as $10r + 6g = 70$ means "10 roses and 6 gardenias cost \$70"). Hence, it is reasonable that providing multiple choice options will increase the likelihood of detecting students who think this way.

Garden and Wheels were designed to be parallel items. Figure 4 provides data on students' responses on these two items; the axes are arranged so that students choosing consistently on these items are on the diagonal from left to right.

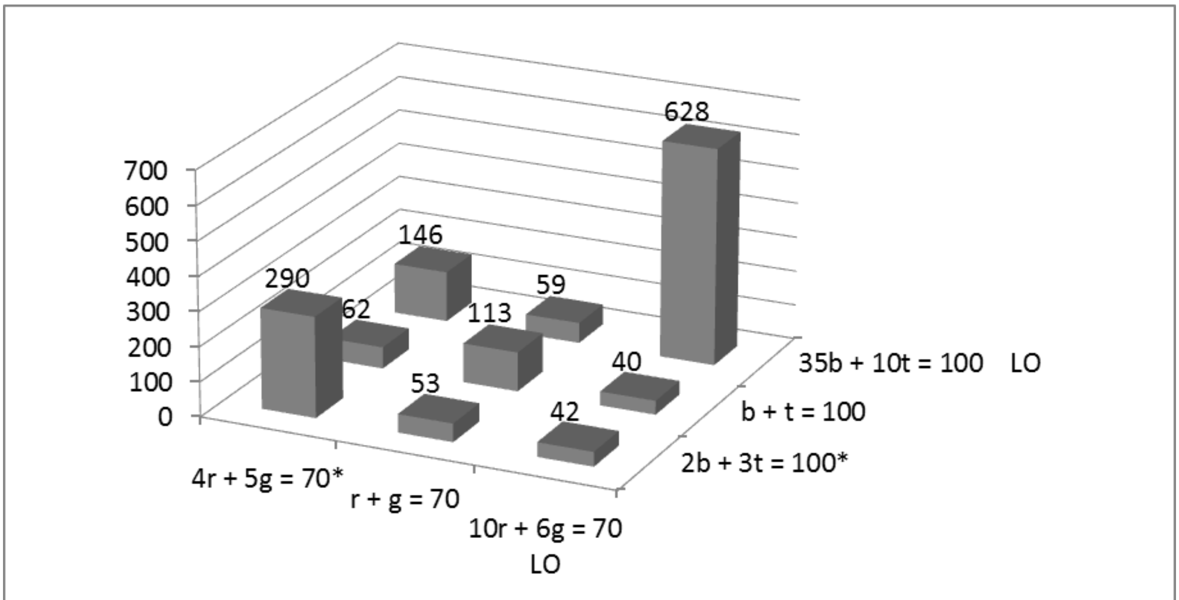

Figure 4. Students' combined responses to Garden and Wheels (n = 1,433).

While 290 students (20%) chose correctly on both items (the left-most column in Figure 4), the right-most column shows that over 620 students (44%) chose $10r + 6g = 70$ and $35b + 10t = 100$ which are the LO options discussed above. Breaking this down by year level; this is 46% of the Year 7 students, 44% of the Year 8 students and 34% of the Year 9 students. Taking account of the earlier comments about the Year 7 data, it is noteworthy that only 22% of the Year 8 students and 32% of Year 9 were correct on both items.

In order to follow students across all three items, a "pattern recognition script" was used on the data. This script has been created to detect common responses by students to a set of items. This data-driven procedure has been found to provide interesting insights into student thinking; see, for example, Steinle et al. (2009).

The six most common response patterns found in this data are listed in decreasing order of frequency in Table 1. The last row (Pattern 6) indicates that only 79 students (i.e. 6% of the sample) chose the correct response on each of these three items. The most common response pattern is Pattern 1; 234 students (i.e., 16% of the sample) chose the three options discussed above indicating *letter as object* misconception. The last column of Table 1 contains the ratio of observed frequency to expected frequency if all students chose randomly on the three items. Pattern 1 has occurred more than seven times what would be expected if choices were random. Pattern 4 (96 students) has correct answers on the last two items, but LO on the first item, indicating that even students with good knowledge seem to be tempted to think about letters as objects occasionally (as found by Warren, 1998).

Table 1
Most Common Response Patterns

Patt-ern	Doughnuts (2,377)	Garden (2,389)	Wheels (2,391)	Freq.	Ratio
1	doughnuts^{LO}	$10r + 6g = 70^{LO}$	$35b + 10t = 100^{LO}$	234	7.3
2	the cost of one doughnut*	$10r + 6g = 70^{LO}$	$35b + 10t = 100^{LO}$	148	4.6
3	the number of doughnuts	$10r + 6g = 70^{LO}$	$35b + 10t = 100^{LO}$	105	3.3
4	doughnuts^{LO}	$4r + 5g = 70$*	$2b + 3t = 100$*	96	3.0
5	one doughnut	$10r + 6g = 70^{LO}$	$35b + 10t = 100^{LO}$	87	2.7
6	the cost of one doughnut*	$4r + 5g = 70$*	$2b + 3t = 100$*	79	2.5

*correct option, LO: Letter as Object

Conclusion and Implications for Teaching

The purpose of this paper was to determine the current incidence of the *letter as object* misconception in algebra in a sample of Australian students some 30 years after the seminal work by Küchemann (1981) in the U.K. The test used was a three-item, multiple-choice, computerised test containing items adapted from Küchemann. Student performance improved from Year 7 to Year 9 on each of the three test items, however, the average facility on these three items was only 40% for the Year 9 students.

If teachers are using this test for formative assessment, then the Year 7 data reported here needs to be interpreted with caution; it will contain some students who have not yet received formal instruction in algebra and hence provides an indication of the thinking that Year 7 students bring to their first algebra lessons in Australia. Exactly 20% of the Year 7 students did not choose any of the responses associated with the *letter as object* misconception, leaving 80% with at least one such response on the three items. Just over 50% of these students had either two or three such responses. Hence, we conclude that between 50% and 80% of Year 7 students bring the *letter as object* misconception to their learning of algebra, which indicates that the 50% to 70% range found in the previous smaller sample was a slight underestimate.

The Year 8 and Year 9 data, on the other hand, provides an indication of the post-teaching prevalence of the *letter as object* misconception. Based on the same criteria as above, between 50% and 75% of Year 8 students and between 40% and 70% of Year 9 students in this sample have this misconception. Thus, our previous estimates (about one half of Year 8 students and about one quarter of Year 9) appear to be underestimates.

It is interesting to note that students who have chosen the options suggesting the *letter as object* misconception in two of these three items, are not choosing an equation based on the *information* given in the question, but are choosing an equation which seems to represent the *solution* to the problem. When teachers provide worded problems for their students to solve, there are likely to be some students who attempt to write an *initial equation* based on the solution to the problem. As noted by Stacey and MacGregor (2000), such students have not grasped the power of algebra to *find* the solutions. Küchemann (1981) noted that this confusion occurred "even with children who did well on the test as a whole" (p. 107).

There is evidence to show that some teachers and textbooks use the letter as object analogy (e.g., Chick, 2009; MacGregor & Stacey, 1997). If teachers believe that it is an appropriate analogy and it is also found in textbooks, then it is likely that students will retain their initial beliefs about letters in algebra standing for objects rather than numbers.

> The SMART test system was designed to make the results of mathematics education research readily available to teachers. As well as the diagnostic information about each of their students in the specific topic, teachers are provided with explanations of the diagnoses and teaching suggestions for dealing with misconceptions and for taking students to the next level of understanding. We expect that this information, in the context of the results of their own students, will increase teachers' pedagogical content knowledge in the particular topic. As Holmes, Miedema, Nieuwkoop, and Haugen (2013) note in their study with teachers: "All in all, identifying and correcting misconceptions, not mistakes, is a skill well worth developing" (p. 40). Likewise, Russell, O'Dwyer, and Miranda (2009) conclude, "this study suggests that the use of diagnostic assessment systems, such as the DAAS, promises to enhance teaching and learning by enabling teachers to more effectively assess student understanding in a timely manner, diagnose misconceptions, and then help students develop their understanding so that a given misconception is no longer held" (p. 423).

Support for explicit classroom discussion of incorrect student work (including misconceptions) is provided by Booth, Lange, Koedinger, and Newton (2013). They compared the progress of students receiving various instruction and noted,

> The present study…suggests that receiving incorrect examples can be beneficial regardless of whether it is paired with correct examples. This finding is especially important to note because when examples are used in classrooms and in textbooks, they are most frequently correctly solved examples. In fact, in our experience, teachers generally seem uncomfortable with the idea of presenting incorrect examples, as they are concerned their students would be confused by them and/or would adopt the demonstrated incorrect strategies for solving problems. Our results strongly suggest that this is not the case, and that students *should* work with incorrect examples as part of their classroom activities. (p. 32)

Preliminary evidence of the success of the SMART test system is provided in Steinle and Stacey (2012). Teachers are requested to complete surveys after using a SMART test. One of the multiple-choice survey questions is: As a result of using this quiz have you learned something useful for you as a teacher? Of the 127 responses to this question, 92% answered either "Yes, very valuable learning" or "Yes, useful learning". Another question probed the effect on teaching practice: Did you adjust your teaching plan as a result of the diagnostic information? Of the 124 responses to this question, 87 (70%) indicated that they did adjust their teaching. We expect that teachers' use of this system will lead to improved teaching and learning as they take steps to either avoid misconceptions (such as not using unhelpful analogies) or to help students to leave them behind (by discussing them explicitly in classrooms).

Acknowledgements

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Developing Interactive ICT Tools for the Teaching and Learning of Vectors at A-Level

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The aim of this qualitative study is to embed ICT in the teaching and learning of A-level vectors. Initially, students' learning difficulties and misconceptions in A-level vectors were diagnosed. Then, based on the difficulties identified, concepts in vectors together with interactive tools were developed and integrated in a webpage to enhance the teaching and learning of vectors at A-level. Finally, the tools developed were exposed to trainees for feedback and evaluated using the framework proposed by Pillay and Clarke (2008). The tools met the following criteria: learner focus, integrity, usability, and accessibility.

The world is experiencing a major shift in educational practices under the umbrella of Information Communication and Technology (ICT) enabled learning environment. Many studies have shown that the use of ICT in the teaching of Mathematics tends to improve learning, motivate and engage learners, promote collaboration, foster enquiry and exploration, and create a new learner centered learning culture (Glenn, 2004; Ng, 2005; Resta, 2002; Zhu, 2003).

Countries like United States, Australia, and the U.K. have developed high quality of online curriculum content for schools using dynamic and interactive software. This research project targets the creation of a dynamic online tools for A-level vectors for both teachers and students. There are hardly any online mathematical tools with connected pedagogical knowledge and technological tools suitable for the teaching and learning of A-level vectors for the Mauritian context. This study is an initiative to kick start research on the use of interactive ICT tools in the teaching and learning of mathematics at the secondary level in Mauritius.

During the last few years, the average pass rate in A-level Cambridge mathematics examination was close to 86 % with only around 54 % of the candidates managing to get at least a grade C. These figures show that students are encountering difficulties to perform well in A-Level mathematics. Some of the chapters in which students faced difficulties are integration, complex numbers, differential equations, and vectors. This study will focus on vectors.

Learning of vectors represents an important step in developing students' ability to solve problems. Knowledge of vectors is crucial both in solving problems from other chapters of mathematics and in other subjects like physics. However, students at A-level encounter difficulties in understanding and applying the concepts of vectors especially in three-dimensional space.

Both teachers and students have often commonly reported that the concepts of vectors are abstract and hard to apply. Every year, the general comment from the chief examiner of A-level Cambridge Mathematics examination includes the remark "Candidates have difficulties to work with the vector question and generally show poor mastery of its concepts". Examination reports also commonly include comment like the main difficulty

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of the candidates is that they do not grasp the geometry of the situation and work with irrelevant pairs of direction vectors.

This problem is also specified by many teachers who believe that students are not able to understand and visualize the physical geometry of vector questions. Such students are unable to convert the semantic structure of the vector question into a good visualization of the physical problem of the question, especially when both lines and planes are involved.

This study targets the arousal of visualization sense in students so that they can convert verbal abstract vector questions into visually sound physical problem using ICT. It also helps to promote conceptual understanding and will help in improving performance in A-level, as displayed by ICT-based research studies (BERA Professional User Review, 2014).

Aims

The general aim of this study is to embed ICT in the teaching and learning process of the A-level Vectors to fulfil good pedagogical principles, that is, to make teaching and learning more engaging, motivating and interactive. Moreover, ICT-based teaching promotes conceptual understanding and retention ability. The following objectives are targeted in this research work:

- To diagnose students' learning difficulties in A-level vectors.
- To develop interactive tools to enhance learning of vectors in A-level.
- To implement and evaluate the teaching and learning tools developed.

Methodology

A mixed-method approach was used and the study was scheduled on different phases. The participants were pre-service trainee educators from the Mauritius Institute of Education (MIE) and A-level students.

During Phase 1, the research team reviewed the literature and Cambridge reports on the difficulties faced by teachers and students with respect to A-level Vectors. In addition, the existing ICT tools for the teaching of mathematics was documented. In parallel, a questionnaire, based on A-level vectors, was designed and administered to 45 MIE trainees. The sample contained 26 trainees enrolled on Teacher Diploma Secondary (TDS) and 19 Trainees enrolled on Bachelor in Education (BEd) programmes. The data collected gave an insight of the difficulties (in key concepts) faced by trainee teachers.

In Phase 2, based on the findings of Phase 1, the research team designed appropriate lessons (instruments) to develop interactive tools, which hold pedagogical promise on A-level vectors. The content area vector was divided into three subsections, three dimension vectors (including coordinates in 3D), lines and planes. The whole work was collaboratively vetted by the research team. The use of Tablets PC was also used by research team in the development of the teaching tools. The help of a graphic designer was needed for the setting up of the electronic page. The learning tools cater for different learning styles and allow the user to (i) review their previous knowledge, (ii) interact with the tools so that they can create or develop new ideas for better understanding, and (iii) solve problems on the concept to which they are exposed.

In Phase 3, once the learning resources developed, they were piloted with pre- and in-service educators and A-level students, and refined accordingly. The tools will also be accessible through Tablets PC. The webpage will soon be uploaded on the MIE website to provide access to everyone. The webpage has been designed so that it is user-friendly. A mathematics community will be developed to promote discussions and sharing of ideas between educators and students among others.

Evaluation of the Teaching Tools

The teaching tools were evaluated using the framework "evaluating ICT-based materials" proposed by Pillay and Clarke (2008). The framework consists of two phases. The first one deals with the pedagogical principles that guides the educational soundness of ICT-based tools. Under this phase, the framework considers the following criteria: learner focus, integrity, usability and accessibility of the tools. These criteria for ICT-based tools are expected to successfully promote learning. The second phase of the framework focused on the design, use and impact of the ICT-based tools. The design evaluation criteria relate to the quality of the educational soundness of the tools, the use of evaluation criteria relates to the factors influencing the utilization of the tools, and the impact of evaluation relates to the learning outcomes of the students. The framework is displayed next.

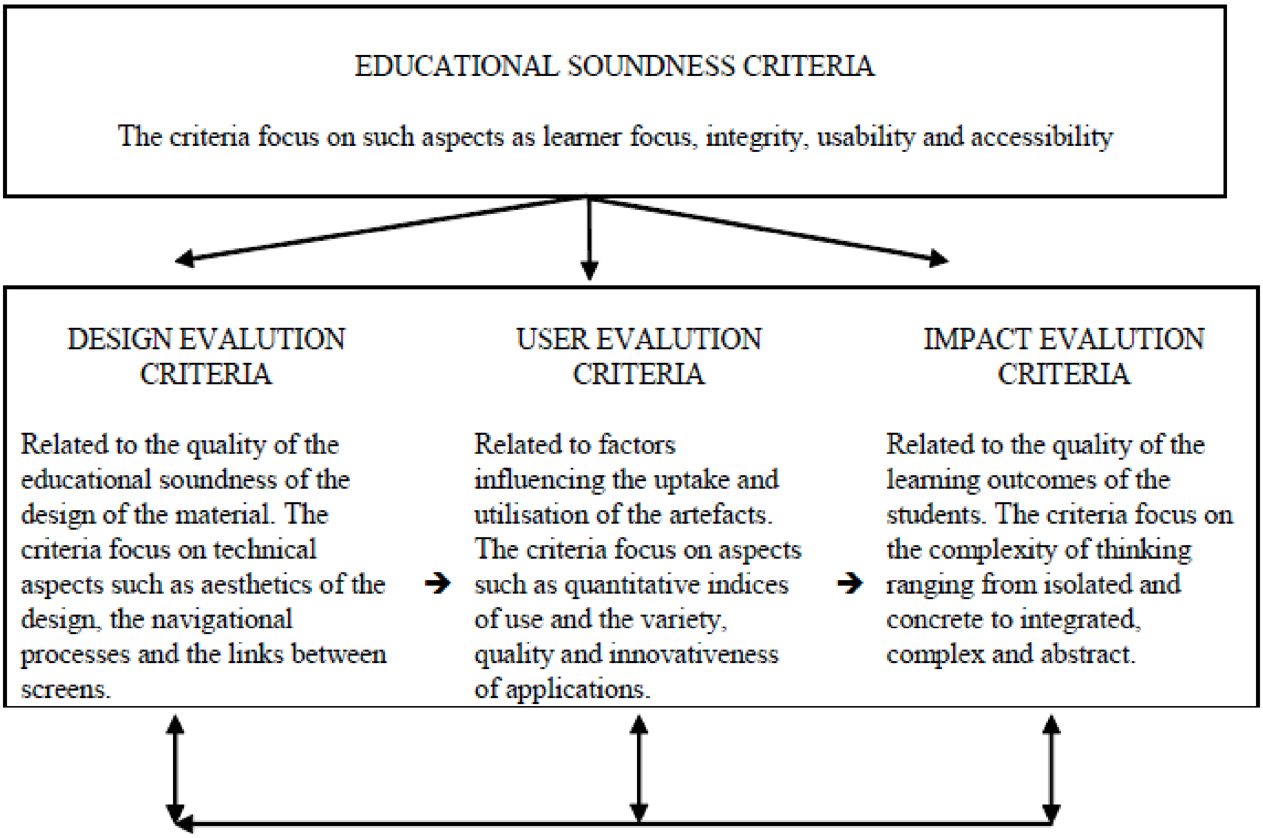


Figure 1. Framework for evaluating ICT-based learning materials (Pillay & Clarke, 2008).

Analysis

Learning Difficulties in A-Level Vectors

A questionnaire developed and administered to gauge the difficulties in A-level Vectors by MIE trainees (TDS and BEd). The findings from the questionnaires are surprising. Many questions were left unanswered while most of the solutions provided, for the questions attempted, were wrong. A major issue found in Cambridge reports is the lack of diagram in solutions to problems on vectors. Many of the difficulties faced by the trainees matched those found in the Cambridge reports. These observations demonstrate that the trainees still faced difficulties in vectors and this is quite surprising, in particular for TDS and BEd trainees who are aspiring mathematics educators. A sample for one question is next presented.

Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find angle ABC.

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D.

Figure 2. Sample question (June 2011, p. 1, Question 8).

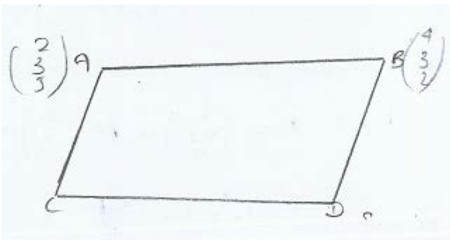
Extract of Examiner's Reports

This caused most candidates difficulty and there were only a small number of correct solutions. Most candidates did not realise that vector *OD* could be calculated directly from *OA* + *AD* and that *AD* = *BC* since *ABCD* is a parallelogram. Confusion often resulted from the fact that candidates were not prepared to sketch a diagram. Following the observations made from an examiner's report, a similar question was set to verify whether students are still facing difficulties with such problem. Extracts of trainees' work are shown below.

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}.$$

The point C is such that ABCD is a parallelogram
Find the position vector C relative to the origin.

$\overrightarrow{OC} = \begin{pmatrix} 10 \\ 1 \\ 5 \end{pmatrix}$ Position vector of C $= \dfrac{10\hat{i} + \hat{j} + 5\hat{k}}{\sqrt{10^2 + 1^2 + 5^2}} = \dfrac{10\hat{i} + \hat{j} + 5\hat{k}}{\sqrt{126}}$



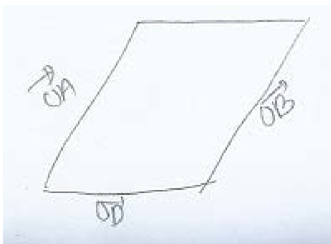


Figure 3. Extracts of trainees' work.

The above question (Number 3) tested trainees' knowledge of simple vector in 3D. These concepts have already been learned in Additional Mathematics at O level but still many respondents could not find the position of C given the vectors of A, B and D. Some were looking for the unit vector of C while some did not even label their diagram properly as can be seen in the above extracts. The findings from the survey confirmed those observed past examination reports.

It was disappointing to see that trainees did not know the basic concepts learned in vectors/coordinates, namely how to write coordinates in 3D. Base vectors or column vectors were used instead. Trainees demonstrate much difficulties on concepts such as scalar product, unit vectors, parallel and perpendicular vectors. Several wrong answers are due to incorrect calculations.

The trainees surveyed also faced difficulties in writing the general vector equations for lines and planes. Worst, many could not explain the notations used in the vector equations.

Trainees had difficulty to find the angle between two vectors. Some could not even recall the formula. Many of them did not realise that $\cos 90^{\circ}$ is equal to 0. In addition, they do not even know the meaning of scalar product of vectors.

Many trainees do not know how to verify whether two lines intersect or not. Many do not even know how to find the point of intersection in case it exists. Many could not even find the vector equation of a line. Incomplete answers were frequently observed.

Developing Interactive Tools to Enhance Learning of Vectors in A-Level

Following the identification of learning difficulties on vectors, lesson notes were written and interactive ICT tools were developed. Geogebra software was used for the development of the interactive tools, which are expected to enhance the teaching and learning of Vectors. The process of developing the webpage was time consuming. The tool will be available soon on the MIE website. Two snapshots of the webpage are shown below.

Figure 4. An example of the interactive tool.

6.5.1 Geogebra link: Point of intersection of line with plane

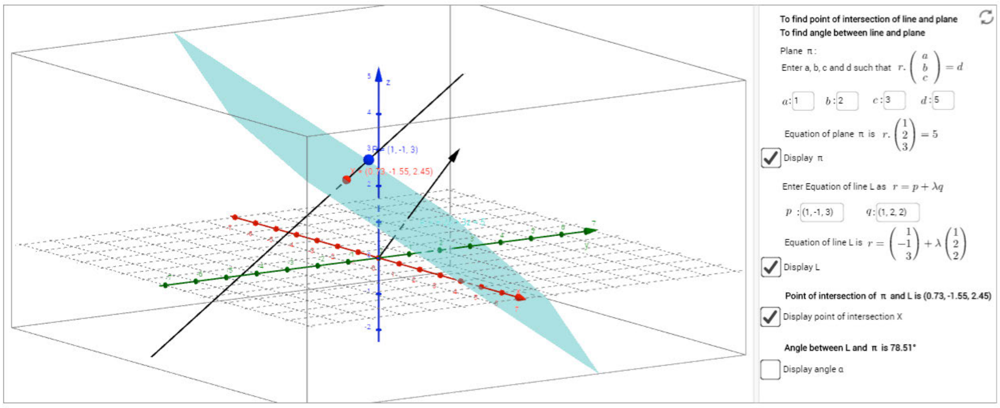


Figure 5. Second example of interactive tool.

Implementing and Evaluating the Teaching and Learning Tool Developed

Once the interactive tool was ready, it was presented to MIE trainees (enrolled in TDS and Post-Graduate Certificate in Education [PGCE]). Various requests were made by teachers for the availability of the tools, which they found amazing. Some issues were also highlighted and propositions made for the improvement of the tools. During the final stage of the study, the tools were evaluated using the framework "evaluating ICT-based materials" proposed by Pillay and Clarke (2008). The framework consists of two phases. The first one deals with the pedagogical principles that guides the educational soundness of ICT-based tools (that is, learner focus, integrity, usability and accessibility of the tools). The second one, the evaluation criteria, relates to the quality of the educational soundness of the tools (that is, Design, Use and Impact). These criteria for ICT-based tools are expected to successfully promote learning. A second questionnaire was developed (to evaluate the tools) and administered to PGCE FT, PGCE PT, and TDS.

Indices were created for each of the following: Learner Focus, Integrity, Usability, Accessibility, Design Evaluation, User Evaluation and Impact Evaluation. For example, the index (cumulated rate) for Learner Focus varies from 5 to 25 as it includes five items that are rated from 1 to 5. Similarly, the indices for the other variables are as follows: Integrity, 4 to 20; Usability, 5 to 25; Accessibility, 2 to 10; Design Evaluation, 3 to 15; User Evaluation, 3 to 15; and Impact Evaluation, 1 to 5. The frequency of the indices is displayed below. Based on the rate provided by the respondents, the tools under study were found to meet the following criteria:

Learner focus 84.4% of the respondents provided a cumulated rate of at least 20 out of 25.

Integrity 93.7% of the participants gave a cumulated rate of at least 15 out of 20.

Usability 78.1% of the respondents assigned a cumulated rate of at least 20 out of 25.

Accessibility 96.8% of the participants provided a cumulated rate of at least 7 out of 10

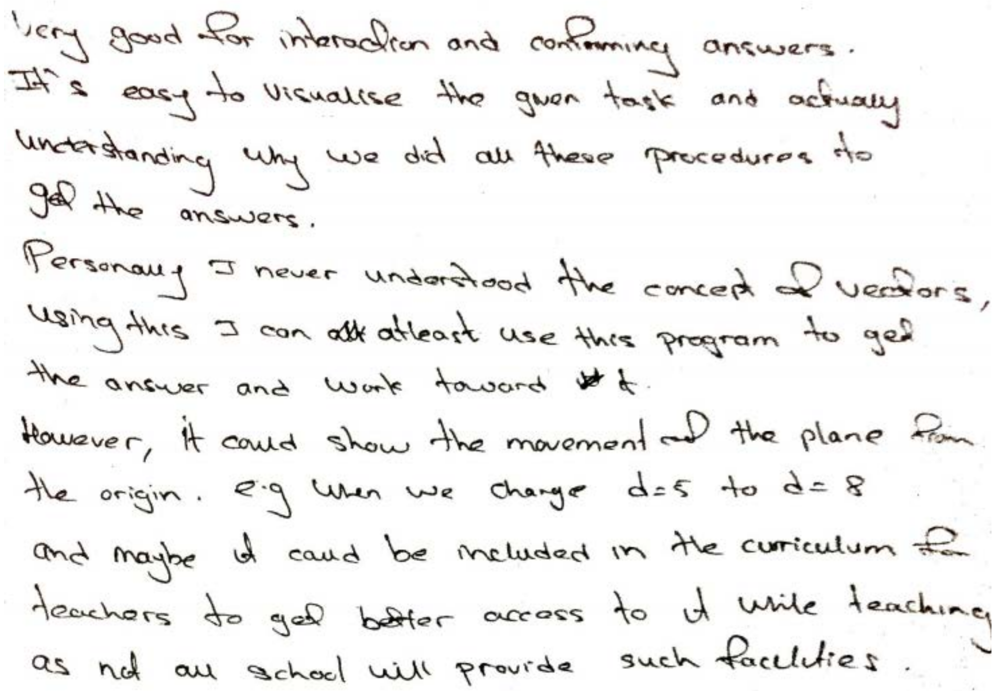
The second phase of the framework focuses on the design, use and impact of the ICT-based tools. The design evaluation criteria relate to the quality of the educational soundness of the tools, the use of evaluation criteria relates to the factors influencing the utilization of the tools, and the impact of evaluation relates to the learning outcomes of the students. Based on the cumulated rate provided by the trainees and educators, the following Evaluation criteria were met:

Design 83.9% of the respondents provided a cumulated rate of at least 10 out of 15.
User 92.3% of the participants gave a cumulated rate of at least 10 out of 15.
Impact 80.6% of the participants provided a cumulated rate of at least 4 out of 5.

The findings are encouraging with positive feedback. It was found that the tools developed met the following (pedagogical) criteria: learner focus, integrity, usability and accessibility of the tools. The tool developed also met the three evaluation criteria: design, use and impact.

The participants who have been exposed to the interactive tools by now are very satisfied with visual aspect and the interactivity of the tools. The latter will be uploaded on the MIE website to provide access to all teachers and students in the country. The interactive ICT tool will be presented to colleagues during an in-house session and at least one workshop will be conducted to expose the tools to educators from various secondary schools. More sessions may be organized to increase the awareness of the existence of the interactive webpage. This interactive webpage is a first of its kind and will be the baseline for many others.

Feedback. Written feedback were also gathered from a TDS FT cohort. The feedback was useful for the refinement of the tool. Most feedback was very encouraging. A sample of two extracts is presented next.



Very good for interaction and confirming answers.
It's easy to visualise the given task and actually understanding why we did all these procedures to get the answers.
Personally I never understood the concept of vectors, using this I can atleast use this program to get the answer and work toward it.
However, it could show the movement of the plane from the origin. e.g when we change d=5 to d=8 and maybe it could be included in the curriculum for teachers to get better access to it while teaching as not all school will provide such facilties.

Figure 6. Sample extracts (T21).

Conclusion

This project was initiated following observations made about poor performance in mathematics at A-level and instrumental understanding of the topic of vectors by MIE trainees. The aim of the study is to embed ICT in the teaching and learning process of the A-level vectors that fulfil good pedagogical principles, hence making teaching and learning more engaging, motivating and interactive. Moreover, ICT-based teaching promotes conceptual understanding and retention ability.

The study was conducted in three phases and used mixed method approach. In the first phase, the aim was to diagnose students' learning difficulties and misconceptions in A-level vectors. As such, an analysis of Cambridge reports of A-level Mathematics (Syllabus 9709) was undertaken to find out the main difficulties faced by students in A-level Vectors. Various difficulties were noted but only a sample presented in this paper. Based on the difficulties identified, interactive tools were developed and integrated in a webpage to enhance the teaching and learning of vectors at A-level. Finally, the tools developed were evaluated using the framework proposed by Pillay and Clarke (2008). It was found that the tools developed met the following (pedagogical) criteria: learner focus, integrity, usability, and accessibility of the tools. The tool developed also met the three evaluation criteria: design, use and impact; and received positive written feedback from respondents.

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The Modelling Process and Pre-Service Teacher Confidence

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Many teachers and pre-service teachers of mathematics lack experience with teaching methods, such as mathematical modelling, that require a conceptual learning and problem solving approach. To address this problem, this paper presents a study of a method – the Enhancement, Learning, Reflection (ELR) process – that has been designed to improve pre-service students' confidence in teaching mathematics, with a particular focus on the use of modelling as a teaching method. Results from the case study show that the PST participants involved in the ELR process did indeed experience an increase in confidence in their ability to present the modelling concept to a classroom of high school students.

Teachers' confidence in their own mathematical abilities – or lack of it – can have a powerful effect on their students (Laursen, Hassi, & Hough, 2016). For pre-service teachers (PSTs), confidence is strongly shaped by their own learning experiences in the mathematics field. This means for many PSTs, because past teaching methods may have emphasised procedural skills over conceptual learning and problem solving, teaching situations that require a more laissez-faire approach often lead to increased anxiety and therefore a lack of confidence (Laursen et al., 2016; Shilling, 2010).

Indeed, despite the increasing number of suggested methods for addressing the adequate training of PSTs, a lack of confidence amongst teachers of mathematics and the adverse effects this has on student learning and engagement remains a problem in Australia (Hamlett, 2009; Victorian Auditor-General, 2012; Yeigh et al., 2016). To foster the development of greater confidence in the area of mathematics education, PSTs arguably need to be offered the chance to engage in deeper, more reflective learning opportunities that also encourage self-reflection, novel ways of thinking and the utilisation of new or unfamiliar teaching methods. One approach that offers these learning opportunities is inquiry-based-learning (McGregor, 2016; Yoshinobu & Jones, 2013). In contrast to more traditional lecture-as-instruction methods, an inquiry-based-learning classroom passes mathematical authority to the student (Trigwell, 2012). This means the role of the inquiry-based-learning educator thus changes from a prescriptive "information giver" to a facilitator who poses questions and guides students' construction of ideas.

One field of mathematics that reflects those tenets of inquiry-based-learning, but which is often categorised under different name, is modelling. Mathematical modelling can be defined as a "process of representing real-world problems in mathematical terms in an attempt to understand and find solutions to the problems" (Ang, 2010, p. 53). Consistent with the inquiry-based-learning approach, as a learning process modelling intends to present students with novel data and requires the learner to explore relationships within that data (McGregor, 2016). Modelling also helps learners (both students and PSTs) to both express and adapt their current ways of thinking in order to read, interpret, and then develop useful tools/models for solving specific problems. It can also help to awaken

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 93–100). Melbourne: MERGA.

critical and creative senses which can thus provide the leaner with better comprehension of mathematical concepts (Atlay, Ozdemir, & Akar, 2014; Biembengut & Hein, 2013).

From a teaching perspective, various studies have shown that many teachers have difficulties in understanding modelling and therefore avoid using modelling problems in their classrooms (Atlay et al., 2014; Pereira de Oliveira & Barbosa, 2013; Thomas & Hart, 2013). This is largely due to lack of experience and knowledge related to the pedagogical issues such as how to manage the process (Biembengut & Hein, 2013; Borromeo Ferri & Blum, 2013; Ng, 2013), as well as the reality that when teaching modelling, the teaching process becomes more open and less predictable (Blum & Borromeo Ferri, 2009; Thomas & Hart, 2013) therefore requiring teachers to have confidence in their mathematical abilities (Atlay et al., 2014). Blum (2015) suggests that one way of providing future teachers with the necessary professional knowledge is to offer specific modelling courses which also include compulsory own teaching experiences as a required component of their education degree.

In current teacher education courses, the specific development of modelling skills are often either missing or only briefly touched upon in the mathematics education components (Biembengut & Hein, 2013; Pereira de Oliveira & Barbosa, 2013; Villarreal, Esteley, & Smith, 2015), yet modelling can serve many functions consistent with an inquiry-based-learning approach. This paper presents a case study where modelling is used as a method to develop PSTs' confidence in teaching mathematics, and proposes that such skills be a core component of the preparation of mathematics teachers.

Method

To improve the training of teachers at the university level by addressing the lack of confidence in science and mathematics instruction among teachers in secondary Australian schools, six universities across Australia have been participating in an Office of Learning and Teaching project: *It's Part of my Life: Engaging University and Community to Enhance Science and Mathematics Education*. An outcome of the project is a new university teaching method, the Enhancement-Lesson-Reflection process (ELR), which has been designed specifically to address PST confidence. By utilising conceptual learning and problem solving methods, the ELR process teaches PSTs how to take a student-focused learning approach. The process involves engaging PSTs in multiple, repeated sessions that focus on learning and planning (enhancement), teaching (lesson), and feedback and reflection (reflection). This paper reports only on the research completed at the University of Southern Queensland (USQ) where modelling was used as the teaching method.

Participants and the Enhancement-Lesson-Reflection (*ELR) Process*

Nine PSTs participated in 2015 USQ *It's Part of my Life* program. The PST participants were 2nd-, 3rd-, and 4th-year students studying to become middle- or high-school mathematics teachers at either the Toowoomba (five PSTs) or the Springfield (four PSTs) campuses. Four of the PSTs were males and five were females.

Engagement in the USQ *It's Part of my Life* program required the participants to attend a number of ELR sessions across the teaching semester. All participants were required to attend the introductory enhancement session at the beginning of the program and the group feedback session at the conclusion of the program. In the introductory session, the PSTs were taught about the concepts and theories behind mathematical modelling and were then presented with a mathematical modelling problem to which they had to develop a solution.

In this regard, they were experiencing the mathematical modelling process from the perspective of a student. This introductory session was presented by a visiting educational expert whose area of expertise was the teaching of mathematical modelling. The group feedback session held at the end of the semester required the students, as a group, to reflect on the ELR process and its impact on their teaching and learning experiences. Throughout the program, each participant was also required to: attend an enhancement session with an expert mentor(s) to plan their lesson, teach one session, and engage in a reflection session follow up with their expert mentor(s) the week following their teaching experience. They were also asked to observe and engage in the feedback/reflection sessions for at least three of the other lessons taught by their peers. There were six sessions in Toowoomba and five sessions in Springfield.

The students to whom the Teaching PSTs presented their lessons were Year 9 and 10 students from local high schools attending on-campus sessions. A total of 25-40 students participated in each session. The teaching sessions lasted two hours and were divided into three segments. The first 15 minutes were used as an introduction and time for the students to meet the teaching team over refreshments. Over the next 90 minutes, the students were presented with a real-world problem which they then, in small groups, worked to: devise a group-generated formulation to the presented problem; discuss assumptions and variables; develop a mathematical solution; model possible solutions; and interpret the real world meaning with further model refinement (Stillman, Galbraith, Brown, & Edwards, 2007). The final 15 minutes included a conclusion and collection of survey data. The role of the Teaching PSTs was to lead the main 90-minute segment of the lesson. With the assistance of an expert mentor (a university mathematics lecturer, a practicing mathematician, or a combination of the two), the Teaching PST presented the students with a real-world problem and then guided them through the modelling process.

Data Sources and Analysis

Following a mixed-method approach, a combination of a qualitative and quantitative data collection method was utilised. The quantitative method was used to investigate self-reported changes in PST confidence using a pre- and post-experience survey, and the qualitative interview method was used to further probe by interview how the participants perceived their confidence had changed following their classroom teaching experience.

Confidence and Competence Checklist (CCC) Survey

The CCC survey was specifically developed for this study to measure PST confidence. The survey was first designed to be based on the Australian Institute for Teaching and School Leadership’s (AITSL) seven professional standards for the domains of teaching. Piloting of the survey led to its refinement to become more focused on personal aspects of confidence in the teaching situation. The refined CCC survey includes eighteen questions and asks respondents to indicate, on a five-point Likert scale, how confident they felt about teaching with regards to five aspects of the teaching experience: lesson planning, classroom presentation, ability to centre the lesson on student needs, lesson management, and ability to self-evaluate.

To determine whether the ELR process was effective in changing the participating PSTs’ perceived confidence, the CCC survey was completed before and after the PSTs performed their allocated teaching task. Data from the CCC pre- and post-surveys were combined into five confidence factors to reflect the five aspects of the teaching experience on which the respondents were asked to reflect. Using a Wilcoxon matched-pairs signed-rank test, each of these factors was analysed to assess for changes in confidence using the research question: Did students become more confident after the ELR process in teaching mathematics? The Wilcoxon matched-pairs signed-rank test is a non-parametric, statistical hypothesis test used when comparing two matched samples or repeated measurements on a single sample to assess whether their population mean ranks differ (McDonald, 2014).

Post-Teaching Session Recorded Debrief

At the conclusion of each teaching session, the Teaching PST was asked to reflect on their experience in an audio-recorded debrief. During this debrief the PSTs were asked to reflect on: how the enhancement sessions contributed to their confidence in the lesson they taught, how feedback from prior sessions influenced their confidence, and how the lesson itself may have impacted on their confidence. These semi-structured interviews were completed directly after the Teaching PST's teaching session to enhance the ability to capture participants' immediate feelings about their performance. For each campus, the person conducting the interviews was the main academic responsible for the study on that campus. Using a grounded theory approach (Glaser & Strauss, 1967), the interview recordings were analysed using both manifest and latent content analysis techniques. This meant the data were analysed for both the appearance of a particular word or content (Potter & Levine-Donnerstein, 1999), as well as for the meanings implied through the communications (Holsti, 1969). The aim of the analysis was to further assess whether PST confidence changed or improved as a consequence of the ELR process.

Results and Discussion

Confidence and Competence Checklist (CCC) Survey

The five pre-defined factors designed to assess PSTs' feeling of confidence in their teaching ability were tested for reliability using Cronbach alpha to ensure all factors had a reliability value of approximately 0.7 or higher (see Table 1). The factor "ability to self-evaluate" was omitted because in the post CCC survey, five students stated that they were not able to rate this factor.

The Wilcoxon matched-pairs signed-rank test was used to test for changes in confidence. It assessed how confident the PSTs felt at the beginning of the ELR process compared to how confident they felt following their classroom teaching experience. The results of the Wilcoxon matched-pairs signed-rank test shows that for the factor "presentation skills", PSTs experienced a highly significant increase in confidence. For the factors "student learning" and "effective planning", there is moderate evidence to suggest an increase in confidence, while for the factor "lesson management", there is no evidence to support an increase in confidence.

To test for an increase in overall confidence, the mean and standard deviation for each of the four factors being examined were calculated (see Table 2). A higher mean ranking emerged for all four factors for the post CCC survey in comparison to the pre CCC survey, indicating a general increase in PSTs' confidence after they had undertaken the teaching task. A Wilcoxon matched-pairs signed-rank test further confirmed this overall increase in confidence ($Z = 2.194$, $p = 0.014$, one-tailed).

Table 1
Results of the Cronbach Alpha and Wilcoxon Matched-Pairs Signed-Rank Test for the Four Factors Examined

Survey Factor	Cronbach alpha	Test Statistic	*p*-value
Effective planning	0.694	Z = 1.897	0.029*
Presentation skills	0.825	Z = 2.224	0.0075**
Student learning	0.967	Z =1.703	0.0445*
Lesson management	0.953	Z = 1.160	0.123

* Significant at the 5% level. ** Highly significant

Table 2
CCC Pre- and Post-Survey Factors

Survey Factor	*N*	Minimum	Maximum	Mean	Std. Deviation
Effective planning (pre)	9	3.40	5.00	4.0000	0.47958
Effective planning (post)	9	4.00	5.00	4.3778	0.36667
Presentation skills (pre)	9	2.75	4.50	3.5833	0.55902
Presentation skills (post)	9	4.00	4.50	4.2222	0.23199
Student learning (pre)	9	1.67	5.00	3.4815	1.04231
Student learning (post)	9	3.00	5.00	4.1111	0.60093
Lesson management (pre)	9	2.00	5.00	3.5926	0.92463
Lesson management (post)	9	3.00	4.33	3.9259	0.40062

From the perspective of the ability for the ELR process to increase PST confidence, the results from the CCC survey provide evidence that for the 2015 cohort engaging in the *It's Part of my Life* project, the program was successful in its aims. It also provides evidence to support the argument for the use of problem solving as an approach that may offer mathematics PSTs deeper, more effective, and more relevant learning opportunities that increase their confidence to generate, explore, and analyse unfamiliar ideas.

Post-Teaching Session Debrief

When asked to reflect on how the enhancement sessions and/or how previous feedback had contributed to how confident they felt when teaching their classroom lesson, all but two of PSTs responded positively. The general consensus was that both the enhancement sessions and the individual feedback were important for helping the PSTs to identify those elements specific to their own teaching practices that they may have needed to focus on or improve due to confidence reasons. One participant, for example, identified the need to improve her confidence in speaking to a group, and from the enhancement sessions felt more ready to deliver her teaching session:

> I have learned a lot from it (the enhancement session). The hard part for me is the practice of talking and it has shown me how when I deliver my lesson I (need to) try to make my voice louder.

Another participant admitted that while the enhancement session made her more nervous, it also made her feel more confident as it made her think about things she had not previously thought about with regards to teaching her lesson. Another concurred, explaining that the enhancement session made her realise that she needed to make sure she was fully prepared for her classroom experience and this included all the small touches, such as making sure their spelling was correct in classroom handouts and PowerPoint slides. In this participant's words:

> The enhancement sessions contributed a lot to my confidence. If I had to do it (the teaching session) without that assistance I would have been lost.

Three of the PSTs mentioned that an important lesson that came from the enhancement sessions, and which subsequently contributed to increasing their feelings of preparedness for the classroom teaching experience, was the realisation that in teaching mathematical modelling flexibility was a key teaching quality. This is because mathematical models often have multiple ways of approaching the problem and success comes from being able to manage unknowns as they arise. As explained by the respondents:

> With building my confidence I could see [from the enhancement sessions] that unforeseeable problems are inevitable and that you can still basically have a successful session. I had been pretty nervous dealing with the unknowns but coming in today [for the teaching session] on the back of the other sessions, it wasn't that difficult.

> It [the enhancement sessions] helped me to understand that in maths modelling you don't need an exact answer.

> The enhancement sessions were really important for my confidence and broadening my ideas. I learnt it is [more] about extending the kids and that helped guide me.

With regards to how the actual classroom experience impacted on perceived confidence, all but one PST expressed feeling more positive as the lesson progressed. As explained by one respondent:

> When the students got really involved I settled down and my enthusiasm increased as I fed off their enthusiasm.

Four of the students mentioned feeling more confident in their own mathematical and modelling abilities as a consequence of undertaking the ELR process. Here, statements related to ability were important indicators for the perceived confidence of the respondents:

> The enhancement process improved my confidence in how to do maths modelling. And it improved my confidence in being able to do this again in the future.

> The feedback helped my confidence because if I know what I can improve on from positive and negative feedback, then I can work to improve [on those things].

> By constantly updating my [chosen classroom] problem by going through the modelling process, the modelling process really helped me with planning the lesson. In the end, I thought it worked really well and I was very happy with end result.

> The process taught me a lot about trying to get the kids more engaged. It helped me to focus on the maths side of things rather than focussing [too much] on the modelling process. It taught me to try to get the kids engaged without giving them too much. It taught me to make it [the problem] real life with lots of variables and to facilitate rather than telling them [the students] how to do it. It really taught me to change the way I teach.

The two of the participants who felt the enhancement and feedback sessions had impacted negatively on their confidence levels described feeling overwhelmed after the enhancement sessions. This feeling then led them to doubt their confidence in being able to teach the modelling subject matter. On both occasions, however, once the student had further discussions with a university mathematics educator about the experience, they then felt more confident again and more ready to undertake the task.

Conclusion

The ELR process has been developed through the collaborative efforts between six Australian universities to address the need to develop in PSTs an improved sense of confidence in the classroom. This paper reports the results of the USQ's 2015 iteration of the *It's Part of my Life* program, which utilised modelling as a teaching method.

This case study has provided evidence that the PST participants involved in the ELR process experienced an increase in confidence, particularly in their ability to present the modelling concept to a classroom of high school students. The results thus demonstrate the potential positive application of the ELR process as a teaching method that may be adopted by universities to address issues related to PST confidence in the classroom and personal efficacy in the realm of mathematics education.

While the results from this single case study provide only a snapshot of the potential application of the ELR process as a teacher preparation method, a broader understanding of the ELR process will be gained once these results are combined with the results that are emerging from the other universities participating in the process. Combined results from multiple iterations of the program at USQ will also produce findings that are more generalisable; however, this paper does not purport to present a fully developed university teaching method. Instead, the aim was to show how the trialled application of the ELR process at USQ has already generated some positive results.

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Re-Examining a Framework for Teacher Identity as an Embedder-of-Numeracy

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Research interest in numeracy is growing as a result of increased understanding of the impact of low levels of numeracy. However, there has been little research on factors that influence how teachers implement learning from professional development interventions to support teachers to promote numeracy learning. This paper reports on how a theoretically developed framework for identity as an embedder-of-numeracy was re-examined through empirical research. Additional factors were added to the framework and each factor included in the framework was explicitly defined. The framework seems to capture the complexity of a teacher's identity in this context and is amenable to empirical research.

There is increasing interest in research on numeracy (or mathematical literacy), both in Australia and internationally (Geiger, Goos, & Forgasz, 2015). One of the reasons for this interest is the growing understanding of the impact of low levels of numeracy on productivity, as a result of globalisation and technological changes, and on the economic and social well-being of individuals (Organisation for Economic Co-operation and Development, 2013). While numeracy capabilities continue to develop beyond school, there is an important role for schools in equipping students with the capacity to cope with the mathematical demands of life in the 21st century.

An across the curriculum approach has been taken in Australian schools, with numeracy identified in the Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority, n.d.) as a general capability to be developed in all school subjects. This approach utilises subjects other than mathematics to provide meaningful contexts for students' numeracy development (Steen, 2001). However, it is necessary for teachers to attend explicitly to numeracy learning demands and opportunities in the subjects they teach for this approach to be successful. Previous research has investigated professional development interventions to support teachers to promote numeracy learning (e.g., Goos, Geiger, & Dole, 2014). However, these studies have tended to focus on the effectiveness of the interventions without considering how teachers' knowledge and affective attributes, social interactions, and environmental factors shape the way in which they respond to ideas promoted through these interventions. This issue was addressed in a study that used teacher identity as the analytic lens to identify ways to support teachers to promote numeracy learning across the curriculum.

One of the outcomes of the study was a framework for *identity as an embedder-of-numeracy* that was developed theoretically and then re-examined and revised through empirical research. The development of the initial framework has been reported on previously (Bennison, 2015a). This paper extends this research by presenting the revised framework in order to more fully address the following research question: What factors that contribute to a teacher's identity influence his/her capacity to promote numeracy learning across the curriculum?

Background to the Study

The study aimed to identify how to support teachers to promote numeracy learning in

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subjects across the curriculum. It was conducted in two interrelated phases: a theoretical phase and an empirical phase. The theoretical phase informed the design of the empirical phase, which in turn contributed to re-examining the outcomes of the theoretical phase.

The Theoretical Phase of the Study

The theoretical phase employed an extensive review of literature to propose a sociocultural approach to addressing the study's aims. This approach included using teacher identity as an analytic lens and identifying factors that might contribute to shaping the identity of a teacher in the context of promoting numeracy learning through the subjects they teach. One of the outcomes of this phase of the study was to propose a framework for identity as an embedder-of-numeracy (Bennison, 2015a).

The framework for identity as an embedder-of-numeracy is underpinned by the understanding that:

1. being numerate involves having the *dispositions* that support the *critical use* of *mathematical knowledge* and appropriate *tools* (representational, digital, and physical) in a range of *contexts*: five dimensions of numeracy seen in the numeracy model developed by Goos at al. (2014); and
2. an effective way for teachers to promote numeracy learning is to enhance discipline learning by embedding numeracy into subjects across the curriculum (For an example of how attention to numeracy can enhance learning in history, see Bennison, 2016).

Five Domains of Influence were used to organise the framework: Life History, Knowledge, Affective, Social, and Context. Factors that have previously been shown to influence a teacher's identity were included where it could be argued that these factors were likely to influence how teachers promote numeracy learning through the subjects they teach. For example, the factor, attitudes towards mathematics, was included in the Affective Domain because of the phenomenon of maths anxiety experienced by many pre-service teachers (e.g., Hembree, 1990). The resulting framework is summarised in Table 1.

Table 1
Conceptual Framework for Identity as an Embedder-of-Numeracy (Bennison, 2015a, p.15)

Domains of influence	Characteristics
Knowledge	Mathematics content knowledge (MCK)
	Pedagogical content knowledge (PCK)
	Curriculum knowledge (CK)
Affective	Personal conception of numeracy
	Attitudes towards mathematics
	Perceived preparation to embed numeracy
Social	School communities
	Professional communities
Life History	Past experiences of mathematics
	Pre-service program
	Initial teaching experiences
Context	School policies
	Resources

One of the limitations of the framework for identity as an embedder-of-numeracy is that it is static and cannot provide insights into identity formation and possible trajectories of this identity. For this reason, Valsiner's (1997) zone theory was employed in the study to understand how the factors that influence this identity interact to produce particular identities and how these might change over time. This aspect of the study has been reported elsewhere (Bennison, 2015b).

The Empirical Phase of the Study

The empirical phase of the study was conducted over a two year period (2014-2015) and employed case study methodology (Stake, 2003). Participants were eight teachers from two Australian secondary schools who were recruited because they were participating in a larger project (Numeracy Project). The experience of the teachers ranged from early career to very experienced and the subjects taught were English, history, science, and mathematics. The teachers' participation in the Numeracy Project meant that they had access to a range of activities to support them to promote numeracy learning in the subjects they taught. Data collected during school visits included interviews and lesson observations. Interview transcripts were analysed using content analysis that employed Valsiner's (1997) zone theory as the theoretical framework. Teacher's personal conception of numeracy and the tasks that were used in observed lessons were analysed in terms of the dimensions of Goos et al.'s (2014) numeracy model (For further details of the research design and methods, see Bennison, 2015b).

Revising the Framework for Identity as an Embedder-of-Numeracy

Several modifications were made to the initial framework for identity as an embedder-of-numeracy (Bennison, 2015a) in light of the findings from the empirical phase of the study: Additional factors were added to the Knowledge and Affective Domains, some of the factors were re-named to better reflect what was meant, and the Life History Domain was placed first because factors that contribute to other domains are shaped by factors from this domain. Furthermore, each factor that was included in the framework was explicitly defined. The revised framework is presented in Figure 1. It is not possible within the space limitations of this paper to fully describe how the inclusion of each factor was supported by the literature and empirical phase of the study. Consequently, attention will be given to the changes made to the framework and the definitions of each factor.

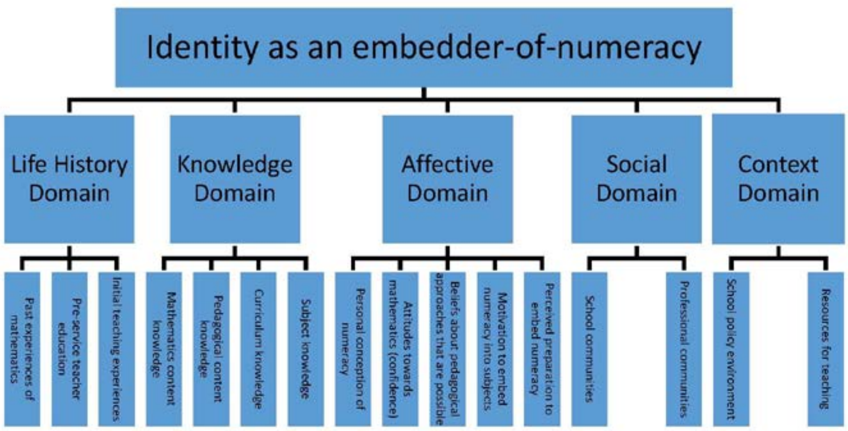

Figure 1. A framework for identity as an embedder-of-numeracy.

Life History Domain

Past experiences contribute to identity (e.g., Phillip, 2007). Consequently, many factors that influence how teachers promote numeracy learning are likely to have been shaped by their past experiences. The Life History Domain in the initial framework included three factors: *past experiences of mathematics*, *pre-service teacher education* (named as pre-service program), and *initial teaching experiences*. No changes were made to this domain as a result of the empirical phase of the study and the included factors were defined in the following manner:

- Past experiences of mathematics: nature (positive/negative) with mathematics and opportunities (both formal and informal) to develop competency with the inherent mathematics in the subjects taught.
- Pre-service teacher education: opportunities during pre-service teacher education to learn about how numeracy can support subject learning and develop pedagogical content knowledge for numeracy.
- Initial teaching experiences: opportunities to engage with an across the curriculum approach to numeracy early in career.

One of the findings of the empirical phase of the study was the limited opportunities for participating teachers, even the most recent graduates, to develop the knowledge for addressing numeracy across the curriculum during their pre-service teacher education. Although not widespread, there have been courses in some pre-service teacher education programs that address numeracy for some time (e.g., Groves, 2001). However, this area of pre-service teacher education will be addressed in the near future in light of the recommendations of the Teacher Education Ministerial Advisory Group (TEMAG, 2014).

Knowledge Domain

A teacher’s knowledge is an important part of his or her identity (e.g., Van Zoest & Bohl, 2005). Several types of knowledge are needed for teaching (Shulman, 1987), but only three types were initially included in the Knowledge Domain: *mathematical content knowledge* (MCK), *pedagogical content knowledge* (PCK) and *curriculum knowledge* (CK). Following the empirical phase of the study, *subject knowledge* was added and these types of knowledge were defined as follows:

- MCK: Level of expertise in the mathematics inherent in the subjects taught.
- PCK: Capacity to design effective numeracy tasks.
- CK: Capacity to identify numeracy learning opportunities and make connections between numeracy and subject learning.

Subject knowledge was seen to encompass content, pedagogical and curriculum knowledge of the subject taught. This was added as a single factor to the Knowledge Domain because the interest in the study was on promoting numeracy learning, not on teaching the subject per se. Two of the teachers in the study were teaching science out of field. While one had completed some tertiary science courses, the other had no formal post-secondary science education. This second teacher, a qualified mathematics teacher, conceded that she needed to learn the science content she was teaching and was not seeing the relationship between numeracy and learning in science:

> I'm not science trained, so, I mean I'm comfortable with the maths more than the science, so maybe I'm not seeing the links as much as somebody trained in science would because I don't know the content beyond that curriculum that I'm studying to deliver.

Conversely, another teacher, who was a qualified history teacher, recognised the importance of providing students with opportunities to develop numeracy-related historical skills such as using timelines, maps and graphs, even if this was at the expense of covering historical content:

> I've really pushed, and a lot of teachers have, to reduce the amount of content we teach and focus on skills because at the end of the day a student can Google when Balboa found the Pacific Ocean but if they can't read a timeline or read a map or construct a graph then, you know, they've lost significant skills.

Not being able to see the links between numeracy and learning in a subject may be the result of a lack of subject knowledge (content, pedagogical, and curriculum). Thus it could be argued that this type of knowledge influences how teachers promote numeracy learning, and so contributes to their identity as an embedder-of-numeracy. However, it should be noted that there is some overlap in the four types of knowledge included in the framework.

Affective Domain

The Affective Domain proved to be the most challenging to conceptualise, possibly because affective attributes cover a broad spectrum (e.g., Phillip, 2007) and the identified factors could also be considered as part of other domains. Three factors were initially included in the Affective Domain: *personal conception of numeracy*, *attitudes towards mathematics*, and *perceived preparation to embed numeracy into subjects*. Following the empirical phase of the study two additional factors were added: *motivation to embed numeracy* and *beliefs about pedagogical approaches that are possible*. The first two of these factors were defined in the following manner:

- Personal conception of numeracy: Belief about what numeracy encompasses.
- Attitudes towards mathematics: Level of confidence with the inherent mathematics in a subject.

The limited opportunities for the teachers to participate in courses during their pre-service teacher education or professional development post-graduation that explicitly addressed promoting numeracy learning across the curriculum meant that the factor, perceived preparation to embed numeracy, was not explored during the empirical phase of the study. For this reason, no definition is provided but it could be seen as related to confidence in dealing with numeracy in the subjects taught.

Teachers who could be described as embedders (Thornton & Hogan, 2004) see numeracy as enriching understanding in the subjects they teach, and so have incentive to promote numeracy learning. In contrast, teachers who find it difficult to make connection between numeracy and subject learning may see numeracy as something extra to be added (e.g., Carter, Klenowski, & Chalmers, 2015). For these latter teachers, it could be argued that making changes to their practices is only likely if they come to see a benefit to student learning in the subjects they teach, thereby making the changes worthwhile (Gresalfi & Cobb, 2011). For example, an early career history teacher placed much greater emphasis on the numeracy learning opportunity provided by timelines in the second year of the study than she had done in the previous year and was able to make explicit links between numeracy and learning in history:

> I think numeracy is used more to help students understand concepts. So next lesson we have lots of data, population statistics, we look at [pause] pie charts and stuff about how many of the Indigenous population were left after the Spanish arrived and that kind of stuff and it helps them to understand how devastating the arrival of the Spanish was. And so I think the numeracy has been used to push, to help students understand concepts.

While there are many possible explanations for this teacher's change in practice (e.g., her increasing experience as a history teacher), it is not unreasonable to suggest that motivation for this change stemmed from recognising that explicit attention to numeracy enhanced learning in history. Thus, motivation to embed numeracy into a subject could be seen as contributing to a teacher's identity as an embedder-of-numeracy.

The interactions teachers have with students can influence whether or not they are prepared to expend the emotional energy to employ pedagogical practices that promote numeracy learning. These interactions lead to beliefs about pedagogical approaches that are possible with particular groups of students and are related to teachers' self-efficacy and agency (Bandura, 1977). Two of the teachers in the study mentioned how their perceptions of students influenced the pedagogical approaches they felt able to utilise in lessons. For example, one teacher reported sensing that the students in her science class needed structured activities and as a result did not allow students to work in groups on a more open task:

> [My students] struggle with any activity that is out of the ordinary, or out of their routine or involves them in having less guidance. They tend either go, "Oh, that's too hard" and switch off … anything outside of their routine just kind of scares them and rather than failing they'd rather not try.

Consequently, beliefs about pedagogical approaches that are possible were seen to contribute to a teachers' identity as an embedder-of numeracy.

Social Domain

Identity development involves participation in communities (Wenger, 1988), so it could be argued that teachers' participation in school and professional communities contribute to how they promote numeracy learning. For the purposes of the study, *school communities* were restricted to the interactions that teachers have with students, colleagues and administrators. Following the empirical phase of the study, interactions teachers have with students were seen as likely to influence teacher beliefs about the pedagogical approaches they could employ and were included in the Affective Domain. *Professional communities* can offer opportunities for teachers to engage in learning, and therefore contribute to how they promote numeracy learning. The two factors included in the Social Domain were defined as follows after the empirical phase of the study:

- School communities: Interactions with colleagues and school administrators related to the meaning of numeracy and who is responsible for numeracy learning.
- Professional communities: Interactions with others in professional associations and professional development activities (including research and development projects) related to promoting numeracy learning.

Context Domain

Practice and identity are related (Wenger, 1998), so affordances and constraints on practice within teachers' professional contexts can influence the ways in which they promote numeracy learning. The *school policy environment* and *access to appropriate resources* for teaching seemed pertinent (named as school policies and resources in the initial framework, see Table 1), and so these were included in the Context Domain. Following the empirical phase of the study, these factors were defined as:

- School policy environment: Curriculum initiatives and accountability measures related to numeracy.
- Resources for teaching: Access to representational, physical, and digital tools needed to support numeracy learning.

Concluding Remarks

The situated nature of identity (Wenger, 1988) makes it possible to theoretically develop a framework that encompasses factors likely to contribute to a teacher's identity in a given situation. This approach was used to develop a framework for identity as an embedder-of-numeracy (Bennison, 2015a) in a study that sought to identify ways to support teachers to promote numeracy learning across the curriculum. An empirical phase in the study allowed the framework to be evaluated and led to several revisions that are reported in this paper. Furthermore, evidence from this phase of the study indicates that the framework allows the complexity of a teacher's identity in this context to be captured, and yet overcomes some of the difficulties of using a more complex framework such as Van Zoest and Bohl's (2005) framework for mathematics teacher identity in empirical research. The empirical phase of the study was limited to eight teachers in two schools who taught a small number of subjects. Further evaluation of the framework could be undertaken by extending the research to more teachers, schools, and subjects.

The focus of the study reported in this paper was identifying ways to support *practising* teachers to promote numeracy learning through the subjects they teach. The teachers who participated in the empirical phase of the study reported that their pre-service teacher education programs had provided limited opportunities to develop the capacity to promote numeracy learning. In light of this finding and the imminent changes to pre-service teacher education programs as a result of the Teacher Education Ministerial Advisory Group (2014) recommendations, there is a need to investigate how best to prepare pre-service teachers so that they are able to attend to numeracy demands and opportunities in ways that enhance subject learning.

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Privileging a Contextual Approach to Teaching Mathematics: A Secondary Teacher's Perspective

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This paper focuses on using sociocultural theory to support a context-based approach to teaching mathematics. A goal of the research was to explore the opportunities-to-learn of using a context-based approach to enrich student engagement with mathematics. This paper focuses on the practice of one female secondary school teacher as expressed through an interview transcript. Data were analysed using a participation framework. Findings suggest that aspects of a context-based approach to teaching mathematics can be used by teachers to promote student engagement with mathematics in the secondary classroom.

The contexts in which mathematics tasks are framed play an important role in the development of student mathematical competencies (Organisation for Economic Co-Operation and Development [OECD], 2009). The Programme for International Student Assessment (PISA) recognises this importance and, as part of its assessment of 15 year olds mathematical competency, utilises real world type problems which require quantitative reasoning, spatial reasoning or problem solving (OECD, 2003). A context suitable for such a mathematics task according to PISA strives to be relevant to students, to shift between the mathematical and the everyday, and to be relevant to the production of a task solution (OECD, 2009). Through the use of such contexts, students may develop important mathematical competencies related to representation, reasoning, argumentation, and communication.

The emphasis on task context when teaching classroom mathematics has been used in varying degrees to locate the learning of mathematics within the social practices of the classroom (Cobb, Stephan, McClain, & Gravemeijer, 2001), to locate the knowing of mathematics within the resources of a mathematical community (Duguid, 2005), to situate the classroom as a community of practice (Hung & Chen, 2007), and to diagnose the difficulties students have when doing influential, large-scale mathematics assessment (Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014).

The social practices of the classroom may be said to encompass the resources and norms its members share when doing mathematics (Cobb et al., 2001). The resources of a mathematical community may be said to include the conventions and relationships that are pertinent to a community, for example economists, when knowing mathematics (Duguid, 2005; Hung & Chen, 2007). As such, knowing and doing mathematics needs to provide opportunities for students to apply mathematics to everyday life and to provide possibilities for mathematisation, that is, opportunities for students to organise information mathematically and to use their pre-existing knowledge and experiences to use that information to complete a mathematics task (Wijaya et al., 2014).

In other words, an understanding of context and its importance to knowing and doing mathematics needs to encompass the social contexts in which students do mathematics, the norms which privilege certain ways of thinking and knowing within those contexts, and the pedagogy that enables students to participate in those contexts. As such it is important that future research explores what opportunities-to-learn students are offered when engaging in context-based mathematics tasks. This paper provides insights into what these

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opportunities-to-learn may look like by exploring the thoughts of a female secondary teacher as she 'talks about' her class doing context-based tasks in her secondary mathematics classrooms.

Theoretical Framing

A context-based approach to teaching and learning is congruent with the writings of theorists such as Vygotsky who described the emergence of human development from social-cultural contexts. According to Vygotsky (1987), learning results from people participating in contexts where semiotically-mediated social interaction is facilitated through the use of psychological tools such as those associated with the signs and symbols (including language) of mathematics. Learning results from participation in shared practice and is mediated by the sociocultural, that is, by those tools constructed by others (e.g., algorithms, mnemonics, formulae, norms etc.) in which they share their ways of knowing and doing. It is through participating in historically and culturally situated contexts that a student's relationships to others, to activity, and to the world may be transformed over time to show congruence with the ways of knowing and doing of mature communities of practice - mathematicians, scientists, historians, etc. (Lave & Wenger, 1991). From this point of view, learning may be considered to be an independent process, but it proceeds through contexts that have social, cultural and historical dimensions. However, participating autonomously in classroom mathematical activity requires that teachers link students' individual mathematical thinking (representations) to the cultural, historical dimensions of mathematical practice through the provision of learning opportunities that are meaningful to students (Cobb et al., 2001).

One way of investigating what opportunities-to-learn students are offered when engaging in context-based tasks is to examine 'talk about' a classroom community (Lave & Wenger, 1991). One source of "talking about" a classroom community that fulfils this requirement is a teacher interview. This paper provides insights into the opportunities-to-learn provided to students in a secondary classroom that uses context-based tasks by exploring the utterances of a female secondary teacher as she 'talks about' her class doing mathematics.

Method

Classroom Context

The interview focuses on this teacher's experiences in Year 11 and 12 classes where students were required to model and communicate solutions to real world type tasks situated within a problem-centred curriculum (for an example of these tasks see Redmond & Sheehy, 2009). In order to operationalise this curriculum this teacher had been using a sociocultural tool, Collective Argumentation (see Brown, 2017), to engage her students in the learning of mathematics. In brief, Collective Argumentation is a sociocultural approach to teaching and learning mathematics. Presented with a task framed in a problem-solving context, students are required to individually represent a solution to the task, join with a group to compare, explain and justify that solution and reach consensus within the group to a solution that can be presented to the class for discussion and validation.

Research Participant

The teacher who is the focus of this article, Jill, is a teacher with significant experience in the secondary mathematics classroom. Having taught in a number of different national and international educational jurisdictions, Jill was based in a middle-class Independent College situated in a major Australian city.

Interview Context

The 30-minute interview was conducted by a Research Assistant and took place in a mathematics classroom after school hours. Jill responded to a set of questions that were general in nature to assist reflection on issues related to teaching mathematics. The interview was situated within a larger study conducted over a three-year time frame that involved university educators working with school teachers of mathematics from schools located in South-East Queensland to bring about and reflect upon change in the way they teach mathematics.

Analytic Framework

Jill's interview was transcribed and subjected to a form of analysis derived from methods associated with sociocultural theory. This form of analysis centres on the broad categories of *location* - how the classroom is organised, *relationships* - the roles and responsibilities visible in the classroom, *content* - type of knowledge privileged in the classroom, *pedagogy* - what the teacher and students do in the classroom, and *assessment* - what is valued in the classroom (Vadeboncoeur, 2006). This paper focuses on the opportunities-to-learn in a secondary classroom that uses context-based tasks.

Analysis and Discussion

Location: Forward Thinking Versus Back Lashes

For Jill, the mathematics classroom emerges as a "forward thinking" space where students need to be prepared to apply "what they know" to "novel situations", a space where teachers need to provide students with 'opportunities' to be 'active participants' in a 21st Century society that has yet to be envisaged.

> I think we need to develop kids in 10 to 15 years for jobs that don't even exist, so we are really getting kids to be forward thinking. They have to be able to transfer what they know to novel situations because who knows what they are going to be doing in 20 years' time when they are finished school, uni, and then have a career. We need to be providing kids the opportunities to be active participants of a society we don't know, or can't even envisage because it's not here yet.

However, Jill worries that the present society in which the classroom is located will diametrically oppose this view of teaching mathematics, prepared to judge students as being "no good" at mathematics if they can't quickly perform mental calculations, even though they are "using" their mathematical knowledge in ways that "make sense of the world" in which they live.

> I worry that we get back lashes about kids not being able to do percentages and kids not being able to do this, that and the other, and it frustrates me because there's two very different opposing views there I see. We (teachers) beat them (students) around the head 50 times because they can't find 33.3% of 245. But if you (teachers) give them five minutes they probably can, but they can't do it in their head.

> It frustrates me… because it's such low level, this stuff that they (society) are saying, they (society) are actually missing the idea that these kids are actually really thinking in quite a sophisticated manner. They (students) are using all of this stuff that they (society) are saying that they can't do but they (the students) are using it in a way that actually makes sense of the world, real life problems. But because they can't do it instantaneously, that they (the students) are no good.

As suggested in the above analysis opportunities-to-learn in Jill's classrooms are expressed in the form of a tension between her classrooms and the society in which they are located. References to feeling 'frustrated' about what Jill interprets as society's privileging 'low level' expectations of what it means to know mathematics over students using mathematics to make sense of the real world echo the tension that exists between classrooms that privilege authentic contexts of learning, that is, those that mirror the kinds of practices evident in professional communities (e.g., economists), over contexts that mirror the skills of the everyday workplace. According to Hung and Chen (2007), such tension can be productive if it is dialectical in nature rather than diametrical, that is, where the transfer of high level processes, such as analysing, synthesising and modeling, to novel real-world situations occurs simultaneously with the development of skills such as mental computation. Jill's understanding of the need for this complementarity is evident in the relationships she sees in her classroom.

Relationships: Talking Versus Telling

For Jill, building relationships with students is about giving students opportunities to learn through "talking to them" for the purpose of finding learning contexts that the students have "access to" and are better able to "contribute" to their learning of mathematics in terms of "confidence", others "interpretations" of their mathematical potentials, and through group interactions where students are more on "par" with each other.

> By talking to them, and it's just like, there are some boys that are in the class that like skateboarding which I just hate and so for me to try and find a context, that takes a lot of effort, but because they (the students) do have that access to the background information (skateboarding) they are certainly able to contribute more, which builds their confidence which builds, you know, other kids, I suppose interpretation of what they can build (mathematically model), or bring to the group which, you know, changes the dynamics of the group, you know all that sort of stuff.

For Jill, the success of the group relationships in her classrooms is dependent on the context in which the mathematics is situated.

> Sometimes I think the context, the start, the question that you (the teacher) use will, may, I don't know, keep the kid at that weak point, which you (the teacher) think they are, or it could actually give them an opportunity to join with the rest of the group and be on par, I don't know. They (the students) work really well as a group and they don't just work out who's, … like I am just trying to imagine now my Year 12s. They know that different people have strengths on different areas and so depending on the context, they will very quickly work out who they can conference with in order to check what they are doing in different sections. So, there's people who are really, really good at the language and the communication and justification of the solution. There are people that are really, really good at the mathematics. There are people that are good at like those questions that require high initiative and really complex thinking.

Through "talking" with students, Jill's understanding of what it means to be a teacher has "changed" and she expresses a "wonder" that some may think that she is not doing her "job".

> So, I think my understanding of teacher has certainly changed and I think that I wonder if a lot of people would think that I am not doing my job. Do you know what I mean, I know I am, but

> because I am not sitting up the front, because I am not telling the kids what to do. I am evoking an argument because of the context I chose, and the kids are arguing 'cause they don't agree with each other and they are using mathematical language they are using the mathematical equations.

Relationships in Jill's classrooms require an expansion of what it means to be a teacher from being an expert who can transmit knowledge to include being an evoker of mathematical argumentation where students, because of the contexts in which the mathematics is situated, are 'arguing' over ideas, using 'mathematical language' and its associated tools, for example, 'equations'. Such an expansion of the understanding of teacher is necessary for engaging students in authentic opportunities-for-learning that privilege conceptual mathematical conversations and reflection on those conversations (Cobb et al., 2001). As such, this expansion in understanding of what it means to be a teacher has become the cornerstone of the content Jill chooses to promote opportunities for her students to learn.

Content: Open-Ended Versus Single Solution

The learning experiences that Jill uses to teach mathematics in her classrooms privilege tasks that are "open-ended", that provide students with multiple entry points to access the mathematics, and that utilise contexts that provide all students with opportunities-to-learn even those who she once considered "weak" in the subject domain. When asked, "How have the learning experiences that you provide your students changed?", Jill responded,

> Oh, way, way different, they are much, more open-ended, they are not single solution oriented at all. There is a lot more entry points (in the tasks). Lot's more hooks for kids to take in. Cause right at the beginning (of a unit of work) you kind of know who your weak kids are. But once again I am now wondering whether that isn't something that I am conditioned (to do). Like I used to think that the kids would only get to this point. Am I pin-pointing kids who I think are weak who may not be, who may have just been (slow) because the context that I use, wasn't right for them? If I changed context would they be better? So, I wonder if, by changing context sometimes those weak kids you know will not actually be (weak), but have better access to the maths because of the context, so I try to mix it up a bit. I try and get a context that I find is interesting, certainly to me, if not to me then I try and find interests of the kids then I try and scope it (the task).

When "scoping" tasks for her students, Jill takes into consideration what students think "didn't work" and what "would work better" and tailors the content of her mathematics lessons to suit the "strengths" and "weaknesses", the "likes" and "dislikes" of her different classes of students.

> The kids are actually, that I teach, are really good, they are very open about what they like and what they don't like and they will try something new and they will be very honest about, nope that didn't work, this would work better if. So, I am more, um, I suppose open to the nuances of the class and changing what I do depending on their strengths and weaknesses and likes and dislikes. So, what I do in one class isn't necessarily what I do in another just because of the kids that you teach in the group. That would probably be where it has changed the most, whereas before I just had one (lesson), you know you set your lesson up into three bits and knock yourself out.

This approach to scoping the content of mathematics lessons is congruent with the PISA stages of mathematisation necessary for facilitating student comprehension, transformation, processing, and encoding of mathematical tasks (Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014). In other words, employing content in the mathematics classroom situated within contexts which are familiar to and of interest to students assists students to identify and select information and to choose mathematical knowledge and procedures relevant to completing a particular mathematical task. It is not surprising, therefore, that Jill's pedagogy as expressed in her interview privileges student participation and engagement with mathematics.

Pedagogy: Drawing Forward Versus Limiting

Teaching emerges from Jill's interview as having to do with "drawing forward" students by privileging learning outcomes that are "better" for students, "different" to what the students have had in the past and that differentiate the classroom in terms of learning "styles", "questions" posed, and "thinking routines".

> I try and get some outcomes that are better for kids, or different for kids, or maybe even not so much look at the high end, but look at the gamut of activities and see what I can direct different styles, different questions, different thinking routines to get the kids to draw forward.

Using a context-based approach to teaching as facilitated by Collective Argumentation (CA) has provided Jill with the tools to provide her students with learning-opportunities that provide them "time to think", to "go" with that thinking, and to take "different paths to achieving the higher expectation" of what it means to participate in the mathematics of the classroom.

> The big thing that I noticed with CA, my expectations of where I thought, you know you have a big list of four things that you want to do in a lesson, where the kids got to, you know I always thought they will be able to do this, this and this and then I will have to help them with the following things, whereas, by not sort of, by allowing them that time to think at the beginning (represent individually), it sort of really made me think that I was limiting what I was doing by how I was teaching the class. I mean that has definitely changed, I have much higher expectations and I actually think a lot more about how the kids are responding and um lay out different paths for them to take and then go with what they think.

As implied in the above interview extracts, using an approach to teaching such as CA that facilitates context-based teaching is characterised by a strong connection to the sociocultural context of the classroom, that is the roles assigned to teacher and students, the social relationships that emerge when those roles interact and by the demands of the practice of teaching and learning (Hung & Chen, 2007) in this case mathematics. For Jill, these characteristics permeate assessment in her classrooms.

Assessment: Continuous Versus Rigid

For Jill, assessment seems to be concerned not only with the "rigidity" of a "high stakes" regime, but also with "reflections" and mathematical "performances" mediated through a "context" based approach to the "transfer" of understanding.

> I use continuous assessment all the time. And when I say, we do reflections and performances and when I say assessment practices, we have high stakes assessment, which is rigid across the year level that can't be changed and it's the same for everyone and that's fine. There's not a lot we can do about that because we have external assessment, but the other assessment that we do is continuous. Once a fortnight we'll do, it could be just a question, it could be a complex question requiring the kids to actually put all of the stuff they have done in the last few weeks together and apply it in a context to see if, for me, to see if they have got the ability to transfer. Do they know it well enough, have we explored it well enough, so they can actually now hook in (their understanding) and transfer to a different context?

Reflections for Jill go beyond "telling what you did" to linking understanding to the "usefulness" of mathematical knowledge in other "subjects" and to "exploring misconceptions" as a means of providing "feedback" to students.

> Sometimes it's a reflection and of the things (assessment) that I have used it is probably best, but it takes the longest to get the kids to do it well. Are the reflections where, you know in the first three or four months they'll just tell you what they did and, you'll be like, well I was there too, I know what happened. It's more a question of how you have understood what you have done, what it (your understanding) links to, where could we be going with it, where else is it useful in any other subjects, because the kids just see maths in maths and they never draw a graph ever in science or anything like that, which is crazy. But so, my assessment practice has changed. Then based on what I get from those reflections, those misconceptions can come up and then we will go and explore them. So, it's a bit of a feedback loop.

Using assessment in a continuous fashion as described above is in tune with viewing teaching and learning as sociocultural practice, where learning is viewed as continuous participation in the practice of a community (e.g., mathematicians) rather than as being an abrupt movement from novice to knower evidenced through the successful completion of an external exam (Lave & Wenger, 1991), a view of assessment consistent with a context-based approach to teaching and learning mathematics.

Conclusion

This paper has explored a secondary teacher's interview script relating to implementing a context-based approach to providing students with learning-opportunities in the subject domain of mathematics. As reported by Jill this approach has the potential to focus the teacher on actively preparing students for a 21st Century society that is continuously changing. In order to do this, teachers need to be flexible in their view of mathematics teaching and learning. However, implementing such a view in the classroom can be demanding as many in society may not consider that the teacher is doing their job as they are not up the front of the classroom telling the students what to do.

For Jill, promoting talk in the mathematics classroom provides the teacher with opportunities to form productive teaching-learning relationships with students where she can model mathematics concepts, ask inquiry type questions and provide feedback in a manner that assists students to build confidence, to bring their ideas to the class, and to contribute to the mathematics of the classroom. Content in the mathematics classroom, according to Jill, is focused on exploring the assumptions upon which ideas are based and on providing students with multiple points of entry to doing the mathematics. Content knowledge is not based on the language of "right" or "wrong" but rather on the appropriateness of ideas, concepts, and procedures to their context of use.

According to Jill's interview, pedagogy is concerned with bringing about the "best" outcome for each student, catering for different learning styles, posing questions that interest students and with 'drawing' students into the mathematics of the classroom through privileging different thinking routines. Assessment is not about the "rigid" measurement of student performance, but about providing students with continuous opportunities to reflect on their mathematics, to apply their understanding to different contexts, to see mathematics as being useful, and to explore misconceptions that come about in the knowing and doing of mathematics.

As referred in the above analysis of Jill's interview, there is a tension between what the community (parents, students and government leaders), think mathematics is and how it should be taught and the way Jill described how she develops an understanding of the mathematics in her classroom. Her concern about students not being given sufficient time to develop their understanding of the mathematics is a real concern. From Jill's utterances, it would appear that society seems to value more those aspects of mathematics that are able to be recalled quickly to obtain a correct result. However, the questions found in highly influential forms of assessment such as PISA require students to be able to think mathematically and to be able to make links between ideas and concepts. It is our argument, that the opportunities-to-learn that are afforded when teachers use a context-based approach to teaching mathematics, such as CA, privilege the development of student understanding that is facilitative of making links between ideas and concepts, between the everyday and the mathematical. Such an approach to providing learning-opportunities for students is in line with the work of researchers such as Cobb et al. (2001) and Hung and Chen (2007). However, before the use of such context-based approaches can spread to other secondary mathematics classroom, in Jill's words, "we need to let go a little bit of this rigorous view of mathematics that they (the students) have to know how to do everything".

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Partial Credit in Multiple-Choice Items

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Multiple-choice items are used in large-scale assessments of mathematical achievement for secondary students in many countries. Research findings can be implemented to improve the quality of the items and hence increase the amount of information gathered about student learning from each item. One way to achieve this is to create items for which partial credit can be given when students select particular incorrect options. To improve the items in this way requires a critical analysis of how the items contribute to the measure of student achievement as well as extensive knowledge of the test construct.

The inclusion of multiple-choice (MC) items in assessments of mathematical understanding appears set to continue as these items are deemed to be efficient and cost-effective for the collection of evidence of student achievement according to Betts, Elder, Hartley and Trueman (2009). For the same amount of test time a broader range of content can be covered with MC items than with other types of items. However, the quality and amount of information collected about student learning can be further improved without asking more of students who respond to these types of items. Such improvement could increase the accuracy and detail of the measures of student achievement.

Multiple-choice items consist of a statement or question, known as the stem, followed by a series of numbers, words, phrases or sentences which might complete the statement or provide the answer to the question in the stem. Typically, one of these options is correct and is known as the key while the other incorrect options are called distractors. The key is generally awarded one mark and zero is allocated for all distractors or missing responses; this is described as dichotomous scoring.

Students who do not fully comprehend an item’s content may have still developed some knowledge and understanding and can be considered to have partial knowledge of the concept tested in the item. When items are scored dichotomously there is no score for partial knowledge but some scoring procedures enable partial credit to be allocated.

Partial Knowledge

There are various descriptions of partial knowledge in the research literature and for Bush (2001) it involves the selection of more than one option and is identified as *liberal* MC. Candidates are allowed to select more than one answer if they are uncertain of the correct one. For four options, the candidate scores three marks if they select only the key, two marks for two options, and one mark for three options. Bush found that it took much longer to answer the questions, the instructions needed to be very clear and the weaker students did not like the format.

Partial knowledge was described by Bond et al. (2013) as the ability to eliminate some but not all of the incorrect answers. Such elimination was positively scored but the elimination of the key attracted a negative score. Bond et al. found that students preferred this form of elimination marking to the traditional single answer selection and reported that they found it less stressful and were not distracted by thinking of ideal tactics to maximise their scores.

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The answer-until-correct approach was described by Frary (1989) as a means of rewarding partial knowledge. Students would select options until they were correct, and they scored according to the number of attempts to identify the key. At the time, this method was costly to supervise and correct and while the use of computers would make such a process more efficient, little evidence of its current use has been found in the research literature.

In this review of scoring methods, Frary (1989) reported on a method by which options were weighted and the students would score according to which option they selected. The value of an option was determined by experts but it was deemed difficult to explain the process to examinees. Briggs, Alonzo, Schwab, and Wilson (2006) studied a similar scoring process in which each option was linked to a developmental level and even though this method appeared to work for MC items in Science, it is difficult to determine how several different levels could be planned for the options for each MC item in Mathematics.

Further scoring processes which involve option weighting are described by Diedenhofen and Musch (2015). The options are weighted after the students have completed the test and the responses are analysed by examining the correlation between the frequency of option choice and total score. Diedenhofen and Musch reported increased test reliability and validity with such scoring but suggested it is not suitable for easy items.

Proportional Reasoning

Proportional reasoning has been described by Siemon, Bleckly, and Neal (2012) as a key concept for students in early secondary, "without which, students' progress in mathematics will be seriously impacted" (p. 22). It pervades all areas of mathematics for lower secondary students and underpins many aspects of the upper school curriculum including similarity, trigonometry, functional relationships and algebraic formulation. For Siemon et al. (2012), proportional reasoning "involves recognising and working with relationships within relationships (i.e., ratios) in different contexts" (p. 32).

According to Lamon (1993), students can demonstrate proportional reasoning when they understand equivalent ratios and the invariance of relationships, even though they are unable to represent the relationship using mathematical symbols. Proportional reasoning, as the ability to reason when using proportions, or to solve problems when the relationship between quantities or variables is proportional, is the definition underpinning this study. The skills and concepts that students need to develop to be able to solve a range of problems to demonstrate sound proportional reasoning include understanding and manipulating ratios, rates, fractions, decimals and percentages.

The acquisition of the skills for sound proportional reasoning occurs over considerable time and from some research studies it has been possible to identify some of the stages in this development. Students learn some aspects of a concept before being fully competent and they may be described as having partial knowledge of the concept. Such partial knowledge can be used to create distractors for MC items which can be scored with partial credit. For some of the concepts necessary to develop sound proportional reasoning there are misconceptions that are commonly held by students and these can inhibit the ability to demonstrate other skills and understandings.

Many of these misconceptions have been described in the research literature and some of these are considered in this study. Partial knowledge can be demonstrated when students recognise the need to increase or decrease a quantity but make additive errors, using addition rather than multiplication to solve proportion problems. Misailidou and Wiliams (2003) found that additive errors were the most common errors made by students aged 10 to 13 years as they solved problems on proportional reasoning. Another demonstration of partial knowledge occurs with the use of absolute rather than proportional change. Students can increase or decrease quantities but use a fixed amount rather than a proportional amount and in this type of situation would use \$15 to represent a 15% increase regardless of the starting amount. Students may also recognise an increase in size or shape but have the scale factor incorrect as when moving from linear measure to area measure.

Student development of understanding fraction operations comes in stages and a common error made by students is described by Behr, Wachsmuth, Post, and Lesh (1984) in their study where 30% of the students added the numerators and denominators to find $\frac{1}{2}+\frac{1}{3}$, giving the answer as $\frac{2}{5}$ instead of $\frac{5}{6}$. It could be concluded that another common error that students would make is to identify $\frac{2}{4}$ as double $\frac{1}{2}$.

The use of MC items to assess mathematical understanding can be improved and one way to do this is to provide a score for the partial knowledge of a concept as shown by a student when they select a distractor which shows better understanding than other distractors. Writing such distractors requires an analysis of research findings to identify ways by which students develop concepts. It is hypothesised that providing partial credit produces a more accurate measure of student ability than dichotomous scoring and allows a more efficient use of the MC items for assessment.

Methodology

To collect data for the analysis a test of sixty MC items was designed, created and implemented. Items were written using the content and proficiencies of the Western Australian curriculum in Mathematics for Years 6 to 8 (School Curriculum and Standards Authority [SCSA], 2016). The test consisted of six blocks with ten items in each block and all students were given the first block of ten items, written at the standard Year 8 level. Students were then randomly allocated to two of the other five blocks, each of which was written for a different standard, for example, Year 7 above the standard. While 860 students completed then ten items at the Year 8 standard, between 327 and 360 students completed each of the other blocks of ten items.

The items were designed to test the skills and understandings deemed necessary for the development of sound proportional reasoning which included aspects relating to decimals, percentages, fractions, proportions, ratios, rates and linear relations. For each item there were four options, the key and three distractors. One distractor was written to attract students who knew something, but not everything about the item's content and hence allowed partial knowledge to be demonstrated. This partial knowledge was deemed to be worth some credit but not as much as was awarded for the selection of the key. The author's experience as a Mathematics teacher and results of studies reported in the research literature were used to inform the creation of distractors to be awarded partial credit.

Items 9, 56, 15, and 38 are presented in Figure 1 and relate to percentages, rates, fractions and ratios respectively. For Item 9, the distractor designed to elicit greater information is Option b and students who select this option could be thinking of absolute rather than proportional change. For Item 56, it was thought that students who were not competent in adding fractions would select Option c. For the rates described in Item 15 students not recognising that the smaller floor was a quarter of the area of the larger floor, might be able to demonstrate partial knowledge by recognising that there is a factor of two in the linear measure. The distractor in Item 38, Option c, was created to allow students who are using "additive" thinking rather than using proportional reasoning to adjust a ratio, to be awarded credit for their partial knowledge.

Item 9

The **correct** answer in a student's homework was $744.

The question could have been

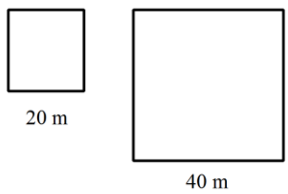
a. Increase $600 by 24%
b. Increase $700 by 44%
c. Decrease $700 by $44
d. Decrease $800 by $166

Item 56

Jon's pancake recipe requires $1\frac{3}{4}$ cups of flour. How much flour will Jon need when he **doubles** the recipe?

a. $3\frac{1}{2}$ cups
b. 3 cups
c. $2\frac{6}{8}$ cups
d. $2\frac{1}{2}$ cups

Item 15



Two square gym floors need to be polished. The time estimate for the larger floor is 8 hours.
If the floors are polished at the same rate, then the time needed for the smaller floor is

a. 16 hours
b. 8 hours
c. 4 hours
d. 2 hours

Item 38

Daniel has two dogs: Benson who weighs 10 kg and Shamrock who weighs 15 kg.

Daniel gives them treats according to the ratio of their weights.

If Daniel gives Benson 12 treats, how many should he give to Shamrock?

a. 24
b. 18
c. 17
d. 12

Figure 1. Multiple-choice items from the online test.

Approval to conduct the test in Western Australian schools was obtained from the University of Western Australia (UWA), the Department of Education, Catholic Education and the Association of Independent Schools. The Year 8 students who volunteered to sit the test came from twelve different secondary schools and there were at least three schools from each sector. Given the nature of the investigation and the proposed analysis, as well as the number of students who volunteered it was not considered necessary to confirm that the sample obtained was representative of Year 8 students in the state. The online test was conducted in November 2016, a time by which the Year 8 curriculum would have been covered for most students in these schools. The survey platform which supported the creation and delivery of the online test is one licensed to UWA (Qualtrics, 2016).

The software program RUMM2030 (Andrich, Sheridan, & Luo, 2015) was used in the application of Rasch Measurement Theory to the results. Three different analyses were conducted. First, all items were scored dichotomously with one mark allocated for the selection of the key and zero otherwise. For the second analysis with polytomous scoring, two marks were allocated for the key, one mark for the distractor created to be awarded partial credit, and zero otherwise. For the third analysis, scoring was dichotomous or polytomous.

After the second analysis, an examination of the category probability curves indicated that polytomous scoring was not working for all items. These curves are provided in Figure 2 for the four items described earlier. For Items 9 and 56 there was no range of location on the continua for person ability (variable on the horizontal axis) where the probability of obtaining a score of one was higher than for all other scores. This indicates there is insufficient information to justify awarding a score of one and hence acknowledge partial credit. In these two items, the thresholds, which are the locations at which the probability is equal for adjacent scores, are said to be disordered. Evidence of ordered thresholds is seen in Figure 2 for Items 15 and 38. In Item 15 the first threshold (-1.4) is less than the second threshold (1.9). At the first threshold, the probability of scoring zero is equal to the probability of scoring one and at the second threshold, the probability of scoring one is equal to the probability of scoring two. The 35 items in which the thresholds were disordered were rescored dichotomously for the third analysis and polytomous scoring was retained for the other 25 items for which the thresholds were ordered.

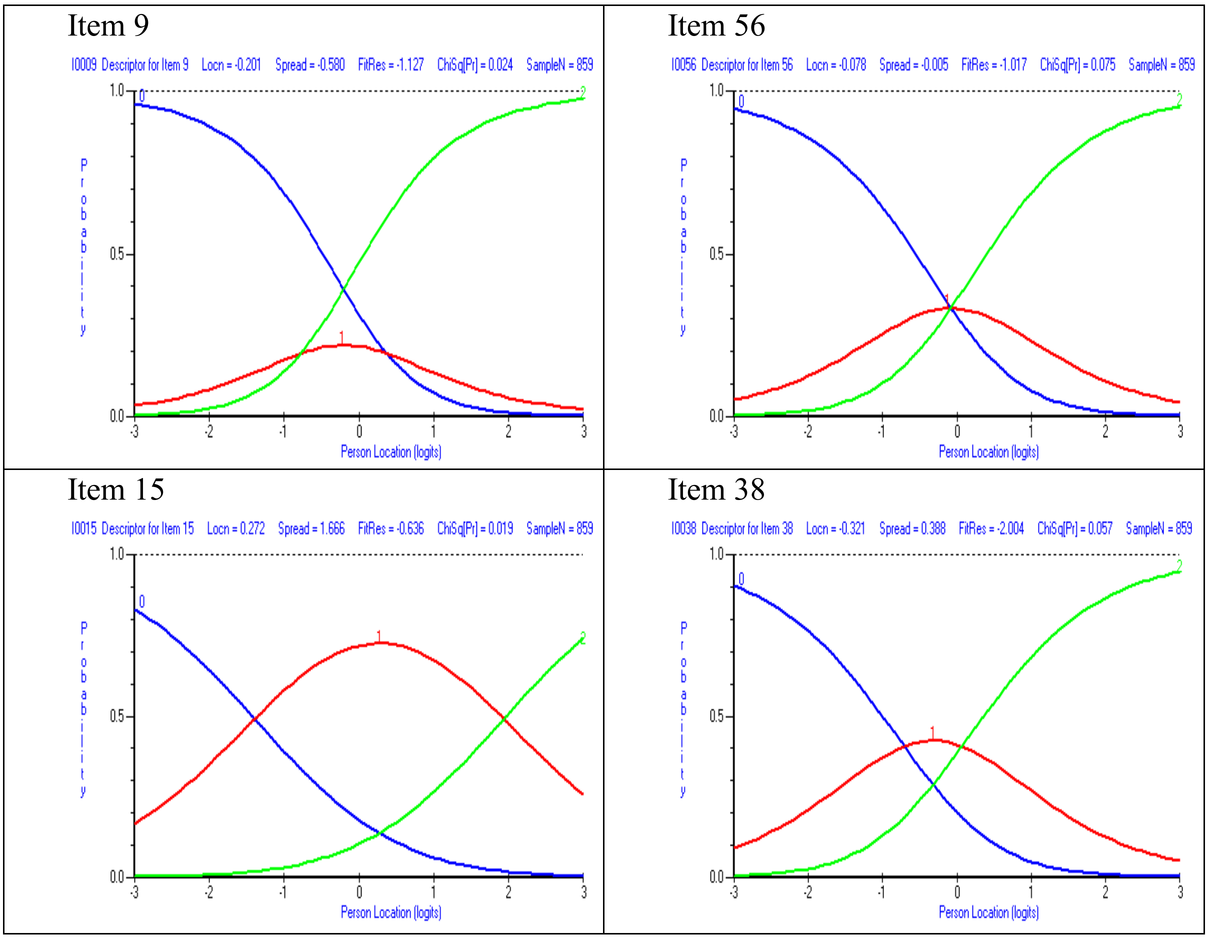


Figure 2. Category probability curves for multiple-choice items.

Results

For 25 of the 60 items, the use of polytomous scoring indicated that thresholds were ordered and this supported the proposal that students demonstrating partial knowledge could be rewarded with a score for a particular distractor other than the key. A comparison of the items where thresholds were ordered with those that were unordered has not shown any pattern that would allow a priori prediction of suitability for polytomous scoring. Items with ordered thresholds were not located in any particular area of person ability, nor concentrated in any of the particular content areas of proportional reasoning. It appears that each item needs to be analysed individually to identify the reasons why the thresholds were not ordered and why the proposed existence of partial knowledge was not confirmed.

For Item 9, the expectation that students could demonstrate partial knowledge of percentage increase with the selection of Option b was not realised. It is suggested that on the developmental pathway for most students, knowing that 44% of $700 is not $44 is learned before, or is easier than knowing that subtracting $166 from $800 is not $744. With fraction doubling in Item 56, scoring the Option c, the one designated as partial knowledge, did not appear to be justified indicating that this type of doubling is not a stage on the learning continuum.

For Item 38, where the thresholds were ordered, the selection of Option c suggests that the students may have considered that the absolute difference between the weights applied also to the difference in the ratio. Option d does not give the students the opportunity to show that they know the number of treats must increase and for Option a, the students recognise the increase but realise that the number of treats cannot be double. The type of additive thinking associated with the selection of Option c is considered as a stage in the development of proportional reasoning and for this item the award of a score for the partial knowledge is justified.

Students recognised the direction of the change in Item 15 and used the factor of 2 which was supplied for the linear measure when they selected Option c. They managed all aspects of the concept of change except for the recognition of the correct factor.

A comparison of the scales for student achievement as shown in Figure 3 indicated that awarding partial credit affected the significance of the gender differences as well as the measures of student achievement. The difference between male and female achievement was significantly higher for males with all analyses but the level of significance (p = 0.0403) was less when scoring for partial knowledge than when all items were scored dichotomously (p = 0.0105). The mean person location increased by 0.417 from -0.316 to 0.099, with the award of partial credit but the increase was greater for females than for males, 0.437 compared to 0.385.

The distribution of persons in Figure*s* 3 and 4 shows a considerable shift to the right at the lower end of the ability scale from the first to the third analysis. This result supports the expectation that the award of partial credit provides a higher level of achievement for persons of lower ability. This movement is not evident for persons of higher ability. The measurement scale appears to be more condensed with higher frequencies of person ability in the middle locations. Further evidence of the narrowing of the scale is seen in the lower standard deviation for both genders and this supports the idea that the overall variation of achievement is reduced when partial knowledge is rewarded in MC items.

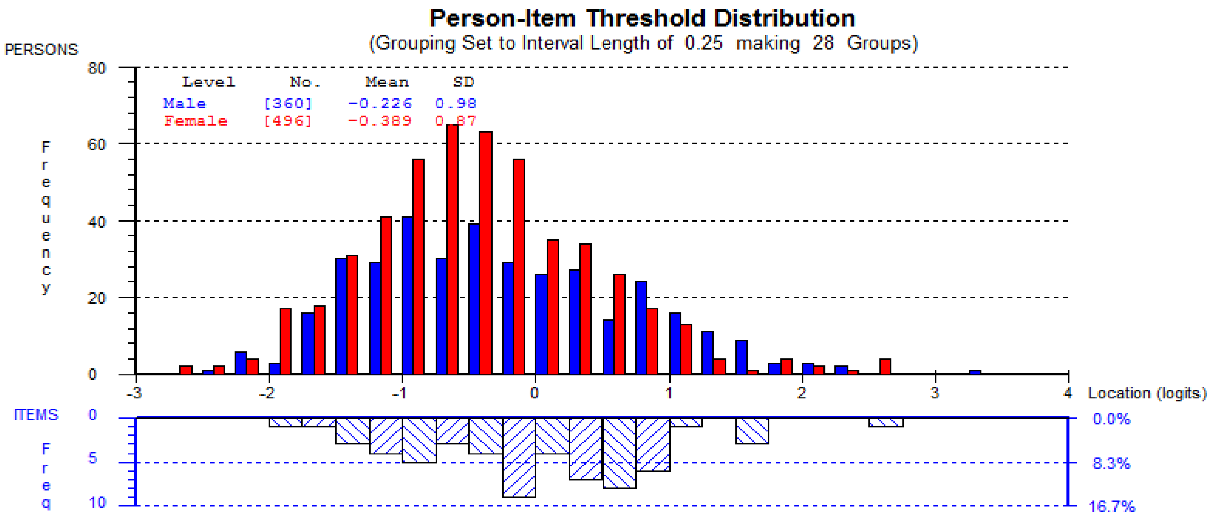


Figure 3. Person-Item distribution showing gender differences for first analysis.

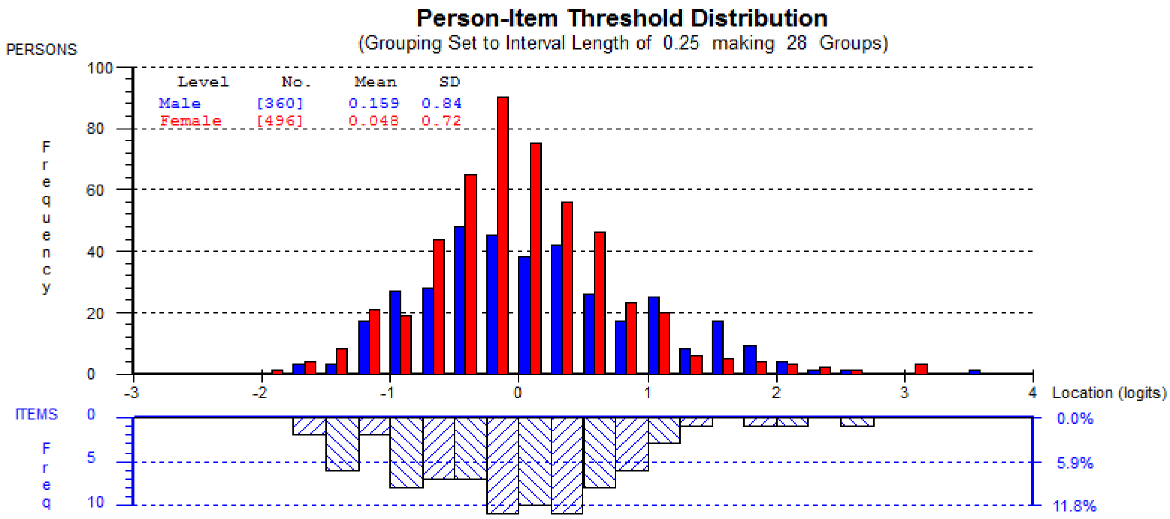


Figure 4. Person-Item distribution showing gender differences for third analysis.

Conclusion

For the allocation of credit for partial knowledge when using MC items in the assessment of mathematical understanding, there are two important considerations. First, it is desirable to have a sound awareness of the development in student understanding of the test content to be able to create items with options that reflect different levels of student ability. Second, it is necessary to critically analyse the students’ responses to the item to confirm that the proposed partial knowledge represents a stage on the continuum of learning. More accurate measures of student ability can be made if credit can be given for partial knowledge and to do this with MC items allows more information about student learning to be gathered without increasing the demands on students taking the test. While greater effort is required to create such items, the time required to complete the test and the behaviour of the students in selecting the best option are not affected.

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How Might the Use of Apps Influence Students' Learning Experiences? Exploring a Socio-Technological Assemblage

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In this paper, we report on primary-school students' views of their learning experiences when they engaged with mathematical phenomena through apps. The students commented on how they used a range of digital tools within the apps to solve problems, and we consider how the affordances of the mobile technologies, including multi-representation, dynamic and haptic, might influence the learning experiences. In particular, we focus on the interplay between the affordances of the mobile technologies with other social and pedagogical aspects, and ask how the assemblage of social and technological entities might influence mathematical learning experiences.

Mobile technologies (MT) populate our social and occupational landscapes. Their presence is ubiquitous, including in educative settings. Their low instrumentation and ease of operation, coupled with the interaction being focused primarily on touch and visual elements, make using them intuitive for learners. Synonymous with MT in educative settings is the use of educational apps. These vary in quality regarding their mathematical and pedagogical approach (Larkin, 2015), evoking questions regarding the appropriateness of the content and pedagogical approaches of some apps (e.g., Philip & Garcia, 2014). However, if MT are a relatively enduring element in mathematics classrooms, their potential to enhance mathematical learning requires examination. Previous research has suggested that the affordances of digital technologies, including MT, have the potential for offering fresh approaches to engage with mathematical concepts and processes, and for re-envisaging various aspects of mathematics education (e.g., Borba & Villareal, 2005; Calder, 2011).

In this paper, we explore this potential by examining students' views on the use of iPad apps, as an example of MT, and we consider how affordances inter-relate with other entities, both social and technical. We present data from a larger study on the use of iPads, in two primary school settings. The aim of the project was to engage in a co-inquiry with teachers into the ways apps might enhance student learning in mathematics. Through discussion with the teachers, a question arose in relation to the use of screen-casting to record calculation strategies, and how the affordances of the iPad app might have presented an alternative learning experience to the use of pen and paper. It was suggested that, in order to examine the learning experience provided by the iPad, we needed to look further at the inter-relationships between the learner and the digital medium through an assemblage of entities, in particular the social and technical entities involved in a learning experience. The question examined in this paper was: How might the assemblage of social and technical entities influence the learning process?

Affordances

Building on the notion of affordance as the inter-relationships between the learner and the environment (Gibson, 1977) or the user and the artefact (Brown, 2005), we acknowledge how the digital medium exerts influence on the students' approach, whilst the students' existing knowledge guides the use of the technology. The students' engagement

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with the pedagogical media is influenced by the actual digital medium, but this in turn also influences the medium (Hoyles & Noss, 2003), and hence the learning experience is fashioned in distinctive ways. For example, one affordance frequently associated with digital environments is the aspect of multiple representations. The ability to link and explore visual, symbolic, and numerical representations simultaneously in a dynamic way has been recognised extensively in research (e.g., Ainsworth, Bibby, & Wood, 1998; Calder, 2011). Apps that enable screen-casting, the digital recording of the computer screen, along with voice recording, introduce a further modal affordance in creating an aural representation to which students can listen. It allows students to record individual or group presentations of mathematical processes, strategies, and solutions.

Research has further shown that iPads and apps foster experimentation allowing space for students to explore (Calder & Campbell, 2016). The apps' affordances of interactivity and instantaneous feedback foster the learners' willingness to take risks with their learning (Calder & Campbell, 2016). Other researchers contend that the affordances of digital technologies, coupled with the associated dialogue and social interaction, may facilitate students learning to pose problems and create explanations of their own (Sandholtz, Ringstaff, & Dwyer, 1997). They allow students to model in a dynamic, reflective way with other learners, mediating the language evoked through interaction with each other, the digital media, and the mathematical ideas, and hence influencing the learning experience (e.g., Calder, 2011).

Therefore, a theoretical perspective that acknowledges the inter-relationship of the multi-modal representations of the iPad with the learner, along with the mathematics and social interaction with others, could help to draw these entities together and help to understand how the use of iPads might influence the learning experience in primary mathematics classrooms.

Socio-Technological Assemblages

Delanda's (2006) assemblage theory explored how inter-relationships are merged to form a social complexity. The social complexity as a whole emerges from heterogeneous parts, and the properties of the whole emerge from the interaction between the parts. We relate Delanda's notion of assemblage with other theoretical perspectives suggestive of collectives. For example, Borba and Villarreal's (2005) perspective saw understanding emerging from the reconciliation of re-engagements of the collectives of learners, media, and environmental aspects with the mathematical phenomena. Borba and Villarreal contend that as each engagement re-organises the mathematical thinking and initiates a fresh perspective, it in turn transforms the nature of each subsequent interaction with the task. The digital media influence the engagement and ensuing dialogue in particular ways, which, with self-reflection or further dialogue with others, transform the learners' perspective (Borba & Villarreal, 2005). The learners then re-engage with the task from this new perspective. This iterative process continues until some form of shared negotiated understanding occurs (Calder, 2011).

From both Delanda's (2006) and Borba and Villareal's (2005) perspectives, the whole, or in the case of this paper the learning experience, becomes the articulation of discursive and non-discursive elements of objects and actions (Delanda, 2006). However, a key distinction is Delanda's proposal that all entities and relationships, whether social or non-social, are ontologically and epistemologically indistinct. As such, knowing or understanding within the learning experience is no longer a means of representing or reflecting on new knowledge, but one of interacting with and creating new knowledge.

Delanda's philosophy proposed that social and non-social are ontologically indistinct; they are both composed of assemblages. Social assemblages may be codified through language, whereas non-social or technical may not. However, the very use of the technical can be seen as expression, and this is illustrated through the multi-representational and multi-modal affordances of the iPad. The learner may select representations as a way of expressing and creating knowledge; the learner may also use hand actions with the touch interface of the iPad screen, again as a way of expressing and creating knowledge.

Despite philosophical distinctions, these two perspectives of merging learners with digital media within a situated learning experience point to the notion of an assemblage of digital and social elements, which we term a socio-technological assemblage. In this paper, we report on students' perceptions of the use of several mathematics apps and investigate how the students perceived the learning opportunities presented by the apps. The students' perceptions are then analysed in relation to the notions of affordances and to the notion of socio-technological assemblages, in order to investigate the influence of the iPad, as an example of MT, on learning experiences.

Methodology

In the research for the larger two-year project, we used an interpretive methodology related to the building of knowledge and the development of research capability through collaborative analysis and critical reflection of classroom practice and student learning. The research design was aligned with teacher and researcher co-inquiry, whereby the university researchers and practicing teachers work as co-inquirers and co-learners (Hennessy, 2014). In the first year of the project, three teachers, all experienced with using MT in their programmes, were involved in the study. The schools were situated in a provincial city. One teacher taught a Year 4 class (7- and 8-year-olds) in a school that used a bring your own device (BYOD) approach, while the other two teachers team-taught in a combined Year 5 and 6 class (9- to 11-year-olds) in a school with one to one provision of iPads (80 students in total). Data, obtained through different sources (focus group interviews, classroom observations, interviews with teachers, and blogs), were analysed using *NVivo* via a mainly inductive or grounded method to identify themes. While the researchers identified the initial themes and codes, refinement occurred through joint critical reflection between teacher practitioners and academic researchers in research meetings. Two themes that were focused on in the research meetings were affordances and socio-technological assemblages.

In this paper, we present data from student focus group interviews and individual blogs in order to investigate the students' views in relation to their learning, and to explore how the two notions, affordances and socio-technological assemblages, may help to examine how the use of iPads have influenced the students' interactions with mathematical ideas. The student blogs were obtained partway through the first year of the project. Focus group interviews with the students were conducted at the beginning and end of the years. Prompts were given to support students in writing their blogs, and semi-structured interview schedules were developed for group interviews with six students from each class.

Results and Analysis

Student Blogs

In the student blog data, references were made to the features and affordances of the apps and iPads, how these influenced the learning experience, and the ways understanding might emerge. Several blog entries referred to the multimodal affordances of screen-casting.

> You can record your learning and you can see what stage you are working on and instead of writing in our book we can just record our voices and upload it to Google classroom!

> It helped to solve my problems - by using Explain Everything you can record and pause and think about what you're saying.

The ability to record, review, and then edit also engendered confidence, hence the affordance of the app to encourage risk-taking and exploration.

> I can confidently record my voice and… [I] feel okay with others hearing my recordings.

The visual dynamic representational affordance was suggested with students using *Hopscotch* and *Multiplier* apps:

> Hopscotch is a big hit, as you can see your creations moving around.

> Multiplier helps me because it shows what it looks like so I know how to do it.

The use of the *Multiplier* app also illustrated the haptic affordance, as the students used their fingers to draw out the matrix and had both visual (including colour coding) and numeric representations simultaneously linked together. It was a multi-levelled problem-solving environment with a tap used during the experimentation and review stages:

> You tap on which one you think is correct.

The haptic affordance was mentioned in other blogs:

> I like to touch things [and move them] and: You could draw over the answer.

The use of programming apps with Sphero robots made strong physical and visual connections too, especially in geometry. At times, the connection between hand movements and movement of the Sphero (small robots) was mediated by the app.

> Tickle helped me programme robots to draw triangles.

> We used this app (Tickle) to learn about making shapes, angles, and vertices.

Students made further comments related to a mixed use of pedagogical media. For example, the students were comfortable moving between their iPad and more traditional media

> I can still switch back to my book easy and it's still easy to use apps.

Key viewpoints from the student blogs referred to the dynamic multi-modal representations, hand actions and the haptic, risk taking, and exploration. Students also referred to mediation through programming and the use of different pedagogical media.

Student Interviews

These key viewpoints were reiterated in the student interviews, both at the beginning and end of the year, but the students' responses also extended their perspectives. Students again referred to drawing on the iPad screens or of tapping to select a tool, hence indicating the haptic affordance of the iPad. In addition, they commented on how the iPad made their work "easier and tidier."

Again the use of screen-casting to record their solution strategies was a key feature. The students talked of video-recording themselves doing maths, and recording their working. As one said,

> It's just like making a movie for maths.

Hence, students were indicating the multi-modal representational affordances of the screen-casting app. The opportunity to record their voices whilst writing and drawing seemed important. One student commented that it was

> hard to explain without writing down. You can write it down as well as explaining it while you're recording.

The use of multiple modes simultaneously supported this student in expressing his thinking. The opportunity to pause and edit recordings also appeared to be significant in supporting students in expressing their thinking.

> The cool thing is that you can actually pause it and then think about what you're going to do.

Students also commented on the assurance that they had a correct solution, and hence had confidence in their strategies to start recording. Others referred to the opportunity to use the different visual and dynamic representations on the iPad and how these introduced them to new strategies.

> I like learning new strategies, using a number line and place value.

> I learnt how to use the reversing strategy on the number line.

Students also referred to opportunities for collaboration with their peers and how they worked on a mathematical idea together.

> I like working with my friends and then recording our voices like working out an equation together.

> Working in a group helps us work independently a bit more because when it's in a group they're all giving us ideas.

The opportunity to share documents was also seen as a way of collaborating.

> You can share docs with someone – so you can tap on the same thing, so you get ideas quicker.

Collaboration also provided the opportunity to explore and experiment with mathematical ideas

> Luke asks me to work with him because we like to help each other out and solve things – so if we don't get something, we try and work it out.

The responses from students in the interviews suggested further viewpoints in relation to the use of multiple modes in expressing and creating their thinking, visual and dynamic images in learning key concepts, and peer collaboration in sharing ideas and in exploring or working out new knowledge.

Discussion

Student responses in the interviews and blogs acknowledged the potential of the iPad in manipulating objects dynamically on-screen. Students spoke of acting directly with the object and referred to tapping or drawing on the screen. The screen-casting feature was seen to introduce multiple modes and representations as students worked simultaneously with dynamic visual recordings (drawing, manipulating digital tools, and writing symbols and words), along with speech, to create a dynamic aural-visual representation. The coding apps *Hopscotch* and *Tickle* were used to connect numeric and symbolic representations in the coding with the physical movements of the Sphero and the creation of geometric shapes. Although the movements were mediated by the coding process, the students commented on the connections between the movements and their learning. Furthermore, the students referred to collaborative working with their peers. They indicated how ideas could be shared and worked on together. The students also mentioned how non-digital and digital technologies were used together.

In sum, student comments were suggestive of inter-relationships between the multi-modal affordances of the iPad, along with other non-digital entities including peer interaction and other pedagogical media. These inter-relationships are interpreted through the notion of an assemblage, where social and non-social become merged.

In relation to Delanda's (2006) assemblage theory, the learning experience is viewed as a social complexity constituted of heterogeneous entities. Students' comments were suggestive of social assemblages such as the use of verbal language when communicating with each other or voice recording. However, students also communicated through tapping on the screen or in sharing a document. Students also referred to use of hand actions when using *Multiplier* or *Tickle*. As such, the technical materiality, that is, the multi-modal affordances of the iPad, were used by the students to communicate and express ideas. Social and non-social could be seen to merge in line with Delanda's theory, and the learning experience became a means of interacting with and creating new knowledge in ways that were determined by the features of the iPad as well as through other media and communication.

Borba and Villarreal's (2005) perspective focused on engagement within a collective of learners, media, and the environment. Engagement re-organises thinking and provides fresh perspectives for re-engagement. The students suggested opportunities to interact in collaborative ways to "work it out" and experiment. The students had opportunities to pause recordings in order to reflect before engaging further with the media. In order to interact with the mathematical ideas, the students drew on existing knowledge and affective dispositions to engage with the mathematical ideas, not just through the iPad, but also through a range of social interactions that evoked interpretations or understandings that were negotiated further (Calder, 2011). Not only did the students note the recordings as a way to show their thinking processes in solving a word problem, it appeared that through pausing and editing, the students took time in preparing and perfecting their recordings. They were able to reflect on what had been said, and think about what to say next. Here the students were influenced by the iPad which they then influenced.

From both theoretical perspectives, the multimodal affordances of the iPad can be seen to provide new entities for social and technical to merge as an assemblage or a collective within a learning experience. So, in answer to our initial query about the use of the iPad to record a strategy, we would suggest that when viewed as a separate entity, pen and paper would seem to provide a similar experience to a written recording on the iPad screen. However, when taken within the entire learning experience, in this case creating a screen cast, there are opportunities for social and technical entities to merge in a way that would not be similar to a pen and paper activity. The content and nature of the screen-casting recordings were seen to merge the multiple modes of verbal expressions with drawings and symbols. Students created their own ways of expressing their knowledge. Furthermore, some students developed these recordings collaboratively and acknowledged opportunities that enabled them to share and negotiate their knowledge in conjunction with the multi-modal affordances. Such recordings, compiled individually or collaboratively, would seem to illustrate the notion of a socio-technological assemblage that may influence the perspectives of those who viewed them, as well as those who created them.

Conclusions

Previous research has suggested that MT offer affordances that can reshape the learning experience. In this paper, we aimed to consider how learning mathematics through apps might influence learning experiences by examining students' views in relation to the idea of a socio-technological assemblage. The idea of an assemblage suggests that the same mathematical phenomena can evoke different ranges of social and technical entities when approached through alternative pedagogical media, and so the resulting learning experiences, constituted by the merging of these different ranges of social and technical entities, will differ. The entities explored in this paper were drawn from the students' comments. These acknowledged the process of verbalisation, along with the manipulation of images, drawing, and other representations, and would suggest a new learning experience: an experience situated within an assemblage of inter-related social and technological influences. Further study of the use of MT from an assemblage perspective could help us understand these influences and, consequently, to develop the use of MT to enhance learning.

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Entangled Modes: Social Interaction in Collaborative Problem Solving in Mathematics

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This paper reports a study conducted in a laboratory classroom with the capability to record classroom social interactions in great detail using advanced video technology. The social interactions of student groups during collaborative problem solving were analysed based on transcript data. This analysis suggests that meaning negotiation in mathematics classrooms can be usefully distinguished as social, sociomathematical, or mathematical. We suggest that all three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom and we envisage all three as constitutive of learning.

Introduction

We propose that student socially performed negotiative activities constitute both an essential aspect of the learning process and a key learning product on which more sophisticated intellectual activity is dependent. In order to investigate this proposition, we have employed student collaborative problem solving as a suitable activity by which the negotiative aspects of mathematics learning can be made more visible. Intact classes of Year 7 students (13 years old) with their usual mathematics teacher were filmed in a laboratory classroom completing a sequence of mathematics tasks individually, in pairs and in small groups. Our goal in the research reported in this paper was the identification of negotiative patterns of social interaction within the context of collaborative mathematical problem solving. We see the identification of such patterns as an essential precursor to the modelling and ultimately the optimisation of student collaborative group work and associated learning in mathematics classrooms. This paper reports the first step towards this long-term goal.

Conceptualising this Study

In associating learning with participation in practice, Lave and Wenger (1991) assert that “participation is always based on situated negotiation and renegotiation of meaning in the world” (p. 52). As legitimate sites of situated mathematical practice, classrooms provide settings in which these negotiative processes can be documented. Clarke (2001b) suggested that the presumptions of meaning are community, purpose and situation, since “it is futile to discuss the meaning of a word or term in isolation from the discourse community of which the speaker claims membership, from the purpose of the speaker, or from the specific situation in which the word was spoken” (p. 36). Contemporary social theories of learning accord a central role to the situated construction of shared meaning, through such constructs as the didactic contract (Brousseau, 1986) and sociomathematical norms (Yackel & Cobb, 1996).

Yackel and Cobb (1996) advanced the notion of sociomathematical norms by investigating the particular regularities in the social interactions within mathematics classrooms. They suggested that individuals develop their personal understandings of the social interactions in the mathematics classroom as they participate in the negotiation of classroom norms, some of which are specific to mathematics. Examples of

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sociomathematical norms include: what counts in that classroom as mathematically different, mathematically sophisticated, mathematically efficient, mathematically elegant, and what is considered to be an acceptable mathematical explanation and justification. Such norms can be distinguished from questions of mathematical correctness (mathematical norms), while also being distinct from social norms that govern other forms of social interaction of a non-didactical nature. We follow the European use of “didactical” here, referring to discipline-specific pedagogical concerns (Brousseau, Sarrazy, & Novotná, 2014).

While teachers play an important role in the classroom, peer interactions also appear to be particularly important for student learning. This observation can appear self-evident in many “Western” educational systems, where student-student interactions are an institutionalised aspect of classroom practice, but it is also consistent with research in classrooms in which such interactions are much less frequent. Results from the Learner’s Perspective Study (Clarke, 2006; Clarke, Keitel, & Shimizu, 2006; Kaur, Anthony, Ohtani, & Clarke, 2013), for example, suggest that students across all cultural settings attach particular significance to explanations provided by their peers in all mathematics classrooms where this occurs. Educational reform prioritising collaborative group work is being undertaken in countries such as China and Korea that had previously made very limited instructional use of student-student interaction. In combination, both research and contemporary reform make the investigation of such interactions and their function in the learning process an international educational imperative.

In terms of research design, some studies of social interaction in settings characterised by collaborative problem solving have constrained the social complexity of the situation by using clinical designs focusing on the interactions of individual dyads, frequently triggered by digitally-delivered problems (e.g., Olive & Steffe, 1990; Steffe & Wiegel, 1994). Other attempts to seek structure in the extreme diversity of such social interactions have included comparative studies, in which aspects of instructional setting, culture and social interactive norms can provide the variation needed to reveal underlying structure or consistency of pattern (e.g., Clarke, 2001a, 2006; Clarke et al., 2006). The first approach (clinical designs) compromises validity in the interest of experimental control. The second (comparative studies) relinquishes control over key variables in an attempt to capture social interaction in naturally occurring settings. Design experiments (e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) represent one approach to resolving the tension between the need for control in an experimental environment and the freedom for the participants to interact and behave as they would in a naturalistic classroom setting. The balance between validity, experimental control and the accumulation of a substantial body of systematically generated, structured data continues to pose a challenge for research investigating student learning in social settings.

Research Design

The analysis reported in this paper addresses the research question: What are the foci of the students’ social interactions during collaborative problem solving in this project? The research was conducted in a laboratory classroom situated within the Melbourne Graduate School of Education at the University of Melbourne, Australia. The classroom is equipped with 10 built-in video cameras and up to 32 audio channels. Intact Year 7 classes were recruited with their usual teacher in order to exploit existing student-student and teacher-student interactive norms. Two classes of Year 7 students (12 to 13 years old; 50 students) provide the focus for this report. Each class participated in a 60-minute session in the laboratory classroom involving three separate problem-solving tasks that required them to produce written solutions.

Problem-Solving Tasks

The problem-solving tasks used in the project were drawn from previous research (e.g., Clarke, 1996; Clarke & Sullivan, 1990, 1992; Sullivan & Clarke, 1988, 1991, 1992). All three tasks had multiple possible solutions, included symbolic or graphical elements, and afforded connection to contexts outside the classroom. Despite having these similar features, the content foci of the three tasks are disconnected to avoid carry-over effects between tasks. Task 1 provided students with a graph with no labels or descriptions (Figure 1) with the following instructions: "What might this be a graph of? Label your graph appropriately. What information is contained in your graph? Write a paragraph to describe your graph."

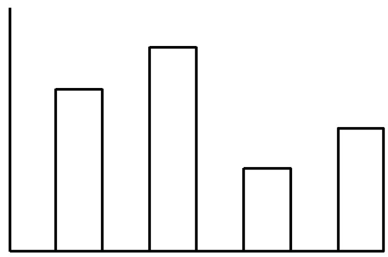

Figure 1. Task 1 stimuli.

Task 2 was specified as follows: "The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer."

Task 3 stated that "Fred's apartment has five rooms. The total area is 60 square metres. Draw a plan of Fred's apartment. Label each room, and show the dimensions (length and width) of all rooms."

The students attempted the first task individually (10 minutes), the second task in pairs (15 minutes), and the third task in groups of four to six students (20 minutes). Figure 2 illustrates the grouping for the three tasks for a single group of students.

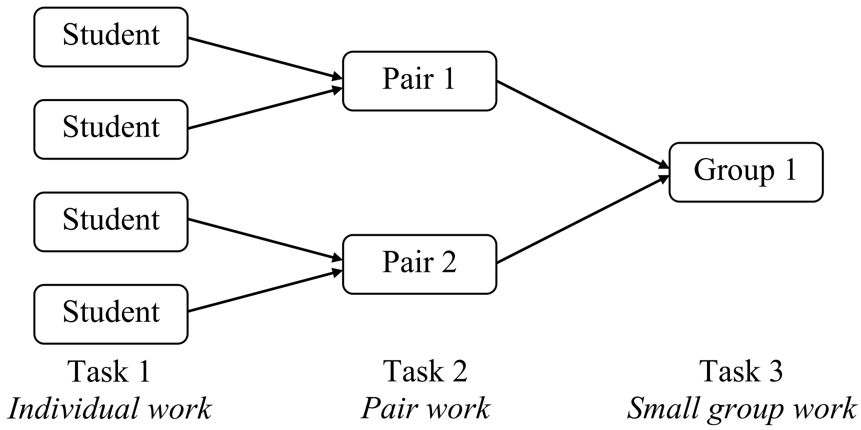

Figure 2. Student grouping for the three tasks.

Findings

As noted above, the analysis reported in this paper concerns the identification of regularities in the negotiative interactions of students engaged in collaborative problem solving. Analysis of the transcripts employed the negotiative event as the unit of analysis (Clarke, 2001b). In this analysis, a negotiative event is defined as “an utterance sequence constituting a social interaction with a single identifiable purpose.” The third task provides the focus of this paper due to the diversity of forms and foci of negotiation evident in the data arising from this group’s interaction during that task.

Analysis of the transcripts of student-student interactions identified three qualitatively distinct foci of interaction, which we have characterised as:

- Mathematical: a concern with mathematical correctness of fact and procedure
- Sociomathematical: a concern with the didactical norms of the classroom
- Social: a concern with social obligations and agency within the group, other than those included in the other two categories.

Below are some illustrative examples of each mode based on the group interaction during Task 3.

Negotiative Event 1 - Mathematical

Pandit: Okay, okay, okay. So, wait, this side’s 20.

Anna: Yeah.

Pandit: This is…

Anna: Wait. Let's just say that's - no, Pandit, it won't work.

Pandit: It does. It does.

Anna: It doesn't. We have to get a 30 there and then look, up to there is 30. Do you have a brain?

Pandit: (Laughs) I have a brain. No. Wait, isn’t that has to times?

Anna: Yeah.

Pandit: Twenty times thirty is like 600.

Anna: Six hundred.

Pandit: It has to be 60.

Anna: Yeah.

Pandit: You did it wrongly. That’s why.

Negotiative Event 2 - Sociomathematical

Anna: Guys, let's actually change the scale.

Pandit: We can't.

Anna: Why not?

Pandit: We're not allowed to change.

Anna: You are. Let's make two centimetre square equals one metre.

Negotiative Event 3 - Social

Pandit: Oh my god. The progress we put in is miserable.

Anna: I don't think they [John and Arman] like you as well.

Pandit: Yeah. I know, right.

Anna: Ha, ha. Both hating.

The first negotiative event involves Anna and Pandit realising that they need to multiply the length and width of the apartment to calculate the total area. Their apartment plan of 20m × 30m had a total area of 600sqm, which was much larger than the 60sqm specified in the task. The focus of the negotiation was on the correctness of the mathematics (Pandit: "You did it wrongly. That's why.").

The second negotiative event involves the students deciding on the scale of their apartment plan. Anna wanted to change the scale from 1 cm:1 m to 2 cm:1 m, but Pandit did not agree with her. The focus of the interaction was based on the students' perceived expectations for task completion (Pandit: "You are not allowed"). The pair was negotiatively revisiting the "rules of engagement" for task completion – that is, the sociomathematical norms prescribing what is and is not a permissible approach to task solution.

The exchange between Pandit and Anna in the third negotiative event about John and Arman's attitude towards Pandit illustrates an interaction focus that is neither about mathematical correctness nor didactical expectations. Their exchange was about the group dynamics in terms of social relations and connected these social relations to the group's productivity.

The three negotiative events illustrate three distinct foci for student-student interaction: mathematical, sociomathematical, and social. Negotiation with respect to each of these appears to employ its own lexicon and can be considered as a distinct mode of interaction. Mathematical interactions and the negotiation of mathematical meaning not only invoke mathematics terminology, but employ distinctive logical connectives, invoking notions of truth (e.g., "is" and "wrong") and also appeal to utility warrants such as "it works". Sociomathematical negotiations are more likely to be phrased in conditional or relativist terms, such as "might" and involve reference to approval or permission. Negotiations of social matters can involve responsibility and expectation (like sociomathematical considerations) but the invoked authority is likely to appeal to moral obligation rather than didactical convention and also to make more frequent reference to affect. The utilisation of each lexicon within an interactional sequence can be thought of as analogous to the familiar phenomenon of "code-switching" documented in classes of bilingual learners. As students shift from one mode to another, they employ language drawn from the relevant lexicon to express themselves.

The excerpts provided give an impression of relatively clear distinction between the three modes of interaction, but, in practice, all three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom. Below is an excerpt that illustrates such entanglement.

John: The area of the … of each room is the same or not same?

Arman: John. Fifteen times four is 60 and 15 times five is 75.

Anna: I don't know.

Pandit: I don't care about that. No, wait. Wait.

John: We care about that - because maybe some rooms are bigger and some rooms are smaller.

Arman: Fifteen times five is 75 and 15 times four is 60.

John: Huh? What?

Arman: Fifteen times four is 60, but we need five rooms. Five, five.

Anna: Twenty, three. Twenty, no, no, no.

Pandit: You do like 10 times 6 or something like that.

Anna: Okay. Let's just do 10 times 6 then.

Pandit: Yay.

Anna: Yay.

Pandit: Oh my God.

Anna: Oh my God.

Pandit: Yeah.

Anna: Yeah, yeah, yeah…

Each person in the dialogue had a different focus, where John was concerned about the task expectation (sociomathematical) in terms of whether the rooms can be of the same size or not, Arman was concerned about John's calculation of the size of each room of 15 sqm which produced an overall size of 75 sqm instead of 60 sqm as specified in the task (mathematical). Neither of the issues raised by Arman nor John was of concern to Anna and Pandit who were focused on working out the overall size of the apartment (mathematical). Anna's parroting of Pandit's utterances towards the end of the excerpt after they reached agreement could either be expressing collegiality or sarcasm towards Pandit (social).

In relation to the unit of analysis, the excerpt can be interpreted as three overlapping negotiative events: one consisting of the interchange between John and Arman focusing on the size of each room; one between Pandit and Anna focusing on the overall size of the apartment; and one concluding exchange between Pandit and Anna that is basically social in focus. Entanglement exists in both the actuality of the overlapping speech (negotiative events) and also in the shifts in focus between one utterance and another. The term "entanglement" also recognises that each pair (John-Arman and Pandit-Anna) is aware of the actions/statements of the other pair, as indicated by Anna's statement: "I don't care." The potential exists for the convergence of the separate negotiations into a further negotiative event involving all four students, but this does not occur within the brief transcribed sequence.

At this point, it is also appropriate to ask, "In what way does access to the video record (as distinct from the transcripts) provide additional insight into the student-student interactions?" From the excerpts above, there is evidence of discord between Anna and Pandit, while John was trying to become involved in the girl's discussion and Arman was shifting between being involved in the task and carrying on an independent conversation. However, it was striking when viewing the video recording, that the interactions between the students appear far less confrontational than might appear from reading only the transcribed verbal interactions. There was frequent laughter between the students and the interactions between Anna and Pandit, and other interactions within the group appear to have a good-humoured character, when one takes into consideration the students' body language, facial expression, and tone of voice. The video record also provides important information concerning whether or not interacting pairs of students were aware of each other's actions/statements and whether a silent student was a "silent participant" in a particular negotiation (Remedios, Clarke, & Hawthorne, 2008) or disconnected from that exchange. In addition, the connection between student utterances and physical actions, not only gestures, but also writing and the drawing of diagrams, can be made with much greater confidence with the support of the video record, with obvious benefits for transcript interpretation.

Discussion and Conclusions

As an entry point to the investigation of the nature of the social interactions evident in the data generated in the Social Unit of Learning project, this paper addressed the research question: What are the foci of the students' social interactions during collaborative problem solving in this project?

The transcript analysis suggests that meaning negotiation in mathematics classrooms can be usefully distinguished as a focus on either social, sociomathematical, or mathematical concerns. As was noted earlier, negotiation with respect to each of these foci appears to employ its own lexicon and can be considered as a distinct mode of interaction. However, to interpret the three modes as a stratification of social process (e.g., in terms of scale (grain size) or a logical or socially-normed interactive sequence) suggests a separation and a hierarchy that is not evident in practice. All three modes coexist in an entangled form in the negotiative interactions documented in the mathematics classroom. Future analysis will involve delineating each of the three lexicons and investigating the nature and consequences of their interaction.

The capability to document classroom interactions continuously and simultaneously in fine-grained detail using multiple cameras and microphones represents a technological advance in classroom research. Such an advance offers possibilities for the structured, rigorous, fine-grained investigation of the complex processes involved in learning in various social settings. This includes the simultaneous documentation of student production of written solutions to problems undertaken collaboratively and the recording of the social negotiative exchanges of which those written solutions are one outcome. A consequence of the type of tasks employed in this study is that student written solutions, supplemented by transcript and video data, are amenable to quite detailed classification indicative of mathematical decisions and associated reasoning in which the students engaged. Such an analysis was not the purpose of this paper, but the connection between multiple data types is a major affordance of the particular research facility.

The distinction between the different negotiative foci (mathematical, sociomathematical, and social) makes visible the dynamics of collaborative problem solving. The entanglement of these three foci within the negotiative dialogue between students is highly significant if we are to understand the dynamics of collaborative problem solving and associated learning. This project envisages all three interactive modes as constitutive of learning and as providing distinct entry points for teacher instructional intervention (or scaffolding). We suggest that each negotiative focus must be accommodated in a social theory of learning and each represents one avenue to improved learning outcomes in our mathematics classrooms. All three must be studied in situ and in relation to each other as they occur in authentic classroom activity. Such postulated interconnectedness poses challenges for theory and for research. This paper offers a starting point for further theorisation and investigation.

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Investigating Teachers' Perceptions of Enabling and Extending Prompts

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Differentiating students' learning needs in primary mathematics classrooms is an issue faced by many teachers. One technique designed to differentiate the level of challenge in mathematics tasks is the use of enabling prompts and extending prompts. We report on survey data pertaining to enabling and extending prompts, and teacher noticing of 37 Year 3 to 6 teachers participating in a project investigating the use of challenging tasks. Data were coded and categorised using grounded theory. The teachers valued enabling and extending prompts when implementing challenging mathematical tasks, and using these prompts stimulated them to notice students' reasoning and mathematical communication.

A challenge for teachers when teaching is to diagnose and notice students' learning and thinking, interpret this information, and decide appropriate opportunities for learning (Hoth et al., 2016). In mathematics education, one model for planning and teaching designed by Sullivan, Mousley, and Zevenbergen (2006) to address the range of mathematical thinking in any classroom is the use of one substantial problem posed to all of the students supplemented by appropriate prompts. These authors emphasised the use of open-ended tasks that create "opportunities for personal constructive activity by students" (p. 498); they also emphasised the use of appropriate sequences of tasks. They coined the terms *enabling prompts* and *extending prompts*. Enabling prompts were designed to support students experiencing difficulty, by allowing students to engage in active experiences related to the initial goal of the task. Extending prompts were for students who completed the initial task readily, and were designed to extend students' mathematical thinking. Sullivan et al. (2006) recommended to teachers that "a decision needs to be made when planning lessons about specific types of barriers that one or more students may experience, and during lessons about difficulties being experienced" (p. 498). This forward thinking and planning was intended as preparation for using an enabling prompt. Similarly, teachers could anticipate which students may benefit from an enabling or extending prompt.

The aim of the study was to investigate teachers' perceived use of enabling and extending prompts when noticing students' mathematical thinking. The following research questions guided the study:

- What were teachers' perceptions about the use of enabling and extending prompts?
- To what extent did teachers notice students' mathematical thinking when reflecting on their lessons?

Background

The position of the project team in the Encouraging Persistence Maintaining Challenge project (EPMC; Sullivan et al., 2014; Sullivan & Davidson, 2014) was the belief that everyone can learn mathematics. In addition, learning mathematics takes concentration and effort over an extended period of time to build the connections between topics, to understand the coherence of mathematical ideas, and to be able to transfer learning to practical contexts and new topics. The project team recognized that students need to be

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encouraged to persist, to concentrate, apply themselves, believe that they can succeed, and make an effort to learn. The project team developed tasks and lessons that were likely to foster such actions and called them challenging, in that they allowed for the possibility of sustained thinking, decision-making, and some risk-taking by the students. Teachers engaging students in appropriate tasks to develop mathematical reasoning was seen as critical (Anthony & Walshaw, 2009; Brodie, 2010).

In the EPMC project, learning was considered to be stronger if students connected ideas together for themselves, and determined their own strategies for solving problems, rather than following instructions they had been given. Essentially, the idea was for teachers to pose problems that the students did not yet know how to solve and to support them to find a solution. The project centred on problems and their effects in classrooms. The features of the challenging tasks (Sullivan et al., 2013) used in the collaboration reported here were based on those developed in the EPMC and they required students to:

- plan their approach, especially sequencing more than one step;
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways;
- engage with important mathematical ideas;
- choose their own strategies, goals, and level of accessing the task;
- spend time on the task and record their thinking; and
- explain their strategies and justify their thinking to the teacher and other students.

At least part of the challenge was the expectation that students:

- record the steps in their solutions;
- explain their strategies;
- justify their thinking to the teacher and other students; and
- listen attentively to each other.

Tasks that were seen to be best were those that address important mathematical ideas, are developmentally appropriate, and with which students can engage with minimal instruction. Such tasks create a challenge for students and require considerable persistence. Engaging with important and complex mathematical ideas requires sustained thinking and considerable effort. Students gain a sense of achievement from overcoming a challenge, and this satisfaction leads to improved self-concept (Sullivan et al., 2013). Others suggest anticipation of students misbehaving in their mathematics classrooms can cause teachers to reduce the challenge of the tasks (Stein, Smith, Henningsen, & Silver, 2009).

Many tasks in the project required students to make decisions on the solution type and solution strategy. The expectation was that students would try to find a solution by themselves rather than seek the support or direction from the teacher, particularly when the solution was not clear. The underlying expectation of the collaboration was that teachers foster persistence in their students. Teachers were encouraged to talk with students about the benefits of persisting and to affirm persistence when they identified it. Noticing persistence was only one of the noticing behaviours the collaboration expected of participating teachers. The teachers were also encouraged to anticipate their students' responses to the tasks based on what they had noticed about students' earlier approaches to the mathematical content. Teachers were also expected to notice the students' actions as they undertook tasks and to respond at the time, monitoring what the students were doing and noticing the mathematical connections students were making.

Several studies indicated that teachers often fail to see the mathematics content of a task from a student's perspective (Fernandez, Cannon, & Chokshi, 2003; Sullivan, Boreck, Walker, & Rennie, 2016), and focus on superficial features of a task or lesson during planning (Choy, 2013). Current research literature reports there are several facets of teacher noticing (Hoth et al., 2016). Examples include: attending to children's thinking, interpreting children's understandings, deciding how to respond on the basis of children's understanding (Jacob et al., 2010), perceiving particular events in an instructional setting, interpreting the perceived activities in the classroom, and decision-making either as anticipating a response to students' activities or as proposing alternative instructional strategies (Kaiser et al., 2013). Deciding how to respond in the moment and the teachers' decision making are what Miller (2011) referred to as situated awareness, and Mason (2011) described as noticing in the moment that enables teachers "to act freshly rather than habitually" (p. 48). The literature suggests a correlation between teacher noticing and use of enabling and extending prompts.

Method

The data were collected from 37 Year 3 to 6 primary school teachers who volunteered to be part of the EPMC collaboration in 2016. The collaboration was an offshoot of the original EPMC project. Participating teachers were provided with two days of professional development. The first day included an explanation of the rationale for the collaboration, the expectations of participating teachers, and an introduction of 15 tasks for potential inclusion in lesson sequences addressing the topic of multiplicative thinking. Each task was documented in a booklet, with the key understandings addressing the intended year level range, curriculum outcome, key mathematical language, pedagogical considerations, a possible introduction, enabling and extending prompts, consolidation tasks (designed for students who need further exploration of the mathematical ideas underpinning the initial task), worksheets, and solutions. The specific information dealing with prompts were:

> ENABLING PROMPT:
>
> Most lessons include a suggestion of an enabling prompt that can be posed to students who have not been able to make progress on the main task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts can involve varying an aspect of the task demand, such as the form of representation, the size of the numbers, or the number of steps. If students have success with the modified task, they can proceed with the original task.
>
> EXTENDING PROMPT:
>
> Some students might finish the learning task quickly. The intention is such students be posed "extending prompts" that extend their thinking on an aspect of the main task.

This information was provided to clarify, for teachers, the intention of the prompts in the task design. On the second day, teachers reported on their experience of implementing the tasks in their classrooms. At the beginning of each professional learning day, participants responded to an online (Qualtrics) survey that included open and closed items. Findings reported in this paper relate to the teachers' responses (Day 2) to the following questions pertaining to enabling and extending prompts, and teacher noticing:

> What were you looking for or thinking about when you chose to use an enabling prompt?
>
> What were you looking for or thinking about when you chose to use an extending prompt?
>
> What is something you noticed about students' mathematical thinking you had not noticed before?

The responses of 37 participating teachers were entered into a spreadsheet, coded, and then categorised through the analysis of data using a grounded theory approach (Strauss & Corbin, 1990).

Results and Discussion

The teachers' results to the open response enabling and extending prompts survey questions are presented in separate sections below, followed by a discussion of self-reported data related to the teaching noticing question.

Enabling Prompts

If a teacher mentioned multiple ideas, each idea was categorised as a separate response. Therefore, the total number of responses (n = 49) was larger than the number of participating teachers (n = 37). Table 1 shows the categories that emerged from the data for the survey question: What were you looking for or thinking about when you chose to use an enabling prompt?

Table 1
Categories of Teachers' Perceptions about Enabling Prompts (n = 49)

Category	Frequency
Potential to assist thinking – help struggling students	13
Ensure entry to problem – anticipate students who might struggle	11
Consider match of student understanding to the task	11
To support and engage – affective reasons	6
Experience success	4
Overcome barrier (students are "stuck" once engaged)	4

Taking the three most frequent categories of responses in Table 1 together paints a picture of the main perceptions teachers had in mind when they were considering enabling prompts. Teachers were aware of the potential of the prompts to help students who were struggling with the main task of the day. Some teachers said that they introduced the main task and watched student behaviours to identify those struggling to "get into the problem." It was at this stage of the lesson that the enabling prompts were used. For example:

> When giving an enabling prompt, I would look for children who were struggling to understand the processes needed to solve the problem.

It was interesting to see that teachers also thought about how to ensure that their students could tackle the main problem. They were not seeing the prompts as an alternative to the problem. They were looking at the way the enabling prompt would create a means of access to the task. For example:

> Something that would help them to do the main problem - usually smaller numbers but the same problem.

The four responses dealing with overcoming a barrier in thinking have connections here. For example:

> When students were "totally stuck", I gave them a simplification "clue" to get them started. Sometimes this was as far as they could go.

Teachers were also considering what they knew about their students' mathematical knowledge in advance of presenting the task and thinking about how the enabling prompt might bridge the two. For example:

> Does this student understand the concepts and do they have the capacity to make the connections they need?

Taken together, these responses show the teachers were thinking about ways to ensure that their students engaged with the main mathematics of the task. A further category of responses added to these findings, as the teachers mentioned affective issues and ways to support and encourage students. For example:

> I was looking for students who showed lack of confidence, students who demonstrated distracted behaviours (in my class they are the kids who don't like maths). How to give those children the confidence to tackle the problem.

These responses show that teachers think about students who cannot tackle the problem due to their attitudes to mathematics. The quoted teacher was aware that a lack of confidence might impact students' persistence with problem solving and that sometimes students who cannot do the problem can become disruptive in the classroom. As Doyle (1986) and Desforges and Cockburn (1987) noted, students sometimes resist tasks that are high in cognitive demand by threatening classroom disorder. The anticipation of students misbehaving in their mathematics classrooms can cause teachers to reduce the challenge of the tasks (Stein, Grover, & Henningsen, 1996; Stein et al., 2009). Allied with teachers' awareness of student negative emotions is their wish to create positive successful mathematical experiences. Another teacher said enabling prompts could help as:

> Making it simpler would empower my students and give them a sense of success – [creating] some light bulb moments.

As can be seen from the responses, teachers were keen to keep their struggling students productively engaged and challenged and, as is reported in the next section, they also considered students who finished the task quickly.

Extending Prompts

The analysis of teachers' responses pertaining to the use of extending prompts fell into five categories (see Table 2). The responses were quite evenly distributed across the categories. The frequency of responses that includes the term *challenge* is not surprising, as it was in the title and purpose of the collaboration. Perhaps teachers were echoing the ideas presented to them when they considered ways in which extending prompts could help them to maintain the level of challenge. Alternatively, they were adopting an orientation to challenge students further.

An interesting category of responses related to teachers attending closely to each student's mathematical thinking, and whether an extending prompt was required. For example:

> [Students] that achieved all the possibilities and could explain their thinking and justify their answers.

The intention of extending thinking was mentioned. For example:

> Extending student understandings - challenge them and put them out of their comfort zone.

The idea that the extending prompt would ask students to apply their knowledge was seen in comments such as:

To see if students could apply their knowledge to other problems.

Table 2
Categories of Teachers' Perceptions about Extending Prompts (n = 33)

Category	Frequency
Taking cues from students and noticing students seeking challenge	8
Extending students' thinking	7
Challenging particular students	7
Matching tasks to student thinking	6
Asking students to apply their knowledge	5

These data raise questions about whether teachers clearly differentiated between extending prompts, which took the task beyond the initial task at its level of difficulty, and consolidation tasks, which were modifications of the initial task at the same level of difficulty. The purpose of the consolidation task was to reinforce or extend students' learning if they had difficulty with the main task of the day (Sullivan et al., 2016).

Teacher Noticing

As reported in Table 2, the teachers took cues from the students and noticed those students who needed an extending prompt. In other words, the teachers noticed the behaviours of their students in the moment in their classroom and decided how to maintain the mathematical challenge. This behaviour, termed by Sherin, Jacobs, and Philipp (2011) as adaptive and responsive teaching, is considered a critical teaching skill.

Another survey question asked: What is something you noticed about students' mathematical thinking that you had not noticed before? Three teachers responded:

> The multiple ways of thinking. Some particular students surprised me with their strategies. This was interesting as it led our discussion.
>
> [Students] tended to get better at it as they completed more tasks. At first they would only solve problems one way but then they realised that there was more than one way. This led to the students looking at different ways without being prompted.
>
> They can think quite differently and have the ability to present various ways of solving a problem.

The teachers, as illustrated by these quotations, noticed a range of strategic thinking, problem solving, resilience, flexibility, and variation in students' mathematical thinking. They also came to recognise that all students can solve challenging problems. Teachers also considered what they noticed in the classroom and reflected on its importance in building their knowledge of the learners and the learning.

Teachers' awareness of the necessity to match the task to the mathematical needs of their students was clear. Self-questioning was reported by one teacher who said, "Was this too easy? Does this extend their mathematical understandings? Does this build their capacity to persist?" There was a shift in the teacher's thinking from considering whether the student could complete the task to whether the task challenged the student's thinking and required persistence. It seems that the prompts served the purpose of stimulating the teachers to consider individual students' thinking and knowledge in relation to the proposed challenging task, to think in advance, and to predict whether the task was a good "fit" for students. This action was seen by Mason (2011) as preparing to notice.

One outcome of this collaboration suggesting teacher change was summarised by the following reflection:

> I have started to give the students the task (on paper) and give them time to figure out what to do without me telling them what to do or how to do it. Giving students enabling/extending prompts has helped their understanding as they are still completing the same tasks - it means I don't have to plan three or four different tasks in one lesson. Getting the students to explain their answers and reasoning has been great. It allows the other kids to hear from one of their peers what I would usually tell them.

This teacher realised the importance of holding back, allowing the students to lead the discussion, which is an example of deciding how to respond in the moment, or situational awareness (Miller, 2011). In addition, teachers were aware that if they noticed students who were "stuck" or who "breezed through" the problem, they had prepared prompts that were designed to help those students.

Conclusion

In relation to the research questions, the findings suggest that the teachers saw value in using enabling and extending prompts to support student learning when one substantial problem is posed to all students. These findings support earlier studies (e.g., Hoth et al., 2016; Sullivan et al., 2006). In our study, the prompts had at least two positive effects on teachers' pedagogies: They allowed teachers to differentiate their teaching and they stimulated teachers' noticing of students' reasoning, strategic thinking, and mathematical communication. However, there appeared to be limited teacher perceptions and understanding of the purpose of the consolidation tasks and extending prompts.

Teachers were also aware of noticing when responding to the interview questions, and some teachers reported that noticing was a difficult component of the lessons. However, the reality of their classrooms required participating teachers to notice the meaningful features of the classroom situation and figure out what to do in the moment. Further studies including lesson observation would provide richer insights into teacher noticing and how this expertise can be fostered. Deciding how to respond in the moment and being aware of student learning is a component of teacher noticing (Stein et al., 2009) but, as the findings of our study suggest, it remains complex and is challenging for teachers to master.

The two days of the research project and professional development suggest that the teachers were able to gain knowledge for implementing the lesson sequence of challenging tasks. There is further research to be done in terms of teachers' ongoing adoption and adaptation of the use of enabling and extending prompts. The extent to which teachers of mathematics can use the idea of prompts as a differentiation technique and devise enabling and extending prompts when teaching with challenging tasks is yet to be fully investigated.

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The Impact of a Measurement-Focused Program on Young Children's Number Learning

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Children form mathematical concepts at an early age, and many of these concepts are linked to informal measurement experiences. However, mathematics education at school is often focused on counting and numbers. A mathematics intervention using a measurement-focused program replaced the usual mathematics program for 40 children entering their first year of school. The results of Counting and Place Value interviews held at the beginning and end of the school year are reported. Findings indicate that a "student-active" measurement-focused program can stimulate the development of children's number knowledge; however, additional counting may benefit children's number skill development.

Children acquire considerable mathematical knowledge before they enter school (Clarke, Clarke, & Cheeseman, 2006). This knowledge is built through experience and exploration in meaningful life contexts. Children's lives are rich in problem solving where children make decisions about number, position, and size in authentic measurement contexts (Clements & Sarama, 2011). In contrast, the mathematics that many children experience on entry to school is heavily number-focused (Benz, 2012). Young children have intuitive and informal capabilities in both spatial and geometric concepts, and numeric and quantitative concepts (Bransford, Brown, & Cocking, 1999). As Clements and Sarama (2011) noted, "children must learn to mathematize their informal experiences by abstracting, representing, and elaborating them mathematically" (p. 968). We believed that children could learn number more meaningfully through a measurement-centred curriculum where authentic experiences required children to solve problems. In this study, we sought to answer the question: What is the impact of a mathematics program based on measurement activities on young children's number learning?

Background

The idea of using a measurement-focused curriculum is not new. In Russia, a curriculum developed by Davydov, Gorbov, Mukulina, Savelyeva and Tabachnikova (1999) was based on findings by Davydov (1975). These authors developed a mathematics program without numbers where the physical attributes of objects were described and compared. It was intended that children should develop, through different activities, an understanding of equality and be able to describe comparisons with relational statements. Davydov described a comprehensive theoretical progression of children's thinking about measurement concepts which was implemented and tested in the Hawaiian program *Measure Up* (Dougherty & Zilliox, 2003). The project started with a generalized approach and then applied the knowledge gained to specific cases. The team worked on ways to deliver the theoretical Russian approach in classrooms. The project identified at least six types of instruction: (1) giving information, (2) simultaneous recording, (3) simultaneous demonstration, (4) discussion and debriefing, (5) exploration guided, and (6) exploration unstructured. The order (1-6) represented a continuum from most teacher-active to most student-active. Sophian (2007) maintained that the relationship between measurement concepts and proportionality supported children to develop deep understandings of

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mathematical structures and properties of number. However, the Russian curriculum approach was not universally admired. It was criticised for its abandonment of numbers, its use of letters as variables, and its focus on early abstraction and generalization (Freudenthal, 1974; Steinweg, 2013).

A contrasting view of the relationship between number and measurement was conceived by Steffe (2010), who viewed discrete quantities and continuous quantities as connected. He named his four counting schemes as discrete quantitative measuring schemes and described four stages: (1) the perceptual counting scheme, (2) the figurative counting scheme, (3) the nested number sequence, and (4) the explicitly nested number sequence. For Steffe, the development of awareness of continuous quantities was analogous to his four stages of an awareness for discrete quantities.

Sophian (2007) questioned the common perspective that "children's thinking begins with the premise of counting, or some form of determining the numerical values of discrete quantities, [and] is the foundation for much of children's developing knowledge about mathematics" (p. 3). She described a contrasting position "that what is most fundamental for mathematical development is not counting or other mechanisms for apprehending numerosity, but rather basic ideas about relations between quantities" (p. 3). It is this comparison-of-quantity perspective that informed the present study.

Clements and Sarama (2009) wrote that "measurement can be defined as the process of assigning a number to a magnitude of some attribute of an object, such as its length, relative to a unit. These attributes are continuous quantities" (p. 163). This definition emphasises the links between number and measurement and the commonalities between discrete and continuous quantities. It was used in the present study.

The intended curriculum (Van den Akker, 2003) is documented in the Australian Curriculum: Mathematics (AC:M; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012), which contains the content strands Number and Algebra, Measurement and Geometry, and Probability and Statistics. The strands are intended to be integrated in practice together with the proficiencies: understanding, fluency, problem-solving, and reasoning. Curriculum outcomes for number and place value in the first year of school (Foundation) focus on counting numbers to 20, counting and subitising small collections, and comparing collections. Measurement outcomes specify "Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning" (ACARA, 2012). While the national curriculum has been implemented since the original comparative data used in this paper were collected, the Victorian curriculum, at the time, specified very similar learning outcomes for students in their first year of school. In this study, we used the Australian Curriculum and investigated a change of emphasis in the implemented curriculum from number to measurement.

Method

A year-long project was conducted with children five to six years of age entering school in Victoria to examine the impact of a mathematics program based on measurement activities on young children's number learning. The implemented curriculum in this experiment emphasised measurement, and de-contextualised number was not taught at all – Number was used to quantify attributes.

Detailed planning was undertaken by two teachers who worked with the 40 children. The program was designed for a school with a commitment to a Reggio Emilia philosophy of education (Rinaldi, 2006) in which the second author (Yianna) worked at the time. The school offered opportunities for children to learn through problem solving, and they were expected to engage with mathematical thinking every day; teachers were expected to interact with children to challenge and extend their thinking. The measurement-based program was characterised as being “student-active” (Dougherty & Zilliox, 2003), where the planned experiences involved unstructured or guided exploration by the children. Daily mathematics focused on measurement tasks and problems. The classroom observations later in this paper illustrate such problems. Teachers created detailed planning documents and observational records were kept of the children’s actions and ideas. Teachers regularly met to discuss the mathematical learning of individuals and the group as a whole.

The study was characterised as design research because it was interventionist, iterative, practical in a real context, and process-, utility-, and theory-oriented (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). The merit of the program was evaluated by looking at the learning outcomes of the children. We report results of analysis of interview data related to Counting and Place Value together with two classroom observations.

The interview protocol used was developed in the Early Numeracy Research Project (ENRP; Clarke et al., 2002). Space constraints allow only the reporting of Counting and Place Value results. The data from the original large-scale research project are used here as reference points in the analysis of the current data. In the present study, the second author interviewed the children at the start of the year, and an independent trained interviewer conducted the end-of-year interviews. All student responses were independently coded as numerically representing the Growth Points of a theoretical framework according to the coding protocols developed by Clarke and his colleagues.

The mean Growth Point (GP) codes of the children entering school for the first time (five- to six-year-olds) at the beginning and the end of their first year at school were calculated for Counting and Place Value. These means were then compared to the three cohorts of reference school (control) data in the ENRP (Clarke et al., 2002) because for three consecutive years data were collected from a representative range of schools matched to the research schools. Reference schools received no experimental treatment and therefore could be considered to represent children on entry to school and at the end of the first year of school in Victoria. From this large dataset, a matched sample was constructed for Yianna’s data. Her school had been a reference school for the original research project. The assessment instrument was identical to that used with Yianna’s students, and the interview was conducted by an independent trained interviewer.

Findings

ENRP Growth Points were defined to describe children’s developing understanding of each mathematical domain. Here, only the two domains will be used. The Counting Growth Points (Figure 1) described the development of children’s counting by ones, as well as by the multiples of two, five, and ten. The Growth Points were concerned with children’s production of number name sequences. However, the Growth Points were also concerned with children making the count-to-cardinal transition described by Fuson (1982) as being able to think about the number sequence to solve problems.

The Growth Points in Counting were devised to articulate the key steps taken by children in developing their understanding of the number sequence. However, these Growth Points do not describe children’s use of counting in addition, subtraction, multiplication, and division problem solving situations. Such strategies are described in separate domains. Only the relevant growth points in the Number domains will be presented in the analysis of the results.

0. Not apparent: *Not yet able to state the sequence of number names to 20.*
1. Rote counting: *Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.*
2. Counting collections: *Confidently counts a collection of around 20 objects.*
3. Counting by 1s (forward/backward, including variable starting points; before/after): *Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.*
4. Counting from 0 by 2s, 5s, and 10s: *Can count from 0 by 2s, 5s, and 10s to a given target.*
5. Counting from x (where x > 0) by 2s, 5s, and 10s: *Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.*
6. Extending and applying counting skills: *Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.*

Figure 1. ENRP Counting Growth Point framework (Clarke et al., 2002).

The mean Growth Point results in Counting and Place Value are presented in Table 1, where the original cohorts for three years are labelled C1, C2, and C3, and Yianna's children are labelled YP. A matched school dataset is reported as MS. These data were collected in the same school over the original three years of the ENRP via a random sampling of the students. The MS students followed the intended curriculum at the time.

Table 1
Comparison of Mean Growth Point Codes for Counting and Place Value

	Counting		Place Value	
	Mar.	Nov.	Mar.	Nov.
C1 (n = 438)	0.78	1.80	0.36	0.93
C2 (n = 504)	0.86	1.74	0.36	0.98
C3 (n = 523)	0.88	1.83	0.34	0.99
YP (n = 40)	0.33	0.93	0.34	0.95
MS (n = 51)	0.42	1.49	0.29	0.88

An examination of the comparative results (Table 1) reveals that Yianna's children came to school with counting knowledge not as sophisticated as most children involved the original research. On average, Yianna's students began school unable to recite the number names to 20 (mean GP 0.33) and by the end of the year, they had improved their rote counting skills less than the control groups (mean GP 0.93). The mean for the state was almost at Growth Point 2, where the child can reliably count a collection of around 20 objects. By the end of the first year of school, on average, the children in this study could recite the number names to 20 but not reliably count collections of 20 objects.

Matched Group Comparisons in Counting

The results of the experimental group (YP) compared to the matched school data (MS) in Counting (Table 2) show that the two groups began the year with very different percentages of children achieving Growth Points 0-1 Almost three-quarters of YP children

(73%) were unable to say the number name sequence to 20. Only 10% of YP children could count a collection of over 20 objects. Clarke, Clarke, and Cheeseman (2006) reported that 39% of Victorian children achieved this skill. The matched control group had 36% at GP2 or above, indicating that they were more aligned with the broader population on entry to school. The poor counting knowledge on entry to school of Yianna's group was thought by their teachers to be due to a combination of factors: high Language Background Other Than English, high non-attendance at pre-school, low socio-economic background, and a large proportion of newly-arrived migrant families. In part, these factors were a stimulus to trying a different approach to mathematics for the children as a focus on counting had limited success in previous years.

Table 2
The Numbers of Children at Each Growth Point in Counting

	YP Counting		MS Counting	
Growth Point	Mar. *n* = 40 (%)	Nov. *n* = 40 (%)	Mar. *n* = 65 (%)	Nov. *n* = 48 (%)
0	29 (73)	14 (36)	36 (55)	7 (15)
1	7 (17)	15 (38)	6 (9)	6 (13)
2	4 (10)	7 (18)	9 (14)	33 (68)
3	0	2 (5)	14 (22)	1 (2)
4	0	0	0	1 (2)
5	0	1 (3)	0	0

The greatest differences exist between the two groups in the end-of-year results. Yianna's children had learned to count in a measurement-focused curriculum but 14 (36%) remain on GP 0, not yet able to state the number names to 20. A further 15 (38%) could verbally count but were unable to count a collection of objects. Only 10 children could count reliably (26%). The two children who could count forwards and backwards from various starting points between 1 and 100, and who knew numbers before and after a given number, exhibited knowledge described in the AC:M as Year 1 outcomes. One student achieved GP 5, showing the ability to count from a non-zero starting point and to count by twos, fives, and tens to a given target (AC:M Year 2). In contrast, at the end of the year, most children in the control group (MS) could reliably count collections – GP 2 (68%).

Place Value

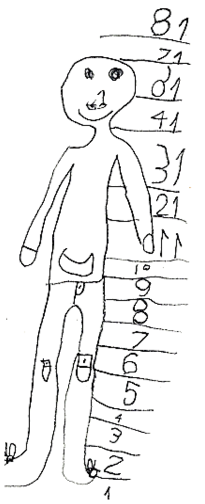
An examination of the beginning and end-of-year data of each of the cohorts for Place Value (Table 1) reveals very similar patterns of results between the three cohorts and the matched sample and the experimental group. By the end of their first year of school, the children interviewed demonstrated that they had a sound understanding of single digit numbers. This relates to GP 1 in the domain of Place Value: Reading, writing, interpreting, and ordering single-digit numbers.

Looking at matched group comparisons in Place Value (Table 3) shows patterns of results in the experimental (YP) and control (MS) groups of Place Value that are very similar. The difference in these data is that by the end of the year, a total of 16% of the children in the measurement-based curriculum group (YP) had extended their knowledge of numbers and the number system beyond single-digit numbers, reading, writing, and interpreting two-digit (GP1) and three-digit numbers (GP2) successfully. Place Value knowledge indicates a developing awareness of the number system as a whole. The "top" (8%) of the experimental group had children who had mastered reading, writing, and interpreting three-digit numbers. This could be attributed to the need to use larger numbers in the measurement context or the removal of a "ceiling effect" of the intended curriculum.

Table 3
The Numbers of Children at Each Growth Point in Place Value

	YP Place Value		MS Place Value	
Growth point	Mar. n = 40 (%)	Nov. n = 40 (%)	Mar. n = 51 (%)	Nov. n = 48 (%)
0	29 (73)	13 (32)	38 (75)	14 (29)
1	10 (25)	21 (52)	13 (25)	29 (60)
2	1 (3)	3 (8)	0	5 (11)
3	0	3 (8)	0	0
4	0	0	0	0
5	0	0	0	0

An examination of the interview results tells only part of the complex story of the intervention. Daily classroom observation data, collected during the measurement-focused program, add to the picture. Two observations about length will illustrate some of the children's thinking.



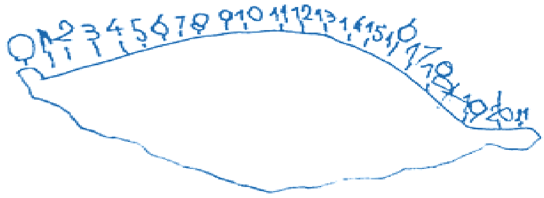
Classroom data: Mitchell's height. In their first week at school, children were invited to consider: How tall are you? How big are your feet? How much do you weigh? These invitations to explore were posted in the mathematics corner of the classroom together with a height chart (in centimetres), scales, Unifx cubes, wooden sticks, assorted blocks, a measuring tape, and a 1 metre ruler.

Mitchell (5 years) (pseudonym) was observed sitting, pencil and paper in hand, looking very busy. School had not officially begun for the day, but Mitchell had decided he would to draw himself against a height chart because he said that he already knew how tall he was.

"Look I'm 17 tall, see, not 18, because the line to my head is at 17." As Mitchell's drawing shows, on entry to school, Mitchell understood that measuring height uses numbers and he knew the number sequence to 17 with only one number missing. For him, the 17 line matched his height. Despite the absence of a 15 in his number sequence, he drew and labelled spaced intervals to illustrate his understanding that he was 17 tall. His counting began at one, and the origin of his measure is not drawn. The labels show that he understood that the count referred to the length of each interval. He carefully matched the line for 17 to the top of his head in the drawing, showing accuracy and an awareness of the end point of the measure. While it is not clear where the notion of 17 arose, Mitchell had in his mind that he was 17 tall and he could clearly show what this meant to him using an approximation of a number line.

Classroom data: Measuring the autumn leaves. The following conversation between Eliza (pseudonym) and her teacher shows how children's attention was naturally drawn to salient features of length measurement.

Eliza: "I don't know how much this leaf is. …..It's 300."
Looked at the millimetres on the ruler she had placed along the length of the leaf
T: "Where did you start your measuring?"
Eliza: "Two."
T: "Why two?"
Eliza: "I don't know. I think I should start at zero."
T: "Where is zero on your ruler?"
Eliza pointed to the zero
T: "So, can you start measuring from zero? Does that help?"
Eliza: "Yes."
T: "Can you measure from the start of your leaf?'
Eliza: "Yes. I will draw a line so I know where the end is."
Eliza traced the outline of the leaf
T: "What are you doing with your leaf?"
Eliza: "I'm putting numbers on something."
T: "What are the numbers for?"
Eliza: "Measuring something. Anything – like shelves, chairs, and leaves."
T: "So why are you putting the numbers along the leaf?"
Eliza: "To know how much the leaf is."



Eliza knows the purpose of numbers in quantifying a length although as yet she has no apparent concept of unit. Her awareness of the origin of the measure was raised in conversation, the number zero became a meaning for her in this context.

Discussion

In an attempt to find a way to make mathematics meaningful for children, a measurement-focused program replaced a counting-based program. The results show that learning achievements in Counting were negatively impacted. More than half of the children in the experimental group were not counting collections reliably by the end of their first year of school. This finding suggests that children entering school – at least those with poor English language skills and a low socio-economic background – seem to need explicit counting practice with the sequence of number names to 20 in addition to experiences with number in measurement contexts. Perhaps the theoretical connection between discrete quantities and continuous quantities as described by Steffe's (2010) counting schemes could also be made more explicit to teachers.

The Australian Curriculum: Mathematics has a Number and Place Value strand at Foundation level that lists five outcomes where counting as quantifying is emphasised. While counting is a key to young children's mathematical futures, children need more than repetitive counting routines and learning through skill-driven tasks (Mulligan & Mitchelmore, 2009). The learning contexts of the measurement-focused program offered children many opportunities to use numbers in relevant and meaningful ways. Children's development of a sense of the number system as reflected in the Place Value results.

The genuine measurement context for number stimulated some children to go beyond the intended curriculum (AC:M) in measurement outcomes. These children could use units to measure and quantify, whereas the intended curriculum specifies only the use of direct and indirect comparisons. This study supports the findings of recent studies (Cheeseman, 2012; Macdonald, 2011; McDonough, 2011) that show that young children are capable of more sophisticated concepts of measurement than the AC:M (2012) specifies.

An implication of this study is that measurement contexts can be productively used with young children to stimulate number knowledge and reasoning.

Like Sophian (2007), we question the perspective that mathematical basis of children's thinking is counting or determining the numerosity of discrete quantities. This study supports a comparison-of-quantity perspective described by Sophian as an effective approach for young learners of mathematics. We are not advocating eliminating counting; we are advocating using counting to determine numerosity in measurement contexts. This study has shown that, in principle, when young children measure, they use numbers in meaningful ways and can acquire number competencies.

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Snapshots of Productive Noticing: Orchestrating Learning Experiences Using Typical Problems

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In this paper, we re-examine the commonly-held notion that typical problems, such as textbook exercises and examination questions, are not useful for orchestrating mathematically-rich learning experiences. Drawing from a larger design-based research project, we present a case study of Alice, a secondary school teacher, who orchestrated a productive discussion by using examination questions. We describe how she perceived and harnessed the affordances of such typical problems before and during her lesson. Findings suggest teacher noticing as a key mechanism to enable teachers to unlock the mathematical potential of such problems.

In Singapore, there has been a recent emphasis on orchestrating learning experiences, "the interaction between the learner and the external conditions in the environment to which he can react" (Tyler, 1949, p. 63), to develop mathematical thinking. We see students' learning experiences as their engagement with mathematical tasks selected, adapted, or designed by their teachers. Orchestrating learning experiences involves teachers providing "opportunities for students to discover mathematical results on their own" or "work together on a problem and present their ideas using appropriate mathematical language and methods" (Ministry of Education-Singapore, 2013, p. 20). Teachers are thus expected to select, adapt, or design tasks and orchestrate learning experiences by engaging students in mathematical activities through these tasks. However, this can be challenging for teachers, considering that it is not just the design of mathematics tasks, but also how these tasks are implemented in the classrooms that matters (Smith & Stein, 2011; Sullivan, Clarke, & Clarke, 2013; Tyler, 1949). We present preliminary results from an ongoing study on orchestrating learning experiences in Singapore. Specifically, we present snapshots of a teacher's productive noticing as she orchestrated classroom learning experiences using tasks developed from typical problems.

Typical Problems: An Untapped Resource

Mathematically-rich challenging tasks as an important vehicle for orchestrating learning experiences (Smith & Stein, 2011; Sullivan et al., 2014). For instance, Smith and Stein (2011) argue that tasks which are of a higher cognitive demand form the basis for engaging students in doing mathematics. Similarly, Sullivan et al. suggest that students' learning experiences are enhanced when they "devise their own methods of solution at least some of the time" (p. 124) to challenging tasks. Despite the affordances of challenging tasks in enhancing learning experiences, there are at least three obstacles that hinder the prevalent use of these tasks in the classrooms: (1) These tasks may be too difficult for many students, and so additional prompts or supports are needed (Sullivan et al., 2014), (2) It is time-consuming for teachers to select, adapt, or design challenging tasks to use, and (3) The inherent complexity of the tasks would involve mathematics from across the curriculum and is best implemented across several lessons, or after a few topics are taught. These demands on teachers' knowledge and time may limit the incidence of such tasks. Moreover, teachers are mindful about the concurrent need to develop

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procedural fluency in students as part of their preparation for tests and examinations. Thus, it is common to use examination-type questions with a more teacher-centered teaching approach in Singapore classrooms (Foong, 2009; Ho & Hedberg, 2005). This preference for using typical problems (standard examination or textbook problems) may reflect teachers' belief that it is "important to prepare students to do well in tests than to implement problem-solving lessons" (Foong, 2009, p. 279), a classroom reality that cannot be ignored.

Here, we define typical problems as standard examination-type questions or textbook-type questions which focus largely on developing procedural fluency and at times, conceptual understanding (e.g., see Figure 2). These questions can be solved more quickly than challenging tasks, and are used frequently in mathematics lessons. Given the omnipresence of such questions in textbooks and other curriculum materials, we see typical problems as an untapped resource that can be used to orchestrate daily learning experiences. Using tasks developed from typical problems to orchestrate learning experiences would position mathematical learning experiences as an integral part of mathematics lessons, and not just reserved for occasional "enrichment" lessons. Preliminary findings suggest that experienced teachers not only see the development of procedural skills with such questions, but also adapt them for different student profiles to develop conceptual understanding. We present Alice as a case study of an experienced teacher to offer insights into how noticing the affordances of a typical problem is critical for orchestrating learning experiences.

Productive Mathematical Noticing

Mathematics teacher noticing is the emerging construct that lies at the heart of these components of teaching expertise. It refers to what teachers attend to and how they interpret their observations to make instructional decisions, or to have a different act in mind (Mason, 2002; Sherin, Jacobs, & Philipp, 2011). Several studies have demonstrated the importance of teacher noticing in enhancing teachers' reflection on teaching to improve their practices (Goldsmith & Seago, 2011; Star, Lynch, & Perova, 2011); while others have highlighted how teacher noticing can enable teachers to respond to students' thinking during lessons (Choy, 2014). More recently, Choy (2016) examines what and how teachers notice when they design tasks to enhance students' reasoning. Even though noticing appears to be part and parcel of teaching, what and how teachers notice may not always lead to mathematically productive instructional decisions. That is, not all noticing is productive.

The FOCUS Framework (Choy, 2015) characterises productive noticing in two ways: having an explicit focus for noticing and focusing noticing through pedagogical reasoning—how teachers justify their instructional decisions or claims about students' thinking using what they attend to. Choy suggests that an explicit focus is useful for supporting teachers to notice relevant instructional details. There are two key aspects of this explicit focus: First, the three components of the didactical triangle, namely the mathematics concept, students' confusion associated with the concept, and teachers' course of action to address students' confusion. Second, the alignment between these three components, that is, whether the teacher's course of action targets students' confusion when they are learning the concept. This alignment between teachers' instructional decisions and students' confusion is not intuitive. Instead, this alignment is mediated by teacher's pedagogical reasoning.

In this paper, we build on extant research on teacher noticing by examining what and how a teacher noticed when orchestrating learning experiences. We focus on what the teacher attended to in relation to the interactions between students, content, and the task. These interactions can be visualised as a socio-didactical tetrahedron (Rezat & Sträßer, 2012). In Figure 1 (left), we follow Rezat and Sträßer (2012) in seeing each face of the tetrahedron as an instantiation of the relationship between a task and mathematics education. For example, the task-students-teacher face represents the interactions that occur amongst teacher, students, and the task. Given our emphasis on teacher noticing, we put "Teacher" as the apex, as seen in Figure 1 (right) to reflect our focus on how the teacher managed these interactions. This paper is framed by the question: How does a teacher's productive noticing of these didactical interactions help in orchestrating learning experiences with typical problems?

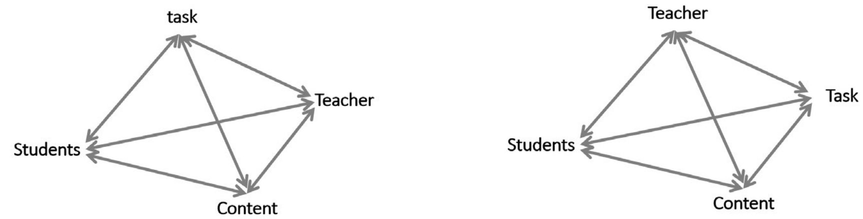

Figure 1. Socio-didactical tetrahedron for using task to orchestrate learning experiences.

Methodology

This ongoing project has adopted a design-based research paradigm (Design-Based Research Collective, 2003) to develop a toolkit to support teachers in noticing and a theory to describe their noticing when orchestrating learning experiences. We engaged in three iterative cycles of theory-driven design, classroom-based field testing, and data-driven revision of the Mathematical Learning Experience Toolkit (MATHLET) to provide a theoretical justification for the analytical frameworks on which the toolkit is based. By engaging with our teacher participants in designing, implementing, and reviewing learning experiences using the MATHLET, we aimed to develop a deeper theoretical understanding of how teachers orchestrate mathematically meaningful learning experiences. Four experienced mathematics teachers from three secondary schools, with different achievement bands and demographic factors, participated in this study. Each teacher designed and implemented a lesson during each design-cycle phase using the MATHLET, which resulted in 12 design cycles in total. Data were generated through voice recordings of planning discussions, pre-lesson discussions, post-lesson discussions, video recordings of lessons, and lesson artifacts. Findings were developed using a thematic approach (Bryman, 2012) together with the two characteristics of productive noticing as proposed by the FOCUS Framework (Choy, 2015).

Snapshots of Productive Noticing

Alice and the Context of Her Snapshots

Alice (pseudonym) is a Senior Teacher at a government-funded school with above-average performance in the national examinations. She has a strong mathematical background and has been actively involved in mentoring novice teachers. We present an analysis of Alice's lesson on matrices for Secondary 3 (Grade 9) students. The syllabus document encouraged teachers to provide students opportunities to apply matrix multiplications to solve contextual problems, and for them to justify if two matrices can be multiplied by checking the order of the matrices. Prior to this lesson, her students had learnt how to multiply two matrices. For the lesson, Alice modified a typical problem and used it in an introductory task to orchestrate a mathematically productive discussion. Students then worked through a sequence of typical problems and presented their answers. Referring to Figure 1, we focus on Alice's instrumentalisation (Verillon & Rabardel, 1995) of the task using a typical problem in relation to the mathematics (teacher-task-content face), her use of the task with students (teacher-student-task face), and her attention to students' thinking about the concepts (teacher-student-content face). In managing these sets of interactions, Alice demonstrated productive noticing of the mathematics to be taught, her students' learning, and the affordances of the typical problem.

Alice's Instrumentalisation of a Typical Problem

For her introductory task, Alice selected a standard examination question (See Figure 2) which comprised of two parts. The first is a routine matrix multiplication involving pre-multiplying a 3×1 matrix by a 2×3 matrix to obtain a 2×1 matrix $\begin{pmatrix} 185 \\ 184 \end{pmatrix}$ as the solution; the second part asked for the meaning of the product in the context. Many teachers would have only focused on guiding students to solve the question as intended. However, Alice seemed to notice the mathematical affordances. During the post-lesson interview, she explained:

> Why I choose this question is because most of the exam style questions are based on solving problems involving matrices. And this question will extend their thinking and help them to transfer mathematical ideas into other representations. This is what I find challenging amongst some students…

Alice highlighted that the question could potentially extend students' thinking in terms of expressing the same information in using different representations involving matrices or otherwise. Students often do not have opportunities to express information using different matrices as the matrices are usually given in the question. As a result, they have limited opportunities to see the connections between arithmetic and matrix multiplication. To achieve her objectives, Alice modified the typical problem as follow:

> Teresa and Robert attend the same school. They keep a record of the awards they have earned and the points gained. Teresa obtained 29 Gold, 10 Silver, and 5 Bronze awards. Robert obtained 30 Gold, 6 Silver, and 8 Bronze awards. They gained 5 points from each Gold award, 3 points for each Silver award, and 2 points for each Bronze award. Find the total number of points that Teresa and Robert gained.

We note that Alice did not include any matrix in her modified question, and this expanded the solution space of the original question. For example, students could solve the problem using arithmetic without matrices. Alternatively, students who see the problem as a matrix multiplication problem would first need to formulate the matrices before deciding on the order of matrix multiplication. This provided opportunities for Alice to emphasise the connections between matrix multiplication and arithmetic which could potentially provide meaning to matrix operations. In so doing, she attempted to develop both conceptual understanding and procedural fluency in the matrix operations.

Example 1

[Nov 2013] Teresa and Robert attend the same school. They keep a record of the awards they have earned and the points gained. The matrices show the numbers of awards and the points gained for each award.

$$\begin{array}{c} \\ \text{Teresa} \\ \text{Robert} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Gold} & \text{Silver} & \text{Bronze} \end{array} \\ \begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Gold} \\ \text{Silver} \\ \text{Bronze} \end{array} \begin{array}{c} \text{Points} \\ \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$.

(b) Explain what your answer to (a) represents.

Figure 2. The introductory problem (original form).

Moreover, Alice highlighted that she chose an “easier” question with a relatable context to start the lesson. Alice also revealed that she considered what students had found confusing from her analysis of students’ errors committed in past year examinations. She weaved these questions as a sequence of tasks in her worksheet. Therefore, her noticing is classified as productive according to the FOCUS Framework: Alice attended to the key concepts in matrices, recognised students’ possible confusion from their mistakes in past year examinations, and modified a typical problem and used it together with a carefully constructed sequence of questions to design a task that addresses students’ confusion.

Alice’s Orchestration of Discussion Using a Typical Problem

Alice demonstrated her recognition of the typical problem’s affordances as she orchestrated discussions during the lesson:

1 Alice: (Walks around the class and comes to Student S1.) Can you write this for me on the board?

2 S1: Ok. (Walks to the whiteboard and writes the following:
$T = 5 \times 29 + 3 \times 10 + 2 \times 5 = 185$
$R = 5 \times 30 + 3 \times 6 + 2 \times 8 = 184$)

3 Alice: (Walks around while waiting for Student S1 to finish writing.) Ok. Most of you have written what [Student S1] has written. 5 points for 29 gold, 3 points for 10 silver and 2 points for 5 bronze. Most of you have written in this manner. The last few days, we have been talking about matrices, right? Would you like to convert this to a matrix problem?
(Looks at Student S2) Have you written it in matrix form? (Student S2 nods and Alice goes over to look at his answers.) Okay. Can you write your answer on the board?

4 S2: (Walks to the board and writes the following.)

$$T = \begin{pmatrix} 29 & 10 & 5 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 185 \text{ and } R = \begin{pmatrix} 30 & 6 & 8 \end{pmatrix} \times \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = 184$$

5 Alice: Any other answers from [Student S2’s] answer? (Walks around the class and selects Student S3’s answer) Can you write this on the board?

6 S3: (Walks to the board and writes the following.)

$$\begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 29 \times 5 + 10 \times 3 + 5 \times 2 \\ 30 \times 5 + 6 \times 3 + 8 \times 2 \end{pmatrix} = \begin{pmatrix} 185 \\ 184 \end{pmatrix}$$

7 Alice: Thank you all three of you. [Student S1] has written using an arithmetic method. Most of you have written in this manner. This one comes very naturally to you, ok? [Student S2] has written Robert and Theresa’s award separately. He has tried to use the matrix method, (points to Student S1’s solution.) Something like this, ok? Let’s check whether the order of matrix is correct or not.
(Alice goes through the method of matrix multiplication and gets the class to check the order of Student S2’s matrices.)

… Ok. Student S3 has written Robert's and Theresa's together so that you only write this matrix once (points to the column matrix [5 3 2]). Don't need to write two times, correct or not? See. Over here. You have to write two times but here, [Student S3] only has to write it once. Let's check the order again…

8 Alice: (After a short time) I would like to bring this problem a little bit further. Notice that Student S3 presented the information this way. Is there another way to represent the same information?

(After some time, Student S4 highlights another possible way.)

Here, we see how Alice orchestrated a mathematically productive discussion (Smith & Stein, 2011). Alice carefully attended to students' answers before she asked for volunteers during the whole class discussion. However, it can be inferred that she was deliberate in her selection and sequencing of students' responses (See Lines 1 to 6). By beginning with an arithmetic solution, Alice connected Student S1's arithmetic operations to matrix multiplications through the sequencing of Student S2's and Student S3's matrix solutions. The reason for using a single matrix multiplication (Student S3's solution) was also made explicit when Alice moved from Student S2's solution to Student S3's using a matrix approach (Line 7) before she highlighted the different ways to express the given information as matrices (Line 8), which was an important idea for the lesson. As highlighted during her post-lesson interview, she knew "that certain students will give these answers" and hence, we see that Alice had anticipated students' solutions before the lesson. Therefore, Alice's noticing is classified as productive because she attended to the different solutions and orchestrated the discussion to highlight the key mathematical ideas.

Alice's Response to Students' Thinking When Using Typical Problems

Another snapshot of Alice's productive noticing could be seen from how she responded to Jason's (a pseudonym) ideas when working on a matrix formulation problem (See Figure 3).

31 Alice: Now we have got the cost to these two outlets already, what about your deliveries? How can you represent your deliveries in a matrix? [Jason], how can you represent the deliveries?
32 Jason: 27 by 25.
33 Alice: 27 and 25, so how to write? Row or column?
34 Jason: Column, 1 by 2.
35 Alice: (Writes column matrix as suggested by Jason) You can write like that right?
36 Jason: Yeah.
37 Alice: You can also write like that. (Points to a row matrix.) Which one do you choose?
38 Jason: The first one (Referring to the row matrix).
39 Alice: The first one? Why the first one?
40 Jason: Depending on the position.
41 Alice: Depending on the position, so [Jason] come and write out for us, what are the two matrices you have chosen such that the product will give you the total cost?

In this excerpt, Alice attended to Jason's ideas (Line 31) and asked questions (Lines 33, 37, and 39) to reveal what Jason was thinking. This reflected Alice's attempt to gain an awareness of Jason's thinking. She refrained from evaluating Jason's incorrect answer (Line 34) and instead, prompted Jason through a series of questions. By engaging Jason to think about his answers, Alice could direct Jason's attention to the need to check the order of matrices. Alice's noticing was productive because she "asked questions to reveal" Jason's thinking (Lines 39 and 41) and built on his understanding (Choy, 2015).

A bakery produces 3 different types of bread : white bread (W), wholemeal bread (M) and multi-grain bread (G). Delivery is made to 2 distribution outlets in the following way:

Outlet A receives	60 loaves of W,	50 loaves of M,	30 loaves of G.
Outlet B receives	40 loaves of W,	70 loaves of M,	20 loaves of G.

The costs of one loaf of W, M and G are \$2.10, \$2.70 and \$2.90 respectively.

It is given that $\mathbf{P}=\begin{pmatrix}60 & 50 & 30\\ 40 & 70 & 20\end{pmatrix}$ and $\mathbf{Q}=\begin{pmatrix}2.1\\ 2.7\\ 2.9\end{pmatrix}$.

(a) (i) Evaluate **PQ**.
(ii) Explain what the answer in (i) represents.

(b) In a particular month, Outlets A and B receive 27 and 25 such deliveries respectively. Form two matrices so that their product will give the total cost of the bread delivered to the 2 outlets. Find the product.

Figure 3. Problem discussed by Alice and Jason.

Concluding Remarks

The three snapshots presented above described how Alice orchestrated the interactions within the teacher-task-content face, teacher-task-student face, and teacher-student-content face of the didactical tetrahedron. First, Alice was able to tap the mathematical affordances of typical problems, and modified them appropriately to address possible confusions faced by her students when learning matrices. Her ability to see and use typical problems beyond their current forms provide an existence proof for the use of such problems to orchestrate discussions. Alice's use of typical problems highlights the critical role of productive noticing in enabling her to do this work (Choy, 2016). More importantly, it goes beyond Choy's (2016) work on productive noticing involving a single task, and extends the study of noticing into the realm of using a sequence of typical problems to bring about mathematically productive learning experiences for students. As seen from these snapshots, Alice not only attended to the mathematical possibilities of typical problems, but also exploited these problems fully during her orchestration of the lesson. The snapshots suggest that Alice had a bird's eye view of how a sequence of tasks is embedded in a lesson, which in turn is embedded in a sequence of lessons within a unit. This highlights her noticing of mathematical connections between tasks, lessons, and units within the curriculum. At the lesson level, she demonstrated the five practices of anticipating, monitoring, selecting, sequencing and connecting during the whole class discussion (Smith & Stein, 2011). Her orchestration of the classroom discussion is reflective of her productive noticing according to the FOCUS Framework (Choy, 2015). This work highlights the mathematical potential of using typical problems to orchestrate learning experiences. Alice's orchestration of this lesson opens an avenue for mathematics educators to support teachers in noticing and harnessing the untapped potential of typical problems.

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The Argument from Matriculation Used by Proprietors of Victorian Secondary Schools Around 1900

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In this paper, we analyse data from the University of Melbourne's Matriculation examinations around 1900. The analyses reveal that many schools cleverly developed and applied strategies so that their Matriculation results would appear to be more impressive than they really were. After "excellent" results had been achieved, the schools advertised their Matriculation "successes" in ways which suggested that the schools' "outstanding" results derived from high-class teaching. In this paper, we argue that these tactics generated artificially high "standards", and that throughout the twentieth century there was a tendency to try to maintain those standards.

Introduction

At the end of the nineteenth century arithmetic, algebra, Euclidean geometry, and trigonometry—but not calculus—formed an unofficial canonical secondary-school mathematics curriculum in all Australian colonies. Some students in secondary schools were prepared for university-entrance examinations conducted by the colonial universities, and courses of study prescribed for those examinations were like those prescribed for students of comparable age in Great Britain who were preparing to enter British universities (Clements, 1979). In Great Britain, however, students intending to proceed to universities tended to remain in school for one or two years longer than colonial students intending to enter local universities—the most typical age for students entering British universities was 19 but, for Australian universities, it was 17. As William Webster, head of Mathematics at Christ's Hospital, in London, told the Taunton Royal commissioners in 1865, the best British schools carried boys into third-year university mathematics, and the best students almost completed most of the mathematics required for a mathematics degree at the University of Cambridge (Great Britain, 1865, see Question 8203).

During the nineteenth century, well-to-do colonists in Australia often wanted their children to qualify for registration with British professional societies (e.g., in Medicine, Law, and Engineering). As a result, Australian colonial universities took steps to ensure that 17-year-old students met minimum qualifications for entrance to major British universities. This resulted in Australian universities defining post-Matriculation courses, and students who passed the local university-entrance examinations at the "pass" level could remain at school, for an additional one to four years, preparing for honours-level post-Matriculation examinations (Clements, 1979).

Post-Matriculation Mathematics in Schools in Victoria in the Early 1900s

Around 1900, the standard of work in post-Matriculation classes of some of the schools in Victoria was very high. In 1884, at Presbyterian Ladies' College (hereafter PLC), for example, 15-year-old Mathilde Monash—a sister of John Monash, who would become a well-known Australian engineer and soldier—sat in post-Matriculation classes and gained honours in French, German, English and Geometry, as well as passes in Algebra, Arithmetic and Physics (University of Melbourne, 1884). She was placed third on the first-

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Melbourne: MERGA.

class honours list for Modern Language, and seventh on the second-class list for Mathematics. In 1885, she again gained places on the same Matriculation class lists, securing second place on the first-class honours list for Modern Languages, and third place on the first-class list for Mathematics. Even though the minimum age for entry to the University at that time was 15 years, and she was now 17, she returned to PLC in 1886, and gained the exhibition (i.e., first place, on the first-class honours list) in Modern Languages. Motivated by the desire to become the first female to secure the Matriculation Mathematics exhibition, she returned to school in 1887, but only obtained equal fifth place on the first-class honours list in Mathematics. In 1889, Ellen Whyte, also a PLC student, gained a place in the first-class honours list in Mathematics. The next year she returned to school and succeeded in creating history by gaining the Mathematics exhibition. One PLC historian wrote: "Great was the jubilation, and many the comparisons of Ellen Whyte with Agneta Ramsay, the English girl who had taken the first place in the Mathematical Tripos at Cambridge" (Fitzpatrick, 1975, p. 105). Whyte's success caused many to question the traditional assumption that women were not as capable as men at mathematics. It also drew attention to the fact that some students returned to school to attempt to win exhibitions even after they had gained first-class honours at Matriculation. This helps to explain why PLC could claim, in an advertisement in 1899, that its students were "carried on to M.A. pass standard in six departments" (Presbyterian Ladies' College, 1899, p. 11).

According to University of Melbourne (1900) Matriculation records, 30 post-Matriculation students presented for honours in Mathematics in November 1900 and, of those, 25 were from the following schools: Melbourne Church of England Grammar School (one student), Methodist Ladies' College (three students), Presbyterian Ladies' College (three students), Scotch College (five students), South Melbourne College (two students), University High School (five students), and Wesley College (six students). Honours Matriculation mathematics classes were mainly offered in the colony's largest, most prestigious colleges.

In fact, the practice of offering free tuition in post-Matriculation classes to students who were already qualified to enter university continued in Victoria until about 1970. "Top" secondary schools would offer scholarships to brilliant students, to encourage them to remain a year or two longer at school before they proceeded to the University of Melbourne (or to some other university). The practice was discontinued in modern times after it was decided that there should be a penalty applied to such students when entrance-scores to universities were calculated. Before that, however, talented young first-up students presenting for Matriculation mathematical subjects were forced to compete against persons who had already obtained honours in Matriculation mathematics. Not surprisingly, the repeaters tended to do better than the younger, first-up students. Subsequently, at the University of Melbourne, students in prestigious "honours" mathematics classes (which studied more advanced courses than, the "pass" mathematics classes) tended to be from well-connected families which had chosen to allow their children to spend more than one year in twelfth-grade mathematics classes. Artificially high standards for secondary-school mathematics were thereby established—and there were leading figures within the mathematical community who wanted to see the old "standards" maintained.

Few parents in Victoria in 1900 would have realised that most of the highest Matriculation honours in mathematics were gained by students who had devoted between two and four years to specialised post-Matriculation study. Typical prospective parents would probably have regarded the fact that students from a certain school regularly obtained high honours as providing overwhelming evidence that that school had very good academic standards. Those parents would have been unlikely to know that at many smaller schools, post-Matriculation classes were an economic impossibility and that students from those schools could not have been expected to obtain the top honours.

A Mathematics Professor's Views on Standards Expected of Incoming University Students

In 1903 Edward J. Nanson, Professor of Mathematics at the University of Melbourne between 1875 and 1922, gave evidence before a Royal Commission inquiring into the state of the University. Nanson, a University of Cambridge graduate, had been Matriculation examiner in Algebra for the University of Melbourne for many years, and the commissioners would have expected him to have a good idea about what might reasonably be expected of secondary-school mathematics students. The following excerpt from Nanson's evidence before the Royal Commission throws light on relationships between school and university mathematics in Victoria at the time. Most of the questions were asked by Theodore Fink, who chaired the Commission.

> *Fink:* Have you formed any ideas as to how much mathematics, without cramming or undue forcing, commencing at appropriate ages, a boy ought to know when he comes up in his seventeenth year?
>
> *Nanson:* I think when a boy comes to the University at that age he ought to know practically all that is required in Pure Mathematics, Part I.
>
> *Fink:* Would that be fixing a higher standard than other universities or other secondary schools?
>
> *Nanson:* I am not familiar with secondary education or school work. What opinion I can give is based on the results of my own students. It seems to me a large percentage are able to get through Pure Mathematics I without coming to lectures. I think in a great many cases they must have done pretty well enough at school to get through.
>
> *Fink:* In 1902, at the University of Melbourne, 57 passed out of 79 who went up for Pure Mathematics I. Of those 57, only 20 thought fit to attend lectures. Either the lectures were not suitable, or they were coached outside, or the subject was too easy?
>
> *Nanson:* Yes.
>
> *Fink:* Is there much difference between Pure Mathematics I at the pass standard and the honour standard in the Matriculation examination?
>
> *Nanson:* There is a considerable difference. There is a very radical difference between honour work and pass work in examination.
>
> *Commissioner Black:* What is the difference between pass and honours in regards to Matriculation?
>
> *Nanson:* The range in Algebra, that is the book work, is practically the same in the two, but the questions that are set for honours work in Matriculation are such that the pass men in Pure Mathematics I would not have the slightest chance of doing.
>
> *Commissioner Black:* The present honour standard in Matriculation is a higher standard than the first-year pass?
>
> *Nanson:* Yes. The standard in Geometry and Trigonometry is not as high as it is in Algebra.
>
> *Fink:* I see Pure Mathematics is essential to the degree of B.A.?
>
> *Nanson:* Yes.

(University of Melbourne, 1904, evidence of
E. J. Nanson, pp. 129–130, Questions 1895–1907)

Questions on the University of Melbourne's Matriculation honours Mathematics papers were certainly difficult. Question 3 on the Geometry and Trigonometry honours paper for the November 1902 illustrates that point. The question stated:

$$\begin{aligned}
\text{If}\quad L &= qr + p(q\cos B + r\cos C - p\cos A),\\
M &= rp + q(r\cos C + p\cos A - q\cos B),\\
N &= pq + r(p\cos A + q\cos B - r\cos C), \text{ and}\\
P &= q^2 + r^2 + 2qr\cos A,
\end{aligned}$$

Prove that $\dfrac{PL + MN}{qr} = (p\sin A + q\sin B + r\sin C)^2$, *A, B, C,* being angles of a triangle.

That question would challenge top mathematics students in schools of any era. The same level of difficulty could be found in questions on all the honours mathematics papers.

The Argument from Matriculation in Victoria, Around 1900

Secondary education in Victoria around 1900 was almost entirely a free-enterprise affair. Anyone wishing to teach in a private or Church-related school was not required to possess any academic or professional qualifications. The colony had never had a teacher-education institution which was directed at prospective secondary teachers, and there was no system of checks on the ways in which schools were run, no government regulations relating to buildings, and no system of inspection of secondary schools. Provided secondary teachers kept to the law, and proprietors filed their annual reports showing the numbers of students in their schools, they could teach what they liked, in whatever ways they liked. There were no government restrictions on the fees that the proprietors could charge parents.

Given this unregulated state of affairs it is not surprising that during to period 1856–1905, when the University of Melbourne's Matriculation served both as the entrance examination for the University and as a public examination for students wishing to enter professions, secondary-school proprietors, teachers, parents and University teachers came to regard results at Matriculation as the most appropriate measuring stick against which the quality of work done in a secondary school could be assessed. Thus, arose what was called the "argument from Matriculation"—that is to say, the argument that the quality of educational experiences offered by a secondary school could be measured by studying the performances of students from that school on the Matriculation examination (see The Argument from Matriculation, 1904, p. 19).

Tactics Used by School Proprietors to Boost their Arguments from Matriculation

A South Melbourne College example. Perhaps the best example of how the argument from Matriculation was presented by school proprietors came in the form of a full-page advertisement for South Melbourne College (SMC), a private school, which appeared in the *Australasian Schoolmaster* of January 1897. The Principal of SMC, John Bernard O'Hara, was a well-regarded Australian poet, and his flair for colourful combinations of words was evident in the advertisement:

SOUTH MELBOURNE COLLEGE

A Splendid Success
A Phenomenal Year

In smaller print, immediately below this heading, was:

> The South Melbourne College has now firmly established its reputation as the premier school in Victoria. The university results for the past four years exceed in brilliance those of any other period of the College, and vindicate the claim of the South Melbourne College to rank as the leading college in the colony.
>
> During the past eight years the College has matriculated over 170 pupils, gaining first-class honours in Mathematics, Physics and Chemistry, as well as numerous scholarships at Ormond College, Melbourne University, and exhibitions under the Education Department.
>
> For the past five consecutive years this College has held the first or second place in the mathematics honours lists at Matriculation.

The remainder of the page was filled with details of results obtained by South Melbourne College students at the November Matriculation examination of 1896. Some of the points made were

1. The South Melbourne College gained the highest number of:
   (a) Passes, viz. 27 (no other school gained more than 24).
   (b) Exhibitions, viz. 3 (no other school gained more than 1).
   (c) Places in class lists, viz. 13 (no other school gained more than 10).
   (d) First and Second class honours, viz. 11 (no other school gained more than 8).
2. The College won the Mathematics exhibition for the third consecutive year, and the Physics and Chemistry exhibition for the second consecutive year.
3. The College gained places in five out of the six class lists. Viz. in (a) Mathematics, (b) Classics, (c) English and History, (d) French and German, (e) Physics and Chemistry. No other college gained places in as many lists.
4. The College presented the only girl who gained honours in Physics and Chemistry, and in the history of Matriculation no girl from any other school has made the class lists in Physics and Chemistry.

(South Melbourne College, 1897, p. 133)

What parents, looking for a suitable secondary school for their child, would not have been impressed by the details provided in that advertisement?

But, if the same parents who read this South Melbourne College advertisement had also read a supplement to the 1896 *Annual Report* of another Melbourne secondary school, University High School, they would have found that Thomas Palmer, the Principal of that school, was also making persuasive claims about the successes of his students at the same November 1896 Matriculation examination. Palmer claimed:

> The number and value of the scholarships and exhibitions gained at the affiliated colleges of the University eclipse anything hitherto recorded in the annals of secondary school education in Victoria. … At the Matriculation examination, 24 of our pupils passed, making a total of 31 for 1896, thus giving us the highest record in the number of passes at Matriculation.

(University High School, 1896, p. 1)

Heads of other secondary schools in Victoria were not willing to allow J. B. O'Hara and Thomas Palmer to fight it out between themselves. An advertisement in the *Argus* in January 1897 for Presbyterian Ladies' College (PLC) pointed out that over the past 16 years 310 PLC students had passed Matriculation, 16 had gained exhibitions, 62 first-class honours, and 50 second-class honours (Presbyterian Ladies' College, 1897). Another advertisement, for Scotch College, reported that for each of the past six years a student from Scotch College had gained the Classics exhibition, and that 17 of the 24 first-class honours given in Classics for that period had gone to Scotch College boys (Scotch College, 1897). *The Camberwell and Hawthorn Advertiser* (5 January 1900) stated that for the period 1893–1898, 116 Hawthorn College boys had passed the Matriculation examination, “this being 18 more than other boys’ school, public or private, in the colony, except one” (Hawthorn College, 1900, p. 3).

Each of the above claims was true. The advertisements testified to the fact that heads of schools knew how to massage Matriculation results so that they appeared to indicate that a school was, academically and pedagogically speaking, a centre of excellence. However, our analyses of three major data sources revealed that the apparently superior results of the schools named above may not have had much to do with the quality of instruction given in the named schools. These data sources were (a) the Matriculation examination records of the University of Melbourne (held in the University’s archives); (b) a folder held by the Ministry of Education containing details of all scholarships given by Victorian secondary schools to state-school pupils between 1894 and 1900; and (c) annual reports of University High School from 1896 to 1900.

Influence of scholarships on Melbourne Matriculation results around 1900. Between 1898 and 1900, inclusive, University High School (UHS) students gained 24 places on Matriculation class lists, including 10 first-class honours (Matriculation entries and results for the University of Melbourne). No other school in Victoria gained as many first-class honours during the same period. What was never known, generally, however, was that 23 of the 24 honours were gained by students on full-fee-paying scholarships awarded by the proprietors. If one considers the number of passes at Matriculation examinations obtained by UHS students, one finds that of the 26 students who attempted to pass the Matriculation examination as a whole, in November 1900, 17 held full scholarships. Of the 17 UHS students who succeeded in gaining an overall pass, 14 held full scholarships.

Five of the seven students from Wesley College who obtained honours at the November 1900 Matriculation examination were former state-school pupils who had won scholarships to the College (see Scholarship Folder, Victorian Department of Education, 1896–1900, and “Entries-Results” for the University of Melbourne’s November 1900 Matriculation examination). In fact, 23 of 48 Wesley College candidates for the November 1900 examination held scholarships, as did 8 of 14 College students who gained an overall Matriculation pass. But, 27 of the 48 Wesley College students who attempted to secure Matriculation passes failed to do so, and at least 11 of the failing students held scholarships. At South Melbourne College, 24 of 43 students who attempted an overall pass succeeded in doing so, and of those 43, 12 had previously gained overall passes.

Sending up very young students to boost the number of Matriculation passes. Another tactic used by some proprietors to swell the number of Matriculation passes gained by their students was to send up students who were 12, 13, or 14 years of age. Although over 90% of students who attempted an overall Matriculation pass in 1900 were 15 or more, and over 70 percent were 16 or more, students of any age could present for the examination.

Allowing only “recommended students” to present for Matriculation. Some schools did not approve of their students sitting for the Matriculation examination under the umbrella of the school unless they had been “recommended” to do so by school authorities. In their advertisements, these schools drew attention to the *percentage* of students *whom they had recommended* who passed the Matriculation examination. Thus, for example, the head of Methodist Ladies’ College (MLC), pointed out that 13 of the 14 MLC students who attempted to pass the November 1899 Matriculation examination succeeded in doing so (“Methodist Ladies’ College”, 1900). Seventeen of the 24 students (“70.8 percent”) from University High School (UHS) who attempted to pass the November 1900 Matriculation examination managed to do so—largely because parents of UHS students who had been prepared for the examination but were deemed to be "unlikely to pass" were informed that "it was not wise for their children to present for the examination" (University High School, 1901, p. 9). In 1900, this policy of "do not present unless recommended" was in place in many of Victoria's secondary schools (e.g., at Grenville College, in the City of Ballarat—see the *Ballarat Courier*, 17 December 1900, p. 4).

Disguising the actual results of a school by publishing "honours lists". Another tactic used by heads of schools in order to give an inflated impression of their schools' Matriculation results, was to publish "Honour Matriculation lists" which contained only the names of students who had passed the Matriculation examination as a whole (see, e.g. the *Xavierian, 1898–1902*; see also, *Ballarat Courier,* 17 December 1900, p. 3, for reports on Grenville College and St. Patrick's College; and Scotch College, *Report*, Christmas 1901, pp. 15–16). The average parent was not aware of what the honour lists entailed and might have thought that the lists included the names of all of the Matriculation candidates from a school—because, after all, the lists often showed Ns (i.e., fails) on individual subjects, as well as Ps (passes) and Hs (honours). Typically, honour lists suggested that schools had gained better Matriculation results than what had, in fact, been the case.

Summary, and Concluding Comments

Around 1900, very few, if any, of those who based their assessment of the efficiency of Victorian secondary schools on Matriculation results would have been even remotely aware of the multitude of factors which, taken together, would have cast doubt on the validity of that criterion. Parents were not in a position to know that results for some schools were much influenced by the performances of post-Matriculation students and former state-school scholarship winners. They would not have been aware, either, that many of the Matriculation passes counted by the proprietors of larger schools were gained by students whom they had pressured to sit for the Matriculation examination two, three, four, and even five times. Again, statistics were not available to the public at that time to show how some proprietors prepared and presented all students in their schools who had any possible chance of passing Matriculation, irrespective of the ages of the students concerned, in order to get as large a number of passes as possible. Objective statistics which showed the number of failures by pupils attending the different schools were not officially reported. At the other extreme, parents had no way of judging the efficiency of schools at which proprietors used the "recommendation technique" so that the highest possible *percentage* of passes might be obtained by students from their schools. And, even if interested persons had been aware of most of the techniques that have been mentioned, they might still have been misled by a detailed "honour list" which appeared to list the results of all students at a school who had presented for a Matriculation examination but which, in fact, contained mostly the results of those students who had done well in the examination.

Proprietors who used any of the above-mentioned tactics rarely let the interested public know how much their tactics influenced the overall results. The tactics were designed to deceive—although, often, the proprietors and principals did draw attention to the fact that *other* "well-performing" schools were using dubious tactics (see, for example, the advertisement for Glenthorpe College, Ascot Vale, in *Argus*, 28 January 1899, p. 14). Proprietors used all their ingenuity in their reports and advertisements to make their schools' Matriculation results appear to be second-to-none so far as quality was concerned. However, an examination of arguments used and statistics quoted, reveals that it was much easier for principals and proprietors of larger schools to "engineer" apparently respectable Matriculation results than it was for principals and proprietors of smaller schools. That raises an interesting question: Was the instruction given in larger schools generally better than that given in smaller schools? It is at least clear from the analyses presented in this paper, and much more detailed analyses have been presented elsewhere (Clements, 1979), that Matriculation results could be interpreted in many ways, and that sometimes apparently-strong results were misleading.

When one considers the success of proprietors around 1900 in using the argument from Matriculation to lure interested parents into enrolling their children at "successful, high-quality" schools, it is hardly surprising that similar arguments continued to be used well into the twentieth century. Some schools came to be regarded as strong mathematical schools, and for that reason some parents chose to send their children to those schools. The relevant question for us all to ponder is this: Is there evidence that similar misleading, but nevertheless persuasive arguments and tactics are still in use in Australia today? If so, what is the impact on school mathematics—especially on how students think about mathematics, about how they study mathematics, and on how teachers plan and teach mathematics lessons?

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That First Step: Engaging with Mathematics and Developing Numeracy

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Many resources have been created with the aim of helping children and adults overcome difficulties with mathematics and to develop or improve their numeracy. However, these are only used once the individual has *decided* to act – to do something to improve their mathematics and numeracy. Unfortunately, someone who knows that they need to improve their mathematics or strengthen their numeracy is not always accessing these resources. In this theoretical paper, I explore reasons why individuals may not engage with resources designed to help them develop their mathematical understandings and numeracy and identify the need to address how to get individuals to take that first step.

It is not a new finding that some individuals do not want to engage with mathematics or choose to develop their numeracy. Avoidance of mathematics is a behaviour of both children and adults. For example, Kemp and Hogan (2000) reported children may take actions to enable them to avoid mathematics. However, this avoidance can contribute to a feeling of failure in regard to the successful completion of mathematical activities (Chinn, 2012), which leads to more avoidance, when the opposite should happen. Negative views towards mathematics can also impact engagement with mathematics (Grootenboer & Marshman, 2016). Avoidance and negative attitudes towards mathematics also will impact the development of numeracy.

Many resources are available for individuals to access to address their mathematical skills and knowledge and to contribute to the development of numeracy. These resources are developed for many members of the community – children (e.g., http://splash.abc.net.au), young adults (e.g., www.khanacademy.org/math?t=classes), and adults (e.g., www.utas.edu.au/mathematics-pathways) – but these may not be accessed by those who would benefit from them (Mac an Bhaird, Fitzmaurice, Fhloinn, & O'Sullivan, 2013). With the opportunity for these resources to address mathematical skills and knowledge and to develop numeracy, often by transforming thinking and behaviours (for example, Boaler, 2013a, 2013b; Callingham, Beswick, & Ferme, 2015), the question of how to get individuals to take the step to access these resources needs to be considered.

Numeracy and Mathematics

Numeracy has been described in a variety of ways since the word was coined by Crowther (1959) to be used as a mathematical equivalent of literacy. He referred to numeracy as "an indispensable tool to the understanding and mastery of all phenomena" (p. 271), stating that it had two aspects, "an understanding of the scientific approach to the study of phenomena… (and) … the need… to think quantitatively" (p. 270). Cockcroft (1982) argued that the word numeracy had changed since Crowther (1959) described it, as "the association with science is no longer present and the level of mathematical understanding to which the words refer is much lower" (p. 11). Cockcroft (1982) proposed that numeracy should attribute two traits to the individual – "an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to

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cope with the practical demands of his everyday life" (p. 11)

Cockcroft's (1982) description provided a stronger emphasis on the link between mathematics and numeracy. The description of numeracy has changed over time to incorporate an aspect of disposition regarding mathematics, such as "personal confidence, comfort and willingness to 'have-a-go' through the use of mathematical or quantitative means" (Australian Association of Mathematics Teachers [AAMT], 1997, p. 14). In their review of how numeracy has been conceptualised, Geiger, Goos, and Forgasz (2015) examined the use of the word *numeracy* (and its doppelgänger, mathematical literacy) and found that it had many aspects and components attributed to it. These included mathematical skills and processes, competencies, communication, interpretation, capacities, understandings, engagement, contribution, and connection, and are evident both in the classroom and in everyday lives.

Ernest (2002) took a different perspective and considered how an individual may be empowered mathematically. He proposed three domains, within the classroom (mathematical empowerment), through status gained via achievement (social empowerment), or through personal identity and their ability to create and use of mathematical understandings. Although not presented as an expanding model, Ernest's (2002) three domains of empowerment differ in terms of the context within which the individual engages with mathematics and the sphere of individual power that results from their engagement. This context can reflect the aspect of mathematical experiences and activities in the real world and everyday life that is considered by many to be critical in the description of numeracy (Geiger et al., 2015). Common amongst the three contexts is confidence and disposition to engage with mathematics, an element considered by the AAMT (1997) as a component of numeracy.

The question is: How can an individual be encouraged to engage with mathematics and develop numeracy – to take that first step? Three reasons are explored below. The first reason concerns how individuals perceive mathematics, as this can impact their engagement with mathematics and the development of their numeracy. Frameworks from Ernest (1989) and Grigutsch, Raatz, and Törner (1998) are used to investigate how individuals' perceptions of mathematics may link to actions in regard to mathematics. The second reason concerns whether mathematics is seen as existing beyond the classroom and as useful in everyday lives (Grootenboer & Marshman, 2016). If mathematics is seen as not useful, then engagement with mathematics and the development of numeracy may not be seen as worthwhile (Di Martino & Zan, 2011). The final reason focuses on individuals' perceptions of whether they are maths-able (Boaler, 2013a). Perceptions of oneself as maths-able depend on experiences and community expectations (Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014), If individuals do not consider themselves as maths-able, they would be less likely to work on their mathematics skills and knowledge or on developing their numeracy.

The Impact of Perceptions of Mathematics

Ernest (1989) and Grigutsch et al. (1998) had similar approaches to describing how individuals may perceive mathematics. Ernest (1989) discussed three philosophies of mathematics: instrumentalist, Platonist, and problem-solving. He described instrumentalist as viewing mathematics as a "set of unrelated but utilitarian rules and facts" (p. 99), Platonist as "a static but unified body of certain knowledge" (p. 100), and problem-solving as "a dynamic, continually expanding field of human creation, a cultural product" (p. 100). Grigutsch et al. (1998) described four aspects: schema, formalism, process, and application, with schema and formalism belonging to a static view of mathematics and process belonging to a dynamic view of mathematics. Benz (2012) described these four aspects as focusing on calculations (scheme), formal characteristics such as terminology (formalism), problem solving through understanding and discovery (process), and practical use (application). Grigutsch et al. (1998) proposed that application was more likely from a dynamic view of mathematics (that is, comprising the aspect of process).

Viewing mathematics as instrumentalist (Ernest, 1989) or static (Grigutsch et al., 1998) would involve seeing mathematics as an external set of rules that need to be recalled or accessed and used in a precise way to generate an answer or solution. Individuals who hold this view of mathematics would focus on recalling rules and using these rules (Ernest, 1989). Seeing mathematics from a Platonist perspective would involve developing conceptual understandings of the knowledge created by others to underpin and connect the procedures used (Ernest, 1989). Finally, seeing mathematics as problem-solving (Ernest, 1989) or dynamic (Grigutsch et al., 1998) would involve considering mathematics to be more creative and evolving, enabling individuals to try different solutions and potentially be less fearful of making a mistake.

Ernest (1989) highlighted the impact that the educator may have, suggesting that the perceptions that educators have will likely influence how they teach mathematics. He proposed that educators who have an instrumentalist view of mathematics may focus on instructing students to learn rules and generate answers, educators with a Platonist view would explain the external mathematical ideas to their students to enable them to make connections between mathematical ideas, and educators with a problem-solving view of mathematics would create activities that enable their students to actively construct their mathematical understandings and encourage them to engage in problem-solving and problem posing. Anders and Rossbach (2015) stated that the educator's beliefs and actions can impact their students' mathematical learning. When considered in terms of learning and teaching mathematics, Attard (2015a) found that educators who utilise and promote problem solving and collaboration are more likely to engage children with mathematics.

Benz's (2012) findings support these connections between teachers' perceptions of mathematics and how they might teach mathematics, with educators who professed a process view having a higher level of agreement towards constructivist approaches to learning and teaching mathematics than educators who agreed with the static aspects of formalization and scheme. Furthermore, Benz (2012) found that educators with a static approach were more likely to focus on the importance of a correct result. This focus may lead to children disconnecting from mathematical activities (Boaler, 2015), which would impact their numeracy. Attard (2013) indicated that greater student engagement resulted when teachers encouraged students' active participation or social interaction during the lesson and incorporated connections to the students' lives (both current and future) in the lesson. In addition, she found that children who were learning mathematics through discussions and cooperative learning stated they enjoyed the activities. All of these actions are aspects of constructivism, and it is the educator with a process view (Grigutsch et al., 1998), which incorporates problem solving and discovery [similar to Ernest's (1989) problem-solving perspective], who is more likely to agree with a constructivist approach to learning and teaching mathematics (Benz, 2012).

Much like those of adults and educators, children's perceptions of mathematics can also vary. McDonough and Sullivan (2014) stated that it is important to find out children's perceptions of mathematics as they may impact the activities in which they will engage. These researchers suggested specific perceptions that coincided with Ernest's (1989) instrumentalist view might link to children determining the teacher is the source of understanding rather than discussions with their peers. In their research, Di Martino and Zan (2011) found that children perceived that mathematics involved the remembering of rules, was uncreative, focused on answers, and was not applicable to life. Again, many of these aspects relate to Ernest's (1989) instrumentalist view. These perceptions of mathematics, as shown with the last of Di Martino and Zan's (2011) findings, can determine whether children and adults see the applicability of mathematics beyond the classroom.

Seeing Mathematics as Useful Beyond the Classroom

Individuals may dislike the need to do mathematics and to be numerate. Grootenboer and Marshman (2016) stated that children do not always see mathematics as being useful outside of the classroom. Even in the classroom, children sometimes state they that they hate mathematics (Bates, Latham, & Kim, 2013) or that do not want to do mathematics and are happier when there isn't a mathematics lesson (Attard, 2013). Some adults also hate mathematics (Grootenboer & Marshman, 2016) and would prefer not to have to do mathematics in their work, such as pre-school teachers (Bates, Latham, & Kim, 2013). It is likely that these people are able to do mathematics in their everyday lives; however, they just do not recognise that what they do is mathematics (Kimball & Smith, 2013) or they fear engaging in classroom mathematics (Grootenboer & Marshman, 2016).

To be numerate requires mathematics to be seen as of use and useful (Geiger et al., 2015), as numeracy is "the application of mathematics to solve real-life problems" (Grootenboer & Marshman, 2016, p. 45). Unfortunately, the mathematics learned and taught at school may not be seen as being of use or usable outside of school (Kemp & Hogan, 2000) and may be a source of "frustration and powerlessness" (Grootenboer & Marshman, 2016, p. 22). Research has shown that mathematics is used in many everyday instances (Northcote & Marshall, 2016), and this everyday use demonstrates the real-world connection of numeracy (AAMT, 1997). It may be the case that children do not identify when and where mathematics is used in everyday life or recognise that they can use mathematics outside of school (Di Martino & Zan, 2011). Being able to see mathematics inside the classroom as useful and having mathematics outside of the classroom made visible may enable children to develop a disposition that leads to more engagement with mathematics and thus the development of their numeracy (Barnes, 2008). It may be necessary, as Barnes (2008) states, to take actions to enable children to recognise mathematics as useful in their everyday lives and to value becoming numerate.

Individuals' Perceptions of Who can do Mathematics

Self, gender, and societal expectations may impact individuals' beliefs about whether they can do mathematics. Galdi, Cadinu, and Tomasetto (2014) found that young children start building gender stereotypes, with children as young as six years of age developing implicit gender stereotypes. Their findings indicated that, although explicit gender stereotypes were not evident for six-year-old girls and boys, six-year-old girls had implicit gender stereotypes, identifying boys as more able at mathematics than girls. Research suggests that these beliefs continue as females become older, with mathematics seen as a masculine and not feminine ability (Solomon, 2012).

An individual's perception of who can do mathematics can be constructed from their environment (Parker et al., 2014). In a school situation, this can include the mathematics textbooks used by educators when teaching mathematics. In their analysis of images contained in a mathematics textbook for students in high school, Norén and Björklund Boistrup (2016) found that images were more likely to promote a passive or consumer orientation for females and an active, producer, or fixer orientation for males. These orientations, they proposed, could be linked to how mathematics might be used by each gender and could feed through to choices later in life.

Individual and societal beliefs and images of who can do mathematics can also impact perceptions about who is capable of engaging with mathematics. Boaler (2013a) referred to the harm that beliefs about who was maths-capable could have on individuals, specifically, beliefs that "mathematics is for select racial groups and men… (and)… the teaching practices that go with it, have provided the perfect conditions for the creation of a math underclass" (para. 4). These perceptions of who can do mathematics may relate to confidence, which is critical as "confidence mediates their capacity to engage in mathematical learning experiences… (and)… will also be influenced by the nature and perceived success of their involvement in mathematical activities in the classroom" (Grootenboer & Marshman, 2016, p. 24).

These beliefs can also contribute to mathematical self-efficacy. Parker et al. (2014) described mathematical self-efficacy as an individual's beliefs about their competence and capabilities in mathematics. They found that mathematical self-efficacy was positively related to achievement in mathematics some two years later. An antithesis to mathematics self-efficacy is mathematics anxiety. Metje, Frank, and Croft (2007) described how the cycle of failure in mathematics and negative attitudes regarding mathematics may connect to avoiding mathematical activities and contribute to mathematics anxiety. In addition, this cycle of failure makes it difficult to help individuals to overcome mathematics anxiety, particularly as mathematics anxiety will reduce confidence for engaging with mathematics. They propose that remembrance of past failures and mathematics anxiety could result in individuals not taking actions to address mathematical skills and numeracy.

Encouraging that First Step

The three reasons why individuals may not improve their mathematical skills and knowledge or develop their numeracy do not impact a specific population or group. Approaches that target these reasons relate to providing different experiences with mathematics that can impact perceptions of mathematics, show that mathematics is useful beyond the classroom, and change who might be seen as maths-able.

Targeting how individuals see mathematics and how empowered they feel may encourage them to take steps to improve their mathematical understandings and numeracy. If the individual views mathematics as instrumentalist (Ernest, 1989) or schema and formalism aspects within the static view (Grigutsch et al., 1998), they may be disempowered mathematically and then disengage from mathematics (Di Martino & Zan, 2011). This is extended by Ernest (2002), who stated that the disempowerment of the individual who views mathematics as an external set of rules occurs as the rules were external and sanctioned by others. However, if the individual views mathematics as process and application (Grigutsch et al., 1998) or as problem-solving (Ernest 1989), it would enable the individual to have choice in what they do when engaging with mathematics and to see mathematics as a creative or dynamic pursuit (Grootenboer & Marshman, 2016). The opportunity to create solutions, rather than follow set rules or procedures that need to be remembered precisely (with or without understanding) may generate engagement (De Martino & Zan, 2011) – promoting individuals to take that first step. However, this would be tempered by self-efficacy and confidence (Bates, Latham, & Kim, 2010). Thoughtful consideration of these opportunities would be needed, as

> Belief change does not occur simply through the presentation of new, desirable beliefs. … belief change usually requires revisiting and reviewing episodes which gave rise to the held beliefs, and then creating new encounters where new and desirable beliefs can be experienced in positive and successful ways (Grootenboer & Marshman, 2016, p. 17).

Targeting the identification of mathematics in everyday life or seeing mathematics completed in a mathematics lesson as applicable in their lives may impact individuals taking that step to develop their numeracy (Grootenboer & Marshman, 2016). Being able to identify that mathematics is everywhere (Barnes, 2008) – in daily household activities (Anders & Rossbach, 2015) and in work (Northcote & Marshall, 2016) – could be powerful in encouraging the individual to take that first step as their recollections of success could move beyond scores in mathematics assessments to a wider range of experiences where they have seen themselves as successful.

Targeting perceptions of who is maths-able may support individuals in identifying themselves as successful in mathematical experiences or as numerate. The provision of images in the educational environment (Norén & Björklund Boistrup, 2016) and other environments that show all individuals involved with mathematics and demonstrating numeracy may encourage individuals to also see themselves engaging with mathematics or being numerate. An explicit use of positive gender priming, where females are shown successfully and actively engaging in mathematics, may also help individuals to see themselves as maths-able. Galdi et al. (2914) demonstrated the impact of providing positive gender priming for six-year-old girls. When presented with an activity that showed girls achieving in mathematics, the results in a mathematics assessment improved.

Boaler (2014) believed that if individuals are "encouraged to believe they can be successful in mathematics... we will have many more confident and capable mathematics learners" (para. 11). This is reflected in the impact of the individual's mathematical self-efficacy – It needs to reflect a belief that the individual can successfully complete a mathematical task (Parker et al., 2014). The educator can have an impact on engagement with mathematics and the development of numeracy, as discussed by Metje et al. (2007). An example of this is described by Boaler (2013b), who discussed how making mistakes in mathematics should be used by educators as a positive learning opportunity, rather than as a future reminder of previous failures (Metje et al., 2007). Creating an environment where mistakes are no longer regarded as a negative (Boaler, 2013a) could unleash the power of learning from mistakes and help ameliorate mathematics anxiety.

Although approaches were connected to specific reasons why an individual may not act to improve their mathematics or develop their numeracy, approaches may address several reasons. For example, providing positive priming can reflect gender and connections to the real world (Norén & Björklund Boistrup, 2016), using problem-solving in the classroom can incorporate context from everyday lives (Geiger et al. 2015), authentic work examples used to demonstrate the relevance of numeracy (Northcote & Marshall, 2016) can extend beyond mathematical empowerment to epistemological empowerment (Ernest 2002), and examples of everyday use of mathematics may provide remembrances of successful and non-anxious engagement with mathematics (Metje et al., 2007).

I will now return to the question of how to get individuals to take the step to access resources. As Di Martino and Zan (2011) proposed, we need to identify the reasons that each individual has for not engaging with mathematics and their numeracy, and then address the reasons with targeted interventions. However, how will individuals needing to improve their mathematics and numeracy engage with these two processes, that is, take that first step?

Conclusion

Individuals of different ages may recognise the need to address their mathematical knowledge or numeracy, but they do not take that step to access resources that would assist them. Targeting perceptions of mathematics, the usefulness of mathematics beyond the classroom, and who might be seen as maths-able may transform individuals' beliefs about mathematics and who can do mathematics (Boaler, 2013a, 2013b) and their views of numeracy (Grootenboer & Marshman, 2016). As Di Martino and Zan (2011) indicated, we need to focus on the individual. Research is needed to, first, find ways to identify the specific reasons why an individual who needs to improve their mathematics or develop their numeracy does not engage with resources designed to help them and, second, create targeted interventions that address those specific reasons. The overarching third area of research is how to get these individuals needing to improve their mathematics and numeracy involved in these processes – the tangible action of wanting to do it – where the individual takes that first step.

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“Maths Inside”: A Project to Raise Interest in Mathematics

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In this paper, we provide an overview of the “Maths Inside” project, funded by the Australian Maths and Science Partnership Program (AMSPP). The overall aim of the AMSPP is to improve uptake and participation of students in mathematics and science at secondary and tertiary levels. In this research project, we aim to improve student interest in mathematics and support mathematics teachers in their professional learning, through provision of rich and investigative learning resources, including video case studies of CSIRO scientists and mathematicians. Data collection on the outcomes of the project is ongoing and will be reported in subsequent papers.

The need for investment in Science, Technology, Engineering, and Mathematics (STEM) education in Australia was a principal part of the former Chief Scientist’s call for investment in STEM (Office of the Chief Scientist, 2013). In that paper, the case was made for the urgent need to strengthen science and innovation as the drivers of productivity, creating jobs and growing the economy in an increasingly competitive international environment. As part of this investment, the Australian Maths and Science Partnerships Program (AMSPP) was announced in the 2012-13 budget as part of a range of initiatives to improve outcomes in the learning and teaching of mathematics and science (Australian Government Department of Education and Training, 2017). Twenty-two projects were funded to the total of $21.6 million. The projects involve partnerships of organisations, in each case led by a university. The “Maths Inside” project is funded through the AMSPP, and partners are the Commonwealth Scientific and Industrial Research Organisation (CSIRO), the federal government agency for scientific research in Australia, the Australian Association of Mathematics Teachers (AAMT), and The University of Technology Sydney (UTS).

The projects funded under the AMSPP aim to build the confidence, capacity, knowledge base, and pedagogical skill of classroom teachers in mathematics and science, and increase the number of school students undertaking mathematics and science subjects to Year 12. A further aim is to improve outcomes for students in mathematics and science.

We developed a project to address these points, the “Maths Inside” project, so-called because the aim of the project is to help teachers and students understand how mathematics is used “inside” science and other areas. The primary aim of the project is to highlight how mathematics underpins many endeavours. “Maths Inside” uses a variety of resources to make visible the mathematics in these endeavours and aims to assist teachers in answering questions posed by their students about the value of the mathematics they are learning.

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The research questions of the project are:

1. What is the role of rich and authentic learning tasks in raising student awareness of the importance of mathematics in society and in their own careers?
2. How do we improve student participation in, and attitudes to, mathematics?
3. How do we best prepare teachers to engage their students in mathematical endeavours?

In this paper, we discuss the problems that we aim to address with the project, examine relevant literature, and then provide an outline of the methodology being used to address our research questions.

The Current Context

The first issue that is concerning governments, educators, and other stakeholders is the declining participation in higher-level mathematics at senior secondary school levels (Barrington & Brown, 2014). This is suggested to be a result of a number of factors. One concerns the ATAR "gaming" that occurs: Students make subject choices that will maximize their ATAR score, often choosing a lower level of mathematics than they are capable of mastering, or avoiding mathematics entirely (Forgasz, 2006; Kennedy, Lyons, & Quinn, 2014; Mathematical Association of NSW, 2013; Pitt, 2015). A second factor is the removal of mathematics as a prerequisite to study particular courses at a tertiary level (Wilson, Mack, & Walsh, 2013). Many students feel that they will be able to "catch up" on those requirements through bridging courses. It is only when they fail subjects in their first year that they realise the problems with this belief.

The second issue is an apparent lack of motivation and negative attitudes towards the study of mathematics. Many students perceive mathematics as being difficult and unengaging, requiring a lot of time and effort to gain mathematical knowledge and skills (McPhan, Morony, Pegg, Cooksey, & Lynch, 2008). Given that students can choose their own pathways in subject selections, there is evidence that this factor is an influence in a significant number of students choosing to avoid mathematics when it is not necessary to achieve their overall academic goals (McPhan et al., 2008). This negative perception of mathematics often has roots in primary school mathematics lessons, which can lead to the phenomenon of mathematics anxiety. Furthermore, some features of mathematics learning that can cause disengagement are suggested to be rote learning and calculations, memory dependence, unrealistic exercises, mathematics tests, and authoritarianism in mathematics education (Frankenstein, 1989). The "Maths? Why Not?" report supports this picture of mathematics being perceived as an uncreative subject (McPhan et al., 2008).

The third issue concerns the teaching of mathematics. One factor is that teachers do not generally have the experience of working in a field that is underpinned by mathematics, and consequently are unaware of the "mathematics inside". This makes it difficult for them to raise student awareness of the importance of mathematics. A second factor is the lack of qualified and experienced mathematics teachers, which gives rise to the need to employ out-of-area teachers, that is, teachers who may lack relevant content and pedagogical knowledge in mathematics. In a review conducted by the Australian Mathematical Sciences Institute (AMSI), and drawing on the 2011 Trends in International Mathematics and Science Study (TIMSS) survey, "Compared to the international average of 12 per cent, a staggering 34 per cent of Australian Year 8 students were being taught mathematics by a teacher without a solid mathematical background" (Wienk, 2016, p. 16). This is significant because the presence of a qualified teacher in mathematics is not only highly influential in students' perceptions of mathematics, but also in their decisions to enrol in high-level mathematics (AMSI, 2012). This concern is evident from the responses that school principals provided in the Programme for International Student Assessment (PISA) 2012 report, stating that they have found it difficult to hire qualified mathematics teachers. (Organisation for Economic Co-operation and Development [OECD], 2014). In 2013, 8.7 percent of schools reported at least one vacancy in mathematics (Wienk, 2016).

A third factor is the type of pedagogy that is used in mathematics classes. Pedagogies that emphasise rote learning, unrealistic exercises, emphasis on algorithms, and decontextualized learning are thought to contribute to problems of uptake. Students find mathematics boring and are easily distracted, rather than engaged and motivated to take part in the lesson. Further, when students learn by repetition of abstract procedures, they "are not only learning an efficient set of procedures, but an esoteric set of practices that are not well represented outside of mathematics classrooms" (Boaler, 2000, p. 4).

These issues provided a rationale for the "Maths Inside" Project. The following literature guided the structure and content of the activities and professional development in the project.

Building Confidence, Capacity, Knowledge Base, and Pedagogical Skill of Classroom Teachers

A teacher plays a strong and influential role in students' engagement and their decisions to enrol in higher-level mathematics, as found in a study involving Year 10 students (McPhan et al., 2008). According to Sullivan and McDonough (2007), two sets of factors must align to promote student engagement in learning, and the role of the teacher is to address these factors:

> The first set of factors include that the students have the requisite prior knowledge, the curriculum is relevant to them, the classroom tasks interest them, and the pedagogies and assessment regimes match their expectations. The second set of factors relate to their goals for learning, their willingness to persist, and the extent to which they see participation in schooling as creating opportunities. (p. 698)

Teachers need to know their students and what choice of classroom tasks will interest them. In order to do this, teachers must have a wide knowledge of pedagogical practice. This comes from training and experience. However, recent studies have shown that in the Sydney metropolitan area, for example, students in Year 7 have lessons taught by a qualified mathematics teacher 50% of the time, while in regional areas, 29% of Year 7 lessons and 43% of Year 9 lessons are taught by qualified mathematics teachers (MANSW, 2013). This is problematic as the "Maths? Why Not?" report (McPhan et al., 2008) indicated that a strong experience of mathematics teaching in junior school (Year 7 to Year 10) increases the chance of a student enrolling in mathematics for senior school in all geographical areas. Without qualified mathematics teachers, there is a stronger chance that students will become disengaged in the junior school years, potentially leading to increased disengagement in mathematics learning and lower rates of participation in higher mathematics in high school (McPhan et al., 2008). Teachers teaching out of area require ongoing professional support, mentoring, and exposure to examples of sound pedagogical practice.

Increasing Participation in Mathematics and Science to Year 12 and Beyond

The PISA 2012 report found that there was a strong link between active classroom interaction and better student engagement, fostering learning environments where students interrogate concepts and problems critically (OECD, 2014).

There is strong evidence that the geographical location of a school has a significant effect on the number of students enrolled in high-level mathematics (McPhan et al., 2008). Rural schools are often disadvantaged as they find it difficult to provide essential resources such as qualified mathematics teachers and quality educational resources (Jones, 2000). Results from the "Maths? Why Not?" report (McPhan et al., 2008) show that a positive junior school experience for a rural student plays a significantly high influence on enrolment in senior school. For the relatively low number of students who do continue on to senior school in rural areas, students are more likely to enrol into subjects that have a more practical focus, instead of theory-based subjects, such as higher mathematics, for senior study (Jones, 2000). This often leads to higher-level mathematics not being offered as a subject choice in rural schools (AMSI, 2012).

Schools in the metropolitan areas are typically larger than rural or regional schools and have more access to resources. A larger school size can have a positive influence in a student's decision to enrol in higher- level mathematics (McPhan et al., 2008); it is argued that this is because schools in the city have more access to qualified mathematics teachers and resources. Metropolitan areas have the highest levels of enrolment in more advanced mathematics subjects (MANSW, 2013).

Improving Students' Outcomes in Mathematics and Science

Student engagement and motivation are necessary for improving student outcomes in mathematics and science. As noted in the four yearly MERGA review, there is growing interest in the study of motivation and engagement in mathematics education research in Australia (Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016). While some authors use the terms interchangeably, Attard (2012) draws on the definition of engagement as a multi-faceted construct initially offered by Fredericks, Blumenfeld, and Paris (2004). She distinguished engagement from motivation because motivation refers to "beliefs and orientations towards schoolwork and learning" (Attard, 2012, p. 10), while engagement is concerned with cognition, affect, and behaviour. A student may in general be highly motivated, but constrained by some feature of the context of the classroom such as the influence of peers, the relationship with their teacher, or the nature of a particular task, from fully participating in all three aspects of engagement with mathematics on a particular day. The intertwining of these constructs and constraints is acknowledged by Fredericks et al. (2004), who state that "definitions of engagement incorporate a wide variety of constructs. For example, behavioural engagement encompasses doing the work and following the rules; emotional engagement includes interest, values, and emotions; and cognitive engagement incorporates motivation, effort and strategy use." (p. 65).

When teachers use active learning methods, students become more engaged and interested in mathematics. Studies (e.g., Freeman, McDonough, Smith, Okoroafor, & Jordt, 2014) have shown that active learning can also increase student performance with a lower failure rate. The PISA 2012 report found that there was a strong link between active classroom interaction and better student engagement, fostering learning environments where students interrogate concepts and problems critically (OECD, 2014).

Attard (2013) reported on a longitudinal case study of students making the transition from primary to secondary school. The same students were interviewed and observed in classroom settings and focus groups. Attard (2013) found that "those pedagogies that fostered substantive engagement with mathematics were those that promoted active participation, academic challenge, and social interaction, and highlighted the relevance of mathematics within students' current and future lives" (p. 583).

In addition to providing tasks that involve active participation, teachers who show their enthusiasm for mathematics and their concern for their students and their learning are more likely to achieve increased engagement in mathematics (Skilling, 2014).

"Maths Inside"

The aim of "Maths Inside" is to provide teachers with classroom materials that are engaging, interesting, and provide a range of challenge. The materials include, and are inspired by, a series of videos made by the CSIRO about their scientists at work in solving problems and inventing new processes and technology to answer questions of value to the Australian community. Examples include the Zebedee device for 3D mapping, the Square Kilometre Array of telescopes, the Patient Admission Prediction Tool, and Bees with Backpacks (monitoring how, when, and where bees travel). The classroom materials to accompany the videos are prepared by the Australian Association of Mathematics Teachers (AAMT). Writers from AAMT liaise with CSIRO and the project team at UTS to ensure that the videos provide sufficient links to school mathematics. This may mean requesting the scientists to use mathematical language that school students will recognise.

Each activity is linked to the Australian Curriculum: Mathematics, making it possible for all teachers, whether trained or out-of-field, to include the "Maths Inside" activities in the school program. Teacher notes are included with the activities so that all teachers have the Knowledge at the Maths Horizon enabling them to more easily engage students (Hill, Ball, & Schilling, 2008) – see Figure 1. Out-of-field teachers often struggle with this aspect of Subject Matter Knowledge.

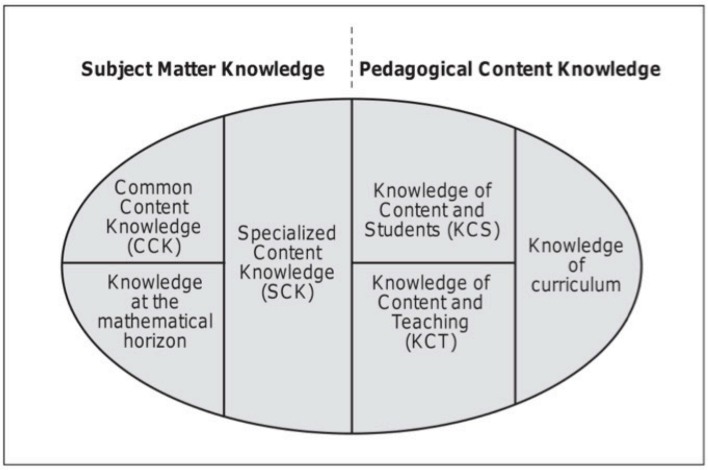

Figure 1. Domain map for mathematical knowledge (Hill, Ball, & Schilling, 2008, p. 377).

The classroom materials are designed to provide teachers with a wide choice of activities. These include group work, individual projects, and hands-on construction, using partially-prepared spreadsheets to investigate scenarios, and some routine calculations. Where the tasks are routine, they are within a theme and context provided by the scientists. The materials are trialled with teachers in professional learning contexts including conferences, and with small groups of students.

The use of active learning methods is the basis of all the classroom materials in "Maths Inside". For example, after viewing the video about the Square Kilometre Array and hearing from astronomers how parabolic dishes collect radio waves, students then build their own parabolic trough solar collectors to cook a sausage, boil water, and melt marshmallows (see Figure 2). The mathematical properties of the reflection of the sun's rays through the focus of their cardboard parabolas are not only visible but also effective, in real-world terms, to provide heat for cooking. Links to the use of inexpensive and readily available technology for cooking in the developing world are made. After the practical and global applications are appreciated, the students more readily engage with the mathematics. The geometric properties of the parabola are then analysed with co-ordinate geometry, making this an unforgettable lesson.

Figure 2. Radio telescopes inspire parabolic cookers (CSIRO, Maths Inside Project Team).

The possibility of humans travelling and living on Mars is a current topic of interest. The videos on Zebedee and the Square Kilometre Array are useful starting points for discussions about communications from Earth to Mars. Speed, distance, and time calculations, based on the relative locations of the planets, are part of the considerations when planning such communications.

The Bees with Backpacks video leads into several possible investigations into the geometry of the hexagon, tessellations, and three-dimensional packing. Students also consider the way that bees communicate through the "waggle dance", involving trigonometry and much hilarity as students perform the dance to tell their friends the direction and distance to a food source outside the hive. Many teachers struggle to find real-world contexts for teaching about ratio; however, the "capture/recapture" method for estimating the sizes of populations can be applied to the bee hive.

Plans are in progress for videos and classroom materials about pollution in the ocean and modelling demands on hospitals, examples of the wide range of real-world problems that CSIRO scientists and data analysts are addressing using mathematics as an essential tool.

Data and Methodology

The effectiveness of "Maths Inside" is being researched in parallel with the development of resources. Surveys from teachers who have viewed the videos at conferences informed the next stage; for example, the project team soon learnt that the initial videos were too long, and that certain kinds of videos would appeal to different age groups. The project team is recruiting schools to participate in ongoing research that will be conducted as a set of case studies with mixed methods for data collection: surveys, observations, and interviews. Baseline data about student attitudes to mathematics and interest in continuing to study mathematics will be collected, and the data will be compared with later survey data (Fraenkel, Wallen, & Hyun, 1993). Members of the team are aware of the limitations of the study, which include the features of case study research (Burgess, 1985) and the need to take a reflexive attitude to the relationships between researchers and participants. In addition, as the schools were self-selected to be involved, generalisations beyond those schools to other situations may be limited.

Conclusions and Looking Ahead

The literature of engagement and motivation in mathematics, especially the importance of providing challenging tasks, demonstrates the usefulness and value of mathematics to students. The value of mathematics has at least two aspects relevant to our ongoing work: the value to society (someone needs to know it to solve real problems), and the value to individual students (If I learn this, I can do…and these careers and jobs will be open to me).

The members of the "Maths Inside" team are planning resources to address those two aspects. Scientists in the videos talk directly about the mathematics that they use to address authentic problems, and the classroom materials draw on those scenarios and provide tasks that show how mathematics is useful to individual citizens. We look forward to reporting on the results of this work in the future.

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Mastery Learning: Improving the Model

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In this paper, we report on developments in the Mastery Learning (ML) curriculum and assessment model that has been successfully implemented in a metropolitan university for teaching first-year mathematics. Initial responses to ML were positive; however, we ask whether the nature of the ML tests encourages a focus on shallow learning of procedures, and whether the structure of the assessment regime provides sufficient motivation for learning more complex problem solving. We analysed assessment data, as well as student reports and survey responses in an attempt to answer these questions.

Mastery Learning (ML) has been defined as "both a philosophy of instruction and a set of methods for teaching and learning" (Groen, Coupland, Memar, & Langtry, 2016, p. 69). Guskey (2010) attributes the development of ML to Bloom (1971). Fundamental to Bloom's (1971) ideas is the belief that most students "can learn a subject to a high level of mastery… if given sufficient learning time and appropriate types of help" (p. 51).

Typically, ML involves the identification within each topic of essential skills and concepts to be learned, and the use of formative tests, as each topic has been studied, either individually as part of a personalised system of instruction (PSI), where students may proceed at different rates, or in teacher-led classes in learning for mastery (LFM), where the class proceeds together. A high standard (70% to 80%) on tests is set as the level necessary to demonstrate "mastery" of each topic. Students who do not reach the mastery level on their first attempt at a mastery test are given extra instruction opportunities and may re-sit the test several times. This approach has been reported as a successful strategy in meeting the challenges faced by university teachers whose students arrive with a range of mathematical preparedness, with many lacking the assumed or recommended prior knowledge for their university studies (Bradley, 2016; Groen, Coupland, Langtry, Memar, Moore, & Stanley, 2015).

A major review of ML was undertaken by Kulik, Kulik, and Bangert-Drowns (1990) who used a meta-analytic methodology to integrate findings from 108 studies of ML initiatives at school and university levels. They concluded that ML approaches resulted in significant positive effects on student learning, as measured by exam performance given at the end of instruction. Where student attitudes to instructional method and to the subject were evaluated, those were also mostly positive. It should be noted that only 15 of the 108 studies were concerned with "college" mathematics. The use of ML and PSI within "Math Emporium" re-designs for developmental mathematics courses for underprepared students at universities in the U.S. has been lauded as a great step forward (Twigg, 2011). In a "Math Emporium", students are required to attend a minimum number of hours in a computer lab using online instructional and testing software. Tutors are on hand to answer questions and to provide guidance. This model is not without its critics, who point to problems in depth of learning (Almy, 2012) and deleterious effects on those who struggle

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with individualised instruction and find themselves isolated and unsuccessful (Cordes, 2014).

The adaptation of ML used in our institution for first-year mathematics classes is described in Groen et al. (2015) and Groen et al. (2016). The model is LFM with traditional lectures and tutorials, not a "Math Emporium". The implementation has evolved over time in response to contextual constraints and to the different priorities of subject co-ordinators and the skills of the teaching staff. While we are committed to ML as a philosophy, and are pleased with the improved pass rates, we realise that all assessment regimes can encourage responses from students that include undesirable behavioural choices. This is often not foreseen when new assessment policies and practices are constructed, and adjustments are required. In this paper, we describe a pilot study and point the way to evidence-based adjustments with the goal of improving student learning. A brief outline of the way ML is currently implemented into two of our largest subjects is provided, followed by a discussion of the concerns that have arisen, the analysis of data undertaken to find evidence for the strength of those concerns, and finally the innovations that we are planning in order to address the concerns.

Mastery Learning in our Subjects

In this paper, we report on ML in two first-year mathematics subjects that are taken by all first-year Engineering students at our institution. The first subject, Mathematical Modelling 1 (MM1), includes complex numbers, vectors and 3D geometry, calculus, first- and second-order differential equations, and modelling using these concepts. The second subject, Mathematical Modelling 2 (MM2), builds upon MM1 and includes functions and calculus of functions of several variables, partial derivatives, optimisation, multiple integrals, and statistics, with an introduction to inference and linear regression. The weekly learning activities of the first subject consist of two one-hour lectures, a one-hour tutorial, and a one-hour computing lab. Eight of the ten computing labs are allocated for mastery tests. The format of the second subject differs in that it has two 1.5-hour-long lectures, one on mathematics and one on statistics. The tutorials and computer labs are similar. The subjects are usually taken in consecutive semesters. First, we report on the cohort of 418 students who completed the first subject in first semester, Autumn 2016, and the follow-on subject in the next semester, Spring 2016. Second, we report on the cohort of 76 students who completed MM1 in Spring 2015 and MM2 in Autumn 2016.

As described in Groen et al. (2015, 2016), ML was introduced in 2014 to improve student learning by ensuring that students mastered the basics of these subjects, which in turn led to improved pass rates. Other positive outcomes included lower examination stress as the final examinations were made optional. Students required at least 80% on each of four mastery tests during the semester, with three opportunities given for students to achieve this level. The tests were taken online under supervision in computer labs. Students were able to see their results for each test as soon as they submitted their answers. Students who achieved 80% to 100% on the mastery tests were allocated marks of 50% to 62.5% on a linear scale towards their final mark for the subject. To earn more than 62.5% and qualify for a Credit, Distinction, or High Distinction, students sat the optional final examination, which tested more complex problem-solving skills.

During 2016, concerns were raised by members of the teaching team that some students were reporting that they found the online mastery tests to be too easy and that preparation to pass the tests did not prepare students for the final examination. There was also a concern that the nature of the tests encouraged surface learning and pattern matching. It was noticed in MM1 that attendance at tutorials dropped as the semester progressed, with low attendance towards the end of semester, when the emphasis was on preparing students for the final examination. In MM2, tutorial attendance dropped significantly after the first few weeks, remaining low for the rest of semester. Participation in the optional final examination in MM2 was lower than anticipated, based on the participation in MM1. To investigate the situation, a plan was put in place to collect and analyse relevant data.

Research Questions, Data Collection, and Analysis

After discussion with the subject co-ordinators of the relevant subjects, the following were formed as our research questions:

1. Is there any evidence that the students' experiences in the ML assessment contribute to a lack of interest and effort in the final examination?
2. What do students say about the perceived level of difficulty in the mastery tests?

The data available for us to investigate these questions consist of:

- All student assessment marks in relevant subjects
- Subject Feedback Survey (SFS) data for relevant subjects (Likert scale questions and open response questions)
- An opt-in survey for MM1 Autumn 2016 (76 students responded, of whom 65 took MM1 in Autumn semester 2016 and MM2 in Spring semester 2016)

Analysis for Students in MM1 Autumn 2016

For the sequence of subjects MM1 in Autumn followed by MM2 in Spring semester, Table 1 indicates the participation levels in the optional final examinations.

Table 1
Final Examination Participation of Students Who Attempted MM1 and MM2 Consecutively in the Autumn and Spring Semesters of 2016

	Did not sit MM2 exam	Sat MM2 exam	Total
Did not sit MM1 exam	36	8	44
Sat MM1 exam	79	295	372
Totals	115	303	418

From the data in Table 1, we see that 372/418 or 89% of these students sat the final examination in MM1, but only 303/418 or 72% chose to sit the examination in MM2. Given that the assessment regime requires students to sit for the final examination in order to access grades higher than a Pass, this is a concerning trend. A possible interpretation is that students in the second subject are less likely to target their efforts towards achieving higher grades. Inspection of the student comments in the SFS for MM2 did not shed any light on this issue.

The data shown in Figure 1 are the distributions of marks in the mastery tests (MT) and the final examinations for students who attempted both examinations. For these students, the distributions reflect some of the differences in the way assessment was organised in each subject. For MM1, students were allowed a second attempt at each individual question in the mastery tests in case they had made a typing error. Many students were able to achieve 100% scores on the mastery tests. This was not the case for MM2. In MM1, students were allowed an A4 sheet of self-prepared notes in the final examination, but in MM2, a formula sheet was provided. Overall, the marks obtained in MM2 were lower than in MM1, and the examination marks in MM2 were skewed to the right.

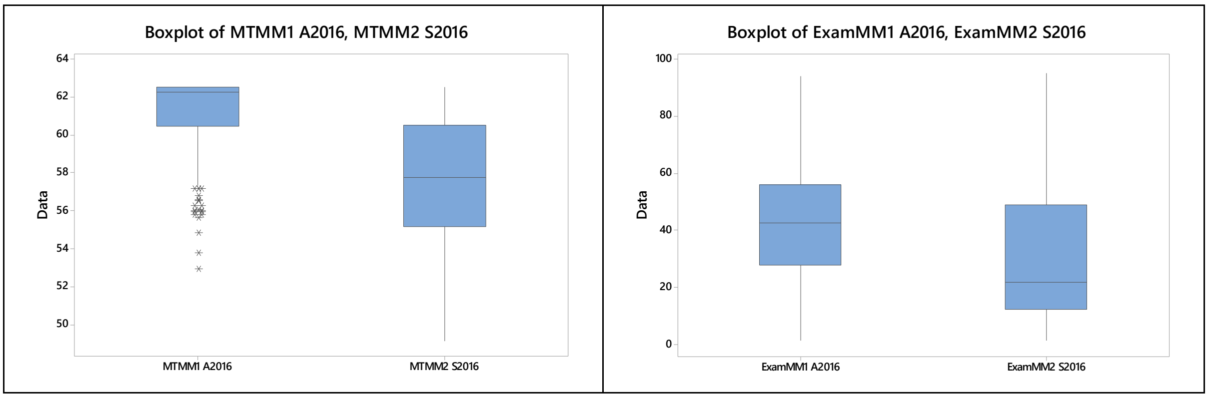

Figure 1. Performance in mastery tests (MT) and in final examinations ($N = 295$).

To further investigate student performance on the final examinations, scatterplots were prepared, which compared mastery test (MT) scores with final examination scores for the 295 students who sat for both examinations in these consecutive semesters (see Figure 2).

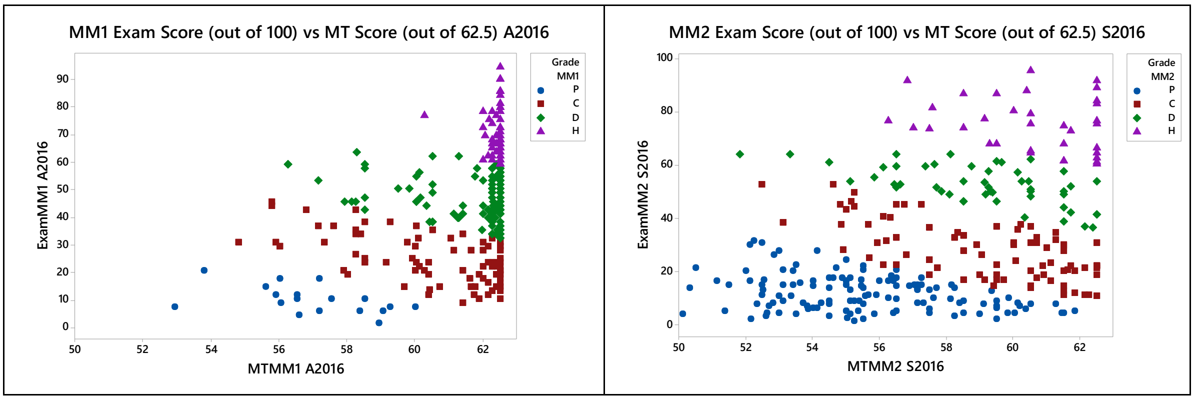

Figure 2. Performance in final examinations vs. performance in mastery tests (MT) ($N = 295$).

Remembering that a larger proportion of MM1 students (than MM2) chose to sit the final examination, we suggest that this may have been a strategic choice, as the marks that students had earned for the mastery tests were so high that only a few more marks on the final were required to achieve a Credit. In MM2, however, the mastery test distribution was wider and more marks were required in the final examination for a Credit. It may be that students in MM2 made a strategic decision not to prepare for or to sit the final examination, if they decided not to aim for a grade higher than a Pass. We can also speculate that, as many students gained full marks on the mastery tests in MM1, they may have assumed that the final examination would be easy. From Table 1, we see that 79 students sat the MM1 final examination, but not the MM2 final examination. Looking at the mastery test scores of these students, we found that many of these students were using third attempts to pass the mastery tests and these were held towards the end of semester. It is possible that these students found that the difficulty of the mastery tests in MM2 deterred them from attempting the final examination, either because they ran out of study time or because they felt the material would be too difficult. It must be remembered that these subjects have large enrolments and it is unlikely that the same motivations regarding the final examinations apply to all students.

The second research question focuses on the perceived level of difficulty of the mastery tests. In the MM1 SFS, students had the opportunity to respond to the questions "What did you particularly like in this subject?" and "Please suggest any improvements that could be made to this subject." While there were many positive comments about the features of ML (that it reduced examination stress, kept students learning throughout the semester, and allowed students to re-sit to improve their marks), three students commented that they found the tests too easy, and each had something different to add to this:

> Also, the mastery quiz assessment is too easy and I don't think should be weighted more than the exam as it kinda gives of a feeling that the content in the mastery test is all we need to know.

> There should be assessment tasks that cover more difficult content instead of the mastery tests, as this will provide a better indication on how prepared we are for the final exam.

> Mastery tests were too easy, at least one or two harder questions in mastery would have pushed people to actually learn how to do difficult questions rather than just breezing though and not needing to work for the final exam. (All comments from MM1 SFS Autumn 2016)

In the MM2 SFS, there were also appreciative comments about the way that ML provided motivation to study throughout the semester. Comments that the tests were too easy were rare; however, it was clear that some students were aware that others had found ways to pass with shallow learning and pattern matching:

> I feel like a lot of fellow students will not have benefitted in the way that they could have because they simply memorised all the variants of the questions in the mastery-tests. (MM2 SFS Spring 2016)

Towards the end of Autumn semester 2016, all students in MM1 were invited to complete an online survey, and this was an opportunity for staff to ask explicitly for feedback about various issues. There were 76 responses to the survey, many of which were incomplete. Table 2 shows the questions relevant to our research questions for this paper.

Table 2
Complete Responses to Optional Survey of MM1 Autumn 2016 Students (N = 60): Questions about Mastery Learning

Question	Percentage agree or strongly agree	Sub-group of 40 who sat both final exams
Doing the Mastery Test improved my confidence with maths in MM1 as the semester progressed.	90%	88%
I felt confident of doing well in the MM1 final exam.	60%	80%
I feel confident that I am ready to undertake MM2.	75%	78%

As shown in Table 2, many students report gaining confidence with mathematics by doing the mastery tests. However, this does not include feeling confident about doing well in the final examination. It should be noted that the sub-group of 40 volunteers were predominantly high-achieving students, not representative of the class as a whole.

What do the survey data reveal about the perceived level of difficulty of the mastery tests? Students were invited to respond in their own words to the question "What did you think about the mastery tests in MM1?" Answers were coded by two members of the research team working together. Codes were chosen to reflect the main opinions expressed. Regarding the ease of mastery tests, four said that they were "too easy" and 13 said "easy". As indicated, 17 of the 40 replies said that the mastery tests were easy. This is reinforced by the fact that many students finished mastery tests well under the time allocated.

Analysis for Students in MM1 Spring 2015

The second cohort described in this paper consists of the 76 students who attempted MM1 in Spring 2015 and MM2 in Autumn 2016. This second group consists mainly of students who took the preliminary subject Foundation Mathematics in Autumn 2015. Table 3 shows the numbers who chose to sit the final examinations in each subject. As for Table 1, we see a drop in the participation in the optional final examination in MM2.

Table 3
Final Examination Participation of Students Who Attempted MM1 and MM2 Consecutively in the Spring 2015 and Autumn 2016 Semesters

	Did not sit MM2 exam	Sat MM2 exam	Total
Did not sit MM1 exam	17	7	24
Sat MM1 exam	14	38	52
Totals	31	45	76

From the data in Table 3, we see that 52/76 or 68% of these students sat the final examination in MM1 but only 45/76 or 59% chose to sit the examination in MM2. This is lower than the 89% and 72% respectively from the larger cohort (Autumn 2016 MM1 to Spring 2016 MM2). A possible reason is that many of these students arrive at university underprepared for mathematics and need all their study efforts just to pass the mastery tests. The final examination was not mentioned by students in the SFS for these subjects, while the majority of the comments about ML were positive.

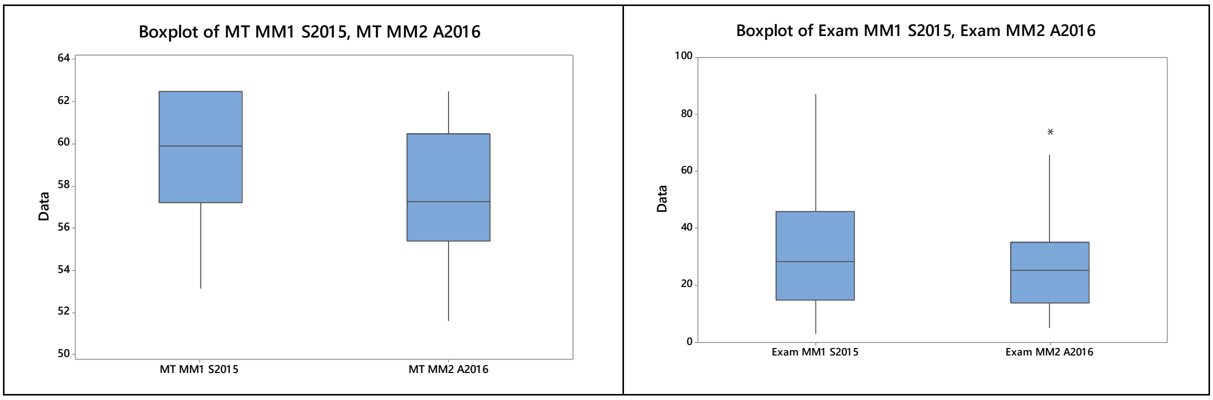

Figure 3. Performance in mastery tests (MT) and in final examinations ($N = 38$).

Figure 3 uses boxplots to show the scores of this cohort on the mastery tests and the final examinations. Comparing this with Figure 1, we conclude that this cohort found the mastery tests and the final examinations more difficult. There is a clear difference in the way that the MM1 mastery test scores of students in the larger cohort (MM1 in Autumn 2016, as shown in Figure 1) are skewed, with many finding the tests easy and scoring close to 100% on mastery tests. This may reflect the fact that many of the students in the larger cohort have studied more advanced levels of mathematics at school, and have an advantage for at least the first two mastery tests. In Figure 4, we note similar trends as in Figure 2. However, the MM1 mastery tests were not found by many students to be easy.

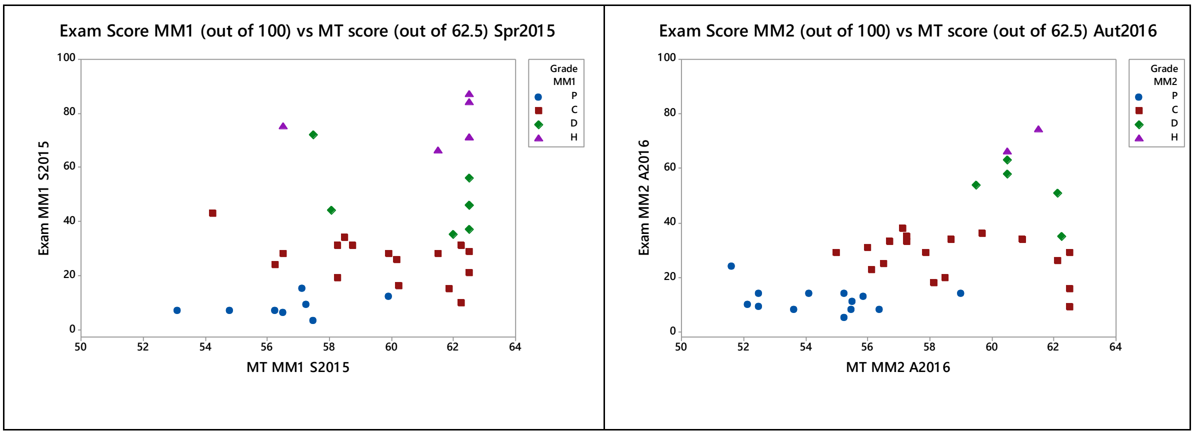

Figure 4. Performance in final examinations vs. performance in mastery tests (MT) ($N = 38$).

In the SFS for the smaller cohort, several students commented that they like the mastery test regime. For example:

> I really liked the mastery exam, each mastery exam tested our knowledge of a specific section of the math topics and thus helped us maximise our knowledge of each topic separately. This really helped as we had an overall understanding of each topic separately and then the final exam tested our knowledge as whole, very neat and knowledgeable setup. (MM1 SFS Spring 2015)

It was heartening to read this perceptive comment:

> I absolutely loved the fact that we got several chances to attempt the mastery tests, which allowed me to learn from my mistakes and be allowed a chance to rectify them… Being allowed the proper opportunity to improve where I lacked is something I will take forward from this subject. Thank you for making maths fun. (MM2 SFS Autumn 2016)

Discussion and Conclusions

Regarding our research questions, we can say the following:

1. A number of students from both cohorts chose not to sit the final examination. This could well indicate a lack of interest in higher grades and an unwillingness to engage with the more difficult material that was tested in the final examinations. The different shapes of the boxplots might also be an indicator. Further investigation is required.
2. In the survey, 17 students said that the MM1 mastery tests were easy or too easy, while none said they were too difficult. In the SFS for both subjects, most comments about the mastery tests focussed on positive features and/or that they were easy.

Implications for Future Practice

A major concern for the teaching team is that many students are not fully engaging with opportunities to learn more complex problem solving and modelling. This material is in the tutorials (which are not well attended, especially towards the end of semester), and is tested in the final examinations. To encourage students to engage more with this material, the following changes are being considered for 2017:

- Making the final examinations compulsory and worth more than the current 37.5% of the overall mark, and making the achievement of some minimum mark in the final examination a requisite for a pass.
- Ensuring that the mastery tests reward effort spent learning about problem-solving, and enlarging the sizes of pools of questions from which the questions are randomly drawn.
- Offering students a weekly test to see if they are prepared for more challenging activities in "advanced" tutorials. These would be problem-based, and encourage collaborative efforts with students working in groups.
- Encouraging students to better utilise existing student study support in the form of U:PASS (UTS Peer Assisted Study Success; University of Technology Sydney, 2017b) and the Maths and Science Study Support Centre activities including the drop-in room (University of Technology Sydney, 2017a).

Our concerns about the nature of the online computer-based mastery tests are not new. Bradley (2016) points out that "It is possible for a student to repeat the problem enough times to learn the procedure for getting the correct answer without mastering the intended mathematical concept" (p. 28). Finding the right balance of activities and assessment rewards to encourage both conceptual and procedural learning is an ongoing task.

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The Interplay Between Pre-service Teachers’ Intentions and Enacted Mathematical Content Knowledge in the Classroom

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Pre-service teachers (PSTs), like practising teachers, enact their mathematical content knowledge (MCK) in pursuit of instructional goals during lessons. In this study, I explored the relationship between six secondary mathematics pre-service teachers’ goals and the MCK that they chose to enact in 10 lower secondary algebra lessons. The findings indicate that PSTs enact stronger aspects of their MCK when they pursue goals that pertain to making mathematical connections rather than procedural mastery. Also, live classroom interactions with confused students can positively impact the instructional goals that pre-service teachers form and the quality of MCK that they enact.

Mathematical content knowledge (MCK) is recognised as a crucial type of teacher knowledge held by effective mathematics teachers. Pre-service teachers (PSTs) begin to develop their MCK specifically for the work of teaching while undertaking their university studies but improvements are needed in the way that this development occurs. A study of over 20 mathematics teacher education programs (including Australia) by Tatto, Lerman, and Novotna (2010) indicated that insufficient attention is paid to equipping PSTs with the MCK needed to provide high quality mathematics instruction. Mathematics education stakeholders in Australia, such as Professor Ian Chubb, past Chief Scientist of Australia (Chubb, Findlay, Du, Burmester, & Kusa, 2012), echo the concerns raised by Tatto et al. (2010). The creation of evidence-based mathematics teacher education programs, which Beswick and Goos (2012) argue are needed in Australia, require teacher educators to possess accurate gauges of PSTs’ MCK and, just as importantly, their willingness to enact certain aspects of that knowledge in front of a classroom of students. To contribute to this area of research, this study investigated the link between the MCK that six secondary PSTs shared with their students during an algebra lesson and the goals behind those actions.

Background to the Study

Specialised subject knowledge was highlighted three decades ago by Shulman (1987) as one of seven knowledge types necessary for teaching. Research undertaken more recently in the field of mathematics education reinforces MCK as one of the foundational knowledge types needed for teaching. In a German study of secondary practising teachers (Baumert et al., 2010), pedagogical content knowledge (PCK) was found to be the better predictor of student results. However, the researchers noted that PCK cannot be developed adequately without strong MCK (Baumert et al., 2010), highlighting the need for studies such as this one to focus on PST MCK. This paper focuses more specifically on the MCK of algebra that secondary PSTs hold and choose to enact in a live lesson.

Scholars have further developed Shulman’s notion of content knowledge needed specifically for mathematics teaching. For example, the regularly cited Mathematical Knowledge for Teaching (MKfT) framework developed by Ball and colleagues (Ball & Bass, 2009; Ball, Thames, & Phelps, 2008) emphasises the need for mathematics teachers to not simply know more mathematics content than students but to hold that knowledge in a qualitatively unique form that is well connected and unpacked. Within the topic of

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algebra, studies either have explored multiple knowledge types for teaching algebra (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) or have focused on a particular type of algebraic knowledge that teachers should enact (Driscoll, 1999).

Guided by the literature, this study explored three types of PST MCK of algebra, namely conceptual knowledge, procedural knowledge, and algebraic ways of thinking. Conceptual knowledge and procedural knowledge are two vital knowledge types popularised by Skemp (1976) and a decade later, Hiebert and Le Fevre (1986). Researchers such as Star (2005) have continued to develop these two knowledge types and it is widely accepted that teachers should ideally be enacting both knowledge types regularly in their teaching. Algebraic ways of thinking (AWOTS) are specific mathematical habits that form the third knowledge type investigated in this study. Here, three AWOTS noted by Driscoll (1999) and Harel (2008) that are of particular relevance to lower secondary algebra are addressed. The first is “manipulating with purpose”, which highlights how one manipulates symbols in algebra to achieve a particular mathematical purpose (Harel, 2008). Harel (2008) also identifies “algebraic invariance” as an important AWOT which recognises that in algebra, a purposeful manipulation can include changing one aspect of a mathematical object (such as the form of an expression) while holding a second aspect unchanged (such as the value of the expression). The third AWOT, “doing/undoing”, reflects one’s ability to understand a process so well in algebra that it can be reversed (Driscoll, 1999).

In this study, putting one’s MCK into practice is referred to as enacting that MCK. A review of mathematics education literature revealed that measures of PST MCK have usually occurred in contexts where the participants enacted their MCK outside the classroom, in the form of written tests and interviews (Ball, 1990; Even, 1993). These measures give a strong indication of the MCK that PSTs may take into the classroom but they do not capture which aspects of that MCK PSTs may heavily emphasise in their teaching or perhaps which aspects they may avoid enacting. Lave and Wenger (1991) argue that knowledge is dynamic and located within a particular community of practice. In the mathematics classroom, teachers need to be able to use their MCK “in the ebb and flow of practice, responding quickly and accurately to student thinking” (McCrory et al, 2012, p. 601). There have been limited studies that have investigated PSTs’ MCK in a live classroom setting (e.g., Borko & Livingston, 1989; Thwaites, Jared, & Rowland, 2011) but as these studies did not focus solely on MCK enactment, the MCK was not systematically reported as it is in this study.

There are limitations in studying only the visible aspects of enacted MCK within a live lesson because the thoughts behind particular actions cannot be observed. Beswick and Goos (2012) argue that research is needed that provides detailed examinations of both the mathematical knowledge and thoughts that underpin teaching actions of PSTs. To capture a more detailed slice of PSTs’ MCK in action in this study, the goals that lay behind PSTs’ MCK related actions were also chosen for investigation. Teachers perform actions during their lessons in pursuit of a particular goal or goals, according to researchers of mathematics teacher decision making (e.g., Schoenfeld, 2010; Simon, 1995). The PSTs’ choice of goal, a critical aspect of the thinking that precedes MCK related actions, can provide additional insights about the MCK that they subsequently enact. Therefore, this study contributes to the limited research undertaken of enacted MCK of algebra and related thinking with the following research questions:

1. What MCK do secondary mathematics PSTs enact in lower secondary algebra lessons?
2. How do PSTs’ instructional goals impact the MCK they decide to enact?

Methodology

The results reported form part of a doctoral study (Daniel, 2015) that investigated six secondary mathematics PSTs' MCK related teaching actions and instructional goals. The PSTs comprised two females and four males aged in their early twenties, in their third or fourth year of study in a Bachelor of Education degree at a regional Queensland University. Ten secondary mathematics PSTs were originally invited by the researcher, who was also their lecturer for a mathematics methods course, to participate in the study. For ethical reasons, the researcher was not made aware of which PSTs had agreed to participate until the completion of the course. All the invited participants agreed to participate in the study but the findings presented in this paper, using aliases to protect the PSTs' identities, pertain only to 6 of the 10 participants who provided data relating to lower secondary algebra lessons during a practicum phase. The lessons focused on manipulating algebraic expressions and solving simple linear equations (six Year 8 lessons) and solving sets of simultaneous linear equations (four Year 10 lessons).

Qualitative data collection techniques were used to capture insights about the PSTs' MCK related thoughts and teaching actions in a live classroom setting. To investigate the MCK that manifested in PSTs' live teaching actions, lesson data were collected from the PSTs' practicum lessons via lesson observations, digital video footage, and lesson artefacts, such as lesson plans. To gather data related to the PSTs' goals, each PST participated in a stimulated recall interview within 48 hours of each lesson observation. Lesson excerpts from the video footage that featured sets of MCK related actions were chosen by the researcher prior to the interview. The excerpts were retained for three reasons: (a) they typified the MCK that the PST enacted throughout the lesson, (b) they featured surprising MCK related actions in the opinion of the researcher, an experienced secondary mathematics teacher, and/or (c) they featured teaching actions where the researcher sensed a PST may provide a particularly rich or insightful reflection. During the interviews, the reduced lesson footage was played to each of the participants who were encouraged to pause the footage and reflect on their thinking at the time. The researcher also paused the footage at particular times and prompted the participants with questions such as, "What were you thinking here?" during the interview. The interviews were videotaped with the video camera focused on a laptop screen containing the lesson excerpts so that the on-screen footage and the participants' retrospective thoughts could be recorded simultaneously.

After the lesson and interview data were collected, the lesson data were reduced further by keeping only those excerpts of lesson footage for which the participants provided commentary during the interview. If a PST did not comment on a set of teaching actions, which happened quite rarely, those actions were discarded for further analysis because it was not possible to discern any connections between the participants' MCK related thoughts and actions. The remaining lesson and interview data were then analysed for evidence of enacted MCK and for any goals identified by the PSTs.

The goals shared by the participants during their interviews were used to partition the lesson data into units of analysis called "episodes". In this study, an episode is defined as a set of one or more MCK related teaching actions that a PST undertakes in pursuit of one or more instructional goals. The researcher noted that during the stimulated recall interviews, the participants either explicitly stated or strongly implied one or more instructional goals as they reflected on their actions in the footage. Across the ten algebra lessons, the researcher discerned 174 goals from the participants' commentary. Those 174 goals, referred to as episode goals, were used to demarcate the footage upon which the participants commented into 137 episodes, underpinned by either one or two episode goals.

The episode data were analysed for the type of MCK enacted by the participants. Literature pertaining to the MCK needed for teaching algebra (e.g., Ball & Bass, 2009; Ball et al., 2008; Harel, 2008; Hiebert & Le Fevre, 1986; McCrory et al., 2012; Skemp, 1976; Star, 2005) was used to develop an initial coding framework to analyse the MCK manifesting in the episodes but the researcher reconfigured several categories and subcategories as patterns began to emerge in the data. The interview data associated with each episode were inductively coded for the instructional goal(s) that the PSTs were pursuing when they enacted certain MCK and then categorised using pattern seeking techniques. It was also noted whether the PSTs' goals appeared to have been formed prior to the lesson or if they were the result of live classroom interactions prompted by an event such as a student question. Finally, cross-variable analyses were undertaken to determine trends in the presence of particular goal types and different types of enacted MCK.

Results

Procedural Knowledge Overshadows Other Types of Enacted MCK

The participants explicitly shared three types of MCK of algebra when teaching, namely algebraic ways of thinking, conceptual knowledge, and procedural knowledge. However, the types of MCK were not enacted in similar proportions because procedural knowledge heavily dominated the teacher talk and written work in the lessons. Figure 1 provides an overview of the MCK that the participants enacted in isolation or in combination in the 137 episodes examined.

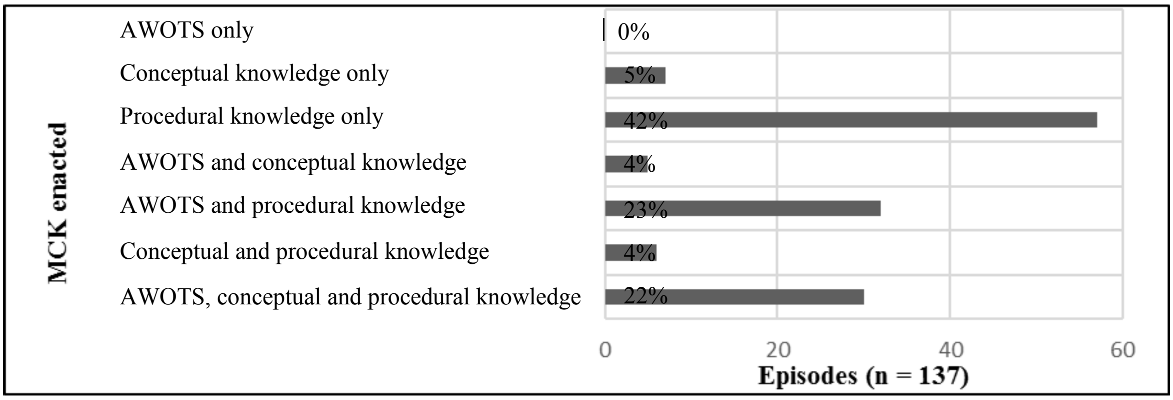

Figure 1. Presence of MCK types in teaching episodes.

The heavy emphasis by the participants on teaching procedures is reflected in Figure 1, which shows that 91% of all episodes (n = 125) evidenced procedural knowledge. The most common manifestations involved the PSTs explaining or questioning students about performing procedural steps. Given the high proportion of episodes with only procedural knowledge (42% of all episodes) and the view that substantial conceptual knowledge and AWOTS enhance the teaching of procedures (e.g., Harel, 2008; Skemp, 1976), the PSTs did not enact conceptual knowledge or AWOTS often enough in their algebra lessons.

When PSTs chose to enact AWOTS, they only did so in combination with another type of MCK which was usually procedural knowledge. The addition of AWOTS in the MCK that the participants enacted tended to support their explanation of procedural steps. One participant, Ben, for example, repeatedly enacted the "manipulating with purpose" way of thinking, which the PSTs enacted more often than any other way of thinking (52% of the 67 episodes featuring AWOTS). In one episode, Ben commented to his students, "What do we want to do to the x? Isolate it. So, you wanna get it on its own." Overall, the AWOTS appeared to be explicitly taught by the participants as a supportive mechanism to assist students to develop procedural mastery, extending the earlier finding that not only were AWOTS not enacted often enough but additionally, not deeply enough by the PSTs.

Conceptual knowledge was the type of MCK that the PSTs enacted the least, featuring in only 35% of all episodes. Unfortunately, across the 10 lessons, no participant spent a significant amount of time specifically addressing a mathematical concept. Rarely enacted in isolation, conceptual knowledge tended to be presented alongside either AWOTS, procedural knowledge, or both knowledge types. The conceptual knowledge that the PSTs explicitly taught consisted of their knowledge of arithmetic operations and of algebraic objects. The participants spoke of the different representations of arithmetic operations (e.g., words and symbols) and conceptual features of algebraic objects. One participant, William, highlighted the concept of equivalence when he spoke explicitly about the meaning of the equals symbol in an equation. He explained to his students as he gestured to the expressions on each side of the equation, $a + 6 = 13$, written on the board, "It means it's like the same on both sides. So, we're saying that this half is the same as this half."

Although the presence of conceptual knowledge in episodes strengthened the mathematical quality of those episodes, all too often conceptual knowledge featured in brief supporting statements that were interspersed amongst longer procedural explanations in the same way that AWOTS were enacted. This resulted in few instances where strong, explicit connections were made between algebraic procedures and related concepts and ways of thinking. An analysis of the participants' instructional goals that lay behind these stronger and weaker MCK episodes revealed important connections between certain goal types and the subsequent MCK enacted. These connections are now described.

Different Goal Types Lead PSTs to Enact Stronger and Weaker Aspects of Their MCK

The PSTs referred to 174 episode goals when they commented on the MCK that they chose to enact in the episodes. Those goals were inductively sorted into nine episode goal types, shown in Table 1, then categorised into two major goal types, namely content focused goals and student focused goals. Content focused goals, comprising the first five of the nine goal types in Table 1, were goals where the participants were intent on teaching particular mathematical content, regardless of their students' understandings, such as a goal to explicitly teach a certain algebraic procedure during a lesson. Student focused goals housed the final four goal types in Table 1 and reflected the participants' desire to present content that aligned with their students' mathematical knowledge, such as a goal to address a particular point of confusion for a student. Table 1 also shows the frequency with which the participants enacted each type of MCK when they pursued each type of goal in an episode. Due to the pervasive nature of procedural knowledge enacted in the majority of episodes, goals referring to procedural knowledge episodes within the table pertain only to those episodes where procedural knowledge was enacted in isolation within an episode so that trends between PST goal types and enacted procedural knowledge can be revealed.

The table reveals that PSTs tend to enact only procedural knowledge in pursuit of the content focused goal types "Develop students' knowledge of procedures" (58% of episodes with this goal type) and "Associate procedure with certain types of solutions" (60% of episodes with this goal type).

Table 1
Type and Relative Frequency of Episode Goals for Episodes with AWOTS, Conceptual Knowledge and Only Procedural Knowledge

Type of episode goal	Total goals for type	Goals for AWOTS (% of total goals by type)	Goals for conceptual knowledge (% of total goals by type)	Goals for procedural knowledge only (% of total goals by type)
Develop students' knowledge of procedures	60	22 (37%)	8 (13%)	35 (58%)
Teach students appropriate use of mathematical language	16	10 (63%)	13 (81%)	1 (6%)
Connect procedure with a concept	12	8 (67%)	8 (67%)	2 (17%)
Associate procedure with certain types of solutions	10	2 (20%)	3 (30%)	6 (60%)
Connect procedure with mathematical purpose	9	9 (100%)	5 (56%)	0 (0%)
Address student confusion	32	23 (72%)	18 (56%)	7 (22%)
Value and/or encourage student contribution	12	4 (33%)	3 (25%)	8 (50%)
Gauge student knowledge	12	4 (33%)	3 (25%)	6 (50%)
Avoid student confusion	11	6 (55%)	5 (45%)	4 (36%)
Total	174	88 (51%)	66 (38%)	69 (40%)

Additionally, when the PSTs aimed to align their teaching with their students' understandings by aiming to "Value and/or encourage student contribution" or "Gauge student knowledge", they again enacted mainly procedural knowledge. These findings suggest that when PSTs focus on teaching their students algebraic procedures in particular or attempt to ascertain their students' knowledge regarding algebraic procedures, their attention is not necessarily drawn concurrently to the concepts and ways of thinking that support those procedures and their teaching remains mathematically superficial as a result.

More encouraging findings revealed in Table 1 pertain to the goals that led the PSTs to enact stronger aspects of their MCK. In this study, stronger MCK refers to the PSTs sharing conceptual knowledge and/or AWOTS in addition to procedural knowledge. Table 1 reveals that two content focused goals that focus on making mathematical connections explicit appear to be constructive ones for PSTs to form. PSTs enacted conceptual knowledge and/or AWOTS in the majority of episodes underpinned by the goals, "Connect procedure with a concept" and "Connect procedure with mathematical purpose". The desire to make a mathematical connection explicit appears to be one that PSTs should be encouraged to prioritise, given the relatively high proportion of episodes with conceptual knowledge and AWOTS that followed. This finding has implications for teacher educators.

A particularly interesting result was the PSTs' enactment of stronger MCK episodes when they attempted to address student misconceptions. Table 1 shows that in the majority of episodes with the goal, "Address student confusion", PSTs enacted AWOTS (72% of episodes with this goal) and/or conceptual knowledge (56% of episodes with this goal). It was noted during the data analysis phase that the majority of the "Address student confusion" episodes occurred spontaneously, prompted by a student error or question that manifested during the lesson. It is therefore quite surprising that some of the PSTs' strongest MCK episodes occurred when they taught unplanned episodes with only moments to form goals. A possible reason for this phenomenon may lie in the PSTs' perspectives of what MCK they believe they need to share with their students. During unplanned episodes, the PSTs based their MCK related decisions, in part, on their interactions with their students. Their own MCK related teaching actions in one episode prompted their students to provide verbal and written contributions in response to those actions. The student contributions, in turn, informed the PSTs' choice of MCK in subsequent episodes, reflecting the cyclic nature of instructional decisions, teaching actions, and live classroom interactions as espoused by Schoenfeld (2010) and Simon (1995) and as illustrated with the following example.

One participant, Sam, presented a careful explanation about how to solve an equation using the backtracking method after having used the balance method in a previous lesson. Sam's explanation made no explicit mention of any AWOTS and, of particular note was his failure to share the purpose of performing the procedure (the "manipulating with purpose" AWOT). After Sam completed his explanation, he was confronted with one student's question, "Is that how we get the answer, Sir?" Sam then offered an explanation, rich in AWOTS, about choosing different methods to solve equations as he confirmed his student's hunch. At this point, the "manipulating with purpose" AWOT was no longer hidden but was highlighted by Sam as a key feature of the procedure. When reflecting on the episode in his interview, Sam laughed at the footage, commenting, "Well I thought, 'Why, why, why was I doing it all this time if not to get to an answer!'". Sam's surprise at having to spell out the point of the procedure was echoed by three other participants during their interviews, suggesting that live exchanges with students can direct PSTs to share aspects of their MCK that might otherwise remain hidden from their students.

Discussion and Conclusion

Existing research suggests that consistently stronger MCK than what the PSTs enacted is needed for more effective teaching. Hence, the findings reinforce calls in Australia for PSTs' MCK to be further developed during their tertiary studies; however, development of only MCK is not enough. The findings suggest that secondary PSTs' MCK related goals do not always lead them to enact the strongest aspects of their MCK of lower secondary algebra. Teacher educators must pay attention to instructional goals that lead PSTs to share weaker and stronger aspects of their MCK in mathematics methods courses and practicums. Those goals can be used by teacher educators as a valuable lens for reflection on the quality of MCK that PSTs enact. In this way, the development of PSTs' MCK related decisions and actions might be developed in a more intentional manner.

This study was limited to one contextual setting, one mathematics topic, and only a small number of participants. The findings are also limited by the loss of data that occurred when participants' enacted MCK that could not be connected to an instructional goal was discarded. The findings do offer, however, a rich slice of both the visible and less visible aspects of secondary PSTs' live teaching practice. The study highlights the methodological benefits of studying the MCK that PSTs enact in live teaching actions alongside the goals that lie behind their MCK-related actions.

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Exploring Ways to Improve Teachers' Mathematical Knowledge for Teaching with Effective Team Planning Practices

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The process of planning mathematics lessons is complex and presents challenges for teachers, specifically in their Mathematical Knowledge for Teaching (MKT). In this paper, I describe findings from case study research in which a Year 1 teaching team engaged in professional reading and used a specific planning proforma to enhance their MKT. Teachers reported feeling more informed in their planning decisions and overall, reported positive changes to their team planning practices which impacted their classroom teaching.

The implementation of the Australian Curriculum: Mathematics (AC:M) offers an opportunity to promote a reform teaching and learning environment that gives students opportunities to develop robust mathematical understandings. In implementing the AC:M, teachers enact an important responsibility in making decisions about "how best to introduce concepts and processes, and how to progressively deepen understanding to maximise the engagement and learning of every student" (Australian Curriculum Assessment and Reporting Authority [ACARA], 2012 p. 19). These decisions, which are often made during planning include, but are not limited to the consideration given to curriculum, tasks, pedagogy, assessment and differentiation. Furthermore, these decisions have the power to directly impact student thinking and learning about mathematics (Kilpatrick, Swafford, & Findell, 2001).

Connected to these decisions is the view that Australian primary mathematics teachers are faced with ever-increasing demands on their planning (Clarke, Clarke, & Sullivan, 2012a). Despite efforts by teachers to plan experiences that encompass the complexities of mathematics teaching, the day-to-day realities of planning for such reform teaching is a difficult task that brings additional challenges, part of which are connected to teachers' MKT (Davidson, 2016). This is especially important given the links between teacher knowledge, effective teaching and student outcomes (Clarke et al., 2002).

An assumption underpinning the research reported here is that effective teaching is preceded by effective planning. That is, teachers are best able to cater for students' needs when they have a clear vision of what they want their students to learn and how they will come to learn it (Hattie & Timperley, 2007). More specifically, teachers' planning decisions are informed by their MKT. In this paper, I describe the findings of a case-study of a Year 1 teaching team's planning practices that supported this group of teachers to improve their MKT and subsequent classroom teaching.

The Connection between Planning and Mathematical Knowledge for Teaching

When planning units of work, Australian teachers often engage in four preparatory actions: (1) reviewing curriculum documents to identify the important ideas, (2) checking available resources, (3) drawing on assessments of student readiness, and (4) drawing on the experience of colleagues (Sullivan, Clarke, & Clarke, 2012b). Based on these four

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actions, teachers establish specific learning goals that inform teachers' choice and sequence of learning tasks which informs their decisions about teaching and assessment. It is reasonable to assume that each of these stages is complex and is shaped, in part, by teachers' MKT. MKT describes the knowledge used by teachers for the teaching of mathematics (Ball, Thames, & Phelps, 2008). It includes two major categories: subject-matter knowledge and pedagogical content knowledge, that is, teachers' knowledge *of* mathematics and their knowledge of *ways* of teaching mathematics.

One aspect of teachers' MKT is the planning decisions that teachers make in selecting the tasks they will teach (Ball, Thames, & Phelps, 2008). While tasks are often viewed as critical in creating potential for student learning (Anthony & Walshaw, 2007), it appears the parameters for selecting such tasks and teachers' capacity to decide on the relevance of tasks is varied (Sullivan et al., 2012a). Furthermore, some teachers experience difficulties in articulating the "big ideas" that inform their teaching (Clarke et al., 2012b). Difficulties by teachers in articulating 'big ideas' was also found by Roche, Clarke, Clarke, and Sullivan's (2014) extensive analysis of primary teachers' unit plans, which showed a high level of variation in the identification and phrasing of key ideas for units of work: an inference being that teachers' understanding of key mathematical ideas will also impact their selection and use of appropriate tasks.

Connected to the way MKT influences planning decisions is the suggestion that teachers may feel constrained to teach in certain ways or use particular tasks if they anticipate negative student responses such as a lack of persistence or risk-taking (Sullivan, Walker, Borcek, & Rennie, 2015b). In response to these anticipated reactions, it has been proposed that teachers may reduce the cognitive demands of the task which has implications for classroom culture (Rollard, 2012) and student dispositions (Dweck, 2000).

However, the provision of lesson planning documentation has shown potential to support teachers in dealing with such constraints (Sullivan et al., 2015a). Sullivan and colleagues (2015a) provided written lesson suggestions to teachers including curriculum links, pedagogical considerations, and task modifications, with teachers reporting that such advice was helpful to their planning and teaching. It should be noted the provision of such documentation was not intended to act as a script but rather to support teachers in anticipating possible directions a lesson might take. Mutton, Hagger, and Burn (2011) refer to this type of planning as "visualisation, rather than planning as a template" (p. 408), and this a focus of the research reported below.

The findings reported below are intended to address the following research question: What approaches effectively support teachers' Mathematical Knowledge for Teaching during team planning and subsequent teaching?

Research Design

The overall project is informed by a conceptual framework that proposes that teacher planning and classroom teaching are a function of their beliefs about mathematics, their knowledge about mathematics and pedagogy, and anticipated constraints (Sullivan et al., 2015b). The focus of the findings presented below is on the aspect of the model regarding the connection between teachers' MKT and their planning decisions.

This study is framed by an instrumental case study design (Stake, 2005) as it is intended to "provide insight into an issue…[and] facilitate our understanding of something else" (p.437). The issue in this case is primary teachers' planning processes, with a focus on team planning. It should also be noted that this case-study is located within the social-constructivist perspective which emphasises the role of personal and shared experiences in the learning process, and assumes a unique relationship between researcher and participants where reality, or “findings”, are co-created between the two and subject to individual interpretations (Creswell, 2007).

Given the complex and idiosyncratic nature of planning, specific criteria were not applied in selecting the case; rather, I sought a team of teachers from the same year level who worked together to plan mathematics on a regular basis, and who were committed to engaging with me as a researcher, in order to explore and improve their mathematics planning. The case in this study consists of the five teachers, most of whom were in their first four years of teaching, in the Year 1 teaching team at a large metropolitan government school servicing a diverse middle to upper class population.

I attended the Year 1 team’s mathematics planning meetings over a six-month period between the end of Term 2 (May) to end of Term 4 (November) as a participant-observer, with each meeting lasting one hour on average. My role consisted of asking questions, providing support, and suggesting approaches to the team’s mathematics planning. Here, I focus on two approaches: professional reading and the use of a specific planning proforma.

In addition to my participation in these meetings and the collection of relevant documentation that formed part of teachers’ everyday work, two surveys (June and November) and a one-on-one semi-structured interview (August) were conducted with each participant. All meetings and interviews were audio-recorded and transcribed. Detailed field notes and a researcher journal were also maintained. These data sources were coded and triangulated to identify emerging themes and provide thick description of the case (Stake, 2005). In terms of survey data, fixed items were analysed with the number of teachers presented as raw figures to give a sense of individual team members’ perceptions about their mathematics planning. Open responses were coded to identify emerging themes. The research questions and conceptual framework were also used to frame the analysis for the various datasets.

Findings

In this paper, I report on the findings in three sections: the first being preliminary findings that led to the identification and trialling of the two approaches that are reported on in the second section. The third section describes teachers’ reactions to the suggested approaches.

The Year 1 Team Before the Intervention

My work with the Year 1 Team commenced during a unit on patterns. During these initial meetings, much time was spent clarifying mathematical language and concepts. For example, in one meeting, approximately 15 minutes was spent clarifying the term “core”: Some teachers referred to it as a “stem”, while others referred to it as a “rule” or “the pattern”. Inconsistent use of terminology added to further confusion about whether the term “core” was intended for both repeating and growing patterns with the team making reference to numerical patterns only such as “2, 4, 6, 8”. It was at this point that I decided to intervene and provide clarification. In turn, this led to a team brainstorm about tasks, with teachers giving examples of tasks they had previously used.

Team members were also invited to complete a preliminary survey of their planning practices. In one item, I asked teachers to identify the most and least helpful aspects of the way their team plans mathematics. In terms of the most helpful aspects, the majority of responses centred on the team's sense of collegiality. For example, one teacher wrote:

> Everyone on the team is enthusiastic about planning and creating the best lessons for students.

In terms of least helpful, time and MKT were two most salient issues. For example, teachers wrote:

> Not always sure of learning sequences and order of areas to teach.

> Getting side-tracked with other discussions.

Overall, what I observed in my initial visits was a "brainstorm, document, reflect" cycle of planning. While these discussions were well-intended, they were often ad hoc, task-centric and mainly informed by teachers' own experiences and web resources, rather than other available text resources such as text books and teacher reference materials.

The Intervening Period

The next stage in the study was to suggest approaches with potential in supporting the Year 1 teaching team in overcoming identified challenges in their MKT. The focus of this paper is exploring the potential of teachers engaging in professional reading prior to planning and the provision of a team planning proforma to promote anticipatory thinking (Mutton et al., 2011).

To inform their upcoming unit on shape the team agreed to read the chapter "Developing Geometric Reasoning and Concepts" (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). This text was selected as it is commonly used in teacher education courses. It is written in a teacher-friendly manner and provides a balance of theory and practice: explaining key ideas and trajectories of learning, as well as providing task suggestions.

To support the team in unpacking the content of the professional reading, a specific planning proforma, intended as a working document, was used. The proforma contained the following sections:

- Key mathematical ideas (Students know and understand…)
- Key skills and strategies (Students are able to…)
- Rich assessment task
- Language focus
- Curriculum standards and relevant proficiencies (Below, At, Above)
- Sequence of development (Summary of learning trajectory)
- Lesson structure
- Individual Lessons including: learning intention, success criteria, launch (introduction), explore (main task with enabling and extending prompts), summary (ways to conclude the lesson) and required materials

The meeting commenced with a general conversation about the reading. Some teachers made comments about their own knowledge including:

> It's that thinking when you're starting to get into this level of thinking, unless you're a really spatial thinker, it takes quite a lot to, I mean, it's not my style of learning. I'm in the zone of confusion.

Comments were also made about the interconnectedness of concepts:

> It was very interesting…Well, just things that I wouldn't have necessarily connected with shape that ended up coming into it. Like, direction and thinking about angles. We're not even calling them

angles - calling them square corners and things like that for the younger kids. I would never have thought that we would touch on that.

Discussions on language were also prominent, such as:

So, it just depends how deep it would go - using that informal language. So, some kids might be using parallel, but some kids might just be saying side-by-side or next to each other.

From these discussions, a natural progression into unpacking relevant curriculum statements and identifying the key ideas and skills ensued. The AC:M (ACARA, 2014) content descriptor for Shape at Year 1 states:

Recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features (ACMMG022)

Taken at face value, this statement can appear quite simplistic. However, upon reflecting on the reading, the team identified the following key ideas which formed the foundation of their unit. These were listed as:

Properties: We can describe shapes by their features.

Visualisation: Shapes can have the same name but look different.

Transformation: We can change the way a shape looks by pushing, flipping or turning it.

Teachers then identified key skills that supported them in gaining clarity on what they would see their students doing such as:

Students can sort shapes that are similar and different, and explain why it belongs/doesn’t belong to that group.

The team also developed a detailed language section including a list of terms such as circle, triangle, orientation, and horizontal, as well as definitions that arose from the discussion. For example, an online maths dictionary was consulted to clarify the following:

In geometry, term 'face' refers to a flat surface with only straight edges, as in prisms and pyramids (e.g., a cube has six faces). Curved surfaces, such as those found in cones, cylinders, and spheres, are not classified as faces. (Eather, 2017).

This was followed by the development of a two-part assessment task to identify student learning needs and was described as follows:

(1) Use at least 2 pattern blocks to make a bigger shape. Draw your shape. Tell me everything you know about your shape. (2) Using some pattern blocks, how would you sort and group these shapes? Explain your groups.

The team conducted the assessment and discussed student responses at the following week’s meeting, using the reading to guide the analysis of student responses. These discussions were critical in informing subsequent decisions on task selection and sequence: in particular, focusing the initial phase of the unit on 2D shapes, rather than teaching 2D and 3D shapes simultaneously. For example, based on the reading and student responses to the pre-assessment, the team decided to focus the first four lessons on exploring properties of 2D shapes in depth, such as properties of quadrilaterals and triangles, which came under the key ideas of “Properties” and “Visualisation”.

As the unit progressed, there was a noticeable shift in the focus of teacher conversations during their planning meetings. For example, by our fifth meeting together, the team was thinking carefully about curriculum statements, together with the key ideas to make careful decision about task-types and accompanying resources. For instance, while discussing a potential lesson on composing and decomposing shapes, the team was experimenting with the available pattern blocks to anticipate student responses and it became apparent that the triangles in the set of pattern blocks could be made to form shapes such as trapeziums and hexagons rather than squares as the team had originally intended.

Teachers also began commenting on how they had been surprised at some of the student learning they had observed. For example:

> …it's quite nice because my kids are really - well most of them are really kind of getting parallel lines and it's…well I never would've thought to introduce like – "that's a *parallel* line…"

The planning cycle evolved over the six-month period, with the team continuing to reference professional readings and make use of a planning proforma to inform their understanding of curriculum statements, key ideas, student assessments, task types, and lesson sequences.

The Year 1 Team After the Intervention

Towards the end of my time working with the Year 1 teaching team, the teachers were invited to complete an exit survey. The survey comprised of both Likert scale-type items and free format questions to which teachers could respond. In one of the survey items, I asked teachers to think about the way their team plans mathematics and indicate the extent of their agreement from strongly disagree (SD) to strongly agree (SA) with the statements before and after their involvement in the project. In Table 1, I present the responses to some of the statements pertaining to MKT and documentation. Given the small number of responses, the numbers of SD, disagree (D), and unsure (U) responses were aggregated as were the responses to agree (A) and SA.

Table 1
Teacher Exit Survey: Response to Survey Items About Teachers' Planning Practices (n=5)

Statement		SD, D, U	SA, A
The way my team plans is helpful to my teaching.	Before	3	2
	After	0	5
The documentation my team produces is clear and helpful to my teaching.	Before	3	2
	After	0	5
I am confident in my understandings of the big ideas I am about to teach.	Before	3	2
	After	0	5
My knowledge of mathematics is good enough that I can plan whatever types of lessons I like for this level.	Before	3	2
	After	1	4
My knowledge of ways of teaching mathematics is good enough that I can plan whatever types of lessons I like for this level.	Before	4	1
	After	1	4

The most positive responses can be seen in the changes to the team's perceptions about how helpful their mathematics planning is to their teaching including the documentation their team produced. Such positive changes were also noted in teachers' confidence in their understanding of the big ideas they were going teach. It is also encouraging to see positive shifts in teachers' perceptions about their MKT, that is, teachers' knowledge of mathematics and ways of teaching mathematics.

Teachers were also invited to describe the impact, if any, their participation in this research project had on their team and individual mathematics planning. All five participants responded, and interestingly, each team member described, amongst other things, the impact of professional reading and documentation as well as feeling more confident in their teaching practice. Such comments included:

> The new planner helps to organise and identify exactly what is happening in each math unit. Raising awareness to use textbooks has also been very beneficial for planning. Also, having a working document helps to keep the team up-to-date.

> I now feel a lot more confident in teaching mathematics and understanding that simple smart activities end up being more beneficial and helpful to me and the students rather than glitzy fun crafty ones.

Overall, teachers' responses indicate their perceived improvements in the way they planned mathematics as a team, which had a positive impact on their perceived success as teachers. These improvements can be partly attributed to their engagement with professional reading and the use of a planning proforma.

Discussion and Conclusion

The findings reported above provide some insights into the ways members of a Year 1 teaching team were able to develop their MKT through effective team planning practices. For this particular group of teachers, engaging in professional reading prior to commencing planning an upcoming unit of work and the use of a specific planning proforma supported teachers in making more informed planning decisions. Teachers also reported positive changes to their classroom teaching.

In terms of the research question, the provision of professional reading and an accompanying planning proforma supported teachers in shifting from a task-centric planning approach to engaging in anticipatory planning practices. Unpacking curriculum statements and key ideas, including mathematical language supported teachers in thinking more critically about student assessments, task-selection, resources and lesson sequence. Most significant were teachers' perceived growth in their MKT, which is an important finding given being "confident in the knowledge of mathematics at the level they are teaching" (Clarke et al., 2002, p. 13) contributes to improved student achievement.

It was also pleasing to hear teachers commenting about their surprises about the sophisticated language their students were using, such as "parallel" and "symmetrical", which has implications for the ways such planning practices can facilitate teachers' high expectations of their students and promote a reform teaching and learning environment.

While two approaches, professional reading and a planning proforma are discussed here, it is important to bear in mind the idiosyncratic nature of planning both to schools and the teams of teachers who work together in those schools. Various factors such as school culture and leadership contribute to any team's ability to plan in an effective and sustainable manner (DuFour & Eaker, 1998). It is also important to consider the "ownership" this team had over their mathematics planning and the implications this might have in a new school year with a new team of teachers.

Given the current priority schools are placing on teachers planning in teams, further research into exploring approaches that optimise this process is recommended.

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Primary School Mathematics Leaders' Views of their Mathematics Leadership Role

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School mathematics leaders play a significant role in leading improvement in mathematics education in schools. An online survey was administered to obtain an overview of the current nature of the role of the school mathematics leader. Responses were received from 56 primary school mathematics leaders from Victorian government schools. Findings based on leaders' views contributed to building a picture of the complex nature of the role. Survey responses suggested the impact of school mathematics leaders was often compromised by lack of time, confidence, expertise and funding. The extent of classroom teaching, teacher content knowledge and principal support also impacted on effectiveness of the role.

Quality leadership in mathematics education is fundamental in improving teacher practice and student outcomes (Gaffney, Bezzina, & Branson, 2014). The actions of leaders, along with the actions of teachers who are skilled, confident, and who have the pedagogical content knowledge to teach mathematics effectively, are critical in improving student learning (Gaffney, Clarke, & Faragher, 2014). Teachers need support to develop the knowledge to meet students' needs and to guide their practice. This support needs to come from leaders who can help them to use evidence of the results of their practice in effective ways (Timperley, 2010). Leaders of mathematics are not always in formal positions such as school principals (Spillane, Healey, & Parise 2009); they are often teachers with the responsibility of leading mathematics in schools. Through their actions they play a significant role by developing connections between leadership and learning and improving teaching practice and student outcomes (Gaffney et al., 2014). As a result, these leaders have the "greatest capacity to bring about positive practical and sustainable change" (Grootenboer, Edwards-Groves, & Rönnerman, 2015, p. 277).

Over the years, teacher leaders in Australian schools who support teachers to improve outcomes in mathematics teaching and learning have been referred to by a variety of terms, including teaching and learning coaches (Department of Education and Training [DET], 2013), numeracy leaders or numeracy coordinators (Cheeseman & Clarke, 2005; Corbin, McNamara, & Williams, 2003), middle leaders (Grootenboer et al., 2015) and more recently in Victorian government schools, Primary Mathematics Specialist Teachers (DET, 2013). These leaders, while building their own mathematics discipline and pedagogical content knowledge and capacity to lead, have the responsibility of supporting the learning of their colleagues by sharing subject expertise and influencing practice (DET, 2013).

For the purpose of this paper, I have used the term *school mathematics leader* to convey what the role entails, which is leading mathematics teaching and learning in schools. The research question addressed in this paper is: What is the current nature of the role of the primary school mathematics leader and what influences the effectiveness of this role?

Literature Review

Cheeseman and Clarke (2006) described the role of numeracy coordinators or school mathematics leaders, and noted their importance in education in primary schools. More

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recently, Faragher and Clarke (2014) investigated how to develop and sustain improvements in numeracy achievement of students living in low-socio-economic communities. They described the value of having dedicated staff roles in numeracy, and also described the support and professional learning that numeracy coordinators or school mathematics leaders provided to staff, as an essential feature for effective outcomes. In addition, Jorgensen (2016) concluded that school mathematics leaders played a key role in developing successful numeracy/mathematics practices in remote and very remote schools.

Gaffney et al. (2014) described the importance of principals and school mathematics leaders working together collaboratively to bring about better learning outcomes in mathematics. Further research by Sexton and Downton (2014) found that school mathematics leaders faced many challenges. While leaders also experienced many successes, they were concerned about sustaining improvement in mathematics education.

The reported research describes the support provided by external advisors or specialist training, which enabled school mathematics leaders to develop their knowledge and skills to support classroom teachers. Such support was instrumental in the success of their research. Faragher and Clarke (2014) reported that, by building on existing strengths and using a research-based model of effective teaching, school mathematics leaders afforded opportunities to support teachers to further develop pedagogical content knowledge. Targeted funding, time for meetings and professional development were found to facilitate the role of the school mathematics leader (Sullivan, 2011). Meanwhile, support from principals who prioritised programs that supported and valued teachers and their professional learning was found to enable effective mathematics leadership (Gaffney et al.,2014).

Although school mathematics leaders facilitated improvements in mathematics education in their practice, many experienced a variety of constraints. Many of the demands and tensions that exist in the role have not always been recognised (Millett, 1998). Some school mathematics leaders display "differing levels of confidence" (Millett & Johnson, 2000, p. 397) and do not always have the resources and the support to carry out the role effectively (Millet, 1998). Also, many school mathematics leaders often make "enormous demands on themselves in terms of curriculum and pedagogic skills" (Millet & Johnson, 2000, p. 396).

Classroom teaching responsibilities, in addition to subject responsibility, contribute to the difficult nature of the role, and can "be a conflicting priority" (Millett, 1998, p. 240). Limited resources or funding can add further tensions. In some schools, despite the school mathematics leader role being seen as integral to the success of improvement in mathematics, there is often no specific, or targeted funding for such a role (Jorgensen, 2016).

Inadequate time allocation is another common constraint of the school mathematics leaders' role, which creates pressure (Cheeseman & Clarke, 2006; Millett & Johnson, 2000). A major factor that influences the success of the role, is lack of time to visit classrooms to gain an overview of mathematics in schools, and to support teachers (Millett & Johnson, 2000). Many school mathematics leaders devote much of their own time to carry out the role, using spare moments in the day to meet informally with teachers (Nias et al., 1989, as cited in Millett, 1998).

The existing research describes the importance of the role of school mathematics leaders and the complexity of the role. In addition, research has shown actions of leaders can influence teachers' practice, which in turn influences students' mathematical learning.

The current research seeks to paint a picture of the everyday work of the primary school mathematics leader in Victorian government schools.

Methodology

A survey was designed to collect data about the current nature of the role of the primary school mathematics leader and aspects that influence effectiveness of the role. The school mathematics leaders that were surveyed were recommended by consultants, university staff, leading teachers or principals, and were sent a link to the survey by email. Survey data were collected online between August and November 2016. Responses were received from 56 primary school mathematics leaders, from schools of different sizes with a range of socio-economic backgrounds. Of the respondents, 48 (86%) were female leaders and eight (14%) were male.

There were 23 questions in total. The survey was designed to be completely voluntary with 20 multiple-choice or short response options that could be added to and three open-response questions, taking no more than 10 minutes to complete. Questions reported in this paper include the years of experience of the school mathematics leader, classroom teaching responsibilities, and the perceived degree of principal support. Three open-response questions provided school mathematics leaders the opportunity to expand on ways they felt they supported teachers: some of the challenges they had experienced, and some of the achievements they felt were significant. This qualitative data was analysed and coded to identify themes.

Results and Discussion

Survey responses are reported in two sections. Firstly, responses to three of the fixed questions will be shared, and then the five most commonly occurring themes from the open response questions will be used to respond to the research question.

School Mathematics Leaders' Experience

To begin it was important to know how many years each school mathematics leader had been in the role. This provided a sense of how well they understood their role, and how many years of experience they were able to draw upon, compared to "newcomers" to the role (Lave & Wenger, 1991).

Table 1
Number of Years of Experience as a School Mathematics Leader (n = 56)

Years of experience	Number of teachers	%
1 - 2 years	20	36%
2 - 3 years	11	20%
3 - 4 years	3	5%
4 - 5 years	5	9%
5 + years	17	30%
Total	56	100%

Table 1 shows 17 (30%) school mathematics leaders had been in the role for five or more years, five (9%) for four to five years, three (5%) for three to four years, 11 (20%) for two to three years and 20 (36%) for one to two years. Of the group of 20 (36%) school mathematics leaders, nine (45%) are beginning leaders involved in the state government Primary Mathematics Specialist (DET, 2013) program.

Classroom Teaching Responsibilities

While the role of school mathematics leaders is to support teachers, there are many who have the added responsibility of classroom teaching. Table 2, shows almost three quarters (*n* = 39) of the school mathematics leaders surveyed, were responsible for teaching children in classrooms, in addition to leading school mathematics. This ranged from teaching eight hours a week to a full-time teaching allocation. Nearly half (*n* = 18) of the 39 school mathematics leaders had no allocated time release for the responsibility of leading mathematics. The frustration of teaching full time in a classroom with no additional time release was expressed in this comment:

> The workload in being a leader in the school is enormous and I feel it is almost impossible to be an effective classroom teacher and an effective curriculum leader.

Table 2
Indication of Classroom Teaching Responsibilities (n = 56)

Years of experience	Classroom teaching responsibilities	No classroom teaching responsibilities
1 - 2 years	17	3
2 - 3 years	8	3
3 - 4 years	3	0
4 - 5 years	3	2
5+ years	8	9
Total	39	17

Interestingly, 17 respondents had no classroom teaching responsibilities. Fourteen of these leaders had full-time coaching, intervention and support roles and three combined their mathematics role with other leadership roles. According to one respondent:

> I was a full-time maths specialist at my last school. It was such a privilege. Now they don't have a full-time person driving the change, and I think it makes all the difference.

Whether the role is labelled as a Coach, a Teaching and Learning Leader or a Specialist, it can be argued that leading mathematics in schools is far more effective without added classroom responsibilities, especially a full-time teaching commitment.

Principal Support

The survey sought to gauge the perceived level of principal support because leadership support that links priorities and programs is crucial to the effectiveness of the role of the school mathematics leader (Gaffney et al., 2014). Table 3 shows that 19 (34%) school mathematics leaders rated their principal support as a ten (very supported), 12 (21%) as a nine, 11 (20%) as an eight, six (11%) a seven and eight (14%) a six or below. Three quarters of the respondents (*n* = 42) who rated principal support as eight or more out of ten, possibly felt they were working well with their principal, and that their role was valued and supported. It is important to note that only five (9%) respondents rated their principal at or below the mid-point on the scale of support.

Table 3
School Mathematics Leaders' Perception of Principal Support (n = 56)

Response Scale	1 Not	2	3	4	5	6	7	8	9	10 Very
Frequency	0	0	1	1	3	3	6	11	12	19
Percentage	0	0	2	2	5	5	11	20	21	34

Themes That Emerged from Open Response Questions

Time. Nearly half of the school mathematics leaders (*n* = 26) identified time constraints in relation to implementation of their role. This is due to having multiple dimensions to their work – such as classroom teaching, coaching, providing feedback, supporting planning, providing professional development, managing resources, supporting data analysis and collection, and developing whole school programs. Time is needed to meet and plan with teams, and to work with students and teachers. As one school mathematics leader reported:

> Time to get it all done. Time to give more support to teams and individual teachers.

Frustration was obvious through responses such as:

> Time! (1st challenge). I find it very difficult to be as an effective leader as I would like to be. Extremely frustrating … we don't have enough time to meet and do this during our lunchtime … Even if I was to work more – i.e. full time, I would not be able to have extra time in classrooms to assist with coaching roles, look for new initiatives and implement programs.

Limited time allocation for the role was also an obvious frustration in this response:

> One hour a week time release to plan Mathematics for whole school PD, data analysis, audit and manage maths resources, prepare for team meetings and provide feedback to teachers being coached.

Although almost half of the school mathematics leaders (*n* = 26) indicated that they were hindered by time constraints, they managed to achieve a great deal. This may partly be attributed to the fact that school mathematics leaders often devote much of their own time to carry out the role (Nias et al., 1989, as cited in Millett, 1998).

Leader confidence. Leader confidence (Table 4) was a major theme that emerged from the data. Twenty-three school mathematics leaders expressed varying levels of confidence in their ability to fulfil expectations of the role. Interestingly, a lack of confidence was not only a concern of less experienced teachers new to the role, but was also a constraint expressed by several (*n* = 4) more experienced school mathematics leaders. Of the 23 (42%) school mathematics leaders who expressed lack of confidence, 10 who had been in the role for one to three years, commented lack of training and access to expertise were constraints in relation to the role. One pointed out that she was worried she was not meeting expectations. Another was concerned about: "Being new to leading a curriculum area and knowing what to do as a leader." Often there is an issue in schools with high staff turnover, or when leaders themselves move schools. As one respondent said: "Coming into the school and establishing myself as a leader and building relationships with teachers was challenging."

Table 4
Number of School Mathematics Leaders Who Expressed a Lack of Confidence in Leading Mathematics (n = 23)

Years as a School Mathematics Leader	Frequency	Percentage
1 - 2	11	20%
2 - 3	5	9%
3 - 4	2	4%
4 - 5	1	2%
5+	4	7%
Total	23	42%

Developing credibility as a leader and unclear job descriptions were also mentioned as constraints and added to the insecurity and uncertainty of the role. One respondent highlighted her concerns and lack of confidence in this comment:

> Not having enough time to discuss direction with my principal [but also] strategies to support me as a coach/mentor.

Leader expertise. One school mathematics leader reported her struggle as she began in the role:

> I was completely inexperienced when taking on the role, and whilst I have had complete trust, support and mentoring from my principal, it has been hard to build my own capacity while trying to build others.

However, as she continued to be involved in on-site professional learning twice a week, to build teacher expertise, this school mathematics leader, pointed out:

> My own leadership and pedagogical content knowledge grew significantly.

As school mathematics leaders continue to build their own expertise, they become increasingly confident and more knowledgeable. This was evident in responses by many (n = 32) school mathematics leaders that reported they had played a major role in developing staff use and understanding of assessment data. They provided (n = 39) support with planning effectively and (n = 35) provided regular professional learning to staff. This is how one particular school mathematics leader supported staff at her school:

> Attend the four level planning days for 100 minutes to support teachers in term planning. Organise professional development for the staff from outside experts. Attend the local mathematics network meeting to represent my school. Deliver professional development at least once per term. Head the Mathematics Action team meetings held fortnightly after school (agendas and minutes and actions related). Provide support for other teachers for their mathematics planning. Manage the mathematics budget and order resources for mathematics. Administer DET On Demand assessment program; administer Mathletics; administer other maths related programs such as Maths 300. Prepare documentation to support the delivery of maths at my school including a language continuum and units of work. Organise and deliver an annual Family Maths Night.

Although this school mathematics leader was new into the role, as an experienced teacher, she was enthusiastic and confident and had developed knowledge and skills over time. School mathematics leaders are often seen as the "front line" people who help teachers to improve their teaching of mathematics (Cheeseman & Clarke, 2006).

Teacher knowledge. Teacher knowledge was another theme that featured in the survey responses. Responses from almost half (n = 25) of the school mathematics leaders pointed out that teacher expertise, including teacher content knowledge and pedagogical content knowledge was a constraint when working with teachers. One school mathematics leader explained:

> We have to spend a long time building up pedagogical content knowledge along with effective teaching structure and strategies for maths.

According to Gaffney and Faragher (2010), there are "significant numbers of Australian primary teachers who would benefit from professional learning in pedagogical content knowledge" (p. 74). However, almost two-thirds (n = 35) of school mathematics leaders believed they had supported teachers to build this knowledge through coaching, modelling, mentoring, providing feedback, professional development and sharing academic readings. One school mathematics leader explained that she:

> Built teacher capacity by providing support in planning, running whole-school professional learning sessions, in class coaching and sourcing external professional learning opportunities.

Another commented on opportunities to develop teacher expertise at her school:

> On-site professional learning is held twice a week, … focused on building teacher capacity to use data to design for deep learning. Mathematics has continued to be a priority over the last two years. Teams of teachers plan and teach collaboratively to support and challenge each other's practice. Our graduate teachers are given a one-hour block each term with the Numeracy leader to debrief, goal-set and clarify. All teams have ongoing support during planning and weekly data analysis.

These examples of regular professional development involving school mathematics leaders contributed to building teachers' pedagogical content knowledge and demonstrate how School management structures can support collaboration and team building.

Funding. Finally, almost a quarter of the responses (n = 13) indicated that school mathematics leaders felt constrained by limited budgets, not only in terms of money to purchase equipment, but also to implement programs, to fund professional development, and to provide time release to allow them to work collaboratively with teachers and in teams. Further tensions existed when literacy became the focus for resources and programs rather than mathematics. One leader pointed out when writing became the focus at her school:

> Maths took a step back resulting in slippage from the years [when] we had three dedicated maths specialists.

Conclusion

Data presented has contributed to building a big picture of the current nature of the school mathematics leader role. It is clear that the actions of school mathematics leaders, who support teachers to build their pedagogical content knowledge and develop and embed quality practice, are a vital link in developing the connection between leadership and learning (Gaffney et al., 2014). Recent projects referred to in the literature are evidence of the value of having dedicated school mathematics leaders (Faragher & Clarke, 2014). School mathematics leaders surveyed shared many successes and achievements. However, based on their views, it is also evident that the role continues to have challenges. The extent of classroom teaching responsibility, time allocated to the role, leader confidence, and funding influenced effectiveness of the role. Although most school mathematics leaders believed their principal supported their role, it would appear that many principals did not always prioritise programs and apply sufficient funding to enable the leaders to achieve maximum effectiveness in their role. As a result, many school mathematics leaders experienced frustration, issues with confidence, and a degree of uncertainty in the role.

When schools see the value of school mathematics leaders as integral to developing quality practices in mathematics teaching and supporting staff, “the role can be funded through various means” (Jorgensen, 2016, p. 36). The implication of these data for educational sectors is that limited investment impacts on effectiveness of the role of school mathematics leaders. To fully support teacher development and improved learning outcomes for students, school mathematics leaders need time to do the job, support from principals, and professional development as leaders.

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Historical Perspectives on the Purposes of School Algebra

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In this paper, we identify, from historical vantage points, the following six purposes of school algebra: (a) algebra as a body of knowledge essential to higher mathematical and scientific studies, (b) algebra as generalised arithmetic, (c) algebra as a prerequisite for entry to higher studies, (d) algebra as offering a language and set of procedures for modelling real-life problems, (e) algebra as an aid to describing structural properties in elementary mathematics, and (f) algebra as a study of variables. We conclude with brief commentary on the question whether school algebra is a unidimensional field of study.

Introduction

Published summaries of aspects of the history of school algebra in the eighteenth and nineteenth centuries have mainly comprised analyses of school algebra textbooks written by European or American authors (see, e.g., Artigue, Assude, Grugeon, & Lenfant, 2001; Chateauneuf, 1929; Chazan, 2012; Chorlay, 2011; da Ponte & Guimarães, 2014; Schubring, 2011). No comprehensive international history of school algebra has ever been published. In this paper, we outline a framework which identifies six purposes for school algebra which have emerged over the past 350 years. We also comment on some of the theories which have been proposed for explaining why school algebra has caused so much difficulty for so many learners.

Algebra in Secondary School Mathematics—In the Beginning

Algebra made a late entry into the canonical school mathematics curriculum, having long been preceded by arithmetic and Euclidean geometry (da Ponte & Guimarães, 2014). Ellerton and Clements (2017) have argued that secondary school mathematics first appeared as a definite curriculum component in 1673, at Christ's Hospital, a school for poor children located in central London. In 1693, algebra formally became a component of the mathematics curriculum at Christ's Hospital when that school published a book on algebra which had been written in Latin by a Swiss mathematician, Johannis Alexandri (1693). That book became the algebra reference text for 14- to 16-year-old boys attending the Royal Mathematical School (hereafter "RMS") within Christ's Hospital program. RMS was established in 1673 for boys who would be selected, from among the academically most capable boys in the School, to prepare them for careers in the Royal Navy or in the merchant marine.

In 1694, Isaac Newton recommended to Christ's Hospital that the RMS mathematics curriculum should include "artificial arithmetic"—another name for "algebra" (Turnbull, 1961). Although the school adopted this recommendation, it insisted that the RMS boys should complete 4½ years of formal study in Latin *before* beginning to study "artificial arithmetic". The RMS boys would be required to leave school at the age of 16, and would only be allowed to study mathematics for 1½ years. If one looks carefully at Alexandri's (1693) book—used at Christ's Hospital between 1693 and 1715—it becomes clear that the

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algebra which the boys were asked to study was pitched at a very high level (Ellerton & Clements, 2017). That was true, despite the fact that most of the boys had had no previous instruction in algebra before moving into the RMS program.

The requirement by Christ's Hospital authorities that RMS boys would be allowed to spend 1½ years only studying mathematics, and that that would come after they had spent 4½ years studying Latin, might seem to be bizarre to modern educators. However, the "best" educational thinking of the time insisted that any worthwhile form of post-elementary education should emphasise Latin and Greek. Proceeding from that assumption, the Secretary to the Admiralty, Samuel Pepys (of diary fame), was able to persuade Christ's Hospital that even during the 18 months when the RMS boys would study mathematics they should do nightly exercises in Latin under the supervision of the school's classics masters.

Future implications for school algebra arising from Christ's Hospital's RMS model should not go unnoticed. By adopting Pepys's and Newton's plan, Christ's Hospital made a policy decision that only very capable boys should study mathematics beyond arithmetic, and that the best curricular preparation for studying school mathematics was a solid 4½-year block of Latin, when the boys were aged between 9½ and 14 years. During the 18 months when the RMS boys would be allowed to study mathematics they would be expected to learn, among other things, algebra, Euclidean geometry, plane trigonometry, and spherical trigonometry.

That well-educated people would agree to such an intended curriculum not only testified to the high level of faith in the virtues of a classical education but also to a belief that algebra, geometry and trigonometry would be relatively easy for students who had already mastered the classics. It was also assumed that the study of school algebra should be confined to very capable boys. A similar attitude would prevail in education circles across Europe and in North America throughout the eighteenth century and for much of the nineteenth century, and would be translated to the colonies. In the second half of the nineteenth century in all the early Australian universities—in Sydney, Melbourne, Tasmania, and Adelaide—matriculation passes in Latin, Greek, Arithmetic, Geometry and Algebra were required before a student could graduate, including those who were destined to become clergymen and lawyers (Clements, 1979; French, 1956).

During the 18th century, largely as a result of the example of Christ's Hospital and of the influence of writings on algebra by distinguished European mathematicians like Étienne Bézout (1794), Alexis Clairaut (1746) and Leonhard Euler (1770), algebra filtered into the curricula of many secondary schools (which were often called "academies") in Europe and America (da Ponte & Guimarães, 2014). However, in almost all of the algebra textbooks which were written by highly-regarded mathematicians, algebra was presented in the form of generalised arithmetic. There were exceptions—at Christ's Hospital, for example, James Hodgson (1723) maintained that school algebra was predominantly something which should help school students solve practical problems. Hodgson had been appointed master of RMS in 1709, and he immediately protested against the practice, in the school, of teaching algebra in Latin. Despite the opposition of Isaac Newton, Hodgson convinced Christ's Hospital authorities that RMS students did not know Latin well enough to be able to learn algebra in that language. From 1710, RMS students learned algebra in English.

Although Hodgson recognised that a stipulation that 14-to-16-year-olds should learn algebra from a text written in Latin was foolish, that did not stop him from demanding algebra at a very high level from his students. Early in his *A System of Mathematics,* Hodgson (1723) freely used Newton's version of differential calculus ("fluxions"), and throughout the book he proved theorems using algebraic methods. Undoubtedly, Hodgson's high standards were related to the fact that RMS boys could not graduate unless they passed an intensive verbal examination of their mathematical knowledge conducted by disinterested experts—and it could be argued that those expectations generated unrealistically high standards for school algebra students, both at the time of Hodgson and in the future.

School Algebra in the Eighteenth and Nineteenth Centuries

During the Hodgson era at Christ's Hospital (1709–1755), the Royal Mathematical School came to be recognised within Europe as a leader in school mathematics (Ellerton & Clements, 2017; Hans, 1951). This persuaded other schools, especially those which prepared students to become surveyors, or officers in the navy, merchant marine, or the army, to introduce algebra and trigonometry into their curricula. A top-down model of curriculum development manifested itself, and most of the algebra textbooks used in schools were authored by professional mathematicians (e.g., Bézout, 1794; Bonnycastle, 1788; Clairaut, 1746; Todhunter, 1863; Wolfe, 1739).

The 18th and 19th centuries were marked by massive colonisation programs by leading European nations, and it was not surprising that the colonisers tended to introduce school mathematics textbooks into their colonies which had been written in the "home" nations. Not only were the languages used in those textbooks those of the colonising powers, but also the authors were, almost always, based in Europe. Inevitably, the textbooks were written in a way which suggested that school mathematics should be a culture-free exercise. Even for students in the European homelands, most algebra textbooks were designed to suit the perceived needs of children of elites. The first algebra textbooks used in the "colonies", around the world, were often written from high mathematics vantage points and were unsuited to the needs of indigenous children, of children of slaves, or of children whose command of the forms of language used by mathematics teachers and authors of mathematics textbooks was not strong (Clements, Grimison, & Ellerton, 1989).

Table 1, adapted from Kanbir, Clements, and Ellerton (in press), outlines six "purposes" of school algebra which emerged from analyses of data sets in numerous archives, including handwritten and printed documents. Table 1 is framed historically by the dates given in the first column. The first two named purposes are associated with periods which began late in the 17th century or early in the 18th century. Then, late in the 18th century and early in the 19th century, school algebra began to be linked to entrance requirements of higher-education institutions—even for courses such as divinity, and law. From the second half of the 19th century, there was recognition of a need to introduce a kind of school algebra which could model, and therefore help to solve, real-life problems.

At about the same time as the idea emerged that algebra should facilitate mathematical modelling of real-life situations, there arose a movement by which school algebra should be important in enabling students to recognise the structure of the real-number system. Finally, in the second half of the twentieth century, especially at the time of the "new mathematics", the importance of the concept of a variable, and of the power of that concept to summarise major mathematical ideas and to model real-life situations, began to be emphasised.

Table 1
Six Purposes for Secondary School Algebra (Adapted from Kanbir et al., in press)

Period	Assumed Purpose	Key Writers or Players	Comments
1693–2017	*Purpose 1*: Knowledge essential for higher mathematics and science.	R. Descartes, H. Ditton, I. Newton, G. Leibniz, S. Lacroix	Mathematicians noted that algebra could assist students to comprehend and express concepts and principles in higher mathematics (e.g., in conic sections, trigonometry, calculus, and mechanics).
1700–2017	*Purpose 2*: Generalised arithmetic	L. Bourdon, E. Bézout, L. Euler, N. Pike, I. Todhunter	Emphasis was on the syntax and the semantics of elementary algebra. Solving an equation was equivalent to "finding the unknown number(s)". Sometimes, an axiomatic approach to algebra was adopted.
1800–2017	*Purpose 3*: A pre-requisite for entry to higher studies	Persons defining university-entrance pre-requisite subjects	During the 19th century many countries required school students who intended, subsequently, to enter prestigious universities to succeed in a school subject designated "Algebra." This tended to change after the 1960s, although it is still the case in the USA.
1870–2017	*Purpose 4*: A language for modelling real-life problems	J. Hodgson, S. Lacroix, J. Perry, F. Klein, E. H. Moore	This has often been called the "functional-thinking approach to algebra". Students describe sequences recursively and explicitly, prepare tables of values, and plot and interpret Cartesian graphs.
1870–2017	*Purpose 5*: An aid for describing basic structural properties	F. Klein, N. Bourbaki, C. Gattegno, Z. P. Dienes	Klein applied function concepts, structural ideas, and associated symbolisms to geometry. In the 1950s and 1960s this was incorporated into school algebra. Gattegno and Dienes argued that elementary-school students could learn algebra before arithmetic, and that structural properties should be emphasised.
1960–2017	*Purpose 6*: A study of variables	SMSG authors, R. Davis, D. Chazan	Solving an equation (or inequality) was seen as finding values of a variable which would make an open sentence true, and which would make it false. Tables of values and Cartesian graphs were to be regarded as depicting relationships between variables. Structural properties (e.g., the distributive property), stated in algebraic language, were to be seen as statements involving variables.

A Theoretical Lens for Interpreting the History of School Algebra

Acceptance, both consciously and subconsciously, of the purposes identified in Table 1 can be linked to corresponding changes in the thinking of pure and applied mathematicians. Figure 1 suggests how school mathematics came to be linked with the developments in mathematics made at various times by mathematicians. Figure 1 is to be interpreted in terms of a "lag time" theoretical lens: after pure and applied mathematicians developed new areas of mathematics, some of those areas were ultimately introduced into school mathematics if and whenever that was appropriate.

Thus, for example, in the first quarter of the 17th century, John Napier and Henry Briggs developed the concept of a logarithm, and this was quickly introduced to, and applied powerfully by, navigators and surveyors. Then, from 1673 onwards, RMS school students were taught how to use logarithms to solve problems associated with navigation. From about 1720 onwards, Isaac Newton's fluxions—which Newton had first developed in the 1670s—began to be introduced to RMS students (Hodgson, 1723). Felix Klein, in his Erlangen program of the early 1870s, showed how algebraic structures and functions could be linked and, ultimately, in the 1960s, this idea came to be embodied in school curricula of the new mathematics period. Matrices, which featured non-commutative multiplication, would also find their way into school algebra programs in the 1960s.

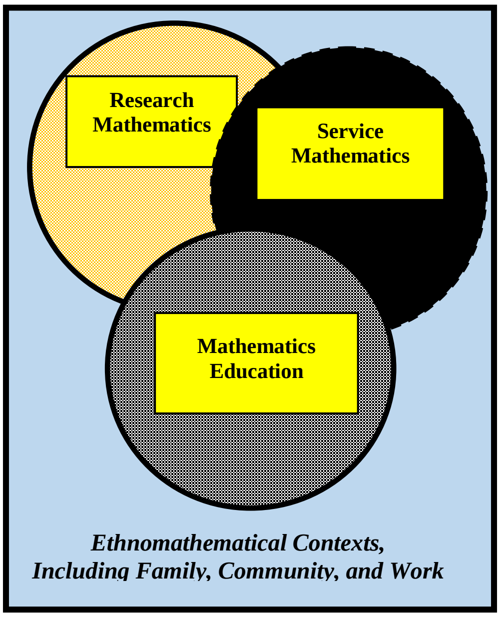

Figure 1. Different ways of "seeing" problems or situations that might relate to mathematics (Clements & Ellerton, 2015, p. 16).

Many aspects of school algebra are not a matter of great concern to mathematicians, and likewise, many parts of the higher forms of pure and applied mathematics are not of much interest to those primarily concerned with school algebra. But, as Figure 1 implies, there are intersections and, in particular, there is an intersection of pure mathematics, service mathematics, and mathematics education. Figure 1 also draws attention to how the purposes of school algebra, and the developments summarised in Table 1, were merged within ethnomathematical contexts, including contexts surrounding families, work situations, and communities in different nations.

Is School Algebra a Unidimensional Field of Study?

If, indeed, the six purposes listed in Table 1 are separable—as we claim they are—then important curriculum questions arise. Are all six purposes important in planning modern school algebra curricula? If the answer is "No," then which should be regarded as appropriate for which students, in which schools, and when, and why? Is school algebra a unidimensional field of study and, if it is not, what meaning should we give to the term "school algebra"? Do current school intended and implemented algebra curricula take sufficient account of the six purposes?

An obvious, and challenging, question is why six purposes, and not five? Or seven? Perhaps there is just one overarching purpose (such as "providing students with a language which will help them to learn to generalise"), which would synthesise the six purposes. Although the purposes identified in Table 1 are important from a mathematics curriculum perspective, it could be the case that there is no obvious best way of synthesising them adequately. One wonders which forms of words, and which conceptual structures, would be needed so that a description would be adequate.

In the 1960s and 1970s, when the "new mathematics" movement was sweeping the world, each Australian state decided to unify its mathematical offerings by creating composite subjects—often called, simply, "Mathematics" (or descriptive titles such as "Mathematics I", "Mathematics II", "Basic Mathematics", "Calculus", "Mechanics", and "Advanced Mathematics") (Clements, 2003). This did not occur in all parts of the world, however—for example, in the United States of America the titles Algebra I, Algebra II, and Geometry remained commonplace in all states. Even today, Algebra I and Algebra II are offered as distinct subjects across the United States—yet, such is the power of labeling that many U.S. students who have completed courses in calculus have told us that they do not think that algebra is involved in calculus (Ellerton & Clements, 2011).

Even in the late 1980s, it was still the case that, in some nations, many students did not study algebra at all (Clements, Keitel, Bishop, Kilpatrick, & Leung, 2013). In the second half of the 1980s in the United States of America, for example, about 30 percent of secondary-school students never studied algebra while still at school, and others studied it for only a short period of time. Yet, in Australia, and in some other nations, algebra was studied by almost all students in Grades 7 and 8 and, often, right through Grades 7 through 12 (Clements, 2003; Usiskin, 1988).

Algebra as generalised arithmetic has been, is, and will continue to be, important. That said, it is possible that the first steps in secondary-school algebra should *not* involve a strong emphasis on operating with algebraic symbols and other representations of varying quantities. Given the great difficulty which many beginning algebra students have always had with the language and written symbols of algebra, one must ask whether it is wise to insist that all seventh- (or eighth-) grade students should be expected to learn to recognise and use the main signifiers which point toward more sophisticated algebraic knowledge and skills (Kanbir et al., in press).

The push toward algebra as functional thinking, as an essential preliminary to being able to model real-life situations, is one that is often emphasised by mathematicians, mathematics educators, and scientists. But, one wonders how often most adults, even those who are well versed in algebra, use algebra to model practical problems—when was the last time you solved a quadratic equation outside of the classroom? On the other hand, graphical representations of data are common in most forms of media, and are much more accessible given today's technology. These representations often summarise relationships between varying quantities, and carry an invitation to express relationships between the variables. The challenge is for secondary teachers to devise ways and means of getting students to the point where they can identify variables relating to situations which are real and of interest to them, to devise learning contexts in which they generate data, and to assist the students to move towards making generalisations about how the data can be related. More than a century ago John Perry, in London, and Eliakim Hastings Moore, in Chicago, strongly favored that approach, but their efforts were thwarted by those who favored a more conservative approach to school mathematics (Kanbir et al., in press).

Too often, the directions of school algebra have been defined by outstanding mathematicians, like Isaac Newton, Felix Klein, and Jean Dieudonné, who have had little experience in teaching "ordinary" school children aged between 10 and 17 years. The new math(s) period generated textbooks which emphasised structure, but the language and symbolisms used in textbooks were so sophisticated that only a small percentage of school students engaged fully with the mathematical texts provided. In recent years, those who constructed the common-core algebra sequence in the United States seemed to think that structural properties (such as the commutative, associative and distributive properties) would be well known by middle-school students, but our experience and research indicates that that is definitely *not* the case (Kanbir et al., in press). Nevertheless, the question remains: Are algebraic structures too difficult for middle- and high-school students? Tzanakis (1991) is one among many to have argued that that is indeed the case.

We are not convinced that school algebra, as it is now being defined in various parts of the world, represents a unidimensional field of study. Studying algebra from a structural point of view may be quite a different thing from studying it from a modelling perspective. Certainly, the two can be combined, but the issue is whether students, teachers, and researchers, who are engaged in efforts to bring together the different strands recognise that although the two approaches may both be labeled as "algebra", they may not be representatives of "the same thing".

Ever since the introduction of algebra into the secondary school curriculum there has always been, among many—perhaps most—middle-school and lower-secondary-school students, a disconnect between the signifier (signs and symbols) and the signified (the mathematical objects which are being considered). We believe that the main issue is a semiotic one—not only do students need to understand what the authors of textbooks write, and their teachers say, about algebra, but they also need to learn how to express themselves using appropriate algebraic language, and to make generalisations of their own.

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Fourth-Graders' Meta-Questioning in Statistical Investigations

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This paper addresses the initial components of an activity in which 4th-grade students engaged in meta-questioning as they created and refined survey questions with the aim of comparing life across two Australian cities. We propose the term, *meta-questioning*, as a core, underrepresented feature of statistical investigations in the primary school. We report on the nature of the students' initial posed questions and their subsequent refined questions, students' justifications for their question refinements, their anticipated data collection, and developments in their question posing skills. Results include a hierarchy of question types posed by the students and how their question types changed with subsequent refinements.

A statistical question is the starting point for any investigation. Yet, posing questions is underrepresented in many curriculum documents (Almonds & Makar, 2010; English & Lehrer, in press; Lavigne & Lajoie, 2007), perhaps because initial statistical questions are often informal and broad and require substantial refining (English, 2014; Whitin & Whitin, 2011). Young students often find it difficult to generate questions that can be investigated or to envision the data that can address their questions (Allmond & Makar, 2010). Lehrer and Schauble (2002) suggested that many of the challenges of posing questions for children can be ameliorated when they are given opportunities to build familiarity with the phenomena being investigated, including talking about and observing the target phenomena. Encouraging young students to evaluate questions posed critically is a foundational statistical process that is often overlooked in mathematics curricula. Yet an ability to deal intelligently with the myriad data in everyday events, including questioning data sources and questions investigated, is essential if citizens are to engage effectively in democratic discourse and public decision-making.

This paper addresses a core component of statistical investigations in the primary school, namely, students' developments in meta-questioning (adapted from Berger, 2014; Driver, 1984). As proposed here and in line with the requirements of a statistical investigation, *meta-questioning* involves (a) identifying the contextual issue to be investigated and posing initial questions, (b) interpreting and reviewing/critiquing initial questions, and (c) anticipating data collection and refining questions (as illustrated in Figure 1). We report on the initial components of an activity in which fourth grade students engaged in meta-questioning as they created and refined survey questions with the aim of comparing life in two Australian cities. The activity was conducted as part of a four-year longitudinal study that is developing third to sixth graders' competencies in modelling with data. Specifically, we address the following:

1. What was the nature of the students' initial posed questions and their subsequent refined questions? How did students justify the changes they made?
2. What forms of required data did the students anticipate for their refined questions?
3. What improvements occurred across the students' successive question refinements?

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Meta-questioning in Statistical Investigations

Statistical investigations are typically described as a four-step process involving posing a question, collecting data, analysing data, and forming and conveying conclusions (e.g., English & Watson, 2015; Konold & Higgins, 2003; Lavigne & Lajoie, 2007; Pfannkuch & Wild, 2003). Typically, such investigations usually do not proceed in such a linear, orderly fashion. As Konold and Higgins (2003) indicated, these phases of a statistical inquiry are interdependent, with a good deal of "backtracking" often taking place (p. 194). That is, experienced researchers consider ahead of time the data they will need, the analyses that might be undertaken, and possible findings. Further refining of initial questions, deciding on data collection and analysis, and how findings will be communicated follow. There is also looking backwards to the initial question(s) as data are analysed and findings produced. In essence, data investigations are complex and require early and continued development of statistical thinking skills. Simply treating an investigation in the primary school as a teacher directed, lock-step process does not adequately develop young students' statistical skills and understandings. In particular, the important processes of meta-questioning, which are often ignored or at best completed by the teacher, require substantially more attention. The words of Hanner, James, and Rohlfing (2002) are particularly germane to this issue:

> Very often, teachers solve all the interesting issues for kids and present them already resolved to children, without giving children the opportunity to grapple with such questions as, "What attributes should we include?", "How many attributes should we consider?" and, "How should they be represented?" When teachers take over these decisions, all that's left is a cut-and-dried graphing or sorting activity, in which teachers have done all the intriguing and motivating thinking ahead of time (p. 106).

As highlighted in earlier studies (Konold & Higgins, 2003; Lehrer & Romberg, 1996), primary school students can learn a good deal about data as they tackle issues that arise in creating statistical questions, in particular when they anticipate implementing surveys with questions they create. By encouraging children to think about how they would answer their posed questions, they become aware of the different responses that might be elicited, and the multiple ways in which their questions could be interpreted, especially if they are worded vaguely or ambiguously. Given the need for more research on statistical question posing and refining, especially in the primary school (Allmond & Makar, 2012; Arnold, 2008; Konold & Higgins, 2003) we argue that special attention needs to be given to the processes of meta-questioning (Figure 1).

Previous research has highlighted the importance of students being able to distinguish between a question that "anticipates a deterministic answer" and one that "anticipates an answer based on data that vary" (Franklin & Garfield, 2006, p. 350). Further emphasising this issue, Konold and Higgins (2003) cite research on the individual and aggregate views in questions students ask. Many of their questions simply require identifying individual perspectives, usually themselves, such as "Who is the oldest?" Konold and Higgins argue that it is not until students are required to respond to questions pertaining to possible differences between two groups that they begin to think about group characteristics rather than those of the individual. To this end, the present activity required students to address the broad contextual issue of comparing life in City A with life in City B.

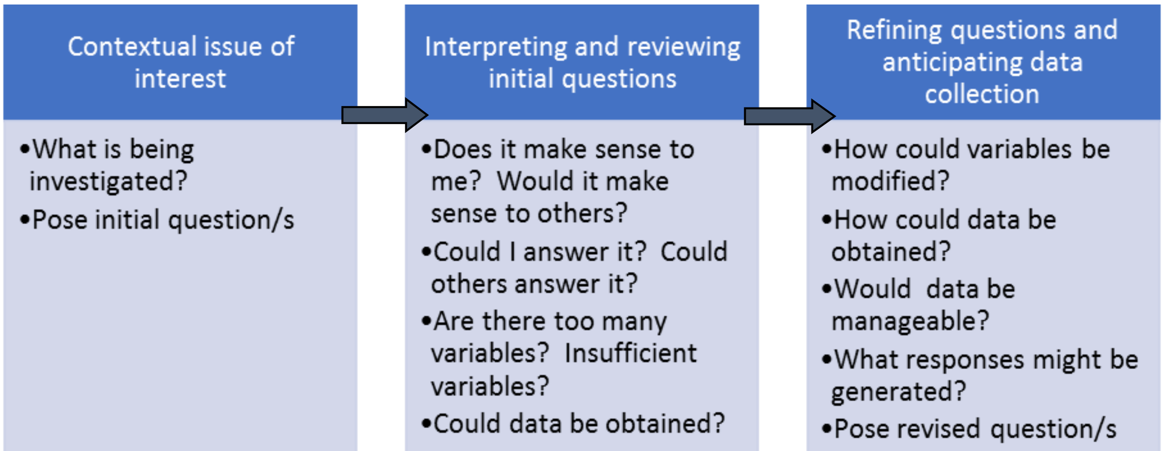

Figure 1. Meta-questioning in statistical investigations.

Method

Participants

Four fourth grade classes and their teachers from two, middle class schools across two Australian states participated in the activity. The school in City A had 56 consenting students (28 in each class) with an average age of 9.8 yrs. The school in city B comprised 33 consenting students (17 in one class, 16 in the other) with an average age of 9.6 yrs.

Research Design

The study adopted a design-based approach involving three main phases, namely, (a) development of activities in consultation with teachers, followed by professional development sessions; (b) teaching intervention implemented by the classroom teachers, together with data collection by researchers, and (c) analyses of data leading to suggestions for future teaching interventions (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

Activity implementation. The overall activity was designed to develop students' skills in (a) posing and refining statistical questions in developing a survey to learn more about their peers in each city (specifically, to compare their respective city lives), (b) collecting, analysing, and representing the data from the survey, (c) identifying variation and trends in the data, and (d) drawing conclusions and inferences while acknowledging uncertainty.

The activity comprised four parts. In the first part of the activity, the students "introduced" one another by sharing photos and providing their inter-state peers with some information, for example, about their family life, hobbies, pets, and sports. Two animal "mascots" were chosen to represent the respective states, with class time spent exploring their features and habitats. In the second part, students studied the feedback from their peers, together with viewing video clips displaying features of each city. In the third part, which is reported here, each student posed three questions that would yield information about life in City A compared with that in City B. The students recorded their questions in the workbooks supplied for each student. Students chose one of their three questions to share with their group (comprising two to three students) and were reminded to check whether their question would "help us to compare life in City A with life in City B." The notion of *variation* was revisited with respect to the purpose of the questions, with students having explored the notion in the previous year's activities.

Students then refined their questions, indicating how their changes improved their original questions. The students were to reflect critically on their improved question and then further refine it. All responses were recorded in the students' workbooks. Part 3 ended with students recording the data they would need in order to answer their refined question. The remaining part of the activity (not reported here) involved the researchers selecting 22 of the students' posed questions (making a few modifications where necessary) for inclusion in an all-class survey. All classes subsequently completed the survey online, followed by students analysing and representing the data, and drawing conclusions and inferences.

Data Sources and Analysis

Data sources included students' responses in their individual workbooks. All whole class discussions were video-taped, while "focus" groups were video- and audio-taped. Focus groups comprising two to four children of mixed achievement levels were chosen in consultation with the teachers. Students' written workbook responses are the primary data source reported here, however, transcripts of focus group work and whole class discussions were analysed in conjunction with these responses.

Content analysis (Patton, 2002) was used by both the first author and an experienced research assistant in initially identifying, coding, and categorising the data recorded in the students' workbooks. In collaboration with the second author and another experienced research assistant, a further cycle of refined coding was undertaken to ensure meaningfulness and accuracy. Transcribed video- and audio-taped data were also analysed to gain greater insights into how students posed, critically analysed, and refined their questions.

In addressing the research questions, we report on (a) the nature of the original questions students chose to refine and their one or two subsequent refined questions, (b) their explanations/justifications for how they refined their original question, and (c) the data that students anticipated would be needed to answer their refined questions.

Results

RQ1: What was the nature of the students' initial posed questions and their subsequent refined questions? How did students justify their improvements to their original questions?

In analysing the students' responses, we considered the features of their initial posed questions with a focus on the question they chose to refine, how they refined their chosen question, their justifications for their changes, and their further (final) refined question. If students posed more than one question within their initial question/s, we recorded only one code for the more appropriate question. The final coding scheme we adopted for both the students' *chosen question to refine* and their *refined questions* follows. Examples and percentages of student responses for each category code are presented.

0*: No response or the content in the original question was subsequently ignored in a refined question.* Sixteen percent of students ($N = 82$) either did not indicate which of their original questions they refined or wrote a refined question that was not related to the original question chosen. All students, however, offered an acceptable first refined question. For the second refinement of the original question—in City A this refinement was optional—45% of students did not provide a response.

1: *The question would generate a simple yes/no response*. Examples included "Does City B have the same weather then [sic] State A?"; "Do you go to theme parks or hotels or go swimming often?" Twenty-two percent of students (*N* = 82) gave a response of this nature for their chosen original question. In contrast, only one student posed such a question for their first refined question and only two for the second refined question.

2*: The question would generate a single numerical or categorical response*. A question of this form appeared superior to the previous type but nevertheless would still produce rather limited data. Such responses included questions of the nature, "How many holidays do you get?"; "What is your favourite food?"; "What is the best thing to do in City B?" Sixteen percent of students posed an original question of this format, with the number increasing to 32% for the first refined question. In contrast, only 7% of students posed a second refined question of this type.

3: *Open-ended questions* that would either (a) generate *multiple responses/lists* (e.g., "What are the typical foods you eat every day?"; "What do you do when it's sunny and what do you do when it's rainy?", and/or (b) ask for an *explanation* (e.g., "Why does City B have theme parks and City A doesn't?"). This third category, which we consider a more advanced form of questioning than the previous types, characterised 41% of the originally posed questions (*N* = 82). This increased to 52% for first refined question, but declined to 39% for the second refined question.

4: *Question gives variables/options to be considered*. Some of the questions coded in this category also included a yes/no query. Sample responses included: "What do you do after you go home after school? Do you do homework or do you play?"; "What is City B's weather like? Is it warm or cold?" This question type was considered the most sophisticated in generating manageable, more substantial data. Only 5% of students (*N* = 82) posed an original (chosen) question of this nature. This improved somewhat for their first refined question, with 15% of students posing such a question. For the second refined question, however, only 6% of the students offered such a response.

In analysing the students' *justifications for their refinements*, we adopted the following categories of codes:

0: *No response or an irrelevant/idiosyncratic response*. Unfortunately, 34% of students could not justify the improvements they made to their original question.

1: *Repeating of question, or the student was satisfied with the posed question, or couldn't refine it.* Eleven percent of responses were assigned to this category.

2: *Recognising the need to change the type of question*. Justifications here included improving on a question that simply requested a "yes" or "no" answer, for example, "My old queschon [question] was a yes or no queschon [question]." Only two students (2%) gave this justification.

3: *Changing qualifying terms* or *removing a simple item of information*. Such responses included: "Well I put the words 'or', 'popliar' [popular] in my question because it might not be your favriout [favourite] but some other people might like it more"; "By saying why instead of would"; and "I deleted different in my question and added are they fun celibrations (celebrations)." Six percent of students gave such justifications.

4: *Adding response options*, such as: "It changed my question because i [I] gave them some options; more choices what they can say to us." Only two students (2%) offered explanations of this type.

5: *Facilitating data collection through (a) requesting more detail from the responder,* and/or *(b) asking for an explanation*, and/or (c) *clarifying the question to get more information or more precise information*, and/or (d) *recognising that the previous question was irrelevant* for the purpose of comparing life in the two cities. Responses of this type included: "I put some more information so that I could get some more data!"; "I would get a variety of answers which is better because you get lots of different opinions."; "There's more than 1 ansewer [answer]"; and "I changed my first question because they were all erelevent [irrelevant] and they wouldn't collect any data from the set question." Forty percent of students gave justifications of this form.

6: *Recognising the need to delimit variables*. Only four students (5%) offered a justification of this form, for example: "It makes it less broad and more speific [specific]"; "Because now there won't be a lot of data"; and "They improved my question because I added a few words to the first question and I got rid of my second question."

RQ2: What forms of data did the students anticipate for their refined questions?

Only students in City B (N = 33) responded to the issue of the data that would be required to answer their refined questions. We coded the students' responses as follows.

1: *Reference to specific data on the question variables*. Responses included: (a) the *type of data*, such as, "The data I would like is temperatures (Numbers e.g., 11° to 13°) during the year (winter, sumer [summer], Autumn, Spring), (b) *lists of data*, for example, "The data I would need is a list of celibrations [celebrations] they have and an answer if they are fun and how they enjoyed it", or (c) *an ordering of data*, for example, "The data I need is the most popular sport to the least popular out of the 5. I also want to know the popular sport cultures." Anticipated data of this nature was given by 42% of the 33 students.

2: Reference to the *contextual issue* (i.e., comparing life in the two cities). Students' responses included: "I need exclusive information relating too [to] their lives and I can see if life in City A is the same as life in City B or completely diffrent [different]"; "I need the data to my question to know if the population in the city is bigger then [than] City B or less." Only 9% of students indicated they would need data addressing the contextual issue.

3. Students offered examples of *possible responses to their questions*, such as*:* "I want them to say two sports/activities they play in winter and why? For example, two activities that they would only play in winter. Maby [Maybe] because it is to [too] hot in sumer [summer] to play the two games they would play in winter." This response was also popular, with 42% of students responding in this manner.

4. Reference to the *nature of questions asked* and *data required* in a survey, such as: "For my question I don't want yes or no, I am aiming to get a [an] explanation. For a good survey, you need a detailed answer. You can also have some varition (variation); Detailed answers, reasons, variation and truth/fact, what type of landscape they live in." Only 2 (6%) of the 33 students offered a sophisticated response of this nature.

RQ3: What improvements occurred across the students' question refinements?

In addressing the third research question we considered the relative changes (ignoring the actual categories of responses) in the students' questions as they progressed from their chosen question, to their first refined question, and then to their second (final) refined question. Only 45 out of 82 students posed three questions (including the original) that could be coded, while 37 students only recorded two questions (original and first refined question). Of the 45 students who recorded three questions, only two did not change their original question and 13 changed once. For the 37 students who only wrote two questions, 34 changed their question.

In coding the relative changes, we considered whether the questions improved, remained the same, declined, or were a combination of these. For those students who offered three questions (N = 45), 53% remained in the same coded category across all questions and 33% showed improvement on the third question or at least improvement on the second question. Fourteen percent remained in the same category or declined as they moved towards their third question (i.e., second refined question). For the 37 students who did not record a third codable question, 59% improved, 32% posed questions in the same category, and 8% posed a question of a lesser quality. Finally, across all students ($N = 82$) for their original and first refined questions, 43% improved, 50% remained the same, and 7% declined.

Discussion and Concluding Points

Meta-questioning is a significant, underrepresented component of statistical investigations, which needs to be introduced early and developed across the primary years. We have reported on fourth grade students' meta-questioning as they created and refined survey questions for comparing life across classes in two Australian states. Of interest were the nature of the students' initial posed questions and their subsequent refined questions, students' justifications for their refinements, their anticipated data collection, and improvements in their question posing as they subsequently refined their questions.

In coding the form of questions students posed, we identified a hierarchy of increasingly sophisticated question types, ranging from a question that would generate a simple yes/no response to one requiring a single numerical or categorical response, through to open-ended questions, and finally questions that offer a set of variables or options to be considered. As students progressed from their original posed question to their first and second refined questions, the use of simple "yes"/no" questions declined considerably. Students' responses suggested that they appreciated that the data generated would be rather limited. Requesting a single numerical or categorical response was more prevalent in the first refined questions, but again the data indicate that the students are capable of rethinking their questions and offering alternatives and feasible justifications. The most common question type was one that was open-ended, where a range of responses would potentially result in a large collection of difficult-to-manage data. The most sophisticated question type, where a choice of variables is provided, was created by only a few students.

Overall, the findings reveal the difficulties primary school students can experience in creating statistical questions, but with opportunities to engage in meta-questioning, as conceptualised in this paper, they can develop key understandings that are foundational to productive investigations. Students need experiences in generating their own questions rather than being supplied with those created by others. Furthermore, when students engage in contextual issues that involve questions regarding potential differences between two groups, they can learn to distinguish between group characteristics that vary and individual perspectives that produce deterministic answers (Franklin & Garfield, 2006; Konold & Higgins, 2003).

Although we did not report on the remaining activity components, we argue that meta-questioning should occur throughout an investigation. In exploring issues of interest, students should reflect on the question/s they are investigating as they collect and analyse data, and consider the conclusions and inferences that can be drawn. Teacher support is important in fostering students' skills and dispositions to engage in meta-questioning, as reflected in earlier studies (e.g., Allmond & Makar, 2010; Lehrer & Schauble, 2002). Clearly, more opportunities are needed for younger students to progress from posing questions that would yield either limited data or too much data, to ones that would generate substantial yet manageable data. The hierarchy of question types we have identified provides one framework for analysing and developing core investigative questioning skills, which remain largely under-developed and under-researched in many primary classrooms.

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Essential Topics for Secondary Mathematics Success: What Mathematics Teachers Think

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In this preliminary study, to inform a larger study where Year 8 students create an online module for peers, I surveyed mathematics teachers (n = 30) on essential mathematics topics: (a) most critical for students' success, (b) most conceptually challenging for students, and (c) in which more fluency is needed, as well as (d) their likelihood of considering an online course as an intervention. Fractions concepts, times tables, and equation solving were most critical for success; students need more understanding of fractions concepts, and more fluency in both fractions concepts and times tables. Online course use addressed teachers' concerns for students in essential mathematics topics.

In this paper, I outline a preliminary questionnaire-based study conducted on what topics mathematics teachers rate as most important and needed for students to succeed in their study of mathematics at secondary school. The results are needed to help inform the focus topic for a larger design-based intervention study in which Year 8 students create an online or e-learning module for peers. In this novel approach, Year 8 students change roles from content consumers to content creators. Learning outcomes and engagement levels will be tracked during the creation process. Also, if mathematics teachers are concerned about their students' understanding or fluency in essential mathematics topics, the questionnaire asked the likelihood of teachers considering an online course or module, as I aim to produce in the major study, to address those concerns.

Background

Disengagement from mathematics by lower secondary students is widespread in Western nations (e.g., Middleton, 2013). In Australia, Martin, Anderson, Bobis, Way, and Vellar (2012) found that disengagement from mathematics in middle years students (Years 6 to 8; n = 1,601) was correlated with the following student and classroom factors: low mathematics self-efficacy, low valuing of mathematics, reduced enjoyment and perceived classroom enjoyment, mathematics anxiety, and perceived classroom disengagement. Balfanz, Herzog, and Mac Iver (2007) noted that course failure in US lower secondary schools "dramatically dampens a young adolescent's perceived control and engagement" (p. 224). Furthermore, systemic secondary school practices in mathematics education may not be meeting the following young adolescents' needs that were mostly better met in primary school: a degree of autonomy, social interaction and relatedness, a close relationship with their teachers, small group work, challenging activities that require higher order thinking (Eccles et al., 1993), and the motivation of a hard, specific group goal (Locke & Latham, 2006). Giving students fresh opportunities to succeed while attention is drawn away from the individual performance and towards a challenging, relevant goal could assist students with concentrating on learning mathematics and not on any previous negative experiences with the subject. To this end, a novel approach has been devised such that a class of Year 8 students works together to produce an e-learning module for peers.

However, the topic on which the inaugural study is created needs to be selected carefully. Ideally, to enhance the project's relevance, value, and wider appeal, the topic

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needs to be one in which other more complex topics depend and is demonstrably essential and therefore valuable to school mathematics, further education, civic life, and the workplace. Enough students need to have some degree of difficulty with the topic such that the finished product, an e-learning module for local peer use, will be seen as a potentially worthwhile and challenging project to work on and a meaningful resource for end-users. In order to willingly commit to the research project, mathematics teachers also need to be able to appraise the topic as worthy of expending effort, time, and resources.

Previous research supports that intervention on the topic of fractions is needed as it is often poorly understood across a broad spectrum of learners: primary school students (Daraganova & Ainley, 2012; Zhang, Clements, & Ellerton, 2015), middle years (Years 5 to 8) students (Clarke & Roche, 2009; Stafylidou & Vosniadou, 2004), more senior high school students (Brown & Quinn, 2006; Kloosterman, 2010), and the general public (Basic Skills Agency, 1997; Reyna & Brainerd, 2008). The Longitudinal Study of Australian Children, Annual Statistical Report 2011 (Daraganova & Ainley, 2012) included primary school teachers' ratings (n = 3,533) of children's numeracy skills (aged 8 to 9 years) and found that a quarter of children had either not yet (7%) or were just beginning (16%) to form an age-appropriate concept of fractions compared to, for example, half that amount either not yet (3%) or just beginning (9%) to form an age-appropriate concept of place value. In an open questionnaire asking Australian middle years students themselves (Years 5 to 8; n = 3562) about their single most important aspiration in mathematics, increased understanding of fractions, decimals, and percentages was the highest response (Wilkie, 2016). However, missing from the literature and to better support the most needed topic in mathematics for middle years students is the standpoint of the mathematics teachers, which this preliminary study aims to help address.

There are other essential mathematics topics that are apposite contenders on which to base a novel intervention study in Year 8 mathematics. Referring to Martin et al. (2012), students need to see the value or relevance of mathematics in order to best engage in the subject. The following essential number and algebra topics are in the Australian Curriculum (AC; Australian Curriculum, Assessment and Reporting Authority, 2016), thereby are relevant to school-based education, and are mentioned in or inferred from the Programme for the International Assessment of Adult Competencies, Australia (Australian Bureau of Statistics, 2013), thereby are relevant to civic and workplace needs: mental computation; multiplication facts (or times tables); estimating; negative numbers; place value of decimals; computing with decimals; percentage of a quantity; percentage change; converting between decimals, fractions, and percentages; repeating patterns; growing patterns; order of operations; and solving equations. Also, the topic of fractions is quite broad and could be split into fractions concepts and computing with fractions.

While it is generally agreed that fluency in multiplication facts is an essential aim of primary education (e.g., Wong & Evans, 2007), it is not clear what importance secondary mathematics teachers hold the automaticity and flexible use of multiplication or times tables facts, and which particular group of facts is important for students to learn or needs extra attention. A recent search in the U.S. database, Educational Resources Information Center (ERIC), of peer-reviewed publications using the Boolean term "and" with the ERIC subjects "multiplication", "computation", and "mathematics instruction" revealed 77 articles, but none of these were research studies or discussion on which multiplication facts students need to be fluent in to recognize factors, support derived strategies, appreciate patterns (like the repeated digit pattern: 11, 22, 33…, of the 11 times table), or calculate commonly encountered multiple quantities quickly, like the number of months in multiple years. Researched support for the benefits of fluently learning multiplication facts only up to 10 x 10, as currently required in the AC, versus learning up to the 12 times tables of yesteryear was not found. An aim of this study is to survey what multiplication facts mathematics teachers deem as necessary for students to fluently learn.

A further aim of this preliminary study is to gauge the likelihood of mathematics teachers using an online or e-learning course — in the form of set of lessons/modules inclusive of competency-based assessment — to address students' deficits in understanding that prevent progress and success. In previous decades, this question would be invalid because the most consistent reason for mathematics teachers not using technology in their classrooms was lack of adequate access to computers (Forgasz, 2006; Zammit, 1992). Now, however, the computer to student ratio in Victorian government schools is nearly one-to-one (1:1.46) in primary schools and better than one-to-one (1:0.94) in secondary schools (Department of Education and Training, 2016). It appears likely that students would engage with a digital resource. Young Australians are avid users of technology, with 99% of 15- to 17-year-olds in 2014-2015 having access to the internet and an average of 18 hours per week use (Australian Bureau of Statistics, 2016). In Adelaide, Paris (2004) found that secondary students usually preferred online supplements to their classroom learning compared to pen and paper-based tasks.

Online courses have multiple advantages for users. They allow for immediate feedback (Butler, Pyzdrowski, Goodykoontz, & Walker, 2008) and easy, global, and rapid connection to other resources (e.g., the digital learning objects repository, Scootle, by Education Services Australia, 2017). Online resources offer asynchronous (anytime) use, mobility, anonymity, and they can be text-based, use multimedia, or be multimodal (Haythornthwaite & Andrews, 2011) and be potentially accessed by unlimited numbers of students. Pertinent to the main study here, digitally composed courses are highly editable and can be quickly and relatively cheaply published either locally or globally.

Use of online courses in U.S. public high schools is widespread, especially for the purposes of regaining credit for failed courses and completing core requirements in the main academic subjects (Clements, Stafford, Pazzaglia, & Jacobs, 2015). In Sydney region secondary schools, Neyland (2011) found a range of attitudes to online courses by computer coordinators, from aversion to sheer dedication. While there is some research on Australian mathematics teachers' beliefs affecting their choices to use technology in general (e.g., Hennessy, Ruthven, & Brindley, 2005; Pierce & Ball, 2009), there is little on their willingness to use online or e-learning courses or modules as interventions where students need extra help in essential topics to progress and succeed in their studies.

There are five research questions for this study:

- Which topics do mathematics teachers rate as critical for success in secondary mathematics?
- In which topics do students need more conceptual understanding?
- In which topics do students need more fluency?
- What multiplication facts do students need to learn?
- What is the likelihood of mathematics teachers offering an online course to their students to increase their understanding of mathematics topics required for success in secondary mathematics?

Method and Data Analysis

A questionnaire was conducted at the 2015 annual Mathematics Association of Victoria conference in Melbourne. Teachers of mathematics were approached and asked to complete a brief questionnaire about the most important topics required for secondary students' success in mathematics. The survey was anonymous, but respondents were asked to indicate the year levels in mathematics they had taught in the last five years. Space was provided for participants to record any further thoughts. Respondents were approached at morning tea break. Teachers were asked to tick the top three mathematics topics in three columns:

1. Topics critical for mathematics success in secondary mathematics
2. Topics in which students need more conceptual understanding
3. Topics in which students need more fluency

The following topics, each with sub-topics in italics, were included as choices: mental computation (including using known facts flexibly, time tables - with a grid to select any or all from 2 to 12, and estimating), negative numbers (computing with negative numbers), fractions (including two subtopics of understanding all fractions concepts and computing with fractions), decimals (including place value of decimals; and computing with decimals), percentage (including percentage of a quantity, percentage change, and converting between decimals, fractions, and percentages), algebra (including repeating patterns, growing patterns, BODMAS [i.e., order of operations – brackets, orders, division, multiplication, addition, and subtraction], and solving equations), and other.

Despite asking participants to tick the top three topics in each column, some ticked more and some less. Three respondents selected more than three topics per column. In these cases, so that no one participant's scores dominated the results, the total score of three was divided evenly across each of that respondent's responses for that column. For example, one participant ticked six subtopics (Only the top three were requested per column), and as such each selected subtopic was assigned 3/6 = 0.5 points. One respondent selected fewer than three subtopics for one column and his or her score was not altered.

A further section asked respondents the following question to rate with yes, no, or maybe: If an online course or module was available to help address the above concerns you have for your students, would you be most likely to consider: using it in your classroom, setting it as homework, or mentioning it as a resource that students can follow up in their own time?

Indicating a concern with the questionnaire, the two subtopics, mental computation and times tables, were quite often ticked ambiguously for the first column, which prompted for the topics critical for secondary mathematics success (6 out of 22 respondents who selected these two topics did not respond clearly, for example, ticking mental computation, but not times tables, then selecting "all" for times tables) and in those cases, each of these two sub-topics was assigned a score of 0.5. This was not an issue for the conceptual understanding and fluency prompting columns that were marked unambiguously for both times tables and mental computation. Reported tallied scores which included averaged data were rounded to the nearest whole number to better reflect the precision of that data.

Results

Thirty completed surveys were collected. All teachers surveyed had taught either upper primary mathematics (Years 5 and 6) or secondary mathematics (Years 7 to 12), with the greatest majority teaching from Year 9 through to senior secondary (Years 11 and 12) mathematics. One secondary teacher had retired more than five years prior to the survey whereas all others had been teaching in the last five years. No respondents had taught mathematics at preschool or at a technical and further education (TAFE) college. One respondent had written "VCE equivalent" for year level of mathematics taught and his or her data were included under "General mathematics".

The mathematics levels taught by number of respondents in the last five years were as follows: Early years to Year 4 (2), Years 5 and 6 (3), Year 7 (9), Year 8 (11), Year 9 (15), Year 10 (18), Foundation or Essential mathematics (4), Further mathematics (16), General mathematics (16), Mathematics methods (17), Specialist mathematics (7), university-level mathematics (7), Technical and Further Education (TAFE) mathematics (0), and online (1).

In answer to the prompt, "I think the following three maths topics… are critical for success in secondary maths: (Tick your top 3)", the highest-scoring topics understanding all fractions concepts (14/30), times tables (13/30), and solving equations (13/30). The clear choice in which the 30 surveyed teachers thought students need more conceptual understanding was fractions concepts. It was selected 16 times as one of the top three. Other choices selected with about half the frequency of the top choice were computing with negative numbers, place value of decimals, growing patterns, and solving equations. The mathematics topic choices for which the 29 surveyed teachers (One respondent did not complete this section) thought students need more fluency were fractions concepts (chosen 13 times) and times tables (chosen 11 times). Computing with fractions and solving equations were the next most frequent choices, as depicted in Figure 1.

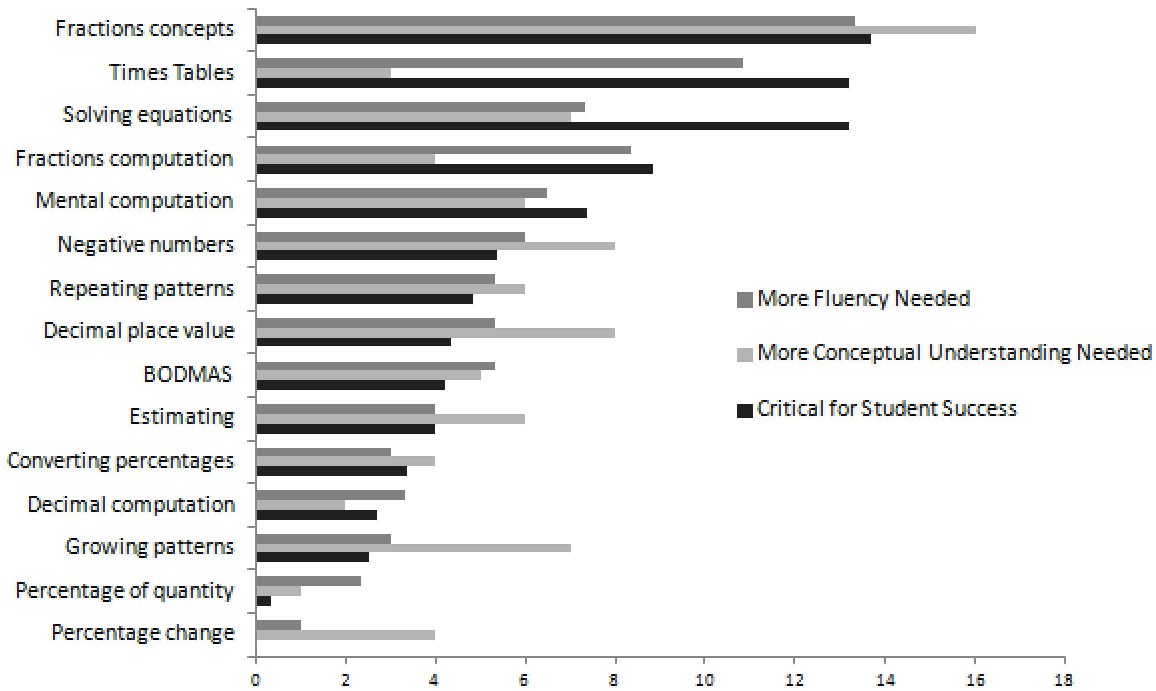

Figure 1. Mathematics teachers' appraisals (n = 30) of essential mathematics topics most critical for success, and in which there are student deficits in conceptual understanding or fluency.

Where times tables were selected across all three categories – more fluency needed, more conceptual understanding needed, or critical for student success – the majority of selections for times tables was "all" (24 out of 33 total selections), meaning here the multiplication factors from 2 to 12. However, in about one-fifth of instances, the selection of which particular times tables required was simply omitted, and one respondent selected factors from 2 to 10 and another indicated that only the seven and eight times tables required further improvements in fluency for students.

Most respondents selected that they would consider using an online course in their classroom to support students with difficulties that they had identified in essential mathematics (18 selected yes, 4 selected no, and 5 selected maybe). Most respondents would also consider setting an online course for homework (yes: 16, no: 4, and maybe: 4) and as a resource for students to pursue in their own time (yes: 16, no: 2, and maybe: 3).

Discussion and Conclusion

The questionnaire results successfully confirmed that fractions concepts are the best choice for the focus intervention topic for the larger study involving the co-creation of an online module by Year 8 students for peers. Fractions concepts were selected as the most frequent choice by secondary and middle years mathematics teachers across all three categories – critical for success in secondary mathematics, more conceptual understanding is required, and more fluency is required. The findings support earlier research (Basic Skills Agency, 1997; Brown & Quinn, 2006; Clarke & Roche, 2009; Kloosterman, 2010; Reyna & Brainerd, 2008; Stafylidou & Vosniadou, 2004; Zhang et al., 2015) that showed that fractions concepts are poorly understood by many students and the general public. Furthermore, the middle years and secondary mathematics teachers' perspectives found here align with that of primary teachers' appraisals (Daraganova & Ainley, 2012) and that of middle years students themselves (Wilkie, 2016) that fractions concepts are often the most problematic for students.

Despite the small number of participants (n = 30), the non-random participant selection, and the questionnaire layout initially prompting ambiguity in the responses between times tables and using known facts flexibly (mental computation), there is, albeit qualified, support for students increasing their fluency in times tables in general and learning multiplication facts with factors up to 12. Most (73%) selections for times tables were "all", meaning here the multiplication factors from 2 to 12. However, the questionnaire did not include the multiplication facts for zero and one, and did not allow for easy, explicit choice between students knowing just up to the single-digit factors (zero through nine), from 0 to 10, the products up to 100 (e.g., 0 x 99 = 0, 3 x 3 = 9, 5 x 15 = 75, 48 x 2 = 96), a concentration on prime number factors and deriving the rest, or any other possibility or combination. Nonetheless, especially in the dearth of information and research on this topic in the literature, it warrants further exploration as to what secondary mathematics teachers regard as the most important multiplication facts for students to be taught, learn, understand, practice, recall, and be able to fluently use, and why.

There was strong support for considering the use of an online course to address concerns that respondents had for their students in essential mathematics topics. This augurs well for finding support from mathematics teachers for the creation or use of an online course in their classrooms and ties in with Paris' (2004) finding that the students themselves prefer web-based rather than pen and paper-based supplements. However, there is only scant research on the use of online or e-learning courses as interventions in Australian high schools. This raises the following questions: What online intervention courses are being used? What criteria do teachers use when selecting an online course for students? For what topics do teachers seek online courses for their students?

If further research with a wider participant base replicates the findings here, that both fractions concepts and multiplication fact fluency are not only vital for secondary mathematics success but also merit remedial intervention, then perhaps a broader-level intervention for improving fraction fluency and conceptual knowledge and multiplication fact fluency in lower secondary school, or earlier, is warranted. Online interventions have the advantage of inexpensively and quickly being made available to unlimited numbers of recipients as long as a central repository or other means for dissemination for such resources is available. Alternatively, the students themselves could, with assistance, create local resources that they need to succeed in their mathematics education to share with peers, and in the process, improve in their engagement with the subject.

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Hypothesis of Developmental Dyscalculia and Down Syndrome: Implications for Mathematics Education

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In this paper, the hypothesis that Developmental Dyscalculia (DD) is a characteristic of Down syndrome (DS) is proposed. Implications for the hypothesis are addressed: If it were to be confirmed that DS implies DD, what would be the consequences for the mathematics education of learners with DS? The use of prosthetic devices to overcome the impaired calculation capabilities of the brain is essential. Fortunately, electronic calculating devices are readily available. Their routine use opens the possibility of studying areas of mathematics that were once inaccessible.

Introduction

Despite improved teaching, family circumstances and higher expectations, learners with Down syndrome (DS) continue to experience severe challenges learning arithmetic (Faragher & Clarke, 2014). These learning difficulties are disproportionate to accomplishments in other areas of the learner's life and persist despite being functionally necessary, of interest to the learner, and with considerable opportunities for regular practice – just the environmental factors usually advocated for effective learning. In addition, these difficulties have been observed around the world and across decades, reducing the likelihood that teaching methods or some other environmental factor is the cause.

Studies of the neurological basis of the development of early number and arithmetical skills (such as, Butterworth, Varma, & Laurillard, 2011; Dehaene, 2011; Dinkel, Willmes, Krinzinger, Konrad, & Koten, 2013; Shalev & Gross-Tsur, 2001) have led to an understanding of processes in typical brain function, and a growing awareness of atypical development. Discovery and description of a specific learning disability named Developmental Dyscalculia (DD) is one outcome of this research effort.

In this paper, the hypothesis that DD is a characteristic of DS is proposed. In raising this hypothesis, current understandings of DD are presented and the symptoms of DD, are described. The research on the development of related areas of mathematics by learners with DS is then summarised before the evidence for the hypothesis of a co-morbidity of DD with DS is discussed. Implications for the hypothesis that DS implies DD follow, focussing on the consequences for the mathematics education of learners with DS.

Background

Before considering disabilities with learning and using mathematics, and how that might affect people with Down syndrome, typical mathematics cognition as scientists have so far revealed is discussed.

Human Perception of Number

Humans have a remarkable capacity to make sense of the world through the use numbers. How our brains do this has been the subject of a great deal of recent research. For

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a review of the field and to note the development in the first decade of this century, the reader is referred to the seminal work of Dehaene (1999, 2011).

> Animal research on number pointed to an age-old competence for processing approximate quantities. This "number sense", which is also present in infants, gave humans the intuition of number. Cultural inventions, such as the abacus or Arabic numerals, then transformed it into our fully-fledged capacity for symbolic mathematics. (Dehaene, 2011, p. x)

Although details are still being refined, research undertaken by psychologists and neurobiologists have revealed a model of human cognition with two systems that are now widely accepted. In Figure 1, a diagrammatic representation has been developed as a synthesis of a number of research descriptions (especially Dehaene, 2011; Lanfranchi, Berteletti, Torrisi, Vianello, & Zorzi, 2015).

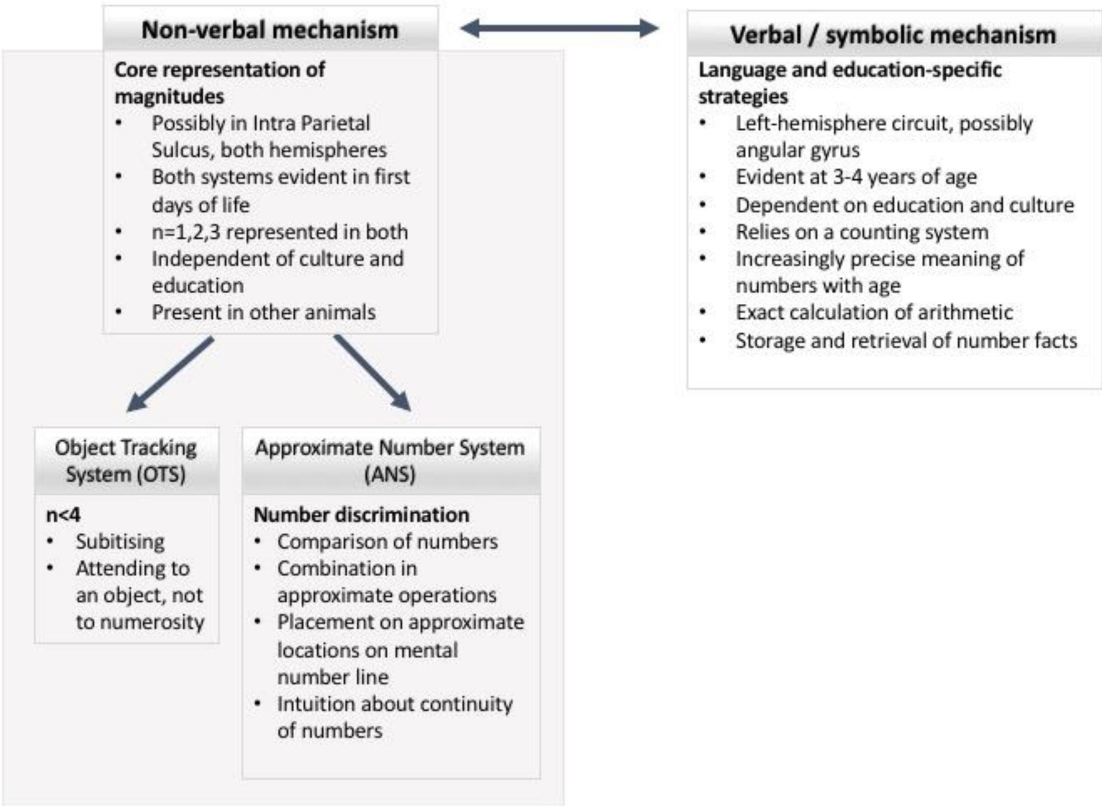

Figure 1. Model of the human mathematics processing systems.

There are two mechanisms – non-verbal and verbal. The non-verbal mechanism has commonly been called 'pre-verbal' however, it continues to function after the development of the verbal mechanism and throughout life, therefore 'non-verbal' is a more apt description. Within the non-verbal mechanism, there are now considered to be two systems – an object tracking system (OTS) and an approximate number system (ANS). These two systems are important in the discussion of mathematics cognition for learners with DS, as it would appear that these may not be completely intact. The OTS is a system that allows tracking of up to four objects in space, without attaching ideas of quantity to them (Xu, Spelke, & Goddard, 2005). The ANS allows approximate discrimination of quantities, including location on a mental number line, with a sense of continuity of numbers.

Comparison and combination of quantities can be undertaken, though not by exact calculation.

Humans share both the OTS and ANS with many other animals. It is the verbal mechanism that humans alone exhibit. At three to four years of age, humans link their quantitative knowledge to language – number words or symbols, building an increasingly precise number meaning, leading on to exact arithmetic and further mathematics (Dehaene, 1999).

Butterworth, Varma, and Laurillard (2011) summarise current evidence from brain imaging research concluding "almost all arithmetical and numerical processes implicate the parietal lobes, especially the IPS [intraparietal sulcus], suggesting that these are at the core of mathematical capacities" (p. 1050). Therefore, the area of the brain implicated in the non-verbal mechanism is the IPS, in both hemispheres. For the verbal mechanism, a major area of the brain is likely to be the pre-frontal cortex or the angular gyrus. These are parts of the brain that are used by the working memory and are engaged when non-routine, automatic thought processes are in action (Dehaene, 2011).

In this section, the model that expresses the current understanding of human number cognition has been presented. This serves as background to consider the system when it does not operate as expected – when calculation is extraordinarily difficult.

Definition and Presentation of Developmental Dyscalculia

It is not a new phenomenon for there to be learners in a primary class that experience significant challenges in learning arithmetic, while having no problems learning other areas of the curriculum. Developmental dyscalculia was coined as a term to describe this condition and was defined several decades ago by Kosc (1974):

> Developmental dyscalculia is a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions. (p. 47)

More recently, Kucian and von Aster (2015) defined DD as "a specific learning disability affecting the development of arithmetic skills" (p. 2) and "a heterogeneous disorder resulting from individual deficits in numerical or arithmetic functioning at behavioural, cognitive/neuropsychological and neuronal levels" (p. 4). DD is described in the Diagnostic and Statistical Manual of Mental Disorders V (American Psychiatric Association, 2013) as a specific learning disorder that cannot be explained by inadequate learning opportunities, and is "characterized by problems processing numerical information, learning arithmetic facts, and performing accurate or fluent calculations" (p. 67). There is a high incidence of co-morbidities with DD such as ADHD. Some researchers (such as Kucian & von Aster, 2015) suggest this could be due to the broad functional and structural differences across the brain observed in individuals with DD. Down syndrome may be another disability paired with DD.

Consensus is emerging from neuroimaging and other behavioural evidence that DD has a basis in neurological impairment (Price & Ansari, 2013). Early evidence for highly specialised areas of the brain performing various aspects of quantity, calculation and other mathematics came from observations of people who had experienced brain injuries (Kosc, 1974).

Diagnosing DD

Growing evidence of the neurobiological basis of DD would suggest diagnosis could be made using neuroimaging techniques. Dinkel and colleagues (2013) describe using functional magnetic resonance imaging (fMRI) to diagnose DD however, research using this technique is in its infancy. Some fMRI studies of people with DS have been undertaken (for a review, see Key & Thornton-Wells, 2012) however, it would appear no studies of calculation have occurred to this point.

In the absence of reliable imaging techniques, diagnosis of DD has been made on the basis of clinical assessments of arithmetic skills (Shalev & Gross-Tsur, 2001). Timed tests are commonly used because the answer alone will not provide the evidence of DD; correct answers may eventually be found but by very laborious or inefficient strategies. For example, in determining the bigger of two sets of objects, people with DD would count both sets of objects to compare rather than being able to know at a glance (subitise). Diagnosis of DD is considered when the "performance in arithmetic is significantly lower than expected for the child's aptitude" (Shalev & Gross-Tsur, 2001, p. 340).

Arithmetical Development of Learners with DS

The mathematical development of learners with DS remains an emerging field of research. Most studies in the area have investigated basic arithmetic, the area of interest in a discussion of DD, and these studies indicate considerable difficulties (see Faragher & Clarke, 2014, for a review of the literature on attainment). Of significance, the areas of difficulty in arithmetic almost completely match the areas of impairment in DD where the evidence exists.

Perhaps as a result of the pervasive view of mathematics as hierarchical, with attainment of basic arithmetic considered a pre-requisite for any further study, research on the mathematical development of learners with DS into areas beyond calculation is rare. Some research has emerged, reporting success in areas such as algebra and coordinate geometry (see, for example, Faragher, 2014; Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010).

Research literature on the mathematics attainment of learners with DS indicates universal difficulties with basic arithmetic. Studies report a range of achievement. However, in none is the achievement on par with matched participants without DS (Faragher & Clarke, 2014). In addition, studies (e.g., Lanfranchi et al., 2015) indicate that arithmetic is a specific difficulty, over and above other difficulties, such as language.

Hypothesis

The hypothesis proposed here is that people with DS experience DD. While occurring in 3-6% of the general population (Rotzer et al., 2009), in the subset of DS, the hypothesis is that DD is comorbid and likely to affect the majority.

Evidence

DS is a chromosomal pattern marked by triplication of some or all of chromosome 21. While it is known that DS affects the brain in a number of areas such as decreased brain volume, including in frontal and occipital lobes and different brain activation patterns (Key & Thornton-Wells, 2012), this remains an area of research. With the invention of less invasive analysis techniques, such as fMRI, there is potential for advances in understanding the neurobiology of people with DS.

At this time, there is no direct evidence from brain imaging studies of DD in DS. Indirect evidence, however, abounds. In recent years, research studies from the field of psychology have begun to explore mathematical cognition in Down syndrome. These fine-grained studies are beginning to shed light on the sub-skills involved in representing quantity and give clear indication of aspects of the non-verbal mechanism that are not operating as they should – that is, there is evidence of DD.

Belacchi and colleagues (2014) studied approximate additions and working memory. The model of number cognition would suggest approximate additions would be part of the ANS and therefore undertaken by the IPS. Working memory is known to be part of the verbal mechanism and to use the frontal cortex of the brain. Their study observed impairment of the ANS with significant impairment of numerosity estimation involving one set. When participants were able to make use of working memory resources (the verbal system), they were successful with estimating the numerosity of additions.

Numerical estimation has been studied by another research team. Lanfranchi and colleagues (2015) researched the ability of people with DS and two groups matched either by mental or chronological age to estimate the location of numbers on number lines. This would be a feature of the ANS. Their results suggest that this aspect is within findings expected for developmental stage of the participants. The paper also reported findings from measures of numerical intelligence and arithmetic knowledge, including statistically significant poorer performance on non-verbal calculations, in which participants had to add or subtract one or more dots from a given set. The operands were in the single-digit range. If four or less, this would involve the OTS. Greater than four would imply the ANS was activated. Their results indicate impairment of the non-verbal mechanism, a marker of DD.

The research team had previously reported findings from related data (Sella, Lanfranchi, & Zorzi, 2013), studying enumeration skills. They note evidence for a specific deficit in the OTS for individuals with DS, which would again suggest DD. Number acuity (the ability to distinguish the larger of two numbers) and the understanding of cardinality was in keeping with mental age.

The work of this research team noted that the groups matched on mental age were very much younger (DS mean age was 14 years; MA matched mean age was five years). The superior lexical performance of individuals with DS was suggested to be due to their longer experience with number words and symbols. This finding would appear to be an indication of the importance and value of education. Dehaene (2011) emphasises the critical role of teaching the cultural tools necessary for moving from the non-verbal to the verbal mechanism which are essential for exact arithmetic. A number of studies have indicated that children with DS can make use of the verbal system (Lanfranchi et al., 2015; Nye, Fluck, & Buckley, 2001; Sella et al., 2013), which employs other parts of the brain than the IPS.

The challenge in this hypothesis is that scientific understandings of DD are still emerging and there continues to be definitional confusion about the condition, its causes, symptoms and impact on learners (Rubinsten & Henik, 2009). A further challenge relates to diagnosis of DD in learners with DS. Timed arithmetic tests are problematic as the results can be confounded by difficulties with completing the tests – understanding what is being asked, recording responses etc. – all of which take time. Observational tests or task-based interviews (Clarke & Faragher, 2014), may provide evidence of the use of inefficient strategies, such as counting small sets rather than subitising. Key here would be the discrepancy between arithmetical achievement and the learner's general achievement profile.

Discussion

Much can be done without linguistic labels (number count words, for example). The major concepts of arithmetic can be accomplished: quantity; comparison; approximate operations. These are the within the non-verbal mechanism. The linguistic labels help, and indeed are necessary, to move beyond the non-verbal system.

Implications of the Hypothesis for Education

Researchers in DD have considered implications of the diagnosis for learners and some propose cognitive training to attempt rewiring the brain. Many also assume that calculation and number sense are essential pre-requisites for further mathematics study. These positions are problematic for learners with DS (and perhaps other learners as well).

Some evidence for the efficacy of cognitive training in ameliorating the symptoms of DD is emerging (Butterworth et al., 2011; Kucian & von Aster, 2015), though intense effort is needed to achieve this improvement. If DD is a person's only cognitive impairment, the learner may well be sufficiently motivated and the potential gains significant enough to justify the effort. For learners with DS who have many other challenges to contend with, devotion to brain training may not be a feasible intervention.

The second implication proposed by others is that calculation is considered to be a precursor to further study in mathematics and therefore, learners with DD will have limitations of further study of mathematics. Even though this has been the accepted and rarely questioned view in the field of mathematics education until recent times (Forgasz & Cheeseman, 2015), there is growing evidence to suggest this is not the case: it is possible for learners with DD to learn other mathematics. Indeed, evidence is available in the work of researchers in DD (Butterworth et al., 2011; Dehaene, 2011).

As noted earlier, some learners with DS have been able to accomplish mathematics from a number of areas, including algebra, trigonometry, and percentages. Each of these students, it must be noted, could not calculate without the support of an electronic calculator.

For learners with DS, the following are alternative implications:

- Learners should use an electronic calculator as a prosthetic device, that is, a device that replaces the function of a part of the body. In this case, a calculator is used to overcome the impaired calculation functions of the brain.
- Explicit and lifelong attention to supporting conceptions of number must be made, making use of a variety of visual supports such as number lines. This encourages and reinforces alternative neural activity.
- Previously so called "functional mathematics" programs for learners with DS need to be fundamentally changed to focus on the use of prosthetic devices such as calculators and electronic banking methods in areas of finance and measurement.
- Learners should not be required to demonstrate accomplishment on basic number work before they are taught mathematics from across the discipline. The content should be from the curriculum of their schooling level and adjusted as needed.

Testing the Hypothesis

The indirect evidence for the hypothesis of Developmental Dyscalculia being a feature of Down syndrome lies in the pervasive nature of calculation difficulties for this population across the world and over decades of research in the field. Direct evidence to confirm the hypothesis is needed and may come from fMRI studies, particularly to examine if the IPS is affected. Alternatively, clinical assessment tools, such as task-based interviews need to be developed that are designed for learners with DS and explicitly probe areas of number development observing strategy use.

Conclusion

Should the hypothesis of DD as a characteristic of Down syndrome be confirmed, the implications for mathematics education are profound. Until now, a great deal of effort has been expended in school mathematics programs trying to teach arithmetic to learners with DS with limited success. This has led to students being taught the same material over and over again, often for all their years of formal schooling. An understanding of the source of the difficulties as being an impairment in calculation, frees those involved in the development of learning programs to include the routine use of electronic calculating devices as a support to enable the teaching of other areas of mathematics. As we await further research from the fields of neurobiology and psychology, mathematics programs can be planned that continue to reinforce understanding of number concepts while teaching year level mathematics content with adjustments.

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Gender and VCE Mathematics Subject Enrolments 2001-2015 in Co-Educational and Single-Sex Schools

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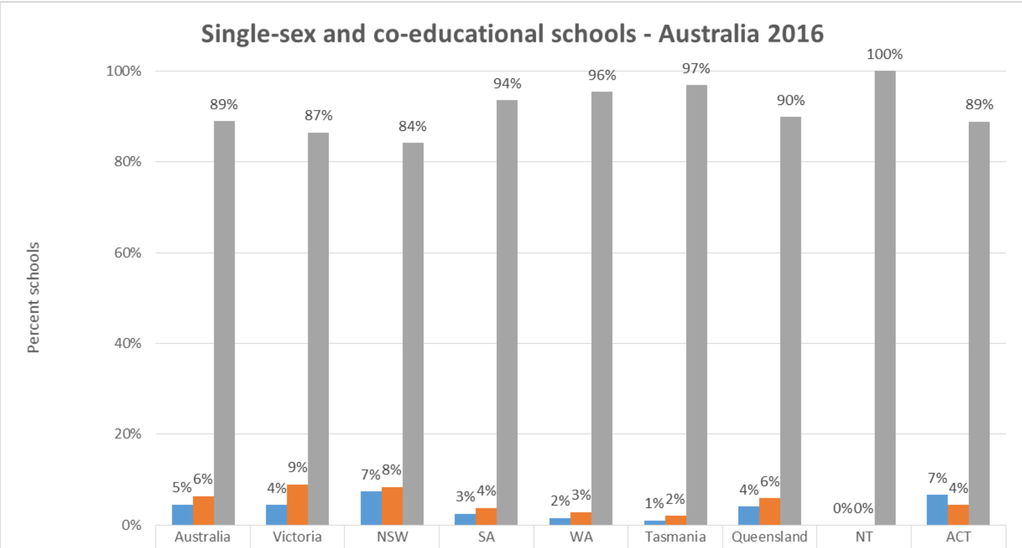
Declining enrolments in advanced level mathematics at the school level are noted with concern. Whether school type (single-sex school or co-education) affects participation in mathematics continues to be debated. In this article we examine, by school type and gender, statistical data from 2001 to 2015 on Victorian Certificate of Education enrolments in the three mathematics subjects offered at that level. Also explored are the choice of, and reasons for, the school setting assumed to promote STEM studies for girls and boys.

Introduction

The debate on the relative merits of single-sex and co-educational schooling for girls and for boys persists in Australia. Passionate protagonists are found on both sides. Whether the context is academic achievement, leadership opportunities, or confidence development, one of the most pervasive views put forward is that single-sex schooling is better for girls, while co-education is better for boys.

As in the past (see Ainley & Daly, 2002), the reality in contemporary Australia is that there are more single-sex schools for girls than for boys. This pattern is more marked in some states than in others (see Figure 1), and in the ACT, the opposite is found. One consequence of having more single-sex schools for girls than for boys is that girls are outnumbered by boys in co-educational schools.



	Australia	Victoria	NSW	SA	WA	Tasmania	Queensland	NT	ACT
Single-sex BOYS	5%	4%	7%	3%	2%	1%	4%	0%	7%
Single-sex GIRLS	6%	9%	8%	4%	3%	2%	6%	0%	4%
Co-educational	89%	87%	84%	94%	96%	97%	90%	100%	89%

Figure. 1. Percentages of single-sex (boys/girls) and co-educational schools in Australia in 2016, by state/territory. [Data derived from https://www.goodschools.com.au/.]

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Single-sex schooling in Australia is predominantly found in the fee-paying sectors of education (Good Schools Guide, 2016). Within the government sector, single-sex schools generally have selective entry, based on academic achievement. While there are some academic scholarships offered in fee-paying schools, those attending them are generally from higher socio-economic backgrounds than students attending government schools.

That school and family backgrounds are major contributing factors to student achievement is widely accepted (e.g., Hattie, 2009). Cobbold (2015) maintained that in Australia, and elsewhere, "school SES has a much larger impact on student achievement than individual family SES" (pp. 4-5). Student prior achievement and confidence levels, expectations of those in the social milieu, and school factors including teachers and subject offerings all contribute to subject choice decisions (e.g., Eccles et al., 1983; Hattie, 2009).

Declining enrolments in advanced level mathematics at the school level (e.g., Barrington & Evans, 2014) and the under-representation of females in these subjects (e.g., Barrington & Evans, 2014; Finkel & Sherry, 2017) continue to be of concern. Forgasz (2016) noted the frequency of claims, and strength of beliefs, that girls attending single-sex schools are more likely than girls in co-educational schools to study mathematics and science subjects. But where is the statistical evidence to support these claims?

In this article, we present statistical data from 2001 to 2015 on Victorian Certificate of Education (VCE) enrolments in the three mathematics subjects offered (specialist mathematics, mathematical methods, and further mathematics) by gender and school type (single sex girls, single-sex boys, co-educational girls, and co-educational boys) obtained from the Victorian Curriculum and Assessment Authority (VCAA).

Our aims in examining the VCE mathematics enrolment data, 2001-2015, were to examine enrolment patterns over time for girls and for boys attending single-sex and co-educational schools, and to determine whether girls and/or boys are more likely to study these subjects if they attend single-sex schools. In addition, to tap current views in Australia about the suitability of single-sex schools for girls and boys to study science, technology, engineering and mathematics (STEM) subjects, we draw on survey data from a larger study about schooling, careers, and STEM pathways.

Previous Research in the Field

Research has been conducted to compare the mathematics achievement of males and females attending single-sex and co-educational schools; attitudes and beliefs have also been investigated. Thien and Darmawan (2016) reported that in 12 countries participating in the first international study of mathematics, "the greater the ratio of single sex to co-educational schools the greater the difference between the sexes in Mathematics Performance, with boys outperforming girls at the 13-year old level" (p. 89).

Lenzer (2006) noted the contradictory findings with respect to girls' mathematics and science achievement and participation in single-sex and co-educational schools. In some studies girls attending single-sex schools, compared to girls in co-educational schools, "are more likely to have confidence or be interested in mathematics and to choose mathematics and or natural sciences as a subject of study later on" (p. 58). But she also reported that "[W]hen students entering single-sex or co-educational schools are matched for background variables, the effect of gender-segregated education on non-traditional subject choice… disappears" (p. 58). Billinger (2008) surveyed single-sex schooling within the US and similarly concluded that the "apparent benefits of single-sex schooling can largely be attributed to selection bias in the pool of students who choose SSE" (p. 402). Thus, school culture appears to be a critical factor implicated in girls' non-traditional subject choice.

The effects of single-sex classes within co-educational secondary schools have also been explored. Leder & Forgasz (1998) reported mixed results on students', teachers', and parents' attitudes to the introduction of single-sex mathematics classes at grade 9 in one Australian co-educational school. "Single-sex classes *per se",* they concluded, "would appear to be too simplistic a strategy to address identified gender inequities in mathematics education" (p. 177). Writing about single-sex classes in the middle years of schooling, Crosswell and Hunter (2012) concluded that "there is no 'right' answer due to the multiple variables that could be playing out in any classpace" (p. 25), and that underpinning "the seemingly simple question of single sex classes in co-education schools, is the much more complex socio-political issue of assumptions about sex and gender" (p. 25).

Australian research on participation in mathematics subjects in co-educational and single-sex schools is scarce. Some work has been conducted internationally, and there are some Australian findings related to STEM participation more generally, and in the physical sciences. Ainley and Daly (2002) reported raw data on physical science participation in single-sex and co-educational schools in Australia in 1998. They found that girls attending single-sex schools were more likely than girls in co-education schools to study these subjects. However, when a multivariate analysis was conducted, this "apparently greater participation… was not statistically significant after allowance was made for other influences that were associated with school gender context" (p. 256). The factors involved in the multivariate analysis included: language background, socio-economic status, earlier school achievement, residential location, and school type.

In summary, the literature is mixed about the benefits of single-sex schooling (or classes) for girls and their achievement and attitudes towards mathematics. Little appears to be known about girls', compared to boys', relative enrolments in senior level mathematics in Australia, nor about females' views and recommendations of school type for boys or girls interested in STEM-related subjects. In this study, we address these issues.

The Study

Methods

The VCAA data. In response to a request to the VCAA, VCE enrolment data for the years 2001-2015 for specialist mathematics, mathematical methods (CAS), and further mathematics, were provided by gender within school type (single-sex and co-educational); permission was denied for a further break-down of the data by school sector (government, Catholic, and independent). Also provided were the number of students within each school type by gender who were eligible to complete VCE in each year, allowing for the proportions of students enrolled in these subjects by gender within school type to be calculated. Analyses of VCE data by gender within school type are unique; the VCAA had not previously been requested to provide data of this kind (Bui, personal communication).

In consultation with VCAA, it was determined that the most effective enrolment comparisons would result from comparing the percentages of students eligible to complete VCE who were enrolled in each subject, that is, not to include students who were studying the subjects as part of their year 11 of the two-year VCE.

For each year, 2001 to 2015, the percentages of students eligible to complete VCE enrolled in each subject were calculated for boys and for girls in single sex and in co-educational schools. These percentages are shown in Figures 2-4 below for each of the three mathematics subjects.

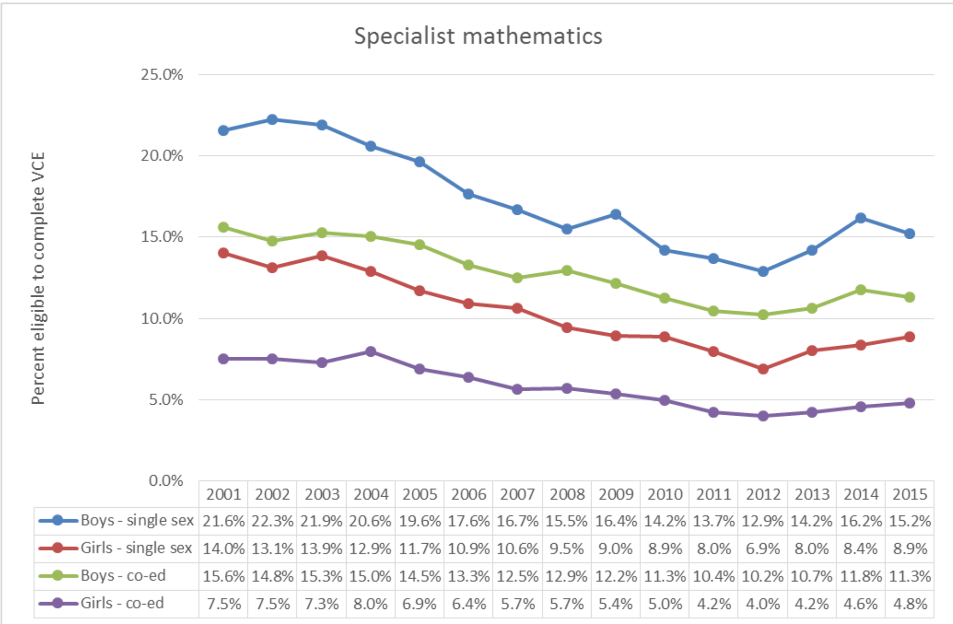
The survey data. The items in which survey participants were asked whether, to promote a boy's/girl's interest in STEM-related studies, they would recommend a single-sex school, a co-educational school, or neither (that it would depend on the child), were of particular interest for this article. Also of interest were the explanations provided for the choices nominated by the respondents.

Results

The VCAA data. Trends in the data for each mathematics subject (see Figures 2 to 4) were examined, and the enrolment pattern findings for each subject are reported below.

Specialist mathematics. The data in Figure 2 reveal that:

- Higher proportions of boys in both single-sex and in co-educational schools study specialist mathematics than girls in single-sex or co-educational schools (that is, boys dominate over girls irrespective of school type).
- The difference in the proportions of boys and girls studying specialist mathematics is about the same in each school type
- A higher proportion of girls in single-sex schools than in co-educational schools study specialist mathematics; the same pattern is evident among the boys.
- Over time, there was a steady decrease in the proportions of boys and girls in both school types studying specialist mathematics until 2012, after which increases for girls in both school types, and inconsistencies among boys in both school types, are evident.



	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Boys - single sex	21.6%	22.3%	21.9%	20.6%	19.6%	17.6%	16.7%	15.5%	16.4%	14.2%	13.7%	12.9%	14.2%	16.2%	15.2%
Girls - single sex	14.0%	13.1%	13.9%	12.9%	11.7%	10.9%	10.6%	9.5%	9.0%	8.9%	8.0%	6.9%	8.0%	8.4%	8.9%
Boys - co-ed	15.6%	14.8%	15.3%	15.0%	14.5%	13.3%	12.5%	12.9%	12.2%	11.3%	10.4%	10.2%	10.7%	11.8%	11.3%
Girls - co-ed	7.5%	7.5%	7.3%	8.0%	6.9%	6.4%	5.7%	5.7%	5.4%	5.0%	4.2%	4.0%	4.2%	4.6%	4.8%

Figure. 2. Percentages of girls and boys eligible to complete VCE in single-sex and co-educational schools enrolled in specialist mathematics, 2001-2015.

Mathematical methods (CAS). The data in Figure 3 reveal that:

- A higher proportion of girls in single-sex schools than in co-educational schools study mathematical methods; the same pattern is evident among the boys.
- Higher proportions of students (both boys and girls) in single sex schools than in co-educational schools study mathematical methods (CAS)

- Over time, there has been a steady decrease in the proportions of boys and of girls in both school types studying mathematical methods (CAS); interestingly the decreases have been greater for girls in both schools types (single-sex: 8.8%; co-educational: 6.2%) than for boys (single-sex: 7.3%; co-educational: 3.9%), and greater in single-sex schools for both girls and boys than for boys and girls in co-educational schools.

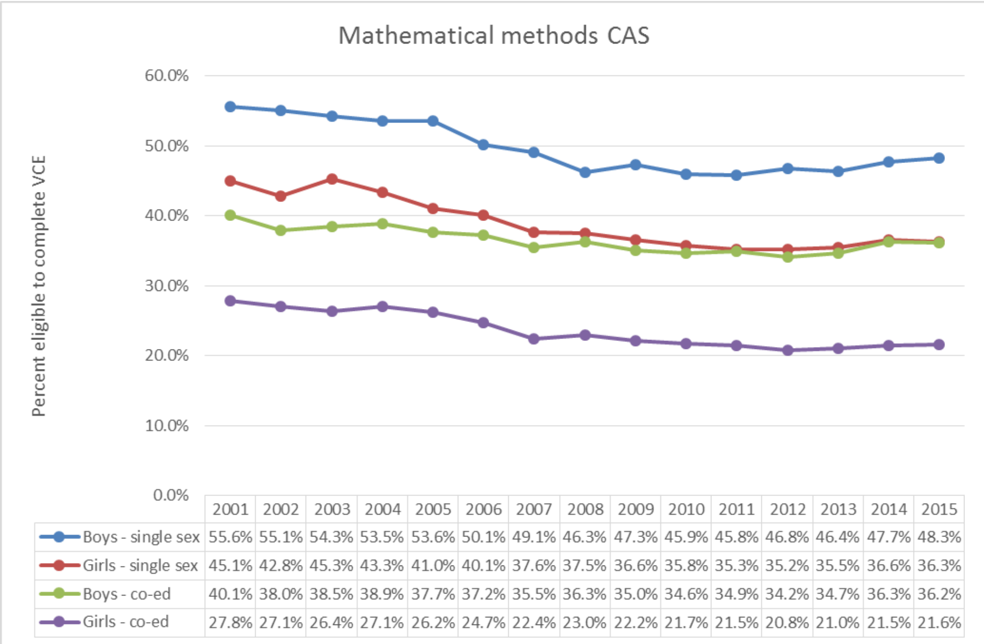


Figure. 3. Percentages of girls and boys eligible to complete VCE in single-sex and co-educational schools enrolled in mathematical methods (CAS), 2001-2015.

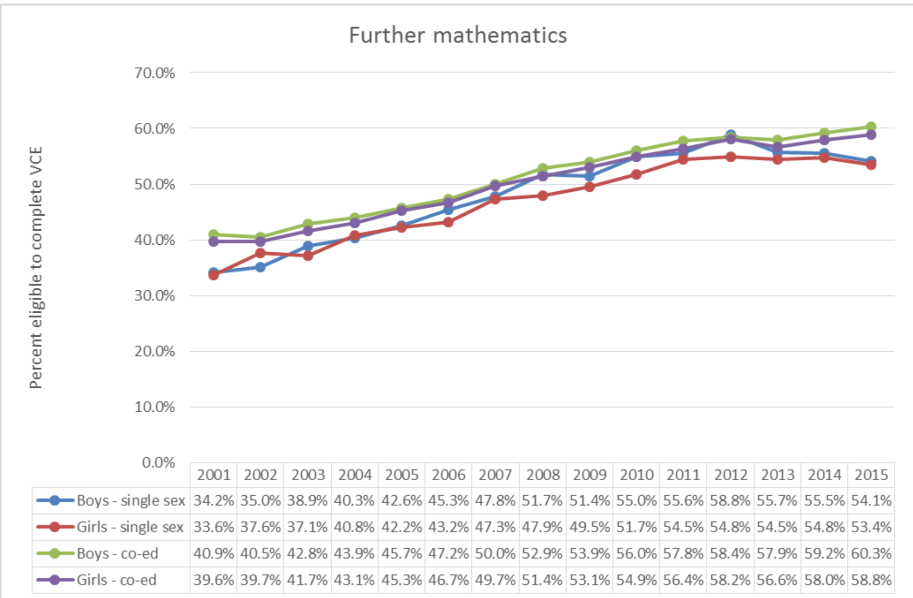


Figure. 4. Percentages of girls and boys eligible to complete VCE in single-sex and co-educational schools enrolled in further mathematics, 2001-2015.

Further mathematics. The data in Figure 4 reveal:

- Similar patterns of enrolments in further mathematics for boys and for girls in both school types
- Over time, the proportions of boys and girls in both school types enrolled in further mathematics have increased at very similar rates.

The survey data. The survey sample comprised over 1,100 females, aged from 18 to over 70. Most had studied mathematics in their final year of secondary school: advanced level (N = 377), intermediate level (N = 472), and elementary level (N = 126) mathematics; some (N = 89) had not studied any mathematics. Consistent with the focus of the larger study on single-sex schools, the majority of respondents (N = 964) had attended a single-sex school and a smaller number (N = 164) a co-educational school.

As can be seen from the data in Table 1, almost half of the female respondents thought that a single-sex school setting would promote STEM-related studies for girls, compared with 14% who thought this was the case for boys.

Table 1
School Setting Thought to Promote STEM-Related Studies

Recommendation	For boys	For girls
Single-sex school	138 (14%)	427 (43%)
Co-educational school	98 (10%)	79 (8%)
Either, depends on child	739 (76%)	485 (49%)
Total	975	991

Whether the type of school the respondents themselves attended seemed to influence the school setting they nominated can be gauged from the data in Table 2.

Table 2
Recommendation of School Setting by Respondents' Own Schooling

	Recommendation	Attended co-educational school	Attended single-sex school
To promote a boy's interest	single-sex school	10 (7%)	128 (16%)
	co-educational school	32 (22%)	66 (8%)
	either, depends on child	107 (72%)	632 (77%)
To promote a girl's interest	single-sex school	27 (18%)	400 (48%)
	co-educational school	35 (24%)	44 (5%)
	either, depends on child	87 (58%)	398 (47%)

It can be seen in Table 2 that a higher proportion of those who attended a single-sex school considered single-sex schools (16%) as more suitable than co-educational schools (8%) to promote a boy's interest in STEM-related studies, while a higher proportion of those who attended a co-educational school thought boys would benefit from attendance at co-educational schools (22%) than single-sex schools (7%). The differences in the settings nominated were statistically significant (χ^2 = 30.09, p<.001, effect size, V=.18).

A comparable pattern can be seen in Table 2 for promoting girls' interest in STEM. Of those who attended single-sex schools, a higher proportion nominated single-sex schools 48%) than co-educational schools (5%) to promote girls' interest in STEM. Of those who had attended co-educational schools, a higher proportion recommended co-educational schools (24%) than single-sex schools (18%) to promote girls' interest in STEM. The different patterns nominated were statistically significant (χ^2 = 81.55, p<.001, effect size, V=.29). Also noteworthy are the smaller proportions of those attending single-sex and co-educational schools who nominated "could be either" for girls (47% and 58% respectively) than for boys (77% and 72% respectively).

As indicated earlier in the paper, respondents were also asked to provide the reason(s) for their choice of school setting to promote STEM interest for girls and for boys. The explanations of those whose recommendation for boys and girls differed were of particular interest. Space constraints allow only a small but representative set to be included here.

To promote a BOY'S interest in STEM	To promote a GIRL's interest in STEM and
Attended single-sex school; advanced and intermediate maths in final year of school	
Either, depends on child	*Single-sex school*
Boys are seen as more naturally gravitating towards these subjects. In fact, although I am pronouncing on matters about which I know next to nothing, I would have thought that a boy in a single sex school might have more difficulty pursuing humanities. Whether the child is in a single sex school or a co-ed school (and therefore, perhaps, opinions of their peers about their choice of subjects) probably has much less significance from a gender perspective.	Girls are rarely told these days (I hope) that 'girls don't do that', but that doesn't mean that the subtle societal messages don't do a damn good job of making sure girls 'know' that STEM subjects are not feminine, and what's more, that femininity as defined by society is an overarching goal. I recall being encouraged at a single sex school to take STEM subjects because I was smart, and good at them, and perhaps I felt that I should take them in case I needed them.
Attended single-sex school; advanced and intermediate maths in final year of school	
Co-educational school	*Single-sex school*
Look at industry - males don't seem to need any consideration here - system seems to be working for men in STEM.	I think girls benefit from a single sex schooling system where they are given the tools and ideological foundation to believe they can achieve anything - before having to identify with the gender bias and inequalities that exist in STEM.
Attended single-sex school; advanced and intermediate maths in final year of school	
Either, depends on child	*Single-sex school*
Boys don't get told they are not good at maths or science so I think choice of school is not as important	Peer pressure and gender stereotypes are more likely to arise at a co-ed school
Attended co-educational school; intermediate mathematics in final year of school	
Either, depends on child	*Single-sex school*
Each child learns differently and is to be nurtured for their individual learning style	Girls I have observed in 15 years plus teaching are more confident and driven in a single sex setting

Summary of Findings

Higher proportions of boys in single-sex and in co-educational schools than girls in single-sex and in co-educational schools are enrolled in specialist mathematics. While for specialist mathematics there was a higher proportion of girls from single-sex than co-educational schools enrolled, the same was true among boys in the two school types. Higher proportions of girls and boys in single-sex schools than in co-educational schools were enrolled in mathematical methods CAS. The proportions of students enrolled in further mathematics is virtually identical among boys and girls in single-sex and co-educational schools.

It is too simplistic to conclude that the gendered setting of the school alone contributes to the differences found, particularly considering that the same proportions of boys and girls in both school types were enrolled in further mathematics. Yet from the explanations provided for the preference expressed for a single-sex or co-educational school to promote STEM-related subjects it can be seen that respondents were influenced by their own school history and that, among this group of generally well-educated females, the belief that girls more often than not benefit from attendance at a single-sex school persists.

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A Secondary Mathematics Teacher's Perceptions of her Initial Attempts at Utilising Whiteboarding in her Classes

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Excellent mathematics teachers establish learning environments that encourage students to actively engage with mathematics and foster co-operative and collaborative learning. Whiteboarding, using an erasable surface on which to work and share ideas, has been shown to increase student engagement, collaboration, and higher-order thinking. We report on one teacher's experiences as she introduces whiteboarding into her secondary mathematics classroom. The teacher reports increased student confidence and collaboration and we see a shift in her focus from concerns about classroom management, to a passionate recommendation to use whiteboarding in mathematics instruction.

Introduction

The Australian Association of Mathematics Teachers (AAMT) Standards (2006), in defining excellence in mathematics teachers' professional practice, maintain that excellent mathematics teachers maximise students' learning opportunities by: actively engaging students in their learning; establishing learning environments which encourage students to engage with mathematics; fostering co-operative and collaborative learning; expecting and encouraging students to "have a go"; encouraging high quality verbal and written communication; monitoring students' practice to plan for future learning experiences, and to map and report on students' progress. These teachers utilise classroom organisation and teaching strategies which best meet students' learning needs. This paper reports on one teacher's experiences as she introduces whiteboarding into her mathematics classrooms, in an effort to inspire her students to "embrace maths in terms of their future goals and make maths as exciting as possible for them" (Gail, Individual Interview).

Whiteboards and Whiteboarding

The term *whiteboarding* has been in use since the 1990s, and has been defined as "the action or process of using a whiteboard, especially as a means of collaborating with others" (Oxford University Press, 2016). While within educational settings whiteboards can be used for students to display their work, whiteboarding is essentially an active learning process involving students communicating, taking risks, making mistakes, sharing, modifying and evaluating their own and each other's' ideas (Wenning, 2005).

The whiteboards used for whiteboarding can be virtual or real, electronic or not, hand held, mobile on wheels or easels, mounted on a table top or a wall (they may be the table top or wall) and range in size from very small to the size of a classroom wall. Whiteboarding, in fact, does not require a whiteboard at all. Any easily erasable, easy to write on surface will do; blackboards certainly fit the bill, as do windows, or electrostatic plastic film. The essential features of whiteboards are the readily shared space they provide for recording ideas, their ease of use in communicating these ideas, and the flexibility they provide for writing, erasing and modifying responses.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 261–268). Melbourne: MERGA.

The literature describes the utilisation of whiteboards, in various forms, across a variety of educational settings and disciplines to support collaborative learning, to support the development of higher order thinking, to improve the quality of classroom discourse and to provide the teacher with insights into students' thinking.

Mini whiteboards (used in primary and secondary schools across a range of disciplines), while not generally associated with collaborative whiteboarding, are useful for increasing classroom interactivity (Beauchamp & Kennewell, 2008), engaging all students in classroom discussions and enabling teachers to gain immediate, simultaneous feedback from all students and insights into their understanding (Wiliam & Leahy, 2015; Swan, 2006). Their appeal to students involves their non-permanence, which appears to encourage them to take learning risks and to engage quickly with the learning task, understanding that they can readily modify their responses with no lasting record of their work (Swan, 2006).

Medium-sized hand-held whiteboards have been used by students working collaboratively in small groups to record their solutions, findings or ideas, which are subsequently presented to, and discussed with, their peers (MacDuff, 2011; Wells, Hestenes, & Swackhamer, 1995; Wenning, 2005;). Wenning (2005) claims that this kind of whiteboarding affords improved classroom discourse through making student thinking visible, enhancing student cooperation, motivating students to become active in their own education; improves communication skills, increases student participation and enhances student learning (whiteboards USA, n.d.). West, Sullivan and Kirchner (2016) found group whiteboarding on medium hand-held whiteboards enabled elementary science classes to jointly generate, share, synthesise and build on their ideas, while building their literacy through improved classroom discourse. Henry, Henry and Riddoch (2006) appreciated the flexibility the whiteboards provided for recording, modifying and discarding students' ideas and the easy access students had to each other's' thinking, which promoted whole class discussion and understanding. MacIsaac (2000) found this method of whiteboarding valuable for actively involving students in large first year physics lectures in workshop-type activities. His students found whiteboarding increased their motivation, concentration and interest, enabled active participation in lectures and promoted deep thinking. They found that whiteboards facilitated the easy recording and sharing of solutions, assisted them in visualising concepts and afforded them opportunities to critically evaluate and support each other's' learning.

Board Rooms

Large, wall mounted, or mobile whiteboards are usually located centrally at the front of classrooms and are often the focal point for the delivery of teachers' notes, or as a teaching space for teacher-led demonstrations, discussions or explanations. However, de-fronting the mathematics classroom by fixing large whiteboards on all available vertical surfaces—creating a *board room*—for the use of students in doing mathematics, is being utilised in several educational settings with remarkable benefits.

The 360 Degree Math program, created by science teacher Sean Kavanaugh, is aimed at improving students' mathematics scores and engagement with mathematics (Antoniades, 2013). As Kavanaugh describes (Antoniades, 2013), the program gets students out of their seats and working on wall-mounted whiteboards, students' thinking becomes visible, the teacher becomes the audience and the students become the performers. Although no research based evidence has been published, Kavanaugh reports that students' engagement with mathematics improved dramatically, as did the school's performance in School District mathematics assessments (Antoniades, 2013).

Liljedahl (2016) utilises board rooms as part of a collection of classroom practices aimed at bypassing normative classroom practices which inhibit students' development of problem solving skills and mathematical thinking. He found that students working on whiteboards were more eager to start, evidenced more discussion, participation and persistence, and recorded more of their exploratory approaches to the problem, than on non-erasable surfaces. Wall mounted whiteboards were the most effective in nearly all of these measures and students were more mobile in sharing their knowledge and seeking support when working on vertical boards.

Seaton, King and Sandison (2014) describe *board tutorials*, classes where university students complete their mathematics problems at boards mounted on walls around the room instead of in their notebooks. This practice was introduced at our university 25 years ago. While the outcomes of this approach have never been researched, it is considered a very successful innovation by tutoring staff. "What a difference! Where tutorials had once been quiet, passive affairs, they were now full of animated, engaged learners and teachers…students interacted with the subject material, collaborated with other students, and interacted with the tutor" (p. 106). Students are no longer anonymous and cannot hide their lack of engagement or understanding as tutors can readily see students' misconceptions and deal with them.

Introducing this approach into secondary school mathematics classes has been of interest to us for the past three years. A pilot study comparing high school students' engagement in a class held in a board room and a class held in a desk room (Sandison, Forrester, & Denny, 2015) found behavioural engagement to be considerably higher in the board room. A teacher interview and student survey indicated that both the teacher and students were very positive, reinforcing the benefits already mentioned.

Having identified the benefits of using whiteboards in several educational settings we are interested in assisting teachers in developing their classroom practices into board rooms to support the engagement and learning outcomes of their students. Therefore, we commenced this study with this research question: What are the perspectives of teachers introducing board rooms into their secondary mathematics classrooms?

Methodology

The qualitative research reported in this paper is informed by phenomenology, which seeks to understand an experience from the perspective of the participants (Kervin, Vialle, Howard, Herrington, & Okely, 2015). The study seeks to investigate the lived experiences of a group of teachers introducing board rooms into their secondary mathematics classes.

Data were collected through individual interviews with each participant to gain background information, followed by four semi-structured focus group interviews with four participants undertaken over a period of six months. Three focus group interviews were undertaken in the fourth term of the school year, the final interview was conducted at the end of the first term the following year. Between the third and fourth interview professional development involving the development of *thinking classrooms* (Liljedahl, 2016) was provided to participants.

Focus group interviews were considered advantageous for this study as they provide interaction among participants and can yield extensive and rich data (Creswell, 2015). The first focus group interview was undertaken prior to, or immediately following, the installation of the board rooms, the second two weeks later, the third a further two weeks later and the fourth nearly five months later. Focus group interviews were audiotaped and transcribed and thematic analysis conducted. This paper presents the findings in relationship to one participant teacher, Gail[1].

Participant

At the commencement of this project Gail had been teaching secondary mathematics for seven years and was teaching five classes: a low ability Year 7; a high ability Year 9; a moderate to high ability Year 10; a low to moderate ability Year 11 General Mathematics[2] and; a low to moderate ability Year 12 General Mathematics 2[3].

Gail is very keen to take on leadership roles within her school and mathematics education community. Gail is an extremely well prepared and organised teacher whose mathematics classes are characterised by control; focusing the first term of every year establishing and maintaining rules, routines and expectations. Gail believes her role as a teacher is to help students understand the importance of mathematics for their future, to inspire them to embrace mathematics and be excited by it. She aims to show students the relevance of what they were learning, often connecting mathematics with other Key Learning Areas. Gail utilised group work, encouraged collaborative learning and aimed to build confidence in her students.

As an alumnus of our university, Gail previously experienced learning in the board rooms as a student and, later as a tutor. Gail's experiences in the board rooms were very positive, indeed, Gail loved the "success behind it" (Individual Interview).

Results

Before Whiteboard Installation

Gail entered the study concerned that the number of students taking mathematics was dropping. She identified the need to try new ways of teaching to motivate and engage students. "I'm all for trying something new, particularly with the low ability kids and getting them engaged and excited about mathematics" (Focus Group 1 (FG1)). From her experiences as a student and tutor, Gail was enthusiastic about using board rooms at school, believing it would allow students to lead the way in their learning, with her role being a guide for them on their "self-discovery journey" (FG1). Her goal was to build students' confidence in mathematics, "breaking down those barriers that the kids have … of maths... If I can get every kid from the bottom to the top to say, 'I can give something a go in maths', I've done my job" (FG1).

Gail had firmly established class expectations and structure, which differed from class to class according to Year level and ability. Students in all classes were seated in rows to enable Gail to give direct instruction and monitor attentiveness. Typically, Gail's lessons involved a computer presentation, followed by checking homework and practice activities. In providing direct instruction, Gail looked to make the mathematics relevant and to ensure student understanding. She utilised group work to encourage peer learning.

Prior to installing the whiteboards Gail was excited by the possibilities the board room would provide to change classroom practice, humorously saying "let's throw out the

[1] A pseudonym

[2] General Mathematics is the lowest level of mathematics for Year 11.

[3] General Mathematics 2 is the lowest level of mathematics examined at the NSW HSC.

desks" (FG1). However, she also had reservations regarding classroom and behaviour management. She was concerned about how she would control students' behaviour while at the boards, how she would avoid students writing or drawing inappropriate words or pictures on the boards and how to accommodate the 28 students in her Year 9 class.

Gail's Main Concern: Classroom Organisation and Management

Despite her reservations, Gail used the whiteboards every day for practice activities after a time of direct instruction and demonstration. Gail attempted to de-front the room by rearranging the desks into groups and standing at a different board each lesson to deliver her initial ten-minute explanation/demonstration of the topic. However, after a few weeks, Gail found that grouping tables was not efficient for her direct instruction time, as she wanted to use the fixed mounted projector. Gail could see no way around this, so rearranged the tables into a U shape, reluctantly re-establishing a front.

Initial classroom and behaviour management concerns led Gail to introduce several behaviour and classroom management measures. These included: the establishment of board rules and associated consequences; controlling and monitoring movement to and from the boards; introducing a "texta licence", which students earned by demonstrating a pre-determined level of understanding, and carrying with it a sense of "privilege" to be working at the boards; students writing their names at the top of their boards to ensure they took responsibility for any writing or drawing; and having half of the large Year 9 class working on the boards at one time, while the other half worked at their desks.

Gail found her initial concerns for classroom and behaviour management unfounded. Having established effective classroom management previously, the transition to board work seemed to flow easily and, in fact, when students were working at the boards she could monitor their behaviour more readily than when they were working at desks. As Gail observed, "you'd be foolish to be off task because I can see what you're doing" (FG1).

The main behavioural change Gail noticed as a result of board work was an increase in classroom discourse and resulting increased classroom noise. While this was notable, it did not concern Gail, commenting: "You can't go in there and expect them to quietly just work on the board." In fact, Gail found the increased conversations a sign that students were engaged with the mathematics and working collaboratively: "So as long as they're talking about "$yx + 3 = \ldots 10$" and how they got to x, I don't care that they're talking" (FG3).

Gail had often provided opportunities for group work at desks and so she was keen to exploit the opportunities the whiteboards provided for collaboration and peer support. She regularly encouraged her students: "if you're stuck, look over your shoulder and see what others are doing" (FG1). In the Year 9 class where only half the class worked on the boards at any time, Gail urged those at the desks to look at the work on the boards, seeing this as "peer demonstration".

Gail's Approach to Lesson Preparation and Delivery

Prior to installing the board room Gail decided to utilise the whiteboards all the time, in her words "I decided to throw myself into the deep end and hope that I could swim" (FG1). However, after installation Gail lacked confidence in how to use them, commenting that she had to force herself, while trialling how to use them. Gail's initial teaching approach was to simply prepare as she would for her normal lesson but ask students to do all their practice work on the boards instead of in their books. This strategy worked well for Gail as it was relatively easy to implement and required changes to classroom management rather than a major shift in her teaching.

Gail sought regular, anonymous feedback from her students and found students were so keen to work at the boards, that she began to consider alternative strategies to get students onto the whiteboards immediately as they entered the classroom, rather than after her normal introductory session. She discussed a few possible strategies at the second focus group and thought she might try pre-printing lesson notes, handing them out without giving an explanation/demonstration time and "let [students] figure it out" (FG2). However, Gail did not report trialling this. After four weeks, while she was convinced that the board room provided the best environment for students to complete their practice activities, she still felt constrained by the need she saw for students to take notes "so they can see a model…of what they're expected to do…that way when they go home they've got something they can flick through, as their summary when they are going to study as well" (FG3).

In the second meeting Gail discussed trialling assessment on the boards and attended the next focus group meeting excited to share her successful first attempts giving a standardised algebra test with differing numbers and pronumerals. While she required students work silently, she encouraged them to look around the room to get ideas and felt this assessment compared favourably to regular pen and paper tests.

Five months on, Gail had been promoted to a different school which had a board room as a common teaching space and introduced whiteboarding to her students and mathematics staff. In Week 5 of Term 1, she attended the *Thinking Classrooms* Professional Development session and started to incorporate problem solving tasks into her lessons. Her practice was continuing to develop and at the end of Term 1 (FG4) she commented:

> I'm finding from this experience [focus group discussions] and talking to Peter [Liljedahl] as well, I love, like, riddles and puzzles and things like that and I'm finding I'm searching more and more for them and the more I give it to the kids…as a reward at the end, they kind of get their work done to say, "well, where's the riddle?". I started to research open ended riddle stuff to match what I'm doing. So when I did finance with my…Year 12 class, and Year 11 classes, I put the tax man [problem] in there, 'coz it's finances. And then…when I did a bit of algebra, I put the Einstein's riddle for them to try and solve. So those sort of problem solving are now starting to creep more and more into my teaching as well.

Gail's Perceptions of her Students' Responses

Early in the study, Gail reported differing initial reactions to board room lessons from different classes. Her Year 12 "absolutely love[d] it in there" (FG1), while her Year 9 students were reluctant; "You can't be serious, you can't expect us to do this!" (FG2). However, by the end of the lesson, they appeared to be enjoying themselves and later when asked to evaluate the board lessons most spoke positively, saying, "we're getting more out of it" (FG2).

Over the next few weeks, Gail felt she was building confidence in all her students, they were enthusiastic to do mathematics on the boards and were enjoying their mathematics experiences. While she became concerned that some students were not paying adequate attention to her introductory presentations/explanations, because they were so keen to get up to work at the boards, she realised that they were helping each other and collaborating at the boards and not missing out.

Gail was encouraged and surprised by her students' responses:

> I think my biggest surprise…going into this project initially, I thought yep, top kids are going to benefit from this, bottom kids not so much and I found that was certainly reverse to the truth. The

> bottom kids are getting in there again, having a go, whereas the top kids I've had to split—because they're such big classes—half-half [so they do not get as much time on the boards] but overall the student feedback was very positive. (FG3)

Gail's Conclusion: Throw Yourself Out There…See What Works!

Following Gail's promotion to a new school where she no longer had whiteboards in her own room but had to book a common space, her enthusiasm for whiteboarding continued to grow. Gail used the board room as much as possible (often sending a student down mid-class to check if it was free), but found for her smaller Year 12 classes she had enough space on her own whiteboard. This loss of her own board room led Gail to comment, "having the opportunity to be in the deep end and then ripped out of the deep end and put into the shallow end, I want my deep end I would have stayed there" (FG4).

Gail is now passionate about using board rooms in the instruction of mathematics. She no longer talks of "texta licences", nor is she concerned about classroom management or behaviour issues. She has experienced success with students of all abilities and this now drives her to share practice with others and encourage the use of board rooms. She encourages others to trial whiteboarding, to 'throw yourself out there…and see what works', to persevere for at least six weeks to get a sense of best use and benefits.

> I feel if this is the direction that we are going to go as a mathematical society, this is bigger than just … us who are a couple of teachers. We need to get the deputies and principals on board and when we throw teachers in with the whiteboards and start putting them into the schools they will have no choice but to start using them and when they have no choice, that's when they start to develop themselves and it's that whole regenerating interest in what you're doing. (FG4)

Discussion, Conclusions, and Implications

While Gail's initial conversations focused on classroom management, the benefits of teaching in a board room and whiteboarding, as highlighted in the literature, were emerging from her reflections. The board room made students' behaviours and thinking visible (Liljedahl, 2016; Seaton et al., 2014), enabling Gail to easily monitor her students' behaviour and mathematical thinking. In line with the literature and in keeping with excellent pedagogy as defined by AAMT Standards (2006), active participation, classroom engagement, discourse and collaboration immediately increased in all of Gail's classes, regardless of Year or ability level (Liljedahl, 2016; Sandison et al., 2015, Wenning, 2005). The ease of student mobility (Liljedahl, 2016), the opportunities the boards afforded to access each other's' thinking, explore ideas, and easily modify or discard them (Liljedahl, 2016; Swan, 2006; Wenning, 2005; Wiliam & Leahy, 2015) seems to naturally motivate students and induce on-task talking, behaviour and collaboration (Sandison et al., 2015).

Although Gail's enthusiasm to "dive in" to whiteboarding was palpable, her initial concerns and efforts were focused on classroom management. An area of strength in her classroom practice, it was interesting that her determination to maintain classroom control dominated her initial preparation. The literature on whiteboarding and the use of board rooms does not discuss this as a constraint or challenge, and it would be worthwhile investigating if this is a common issue for teachers considering or implementing whiteboarding into their classroom practices.

Knowing where to start in board room teaching was not clear to Gail, despite being a thoughtful, motivated and well-prepared teacher. Her determination to trial whiteboarding led her to continue with her normal practice while simply switching from book work to board work for practice activities. This was a successful strategy for Gail. However, once she was confident in her ability to manage her students' behaviour, Gail began to experiment with her pedagogical practices, responding to students' feedback, needs and behaviours, her professional discussions in the focus group meetings and the *Thinking Classrooms* professional development. Gail explored whiteboard assessments and began to introduce more engaging, non-routine problem-solving riddles and puzzles into her lessons, trying to link them meaningfully with syllabus-based content. The identification or development of non-routine problems linked to the curriculum content can be time consuming and difficult. Gail benefitted from the professional development provided by the focus group interviews and offered in this research. In exploring the experiences of teachers in their early efforts at whiteboarding, their approaches, successes, challenges and obstacles we are looking to gain a clearer picture of the types of support needed to ensure success.

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The Development of Addition and Subtractions Strategies for Children in Kindergarten to Grade 6: Insights and Implications

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This paper provides insight about the development of addition and subtraction strategies for nearly 22,000 Australian primary school children in 2016. The children were each assessed by their teacher using a task-based assessment interview that identified the strategies they used to mentally perform addition and subtraction, and matched these to a growth point framework. The findings highlight the broad distribution of strategies used by children in each grade level and suggest that few children, including those in Grade 6, reach Growth Point 6 that involves the mental calculation of two-digit and three-digit numbers. These findings have important implications for classroom teaching and professional learning.

Mathematics education provides knowledge that ultimately empowers people to access further education, employment and active citizenship. *Numeracy Now* is a system-wide strategy launched by a Catholic diocese in New South Wales to transform the learning and teaching of mathematics so that all students might thrive mathematically.

Key approaches for the initiative were: developing the mathematics instruction leadership of primary school principals and curriculum leaders; classroom teachers using the Mathematics Assessment Interview (MAI; Gervasoni et al., 2011) and an associated framework of growth points (Clarke et al., 2002), using this data to guide school-based reform supported by system consultants and using the Extending Mathematical Understanding intervention program (Gervasoni, 2015) in the second year of schooling and beyond to provide intensive specialised instruction for children who were mathematically vulnerable.

This paper reports on one aspect of this initiative in order to highlight issues associated with children's development of addition and subtraction strategies for calculating. This aspect was the use of the assessment interview and an associated framework of growth points to determine the Addition and Subtraction Strategies growth points reached by all children across the system in 2016. The insights gained and the implications for classroom instruction and teacher professional learning will be discussed with the view to recommending approaches for enhancing mathematics learning for all.

Using Assessment Interviews to Identify Children's Number Knowledge

Clinical assessment interviews are widely used by teachers in Australia and New Zealand as a means of assessing children's mathematical knowledge. This follows three large scale projects that informed assessment and curriculum policy formation in Victoria, NSW and New Zealand: *Count Me In Too* (Gould, 2000) in NSW, the Victorian *Early Numeracy Research Project* (Clarke et al., 2002), and the *Numeracy Development Project* (Higgins, Parsons, & Hyland, 2003) in New Zealand.

A common feature of each of these projects was the use of a one-to-one assessment interview and an associated research-based framework to describe progressions in mathematics learning (Bobis et al., 2005). Teachers participating in each project indicated that the benefits of the assessment interview, though time-consuming and expensive, were

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 269–276). Melbourne: MERGA.

considerable in terms of creating an understanding of what children know and can do, and for planning instruction. Indeed, an important feature of clinical interviews is that they enable the teacher to observe children as they solve problems to determine the strategies they used and any misconceptions (Gervasoni & Sullivan, 2007). They also enable teachers to probe children's mathematical understanding through thoughtful questioning (Wright, Martland, & Stafford, 2000) and observational listening (Mitchell & Horne, 2011).

The insights gained through these assessments inform teachers about the particular instructional needs of each student more powerfully than scores from traditional pencil and paper tests, the disadvantages of which are well established (e.g., Clements & Ellerton, 1995). Bobis et al. (2005) concluded that one-to-one assessment interviews and associated frameworks assisted to move the focus of professional learning in mathematics from the notion of children carefully reproducing taught procedures to an emphasis on children's thinking. It is broadly accepted that the traditional focus on taught procedures for calculating can negatively impact on children's number sense (Clarke et al., 2006) and may impede children's development of powerful mental reasoning strategies for calculating (Narode, Board, & Davenport, 1993). Thus, because of the deep insight about children's mathematical knowledge gained through the use of one-to-one assessment interviews, the Mathematics Assessment Interview was chosen as the assessment tool for the *Numeracy Now* initiative. It was anticipated that the data obtained would provide important insights for teachers and that the cohort data would provide a rich snapshot of children's addition and subtraction strategies for principals and system leaders.

The Mathematics Assessment Interview and Growth Points

One aspect of the *Numeracy Now* initiative is the annual assessment of nearly 22000 Kindergarten (five-year-old) to Grade 6 (11-year-old) children's whole number knowledge at the beginning of each year using the Mathematics Assessment Interview (MAI). This assessment tool is a refinement of the original Early Numeracy Interview (Department of Education Employment and Training, 2001) developed as part of the *Early Numeracy Research Project* (ENRP; Clarke et al., 2002).

The Mathematics Assessment Interview is a clinical interview with an associated research-based framework of growth points that describe key stages in the learning of nine mathematics domains. The principles underlying the construction of the growth points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics (e.g., Fuson, 1992; Gould, 2000; Mulligan, 1998; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000); reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of children who may be mathematically vulnerable.

The growth points form a framework for describing development in nine domains, including four whole number domains: Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The processes for validating the growth points, the interview items and the comparative achievement of children in project and reference schools are described in full in Clarke et al. (2002).

To illustrate the nature of the growth points, the following are the growth points for Addition and Subtraction Strategies. These describe the strategies children use to calculate.

1. Counts all to find the total of two collections.

2. Counts on from one number to find the total of two collections.
3. Given subtraction situations, chooses appropriately from strategies including count back, count-down to and count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding nine, build to next 10, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2013). They enable teachers to: identify the zone of proximal development for each child in each domain so instruction may be customised and precise; identify any children who may be vulnerable in a given domain; and highlight the diversity of mathematical knowledge in a class. The whole number tasks in the MAI take between 15-25 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total, and given success with a task, the teacher continues with the next tasks in a domain (e.g., Addition and Subtraction Strategies) for as long as the child is successful. If the child cannot perform a particular task correctly, the interviewer moves on to the next domain or moves into a detour, designed to elaborate more clearly any difficulty a child might be having in a particular area. To illustrate, Figure 1 shows three questions from the GP6 Addition and Subtraction Strategies section of the interview. Words in italics are instructions for the interviewer, and the symbols and arrows indicate which question to ask next. Teachers report that the assessment interview provided them with insights about children's mathematical knowledge that might otherwise remain hidden (Clarke, 2013). This was an important reason for using this assessment instrument as part of the *Numeracy Now* strategy.

Assessing Children's Whole Number Knowledge

The data examined in this paper were collected from all 56 Catholic primary school in one of the three Catholic Dioceses in Sydney. Participants included 21,884 Kindergarten (five-year-old) to Grade 6 (11-year-old) children who were assessed at the beginning of 2016 by their classroom teachers using the *Mathematics Assessment Interview* (MAI).

The teachers followed a detailed script to present each task to children and recorded their responses and strategies on an interview record sheet. These responses were analysed and coded by the teachers using a scoring rubric to determine the growth points children reached in the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The use of a detailed script, record sheet and scoring rubric were important for increasing the validity and reliability of the assessment data. Further, all school leaders and teachers had participated in professional learning focused on understanding and implementing the MAI and growth point framework. This also increased the trustworthiness of the assessment data across schools.

25) Multi-Digit Strategies
I am going to show you some questions. Tell me the answer**.**
Show the white cards for the following questions ***[at any stage]:*** ☹ → **Section D Q29a**
a) 68 + 32 **b)** 25 + 99 **c)** 100 – 68

For the next two questions (d) and (e) read the questions (no cards provided).

d) half of 30 **e)** double 26

26) How Many Digits?

a) *Show the blue card with 134 + 689.*

Please read the card to me. Is the answer to this more than 1000 or less than 1000?

Please explain. ☹ → **Section D Q29a**

b) *Show the blue card with 1246 – 358.*

Please read the card to me. Is the answer to this more than 1000 or less than 1000?

Please explain.☹ → **Section D Q29a**

27) **Estimating and Calculating Addition**

a) *Show the yellow card with 347 + 589.*
Please read this to me. Please estimate the answer to this.
(*If necessary, prompt:* "What would the answer be 'round about'?")

No estimate, or one outside the range 800-1000 → **Section D Q29a**

b) Can you work out the exact answer to this in your head? *(936)*

If "yes", encourage the child to try to do so. If not successful (or if the response to the question in part c) is "No"), make the following request:

c) Please use the pencil and paper to work it out <u>any way you like</u>. ☹ → **Section D Q29a**

Figure 1. Three MAI tasks for Addition and Subtraction Strategies Growth Point 6.

The growth points determined for each student were checked, entered into an Excel spreadsheet in each school and collated by system leaders. These data were next analysed by the research team, according to the ethical guidelines approved for the research, to determine the percentage of children on each growth point in each whole number domain and grade level. Only the growth point data for the Addition and Subtraction Strategies domain are reported in this paper.

Insights about Children's Addition and Subtraction Strategies

Examination of grade level growth point distributions for the Addition and Subtraction Strategies domain across the first seven years of schooling provides a rich picture of the development of children's knowledge and strategies, and enables associated insights and issues to be identified. Figure 2 shows the growth points for 21,884 children from Kindergarten to Grade 6 in 2016. This includes all children attending primary schools in the Diocese.

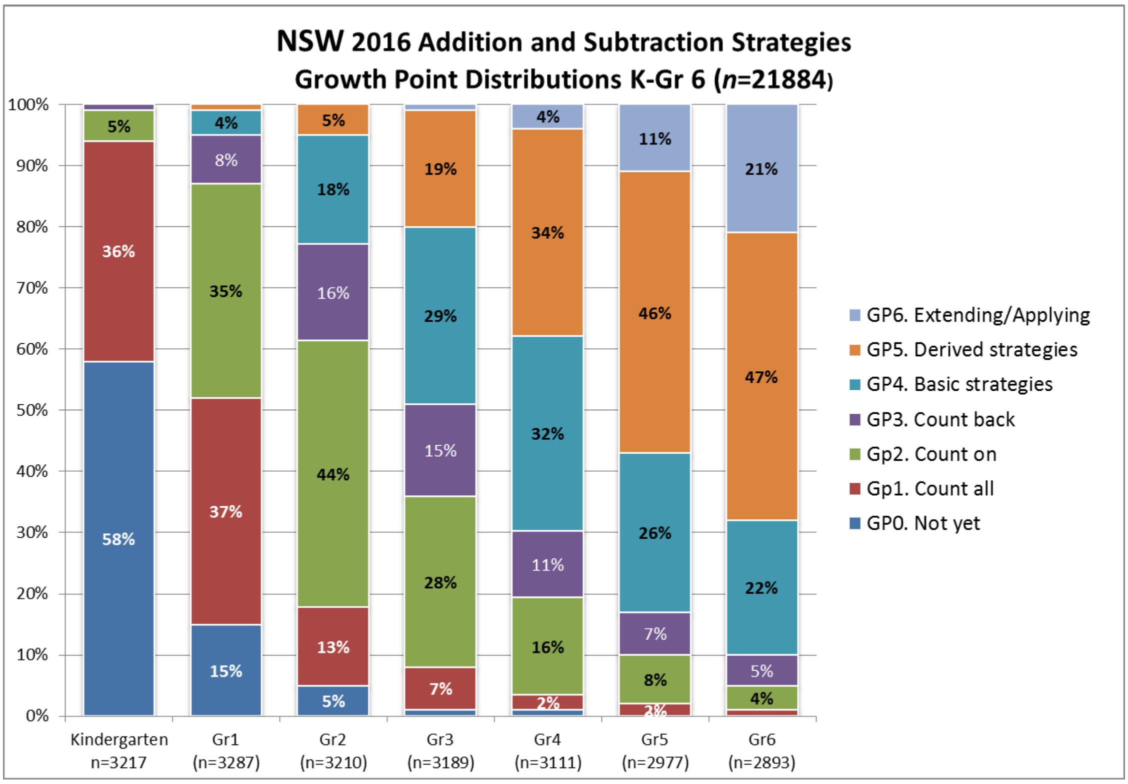

Figure 2. Addition and Subtraction growth point distributions for children in Grades K-6.

Four key insights concerning children's development emerge from analysing the growth point (GP) data:

1. Progress from one grade level to the next is evident with the median growth point increasing from Kindergarten to Grade 5 by one growth point each year, at which point the median growth point increase ceases;
2. There is a wide spread of growth points in each grade level, particularly from Grade 1 to Grade 6;
3. The 80th percentile growth point (GP5-derived strategies) does not increase from Grade 3 to Grade 5; and
4. The 20th percentile growth point does not increase from Grade 2 to Grade 3 (GP2-count-on) and from Grade 5 to Grade 6 (GP4-basic strategies).

A key focus for learning and teaching in addition and subtraction is children's development and use of basic and derived strategies for calculating as opposed to counting-based strategies. Analysis of the systems data highlights three issues associated with children's development of these strategies:

1. Large groups of children from Grade 3 to Grade 6 use counting-based strategies to calculate (GP 1, GP2 and GP3), including 51% of beginning Grade 3 students;

2. The majority of Grade 4 to Grade 6 children successfully use basic or derived strategies for calculating with one-digit numbers, such as for doubles, tens facts and near doubles; and
3. Very few Grade 4 to Grade 6 children reach GP6, which involves mental calculations with two- and three-digit numbers, such as 24 + 99 or 100 – 68.

Discussion

The findings presented in the previous section highlight several important issues related to the instructional needs of children in the Addition and Subtraction Strategies domain. First is the broad distribution of growth points in each grade level, and the wide distance between the lowest and highest growth points in each grade level. This shows that children in each grade have diverse learning needs, and underscores the complexity of classroom teaching and of teachers responding to children's diverse knowledge and experiences. It is clear that there is no teaching "formula" that will meet all children's needs. Thus, it is essential that classroom teachers are (1) able to identify children's current knowledge in order to plan suitable learning opportunities for them, and (2) have the ability to differentiate learning opportunities for individuals according to the range of knowledge represented in a teaching group.

The second point is that children's progress through the growth points from one grade level to the next is clearly obvious in most grade levels. However, it is also clear that progress from one growth point to the next is more challenging at some points for certain groups of students. For example, progress from GP5-GP6 appears to be challenging for many children. Knowledge about these challenging points in children's learning and about how to assist children to reach them is necessary to enable teachers to be most effective, and may be a useful focus for professional learning programs.

A third issue highlighted by analysis of the growth point distributions is the significant number of children using counting-based strategies beyond Grade 2. Counting strategies add considerably to the cognitive load in children's working memory and may lead to these children having difficulty accessing many of the concepts explored in their classrooms when the ability to mentally calculate is assumed.

A fourth point arising from the analysis is that few children reached the highest growth point (GP6), even by Grade 6. On one hand, this provides confidence that the MAI is a useful assessment tool throughout the primary school. However, this also emphasises the importance of teachers creating learning environments that enable all to thrive and reach the higher growth points. It appears that this is not currently so for many children.

Finally, the analyses suggest that several growth points represent challenging aspects of learning in addition and subtraction. These include using basic and derived strategies as opposed to counting-based strategies in addition and subtraction, and mentally calculating with two-digit and three-digit numbers. It is recommended that professional learning opportunities for teachers focus on these challenging growth points and associated powerful pedagogical actions and tools that will assist all children to learn these strategies.

Conclusion

The findings presented in this paper suggest that meeting the diverse learning needs of children is a challenge, and requires teachers to be knowledgeable about how to identify each child's current mathematical knowledge and to customise instruction accordingly. This calls for rich assessment tools capable of revealing the extent of children's knowledge and strategies, and an associated framework of growth points capable of guiding teachers' curriculum and pedagogical actions. Growth points help teachers to identify children's zone of proximal development in order to design effective learning opportunities for children, and in order to adjust activities to increase engagement and remove features that create barriers to learning. Thus, reference to a framework of growth points helps to ensure that instruction for children is closely aligned to their initial and on-going assessment, and is at the "cutting edge" of each child's knowledge (Wright et al., 2000).

A particular issue highlighted through examining the system date is that few children reach GP6 which requires them to mentally calculate with two-digit and three-digit numbers. Only 21% of beginning Grade 6 students reached GP6. This contrasts to a longitudinal study by Clarke et al. (2006) who used the same assessment interview and growth point framework to measure children's progress following the *Early Numeracy Research Program*. In their study, 22% of children reached GP6 at the end of Grade 4 and 59% at the end of Grade 6.

Researchers have long since noted that, despite curriculum advice to the contrary, many teachers introduce children to formal written methods for adding and subtracting two-digit and three-digit numbers in Grade 2 and Grade 3 (e.g., Narode, Board, & Davenport, 1993). The evidence presented in this paper suggests that this is before most children mentally calculate with numbers in this range. We hypothesise that this leads many children to approach calculations in a procedural way and may curtail the development of their number sense and flexible mental strategies for calculating with larger numbers. We suggest that teachers introduce children to formal written methods after they reach GP6 and can flexibly and creatively calculate mentally with two-digit and three-digit numbers. This aligns with NSW syllabus guidelines.

> From Early Stage 1 there is an emphasis on the development of number sense, and confidence and competence in using concrete materials and mental, written and calculator techniques for solving appropriate problems. Algorithms should be introduced after students have developed mental strategies for computing with two- and three- digit numbers. (NSW Board of Studies, p. 34)

We suggest that formal written methods should be reserved for calculations that are too complex to perform mentally. A later introduction of written methods may increase the conceptual understanding that children can apply to solving more complex calculations in efficient and flexible ways. We also recommend that teachers focus more intensely on children's development of basic and derived strategies in the first three years of primary school as a means of reducing many children's persistent use of counting-based strategies. The initial use of ten-frames, structured bead-strings and arithmetic racks to model, describe and simulate and discuss these basic and derived strategies may be a useful approach.

Assisting children to learn mathematics is complex, but teachers who are equipped with the pedagogical knowledge and actions necessary for responding to the diverse needs of individuals are able to provide children with the learning opportunities and experiences that enable them to thrive. With respect to children learning efficient, flexible strategies for mentally calculating with two-digit and three-digit numbers, the findings presented in this paper call for new pedagogical approaches in the schools included in this study.

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Teaching Fractions for Understanding: Addressing Interrelated Concepts

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The concept of fractions is perceived as one of the most difficult areas in school mathematics to learn and teach. The most frequently mentioned factors contributing to the complexity is fractions having five interrelated constructs: part-whole, ratio, operator, quotient, and measure. In this study, we used this framework to investigate the practices in a New Zealand Year 7 classroom. Video recordings and transcribed audio-recordings were analysed through the lenses of the five integrated concepts of fraction. The findings showed that students often initiated unexpected uses of fractions as quotient and as operator, drawing on part-whole understanding when solving fraction problems.

Fractions are notoriously difficult for students to learn and present ongoing pedagogical challenges to mathematics teachers (e.g., Behr, Lesh, Post, & Silver, 1983; Siemon et al., 2015). They are crucial, however, to students' future understanding of concepts such as proportional reasoning that are necessary not only for deeper mathematical understanding but also to support daily activities. These difficulties are often observed across all levels of education beginning from early primary years (e.g., Charalambous & Pitta-Pantazi, 2006; Empson & Levi, 2011; Gupta & Wilkerson, 2015). Different reasons have been identified for these difficulties, particularly in primary school. For example, fraction understanding is underpinned by larger mathematics cognitive processes including proportional reasoning and spatial reasoning (Moss & Case, 1999). In relation to having different notions of fractions, Hackenberg and Lee (2015) showed that limited understanding of particular aspects of the different meanings of fractions affects the ability of students to generalise and to work with fraction concepts. Similarly, Siemon et al. (2015) indicated that learning fractions is difficult because they are commonly used to represent a relationship between numbers rather than an absolute quantity.

Various studies considered the existence of interrelated fraction concepts as a major factor contributing to the difficulty of developing fraction understanding (e.g., Behr, Khoury, Harel, Post, & Lesh, 1997; Behr et al., 1983; Charalambous & Pitta-Pantazi, 2006; Siemon et al., 2015). Kieren (1976) was one of the earliest researchers to recommend that fractions be conceptualised as a set of interrelated constructs (ratio, operator, quotient, and measure) in teaching fractions for understanding. Behr et al. (1983) further extended Kieren's (1976) ideas of interrelated constructs of fraction and developed a theoretical model for learning by adding one additional construct (part-whole).

In this study, we used Behr et al.'s (1983) model of interrelated constructs of fractions to analyse a single lesson in a New Zealand middle school. Hence, this study is guided by the research question: Which constructs of fractions were reflected in the teacher's and students' discussion and use of language? The importance of this study lies in the use of Behr et al.'s (1983) model to analyse classroom interactions through language use.

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Background

Behr et al.'s (1983) theoretical model of interrelated concepts of fraction provides a way of considering pedagogical emphases. In the following section, we describe the five interrelated concepts of fraction and their classroom implications for teachers and students.

Part-Whole Concept

The part-whole construct of fractions is defined as a situation in which a continuous quantity or a set of discrete objects is partitioned into parts of equal size (Behr et al., 1983; Siemon et al., 2015). This representation is commonly used in the teaching of fraction concepts because it is assumed that students' initial intuitive experiences of fractions are derived from fair sharing (Siemon et al., 2015). The part-whole concept of fraction helps to answer the question "How much of an object or set is represented by the fraction symbol?" Although the part-whole concept of fraction is considered fundamental for developing an understanding of fraction concepts (Behr et al., 1997), it has limitations (Siemon et al., 2015). For example, the "out of" relationship between the whole and parts can only apply to proper fractions (4 out of 28 makes sense but 44 out of 28 does not). Therefore, addressing the other fraction concepts is important to develop a deep understanding of fractions.

The Ratio Concept

The concept of ratio is related to a comparison or relationship between two quantities in a given order rather than being a number by itself (Behr et al., 1983; Charalambous & Pitta-Pantazi, 2006). The fraction notation 3/5 may also represent a ratio. For example, in a class of six boys and 10 girls, the ratio of boys to girls is 6 to 10, which is equivalent to 3/5. That is, for every three boys, there are five girls. The ratio interpretation of fractions does not involve the idea of partitioning and is hence conceptually different from the part-whole and quotient concept (Reys et al., 2012).

The Operator Concept

According to Charalambous and Pitta-Pantazi (2006), "the operator concept results from the combination of two multiplicative operations or as two discrete, but related functions that are applied consecutively" (p. 4). It is often indicative of multiplication (Behr et al., 1983; Siemon et al., 2015), particularly the interpretation characterized as "taking a part of the whole", such as one-quarter of a whole number. To master the operator concept of fractions, Charalambous and Pitta-Pantazi (2006) suggested that students could be engaged by multiplying or dividing fractions in a variety of ways (e.g., 3/4 should be interpreted either as 3 × [1/4 of a unit] or 1/4 × [3 units]).

The Quotient Concept

The quotient concept is fraction as division (Park, Güçler, & McCrory, 2013). The fraction 1/4 results from dividing 1 by 4. This interpretation of fractions is often ignored in classrooms (Park et al., 2013) despite providing a firm foundation for students to rename and compare fractions as decimals (Behr et al., 1983; Siemon et al., 2015). This construct also provides an opportunity for students to recognise that a fraction may have an infinite number of equivalent forms.

The Measure Concept

The measure concept of fraction can be interpreted as numbers that can be ordered on a number line. This notion is important for adding and subtracting fractions. For example, if two fractions with unlike denominators are interpreted as a measure (i.e., as distances from zero on the same scale), then they can be added or subtracted as measures only if they have the same units (Charalambous & Pitta-Pantazi, 2006; Siemon et al., 2015).

Generally, students must be comfortable with all of these interpretations of the fraction to have deep fractional understanding, and they must be able to do so without confusing whole number characteristics with fraction characteristics. In addition, understanding of fractions depends on gaining an understanding of each of these different meanings, as well as of their confluence (Behr et al., 1997).

Method

The Participants

As part of a larger study being conducted in Australia and New Zealand, 11 Year 7 students and a mathematics teacher participated in the study. According to the New Zealand Curriculum (Ministry of Education, 2007), students at this year level are expected to understand that the value of a fraction of an amount depends on both the fraction and the amount (for example, 1/3 of 180 = 180 ÷ 3 = 60), and to apply additive and multiplicative strategies flexibly to fractions, such as ratios. The students were from a very low socio-economic middle school in New Zealand that was participating in a local numeracy research project in which the class was divided for instruction, with about half the group working intensively with the teacher while the remaining students worked independently on set work. As part of that project, they were used to having visitors to their classroom, and to being video-recorded, and welcomed researchers. Following ethics approval, students were informed of the study and their consent was obtained.

Data Gathering and Analysis

A single hour-long lesson was video-recorded and analysed using the interrelated theoretical model of fraction constructs (Behr et al., 1983) to analyse the dialogue between the teacher and students in the classroom context. The analysis focused on identifying each of the five different fraction concepts as the lesson unfolded and the teacher responded to students' comments and ideas. The description for each fraction construct and related concepts are shown in Table 1, which was used to guide the analysis process.

The observation data were coded by watching the video-recorded lesson and using the audio transcript. Based on the descriptions shown in Table 1, notes were made when evidence was observed, and a frequency table was prepared to indicate concepts of fraction reflected in the teacher's and students' discussion and use of language. Barron and Engle (2007) suggested that video-audio transcription coding should be iteratively revised until the transcripts eventually provide a reliable record of what the researchers view as the most relevant aspects of the video for their research questions. Consistent with the advice of Barron and Engle (2007), the analysis emphasised the identification of fraction concepts as described in Table 1 through the language used by the teacher and students as the class worked on solving fraction problems.

Table 1
Fraction Constructs, Relevant Concepts, and Assessment Strategies

Fraction constructs	Relevant description/concepts
Part-whole	The process of partitioning, parts are equal. Relationship between the whole and the parts
Ratio	Concept of equivalence, comparison or relationship between two quantities
Operator	Division, or multiplication operation on fractions, taking a part of the whole
Quotient	Addition or division operation on fractions. A single rational number derived from dividing the nominator by the denominator
Measure	Order or identify a number represented by a certain point on the number line

The Classroom Context

After introducing the lesson topic, students were given two consecutive tasks to complete in groups of three or four. First, they were asked to sort a set of the fraction strips in ascending or descending order. The teacher posed follow-up questions while students were sorting out the fraction strips, such as “What can you tell me about ordering?”, “Why is it a smaller fraction?”, and “What is the relationship between the bottom and the top fraction?” The top and the bottom could be confusing terms to use; however, the students were able to later understand the terms as nominator and denominator of a fraction.

After completing the first task, students in groups were asked to choose and solve one of three problems. All students had to be prepared to show their group’s solution on the board and to answer any questions asked by the other students. The context of the problem was a visit to McDonalds following a sports game. The teacher ensured that the students were familiar with this context before asking individual students to read the fractions questions. The three questions were of increasing difficulty and complexity:

1. 4 burgers were ordered and half of these given to another person.
2. 16 burgers ordered and one-quarter given to another person.
3. 40 burgers ordered and five-eighths given to another person.

The third question was clearly much more complex and intended only for competent students, although any of the groups could have attempted it.

Results

Table 2 summarises the frequency of the observed concepts of fraction reflected in the teacher’s and students’ discussion, and use of language. Illustrative examples are taken from the teacher’s and students’ discussions while completing the two tasks.

As shown in Table 2, the most frequently observed fraction concept reflected in the teacher’s and students’ discussion, and use of language was part – whole (F = 40) whereas measure (F = 4) was the least.

Based on the requirements of the first task, all the students were able to order the fraction strips. As they did so, the students (S) discussed the questions posed by the teacher

(T). The conversation between the teacher and the students shown below is an illustrative example of the part-whole concept of fractions.

T. So, can you make a connection with this one right at the top and the one – and maybe one of the others? Anyone?
S: That's one whole.
T: That's one whole. And what's down here?
Students provide different answers.
T: Yes. One whole is ten tenths or twelve, what?

Table 2
Frequency of Observed Fraction Concepts

Fraction concept	Frequency	Example
Part-whole	40	Identifying the relationship between the one whole fraction strip and the parts (1/10, 1/8 etc.); partitioning the whole into a given number of parts in Task 1
Ratio	8	The concept of equivalence was discussed while identifying the whole and parts, such as "1 whole is 10 tenths or 12 twelfths"
Operator	32	Finding 1/4 of 16 is similar to multiplying 1/4 by 16
Quotient	10	The students understand that the value of a fraction of an amount depends on both the fraction and the amount, for example, 1/2 of 4 hamburgers is 2
Measure	4	Arranging fraction strips from biggest to smallest fractions, and representing one-quarter as 1/4

Another group was engaged with the concept of equivalence (ratio), connecting the denominator of a fraction with the size of the fraction strips, illustrated by the following conversation. In this conversation, the teacher intended to demonstrate, for example, two smaller strips of 1/4 are equivalent to 1/2 or the ratio of 2/4 is equivalent to the ratio 1/2.

T: What is the relationship between the size of the square and the bottom number?
S1: The pieces get smaller.
T: Right. And why do you think that's happening?
S1: Because there's [indecipherable] there's more on that one.
T: So, talk about the pieces are getting smaller and the numbers are increasing. So, what do we mean? Try and make a connection there.
S2: The bottom number.
T: The bottom number. So, we're thinking about the bottom number. Does that bottom number have a name?
S3: The denominator.
T: Denominator. Yes. Alright. So. Interesting. So, I can gather from this little discussion that we've had that some people know about numerator and denominator. Some people have recognised that the pieces are getting smaller and the numbers are increasing.

These discussions show that the process of partitioning, the relationship between part and whole, and the concept of equivalence and equivalent fractions were addressed within a single task of organising fraction strips by size.

After finishing the first task, the student groups were engaged in solving their chosen problem in the second task. Each group presented their answer to the rest of the class. The first group presented their solution for the first problem on the board as shown in Figure 1.

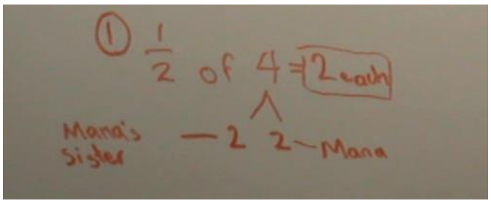

Figure 1. Students' work showing the concept of operator and quotient.

Watching the recorded video and as shown in Figure 3, the students showed that they understood that the value of a fraction of an amount depended on both the fraction and the amount and recognised that one-half of four is equivalent to two, implicitly using both the operator concept and the quotient idea. The operator concept of fraction was also observed in the work of another group (Figure 2). The teacher asked what operation to use. The students redefined one-quarter of 16 as multiplying 1/4 by 16. In addition, they were able to define multiplication as repeated addition to check their answer (16 = 4 + 4 + 4 + 4), making connections to prior mathematical knowledge.

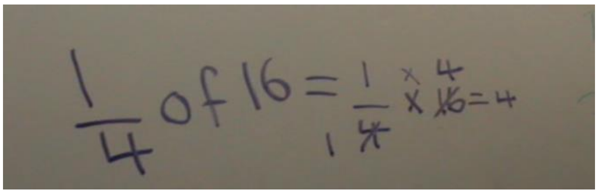

Figure 2. The operator concept of fraction.

The quotient concept was also observed using division, as in this conversation.

T: So, you're imagining that you've got 40 hamburgers. Yes. Okay. And what's the connection to the eight?
S. You're dividing.
T: Okay. Dividing what?
S: To five groups.

In this example, the students were showing the connections between 40, eight, and the result of five groups; that is, dividing 40 by eight resulted in five. They did not use the language of sharing, which is a part-whole notion.

The measure concept was also seen but to a lesser extent and only through the teacher's dialogue, whereas the other ideas were often initiated by the students. For example,

T: Four. What does four quarters look like as a number? Well done. So, there's a quarter in each part. What does four quarters look like? So, this is what a quarter looks like. What does four quarters look like as a number?
S: A whole.
T: It looks like a whole. Yeah. But what does it look like as a number?

This dialogue between students and the teacher indicated the concept of the part-whole relationship that is four quarters form one whole and implicitly considered the idea of a fraction as a number that is the measure concept.

Discussion and Conclusion

Given the nature of the problems posed by the teacher, it is not surprising that the measure concept was only tangentially observed. Of interest, however, was that the students initiated many of the concepts, reinforced by the teacher's questioning. Students seemed to move seamlessly between the ideas, sometimes implicitly drawing on different ideas within the same sentence. The lesson started with a reinforcement of the part-whole concept as commonly practiced in teaching fractions concepts because it is assumed that children's initial intuitive experiences of fraction are derived from fair sharing (Siemon et al., 2015; Strother et al., 2016). In this study, however, the teacher was able to draw out students' understanding of fraction concepts by addressing the other interrelated ideas (ratio, operator, and quotient) through engaging students with practical problems and using manipulatives, and an emphasis on discussion and students explaining their thinking. Similar to other studies (e.g., Charalambous & Pitta-Pantazi, 2006; Siemon et al., 2015), the present study showed that the most frequently observed fraction concept reflected in the teacher's and students' discussion, and use of language, was part-whole whereas measure was the least. This could be reflected in the students' competency with fraction concepts. For example, Charalambous and Pitta-Pantazi (2006) showed that students were successful in tasks related to the part-whole construct and least competent with tasks corresponding to the measure construct.

Fraction concepts are often taught using procedures and memorisation rather than having students develop their own understanding (Siemon et al., 2015). The use of manipulatives in teaching fractions with students working in small groups, and discussion and questioning among students explicitly encouraged, allowed the students to draw on other knowledge, such as repeated addition and "tables" knowledge, and relate this to fraction understanding. Students may, however, have been able to better see the links to ratio and measurement concepts of fraction if they were provided with additional manipulatives such as a thin strip of paper. As suggested by Reys et al. (2012), the strip of paper could be folded into halves, quarters, and so on, and later, students could use length partitioning to represent fractions as points on a number line.

As Tobias (2013) has shown, if the language used by the teacher is incorrect or confusing to explain fraction concepts, students may continue to use the teacher's incorrect language to describe fractions, and not understand the concepts clearly. For example, questions using language such as "bottom" and "top" number could be potentially limiting for students because "bottom" and "top" don't clearly describe fraction concepts.

This small-scale study showed that the use of interrelated fraction concepts in conversation might have implications for mathematics teachers' pedagogical and assessment strategies. For students to have a good understanding of fraction concepts, teachers need to include different constructs of fraction concepts in their pedagogical and assessment approaches (Siemon et al., 2015). Such use needs to be deliberate and focussed, whereas, in this study, the usage of different concepts arose informally through classroom discussion. Charalambous and Pitta-Pantazi (2006) used the model as a reference point to investigate students' notions of the different constructs of fractions through administrating a test. Other studies used Behr et al.'s (1983) model to analyse students understanding of one element of fraction concepts (e.g., Empson & Levi, 2011; Mitchell & Horne, 2009). This study has shown that the same model can be applied to normal classroom discourse.

Overall, this small-scale study showed that although students' intuitive understanding of fractions as part-whole provided important prior knowledge, they also used language consistent with other concepts of fraction, such as ratio, operator, and quotient, with teacher encouragement. Perhaps a next step is to make these concepts explicit to the students by developing the language use of the more unfamiliar terminology. In this study, similar to Kieren's (1976) suggestion, the ratio, operator, and quotient concepts of fraction were often reflected in part-whole contexts during the dialogue between the teacher and students. However, because of the limited scope of the tasks that the students completed, there were limited opportunities for students to use measurement concepts. The study could have more to say on the measurement concepts if the students were engaged in more diverse activities.

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Teachers' Understanding and Use of Mathematical Structure

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In this paper, we examine junior secondary mathematics teachers' understanding of mathematical structure, and how they promote structural thinking in their teaching. Five teachers were surveyed, and three were interviewed and the observed teaching a junior secondary mathematics class. Results showed that teachers have conflicting understandings of structure, and their perceived understandings, obtained from survey data, were not reinforced by their interview responses or observations of their teaching. Analysis of connections, recognising patterns, identifying similarities and differences, and generalising (CRIG) components from observation data showed a lack of attention to structural thinking.

Recent results on the Trends in Mathematics and Science Study (TIMSS; Mullis, Martin, Goh, & Cotter, 2016), and OECD Programme for International Student Assessment (PISA; Thomson, De Bortoli, & Underwood, 2016) reveal Australia's decline on international mathematics tests rankings, which has caused concern about the state of mathematics teaching and learning. Lokan, McRae, and Hollingsworth (2003) and Vincent and Stacey (2008) asserted that Australian mathematics teaching was dominated by procedural pedagogical practices, reinforcing that Australian mathematics teaching concentrated on the use of textbook exercises and worksheets, which resulted in instrumental learning (Skemp, 1976) instead of conceptual understanding (Hiebert, 1986).

Recently, there has been an increasing research interest in mathematical structure and structural thinking. Mason, Stephens, and Watson (2009) defined mathematical structure, referred to here as structure, as "the identification of general properties which are instantiated in particular situations as relationships between elements" (p. 10). Mason et al. (2009) argued that students involved in structural thinking receive an intrinsic reward, and that teachers' awareness of structural relationships transforms students' thinking and disposition to engage. They claimed that structure is essential to mathematics teaching and learning as it relates procedures and concepts to promote structural thinking.

Recent studies have focused on describing mathematical structure across early childhood and secondary students (Mulligan & Mitchelmore, 2009; Stephens, 2008) and out-of-field mathematics teachers (Vale, McAndrew, & Krishnan, 2011), supporting a need for further research in mathematics teachers' understanding of structure.

Research Questions

Three main questions were addressed in this study:

1. How do mathematics teachers demonstrate an awareness of mathematical structure?
2. How do mathematics teachers promote structural thinking when teaching mathematics?
3. Is there a discrepancy between what teachers say and do concerning mathematical structure?

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Literature Review and Theoretical Framework

The notion of structure is found in the Australian Curriculum: Mathematics (ACM; Australian Curriculum, Assessment and Reporting Authority, 2015) through the proficiency strands of understanding, fluency, problem solving and reasoning. In the NSW K-10 Mathematics Syllabus (NSW Board of Studies, 2012), produced in response to the ACM, structure can be identified in the working mathematically processes of communicating, problem solving, reasoning, understanding, and fluency.

The concept of mathematical structure has a long history in mathematics education research, but it is not a term very familiar to teachers. Taylor and Wade (1965) acknowledged that "structure" occurred frequently in mathematics education literature. Fischbein and Muzicant (2002), Stephens (2008), and Mason et al. (2009) all identified it as synonymous with relational thinking (Skemp, 1976). Barnard (1996) described it in terms of cognitive units or blocks of information, while Mulligan and Mitchelmore (2009) associated it with young children's ability to recognise patterns and relationships.

Effective mathematics teaching must include attention to structure. Pedagogical Content Knowledge (PCK; Shulman, 1987) and Mathematical Content Knowledge (MCK; Ball, Thames, & Phelps, 2008) both demonstrate the importance of structure in pedagogy and content knowledge in mathematics teaching. Structure connects mathematical procedures and concepts that are integral to PCK and MCK. Teachers who embed structure in their lessons develop students' mathematical understandings more coherently and with depth. Vale et al. (2011) educated practising teachers to appreciate structure. In a study of secondary mathematics teachers, Cavanagh (2006) found that they did not identify with working mathematically, but did use components of working mathematically to develop students' structural thinking.

The present study draws on the theoretical framework developed by John Mason and colleagues. Mason (2003) was "interested in the lived experience of mathematical thinking" (p. 17). His personal awareness in thinking mathematically made him attentive towards the form and structure of learners' mathematical thinking. Mason et al. (2009) later noted that structural thinking occurs on a continuum, and that it is difficult to identify student mathematical thinking as it exists between a single idea, or an idea related to a set of properties. Mason et al. (2009) identified five forms of structure that allow structural thinking to be observed: holding wholes (gazing), recognising relationships, discerning details, perceiving properties, and reasoning.

From these forms, the first author developed four observable components of structure to identify teachers' understanding of structure from their utterances when teaching. The first form of holding wholes (gazing) is interpreted as *connections* to other mathematics learning because gazing makes connections to the whole. The second form of *recognising* deals with mathematical relationships in patterns. The third, discerning details, is acknowledged as *identifying* similarities and differences. The final forms, perceiving properties and reasoning, were combined to form the single component of *generalising,* which relates mathematical ideas to a whole. Overall, the generalising component was considered to have greater relevance to this research.

These components of *connections* (C), *recognising patterns* (R), *identifying similarities and differences* (I), and *generalising* (G) will be referred to by the acronym CRIG. The connections component considers how we think about mathematics and making connections between present, prior, and future learning. Connections require recalling mathematical knowledge and adapting it to new knowledge. The NSW K-10 Mathematics Syllabus recognises that "students develop understanding and fluency in mathematics through inquiry and exploring and connecting mathematical concepts" (NSW Board of Studies, 2012). This syllabus has as one of its outcomes for working mathematically that a student "communicates and connects mathematical ideas" (NSW Board of Studies, 2012, outcome MA4-1WM).

Recognising patterns is identified extensively throughout mathematics education as critical to mathematics learning and the development of relationships. In the NSW mathematics syllabus (NSW Board of Studies, 2012), patterns are associated with the Number and Algebra strand as: "Develop efficient strategies for numerical calculation, recognise patterns, describe relationships". In Stage 4, a Number and Algebra content strand outcome is: "Create and displays number patterns" (NSW Board of Studies, 2012, outcome MA4-11NA). It also states: "Students develop efficient strategies for numerical calculation, recognise patterns, and describe relationships" (NSW Board of Studies, 2012, p. 18).

Learning mathematics includes developing skills in identifying similarities and differences; differences could be equal or unequal, bigger or smaller. In the NSW mathematics syllabus (NSW Board of Studies, 2012), Stage 4, Number and Algebra outcome MA4-4NA states, "compares, orders and calculates with integers".

Generalising was identified by Mason et al. (2009), who wrote that an appreciation of structure is supported by experiencing generality. The NSW mathematics syllabus K–10 (NSW Board of Studies, 2012) Number and Algebra strand includes experiencing generalisation. Concept formation is a process that involves generalising, and the movement of the concrete towards abstraction is associated with structure.

Analysing teachers' awareness of structural relationships is difficult, but recognising structure can be considered through teachers' talk about structure and where it occurs in their utterances when teaching mathematics. The CRIG components form a basis for identifying explicit characteristics, in analysing teachers' awareness of structure and the promotion of structural thinking in their teaching.

Methodology

Context and Participants

The study took place in a comprehensive secondary Catholic boys' school of approximately 300 students in metropolitan Sydney. The principal gave permission to conduct the research, and the head teacher of mathematics confirmed that all eight mathematics teachers at the school would participate. These teachers were invited to join the study, but only five teachers participated in the survey, and three of the five were subsequently selected as case studies, interviewed, and then observed teaching mathematics. Of the three teachers, two were women and one was a man, and their teaching experience ranged from 3 to 17 years. The least experienced teacher had mathematics as her first subject, the male teacher was head of mathematics with a PDHPE background, and the third teacher was science trained.

Instruments

The first instrument was a survey, hosted on SurveyMonkey, designed to identify the teachers' understanding of structure. The survey contained 22 statements that were anonymously answered using a 5-point Likert scale (Disagree, Partially Disagree, Neither Agree Nor Disagree, Partially Agree, Agree). The survey statements were grouped into four categories: mathematical pedagogy and content, mathematical structure, CRIG components of mathematical structure, and structural thinking.

The first author conducted individual semi-structured interviews with each of the three teachers as the second form of data collection, and these lasted about 10 minutes. The interview questions were intended to glean further information that expanded on the survey responses. For consistency purposes, it was necessary to give teachers a definition of structure. Each teacher read the following passage before the interview began:

> Some authors describe mathematical structure as the building blocks of mathematical learning. Mathematical structure can be found in connecting mathematical concepts, recognising and reproducing patterns, identifying similarities and differences, and generalising results. Students who perform structural thinking use these skills without always considering them when solving problems. Many students need to be taught these skills when introduced to concepts as a reminder of how to think mathematically.

The interviews were recorded on a mobile phone, and transcribed by the first author to a Word document and copied to NVivo. Next, the three teachers were each observed teaching three consecutive 50-minute junior secondary mathematics lessons over a one-week period. The first author identified and recorded each of the teachers' utterances that referred to a CRIG component. These were entered into an observation template in a Word document and then copied to an Excel spreadsheet file.

Analysis

The survey data were analysed with percentage breakdowns of the Likert scale scores for each statement (scores ranged 1 for Disagree to 5 for Agree).

Interview responses were first analysed to identify how teachers' comments demonstrated levels of awareness of structure. The first author categorised words and phrases made by the teachers as being either specific or nonspecific. A specific statement was one that related directly to mathematics, such as "Doing series and sequences, I took them back to tables of values", and a nonspecific statement had no direct impact on the mathematics teaching, such as, "Recognising similarities and differences, I do that". Subsequently, the transcripts were re-coded to categorise teachers' responses according to the four CRIG components. A content analysis, to categorise concepts through words and phrases that referenced structure, was then conducted to identify three major themes recognising students' structural thinking, student engagement when thinking structurally, and the benefits of structural thinking.

Data from the lesson observation templates were entered into an Excel spreadsheet to allow for allocating and filtering of teachers' utterances into the four CRIG components. Next, two sub-categories were created. The first subcategory identified an utterance as a high or low level of structural thinking. Any utterance that promoted a higher level of structural thinking was coded as analytical. Utterances that were weak in structural thinking were coded as superficial. For example, "What does the denominator tell us about the fraction?" was coded as analytical, while "We never add the denominators" was coded as superficial.

The second subcategory of utterances identified whether teachers focused on concepts or procedures when attempting to promote structural thinking. Two new codes were introduced. The first was the concept domain, where utterances explained or questioned the way something was done, such as: "Dividing by a quarter is the same as multiplying by…". The second code was the content domain in which teachers' utterances were procedural or topic-oriented, such as what to do to solve a problem: "What you do to the bottom you do to the top".

Results and Discussion

Survey results shown in Table 1 indicate that teachers believed that they possess a high level of awareness of structure, as averages for all the statements were close to the maximum.

Table 1
Survey Question Group Averages from Likert Scale Responses

Group	Questions	Survey classification	Average
1	1–6, 20	Mathematical structure	4.56
2	7–11	CRIG components of mathematical structure	4.56
3	12–19	Structural thinking	4.18
4	21–22	Mathematical pedagogy and content	4.60

The teachers' interview responses reflect varied and individual interpretations of the meaning of structure, such as the building blocks of knowledge, organisational features of a lesson or curriculum, or as the structure of a solution.

Table 2 contains the frequencies of specific and nonspecific responses made in each of the four CRIG components from the interview questions. Statements related to connections were more frequent than any of the other components; however, all these statements were nonspecific. No assumption was made that a CRIG response represents a teacher's awareness of structure; in fact, the high number of nonspecific connections statements indicates a lack of structural awareness.

Table 2
Frequency of Teachers' CRIG Specific/Nonspecific Responses to Interview Questions

CRIG component	Specific/nonspecific example	Frequency
Connections	Specific	0
	Nonspecific	13
Recognising patterns	Specific	1
	Nonspecific	8
Identifying similarities and differences	Specific	1
	Nonspecific	2
Generalising	Specific	2
	Nonspecific	5

Lesson observation data in Table 3 show the frequencies of utterances. CRIG components are first identified, then the first subcategory (analytical or superficial), and then the second subcategory, domain (concept or content). Except for generalising, across the CRIG categories, there were fewer analytical/concept utterances compared to the dominance of superficial/content utterances. This indicates that reference to, or the promotion of, structural thinking was infrequent. Analytical/content utterances had the lowest frequency across all CRIG categories, indicating teachers' references to procedures were fewer when structural features are identified.

Table 3
Teacher Utterances as Combined Categories Frequency

CRIG component	Name of utterance	Domain	Frequency
Connecting	Analytical	Concept	14
		Content	3
	Superficial	Concept	4
		Content	17
Recognising	Analytical	Concept	6
		Content	2
	Superficial	Concept	4
		Content	21
Identifying	Analytical	Concept	3
		Content	3
	Superficial	Concept	14
		Content	32
Generalising	Analytical	Concept	37
		Content	11
	Superficial	Concept	22
		Content	34

Table 4 presents the lesson observation data without the CRIG components. To identify teachers' awareness of structure, the frequency of analytical/concept utterances was considered. This was close to one quarter of the total number, indicating some attempts by teachers to promote structural thinking and suggests that the teachers in the study were not structurally aware. To see if teachers were promoting structural thinking, responses during the interviews were reviewed. Only one teacher showed an awareness of structure in the interview. The attempts to promote structural thinking observed during the lessons reflect a discrepancy between teachers' understanding of structure and how they used it when teaching mathematics.

Table 4
Frequency of Concept/Content Statement as Analytical/Superficial

Analytical/superficial	Concept/content	Frequency
Analytical	Concept	60
	Content	19
Superficial	Concept	44
	Content	104

The research questions focused on what teachers say they know about structure and how their actual teaching promotes structural thinking identified through an analysis of their utterances. The inconsistency between the results in the survey, the interview, and classroom observation provides incongruous answers to these questions. Data from teacher observation responses shows that their descriptions used to identify structural thinking was inconsistent with the survey and interview data.

Survey results indicated that teachers felt they were aware of what structure means, but interview descriptions of what they described as structure contradicted this. The interviews gave no conclusive evidence that teachers understood what was intended by mathematical structure. Observation data revealed limited attention to mathematical structure. Procedural understanding, indicated by utterances that inhibited structural thinking, occurred more often than conceptual understanding utterances, possibly impeding the promotion of structural thinking in teaching and learning.

A comparison between teachers' interview responses with utterances made when teaching showed that the interview comments were weak in structural awareness, yet about a quarter of their classroom utterances promoted conceptual understanding. This suggests that the teachers may unknowingly promote structural thinking.

Further comparison between the interview and observational data showed a shift in the teachers' attention to individual CRIG components from the interview to the classroom observation. In the interviews, teachers identified connections and recognising patterns, but in the classroom, their attention was aligned with generalising. This creates ambiguity in teachers' awareness of structure when expressing themselves in an interview as compared to what they say when teaching mathematics. Overall, these data indicate that teachers may not have a deep understanding of structure. The benefits of structural thinking are acknowledged, but are not substantial when teaching mathematics.

In Cavanagh's (2006) study, it was found that mathematics teachers did not have a deep understanding of working mathematically processes when teaching mathematics, even though they taught some working mathematically processes. Working mathematically processes have components of structure embedded throughout, so some similarities can be drawn between these studies. When it comes to teaching components of structure, teachers believe that they are utilising structure, but they are not aware of the complex components of structure.

Mason et al. (2009) espoused the importance of students engaging in structural thinking to be able think deeply about mathematics and argued that this will happen when the teacher is structurally aware. All teachers of mathematics need to be aware of the role of structure in developing their mathematical knowledge and pedagogical practices. Effective delivery of mathematical content, either procedural or conceptual, can increase structural thinking when the teacher is aware of this structure. Teachers' awareness of structure can support all stages of the structural thinking continuum and overcome what Mason et al. (2009) called the "mythical chasm" between procedural and conceptual understandings of mathematics learning.

Conclusions and Further Research

This study has shown that there is a critical need to improve teachers' understanding of structure to encourage structural thinking skills and this needs to occur early in their career, and primarily in teacher education programs. The research reported here has formed a basis for the first author's broader doctoral study into pre-service teachers' noticing of structural thinking. This new research will attempt to develop pre-service teachers' noticing of structural thinking and awareness of structure through a community of inquiry. The pre-service teachers will be involved in teaching Years 5 to 8 mathematics. How these pre-service teachers' notice structural thinking will be explored to see if their understanding of structure improves and how their pedagogical practices change to promote structural thinking.

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Initial Teacher Education Students' Reasons for Using Digital Learning Objects When Teaching Mathematics

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A current issue in initial teacher education (ITE) in Aotearoa, New Zealand is how students can best be supported to use digital technologies for mathematics teaching. While many ITE students are familiar with digital technologies for personal use, they are less likely to know how to incorporate them into the mathematics learning process. Supporting ITE students to become more critical, knowledgeable, skilled, and confident about using digital technologies was the main aim of the study. Forty second-year ITE students were surveyed about the Digital Learning Objects promoted by the Ministry of Education that they would choose to use to teach area measurement. Several different reasons were reported.

In Aotearoa, New Zealand, mathematics education in primary schools has been identified as a priority learning area. Teachers have been encouraged to implement a variety of pedagogical strategies to enhance the teaching and learning process (Anthony & Walshaw, 2007; Education Review Office, 2013). The two national curriculum documents – The New Zealand Curriculum (Ministry of Education, 2007 – for English medium classrooms) and Te Marautanga o Aotearoa (Ministry of Education, 2008 – for Māori medium settings) – indicate that using ICT could be really useful for learners. For instance, the Ministry of Education (2007) states that "Schools should explore not only how ICT can supplement traditional ways of teaching but also how it can open up new and different ways of learning" (p. 36).

To support teacher engagement in New Zealand with digital technologies, Te Pātaka Matihiko Our Digital Storehouse provides a gateway to a collection of digital learning objects (DLOs) produced by The Le@rning Federation (TLF), an initiative of Australian and New Zealand government departments (https://nzmaths.co.nz/about-learning-objects#LO2). The DLOs are interactive and designed to support student learning of key mathematical ideas from Year 1–10.

Research indicates that learning to use digital technologies effectively in mathematics education is complex and requires careful consideration and development of technological, pedagogical, and content knowledge expertise (Attard, 2013; Calder, 2017). In New Zealand, there has already been some exploration of using digital technology for learning mathematics (Calder 2011; Ingram, Williamson-Leadley, & Pratt, 2016). International research by Handal, Campbell, Cavanagh, and Petocz (2016) found that ITE students require numerous opportunities to explore and appraise digital technologies in order to make decisions about their worth as part of the teaching and learning process.

Area measurement is a core component of the mathematics and statistics learning area (Ministry of Education, 2007, 2008). Teachers need to be knowledgeable about key measurement ideas such as area because developing an understanding of this topic can be challenging for some children (Huang & Witz, 2011; Muir, 2006).

Teachers' mathematical knowledge for teaching can also affect the way they use technology (Orlando & Attard, 2016). Positive self-efficacy in their own mathematical practices means that ITE students can focus more on developing an appreciation of the affordances of digital technology for primary school learners.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 293–300). Melbourne: MERGA.

Edson and Thomas (2016) suggest that as well as providing ITE students with resources, teacher educators can offer students opportunities to:

- “construct their own mathematical knowledge as learners” (p. 233);
- “build robust knowledge” (p. 233) of how to use digital technology for teaching mathematics; and
- practise in the field. Practicum is an opportunity to safely take risks and try out resources and tools.

According to Skemp (2006), for deep mathematical understanding, it is necessary for learners to have both conceptual and procedural knowledge, and know how to apply these in different situations. However, often in mathematics education, children are capable of completing calculations correctly and yet have difficulty in understanding what the calculations mean, and why those particular numbers are appropriate in a given context. These difficulties arise because children learn the rules or algorithms or formulae to derive the correct answer, yet have little or no conceptual understanding of why the procedure works (Skemp, 2006). Muir (2006) and Huang and Witz (2011) state that when children are learning about area measurement, they should be offered opportunities to develop conceptual understanding.

Effective teachers understand that their pedagogical practice must be varied so that all learners are provided with maximum opportunities to access, construct, and consolidate their mathematical thinking. Digital technologies can allow children to explore mathematical ideas in visually interesting and interactive ways. Learning can be enhanced by the manipulation of objects and the exploration of patterns and relationships (Calder, 2017).

Research Design

Mishra and Koehler’s (2006) TPACK (Technological Pedagogical and Content Knowledge) framework is a model for understanding how teachers learn to use and evaluate the value of digital technologies within the context of their own practices. Interactions between content, pedagogy, and technology forms the core of the model. Koehler, Mishra, and Cain (2013) argue that if teachers are to grow their technological know-how, it must be done in tandem with trials in their own classrooms for it to be meaningful and have lasting effect. In other words, as technological pedagogical content knowledge grows, pedagogical content knowledge shifts, influencing actual practices that positively affect the nature and quality of tasks that learners engage in (Assude, Buteau, & Forgasz, 2010).

Sutton’s (2011) research concludes that preservice students need authentic learning experiences using technology as part of their teacher preparation programmes. It is vital that lecturers model digital technology use in content areas that ITE students will be teaching. Without such experiences, ITE students have difficulty appreciating the relevance of digital technologies in learning contexts, or reflecting on and retaining the digital technology skills and knowledge they acquire.

Although we were aware that our ITE students appeared “tech savvy”, we suspected this was superficial in many cases, since their digital use was mainly confined to social networking and retrieving information (Starcic, Cotic, Solomonides, & Volk, 2015). We therefore designed a project to investigate how we could best support our students to teach effectively with digital learning objects (DLOs). To that end, we constructed a project based on TPACK principles, and used a mix of qualitative and quantitative tools to find out what their experiences of using DLOs meant to them. Our overarching questions were:

- What are ITE students' perceptions about the digital learning material and their use that they explore as part of their mathematics education course?
- What do the mathematics teacher educators learn about ITE student appropriation of digital technologies for their emerging pedagogical practice in mathematics?

Method

For the larger study, we used surveys, field notes, and evaluations of our own classroom observations. The participants were 40 ITE undergraduate students in a second-year compulsory mathematics education paper and three lecturers. In framing the project's methods, we turned to Ulvik (2014), who had extracted a number of action research principles used in ITE. We used some of her key principles to inform our research design. These also linked to the TPACK (Mishra & Kohler, 2006) model that combines technology, pedagogy, and content knowledge for developing teacher knowledge and confidence when using digital technologies for teaching and learning. Focusing on one aspect at a time was crucial to avoid too many variables, which is why we focused on one component of mathematics, area measurement. We also saw students as important sources of information, which led to surveying them about their perspectives on the use of the DLOs. This also meant they could articulate their experiences and learning that, in turn, would inform our pedagogical and technological practices regarding the teaching and learning of area measurement.

Across the wider project, we observed students working in groups, administered paper-based surveys, and reviewed students' own teaching unit plans and reflections. Another method included reflective notes and conversations between the two mathematics teacher educators (who were not confident about using digital technologies in mathematics education), and the teacher educator colleague who acted as the digital technology and action research adviser. This latter method helped us review our actions and interim explanations, as well as track our own confidence in trying out different digital technologies.

We created a learning opportunity in which our ITE students explored DLOs related to area measurement ideas deemed appropriate for primary-school aged classrooms. The class spent two to three hours on an investigative task exploring relationships between area and perimeter, considering both the value of such a task and the pedagogical implications of an investigative approach. The task consisted of the following scenario:

> Holly just got a new lamb. Her mum offered to help her make it a moveable pen. They found they had 24 metres of fence netting.
>
> What shapes and sizes of pen could they make for the lamb?
>
> Holly is worried the lamb won't get enough to eat. Justify what shaped pen would give the lamb the most grass.

Students used grid paper and measurement materials to explore the problem. They could start with any shape and discuss their thinking with others while writing and explaining their findings. The third lesson was a 45-60 minute session in a computer lab exploring selected DLOs on nzmaths.co.nz. The DLOs were about finding the area measurement of rectangular, compound, and various triangular shapes that targeted the Measurement and Geometry strand of the New Zealand Curriculum Levels 3 and 5 (children aged 10–14).

In this paper, we focus on one task from the survey. After identifying which DLOs they would use for teaching area measurement, the ITE students were then asked to "List three reasons for your choices". Their responses to this task are the focus of this paper. Other questions included asking the ITE students to identify concepts and skills they thought the DLOs might support children to learn, identify any reasons they might have for not using the DLOs and offer suggestions to a colleague as to how the DLOs might be used.

Data Analysis

Given that TPACK (Mishra & Koehler, 2006) principles helped shape the structure of the project, it was fitting to use it as an analysis lens. In the wider project, we considered the pedagogical content knowledge students and lecturers were learning, alongside any growing technological pedagogical content knowledge. Analysis took the form of categorising frequency counts of responses, which helped identify patterns and trends.

Results

After exploring the digital learning objects, students completed a written survey that consisted of seven questions, although only one is examined here. Student responses (n = 40) were transcribed and coded from 1–40. Transcripts were then read by the first author to identify the key themes that emerged from students offering reasons for their choice of the DLOs they would use for teaching and learning measurement. Six key themes were identified with the frequency of each one calculated. Data and themes were then presented to the other two authors and checked for consistency.

Table 1 presents the themes and examples of students' reasons for the DLOs they would select to use for teaching. The numbers in brackets indicate which participant's (numbers 1-40) idea is used.

Table 1
Key Themes for ITE Students' Choices and Frequency of Responses

Key themes	Frequency	Examples of student responses (participant number)
Simple/easy with opportunities to reinforce learning	11	… easily manipulate shapes (3)
		Level 3 tasks are worthwhile as it's simple but students will still learn concept of area (20)
		These help reinforce … the area of rectangles and triangles (22)
		Helps students consolidate their learning (31)
		…explains what to do when you're wrong without giving the answers (35)
		Allows children to do more examples (39)
Understanding	9	Area of triangles helped students understand why the formula is different to the area of a square (12)
		Understand what a m2 is (22)
		…activities break the task down so it can give the students a basic understanding on the maths (28)
		Basic understanding of the concept and rules of

Key themes	Frequency	Examples of student responses (participant number)
		area (29)
Working with different shapes	9	By breaking down the shape into blocks, it is likely to make the concept easier for children (10)
		Provides children with different shapes and different approaches or methods to calculate the areas (10)
		Gives children the ability to work with different shapes… Lets children break down the shapes (11)
		Able to explore ways of dividing shapes (24)
Engaging and interactive	9	The game style makes it fun and engaging for students (5)
		It is an interactive activity which allows children to easily figure out questions (11)
		Engages children through cartoon….. relevant to their interests (13)
		More interactive for children (23)
Use formulae	4	Teaches how to use formulas (13)
		Because it gives you options to find the correct formula. Students struggle with identifying the correct formula so this is great (15)
		… to find individual shape areas using relevant formula (24)
Visual	4	Great visual aid in learning (23)
		Suited for visual learners (24)
		It gave me a visual idea of how to split the shape & do the working out (37)
Others	5	They show progressions to the final result (16)
		It applies basic facts to answer complex problems (16)
		I could problem solve; it wasn't done for me (19)
		Makes children estimate before working it out (39)
		Because it gives the students a chance to be exposed to mathematical language (40)

Discussion

It is pleasing to see that the ITE students' reasons for the selection of DLOs that they would use for the teaching and learning of measurement covered a wide range of ideas. Students appeared to consider that the digital objects they explored could play a role in enhancing learning experiences about area measurement for children.

Learning about measuring area needs to be made as simple as possible for learners because it can be a complex topic and confuse many (Huang & Witz, 2011). It is incumbent, therefore, on teachers to provide a variety of opportunities for learners to access, construct, review, and consolidate their mathematical thinking as simply and clearly as possible. It is noted that 11 of these ITE students expressed support for this idea and considered that the DLOs that they selected were able to provide such opportunities for this to occur.

Another factor highlighted by some students was how the DLOs could support children to actually understand mathematical concepts and/or formulae. The notion of children developing a conceptual understanding of mathematical ideas such as area measurement is important for teachers to consider if they are to become effective practitioners (Anthony & Walshaw, 2007; Huang & Witz, 2011). Table 1 shows nine students' responses indicating that they thought it important for children to understand how or why a formula arises, what a square metre actually means in terms of covering a space, and what processes and strategies are involved in measuring area. Ultimately, learners will use formulae to accelerate their calculating, as a further four students noted in a separate theme in Table 1, but this should be based on understanding how such formulae are derived (Huang & Witz, 2011; Skemp, 2006). Developing an appreciation of the role that digital technology can play in facilitating such learning, and understanding key measurement content so that their pedagogical practice is maximized, are crucial for teacher TPACK to grow (Assude et al., 2010; Koehler et al., 2013; Mishra & Koehler, 2006).

Nine responses about the usefulness of working with compound shapes indicated that some of these ITE students have an appreciation of the importance of being able to find the area of a variety of shapes. This idea corresponds to the demands of curriculum documents in New Zealand for middle and senior primary school learners (Ministry of Education, 2007, 2008) and research by Huang and Witz (2011). It is heartening, firstly, that these ITE students appreciated the strategy of breaking up compound shapes into rectangles and/or triangles to calculate smaller manageable areas that could then be added together, and secondly, that they saw value in the DLOs for supporting this learning.

Engaging children in mathematical activities is important for their learning, and ITE students need to develop skills in considering factors that encourage mathematical engagement. Nine ITE students in this study felt that their choice of DLOs would support children's engagement with the mathematics ideas because of the contexts they provided. For example, using a cartoon for learning or perceiving a task as a game was seen as helpful for encouraging children's participation. This thinking aligns with Attard (2012) and Calder (2011, 2017), who suggest that presenting children with tasks in interesting contexts using digital technology can provoke interest and interaction.

Calculating the square metres of a classroom floor, and the use of grid paper where squares are covered by an item or used to capture the surface area of a shape, help children to "see" the task at hand and develop understanding of square units (Muir, 2006). Examples of comments from four ITE students in Table 1 indicate appreciation of the DLOs also providing opportunities for visual exploration (Calder, 2017) of shapes for learning about area.

In the final theme (Other), ITE students also suggested further possibilities that the DLOs offered for children's learning. These included opportunities for problem solving, using basic facts, making estimates, progressing to a final result, and learning appropriate mathematical language. It is encouraging to see ITE students' awareness of such issues in mathematics learning (Anthony & Walshaw, 2007).

Limitations and Conclusion

Research shows that not all children learn mathematics in the same way (Anthony & Walshaw, 2007; Attard, 2012). The findings from this study indicate that these ITE students consider the DLOs can offer another avenue for learning about area measurement.

There are a number of limitations in this study. The time given to students to explore the DLOs and make key decisions as to what they would use with learners was limited. The findings in this paper also only address one of the key questions that students were asked to respond to in the survey, and interviews with students would have helped explore their reasoning in more depth. Furthermore, the exploration of the DLOs was positioned after students had examined these ideas in a more traditional pencil and paper setting.

Reversing the order of the research process is one pedagogical implication for further ascertaining insights into ITE student thinking about the use of DLOs for teaching and learning mathematics. Implications for initial teacher educators also include providing ITE students with ample time to explore the use of the DLOs in their pedagogical practice, thereby supporting the development of student TPACK expertise. These aspects go hand-in-hand with careful pedagogical designs that focus on when these area measurement DLOs are explored. In further research, the survey used in our study could be complemented with interviews including ideas from students as to how mathematics education ITE programmes might further enhance their development as teachers.

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Peer Observation as Professional Learning about Mathematical Reasoning

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Mathematical reasoning features in curriculum documents around the world, but is understood and enacted poorly by teachers in classrooms. We explore teachers' noticing of reasoning during observed lessons. Two teams of primary teachers in Canada and Australia worked to plan, deliver, and observe lessons intended to include reasoning. They observed each other teaching a lesson that was planned with the assistance of a researcher, and later, a researcher observed each post-lesson discussion. Given the reported benefits of teachers' noticing of reasoning during peer-observed lessons, targeted professional learning support is required to further enact teachers' peer discourse to facilitate mathematical reasoning.

Mathematical reasoning is at the forefront of efforts to reform mathematics teaching and features in the curriculum documents world-wide (e.g., Australian Curriculum Assessment and Reporting Authority [ACARA], 2017; Ministry of Education Province of British Columbia, 2007). "[Reasoning is the] capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising" (ACARA, 2017, p. 5).

Reasoning plays a critical role in learners' capacity to make sense of mathematics (Goos et al., 2017; Stiff & Curcio, 1999), but changing requirements of curriculum documents contribute to teachers' uncertainty. Clarke, Clarke, and Sullivan (2012) reported that "teachers incorporate some aspects of reasoning into their teaching, but not others" (p. 32) and further that "teachers need support in identifying tasks that prompt reasoning" (p. 32). Loong, Vale, Herbert, Bragg, and Widjaja (in press) found that most primary teachers in their study had limited understanding of the nature of mathematical reasoning before engagement in a professional learning program. Despite these difficulties, teacher enactment and student experiences of reasoning in Australian and Canadian classrooms remain relatively unexplored.

The focus of this paper is on what the teachers noticed about mathematical reasoning and its teaching during the enactment of the lesson they had prepared in their peer learning teams to embed reasoning purposefully, thus attending to the research question: What is the impact of professional learning through peer observation to foster a mathematical-reasoning-based pedagogy?

Further discussion of the literature informing this study can be found in the Background section, followed by details on the conduct of the study in the Methodology section. Thematic analysis of these data is presented and discussed in the Results and Discussion section. Finally, in the Conclusion, the results are summarised.

Background

This section commences with a discussion of the growing significance of mathematical reasoning in curriculum documents and teaching practices. Next, we explore the teachers' professional noticing, with a focus on the benefits and challenges of peer-observed lessons.

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Reasoning

As a discipline, mathematics requires reasoning for deep understanding of mathematical content and for the validation of ideas (Bragg, Herbert, Loong, Vale, & Widjaja, 2016); with reasoning being "the glue that holds everything together, the lodestar that guides learning" (Kilpatrick, Swafford, & Findell, 2001, p. 129). Reasoning is a key component of mathematical proficiency. It is gaining prominence in mathematics curricula around the world, acknowledged as necessary in the learning of mathematics across all years of schooling (Kilpatrick et al., 2001; Vale, Widjaja, Herbert, Bragg, & Loong, in press). Reasoning is an explicit requirement of the Australian Curriculum, which emphasises that reasoning requires action from the learner. These actions are promoted through classroom practices as "students learn to give explanations and justifications when teachers provide tasks that require them to investigate mathematical relationships" (Goos et al., 2017, p. 37).

For teachers of mathematics to promote an increasingly sophisticated capacity for logical thought in their students, they must be able to notice and identify when reasoning is taking place, but noticing reasoning is not easy for teachers (Bragg et al., 2016). This current study provided opportunities for focussed peer observation of reasoning (along with team lesson planning and post-lesson discussion) as vehicles to enhance teachers' noticing of reasoning in classrooms.

Developing Noticing of Reasoning through Observation

Professional learning requires developing sensitivity to notice and providing opportunities to learn from experience to inform future practice, since what teachers notice and focus their attention on when observing lessons is critical for pedagogical learning (Mason, 2010). "Learning to notice does not just involve noticing aspects of teaching that before went un-noticed but also includes the sensitivity and inclination to be aware" (Nicol, Bragg, & Nejad, 2013, p. 370). Hence, focussing the teachers' awareness on key aspects of a lesson is essential to providing opportunities for professional learning.

One effective strategy for teachers' professional learning is observing, reflecting on and discussing lessons taught by experienced teachers or coaches (Casey, 2011; Clarke et al., 2013). Demonstration lessons provide teachers with opportunities to notice and are effective when teachers' observation is purposeful and focussed on children's thinking leading to learning objectives (Clarke et al., 2013). In an earlier study by the authors and colleagues (Bragg & Vale, 2013), demonstration lessons were employed to model the promotion and development of children's mathematical reasoning and these lessons were successful in assisting teachers to identify aspects of reasoning (Herbert, Vale, Bragg, Loong, & Widjaja, 2015). However, logistically and financially, demonstration lessons are not always sustainable, as the employment of expert teachers to conduct the lessons and facilitate discussions is required, along with relief teachers to provide time out of class for teachers to undertake observations, reflection, and discussion. Also, Jaworski (1998) stressed the importance of a teacher experiencing and tackling a problem themselves thus "providing more power and control" (p. 19), and teachers who lack the authority to make pedagogical decisions become frustrated (Warfield, Wood, & Lehman, 2005). Therefore, we sought an alternative model for professional learning that was potentially more sustainable and would provide the teachers with autonomy.

The Department of Education and Early Childhood Development (DEECD, 2004) recommended peer collaboration for improved student and teacher learning, encouraging peer observation to enhance improvements in student outcomes. "One of the most effective ways to learn is by observing others, or being observed and receiving specific feedback from that observation. Analysing and reflecting on this information can be a valuable means of professional growth … [p]eer observation promotes an open environment where public discussion of teaching is encouraged and supported" (DEECD, 2004, p.11).

Dufour and Eaker (1998) found that peer observation was effective in improving student outcomes, while Johnston and Cornish (2016) suggested a number of advantages of peer observations, such as offering feedback for improved student learning, reducing isolation, promoting teachers' self-reflection, encouraging teacher conversations, and providing exposure to a range of teaching approaches. Further, Wilson (2013) asserted that peer observation has potential to improve teaching by giving and receiving constructive feedback from colleagues, thus cultivating collegiality. Such discussion and reflection can facilitate a taken-as-shared understanding of the nature of good practice and hence moves towards improvements in teaching (Byrne et al., 2010).

A barrier to the success of peer observations can be a perceived imbalance of power (Gosling, 2002), especially where observation is utilised as a mechanism to evaluate teaching performance or under-performance or to appraise individuals (Byrne et al., 2010). Further, peer observations require colleagues to observe each other teaching and provide feedback on their observations for improvements to practice to occur, but this can be confronting for many teachers who are used to teaching in isolation, unobserved by peers. Therefore, successful peer learning communities must be comprised with non-judgemental, cooperative colleagues who share an equal status and seek to build an atmosphere of trust (Schuck, Aubusson, & Buchanan, 2008) and respect (Wilson, 2013).

Methodology

This paper reports on the second phase of a larger study that developed a professional learning program to foster greater understanding of mathematical reasoning and its facilitation in primary classrooms. This phase explored the use of peer observations following collaborative peer and researcher planning, with post-lesson discussions.

The two primary schools selected for this study—one regional Australian school and the other a suburban Canadian school—had been participants in the first phase of the study. The sample was convenience-based because the two teams of teachers agreed to continue into this stage of the research when approached by a researcher. This paper reports on two teams from the project, one Australian team of three Grade 5 teachers, and the other team comprised of a Grade 2/3 and a Grade 3 teacher from Canada.

Initially, the researchers conducted whole-school presentations to report findings from Phase 1 of the project and provide further resource materials generated by the larger project team. Each of us then participated in a meeting with a team of teachers to plan forthcoming mathematics lessons with a goal of ensuring aspects of mathematical reasoning were embedded into the lessons. Both teams planned one lesson to be taught in their own classrooms. The Australian team, who were experienced in planning lessons together, developed the same lesson to be taught across all three classrooms; while the Canadian team who were not used to planning together developed two different lessons to suit the needs of their classes. The researchers acted as participant-observers, contributing to the planning only when responding to teachers' questions or requests for advice.

Each teacher in the relevant team then taught a lesson, with other members of the team observing. The observing teachers completed an observation schedule and reflective notes. After each teacher had taught a lesson, the team met with a researcher to reflect on students' reasoning noticed in the lessons, actions to support reasoning, and the outcomes of the lesson. These post-lesson discussions were audio- and video-taped, and student work samples and notes from the observation were kept.

The recordings from these post-lesson discussions were transcribed and entered into *NVivo* (QSR International, 2017). Data analysis included an initial reading of the transcripts jointly by the researchers to establish themes and coding categories. The researchers independently coded the transcripts from one school with inter-judge reliability confirmed through verification of each other's coding. The transcripts were then scrutinised to refine themes into finer categories.

This paper reports on seemingly typical data from the two post-lesson discussions with the two teams.

Results and Discussion

The purpose of the current study was to better understand the impact of peer observation to foster mathematical reasoning-based pedagogy. The themes arising from the teachers' post-demonstration discussion, are summarised as: *Reasoning, Opportunities, Observations, Language, Content, Challenges*, and *Connections*. The 27 comments coded under the theme of *Observations* are the focus of this paper, as these data offered the best insights into the teachers' impressions of the peer observation process. Under *Observation*, three categories were established: Advantages, Disadvantages, and Noticing reasoning.

Advantages

As expressed by the teachers, advantages of peer observation included constructive feedback from peers to suggest reasoning opportunities, an exposure to varied teaching practices, cultivating collegiality and a respectful environment, a "stronger connection to the process" over alternative professional learning experiences, and opportunities to become aware of previously unnoticed student interactions. Teachers felt that peer observation assisted them to develop strategies for orchestrating class discussions eliciting reasoning. The following quote demonstrates the observing teacher's noticing and sharing possible improvements for her peer to promote reasoning in the future.

> Kate: … I was interested because you picked the kids that showed their answer and I was like, "Okay." But I was thinking, "What if you would have—before you did that—say, 'Who would like to show me why it's why or why not?'"

Conversely, some teachers took up the opportunity peer observation offered to learn from their colleagues through focussing on teacher actions (as suggested by Clarke et al., 2013).

> Sue: I did a lot of work with Dan a couple of years ago with this style of maths and I deliberately went last. It was great to be able to watch Cath and Dan. Probably I didn't think my language, I wasn't prompting as well as Dan and Cath were, because I wasn't tuned into maths.

Johnston and Cornish's (2016) suggestion that peer observation offers exposure to diverse teaching approaches was evident when the observing teachers actively engaged in noticing good pedagogical practice, although had been unrecognised by those teaching at times.

> Amy: The neat thing that Kate did as they were doing that … [she] kept identifying the strategy they were using. So, she named the strategy.
>
> Kate: I don't even know I do that.

> Amy: It's like "Did you realise you're using multiplication?" "Oh, all right you're using …" so it was good just to identify what it was.

Consistent with Wilson's (2013) claim that peer observation develops collegiality, we also found evidence of this in the comments from the teachers in this study. The following quotes indicate enjoyment and effectiveness in working together.

> Researcher: It's nice to see you two both excited about it.
>
> Amy: It was, it was fun.
>
> Kate: We enjoyed it.
>
> Cath: It's much easier to work with other people … We expressed how valuable we thought it was and so there has been talk about us doing that—I'm not sure—once a month, or … increasing emphasis on peer observations.

The views expressed here are also reflected in Wilson's (2013) comment that peer observation "can foster an environment in which reflecting on our own practice is valued" (p. 46). Schuck et al. (2008) stressed the importance of peer-learning teams made up of supportive, cooperative colleagues sharing equal status working towards a common goal. Participants articulated the value they perceived in working in this way with peers for a clearly defined purpose.

> Dan: [Observing] was good, too, in a non-threatening environment because we had a purpose.
>
> Sue: But I think that's probably easier for us because we all know each other really well … Imagine if you did that weekly or fortnightly … for coaching.
>
> Cath: It definitely made you stay focussed and on task with the reasoning whereas I think if, for instance, if you had just come and asked me to do that lesson it

Some participants commented that peer observation was better for developing their understanding of mathematical reasoning than demonstration lessons with a pre-prepared lesson plan. As noted by Warfield et al. (2005), the teachers had the authority to make decisions, developing and adapting their own lesson for their own purposes, with a researcher's assistance, in embedding reasoning opportunities that they had agreed on.

> Cath: Planning with each other and observing each other was much better than just watching and trialling the demonstration lesson.
>
> Amy: And I just thought the value, I'm—yeah, I see it, I see it, I see it.
>
> Sue: I think it's great—I like it.

Peer observation gave participants the opportunity to listen to students' conversations that they might normally miss in teaching a lesson and managing a class.

> Kate: You know what was interesting—you didn't hear it—was one of the conversations the kids were having when they were partnering together…so I listened to each of their reasons.

This point does not appear to have been raised in previous research.

Disadvantages

Despite the number of advantages noted by the participants during the post-lesson discussions, two disadvantages were raised: personal discomfort and financial cost. Two participants commented about being nervous when being observed.

> Amy: I was nervous having Kate come into my room and watch me teach.
>
> Sue: We were still a little bit anxious about it but it had a much more natural feel to it [than demonstration lessons].

We are mindful of Schuck et al.'s (2008) advice about creating an atmosphere of trust to overcome the teachers' personal discomfort, so it is interesting that these teachers were initially nervous being observed by supportive colleagues. "Mutual respect and responsibility are, therefore, important foundations for long-term improvements to practice" (Byrne et al., 2010, p. 216).

Another disadvantage related to the difficulties of funding time release.

Cath: It was really powerful those couple of years ago when Dan and Bonnie did have release to help with planning but it meant that they could also release someone to go and watch someone else teach. If there is a budget for it.

Noticing Reasoning

Peer collaboration in the planning, observation, and debriefing of the lesson offered opportunities for the teachers to notice and discern students' reasoning. Byrne et al. (2010) also noted the importance of conversations to facilitate learning by fostering reflection.

Amy: It's very, very valuable, but you need to be very, very calculated in the wording that you choose. Whether it is 'Show me' or 'Convince me' or 'Show me another way', 'What's another strategy?' and realising that they might have not had a strategy. This was something that I picked up from Kate's lesson because one of her students said, "Is it like when we did this?" Then Kate said, "Oh you made a connection to ..." and built on that strategy. Now I have strategies to build on that we've actually discussed as a class. So, the reasoning is really, really important, and taking the time to find out why you think that.

The shared language drawn from the peer observation lesson reinforced the employment of reasoning prompts. Further advice for improving pedagogical practices and language to cultivate reasoning were noted by peers, who indicated missed opportunities to promote reasoning.

Kate: Yeah that's the one thing I was going to say—because you kept saying "Tell me". And I thought that's where you would have said, "Show me."

Teacher's learning to notice is more than observing previous un-noticed behaviours, but becoming more aware and sensitive of classroom interactions (Nicol, Bragg, & Nejad, 2013). During peer observation, participants noticed changed behaviour when reasoning was embedded in the observed lesson, such as the expectation that students would have the freedom and obligation to explain their thinking.

Amy: They had the freedom to borrow from one another.

Kate: They had the freedom they could borrow from somebody else.

Additional positive behaviours noticed by the teachers as a result of injecting reasoning into the lesson can be seen in the following quotes: respect for each other; and, higher levels of engagement and reasoning.

Amy: What was really amazing, I was blown away by how they listened to each other. They can be very disrespectful to one another and they really did listen and nobody said, "No, you're wrong"

Amy: I just couldn't get over how they were focussed the whole time.
Kate: Yeah, they were focussed the whole time.

Amy: I wouldn't have got that from him otherwise. I would have just thought he couldn't tell me why it was right or wrong. I thought, "That was some pretty good reasoning."

Some observers expressed surprise at what they heard the children expressing.

Amy: Because you were allowed to have a completely different opinion; there was no one thing I was looking for. I forget that and I just saw the power of it today, of giving them that freedom. I just thought to myself, “I just saw four or five kids contribute to something that I would never have thought they would have.”

Kate: Aiden just shook his head after he listened to Lucy. He goes “Boy was I wrong!” So, I said, “Oh so Lucy convinced you?” He goes, “Yep!”. So, I thought that was really interesting.

The results of this study confirm the benefits of peer observation identified in the literature as a means of professional learning, such as awareness of diverse teaching approaches, fruitful discussion, supported reflection, autonomy, and collegiality. In addition, we have recognised in the data the importance of teachers tuning into children’s conversations about their mathematical reasoning as something that had previously gone un-noticed and may not have been noticed without the support of peer observation.

Conclusion

Whilst mathematical reasoning is emphasised in curriculum documents around the world, little is known about teacher professional learning in implementing this new focus in the curricula. This focus is important because the ability to reason is fundamental in connecting mathematical ideas (Goos et al., 2017). Both in the larger and current study, we have responded to the call to provide teachers with more support in embedding a range of reasoning actions in their mathematics lessons (Clarke et al., 2012).

In this study, peer observation of lessons was trialled as a means of professional learning to build teachers’ understanding of reasoning. Two groups of teachers met together to plan a lesson based on an expected content area but with a particular focus on ensuring reasoning was also fore-fronted. They observed each other teaching the lesson, then met to consider the lesson and its outcomes.

Participants discussed the value of this form of professional learning, commenting on peer observation raising opportunities to listen to students’ conversations and observe pedagogical strategies for eliciting reasoning. They noticed changes in student behaviour when reasoning was encouraged and valued. Peer observations also served to cultivate collegiality, with autonomy and ownership over lessons they prepared. These advantages indicate that this form of professional learning for building teachers’ capacity to embed reasoning in their mathematics lessons warrants further research.

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Exploring Reasons Why Australian Senior Secondary Students Do Not Enrol in Higher-Level Mathematics Courses

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In this research paper, I present the reasons why senior secondary students elect not to enrol in a higher mathematics course. All Year 11 and Year 12 mathematics students within Western Australian secondary schools were invited to participate in an online survey comprised chiefly of qualitative items. The key reasons espoused by students include an expressed dissatisfaction with mathematics, the opinion that there are other more viable courses of study to pursue, and that the Australian Tertiary Admissions Ranking (ATAR) can be maximised by taking a lower mathematics course. In addition, student testimony suggests that there are few incentives offered for undertaking a higher mathematics course.

Mathematics has been heralded as a critically important subject for students to undertake (McPhan, Morony, Pegg, Cooksey, & Lynch, 2008; Office of the Chief Scientist [OCS], 2014; Sullivan, 2011). This importance has been argued largely on the basis of students learning key interdisciplinary knowledge such as science, technology, and engineering (Ker, 2013), and to use this knowledge base to add intellectual value to new technologies, drive innovation and research capacities, and to help Australia compete globally (Australian Academy of Science [AAS], 2006). Furthermore, failure to produce a workforce with sufficient training in mathematics is considered a national concern for the economy of Australia and for keeping Australia as a competitor in the technological world (AAS, 2006; Hine et al., 2016; Maltas & Prescott, 2014; Rubinstein, 2009).

The importance of mathematics is also highlighted within tertiary study, where researchers suggest that university success depends on the level of mathematics studied at secondary school (Nicholas, Poladin, Mack, & Wilson, 2015; Rylands & Coady, 2009). More specifically, findings from various studies indicate that students who undertake higher-level mathematics courses at a secondary level tend to outperform their counterparts who undertake a lower-level mathematics course (Anderson, Joyce, & Hine, in press; Kajander & Lovric, 2005; Sadler & Tai, 2007). Despite this acknowledged importance, the number of students enrolling in higher-level and intermediate secondary school mathematics in Australia is declining (Barrington & Evans, 2014; Kennedy, Lyons, & Quinn, 2014; Wilson & Mack, 2014).

While most Australian universities have dispensed with subject prerequisites for degree programs (Maltas & Prescott, 2014; Nicholas et al., 2015), the phenomenon of declining enrolments is also experienced within tertiary mathematics courses (Brown, 2009; OCS, 2012). At the same time, there has been a reported increase in first-year university students lacking the appropriate mathematical background to complete courses in various disciplines (Poladian & Nicholas, 2013; Rylands & Coady, 2009; Wilson et al., 2013). Studies conducted in New South Wales and South Australia have identified why Australian students enrol in higher-level mathematics courses (Mathematical Association of New South Wales, 2014; McPhan et al., 2008), but there are few reasons proffered as to why capable students do not enrol in these courses. More recently, some researchers in Queensland have identified that capable students do not enrol in senior calculus mathematics courses due to a limited understanding of the relevance of mathematics (Easey & Gleeson, 2016) or the removal of Mathematics C (an advanced mathematics

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course in Queensland) from university prerequisite lists (Jennings, 2014, 2013). Additionally, there is no research available that seeks to explain the declining student enrolments in a Western Australian context.

Research Aims and Significance

The aim of this research is to explore the perceptions of Year 11 and Year 12 Australian Tertiary Admissions Ranking (ATAR) mathematics students in Western Australian schools as to why they believe that senior secondary students do not enrol in a higher-level mathematics course. The ATAR is a percentile score that denotes an Australian student's academic ranking relative to his or her peers upon completion of secondary education. This score is used to predict a student's suitability for particular university courses and, ultimately, for university entrance. The research itself builds on the findings of a previous study (Hine, 2016) in which I investigated the perceptions of Heads of Learning Area: Mathematics (HOLAMs) as to why they felt that capable senior secondary students do not enrol in the two highest mathematics courses. HOLAMS indicated perceptions of student awareness that two mathematics courses are not needed for university entrance, there are other viable and less rigorous courses of study available, and students can maximise their ATAR score without completing those mathematics courses.

It is hoped that findings from this research project may be of particular interest to secondary and tertiary mathematics educators in Western Australia, and more broadly to mathematics educators across Australia. The overarching guiding question to be explored is: What are the factors that influence Year 11 and Year 12 ATAR students' decisions not to enrol in higher-level mathematics courses in Western Australian secondary schools? This research is a predominantly qualitative study designed to give a snapshot (Rose, 1991) of the students' perceptions regarding this phenomenon.

Methodology

This study was interpretive in nature, and relied principally on qualitative research methods to gather and analyse data about why Year 11 and Year 12 ATAR mathematics students feel that senior secondary students do not enrol in higher-level mathematics courses. All Year 11 and Year 12 ATAR mathematics students in Western Australian secondary schools were invited to participate in the study. Participants registered their perceptions through the completion of a single anonymous, online survey comprising 12 five-point, Likert scale items (Q3) and two open qualitative questions (Q4 and Q5). The survey items were developed from the findings of a previous study (Hine, 2016) as well as from current literature (Barrington & Evans, 2014; Kennedy, Lyons, & Quinn, 2014; Wilson & Mack, 2014). The 12 Likert scale items required participants to the extent to which they felt that senior secondary students did not enrol in a higher mathematics course (1 = Strongly Disagree, 2 = Disagree, 3 = Undecided, 4 = Agree, 5 = Strongly Agree). The two open-ended questions asked participants to elaborate on their responses to the Likert scale items and to make any further comments regarding why they felt that senior secondary students did not enrol in a higher mathematics course. Additional demographic information of participants was obtained through a series of closed questions regarding gender, year level, the mathematics courses currently enrolled in (e.g., Applications, Methods, Specialist), type of school (e.g., secondary 7-12), gender composition of school (e.g., co-educational), and location of school (metropolitan or regional).

Participants

In Western Australia, there are 168 secondary schools (36 Catholic, 52 Independent, and 80 Government) offering Australian Tertiary Admissions Ranking (ATAR) mathematics courses to Year 11 and 12 students. These courses are Mathematics Applications, Mathematics Methods, and Mathematics Specialist (School Curriculum and Standards Authority, 2016). All Year 11 and Year 12 students enrolled in these purposively sampled schools were invited to participate in the research, and a total of 1,351 students from 26 schools gave their consent to participate. The demographic information of the participants is provided in Tables 1, 2, and 3.

Table 1
Summary of Participants' Demographic Data (by Gender and Year Level)

Gender	Year 11	Year 12	Total
Male	278	212	490
Female	455	406	861

Table 2
Summary of Participants' Demographic Data (by School Location and Composition)

School composition	Metropolitan	Regional	Total
Coeducational	737	113	850
Single Gender	501	0	501

Table 3
Summary of Participants' Demographic Data (by Mathematics Course and Gender)

Course(s)	Male	Female	Total
Applications	264	554	818
Applications and Methods	7	9	16
Methods	109	288	397
Methods and Specialist	58	62	120

Data Analysis

Qualitative data from the 1,351 completed surveys were explored using a content analysis process. According to Berg (2007), content analysis is "a careful, detailed systematic examination and interpretation of a particular body of material in an effort to identify patterns, themes, biases and meaning" (p. 303). After the two open-ended questions had been examined for themes, patterns, and shared perspectives, I analysed the data according to a framework offered by Miles and Huberman (1994), which comprises the steps: data collection, data reduction, data display, and conclusion drawing/verification. The themes drawn from the qualitative data are displayed in Table 5. For responses to the Likert scale items, descriptive statistics (weighted mean) were used to analyse collected data.

Findings

For the Likert scale items, the number of participants registering a scale rating (i.e., 1 - 5) and the weighted mean for each question item has been included. Within Table 4, a higher weighted mean represents stronger agreement with the question item, while a lower weighted mean represents stronger disagreement. In descending order, the five question items "Other courses are more viable/more attractive", "Dissatisfaction with mathematics", "Maximise ATAR without higher maths", "Higher mathematics not scaled", and "Not needed for university entrance" registered the highest weighted means. At the same time, question items "Not offered at our school", "Gender-related issues", and a "Lack of qualified staff" received the lowest weighted means.

Table 4
Responses to Likert-Scale Question Items

Question item	1	2	3	4	5	Weighted mean
Other courses more viable/attractive	38	112	262	549	383	3.83
Dissatisfaction with mathematics	99	213	467	413	152	3.22
Maximise ATAR without higher maths	94	228	489	404	128	3.18
Higher mathematics not scaled	200	250	315	278	301	3.17
Not needed for university entrance	160	303	322	377	185	3.09
Compulsory subject selections	324	305	366	243	101	2.62
Friends doing the same courses	343	373	355	220	52	2.45
Dislike the teachers	415	328	318	187	95	2.42
Timetabling constraints	485	360	308	138	43	2.17
Lack of qualified staff	707	262	201	100	67	1.92
Gender-related issues	863	228	170	41	39	1.63
Not offered at our school	1098	92	95	26	27	1.34

Table 5
Summary of Extended Answer Questions (Responses to Questions 4 and 5)

Key Themes	Question 4	Question 5	Total
Dissatisfaction with mathematics	215	558	773
Other courses are more viable/more attractive	108	282	390
Higher mathematics courses are not scaled sufficiently	102	60	162
Not needed for university entrance	60	73	133
ATAR can be maximised taking a lower maths course	76	55	131
Not needed for future life or career	33	72	105
Dissatisfaction with higher mathematics teachers	52	46	98

For Questions 4 and 5, the most commonly proffered qualitative responses included a dissatisfaction with mathematics, a decision to enrol in more attractive or viable courses, and a perception that mathematics is insufficiently scaled as a Year 12 course (see Table

5). These qualitative responses (which have been summarised in Table 5 with other responses) will now be explored.

Dissatisfaction with Mathematics

Participants asserted that the chief reason that secondary students did not enrol in a higher mathematics was due to a dissatisfaction with mathematics. Such dissatisfaction was registered via a variety of associated themes, including a perceived discrepancy between the complexity and workload of Applications and Methods courses, an acknowledged mismatch between effort and reward, a lack of confidence to study a higher mathematics, and an expressed lack of interest or enjoyment in the subject. The most frequently expressed theme by participants was the perceived discrepancy between Mathematics Applications and Methods courses, particularly in terms of overall workload and complexity of content (Q4: 139/215, Q5: 395/558). For instance, one participant reflected on this perceived discrepancy between courses:

> I was previously enrolled in Methods, however I found it extremely hard. I had never received such low scores in maths. Now being in Applications, I have noticed that the topics studied are completely unrelated to Methods. It's not necessarily that Methods students are learning a harder level of math, they are learning a completely different topic which is harder to understand. I didn't see how what we learnt applied to real life like the topics we learnt in Applications do. I think there needs to be a bit of consistency in the topics. I also found Methods stressful as we went through the topics very fast.

From those participants asserting that students' dissatisfaction with mathematics stemmed from a perceived discrepancy between Applications and Methods courses, many proposed that an "in-between" course needs to be developed and offered to students. According to those participants, such a course would contain a considerable amount of content common to both Methods and Applications courses, and pitched at a level of difficulty in between those courses.

Other Courses are Viable/More Attractive

The second most common assertion participants made was that secondary students tend to enrol in those courses of study that appear to be more viable or more attractive than a higher mathematics course. In particular, participant responses regarding "course viability" or "course attractiveness" were further classified into the following associated themes: Students chose a "lower" mathematics course in order to excel at it, observed that lower courses were less stressful to undertake, rationalised that undertaking a lower mathematics course translated into less time studying mathematics and more time to allocate to other ATAR courses, and decided to broaden the variety of ATAR courses studied. The most commonly occurring theme was that students felt that undertaking a lower mathematics course required them to devote less time to mathematics study and to set aside more time to successfully complete other ATAR courses (Q4: 43/108, Q5: 123/282). To illustrate, a participant stated:

> I feel as though I prefer to do really well in Applications than have to struggle through Methods with only satisfactory results. It also means I can put more effort into other subjects as I am not having to spend hours and hours of my time doing maths each week.

Another participant advanced this statement, rationalising how taking a lower mathematics course translated into increased time for other courses and a higher ATAR overall:

> I think that people don't choose higher maths because the[se subjects] are subjects that require an increased amount of time and effort. You have to weigh up whether or not doing very well in Applications is going to be better for your ATAR than just doing average in Methods. I know for me, I would love to take a higher level maths; however, I wouldn't have time with my other subjects to do as well, and higher maths [subjects] generally don't get scaled enough. So overall it would be detrimental to my ATAR.

A further concession made by many participants was that on top of the perceived extra effort and workload associated with higher mathematics courses, taking a lower mathematics course would not only increase their ATAR score but improve their chances of being accepted into their desired university degree course.

Higher Mathematics Courses are not Scaled Sufficiently

Several participants (Q4: 102, Q5: 60) intimated that the reason that students do not enrol in a harder mathematics course was due to insufficient scaling or incentives. For example, one participant reinforced some previous key findings by arguing "Higher mathematics courses are not scaled enough. The difference between Applications and Methods in hardness is not compensated by scaling. People are better off doing Applications in terms of time spent on the subject and difficulty". Other participants felt that by completing the Mathematics Applications course instead of Mathematics Methods, their mathematics result would be impacted greater by scaling measures. To illustrate, a participant hypothesised:

> If I dropped down to Maths Applications due to the impractical scaling of the two maths subjects (Methods and Applications) I could achieve a better ATAR by getting much higher results which are only scaled down a small amount instead of getting mid-range results which scale up by a small amount. This is seen by many students [who] I know drop down in both the current Year 12 cohort and the Year 11 cohort, this is not rational as harder maths courses are not rewarded per se for their extra effort.

There were some participants who drew attention to the 10% bonus marks offered by the School Curriculum and Standards Authority (SCSA) to Year 12 students completing Mathematics Methods or Mathematics Specialist courses from 2017 onwards. One participant stated:

> Especially for this year, Methods and Specialist will not be given the 10% additional bonus if it is in your top score. Those harder subjects are not scaled much so the same amount of effort required a 65 in Methods could get a 90 in Applications, allows the people who do easier maths to get a higher ATAR…please explain how that is fair at all?

All participants who voiced concerns over insufficient scaling or incentivisation of higher mathematics courses based their reasoning upon a perceived difference in difficultly between courses (e.g., Methods and Applications), a drastically different scaling method to be used for easier or more difficult courses, the maximisation of the ATAR by taking the easier mathematics course, and the incentive offered to students from 2017 onwards. Irrespective of reason, all participants expressed that scaling procedures influenced their decision not to enrol in a higher mathematics course.

Conclusion

The purpose of this research paper was to outline reasons why Year 11 and Year 12 ATAR mathematics students in Western Australia do not enrol in higher-level mathematics courses. I identified three key findings via Likert-scale items (Table 4) and open questions (Table 5) for further consideration. First, students indicated dissatisfaction with the perceived discrepancy in difficulty of Methods and Applications courses currently offered in Western Australian schools. Aside from the apparent "jump" in content complexity between these courses, students feel that the time and effort spent on undertaking a more difficult course (i.e., Methods) is unrewarded. At the same time, students suggested that the creation of a mathematics course whose level of difficulty lay in between Methods and Applications would assist in reducing the current discrepancy and consequently encourage more students to enrol in it.

Second, students feel that undertaking an easier mathematics course will allow additional time to focus on other ATAR courses. The themes associated with this finding suggest that students are interested in adopting a balanced approach to their studies where they can apportion a similar amount of time and effort to mathematics as their other ATAR courses for maximal reward. Additionally, there appears to be an expressed need by students to feel confident in the mathematics course they take; this confidence is brought about by choosing a course where the content can be mastered and the level of stress associated with such mastery is not atypically high compared with other ATAR courses.

Third, students believe that there is an insufficient reward offered for taking a higher mathematics course. For the most part, students nominated that the scaling procedures or a lack of incentivisation deterred them from enrolling in a more difficult course. Interestingly, at the time of data collection, neither the Year 11 nor Year 12 students involved in the study had any foreknowledge of how the scaling process in Western Australia had worked for previous Mathematics Applications, Mathematics Methods, and Mathematics Specialist student cohorts; they would become the first and second cohorts, respectively. Some Year 12 students lamented that in 2017 – when they had completed secondary schooling – they would miss out on the incentive offered by the Tertiary Institutions Service Centre (TISC) to students completing Mathematics Methods and/or Mathematics Specialist courses. Students completing either the Methods course or both Methods and Specialist courses will receive a 10 percent bonus of their final scaled score in those courses (TISC, 2016).

This study builds on the previous research conducted in Western Australia regarding student enrolments in senior secondary mathematics courses (Hine, 2016), in that it sought to engage the student voice. The findings outlined illustrate various tensions regarding students' decisions not to enrol in a higher-level mathematics course. These tensions appeared to focus more on the students' short-term goals (e.g., achieving a higher ATAR in an easier course for reduced effort and stress) rather than on the mastery of mathematical concepts required for a career or for further study. Based on these findings, future research efforts could be directed at asking the Year 11 and Year 12 participants the extent to which they feel their choice of secondary mathematics course prepared them adequately for the future (i.e., a longer-term goal). Other efforts could focus on a replica study in the next few years, especially once the bonus marks system for Methods and Specialist has been introduced.

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Does (Problem-Based) Practice Always Make Proficient?

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The current study compared the rate at which problem-based practice increased the use of retrieval-based strategies for students identified as displaying accurate min-counting with students identified as displaying almost proficient performance. The findings supported the prediction that the rate at which problem-based practice promoted retrieval use was lower for students in the accurate min-counting group; in fact, it had no effect on their retrieval development. Implications for teaching practice are discussed, in particular, the notion that such students may require exposure to different problem representations (e.g., visual imagery) to move them away from conceptualising addition as counting.

Education standards indicate that children will solve most simple (single-digit) addition problems using retrieval-based strategies by second or third grade. Retrieval-based strategies encompass direct retrieval as well as decomposition strategies, where a number is partitioned to make use of a retrieved fact (e.g., 3 + 4 = 3 + 3 + 1 = 6 + 1).

It has slowly come to light that many children are not solving simple addition problems in ways that match curriculum expectations. Children's use of retrieval is considerably lower than expected in second and third grade (Cowan et al., 2011; Geary, Hoard, Byrd-Craven, & deSoto, 2004), and their use of min-counting is much higher than expected (Cumming & Elkin, 1999). This is a concern for educators given that the frequency with which retrieval-based strategies are used to solve simple addition problems has been found to be strongly associated with (i) maths achievement (Geary, 2011a), (ii) understanding key principles such as the commutativity of addition and the complementary relationship between addition and subtraction (Canobi, 2009), and (iii) the use of flexible mental strategies for adding and subtracting multi-digit numbers (Carr & Alexeev, 2011).

Hopkins and Bayliss (2017) found that in seventh grade, around 35% of students were still reliant on min-counting to accurately solve simple addition problems. This group of students was referred to as displaying an accurate min-counting pattern of performance and was distinguished from students who displayed (i) an inaccurate min-counting counting pattern, (ii) almost proficient performance (i.e., the predominate use retrieval-based strategies), and (iii) proficient performance (the exclusive use of retrieval-based strategies). Students in the accurate min-counting group showed lower achievement in maths compared to students who predominately or exclusively used retrieval-based strategies. Hopkins and Bayliss (2017) suggested that students in the accurate min-counting group had not developed problem-answer associations in memory that allowed them to directly retrieve answers or use retrieved facts as part of a decomposition strategy because they had a high confidence threshold for using retrieval. However, it was not clear if min-counting was the most efficient strategy these students had access to in their strategy repertoire or if they simply had chosen to min-count on the one occasion they were assessed. The aims of the current pilot study were to test the assertion that such students do not use retrieval-based strategies because of a high confidence threshold for using retrieval.

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Background

Children generally learn retrieval-based strategies for solving single-digit addition problems as a result of problem-based practice; that is, practice solving single-digit addition problems using strategies of choice. Children may initially solve simple addition problems using counting-all strategies but with continued practice, children will abandon the use of less efficient strategies and replace them with the min-counting strategy (where the smaller addend is counted on), direct retrieval and decomposition strategies (Canobi, 2009; Hopkins & Lawson, 2002; Siegler & Jenkins, 1989).

The transformative role practice can have on strategy use is sometimes overlooked in educational literature. For example, in a report by the U.S. National Research Council (2001), it was stated that, "The role of practice in mathematics, as in sports or music, it to be able to execute procedures automatically without conscious thought. That is, a procedure is practiced over and over until so called automaticity is attained" (p. 351). While practice can automatize the execution of a strategy, this is not the only role of practice. Specifically, the benefits of problem-based practice extend beyond automaticity to encompass evolution in strategy use (Hopkins & Lawson, 2002).

Problem-based practice is arguably the most important teaching approach for strategy development as it leads to the use of multiple strategies and promotes adaptive strategy use (Verschaffel, Luwel, Torbeyns, & Van Dooreen, 2009). Other approaches include strategy-based practice, where students are directly taught a strategy like min-counting and are required to practice using the particular strategy (e.g., Fuchs et al., 2010), and fact-based practice, where students rehearse each problem with the correct answer (e.g., Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009).

However, not all children develop accurate retrieval-based strategies when exposed to problem-based practice. Two explanations are commonly put forward elucidating why this is so. Firstly, it is suggested that children make too many errors during problem-based practice as a result of losing track of the count (e.g., Geary, Bow-Thomas, & Yao, 1992). Frequent errors form competing (incorrect) problem-answer associations in memory leading to retrieval errors or the continued use of backup strategies (Shrager & Siegler, 1998). Secondly, it is contended that some children have a learning disability, characterised by particularly low mathematics achievement and retrieval deficits (e.g., Fuchs et al., 2010; Geary, Hoard, & Bailey, 2012). These deficits appear to be related to difficulties inhibiting irrelevant information during the retrieval process (Geary, 2011b). However, neither of these explanations can explain an accurate min-counting pattern of performance. Children in this group rarely make mistakes and, although low achieving as a cohort, are generally not among the lowest achieving students (Hopkins & Bayliss, 2017).

There is a third explanation, however, which can be used to explain an accurate min-counting performance pattern; confidence. Shrager and Siegler (1998) argued that although practice using a back-up strategy builds the associative strength of an answer in memory, the answer will only be stated if its associative strength exceeds an individual's response threshold for determining confidence in its correctness. Students with a high confidence threshold for retrieving answers will need more practice before becoming sufficiently confident to use retrieval, compared to students with a less restrictive confidence threshold. Bailey, Littlefield, and Geary (2012) found that boys and girls displayed different developmental paths towards using retrieval for simple addition and suggested that girls have a higher confidence threshold for retrieval than boys. Similarly, Hopkins, and Bayliss (2017) found that students in the accurate min-counting group were more likely to be girls.

In the current study, we compared the rate at which problem-based practice increased the use of retrieval-based strategies for students identified as displaying accurate min-counting with students identified as displaying almost proficient performance. We predicted that this rate would be less for students in the accurate min counting group because students have a higher confidence threshold.

Method

Participants attended one primary (elementary) school located in Melbourne, Australia. Initially 94 students were assessed on how they solved single-digit addition problems. Students were in Grade 2 (n = 14), Grade 3 (n = 20), Grade 4 (n = 24), Grade 5 (n = 15), and Grade 6 (n = 21). Students were individually withdrawn from class and assessed on how they solved 36 single-digit problems on a trial-by-trial basis, using a combined method of observation and self-report. Immediately after each problem was solved, the child explained to a research assistant (RA) who had observed them how they had solved the problem. This method for assessing strategy use has been shown to be a reliable and valid approach (Reed, Stevenson, Broens-Paffen, Krischner, & Jolles, 2015; Siegler, 1987) and used extensively in studies of single-digit addition skill (e.g., Canobi, 2009; Geary et al., 2004). Reaction times (RTs) for each trial were recorded, representing the time the problem was displayed to the time when the child gave their answer. Mean RTs separated by strategy use are reported for correct trials in Table 1. These data corroborated children's self-reports with RTs to correct retrieval trials generally under three seconds. The problem set encompassed 36 single-digit problems written in the form $a + b =$; where $a \leq b$ and $1 < a, b \leq 9$.

Table 1
Strategies Reported During the Initial Assessment with Corresponding RTs for Correct Trials

Strategy	Example	No. correct trials (%)	Mean RTs (*SD*)
counting-all	3+4=1, 2, 3, 4, 5, 6, 7	52 (1.7)	13.1 (7.2)
counting-on-right	3+6=3: 4, 5, 6, 7, 8, 9	90 (2.9)	10.8 (7.2)
min-counting	3+6=6: 7, 8, 9	1,074 (34.5)	6.4 (9.0)
decomposition	4+5=4+4	106 (3.4)	5.4 (4.3)
other	2+4=2+2+2; 3+3=2x3; 3+5=4+4	55 (1.8)	6.5 (5.8)
retrieval	7 (*just knew it*)	1,735 (55.8)	2.3 (1.4)

Note. No. of correct trials was 3,112, representing 92% of trials.

Results of this initial assessment were used to cluster students into groups based on performance characteristics. Students who always used retrieval-based strategies were first identified (proficient group), followed by students who frequently used a counting-all strategy (inefficient-counting group). A k-means clustering technique was used to group the remaining students into three clusters, similar to Hopkins and Bayliss (2017). These clusters included (i) students who exhibited typical retrieval development (almost-proficient group), (ii) students who frequently used min-counting and were accurate (accurate-min-counting group), and (iii) students who frequently used min-counting and were inaccurate (inaccurate-min-counting group). Performance characteristics for each group are summarized in Table 2.

Students in the accurate min-counting group were distributed across grade levels: Grade 2 (*n* = 6), Grade 3 (*n* = 12), Grade 4 (*n* = 9), Grade 5 (*n* = 6), and Grade 6 (*n* = 6). This performance pattern was the focus of investigation. Students in the almost-proficient group exhibited more typical retrieval development consistent with curriculum expectations, and used for comparative purposes. Students in this group were distributed across grades as follows: Grade 2 (*n* = 1), Grade 3 (*n* = 7), Grade 4 (*n* = 8), Grade 5 (*n* = 7), and Grade 6 (*n* = 12).

Table 2
Percentage for Trials Indicating Strategy Frequency and Accuracy (in Parentheses) by Group

Group	No. students	Retrieval	Min-counting	Decomp. + other
Proficient	7	91.7 (98.8)	0	8.3
Almost-proficient	35	67.7 (98.2)	25.7 (96.7)	4.5 (85.0)
Inaccurate-min-counting	8	58.3 (93.6)	27.4 (58.9)	1.0
Accurate-min-counting	39	35.5 (96.5)	54.8 (90.1)	6.5 (84.0)
Inefficient-counting	5	19.4 (92.2)	11.1 (77.4)	4.4

Note. Accuracy not shown if based on fewer than five students.

Participants in the study included six students: three students who were randomly selected from the accurate-min-counting group and three students who were randomly selected from the almost-proficient group. Participants were monitored as they engaged in problem-based practice for 15 consecutive school days (where possible). Each day, students were individually withdrawn from class and strategy use was determined using the same combined method of observation and self-report, and the same 36-problem set, described above. To estimate the rate at which problem-based practice leads to retrieval development, a regression line was fitted to each students' data representing the number of problems correctly retrieved each practice session. It was not clear if students' errors should be corrected during the study as corrective feedback could influence retrieval development differently for students with different profiles. For this reason, the influence of corrective feedback was controlled for using a multiple-baseline design. During the uncorrected condition, no feedback was given. During the corrected condition, the RA would respond with "let's check that" if an incorrect response was given. She then proceeded to model the correct use of the same strategy that the child had reported using.

Results

Participants were monitored each day for 15 consecutive days. Area graphs in Figure 1 illustrate the range of strategies correctly applied and the number of errors made at each time interval (i.e., practice session) by each child, as well as the influence of corrective feedback. Numbers along the y-axis indicate the number of problems. Numbers along the x-axis indicate consecutive school days. A dotted line separates the no feedback condition from the corrective feedback condition. Data are missing for Kyle and Karen due a computer fault.

Visual inspection of graphs in Figure 1 suggest that participants selected from the accurate min-counting group did not benefit from problem-based practice in terms of improved correct retrieval; whereas, two of the three participants selected from the almost proficient group did show marked improvement in correct retrieval. The third child in the accurate min-counting group (Karen) did not exhibit performance consistent with her group: she made frequent errors, particularly min-counting errors (21.2%) rather than retrieval errors (0.8%). In addition, the third child in the almost proficient group (Casey) did not benefit from problem-based practice, appearing to use retrieval less frequently during the corrective feedback condition.

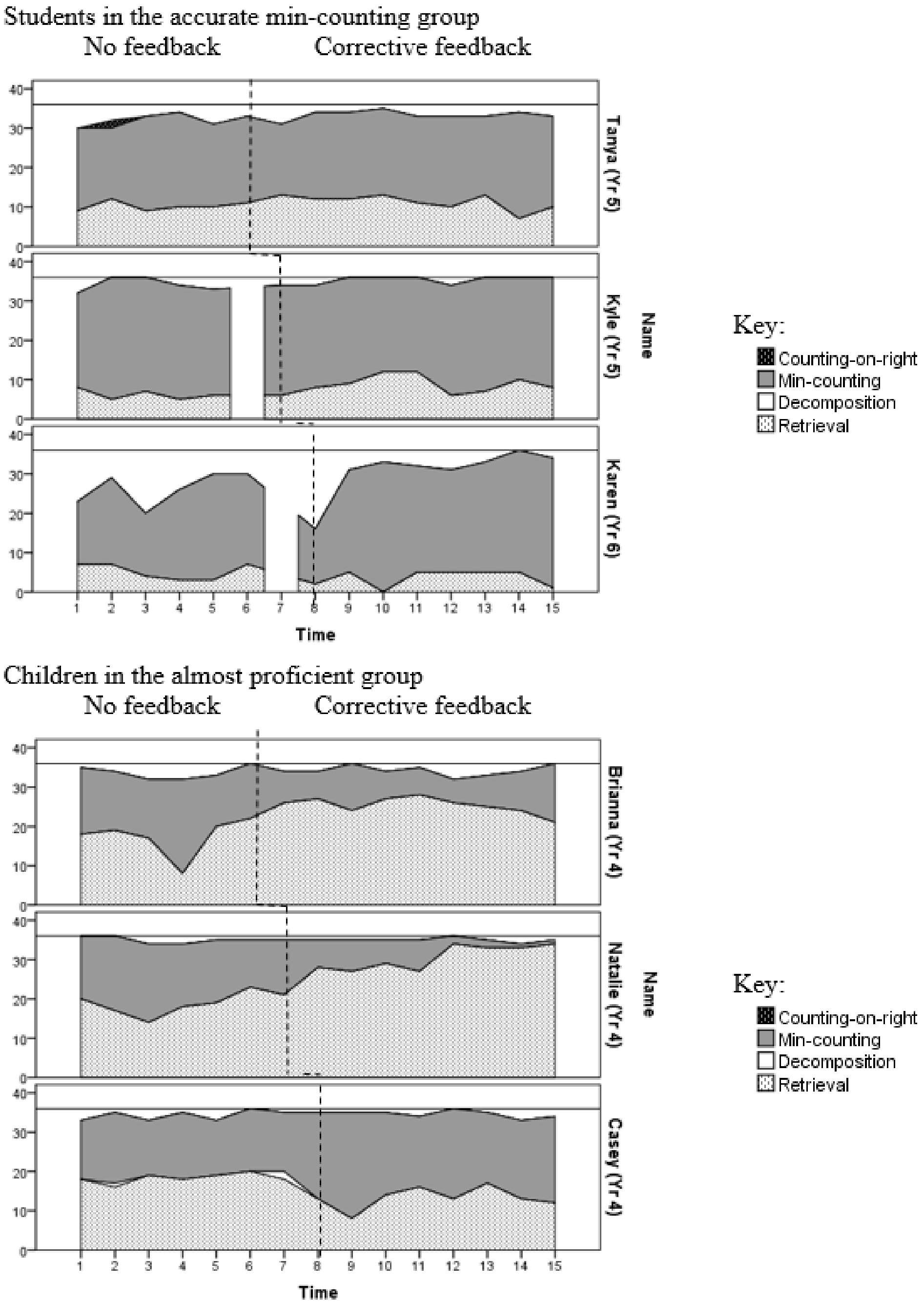

Figure 1. Area graphs displaying the strategy mix and errors across time by participant.

Table 3
Changes in the Frequency of Correct Retrieval as a Result of Problem-Based Practice

	Parameter	Estimate*	SE	t	p
Tanya	Intercept	10.74	.98	10.92	.000**
	Slope	.011	.12	.10	.922
Kyle	Intercept	5.9	1.18	5.00	.000**
	Slope	.23	.18	1.80	.10
Karen	Intercept	5.6	1.17	4.79	.000**
	Slope	-.17	-.37	-1.36	.199
Brianna	Intercept	16.56	2.38	6.95	.000**
	Slope	.70	.26	2.66	.020***
Natalie	Intercept	13.9	1.35	10.24	.000**
	Slope	1.41	.15	9.49	.000**
Casey	Intercept	19.49	1.59	12.27	.000**
	Slope	-.46	.18	-2.64	.020***

*Unstandardized coefficients **$p < .001$ ***$p < .05$

Results of the regression analyses confirmed these patterns (see Table 3). Specifically, it was revealed that two of the three students whose performance was consistent with an accurate min-counting pattern (Tanya and Kyle) did not benefit from problem-based practice and their frequency of correct retrieval remained constant across sessions. By contrast, two of three students whose performance was consistent with an almost proficient pattern (Brianna and Natalie) benefited from problem-based practice with an estimated rate of increase of 0.7 and 1.4 problems per practice session respectively (as indicated by slope estimates in Table 3).

Discussion

A considerable proportion of students are not using retrieval-based strategies for solving single-digit addition problems when they are expected to do so (Hopkins & Bayliss, 2017) and these students are likely to show lower achievement than their peers who do use retrieval-based strategies (Geary, 2011a; Hopkins & Bayliss, 2017). This pilot study investigated if students who were identified as predominately using an accurate min-counting strategy for simple addition benefitted as much from problem-based practice as peers who showed higher retrieval use. The findings supported the prediction that the rate at which problem-based practice promoted the retrieval-based strategies was lower for students in the accurate min-counting group; in fact, this type of practice had no effect on retrieval development for these students. This is an important finding as problem-based practice is the principal type of practice used to promote retrieval, and retrieval is thought to be “dependent on sufficient and appropriate practice” (U.S. Department of Education, 2008, p. xix).

While it is recognised that students who make frequent errors with backup strategies are less likely to benefit from problem-based practice (Shrager & Siegler, 1998), the findings of this study indicate that confidence may also be a factor that hinders retrieval development. The notion of a high or restrictive confidence criterion has been referred to many times in the literature (e.g., Bailey et al., 2012; Geary et al., 2004; Shrager & Siegler, 1998) but this idea has remained largely unexplored empirically. Findings from this study suggest that further research examining this factor may prove to be important for research and teaching practice. Further research is needed to test the assumption that confidence is restricting retrieval use for these students, rather than another factor such as *Einstellung*; that is, a disposition towards applying a familiar strategy (Bilalić, McLeod, & Gobet, 2010). This is the focus of a current research project involving the authors.

The findings of this study also indicated that the methods adopted for categorizing and selecting students from groups based on what strategies they use and how accurate they are need to be refined. At least one participant did not exhibit performance characteristic of her group. For the purpose of testing predictions, particularly using a single case study design, it would be more efficacious to purposefully select students from each group (rather than randomly select them) so that they represent typical performance for that group. There was also some evidence that corrective feedback can inhibit retrieval use and so larger-scale studies will need to consider feedback in their design.

Despite these limitations, findings from the cluster analysis of data collected using the initial assessment were consistent with that found by Hopkins and Bayliss (2017) using a separate sample of students. This analytical approach shows much potential. A similar cluster method has been used to identify cognitive subtypes of mathematics learning difficulties (Bartelet, Ansari, Vaesson, & Blomert, 2014) and profiles for core numerical competencies (Reeve, Reynolds, Humberstone, & Butterworth, 2012).

Importantly, the current study goes one step further than identifying different groups of students and elucidating their performance profile. Specifically, it presents some initial evidence indicating that students with different profiles might in fact respond differently to a particular type of teaching practice. This has definite implications for classroom teachers. For example, students identified as almost proficient in simple addition are likely to be able to continue to improve their capacity to directly retrieve answers through conventional problem-based practice. We are not suggesting that this group can be ignored by teachers, but rather, that the main role of teachers would appear to provide sufficient opportunities for them to engage in problem-based practice. By contrast, it is likely that students with an accurate min-counting performance profile require exposure to a variety of different problem representations in an attempt to move them away from conceptualising addition equations primarily as "counting problems". It is worth noting that a subitising-based intervention has shown promise in improving retrieval, and reducing reliance on min-counting (Hopkins & de Villiers, 2016). It may be that primary school teachers could consider exposing accurate min-counters to additional games and activities involving subitising and other number visualisation tasks. These students might then be able to apply such imagery to aid retrieval when confronted with addition problems, and in time, as retrieval improved, perhaps start to simultaneously benefit more from engaging in problem-based practice. These ideas provide new directions for both classroom-based practice and future research.

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Explicitly Connecting Mathematical Ideas: How Well Is It Done?

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Multiplicative thinking is a critical stage of mathematical understanding upon which many mathematical ideas are built. The myriad aspects of multiplicative thinking and the connections between them need to be explicitly developed. One such connection is the relationship between place value partitioning and the distributive property of multiplication. In this paper, we explore the extent to which students understand partitioning and relate it to the distributive property and whether they understand how the property is used in the standard multiplication algorithm.

Multiplicative thinking is one of the "big ideas" of mathematics (Hurst & Hurrell, 2014; Siemon, Bleckley, & Neal, 2012) and it underpins many important mathematical concepts required beyond primary school years. Multiplicative thinking could be described as a complex set of concepts which are interrelated and linked in various ways (Hurst & Hurrell, 2016). The Australian Curriculum, Assessment, and Reporting Authority (2017) describes the proficiency of Understanding in terms of a clear and strong knowledge of "adaptable and transferable mathematical concepts" which enables students to make connections between concepts that are related. In short, it could be said that the development of multiplicative thinking depends largely on knowing about the links and relationships between ideas in order to understand why procedures work as they do.

Several researchers (Clark & Kamii, 1996; Siemon, Breed, Dole, Izard, & Virgona, 2006) have noted that an inability to think multiplicatively greatly hinders the development of higher level concepts such as fractions, proportional reasoning, and algebra. This underlines the need to understand what constitutes multiplicative thinking and to identify how key elements are linked so that they can be developed in a conceptual and connected way across all school years.

Siemon et al. (2006) defined multiplicative thinking in the following terms:

- a capacity to work flexibly and efficiently with an extended range of numbers (e.g., larger whole numbers, decimals, common fractions, ratio and percent);
- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion;
- the means to communicate this effectively in a variety of ways (e.g., materials, words, diagrams, symbolic expressions and written algorithms).

If students are to work "flexibly" with a range of numbers, we believe that there must be explicit teaching of the many connections within the broad idea of multiplicative thinking. Specifically, in this paper, we explore the link between partitioning based on place value, and the distributive property of multiplication.

Background: Place Value, Partitioning, and the Distributive Property

The distributive property of multiplication could be considered as the basis of the vertical multiplication algorithm that is taught in a range of ways by teachers. The importance of this property cannot be under-estimated and Kinzer and Stafford (2013) note the importance of partitioning, stating that "this kind of reasoning is the first step in moving beyond repeated addition and using the distributive property to make sense of

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 325–332). Melbourne: MERGA.

multiplication" (p. 304). Kinzer and Stafford (2013) also underline the importance of the array in developing an understanding of the distributive property and note that "the distributive property helps students understand what multiplication means, how to break down complicated problems into simpler ones, and how to relate multiplication to area by using array models" (p. 308). Their view supports the Common Core State Standards for Mathematics (NGA Centre, 2010) which also underlines the importance of the link between multiplication and the array, and hence the distributive property.

The importance of the distributive property and the link with partitioning is emphasised by Jacob and Willis (2003) who noted that "part-part whole reasoning with groups also enables children to use the distributive property of multiplication over addition" (p. 7). Also, Norton and Irvin (2007) said that "critical concepts underpinning algebra (e.g., equal concepts, integer study, fractions, the distributive law and general arithmetic computational competency) need to be emphasised in the primary years" (p. 559). The quality of understanding about multiplication that results from knowing about the distributive property is further noted by Young-Loveridge and Mills (2009) who said that multiplication strategies based on partitioning and the distributive property are more advanced than those based on other ideas such as repeated addition.

Kaminski (2002) studied how a group of pre-service primary teachers used the distributive property in flexible ways when solving multiplication exercises. He made an interesting observation that "It was clear that while many students had heard of the distributive law, many were still not clear on its application" (p. 141). Kaminski's sample consisted of pre-service teachers, as opposed to in-service teachers but his observation begs the question as to whether or not the majority of in-service teachers would be clear about how the distributive property can be applied. It also follows that the same situation might apply to realising (or not realising) the connection between place value partitioning and the distributive property.

Methodology

This paper reports on a study that developed from a larger and on-going study into multiplicative thinking of children from 9 to 11 years of age. The original study has been conducted for over three years in Western Australian primary schools and has gathered data from over 1,000 children in eight schools. Two data gathering instruments – a written Multiplicative Thinking Quiz, and a semi-structured interview – have been developed and refined during that time and are used in this current study involving two primary school classes at one school in the south-west of the United Kingdom. The quiz was administered to both classes on the same day under identical conditions. The framework for the analysis of data is based on connections between place value partitioning, the distributive property of multiplication, and the standard written algorithm for multiplication, in order to determine if students have an understanding of those connections, and are able to articulate that understanding. The framework is depicted in Figure 1.

In the Multiplicative Thinking Quiz (MTQ), students were asked a total of 18 questions, five of which are based on aspects of the framework (see Table 1). We wanted to find out the extent to which students demonstrated an understanding of partitioning, were able to identify when the distributive property was correctly applied, and whether they were able to explain why the property worked in terms of partitioning. In short, we wanted to see the extent to which they connected the ideas and then how they used the written multiplication algorithm during the semi-structured interview.

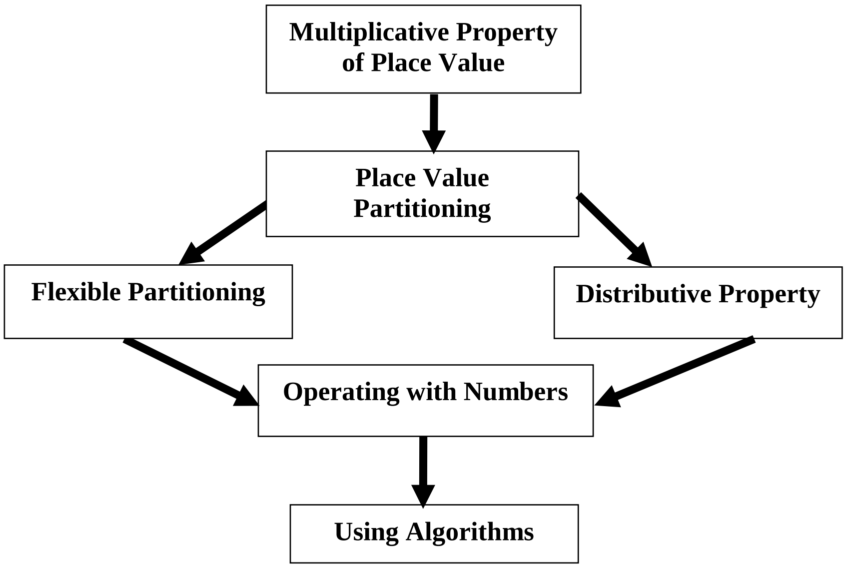


Figure 1. Framework for analysing data from Multiplicative Thinking Quiz.

Data and Analysis

Table 1 presents the responses to the relevant questions from the MTQ. Class A was a Year 5 class (n = 29) and Class B was a Year 6 class (n = 27). Descriptive statistics are used to show the percentage of each class that responded correctly for each question. Several observations can immediately be made from Table 1.

First, while approximately two thirds of the total sample were able to mentally calculate the answer to 6 × 17 (Question 1), a smaller percentage were able to explain their calculation in terms of place value partitioning (Question 2), which is the basis of the written algorithm. However, a similar proportion of students who performed a correct mental calculation were able to use a written algorithm to solve 9 × 15, based on the standard place value partition (Question 3). In the analysis of the quiz responses for Question 3, students needed to indicate that they had ‘carried a four’ to qualify as a correct response. Second, a much smaller proportion of students were able to identify both correct responses to the question about the distributive property. The interesting aspect of this observation is that the mathematical understanding that underpins Questions 2 and 3 is the same as for Questions 4 – partitioning based on place value. Third, a comparatively small proportion of students could explain their choices of answers (Question 5) in terms of what they had already seemed to understand from their responses to Questions 1, 2, and 3. In other words, the majority of students were able to use place value partitioning either mentally or in a two by one digit algorithm, but many of them were unable to connect the same idea of partitioning to identify when the distributive property was correctly applied, and even less could explain that in terms of partitioning. All of the seven students who explained the fifth question in terms of partitioning used partitioning to explain their answers to Questions 2 and 3.

Table 1
Summary of Responses to Selected Questions from the Multiplicative Thinking Quiz

Question from Multiplicative Thinking Quiz	Class A	Class B
1. Used mental computation to obtain correct answer for 6 × 17	62%	67%
2. Explanation of mental computation for 6 × 17 is based on place value partition	45%	59%
3. Use of standard algorithm is correct and shows place value partitioning (i.e., the "carried 4") to solve 9 × 15	66%	70%
4. Identifies both (80 × 3 + 9 × 3) and (90 × 3) – (1 × 3) as the only correct options giving the same answer as 89x3 (Distributive Property)	34%	26%
5. Explanation of above question (about Distributive Property) is based on place value partitioning	10%	15%

The following samples from Student Wesley are indicative of responses for the MTQ questions.

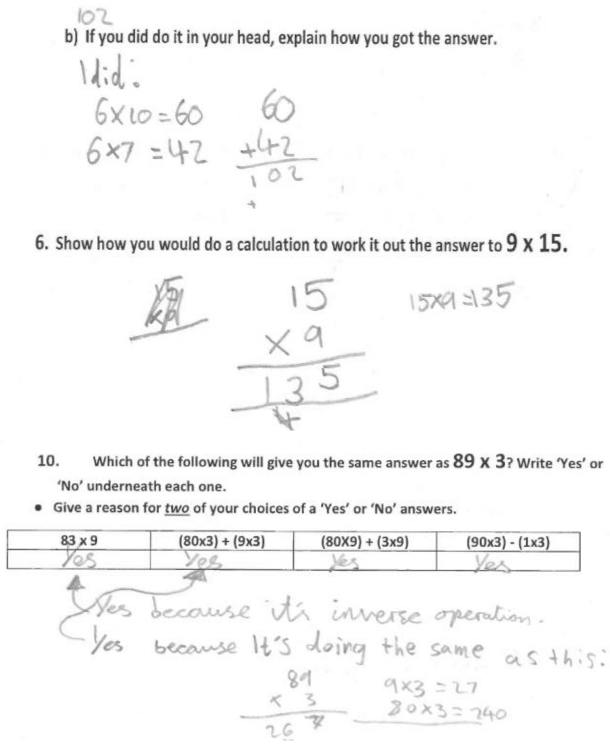


Figure 2. Samples from student Wesley.

Wesley appears to have an understanding of place value partitioning and has given sound examples of it for the first two questions. However, when the question is presented in a different context, he seems quite confused and has mistakenly identified all options as being correct. Wesley has also confused the idea of 'inverse operations' a term that he would have heard at some stage but not fully understood. As well, Wesley did not seem to trust the idea of partitioning as he has used an algorithm to work out the answer to 89 × 3 when there was really no need to do so, if he understood how the property works. During the interview, Wesley used a four-line algorithm to solve 29 × 37. This seems to indicate that he understands how to apply the distributive property as he has identified that there are four elements to the multiplication.

In contrast to the explanations of students who were unable to explain the fifth question in terms of partitioning, the following sample from Student Callum is presented as an example of a satisfactory explanation. Student Callum also displayed some flexibility in his thinking by solving the first example with non-standard partitioning as shown in the second part of the sample.

10. Which of the following will give you the same answer as **89 x 3**? Write 'Yes' or 'No' underneath each one.

- Give a reason for *two* of your choices of a 'Yes' or 'No' answers.

83 x 9	(80x3) + (9x3)	(80X9) + (3x9)	(90x3) - (1x3)
No	Yes	No	Yes

(80×3)+(9×3) yes because you partison. 83×9 No because you are changing the numbers.

b) If you did do it in your head, explain how you got the answer.

12×6 + 5×6 = 102

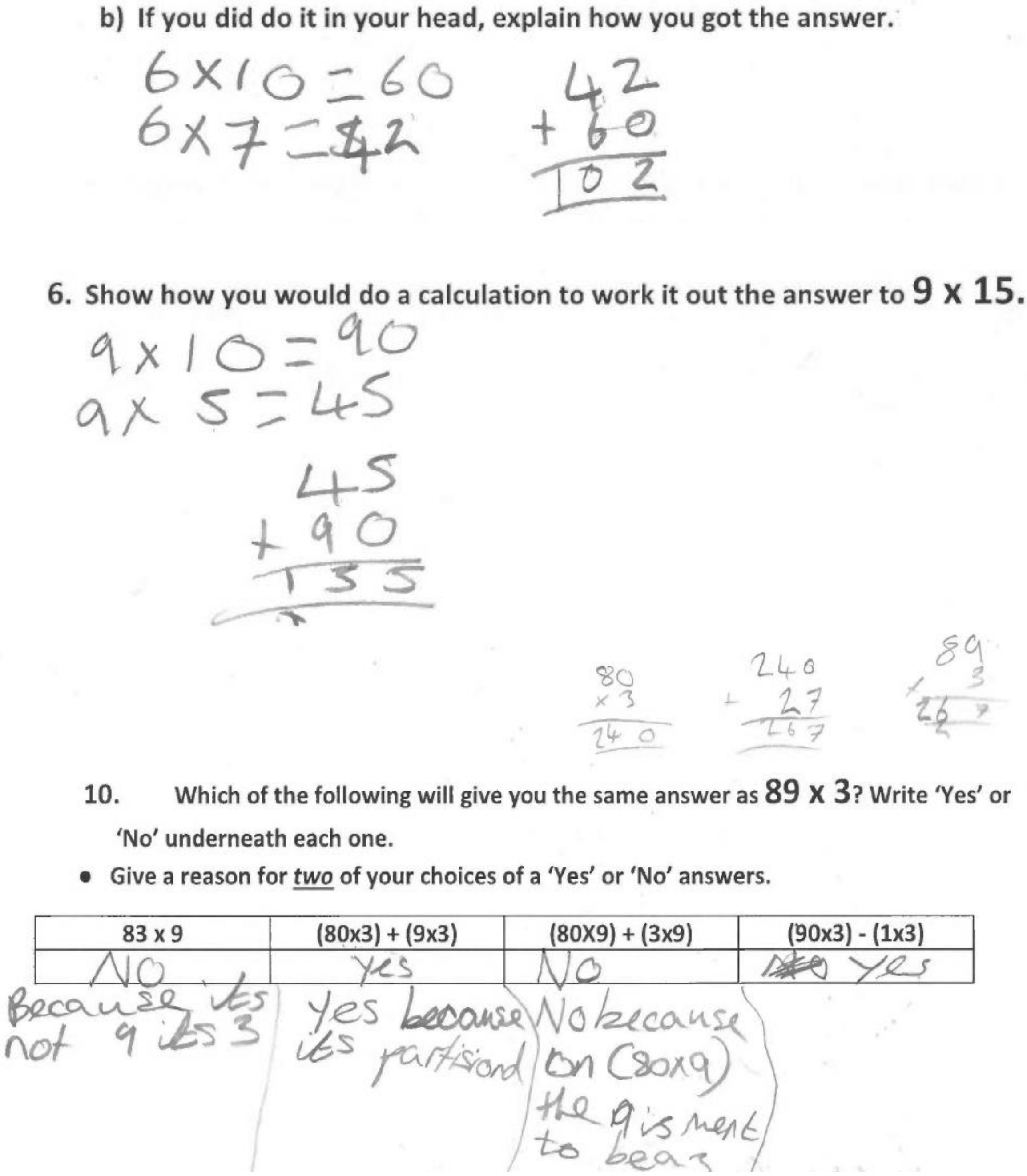
6. Show how you would do a calculation to work it out the answer to **9 x 15.**

9×10 = 90 + 45 = 135

5×9 = 45

Figure 3. Sample from student Callum.

Another point of interest is how some students who used place value partitioning for both the questions about 6 × 17 and 9 × 15, and who also identified the correct choices for the question about the distributive property, still found it necessary to calculate the answer for (80 × 3) + (9 × 3), despite saying that it would give the same answer as 89 × 3. There seem to be a couple of possible explanations for this, as exemplified by the sample from Student Izzy (Figure 4). First, it could be that students who did that did so as a matter of course or habit, in that they accept that they need to use an algorithm for such calculations irrespective of whether they actually need to do so or not. Second, it may be that their understanding is not sufficiently robust – perhaps they need to calculate with an algorithm to prove to themselves that the partition actually works.


b) If you did do it in your head, explain how you got the answer.

6x10=60
6x7=42

42
+60
102

6. Show how you would do a calculation to work it out the answer to 9 x 15.

9x10=90
9x5=45

45
+90
135

80
x3
240

240
+27
267

89
x3
267

10. Which of the following will give you the same answer as 89 x 3? Write 'Yes' or 'No' underneath each one.

- Give a reason for *two* of your choices of a 'Yes' or 'No' answers.

83 x 9	(80x3) + (9x3)	(80X9) + (3x9)	(90x3) - (1x3)
NO	Yes	NO	~~No~~ Yes

Because its not 9 its 3

Yes because its partisiond

No because on (80x9) the 9 is ment to bea 3

Figure 4. Samples from student Izzy.

It is worth considering the work of a student who, in general, did not respond well to the five MTQ questions, as shown in Table 1. Student Francis made an incorrect calculation for the question about 6 x 17, *did* use an algorithm to correctly work out the answer for 9 x 15, but was unable to identify the correct choices for the question about the distributive property. During the interview, the following exchange occurred [with notes by the interviewer]:

> I: [Francis said that (80 × 3) + (9 × 3) would give the same answer as 89 × 3 but when explaining how it worked, he had to actually work out the two parts and took prompting to arrive at the correct answers for each part. He wrote it as a vertical addition]. "Do you need to work it out to prove it?"
>
> F: "Yes".
>
> I: [He was shown the example (50 × 6) + (3 × 6)] "What would it be the same as?"
>
> F: "Fifty-three times . . . twelve . . . no . . . times six".
>
> I: [Francis was shown (70 × 4) + (6 × 4)] "Do you need to work them out or are you happy that they will give the same answer as 76 × 4"?
>
> F: "Yes".

There is a considerable degree of uncertainty about the answers offered by Francis. While he made a computational error in the 6 × 17 question, he did use place value partitioning for that question and also the question about 9 × 15. However, he was unable to apply that knowledge to the questions about the distributive property, both in the MTQ and the interview. This suggests that he has developed partial understanding of the mathematics involved but has certainly not been able to connect the idea of place value partitioning to the explanation of how and why the distributive property is applied.

Conclusion

On the basis of the analysis of data from the MTQ and the interview, it would seem that there are several levels of understanding shown by students in the sample. These could broadly be described as follows:

- Students who understand place value partitioning, use it when calculating answers to multiplication examples (either mentally or written), understand the distributive property, and explain the latter in terms of partitioning.
- Students who understand place value partitioning, use it when calculating answers to multiplication examples (either mentally or written), correctly identify examples of the distributive property, but do not trust the partitioning and need to calculate a product as proof.
- Students who understand place value partitioning, use it when calculating answers to multiplication examples (either mentally or written), but do not apply it to explain how and why the distributive property works.
- Students who demonstrate a partial understanding of aspects of the above three characteristics but whose understanding is incomplete and not consistently applied.

Hence, we believe that there are some clear implications for teaching. First, teaching should focus on establishing the link between standard place value partitioning and the distributive property and this could be successfully developed through the use of the multiplicative array. Second, the written algorithm for multiplication needs to be developed from the grid method, which is based on standard place value partitioning and the array. Third, the specific mathematical language related to 'partitioning' should be incorporated when developing students' understanding of the distributive property. As well, we think it is important for teachers to encourage students to trust the fact that ideas like the distributive property will work when applied correctly. Helping students to make such connections should situate them better when learning how the distributive property informs aspects of algebraic reasoning.

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Exploring Undergraduate Mathematics Students' Difficulties with the Proof of Subgroup's Closure under Operation

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This study aims to explore undergraduate mathematics students' difficulties in their initial encounter with the subgroup test, and in particular in the proof of closure under operation. Subgroup test is one of the first results in an introductory course of Group Theory where students need to cope with the characteristic, for novice students, high level of abstraction. For the purposes of this study, the researcher has used the Commognitive Theoretical Framework. Analysis suggests that students' difficulties are due to various reasons, including the formalism of the definition of group, incomplete metaphors from other mathematical discourses, confusion of the involved structures, and the proof process per se.

Subgroup Test is, more often than not, one of the first routines undergraduate mathematics students need to cope with in their first engagement with Group Theory, where they need to prove its three conditions, namely, non-emptiness, closure under operation and closure under inverses. Often, though, this apparently simple task proves to be an arduous endeavour, partly due to the abstract nature of Group Theory (Hazzan, 2001). A typical first Group Theory course requires a deep understanding of the abstract concepts involved, namely group, subgroup, coset, quotient groups etc. In addition, the deductive way of teaching Group Theory is unfamiliar to students and, in order to achieve mastery of the subject, it is necessary to "think selectively about its entities, paying attention to those aspects consistent with the context and ignoring those that are irrelevant" (Barbeau, 1995, p. 140). Gueudet (2008) suggests that many pedagogical issues emerging in undergraduate Mathematics Education are based on the transition from secondary to tertiary Mathematics, which can still occur in their second year. In fact, student difficulties in Abstract Algebra may be an indication of problematic transition, mainly due to the particular nature of this module (Ioannou, 2012). The aim of this study is to investigate the undergraduate mathematics students' difficulties with the concept of subgroup, and in particular in proving closure under operation, during their first encounter with Group Theory. For the purposes of this study, there has been used the Commognitive Theoretical Framework (CTF) (Sfard, 2008), due to its great potential to investigate mathematical learning in both object level and meta-discursive level (Presmeg, 2016).

Theoretical Framework

CTF is a coherent and rigorous theory for thinking about thinking, grounded in classical Discourse Analysis. It involves a number of different constructs such as metaphor, thinking, communication, and commognition, as a result of the link between interpersonal communication and cognitive processes (Sfard, 2008). In mathematical discourse, objects are discursive constructs and form part of the discourse. Mathematics is an *autopoietic system* of discourse, namely "a system that contains the objects of talk along with the talk itself and that grows incessantly 'from inside' when new objects are added one after another" (Sfard, 2008, p. 129). Moreover, CTF defines discursive characteristics of mathematics as the *word use*, *visual mediators*, *narratives*, and *routines* with their associated metarules, namely the *how* and the *when* of the routine. In addition, it involves

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 333–340). Melbourne: MERGA.

the various objects of mathematical discourse *such as the signifiers, realisation trees, realisations, primary objects and discursive objects. It also involves the constructs of object-level and metadiscursive level (or metalevel) rules.* Thinking "is an individualized version of (interpersonal) communicating" (Sfard, 2008, p. 81). Contrary to the acquisitionist approaches, participationists' ontological tenets propose to consider thinking as an act (not necessarily interpersonal) of communication, rather than a step primary to communication (Nardi, Ryve, Stadler, & Viirman, 2014).

Mathematical discourse involves certain objects of different categories and characteristics. *Primary object* (p-object) is defined as "any perceptually accessible entity existing independently of human discourses, and this includes the things we can see and touch (material objects, pictures) as well as those that can only be heard (sounds)" (Sfard, 2008, p. 169). *Simple discursive objects* (simple d-objects) "arise in the process of proper naming (baptizing): assigning a noun or other noun-like symbolic artefact to a specific primary object. In this process, a pair <noun or pronoun, specific primary object> is created. The first element of the pair, the signifier, can now be used in communication about the other object in the pair, which counts as the signifier's only realization. *Compound discursive objects* (d-objects) arise by "according a noun or pronoun to extant objects, either discursive or primary." In the context of this study, groups are an example of compound d-objects. The (discursive) object signified by S in a given discourse is defined as "the realization tree of S within this discourse." (Sfard, 2008, p. 166). The *realization tree* is a "hierarchically organized set of all the realizations of the given signifier, together with the realizations of these realizations, as well as the realizations of these latter realizations and so forth" (Sfard, 2008, p. 300).

Sfard (2008) describes two distinct categories of learning, namely the *object-level* and the *metalevel discourse learning*. "Object-level learning […] expresses itself in the expansion of the existing discourse attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives; this learning, therefore results in endogenous expansion of the discourse" (p. 253). In addition, "metalevel learning, which involves changes in metarules of the discourse […] is usually related to exogenous change in discourse. This change means that some familiar tasks, such as, say, defining a word or identifying geometric figures, will now be done in a different, unfamiliar way and that certain familiar words will change their uses" (Sfard, 2008, p. 254).

CTF has proved particularly appropriate for the purposes of this study, since, as Presmeg (2016) suggests, it is a theoretical framework of unrealised potential, designed to consider not only issues of teaching and learning of mathematics per se, but to investigate "the entire fabric of human development and what it means to be human" (p. 423).

Literature Review

Research in the learning of Group Theory is relatively scarce compared to other university mathematics fields, such as Calculus, Linear Algebra or Analysis. The first reports on the learning of Group Theory appeared in the early 1990's. Several studies, following mostly a constructivist approach, and within the Piagetian tradition of studying the cognitive processes, examined students' cognitive development and analysed the emerging difficulties in the process of learning certain group-theoretic concepts.

The construction of the newly introduced concept of group is often an arduous task for novice students and causes serious difficulties in the transition from the informal secondary education mathematics to the formalism of undergraduate mathematics (Nardi, 2000). Students' difficulty with the construction of the Group Theory concepts is partly grounded on historical and epistemological factors: “the problems from which these concepts arose in an essential manner are not accessible to students who are beginning to study (expected to understand) the concepts today” (Robert & Schwarzenberger, 1991, p. 129). Nowadays, the presentation of the fundamental concepts of Group Theory, namely group, subgroup, coset, quotient group, etc. is “historically decontextualized” (Nardi, 2000, p. 169), since historically the fundamental concepts of Group Theory were permutation and symmetry (Carspecken, 1996). Moreover, this chasm of ontological and historical development proves to be of significant importance in the metalevel development of the group-theoretic discourse for novice students.

From a more participationist perspective, CTF can prove an appropriate and valuable tool in our understanding of the learning of Group Theory due both to the ontological characteristics of Group Theory, as well as the epistemological tenets of CTF (Ioannou, 2012). Group Theory can be considered as a metalevel development of the theory of permutations and symmetries, and CTF allows us to consider the historical and ontological development of a rather “historically decontextualized” modern presentation of this Theory.

Research suggests that students’ understanding of the concept of group proves often primitive at the beginning, predominantly based on their conception of a set. An important step in the development of the understanding of the concept of group is when the student “singles out the binary operation and focuses on its function aspect” (Dubinsky, Dautermann, Leron, & Zazkis, 1994, p. 292). Students often have the tendency to consider group as a “special set”, ignoring the role of binary operation. Iannone and Nardi (2002) suggest that this conceptualisation of group has two implications: the students’ occasional disregard for checking associativity and their neglect of the inner structure of a group. These last conclusions were based on students’ encounter with groups presented in the form of group tables. In fact, students when using group tables adopt various methods for reducing the level of abstraction, by retreating to familiar mathematical structure, by using canonical procedure, and by adopting a local perspective (Hazzan, 2001).

An often-occurring confusion amongst novice students is related to the order of the group G and the order of its element g. This is partly based on students’ inexperience, their problematic perception of the symbolisation used and of the group operation. The use of semantic abbreviations and symbolisation can be particularly problematic at the beginning of their study. Nardi (2000) suggests that there are both linguistic and conceptual interpretations of students’ difficulty with the notion of order of an element of the group. The role of symbolisation is particularly important in the learning of Group Theory, and problematic conception of the symbols used probably causes confusion in other instances.

Introduction of Abelian group is also an important milestone in the learning of Group Theory. Ioannou (2016a) suggests that students’ engagement with this concept is generally satisfactory. Yet students face certain difficulties, due to the concept of commutativity in the context of the newly introduced discourse, and in the application of the relevant metarules, with particular focus on the how of the routine. In addition, according to a preliminary investigation of students’ application of metarules, it has been identified that in the first steps of Group Theory learning, disengagement with the ‘how’ of metarules occurs quite often (Ioannou, 2016b). Finally, the characteristics of student responses towards learning Group Theory vary, in accordance to their emotions, beliefs and attitudes (Ioannou, 2016c).

A distinctive characteristic of university mathematics is the production of rigorous and consistent proofs. Proof production is far from a straightforward task to analyse and identify the difficulties students face. These difficulties have been extensively investigated for various levels of student expertise. Weber (2001) categorises student difficulties with proofs into two classes: the first is related to the students' difficulty to have an accurate and clear conception of what comprises a mathematical proof, and the second is related to students' difficulty to understand a mathematical proposition or a concept and therefore systematically misuse it.

Methodology

This study is part of a larger research project, which conducted a close examination of Year 2 mathematics students' conceptual difficulties and the emerging learning and communicational aspects in their first encounter with Group Theory. The module was taught in a research-intensive mathematics department in the United Kingdom, in the spring semester of a recent academic year.

The Abstract Algebra (Group Theory and Ring Theory) module was mandatory for Year 2 mathematics undergraduate students, and a total of 78 students attended it. The module was spread over 10 weeks, with 20 one-hour lectures and three cycles of seminars in Weeks 3, 6, and 10 of the semester. The role of the seminars was mainly to support the students with their coursework. There were 4 seminar groups, and the sessions were each facilitated by a seminar leader, a full-time faculty member of the school, and a seminar assistant, who was a doctorate student in the mathematics department. All members of the teaching team were pure mathematicians.

The lectures consisted largely of exposition by the lecturer, a very experienced pure mathematician, and there was not much interaction between the lecturer and the students. During the lecture, he wrote self-contained notes on the blackboard, while commenting orally at the same time. Usually, he wrote on the blackboard without looking at his handwritten notes. In the seminars, the students were supposed to work on problem sheets, which were usually distributed to the students a week before the seminars. The students had the opportunity to ask the seminar leaders and assistants about anything they had a problem with and to receive help. The module assessment was predominantly exam-based (80%). In addition, the students had to hand in a threefold piece of coursework (20%) by the end of the semester.

The gathered data included the following: Lecture observation field notes, lecture notes (notes of the lecturer as given on the blackboard), audio-recordings of the 20 lectures, audio-recordings of the 21 seminars, 39 student interviews (13 volunteers who gave three interviews each), 15 staff members' interviews (five members of staff, namely the lecturer, two seminar leaders and two seminar assistants, who gave three interviews each), student coursework, markers' comments on student coursework, and student examination scripts. For the purposes of this study, the collected data of the 13 volunteers has been scrutinised. Finally, all emerging ethical issues during the data collection and analysis, namely, issues of power, equal opportunities, right to withdraw, procedures of complain, confidentiality, anonymity, participant consent, sensitive issues in interviews, etc., have been addressed accordingly.

Data Analysis

The analysis that follows, aims to explore undergraduate students' difficulties with the proof of subgroup's closure under operation. The application of this condition for a set to be a subgroup was problematic in six (6/13) students' coursework solutions, namely, Dorabella, Leonora, Manrico, Musetta, Francesca, and Tamino's (all pseudonyms), in the solution of the following mathematical tasks:

1. Using the usual test for being a subgroup, prove that for any $n \in \mathbb{N}$, the sets $\{g \in GL\,(n, \mathbb{R}) \colon Det(g) = 1\}$ and $\{g \in GL(n, \mathbb{R}) \colon gg^T = I_n\}$ are subgroups of $GL\,(n, \mathbb{R})$.
2. Suppose X is a non-empty set and $G \leq Sym\,(X)$. Let $a \in X$ and $H = \{g \in G \colon g(a) = a\}$. Prove that H is a subgroup of G.
3. Suppose $(G, \cdot)$ is a group and H, K are subgroups of G. Show that $H \cap K$ is a subgroup of G.

The first difficulty was related to an incomplete object-level learning regarding the distinction between the element of a group and a subgroup. This possibly indicates unresolved problems regarding the definition of the group and its axioms and, moreover, the properties of the elements of a group. There are indications of an incomplete endorsement of the notation used in the exercises, for instance the subgroup $Sym(X)$ and the set X. The unfavourable effect of the problematic metaphors from Linear Algebra regarding the inverse and the transpose of the matrices, and the impact they have in the application of the routine and the solution of the exercises are obvious. For instance, in her solution of Task 1, as it can be seen in Figure 1, Leonora has applied the metalevel rules accurately, since she has applied the test appropriately, showing that she has grasped the applicability and closure conditions of this particular routine as well as its course of action.

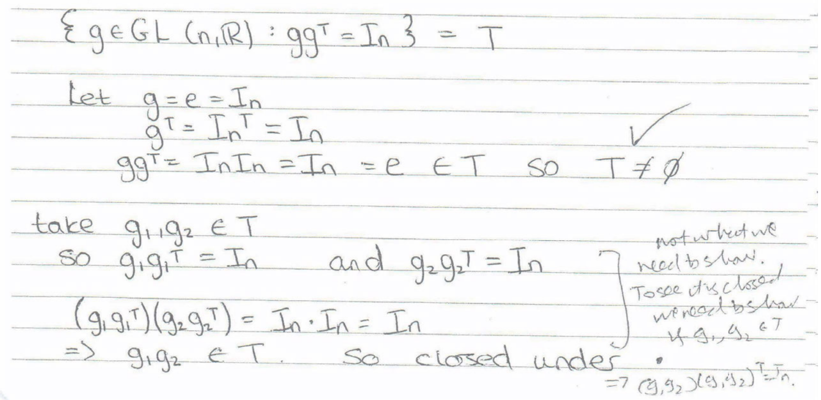


Figure 1. Leonora's solution of Task 1.

She presents her solution in a comprehensive way, using verbal explanations on some occasions. The only inaccuracy occurred in the use of symbolisation in the second example, while proving closure under operation. Instead of proving that $(g_1 g_2)(g_1 g_2)^T = I_n$, she proved that $(g_1 g_1^T)(g_2 g_2^T) = I_n$. This possibly suggests incomplete object-level learning of transposition as well as of the definition of group and the group axioms in particular. In addition, this probably suggests that she may not have realised that $g_1 g_2$ is another element of the group and not a subgroup, and that it has to be considered as such.

Furthermore, in Leonora's solution of Task 2, as seen in Figure 2, there are also indications of an incomplete object-level learning. The first one is revealed in the use of notation, which may have deeper roots relating to the essence of understanding of the elements of the group and their properties. Additionally, she finds it difficult to define the different operations in the different structures. For example, she writes the expression $g(a_1)g(a_2)$, which uses elements of the set X but under operation, which does not operate in X. She has a vague view of what is H and what is X, i.e. that X is a non-empty set and that H is a subgroup of G with a certain condition. At some point, she also writes $a \in H$, which is not true since a is an element of X.

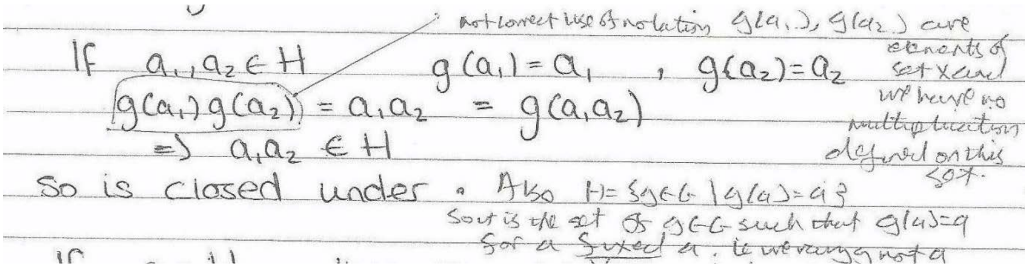


Figure 2. Leonora’s solution of Task 2.

Her incomplete object-level learning regarding the concepts involved in this exercise is clearly expressed in the following interview excerpt.

> I found quite hard, because... I got a bit confused with this um… *Sym (X)* and stuff, but – so I don’t - I started it but then I weren’t sure, whether I was doing it right, so I kind of have stopped, and I’m gonna go ask for help. To like – because I – I don’t like, if I’m doing something and I’m not sure if it’s right, I don’t like to carry on because I don’t want to do it all wrong.

Another example of problematic proof of closure under operation occurred in Manrico’s solution of Task 1. In the second example for closure under operation he does not prove what he is supposed to prove. As the solution in Figure 3 suggests, he rather concludes that $g_1 g_2 \in GL(\mathrm{n}, \mathbb{R})$, instead of proving $(gh)^T = (gh)^{-1}$.

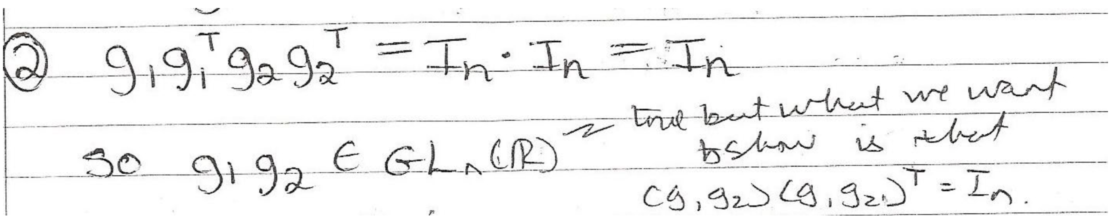
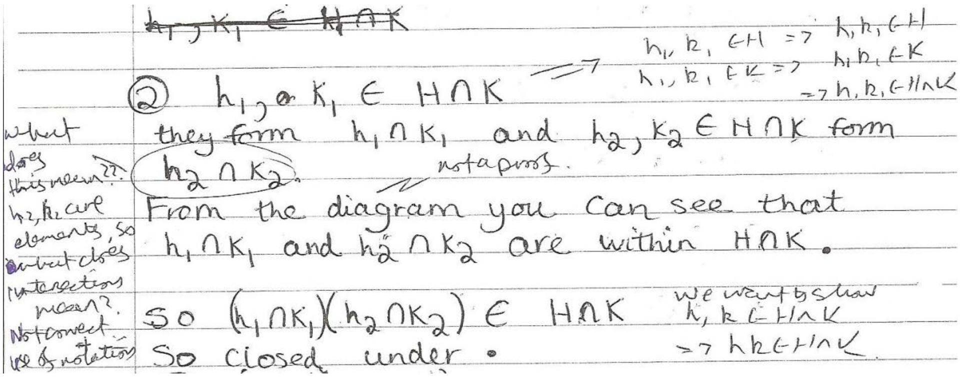


Figure 3. Manrico’s solution of Task 1

In contrast to Leonora’s case, the above excerpt reveals problematic application of metarules, since the inaccuracy is not a result of incomplete object-level learning. The algebraic manipulations are correct in general, yet inappropriate for the context of this task. I would suggest that this inaccuracy is grounded on incomplete metalevel learning, since it is a result of inaccurate consideration of the applicability conditions of the particular routine as well as the “course of action”.

Moreover, regarding Task 3, Manrico’s solution also demonstrates further inaccuracies, as shown in Figure 4. The first relates to the expression $h_1 \cap k_1$ and $h_2 \cap k_2$. There are indications of incomplete object-level learning of the d-object of subgroup as well as the elements of the subgroup.



~~$h_1, k_1 \in H \cap K$~~

② $h_1, k_1 \in H \cap K$ ⇒ $h_1 k_1 \in H$ ⇒ $h_1 k_1 \in H$
$h_1 k_1 \in K$ ⇒ $h_1 k_1 \in K$
⇒ $h_1 k_1 \in H \cap K$

they ~~form~~ $h_1 \cap k_1$ and $h_2, k_2 \in H \cap K$ ~~form~~ $h_2 \cap k_2$. not a proof.

From the diagram you can see that $h_1 \cap k_1$ and $h_2 \cap k_2$ are within $H \cap K$.

So $(h_1 \cap k_1)(h_2 \cap k_2) \in H \cap K$ we want to show $h, k \in H \cap K$ ⇒ $hk \in H \cap K$

So closed under ·

what does this mean?? h_2, k_2 are elements, so what does intersection mean? Not correct use of notation

Figure 4. Manrico's solution of Task 3.

In addition, there are problems with the application of metarules (the well-defined and established, among the mathematical community norms of proving), regarding the use of visual mediators. As Figure 5 shows Manrico has based his proof entirely on visual mediators (in this case Venn diagrams, used as metaphor from Set Theory), a tactic that is not acceptable by the markers.

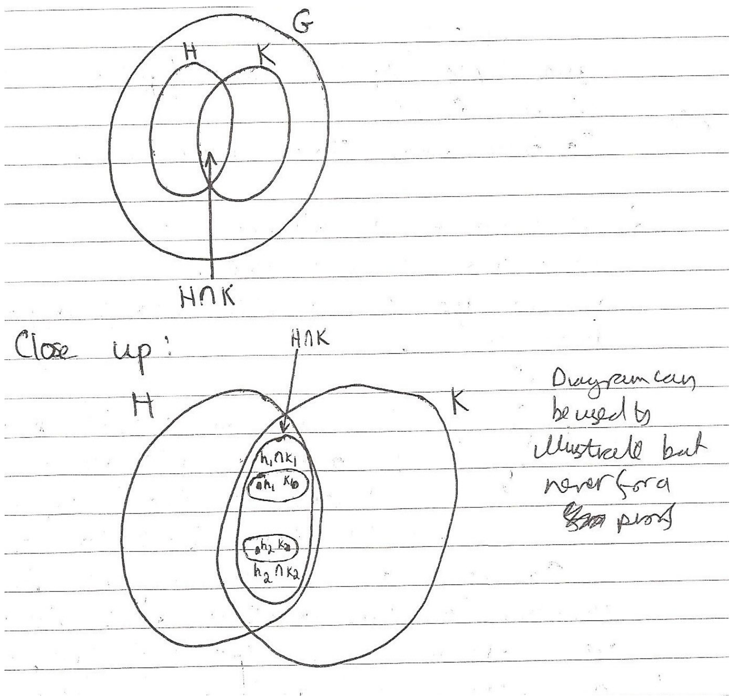


Figure 5. Manrico's solution of Task 3.

Regarding the use of visual mediators as core part of the solutions, there are indications that such use is often linked with lack of confidence or certainty about the quality of the algebraic reasoning. In three students' cases, namely Manrico, Calaf, and Tamino, they make use of visual means of representation, such as Venn diagrams. The use of such visual mediators is not supportive; instead when such approach to solution is applied, these students tend to base the core of their solution on them.

Conclusion

This study's aim was to explore undergraduate mathematics students' difficulties in proving closure under operation, in their initial encounter with the subgroup test. Data analysis suggests that students' difficulties are due to four general reasons. In agreement with Nardi (2000), the formalism of the definition of group requires "decoding" by novice students, due also to the abstract nature of Group Theory (in agreement with Hazzan, 2001). Another difficulty is caused by the problematic metaphors from other mathematical discourses, such as Set Theory and Linear Algebra. Similar to Dubinsky et al. (1994) and Iannone and Nardi (2002), this study highlights students' difficulty to distinguish the different characteristics and requirements that the involved structures have, namely sets, groups, subgroups and their elements. Finally, the last student difficulty that this study reports is related to the process of proof per se, due to incomplete metalevel learning of the relevant metarules that govern the applicability and closure conditions of the subgroup test.

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Is Mathematics Education Worthy? From Mathematics for Critical Citizenship to Productivity Growth

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Public discourse concerning STEM has an increasing influence on mathematics education, yet the exact role that mathematics plays in STEM is hard to define. I compare STEM to numeracy to investigate how mathematics is repositioned in these two discourses. Each is analysed in terms of rhetorics that argue for the worth of mathematics education. While numeracy viewed mathematics as worthy for critical citizenship, STEM argues that mathematics has worth due to its support of the innovation required for productivity growth. Analysis of the rhetorics is argued to support the mathematics education research community response to changes in public and policy discourses regarding mathematics.

The way that mathematics education is discussed in public discourse has shifted significantly in recent years. In the Australian context, the term numeracy is falling out of favour while STEM (science, technology, engineering, and mathematics) gains prominence in government publications (Australian Council for Educational Research [ACER] & Stephens, 2009; Department of Education and Training, 2016). Many authors argue that the pedagogical implications of STEM are not well defined, and the links between the STEM disciplines are not self-evident (van Driel & Clarke, 2016). Rather than trying to define STEM, or explicate the links between STEM disciplines, I take a different approach. Numeracy and STEM are posited to be rhetorical discourses in Australia. By tracking the shift from a rhetoric of numeracy to a rhetoric of STEM, it is hoped that the way this shift repositions mathematics education can be investigated, which may assist the field of mathematics education research when considering responses to STEM rhetoric.

The use of the term "rhetoric" in this paper comes from the work of Sutton-Smith (1997), who defines rhetoric as a "persuasive discourse, or an implicit narrative… [designed] to persuade others of the veracity and worthwhileness of their beliefs" (p. 8). In the context of mathematics education, rhetorics of mathematics education can be seen as discourses that argue for and provide validity to the teaching of mathematics. Sutton-Smith (1997) described seven rhetorics applied to "play" in different times and places. Within each rhetoric, the nature of play was different, and his work enabled shifts in rhetoric to be identified. Sutton-Smith (1997) was able to identify each rhetoric by identifying different ways that discourses presented play as being worthy. I begin my analysis by identifying different arguments for the worth and validity of mathematics education present in mathematics education discourse. Two research questions guide this study: (1) How do the numeracy and STEM discourses argue for the worth of mathematics education? and (2) Has there been a shift in the way that the worthiness of mathematics education is articulated as discourse shifts from a numeracy discourse to a STEM discourse?

Government texts are used as examples of numeracy and STEM discourse. Analysis involves locating arguments for the worth of mathematics education within government texts. A brief summary of the development of STEM within economic discourse is presented. This then informs analysis of policy documents, in which I seek to identify the rhetorics of which mathematics education numeracy and STEM are representative.

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Literature Review

Sutton-Smith (1997) argued that rhetorics can be hard to identify because they are often expressed implicitly. The form of rhetorical analysis that he employed built on the work of literary theorist, Kenneth Burke (1969). Burke argued that rhetoric attempts to bring about change in people. Rhetorical analysis, as a form of discourse analysis, begins by identifying elements of a text which are persuasive. These persuasive elements can then be analysed in order to identify how the worth of the subject of the discourse is being articulated. Mathematics education rhetorics in text, for example, will try to persuade readers to value mathematics education, and will express implicit beliefs regarding why mathematics education is a worthy endeavour. Direct articulations of these arguments are rare, but Skovsmose's (1994) framework for a philosophy of mathematics education directly articulates arguments for the worth of mathematics education. Reviewing Skovsmose's work, six distinct mathematics education rhetorics can be identified, and are summarised in Table 1. Of the six rhetorics identified, only four were present after the numeracy and STEM texts were analysed. Hence, detailed description of last two rhetorics in Table 1 – which are not part of the analysis– has been omitted due to space constraints.

Table 1
Summary of Mathematics Education Rhetorics and their Positioning of Mathematics

Rhetoric	Worth of mathematics education	Positioning of mathematics with other subjects
Mathematics for critical citizenship	Essential to democratic participation	Positioned with literacy
Mathematics for human capital	Provides skills required for participation in the workforce	
Mathematics for technological progress	Underpins capacity for technological development	Positioned with science and technology
Mathematics for global competition	Provides advantage to nations that do it well	
Mathematics as a human right	Component of education for all	Positioned within the broad context of being educated
Mathematics for social equity	Provides access to the knowledge and skills of the privileged	

Skovsmose (1994) argued that some conceptions of education view mathematics as essential for critical citizenship (mathematics for critical citizenship). Without mathematics, people will not be able to participate in democracy effectively; hence, mathematics education is justified in terms of empowering citizens to be active and critical members of their societies. Within this discourse, mathematics is positioned alongside literacy as a basic prerequisite to civic competence. Skovsmose's (1994) second argument for mathematics education relates to human capital (mathematics for human capital). The economy requires workers who have a level of competence in mathematics. Hence, mathematics education is worthy because it fulfils the human capital needs of a society. Skovsmose's next argument connects mathematics and technology (mathematics for technological progress). In this discourse, mathematics is viewed as a foundational element of scientific and technological development. Mathematics is a means to an end; scientific and technological development are the end, so mathematics education is worthy because without it, science and technology cannot progress. Global competition (mathematics for global competition) has also been used to argue for the worthiness of mathematics education (Skovsmose, 1994). In this discourse, a nation's ability to educate its populace in mathematics is seen as providing a "competitive edge" globally. The identification of mathematics education rhetorics (Skovsmose, 1994) does not provide a comprehensive list of all mathematics education rhetorics. Given the constraints of this paper, the six rhetorics identified provide sufficient framing to enable analysis of numeracy and STEM discourses.

Economic Arguments for STEM and Mathematics Education

As STEM is the more current rhetoric that frames education policy that is considered in this study, a brief survey of the economics research literature that relates to STEM is presented. Quiggin (1999) outlines three economic models that have influenced education policy in Australia. Of these, Human Capital Theory (Becker, 2009) has become increasingly influential. Human Capital Theory argues for the economic value of education (as opposed to cultural value, for example), which is evident in the increased incomes of those with higher levels of education (Becker, 2009). Higher rates of pay are argued to increase an economy's productivity. More wealth can be generated per worker in a productive economy, which contributes to society's wellbeing (Conway, 2013). This line of argument has led economists to try to identify which industries have the most potential for productivity growth. For instance, Peri, Shih, and Sparber (2015), looked at how wage growth in major cities in the U.S. was affected by the proportion of the workforce that worked in STEM industries. They found that, "scientists and engineers are responsible for 50% of long-run U.S. productivity growth" (p. S226). This more recent trend in economic analyses that are concerned with addressing productivity growth has been arguing that disciplines that foster technological progress and research and development are more valuable than others (Office of the Chief Economist [OCE], 2015). STEM workers are the best positioned to drive innovation, and "the role of innovation in sustained economic growth cannot be overemphasised" (OCE, 2015, p. 138). This technological progress will enable productivity growth and lead to a better society with improved living conditions (Conway, 2013). Unlike early formulations of Human Capital Theory, recent economic discourse does not argue for the economic benefit of "education", but for the benefit of education only in areas deemed to have the greatest capacity to foster productivity growth – STEM. Mathematics education policy is likely to be affected by this economic discourse, which elevates mathematics education to a position of prominence within education but also makes mathematics education subservient to innovation and productivity growth.

Method

A range of documents have been analysed in order to identify which of the six rhetorics identified from Skovsmose's work are present in discourses of numeracy and STEM in the Australian context. The rhetorical analysis used involves identifying passages of the analysed texts that try to persuade the reader that mathematics education is worthy (Sutton-Smith, 1997). Skovsmose's (1994) work also highlights that the way in which mathematics is positioned in relation to other subjects varies between some rhetorics. Table 1 summarises how each rhetoric argues for the worth of mathematics education and how some rhetorics position mathematics with other subjects. The way that mathematics is positioned in relation to other subjects within numeracy and STEM discourses was also used to argue for the presence of rhetorics that can be identified in this way.

Selection of Documents

I do not seek to provide a comprehensive review of all policy documents relating to numeracy and STEM, as the research questions that guide this paper can be sufficiently addressed without an exhaustive review. Government publications from the state government of Victoria and the Australian Federal Government have been analysed. One document from the Organisation for Economic Co-Operation and Development (OECD) has also been analysed to provide an international perspective. At least one numeracy and one STEM document whose intended audience are teachers has also been included. I view numeracy and STEM to be discourses that are primarily expressed in these types of text.

Table 2
Summary of Documents Included in the Study

Author	Year	Title	Discourse
Council of Australian Governments (COAG)	2008	*National Numeracy Review Report**	Numeracy
ACER, & Stephens	2009	*Numeracy in Practice: Teaching, Learning and Using Mathematics**	Numeracy
Office of the Chief Scientist	2013	*Science, Technology, Engineering and Mathematics in the National Interest: A Strategic Approach**	STEM
Department of the Prime Minister and Cabinet	2014	*Increasing the Focus on Science, Technology, Engineering and Mathematics (STEM), and Innovation in Schools*	STEM
OECD	2014	*OECD Science, Technology and Industry Outlook 2014*	STEM
Department of Education and Training	2016	*STEM in the Education State**	STEM
Office of the Chief Scientist	2016	*Australia's STEM Workforce**	STEM

Analysis

The first level of analysis involves locating arguments for the worth and position of mathematics education in the selected documents. Rhetoric is evident when a discourse seeks to persuade (Burke, 1969). Excerpts were located that seek to persuade the reader that mathematics education has value in each document. Each of these excerpts was then analysed in relation to the six rhetorics identified in Skovsmose's work. A statement such as, "the available evidence points to better educational and labour market outcomes generally for those with good levels of numeracy" (COAG, 2008, p. 1) is an example of an excerpt that was taken to be rhetorical as it states a benefit of mathematics education, in terms of the labour market. The reference to the labour market signifies the presence of the *mathematics for human capital* rhetoric in this example. A selection of these excerpts from numeracy texts are contrasted with excerpts from STEM texts to assess if the rhetorics present in both discourses are different or similar and thus if there is evidence that the worth of mathematics education is articulated differently in each discourse.

The next level of analysis assesses the frequency with which key terms are used in each discourse. Documents that were under 20 pages long were not included in this word frequency analysis. Both numeracy documents and three of the STEM documents (those with an asterisk in Table 2) were long enough and available in pdf format, so their word frequency could be assessed. *NVivo* software was used to rank the frequency of terms used in each of these documents. The key terms were chosen after an initial analysis was undertaken to determine potential rhetorics in each discourse. Frequency of terms could then be used as corroborating evidence that particular rhetorics characterise each discourse.

Results

Evidence of Rhetorics in Numeracy Discourse

Within each document, arguments for the worth of mathematics education could be located. In a document designed to discuss numeracy with the teacher community, the worth of mathematics education is explained as follows: “Numeracy and literacy remain key domains of learning which are essential for success at school… and ensure children are well prepared for future economic and social prosperity” (ACER & Stephens, 2009, p. 3). Within this discourse, numeracy is positioned alongside literacy as being worthy due to its impact on students’ capacity for effective economic and social participation. Social participation is also prominent in COAG’s Human Capital Working Group’s 2008 report on numeracy: “mathematics curricula and pedagogy… need to provide the pertinent mathematical knowledge required of the citizens of today and tomorrow, a knowledge that will result in an ability to choose and use the mathematics learned to meet personal and social goals” (COAG, 2008, p. 3). Mathematics has a benefit for society because, “mathematical literacy for developing human capital has at its heart the economic argument for numeracy education, that is, the needs of society are changing and in order for the country to maintain its lifestyle and economic well-being we need better and more mathematically educated adults and school leavers” (COAG, 2008, p. 4). Schools must teach mathematics because, “if numeracy is about using mathematics effectively to meet the general demands of life at school, at home, in paid work and for participation in community and civic life, then it is clearly the role of the school curriculum – both documented/planned and implemented/enacted – to enable young people to learn to use mathematics to meet these demands” (COAG, 2008, p. 8). The arguments for the worth of mathematics located in these documents position mathematics alongside literacy (reading and writing) as fundamental skills. Citizenship and economic participation are also prominent in these documents. This suggests that two rhetorics are evident in numeracy discourse: *mathematics for critical citizenship* and *mathematics for human capital*.

Evidence of Rhetorics in STEM Discourse

Arguments that specifically focused on the worth of mathematics education were harder to locate in STEM discourse. This may be due to the way that mathematics is positioned in STEM discourse. In some documents, there is evidence that STEM rhetoric positions the four individual STEM disciplines in service to “innovation”. The OECD (2014) argued that, “the skills associated with innovation include specialised knowledge, general problem-solving and thinking skills, creativity, and social and behavioural skills, including teamwork … [this] goes beyond the traditional focus on STEM disciplines, even though these disciplines occupy a prominent position in innovation policies” (p. 237). Policy decisions, such as increasing participation in STEM education, are “seen as a way to increase the pool of individuals able to enter research occupations or undertake innovation” (OECD, 2014, p. 240). Thus, mathematics education (as well as science, technology, and engineering education) are worthy because they serve innovation. Service to innovation is also present in Australian STEM rhetoric: “STEM must ensure a steady flow of new ideas and knowledge. Innovation must turn knowledge into new and better ways of doing things" (Office of the Chief Scientist, 2013, p. 3). This is also coupled with arguments about economic competition as, "STEM skills are essential in creating and turning new ideas and inventions into lucrative, internationally competitive Australian products, services and exports" (Department of the Prime Minister and Cabinet, 2014, p. 1). Literacy is rarely mentioned, but human capital arguments mirror those present in economics research.

These documents position mathematics as subordinate to either innovation or the other STEM disciplines. While citizenship is less prominent, technological development and global competition strengthen rhetoric relating to human capital goals. This suggests that the rhetorics of *mathematics for human capital*, *mathematics for technological progress*, and *mathematics for global competition* are prominent in STEM discourse.

Table 3
Within Document Frequency Ranking of Key Terms

	Ranked frequency of key terms				
Terms	National Numeracy Review (2008)	Numeracy in Practice (2009)	STEM in the national interest (2013)	STEM in the education state (2016)	Australia's STEM workforce (2016)
Mathematics	1st	1st	34th	16th	58th
STEM	-	-	1st	1st	3rd
Numeracy	4th	2nd	-	80th	-
Science	60th	43rd	2nd	19th	6th
Technology	127th	53rd	11th	31st	47th
Engineering	-	-	17th	-	12th
Workforce	105th	-	39th	34th	26th
Literacy	55th	51st	-	108th	-
International	91st	71st	38th	130th	-
Innovation	-	-	5th	93rd	332nd
Industry	-	-	50th	42nd	19th

Frequency of Key Terms across Both Discourses

Key mathematical terms and terms that underlie each rhetoric appeared in different documents with different frequencies. Table 3 summarises differences between documents. In both of the numeracy documents analysed, the most frequently used terms were "mathematics" or related terms such as "mathematical", and the term "numeracy" was also prominent (4th and 2nd most frequent across the two documents). In STEM documents, "mathematics" was not the most frequently used word. It ranked 34th, 16th, and 58th across the three documents. In the two national documents analysed, the term "mathematics" was used less frequently than science, technology, and engineering. However, in the Victorian government document designed for teachers, "mathematics" is used more frequently than other STEM disciplines. Terms such as "STEM" and "science" are more frequently used in the STEM documents, although "science" also featured in the two numeracy documents (60th and 43rd most frequently used terms). More frequent use of the term "science" in STEM documents supports the claim that mathematics is positioned with science, and is usually subordinate to science, in STEM discourse. In the numeracy documents, the term "literacy" is used in relation to reading and writing and is the 55th and 51st most used term in both documents. This provides corroborating evidence the mathematics is positioned more closely with literacy in numeracy discourse. "Literacy" in relation to reading and writing does not feature prominently in the two national STEM documents but is the 108th most frequently used term in the Victorian teacher-focused document. In STEM documents, terms such as "workforce", "industry", and "innovation" are more frequently used than in numeracy documents, supporting the suggestion that the *mathematics for human capital* and *mathematics for technological progress* rhetorics are prominent in STEM discourse in a way that is not present in numeracy discourse.

Discussion

When viewed as rhetorical discourses, the shift from numeracy to STEM in public and policy discourse can be seen as a shift in the way that the value of mathematics education is articulated and the way that mathematics is positioned within education. Both are influenced by the *mathematics for human capital* rhetoric, as both discourses describe how mathematics education has value because of the way in which skill in mathematics enhances "workforces" and "labour markets". The frequency of terms that point to a human capital rhetoric (such as "workforce", "innovation", "business", and "industry") is higher in STEM discourse, however, which suggests that this rhetoric may be more prominent in STEM discourse. The frequency of terms such as "international" suggests that *mathematics for global competition* is also present in both discourses to some degree.

Social participation, civic competence, and a positioning of mathematics alongside literacy is prominent in numeracy discourse. This suggests that the *mathematics for critical citizenship* rhetoric is evident in numeracy discourse. Within this discourse, mathematics and literacy are foundational; thus, specific discussion of mathematics education occurs more frequently than in STEM discourse. The focus on mathematics' impact on students' capacity to engage in democratic processes is also less prominent in STEM discourse.

In contrast, STEM discourse positions mathematics differently. *STEM in the National Interest* (Office of the Chief Scientist, 2013) frequently uses the term "innovation" (5th most frequently used term), and the other STEM documents analysed use the term "innovation" when articulating the worth of STEM. This positions all STEM disciplines in service of innovation – innovation that will drive economic productivity via research and development, and technological advancement. This aspect of STEM discourse can be linked to economic discourses, particularly those about productivity growth. Within STEM disciplines, mathematics' role as foundational is still present. Mathematics is important as it is required in order to engage in science and engineering, and to develop technology. This suggests that the *mathematics for technological progress* rhetoric is present.

As public and policy discourse has moved from numeracy towards STEM, rhetorics have shifted. The idea that mathematics education is worthy because it enables critical citizenship has become less prevalent. Instead, mathematics education is seen to be worthy because of its contribution to technological progress, and this progress is connected to developing human capital and productivity growth.

Conclusion

For researchers who seek to study mathematics education, shifts in rhetoric at a policy level are likely to affect the research in which our community engages. By examining both numeracy and STEM discourses through the frame of rhetorics, it is hoped that our community could gain productive insight into the way in which these discourses change. This study is not a comprehensive review of numeracy and STEM discourses. I am not able to exhaustively investigate arguments for the worth of mathematics education. Instead, I present a way of thinking about the political and economic discourses that shape our field that appear to be absent in most discussion relating to STEM. If these discourses are primarily driven by economic theory that describes the worth of mathematics education without describing how mathematics can be taught effectively, then shifts in rhetoric will not affect pedagogy directly. As a research community, this rhetoric in the public discourse may not match our own beliefs about the worth of mathematics education, but these shifts in rhetoric do influence research and research funding. My own work in the area of STEM (Jazby, 2016) provides an example of how a rhetorical approach to STEM discourse could affect mathematics education research. As part of an engineering-focused project, mathematical problem-solving pedagogy – developed from mathematics education research that exists outside of STEM rhetoric – was able to be applied to a STEM project. Because STEM was viewed as a rhetorical discourse, the worth of the mathematical components of the project needed to be communicated in terms of developing skills that support innovation. This approach helped gain some modest funding of the mathematical problem-solving component of the project. Engagement with STEM rhetoric did not require radical shifts in mathematics pedagogy, but engagement with the rhetoric did enable project goals to be communicated effectively with those outside of the mathematics education research community. Hence, the analysis presented in this study has some pragmatic value relevant to our research community – a community that exists in an environment in which rhetorics for mathematics education are sure to shift post-STEM.

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Grade 10 Students' Mathematical Understanding and Retention in a Problem–Based Learning (PBL) Classroom

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In this study, we examined students' mathematical understanding and retention in a problem-based learning (PBL) classroom. The participants were 48 Grade 10 students, and the data were collected in December of 2016. After the end of the PBL lessons, the Mathematical Understanding of Function Test (MUFT) and the Retention Test (RT) were administered. The findings showed that most students demonstrated mathematical understanding of functions in all components and more than 50 percent of the students could pass both tests by the overall mean scores. Moreover, the overall mean difference between the MUFT and the RT was low, which means that the students had retention.

For several decades, mathematics teaching and learning have undergone a worldwide reform (Young-Loveridge & Bicknell, 2016). Moreover, studying mathematics with understanding is emphasized by several researchers (Stylianides & Stylianides, 2007). Fennema and Romberg (1999) argue that the development of understanding should be the outcome of teaching and learning mathematics. Furthermore, a greater amount of teaching and learning should utilize tasks that provide problem situations in order to promote learning for mathematical understanding. For these reasons, mathematics is best learned when students learn through problem-solving tasks. Students develop understanding when they engage in classroom activities to solve problems.

Mathematical understanding is accepted by some researchers as a procedural process that consists of an ability to carry out action sequences. Other researchers define it as a part of network of connections (Kinney & Kinney, 2002). Specifically, Hiebert and Carpenter (1992) define mathematical understanding as making decisions involving knowledge. However, mathematical understanding can also be defined as a network of ideas or representations about mathematics (Barmby, Harries, Higgins, & Suggate 2007).

One of the fundamental concepts in mathematics for secondary school is that of functions. Nevertheless, many students still have misconceptions about this topic. The Programme for International Student Assessment 2012 showed that the weakest ability of Thai students was in the change and relations topic in mathematics content. This topic is directly related to functions (The Institute for the Promotion of Teaching Science and Technology, 2014). Functions are important for students' learning of advanced mathematical topics such as calculus and advanced concepts of functions. Therefore, students' understanding of functions should be emphasized (Bardini, Pierce, Vincent, & King, 2014). Hollar and Norwood (1999) proposed mathematical understanding of functions with four components as follows: (1) Modelling: a real-word situation using a function, (2) Interpreting: a function in terms of a realistic situation, (3) Translating: translation among different representation of function, and (4) Reifying: the process of concept development that involves transformation from the operational to the structure phase. In this study, we apply Hollar and Norwood's study (1999) as a tool for assessing students' mathematical understanding of functions.

In addition, Kwon, Rasmussen, and Allen (2005) found that students' retention could be considered as a result of understanding mathematical concepts. Retention means students' ability to recall and organize information about mathematics from their memory.

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Therefore, retention can be defined as having a retentive mind (Kundu & Tutoo, 2002). Several researchers use retention intervals in their studies that last between one week and four weeks (Driskell, Willis, & Copper, 1992). The retention interval is the time period from the latest students' learning to a retention test (Rohrer & Taylor, 2006).

One of the 21st century teaching approaches that support both students' understanding and retention is Problem–Based Learning (PBL). Albanese and Mitchell (1993) suggested that PBL is a teaching approach that promotes understanding while students are challenged and engaged with problem situations. Hung, Jonassen, and Liu (2008) noted that retention of content and problem-solving skills could be considered as a result of PBL classroom. PBL is carried out in classrooms through examining real-world problem situations, conducting research, discussing and working in groups, and giving presentations (Othman, Salleh, & Abdullah, 2013). PBL activities are not only provocative but also beneficial for students in an active learning context. The PBL processes employed in this study are adapted from Othman et al.'s study (2013) with five steps: (1) an introduction to the problem, (2) self-directed learning, (3) group meeting, (4) presentation and discussion, and (5) exercises. In this study, we were interested in exploring Grade 10 students' mathematical understanding of functions and retention in a PBL classroom.

Methods

Participants

We employed a mixed-method design to investigate students' mathematical understanding of functions. The investigation was comprised of 48 Grade 10 students (19 boys and 29 girls) from a high school in Chiang Mai Province, Thailand. Nine students with mixed mathematical abilities were selected in order to gain in-depth information from interviewing them about their mathematical understanding of functions.

Instrumentation

The data were gathered using the following instruments: (1) eight PBL lesson plans (100 minutes per lesson, two lessons per week), (2) the Mathematical Understanding of Function Test (MUFT) and the Retention Test (RT; the RT was parallel to MUFT), adapted from Hollar and Norwood (1999), (3) students' reflections, (4) the teacher's notes, (5) classroom observation forms, and (6) students' interview forms. The data were analyzed in both qualitative (descriptive analysis) and quantitative (descriptive statistics) ways.

Procedure

During the PBL lessons, the data were collected for four weeks in December 2016. One of the researchers was the teacher in the PBL classroom. A mentor teacher observed all of the PBL lessons. Moreover, students' reflections, classroom observation forms, and the teacher's notes were used to reflect on students' understanding and teaching. At the end of the PBL lessons, the MUFT was administered in order to examine students' understanding of functions. The MUFT consists of two problem situations. The New Year party problem is an example problem situation from the MUFT. This problem situation was adapted from Mathematics Assessment Resource Service (MARS, 2003) (see Table 1). Nine students were selected according to their mathematical abilities (three high, three average, and three low) to be interviewed about their mathematical understanding of functions. Four weeks after the end of the PBL lessons, the RT was used in order to examine students' retention. We also used video recording and voice recording to provide supporting data for each teaching period and interview.

Table 1
Problem Situation Example (The New Year Party Problem)

The end of the year is near, which means a new year is coming soon. The New Year party is an opportunity to celebrate with family and/or friends. Your mother has a plan for a New Year party. She assigns you to prepare tables and seats in the party. The pattern setting is that all tables are put together in a line; then, a chair is put on the top and the bottom of each table. Moreover, at the ends of the lines of tables, you will put two seats as shown in the example in Figure 1:

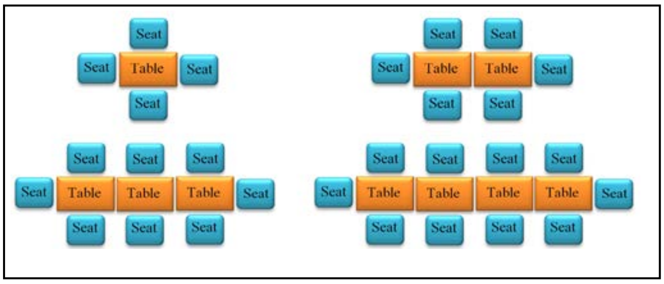


Figure 1. An example problem situation from the MUFT.

Results

Below, the findings are first reported with the students' scores on the MUFT and the RT. Then, information about students' mathematical understanding of functions is aligned with the five steps of the PBL classroom. Finally, details about the MUFT and interviews about students' mathematical understanding of functions in all components are described.

Part 1: The Mean Scores of the MUFT and the RT

The scores from the MUFT and the RT were computed in percentages for reporting the modelling, interpreting, translating, and reifying components and overall mathematical understanding of functions. The differences between the mean scores of the MUFT and the RT in each component ranged from 2 to 10 percent. The students' scores on the reifying component showed the highest mean difference, at 10 percent. The lowest mean difference was the interpreting component, at 2 percent. Moreover, the mean difference in overall understanding was at only 7 percent. This means that the students had high retention. In addition, the mean scores of the MUFT and the RT in overall understanding were 65 and 58 percent, respectively. More than 50 percent of the students could pass both tests by the overall mean scores as shown in Table 2.

Table 2
The Mean, Standard Deviation, and Mean Difference from the MUFT and the RT

Mathematical Understanding of Functions (Components)	MUFT (%)		RT (%)		Mean Difference (%)
	Mean	SD	Mean	SD	
1) Modelling	68	23	62	24	6

2) Interpreting	68	25	66	21	2
3) Translation	66	26	57	22	9
4) Reifying	56	33	46	33	10
Overall	65	23	58	20	7

Part 2: The Students' Mathematical Understanding of Functions in the PBL Classroom

Below, we describe the findings regarding the students' mathematical understanding of functions aligned with the five steps of the PBL process based on students' reflections, teacher's notes, and classroom observations.

1) Introduction to the Problem. In the first step, the students paid attention to the teacher while he introduced the real-world problem situation. Students' reflections showed that they preferred learning through the problem situations because it was interesting and helped them to get a better understanding of the lessons. However, some students expressed that they could not catch all main points; they just got some points of the problem situations. Thus, the teacher sometimes needed to restate the given problem situations to facilitate students' understanding about the problem situations.

2) Individual Work. In the second step, the students analysed the problem situations. Then, they created the problem situation's representations (modelling). Students' reflections showed that the familiarity of the problem situations to their daily lives helped them to be able to create the problem situations' representations. From the classroom observations, most students attempted and were engaged to create the problem situations' representations in their individual work. At the beginning of the PBL lessons, we found that only half of the students could correctly create representations of the problems. However, after the PBL lessons, most students could understand the problem situations quickly and they could correctly provide the problem situations' representations.

3*) Group Meeting.* In the third step, the students continued to analyze and solve the problem situations through small group activities. The students discussed their own ideas with each other about the mathematical understanding of functions in all components. The students' reflections showed that the students liked to meet in groups because it made them feel comfortable sharing their ideas with friends. From the classroom observations, we found that some students did not participate as they should in the group meetings. Therefore, the teacher often tried to motivate the students by asking for their ideas and connecting those ideas with other group members. This helped to increase students' participation in group work. After working together, most students were able to illustrate their understanding in group meetings with less of the teacher's facilitation. For example, the students could make a conclusion about the problem situations' representations from each group member's ideas (modelling), were able to connect the problem situations' representations to the description of a function (interpreting), were able to translate and create the multiple problem situations' representations (translating), and were able to correctly connect the solutions of the problem solutions to new conditions of the problem situations (reifying).

4) Presentation and Discussion. In the fourth step, the students presented their group work to the classroom. In the presentations, the students asked questions when they did not understand the presentations. In discussion, the students could offer their own mathematical understanding of functions by sharing ideas and discussing in the classroom. Finally, the students concluded with the mathematical understanding of functions from their discussions. The students' reflections showed that group presentations and classroom discussion helped them to get a better understanding of functions. From the classroom observation, the researchers found that the classroom discussion was a significant process that enhanced students' mathematical understanding of functions. Especially, most students interestingly discussed about connecting the operational process between the problem solutions and new conditions and concepts of the problem situations (reifying).

5) Exercises. In the last step, students individually worked on the exercises about mathematical understanding of functions. The students' reflections showed that the students could do the exercises and showed their mathematical understanding of functions in all four components. For example, students were able to create exponential equations from the problem situation about bacteria growth (modelling). They were also able to describe number of bacteria at various times (interpreting). Moreover, they were able to translate from exponential equation of bacteria growth to exponential graph (translating). Finally, they were able to understand the exponential shape when bacteria growth rate was changing (reifying).

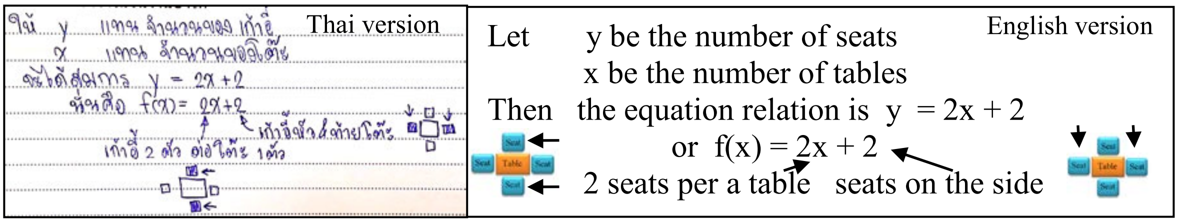
Part 3: The Students' Mathematical Understanding of Functions after all PBL Lessons

At the end of all PBL lessons, the MUFT was administered in order to examine students' understanding of functions. In the following sections, we provide examples that demonstrate the students' mathematical understanding of functions in all four components.

1) Modelling. The MUFT showed that most students could create meaningful models of the problem situations. The students could show complete modelling. For instance, some students' answers illustrated some of the equations for the situations by: (1) defining variables, (2) describing relationships between the variables, and (3) modelling the problem situations. Moreover, some students described modelling by using pictures (see Figure 2).

Figure 2. Example of modelling.

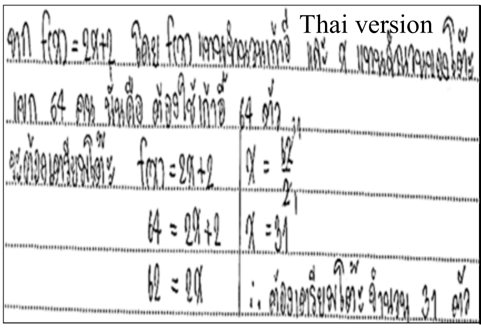
The students' interviews showed that students with high levels of achievement could give examples to clarify their explanations. In addition, they could make interesting references to the problem situations. However, students with average and low levels of achievement could correctly transform the problem situations to modelling, but they could not correctly refer to the problem situations.



2) Interpreting. The MUFT showed that most students could analyze the problem situations and correctly answer the questions about the problem situations. The students were able to find the value of input variables from the given situations (see Figure 3). However, most students couldn't completely describe or interpret the problem situation representations. For example, the students answered that the values of input variables were increased when the values of output variables were increased. Likewise, they did not correctly consider the rates of change and the constants in the problem situations.

Figure 3. Example of interpreting.

The students' interviews revealed that students with high and average achievement correctly described and interpreted situations. The students could specify what they have to do, what they need for interpreting situations, and what they already have. However, we



English version

From f(x) = 2x + 2 where f(x) is the number of seats and x is the number of tables. We have 64 guests so we use 64 seats.

$$f(x) = 2x + 2 \qquad x = \frac{62}{2}$$
$$64 = 2x + 2 \qquad x = 31$$
$$62 = 2x$$
$\therefore$ We will prepare 31 tables.

found from the student interviews that students with low achievement were confused between output variables and input variables. Moreover, they incorrectly solved the equations.

3) Translation. The MUFT showed that most students could present multiple function representations (see Figure 4). In addition, the students had various ways to translate the representations. For instance, they could draw graphs from linear equations, ordered pairs, or by finding the x-intercepts and y-intercepts. Nevertheless, some aspects of the students' representations were still incomplete. For example, some students set inconsistent scales on the x and y axes, some students did not define the x and y axes on the graph or the name of columns in the table, and some students ignored the graph symbols.

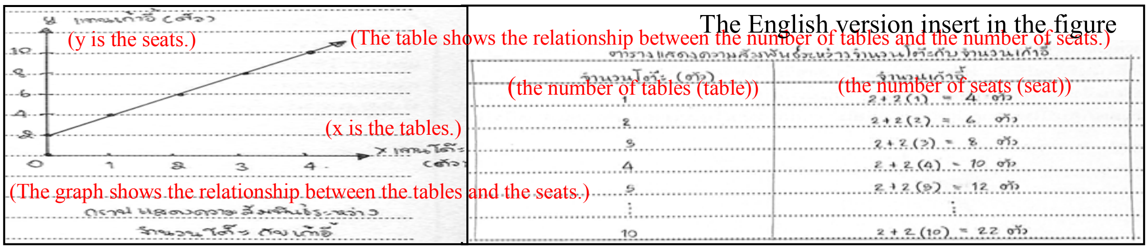
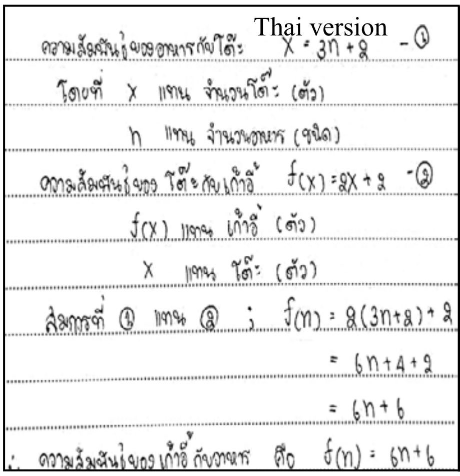


Figure 4. Example of translating.

The students' interviews showed that students with high, average, and low levels of achievement were able to explain how to translate one representation to another representation. However, the responses in the MUFT from low-achieving students were poor and incomplete. The students with high and average levels of achievement still presented their translation of the representations that linked to the problem situations and could explain the pros and cons of each representation.

4) Reifying. The MUFT showed that more than half of the students were able to connect the problem solutions with the new conditions of the problem situations in order to create new concepts. For example, the students could find a relationship between the number of tables (x) and the number of seats (f(x)) from the New Year party problem as the relevant equation $f(x) = 2x + 2$. In addition, the new condition of the New Year party problem is a relationship between a number of tables (x) and a number of food types (n) in the equation $x = 3n + 2$. The problem situation needs the relationship between the number of guests, which equals the number of seats (f(x)) and the number of food types (n). The students replaced $x = 3n + 2$ with $f(x) = 6x + 6$ as the relationship is $f(n) = 2(3n + 2) + 2$ or $f(n) = 6n + 6$ (see Figure 5). Nevertheless, some students' responses showed incomplete solutions, such as incorrectly replacing variables or solving the equations.

Figure 5. Example of reifying.



English version

The relation between the number of food types and the number of table is $x = 3n + 2$ …(1)
When x is the number of table (table)
n is the number of food types (type)
The relation between the number of tables and the number of seats is $f(x) = 2x + 2$ …(2)
When f(x) is the number of tables (table)
n is the number of seats (seat)
Replace (1) to (2); $f(n) = 2(3n+2) + 2$
$= 6n + 4 + 2 = 6n + 6$
∴ The relation between the number of seats and the number of food types is $f(x) = 6n + 6$.

The students' interviews revealed that students with high achievement could explain connections between the problem solutions and the new conditions of the problem situations. They were concerned with more references to the problem situations. In addition, they had several ideas for solving the problems. The students with average achievement were able to identify connections between the problem solutions and the new conditions. However, they could not explain the situations completely. On the other hand, students with low achievement could not identify the connection between the problem solutions and the new conditions.

Conclusion

In this study, we examined Grade 10 students' mathematical understanding of functions and retention in a PBL classroom. The findings showed that the PBL learning context gave students the opportunity to better gain mathematical understanding of functions in all four components: modelling, interpreting, translating, and reifying. Particularly, in the second step of PBL (individual work), the students were able to show modelling components. In the third, fourth, and fifth steps of PBL, the students were able to show mathematical understanding of functions in all four components.

In addition, the overall mean score of the MUFT was 65 percent. Meanwhile, the overall mean score of the RT was 58 percent. Both the MUFT and the RT showed that more than 50 percent of the students could pass the test, according to their overall mean scores. However, the mean scores of both the MUFT and the RT showed that the interpreting component had the highest mean scores and the reifying component had the lowest mean scores. Interestingly, the mean overall difference between The MUFT and the RT was only 7 percent. This showed that the students retained their mathematical understanding of functions.

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Knowledge, Beliefs, and Innovative Curriculum

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In this paper, I report on doctoral research in which I studied the changes in mathematical knowledge and beliefs of two Year 5/6 teachers as they implemented a four-week innovative curriculum unit. Such immersion experiences have the potential to develop teachers' understanding of mathematics in the context of the classroom. Year 6 case study teacher Debbi's experience is discussed in relation to curriculum fidelity and opportunity to learn, in particular the foregrounding of higher achieving students. Debbi's firmly entrenched practices, related beliefs, and affective response to the curriculum presented as the dominant filters for reflection and enaction.

Research focused on the classroom application of a teacher's mathematical knowledge and beliefs and the effect of these on student understanding has highlighted the need for further investigation into professional development strategies that promote deep and flexible understanding of curricular content. Calls for change in perspective from a focus on innovations to those driving them, along with views of teaching as cultural activity, suggest that attempts to develop the skills and knowledge of teachers must be done in the context of the classroom (Doyle & Ponder, 1977; Stigler & Hiebert, 1999). Such professional learning needs to push beyond strategies that promote rudimentary change that is quickly relinquished. To challenge the basic premises of teachers' prior beliefs, to "rock the conceptual boat" (Jones & Nelson, 1992), special conditions are necessary.

This study focused on how teachers held mathematical knowledge for teaching, illuminating the complex relationship between a teacher's knowledge and their beliefs, and the extent to which the use of innovative materials yielded changes in these. Interaction with curriculum and support materials can be thought of as an immersion experience (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2010) that has the potential to build mathematical knowledge for teaching while in the act of teaching. Innovative curriculum tasks put extra demands on the teacher; they tend to be conceptually demanding and unpredictable in nature (no set path of solution). Such features require teachers to make connections between students' informal thinking and the formal knowledge of mathematics, possibly prompting high levels of anxiety in both students and themselves (Stein & Kim, 2010). The major research question underpinning the study was: How do teachers' knowledge and beliefs about mathematics and mathematics teaching change as they teach an innovative mathematics curriculum? Case study methodology (Meriam, 1998) within a constructivist epistemology and an interpretivist theoretical perspective promoted in-depth understanding of the complex interactions and multiple realities of the teachers' world, and the interpretations they made in this context.

The Interconnected Model of Teacher Professional Growth (IMTPG; Clarke & Hollingsworth, 2002; see Figure 1) provided the study's theoretical framework and supported interpretation of mechanisms that trigger change (or growth) in one or more of the four domains of a teacher's world: personal, external, of practice, and of consequence, in this study. It offered a lens through which to describe interaction between domains, the mediating processes of enactment (putting a new idea into action) and reflection, and the resultant (professional) growth. The external domain in this study related to the innovative curriculum unit Some of the Parts (Brittanica, 2006), the Australian Mathematics

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Curriculum, and any other support materials sourced by case study teachers. The domain of practice involved implementation of the innovative unit in the teacher's classroom. Post-lesson reflection formed an important part of the research design, as the aspects that a teacher considers salient are highly subjective, depending on what the teacher values. Teacher-interpreted change by means of post-lesson interviews, therefore, was crucial as such change was "of consequence" for the teacher (Clarke & Hollingsworth, 2002).

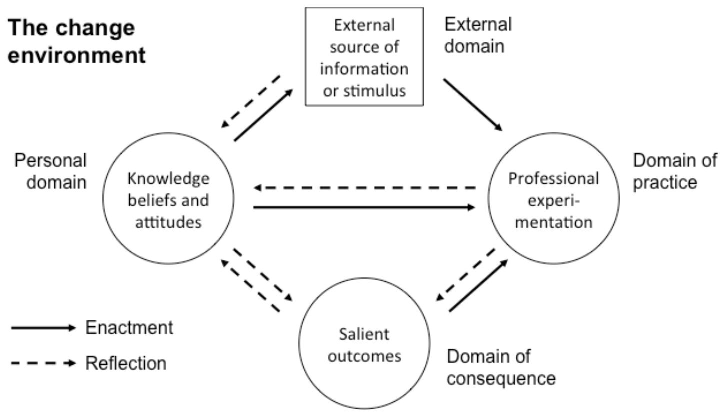

Figure 1. The Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002).

The personal domain of the IMTPG model represents what the teacher knows, believes, or feels in relation to the change occurring in their world. The Mathematical Knowledge for Teaching model (Hill et al., 2008) was used to help focus the analysis of changes in knowledge in the personal domain. Beliefs serve an important function in this model. According to Phillip (2007), a belief *that* is about beliefs, and a belief *in* is about values. Beliefs tend to develop gradually, serving as a filter through which new ideas or approaches are perceived. Newly acquired beliefs then are the most vulnerable (Pajares, 1992). Reflection on such beliefs is necessary, especially when teachers' beliefs are incompatible with goals of the innovation (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997).

Methodology

The case study involved two Year 5/6 teachers as they implemented a four-week fractions unit based on the principles of Realistic Mathematics Education (RME; see, e.g., Streefland, 1991). RME promotes conceptual understanding, progressively using "models of" realistic contexts to develop "models for" abstract representations. Such heuristics are the basis of the Some of the Parts (Brittanica, 2006) unit selected for this research, designed through cross-national collaboration between the University of Wisconsin-Madison and curriculum developers at the Freudenthal Institute. RME assists teachers to establish a classroom culture of "conjecturing, explaining, and justifying" to guide student "reinvention" of mathematics (Gravemeijer, 1999). The instructional design of the teacher support material speaks to the teacher, not through them (Phillip, 2007), outlining in detail the goals of instruction, the mathematics addressed (main concepts explored), comments during instruction, explanation of the models used, possible student responses, and assessment options. Conceptual understanding is promoted through investigation of realistic contexts using models that promote flexible manipulation of fractions, including fraction bars, double number lines, and the ratio table. The unit is organised around key ideas of rational numbers, making it deeply connected to the structures of mathematics, and to the core insights students develop as their mathematical reasoning and understanding develops. For these reasons, the Some of the Parts unit was considered innovative.

Case study teachers volunteered for the research trial. They had at least five years' classroom experience and were both at the same New South Wales government school. The study involved deep observation of knowledge in practice; each teacher was observed for three pre-intervention lessons in their normal routine and then up to 16 lessons during the four-week innovative unit implementation. Individual and group meetings were held with teachers in the term before the Some of the Parts trial to familiarise them with the content and its philosophical underpinnings. Teachers were given the opportunity to read the unit in more depth over the half-year break, and the first week of term was put aside for further clarification if needed.

During both the pre-intervention and innovative unit periods, post-lesson reflective interviews were conducted each day to gauge teachers' perception of the lesson, how it matched their intentions, and moments that they felt were significant. All lessons and interviews were recorded, fully transcribed, and supported by detailed field notes. The Copur-Gencturk (2015) classroom observation record assisted pre-intervention lesson analysis in relation to design and implementation, mathematical discourse and sense-making, task implementation, and classroom culture.

In the pre-intervention phase, teachers completed two questionnaires developed as part of the 17-country Teacher Education and Development Study in Mathematics (TEDS-M; Tatto et al., 2008), to gain an insight into their knowledge and beliefs about mathematics and mathematics teaching prior to the intervention, and as a basis of comparison as the research "story" developed. The first assessed Mathematics Content Knowledge (MCK) and Pedagogical Content Knowledge (PCK), measuring knowledge at least two years beyond the level that the teachers were expected to teach. The second was a beliefs questionnaire presented in a Likert scale of agreement in relation to beliefs about learning mathematics, mathematics achievement, and the nature of mathematics. A background form (adapted from Clarke et al., 2002) was also completed to gain information about the teachers' personal mathematics history and professional development. An in-depth reflective interview was held at the end of the five-week intervention phase.

Data Analysis

Making sense of the data in order to address the research question relied both on direct interpretation and categorical aggregation (Stake, 1995). Constant comparison began with initial observations, background survey information, responses at interview, notes made through direct observation of lessons, and results of the TEDS-M instruments. These data were compared with others, both within and across datasets (Merriam, 1998), allowing consistent refinement. Questions arising from the data, then, were asked during the field experience as well as after its completion. Interviews were fully transcribed and coded using *NVivo 11*. Coding was guided by the overarching research question and connected "sensitising concepts" (Patton, 2002), informed by the relevant academic literature. Preliminary categories located via axial coding helped form a conceptual structural order that facilitated further analysis. The central themes arising from the data formed the basis for the research "story". These themes included curriculum fidelity and planning, opportunity to learn, measures of success, reflection on practice, and growth and change. The first two themes are discussed in relation to Year 6 teacher, Debbi.

Results and Discussion

Debbi was a busy and industrious teacher with around 40 years' experience at the time of this study. She worked hard to plan lessons in her normal routine. Debbi said she "loved mathematics" (5.8.15) and was keen to teach it in a more specialised role to higher achieving students. Debbi indicated on her background survey that she was highly confident in her ability to teach mathematics in general, and to address the needs of high-attaining students. She was less confident to address the needs of low-attaining students but still rated this within the high range (7 on a scale from 1 to 10). Debbi thought the Australian curriculum had been "dumbed down" over the years, and hailed the merits of the "more progressive" U.K. and Singapore, which extended students more effectively (14.7.15). Debbi's lessons were planned for an hour each day in sub-strands (e.g., Monday – Number and Algebra, Tuesday – Fractions and Decimals, etc.).

During the pre-intervention period, Debbi used a variety of engagement activities, as well as conducting her favoured "Mental Maths" sessions for around 20 minutes of a 50-minute lesson. These 10 short-answer questions related to the sub-strand topic of the day; students answered individually and then marked them as a class. As Debbi wrote up the questions, she hinted about how to complete them. These questions were an established part of Debbi's class routine and their hierarchy was well understood by the students.

Amid lessons there were public messages for students about what constituted success in their classroom. Some students were publically praised and rewarded for making mathematical connections and being "smart thinkers" while others were individually named to the class in relation to problem-solving strategies that might match their weaker abilities. Students who got 10 out of 10 in the Mental Maths session had their names recorded on posters up in the room, alongside students who could recite multiplication tables from x2 to x12 in less than 20 seconds. This information was then summarised in a poster that classified students as "champs", "almost a champ", and "more work needed". Students were grouped by achievement based on an assessment at the beginning of the year (with minor ongoing modifications made); classroom tasks and expectations were planned accordingly.

Students seated in tiered groups made it easier to manage questions during class tasks and to provide support, Debbi explained. It also satisfied her desire to accelerate content to abstraction for those students in her top group. Lesson elements were consistently described as being "fun" or "challenging", often interchangeably, suggesting the high value that Debbi placed on engagement and her high-achieving students. This belief was a utopian alternative to that with which she grew up, recalling her own education as delivered "all at one level" (5.8.15).

Debbi's sub-strand per week programming format allowed much time between continuous lessons to modify content and approach, based on what happened the previous week. Debbi considered her current teaching philosophy of "breaking the strands up" as successful, a structure that allowed constant monitoring of students (5.8.15). This belief was embedded early in Debbi's career, making it tightly held. Such trade experience was positively affirmed when Debbi was in control of the lesson design and content, making her feel effective.

Viewing Debbi's pre-intervention lessons through the Copur-Gencturk (2015) observation record revealed differences in the way that the students responded to the tasks, how many tasks were planned for each lesson, and the time dedicated to each part of the lesson (each subtask). The degree to which Debbi perceived that her "top group" was being challenged and her expectations of what the students should be able to complete influenced the types of discourse and classroom culture considerably. The packed and pressured structure of the first two pre-intervention lessons decreased the time available to investigate, to discuss mathematical content in depth, or for students to contribute significantly to class discussion. Debbi's pacing and desire to keep "moving forward" thwarted teachable moments.

As this study aimed to observe possible changes in Debbi's mathematical knowledge for teaching while implementing innovative curriculum, a measure of her personal competence in this area was considered important. In the TEDS-M assessment, Debbi scored 20 out of a possible 29 in questions relating to MCK, and 12 out of 20 for PCK. A third of the difficulties Debbi encountered related to extended reasoning, suggesting that explanations of mathematical ideas may be challenging for her. In the pre-intervention phase, this could also be inferred from the Copur-Gencturk (2015) observation protocol.

Opportunity to Learn and Fidelity to the Innovative Curriculum

Hiebert and Grouws (2007) defined teaching as a bi-directional relationship that includes interactions among teachers and students around content. A student's opportunity to learn is influenced by the teachers' choices of curriculum topic, their goals for learning, the time allowed for classroom tasks, the tasks posed, and considerations about students' entry knowledge. In providing opportunity to learn, teachers make pivotal decisions in relation to interpreting the written curriculum, setting up the features of the task in the classroom and then implementing it to meet the valued learning goals. Opportunity to learn and curriculum fidelity, or alignment of the intended and the implemented curriculum (as represented in the Some of the Parts unit), emerged as related themes in Debbi's classroom.

Opportunity to learn the skills and strategies promoted by the heuristics of RME were influenced by Debbi's interpretation of the unit and the decisions that she made in relation to its implementation. Debbi seemed very concerned about when the Some of the Parts unit was originally published as she had taught in the U.S. in 1993 and used "a lot of textbooks like this, which was considered lazy teaching" (11.6.15). Her colleague, Mark, committed to the recommended pacing and planning of the unit, displaying a trusting relationship between himself, the unit philosophy, and lesson sequencing (Lomas & Clarke, 2016). Debbi's stated intention to "follow" the curriculum, however, was not necessarily complemented by her "trust" in it.

The unit developers stressed an informal approach as the unit lessons began, encouraging teachers to draw upon the everyday experiences of all students without formal reference to fractions. Despite this, Debbi encouraged students to use measurement (rulers) as the problem-solving strategy to decide on fair shares rather than the informal division encouraged by the unit writers. She also introduced procedural cues to assist students with "easy ways" to solve problems, such as, "if the denominator is even, always divide it in half first" (27.7.15), a prompt that became increasingly problematic for her.

At times, Debbi would use the realistic context scenarios given in Some of the Parts, like sharing fruit strips or being shipwrecked on an island and needing to drink from coconut cans, but the connection to the associated model was not fully explored. Possible comparisons of how the mathematical models were the same or different were lost, and opportunities to explore informal problem-solving strategies removed. Debbi was concerned about the "mundaneness" of the models and tasks presented (20.7.15). She taught lessons far more quickly and in less depth than that suggested by curriculum designers. Students were often hurried through their thinking to facilitate this goal. Use of the ratio table and the tasks in Section E (How Far?) where students order benchmark fractions and calculate distances using maps were the aspects of the unit that Debbi enjoyed teaching most, as she thought they were the most appropriate for Year 6.

The early provision of what Debbi called "focus" questions, related to concepts addressed much further on in the curriculum unit, was evidence of Debbi's concern about catering for the stronger students in her class while teaching the innovative unit. It could be argued that such modifications demonstrated Debbi's sound Knowledge of Content and Students (KCS; Hill et al., 2008); she knew her class well and what they would find easy or difficult. This decision was made, however, before students were given the opportunity to interact with the curriculum, the contexts it provided, and the models it promoted.

Prioritising the needs of her higher-achieving students by way of lesson design, pace, and ensuing class conversation skewed the opportunity to learn in favour of her "top group". Debbi said that she was concerned at the level of challenge in the curriculum unit, yet regularly read and interpreted tasks for students to achieve her goal of moving quickly through content. In post-lesson reflection, the significant moments for Debbi generally related to how well her grouping structures were functioning, how many tasks she had got through in a lesson, and the "mastery" that her top group had achieved in relation to the tasks that she had designed herself. Modifications of curriculum are expected as teachers adapt curriculum to the needs of all their students; this is part of a teacher's role in creating opportunity to learn. This was the stated motivation behind Debbi's decisions to choose disparate pieces of the unit. Her focus, however, was on a small group of students.

Grouping of students by achievement was an important part of Debbi's differentiation goal, providing an idealistic alternative to her own as a learner. She was determined to apply this familiar grouping structure philosophy and practice during the innovative curriculum phase, setting up a situation whereby "extra" and "different" work was consistently needed. Debbi's self-concept was significantly tied to her ability to extend her brighter students. It is what she enjoyed, and what she valued. Use of the Some of the Parts curriculum, she feared, would affect their engagement and academic trajectory.

Debbi said that affirmation of her core belief in relation to grouping students was the major benefit of teaching the innovative curriculum. It also made her think hard about how much differentiation was needed in a class. By differentiation, she meant different tasks for each achievement group. Some of the Parts unit tasks were not levelled. Differentiation opportunities depended on the teachers' understanding of their conceptual basis as outlined in the curriculum, and the models that supported them. Suggestions made in the Reaching All Learners section of the unit assist teachers to support and/or extend students, contributing to its potential to provide opportunities for all students to learn. Complementary tasks for each section were also offered. Despite this, Debbi felt that the curriculum experience reaffirmed her belief that weaker students needed rote learning, questioning whether they needed an understanding of fractions at all. Such students would have to rely on procedural knowledge in the future as they "had been doing hands-on concrete aids for seven years and they still haven't got it". She said she was not worried about these students because "they were never going to get it" (3.8.15).

Momentary Change

Beliefs serve as a filter through which new ideas or approaches are perceived, and the challenge to reflect on these needs to be present, particularly when a teacher's beliefs are incompatible with the goals of the innovation (Borko et al., 1997). In her normal routine and during the innovative curriculum phase, reflection on practice was uncomfortable for Debbi. She was more at ease when reflecting on her regular lessons than those during the innovative curriculum. Contradictory, defensive, and/or confusing statements often followed pensive and contemplative moments after each of the 11 lessons taught. Having said this, at the end of the research, Debbi said that the opportunity to regularly discuss her experience with the researcher was beneficial, lamenting that there was little collaboration between herself and the other case study teacher.

Amidst the resistance, Debbi did reflect on the challenges that the innovative curriculum created in relation to her own subject matter knowledge (SMK) and ways to best address student misunderstanding (KCS). The automaticity of Debbi's teaching (as she described it) was challenged, and her anxiety about this was apparent. Frustration about how to explain ideas, confusion about why students did not understand concepts, and even her consistent response about not being able to remember many aspects of the lessons she had just taught pointed to such unease.

Debbi was particularly concerned when she felt that her understanding of the content in Section D (How Much?) was insufficient to continue the lesson. In this section, recipes are used to apply understanding of operations with fractions, and benchmark fractions are used to estimate portions of food measured in grams. Debbi reflected on her inability to make the connection between the title of the unit How Much and the tasks presented. She acknowledged that her confusion might have been caused by the "jumping around" that she did inside the unit. Debbi said that the different models in these tasks had created challenges for all her students, but that she had found their use "very powerful" (27.7.15). The feedback from her "top group" at this time was positive, which was salient for Debbi. In post-lesson discussion, Debbi reflected on her own learning opportunities, citing the need for time to think more deeply about ways to differentiate for her whole class when teaching ratio. Debbi voiced the change she was noticing in her practice: "I am coming to terms with an approach, a different approach… it's changing my habits to some degree… it challenges me, yeah" (27.7.15).

Conclusion

Analysing Debbi's response to the innovative curriculum through the IMTPG model (Clarke & Hollingsworth, 2002), it is apparent that Debbi's belief in her ability to plan and deliver lessons that provided opportunity for high-achieving students to learn was strongly held and resistant to change. It was also where her confidence lay, and her self-concept connected. Debbi did not consider the innovative curriculum a "knowledgeable other" from which to learn, so there was little reflection on her own teaching practice. This embedded belief promoted hand-picking of tasks during the research and thwarted a wider view of the unit goals and associated models. This also prevented opportunities for whole-class investigations and discussion around important key ideas of rational number. There was evidence that the management of such heuristics may have been challenging for Debbi's subject matter and pedagogical content knowledge, causing further anxiety. Fleeting changes of approach occurred when Debbi reflected on the meaning lost for both her and her students through her haphazard approach. In this immersion experience, curriculum fidelity had an observable influence on the opportunity to learn for both Debbi and her students. Beyond curriculum fidelity, it was Debbi's deeply affective relationship with the curriculum that influenced the quality of this experience, as her values became the dominant filter to interpret all others.

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Engaging Pre-Service Mathematics Teachers in Creating Spatially-Based Problems in a 3D Virtual Environment: A CAVE2^{TM} Experience

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The use of multiuser virtual environments for educational purposes is in its infancy but offers potential for exploration of spatial contexts that could not otherwise be experienced. We report on pre-service teachers' experiences in designing learning activities as a result of immersion in the CAVE2^{TM}, a 320-degree, cylindrical 3D virtual environment. Observation of student actions and analysis of student-developed artefacts indicated that 3D and 2D interference impacted the design of immersive learning experiences. We hypothesise that pre-service teachers' capacity to recognise and seize the potential of the CAVE2^{TM} for promoting spatial reasoning is predicated on their own spatial reasoning capabilities.

The use of spatial reasoning (or spatial thinking) is integral to human lived experience via spatial interaction in a three-dimensional (3D) world. Spatial thinking can be considered simply as using spatial memories from life experiences. This can vary from simple tasks like visualising windows in a room, walking along a known path, or estimating size, quantity, or distance of objects, to more complicated tasks, like rotating an object in the mind, drawing a two-dimensional (2D) image of a 3D object, or using a mirror to reverse a car. Spatial reasoning embraces natural abilities, such as navigating the environment and recognising objects or identifying and utilising spatial patterns and their structural relationships that are fundamental to both mathematical and non-mathematical conceptual development (Mulligan, English, Mitchelmore, & Crevensten, 2013). Being able to think spatially is important for our everyday problem solving, and equips school students for success, not only in mathematics, but in science, technology, engineering, and mathematics (STEM) and other subjects (Wai, Lubinski, & Benbow, 2009). The ability to move between 2D and 3D visual environments is a critical part of spatial reasoning skills and competencies (Bruce et al., 2016: Uttal et al., 2002). These skills are important in the real world, such as in interpreting architectural plans and map making/reading.

Located at the University of the Sunshine Coast is a unique learning environment known as the CAVE2^{TM}. This is an immersive environment that enables 320-degree panoramic views of virtual objects and displays. As its name suggests, you step into the cave and are immediately immersed in whatever environment is projected on the walls. You can view projected 3D objects that appear to "hang" in space from whatever angle that is rotated. The virtual space is controlled by one person who sends commands to the screens to provide the immersive experience. The CAVE^{TM} comfortably accommodates groups of approximately 15 people. The carpeted floor invites sitting or lying to maximise the experience if desired. As a new learning space with potential to develop spatial reasoning and thinking, pre-service teachers were provided with a CAVE2^{TM} experience and tasked with developing learning activities suitable for primary school students. As part

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of a larger study investigating multiuser virtual environments and associated technologies, this paper is a report on a scholarly inquiry in progress.

Significance and Background

Spatial reasoning is an important life skill and it is a recognised component of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2016). Yet many people, both teachers and students, find spatial reasoning difficult (e.g., Marchis, 2012). Recent review publications on implementation of the Australian Curriculum have indicated a shift in curriculum foci to strands concerned with spatial reasoning, such as Measurement and Geometry (Atweh, Goos, Jorgensen, & Siemon, 2012). However, it has also been reported that school curricula have only recently begun to recognise the value of teaching that encompasses spatiality (Bruce et al., 2016). Published reports provide blueprints for incorporating spatial reasoning into the curriculum, including a focus on early years education (Davis et al., 2015), and in secondary and tertiary education (Wai et al., 2009). Specific studies have attempted to isolate spatial elements such as spatial visualisation, sense, and orientation (Bruce et al., 2016), whilst others have looked at classroom or other learning environments in a more general or systemic sense (Francis et al., in press). Spatialisation of the curriculum underpins a new vision of how mathematical concepts are formed and connected and how they develop from interaction with the 3D world in both a mathematical way and a non-mathematical way (Davis, 2015).

Defining spatial reasoning is challenging. Definitions incorporate notions of locating, orienting, decomposing and recomposing, navigating, patterning, scaling, transforming, and seeing symmetry. In this paper, we use the definition of the Spatial Reasoning Research Group (of which Woolcott is a member), an international collaboration dedicated to investigating spatial reasoning in educational contexts. Spatial reasoning (or spatial ability, spatial intelligence, or spatiality) is “the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects” (Bruce et al., 2016, p. 2).

Spatial reasoning has a significant role in twenty-first century education (Hegarty, 2014), particularly given its strong correlation to mathematics achievement (Uttal et al., 2013). “There is over a century of research showing strong correlations between spatial reasoning and mathematics performance… and yet we know very little about what this means for educational application.” (Bruce et al., 2016, p. 15). Lowrie, Logan, and Scriven (2012) argued that spatial reasoning needs to be connected with a diversity of mathematics learning environments. Spatial reasoning helps us make sense of the 3D world of objects and space by mentally inserting ourselves into a spatial situation to solve a problem, contextualising the spatial elements. Recent investigations of the use of spatial reasoning in the Australian Curriculum: Mathematics (Measurement and Geometry) by Lowrie and colleagues (e.g., Lowrie, Diezmann, & Logan, 2012; Lowrie & Jorgensen, 2015; Lowrie, Logan, & Scriven, 2012) have suggested that there is considerable potential for application of rich visual or spatial tasks. Despite the potential impact of spatial reasoning on mathematics education, the associated skills and competencies may not always exist in either classrooms or in the teachers operating in those classrooms, in some part due to a focus on number or arithmetic (Mulligan & Woolcott, 2015). In classroom settings, students may be required to draw a 3D object in 2D, make a 3D object from a 2D plan, or to connect a 2D net with a simple 3D object, both with and without the use of digital technologies. These skills and competencies can be improved in both children and adults (Newcombe, 2013; Uttal et al., 2013).

Spatial reasoning experiences have the potential to improve mathematics learning as they enable access to complex mathematical ideas in non-traditional ways. Virtual environments and new technologies are relatively untapped and under-researched resources for developing spatial reasoning. Dalgarno and Lee (2010) report that virtual environments support increased knowledge of spatial representation, opportunities for contextualised and experiential learning, as well as increased motivation, and collaboration compared with 2D activities. Barrett, Stull, Hsu, and Hegarty (2015) report that being provided with experiences where you feel that you are moving, assist with learning the layout of an environment when in a virtual environment. Gregory et al. (2015) have reported on the range of virtual environments available for education. Kennedy-Clark (2011) has reported on pre-service teachers operating in these kinds of environments for science education. Camilleri, de Freitas, Montebello, and McDonagh-Smith (2013) have reported on the engagement value of such experiences compared to that of a traditional classroom or other online experiences. There are no current studies that have explored the impact of a CAVE2™ experience on pre-service teachers' own spatial reasoning and their capacity to use this environment to solve spatial reasoning tasks and create activities.

The focus in this study was in providing learning experiences for pre-service teachers that centred on spatial reasoning, and includes examination of both learning (as understandings) and perceptions. The aim was to develop in the pre-service teachers an awareness of their own spatial reasoning skills and competencies so that they were then aware of their strengths and weaknesses when helping their future students develop spatial reasoning skills in the classroom. We consider the following research questions:

1. How does immersion in a virtual 3D environment impact pre-service teachers' spatial reasoning competence?
2. What is the capacity of pre-service teachers to utilise this virtual environment to design and solve rich spatial reasoning tasks and activities?

The Study

In this study, three groups of pre-service primary teachers (group size approximately 25) were scheduled to undertake their normal two-hour course tutorial in the CAVE2™. Prior to this tutorial, the focus of the course lecture material was on the importance of developing children's spatial reasoning and the role of this within the mathematics curriculum. The CAVE2™ is located within a purpose-built learning space that includes open space outside the actual cave where desks and computers are located. This enables in-cave and out-of-cave learning to flow. The pre-service teachers were guided through tutorial material associated with spatial reasoning, with the cave experience integral to the planned tutorial for that week.

The selected cave experience was a flyover video of the region surrounding the campus, guided by the manager of the Visualisation Facilities. This immersive experience simulated a helicopter or airplane ride of the real local environment within a 65 km radius from the campus site. This flight took in local coastline and mountain terrain. Prior to the immersive experience, the pre-service teachers were asked to think about how this experience could develop spatial reasoning in primary school children. After the experience, they were required to work in groups to develop a mathematical question or problem that they could solve using the CAVE2™ experience. Throughout the tutorial, the participants could move freely in and out of the visualisation space, and also ask the operator to take them to the part(s) of the video that they wished to see again so that they could collect data that they wanted to use to solve or think about their problem.

Data collected and reported here were analysed thematically and came from the following sources:

- Field notes kept by the first author who attended each session
- Participants' questions and some solutions (not all provided their solutions)
- Participants' responses to reflection questions shown in Figure 1

When you were using the CAVE:
How did you identify your problem(s)? What stimulus did you use?
How did you solve your problem(s)? What knowledge or resources did you find useful?
When you were using the map:
How did you identify your problem(s)? What stimulus did you use?
How did you solve your problem(s)? What knowledge or resources did you find useful?

Figure 1. The reflection questions given to the pre-service teachers.

Results and Discussion

The CAVE2TM Experience

The immersive experience appeared to engage all pre-service teachers. They were very keen to step into the cave where they were surrounded by a virtual image of their immediate local environment (i.e., the grassy area outside the building). Students rotated themselves to take in the panoramic view presented to them. They expressed surprise of the realistic nature and that they felt that they were actually standing outside. The cave operator then commenced the "flight" over the buildings and provided students with an aerial view over the university campus. Students identified buildings and other familiar landmarks. For some students, the movement was disconcerting and they experienced some sensations of nausea. They only needed to look down at the floor for a few minutes until this feeling dissipated. No students refused outright to continue with the cave experience due to these sensations. The operator extended the "flight" to take in the local coastline. Students picked out local landmarks and also tried to locate their own houses.

What was noticed by the researcher and tutors was confusion experienced by pre-service teachers due to shapes changing depending on the angle from which they were viewed. For example, at this university, there are straight paths joining buildings on either side of a large lawn. Initially, standing in the CAVE2™ environment, there appeared to be two separate paths as in Figure 2. As the pre-service teachers "walked" through the environment, the paths moved together and joined into a straight path in the centre of the room and then bent the other way if you continued across the room. This generated considerable discussion about whether the CAVE2™ environment is realistic and how it distorts the senses of how the environment should look. There appeared to be general consensus that this would be too confusing for primary school children.

Another example of the CAVE2™ environment impacting students' spatial understanding related to parallel lines. In the lecture prior to the cave experience, the definition of parallel lines had been presented to students with the usual definition as "lines that never meet". At this particular university, its main entrance is via a semicircular road, each side of which then continues as parallel roads ending in carparks. The "flight" over the university enabled students to look at these roads and, due to perspective, the roads appeared to move towards each other. This led to quite a heated discussion with one group, as two students were certain that the roads were parallel from their personal experience, but they stated that they could not be parallel as they appeared to meet in the distance. This caused considerable confusion for students M and D. Student M could not resolve it during the session, even though the tutor discussed other roads, such as long straight country roads that appear to disappear to a point on the horizon. This was extended to discussion about the experience of driving on roads and the fact that the width of the road does not (overly) change so the edges of the road are effectively two parallel lines. At this stage, Student M was unable to reconcile the mathematical definition of parallel lines, as "lines that never meet" with personal experiences in the "real world". To Student M, geometry belonged in a textbook, and she had difficulty changing that view. There was also concern about possible confusion with children in their future classes: "Parallel lines may meet with perspective will then confuse children like M and D confusing their definitions of parallel." (Reflection by one of the pre-service teachers involved in the heated discussion.)

Figure 2. The image of the path between the two buildings from the door.

For the majority of pre-service teachers, the "flight" enabled them to make sense of their suburb and to make the suburb "fit together" with respect to where particular places were located. Whilst viewing the flyover, one said "Wow, I live there" pointing to the screen, "and the creek is just over there! It is not very far away at all. When you drive there or walk there, you have to go all the way around there, which is quite a long way, but it is just there! I had no idea, even when I looked at the map."

Problem Solving in the CAVE2™

For the design and solution of a spatial reasoning activity using the CAVE2™ as a resource, the pre-service teachers typically chose to work collaboratively. They submitted just one set of results for their group and very similar reflections of their conversations. Selected examples to highlight key findings about pre-service teachers' capacity to embrace the immersive environment for teaching and learning purposes are presented here.

Example 1. The CAVE2™ experience led to many students wondering about their local region. One student wrote:

> Sustainability – How has the environment changed, how much has been developed → population growth, reduction in farming?
> How much income was lost as a result?
> → What are the new industries/ forms of income?
> How much 'free space' is left in the Sunshine Coast area?
> →How many more people can move into the Sunshine Coast area?

In this case, the CAVE2™ provided an opportunity for experiential learning (Delgarno & Lee, 2010) by immersing the pre-service teachers into the local environment and enabling them to imagine how their local area has changed over time. These were rich mathematical questions with real-world and cross-curricular links. The pre-service teachers demonstrated spatial reasoning skills and competence in moving between 2D and 3D (Bruce et al., 2016: Uttal et al., 2002) when they identified that solving these problems included mapping over timelines and considering geographical landmarks. However, there was little in-depth thinking of these questions and reflection upon the CAVE2™ resource. The CAVE2™ was not going to be useful to solve the problems as identified as they would need to go elsewhere to find the solution using estimation from mapping data.

Example 2. Many pre-service teachers were not confident or competent in their spatial thinking and found it difficult to relate to 3D problem solving. So, they changed the problem into a more familiar traditional "school-type" 2D problem that they could solve. For example, questions included: "When the perspective of the area shifted facing west and gave a view of the coastline, it activated my wonder of the features of the Sunshine Coast. How high were we flying?" However, the pre-service teachers did not feel competent in solving the 3D problem, writing comments such as: "Having the question from the 3D experience in mind with no way of solving at present time I thought I would solve a similar question relative to the 2D visual". These pre-service teachers solved 2D problems such as: "What is the length of the Obi Obi River from the Kondalilla National Park to the Baroon Pocket Dam?", "How far is it from Narrows to the bakery at Montville?", and "How long will it take me to walk the Great Walk from Kondalilla to Baroon Pocket Dam?"

Example 3. More concerning was the pre-service teacher who admitted in discussions with the researcher that she actually only thought about directions in 2D and that this was probably not a good way to teach it. Admitting her spatial skills were poor led to lots of discussion, and her group decided their problem was, "How could the CAVE be used to educate primary education students?" Acknowledging referring to themselves as the problem, they were keen for suggestions on how to improve their skills.

Example 4. Another group of pre-service teachers, in which three of the four had been in the army, found the experience confronting and had lots of discussion both inside and outside the CAVE2™ trying to resolve their thinking. They did not believe that it was possible or appropriate to be solving these types of 3D problems. One described the problem:

> Without a scale, I decided a problem would not be authentic
> In conjunction with a 2D map, it may work.
> With a 2D map, scale can be worked out. It seems like an expensive YouTube video.
> Engagement is a large factor; however, the problem of scale still plays a factor.

Prior experiences with map reading meant that this pre-service teacher was unable to see past these experiences. Perhaps he was reflecting back on "life or death" experiences where 2D maps had to be carefully studied and memorised prior to entering an area. As a whole, during this tutorial, this group spent considerable time in the CAVE2™ talking to the operator as he suggested ways that perhaps they could develop a scale. For example, at one stage, he showed them particular landmarks and asked them what size they were and how they could use the size of these landmarks to develop an approximate scale. However, they had argued that it wasn't accurate and that it wasn't a real scale. They were reluctant to consider estimation as part of their mathematics problem-solving toolkit. This near-refusal to problem-solve in this space led the researcher to reflect in her field notes: "I don't think they believed that primary school mathematics, or any mathematics, belonged in the 3D world, or perhaps it was more of a concern with using estimation". For them, the problem needed to be situated and solved in 2D by: "us[ing] Google maps or 2D map".

Implications and Conclusion

Pre-service teachers were provided with the opportunity to visit the CAVE2™, a 3D immersive visualisation facility, with the task of developing learning activities suitable for primary school students. While the 3D spatial environment was initially confronting, pre-service teachers generally found ways to utilise this unique resource and think about their personal spatial reasoning competence. For some pre-service teachers, there was confusion around shapes changing depending on the angle from which they were viewed and confusion around their spatial understanding related to parallel lines and perspective. The experience allowed many to make sense of their suburb as they viewed it from above.

Some pre-service teachers were able to demonstrate spatial reasoning competence and design questions that would lead to rich activities with spatial reasoning components, such as: "How has the environment changed, how much has been developed → population growth, reduction in farming?" There were others who were unable to solve their 3D questions and so solved 2D questions such as "What is the length of the Obi Obi River from the Kondalilla National Park to the Baroon Pocket Dam?" There were some pre-service teachers who could not utilise this virtual environment to design or solve rich spatial reasoning tasks and activities either because of their self-identified lack of 3D spatial reasoning skills or because they believed that "Without a scale, I decided a problem would not be authentic".

If pre-service teachers do not have spatial thinking and reasoning skills, they will not be able to help their students develop these skills. Both children and adults can improve spatial thinking with appropriate teaching and technology (Newcombe, 2013; Utall et al., 2013). Therefore, it is important that pre-service teachers are given opportunities to improve their spatial thinking and reasoning skills as part of their university education and are encouraged to continue developing these skills. The CAVE2™ provides a unique way for pre-service teachers to engage with and reflect on their 3D thinking and reasoning abilities and may provide the stimulus to seek to improve their skills. We are continuing to explore whether other resources in the CAVE2™ can be used to develop pre-service teachers' spatial thinking and reasoning. Interestingly, the CAVE2™ fly-over experience in this study resulted in a number of people experiencing motion sickness. This means that the experience has potential to support large-scale spatial learning in the same way as the real experience (Barrett et al., 2015).

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Structure in the Professional Vocabulary of Middle School Mathematics Teachers in Australia

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Members of the Australian mathematics education research community and experienced teachers of mathematics participated in the process of documenting the professional vocabulary of middle school mathematics teachers. This vocabulary, the Australian Lexicon, captures the language in use by Australian mathematics teachers when describing the phenomena of the middle school mathematics classroom. In this paper, we examine the structure of the Australian Lexicon with particular attention given to content, connection, and characteristics of the professional vocabulary available to middle school mathematics teachers in Australia.

A technical or professional language to describe and analyse practice in teaching has been previously reported as lacking or underdeveloped (Grossman 2009; Lampert, 2000; Lortie, 1975). Lampert (2000) has concluded that the lack of opportunities to work collaboratively with peers on the problems of practice result in "a language of practice [that] remains flat or nonexistent" (p. 90). Connell (2009) has similarly observed that the teaching profession's organisational culture does not always support the "the informal processes by which practical know-how is passed to new teachers in on-the-job learning" (p. 223) and that a culture that might do so needs to be purposefully fostered.

Bhatia (2006) argues for studies of professional practice in order to:

- gain a more informed and comprehensive view of the language employed by professionals to describe, represent, interpret and theorise; and
- gain significant understanding of the coherent and social reality of members of the profession.

This research engages in the first of Bhatia's goals. However, with regard to the second, the social reality is inferred through the activities that are named. The Sapir-Whorf hypothesis suggests that our lived experience is shaped by our capacity to name and categorise our world: "We see and hear...very largely as we do because the language habits of our community predispose certain choices of interpretation" (Sapir, 1949, p. 162). If the Australian teachers' conceptions of the mathematics classroom are constructed around activities that they can name, then it may follow that they are unlikely to engage in activities that they cannot name.

Research Methodology

This research has put into practice a methodology that can be legitimately described as "negotiative". It relied, to a significant extent, on the collaborative involvement of members of the mathematics education community. Members of the research community and a select group of practitioners have participated in the process of negotiating the lexicon employed by practitioners (middle school mathematics teachers), whilst the community at large has assisted in the validation of that lexicon.

This research project shares attributes with the discipline of anthropology as its goal is the construction of a cultural artefact. As such, it has some commonality with the aims of the applied ethnographer, as insights – the lexicon employed by middle school

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mathematics teachers – are generated through the perspective of the 'insider' (Hammersley & Atkinson, 1995; Hoey, 2014). In this case, however, the key insiders are drawn from two distinct communities:

- our teacher partners, whose professional expertise both informs and shapes the study; and
- the broad practitioner community, whose input is sought in the process of refining and ratifying the national lexicon.

The emphasis in this methodological approach is thus on allowing critical categories and meanings to emerge from the ethnographic interaction (in our case the encounter involving ourselves with our partners) rather than imposing these from existing models.

Research Questions. The research outlined in this paper has been driven by the following research questions:

1. What are the terms that teachers use to describe the phenomena of the Australian middle school mathematics classroom?
2. What is the significance of the things that are named by Australian middle school mathematics teachers in relation to the phenomena of their classrooms?

One of the immediate and significant products of this work is the documentation of a collection of elements (terms, descriptions, examples, and non-examples) that together make up the Australian Lexicon, that is, the vocabulary used by teachers to describe the phenomena of the middle school mathematics classroom.

Research Design

Research Context. The research outlined in this paper is undertaken as part of a larger project *The Lexicon Project*, being undertaken simultaneously in Australia, Chile, China, the Czech Republic, Finland, France, Germany, Japan, and the U.S.A. Research teams in each country are documenting their teaching community's lexicon: "the vocabulary of a person, language, or branch of knowledge" (Stevenson, 2015), in order to use these as analytical tools to categorise, interrogate and enrich classroom practice, classroom research, and educational theorising. This paper relates only to the Australian Lexicon.

Stimulus Package. A video package of nine mathematics lessons (one from each participating country) was a key catalyst for the initial generation of the key terms in the lexicon. These lessons were selected by each country team to maximise the diversity of activities displayed. Each team contributed video material, time-stamped transcripts and classroom supporting material for one lesson of mathematics at Year 8. The video data files were configured into one viewing window (see Figure 1) – a synchronised display of three videos, arranged as teacher camera, whole class camera, and student camera, with all public utterances shown as English subtitles, and a time counter, to allow for the recording of starting and finishing times of each video excerpt illustrative of a particular term.

Figure 1. The video "three-up" (three camera angles with time-code and subtitles).

Generating Data. The Australian research team is composed of two mathematics teachers of more than 20 years of experience, a recently graduated teacher, and four academic researchers. All research team members viewed the Australian lesson, whilst the remaining eight lessons were assigned to team members using a matrix structure ensuring at least one experienced teacher viewed each lesson and each lesson was viewed by a minimum of four team members. The prompt used for stimulating thought whilst watching the video was, "What do you see that you can name?" A standardised template was used to record any term that came to mind. It was not necessary to identify a video example of each of the terms generated as the primary purpose of the video was to stimulate thinking about classroom events, actions, and interactions and the recollection of associated terms.

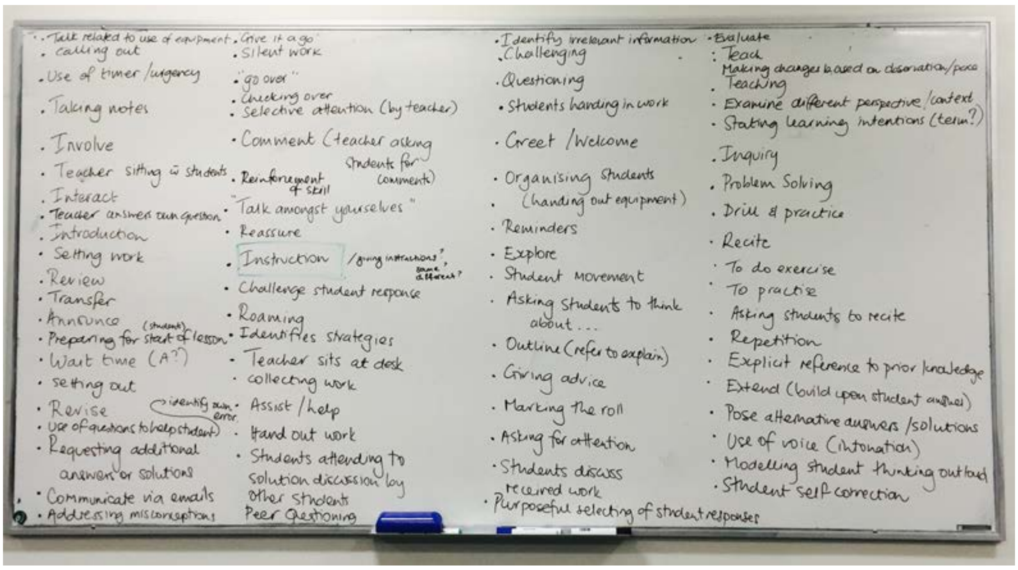

Figure 2. Image of candidate terms or short phrases for inclusion in the lexicon (August, 2015).

Generating the Lexicon. At regular meetings, the research team shared terms, phrases, and short descriptions of familiar activities that were felt to be possible candidates for inclusion in the lexicon (see Figure 2). In order for a term or short phrase to be included in the lexicon, team consensus was required, and if agreement was not reached, authority was accorded to classroom experience. In other words, the teachers on the team were given final say about whether a term was indeed likely to be familiar to teachers. We also found it useful to include two additional categories for the classroom events that did not seem to meet the criteria for inclusion:

- Phrases that are recognizable and readily understood, describing familiar classroom phenomena for which there did not appear to be a single, institutionalized name (e.g., setting a time limit).
- Familiar Activities, those pedagogical activities that are seldom described or referred to, but have a familiar quality to them (e.g., arranging the seating).

We felt it useful to record items falling into these two categories, in part, to anticipate the possibility that these practices might be named by other communities.

The Australian Lexicon

The Australian (middle school mathematics classroom) Lexicon consists of 63 terms that are familiar and in use by teachers in the mathematics education community. Because video played an important role in stimulating recognition of terms, it is possible to illustrate many of these terms with video exemplars.

Assessment	Any activity undertaken by the teacher or a student(s) with the primary purpose of generating information about student learning or achievement.	For example: • The teacher administers a test. • The teacher observes students while they work, making notes on each student's progress. Non-example: • Assigning homework, unless the teacher explicitly indicates that the purpose is assessment.
Practising	The activity of repeating a procedure for the purpose of improving efficiency or accuracy in its use.	For example: • A student solves ten consecutive tasks all involving the addition of fractions. • A student works through the problems on past exam papers. Non-example: • A student attempts to make use of the property of similar triangles in a real-world context for the first time.
Scaffolding	A form of (typically verbal) guidance intended to influence a student's thinking in order to assist them in the achievement of some cognitive task (e.g. solving a problem or learning a new skill).	For example: • The teacher (while walking around the room) asks a student whether the student's current approach is effective (stimulating the student to reflect on their approach). • The teacher asks, "Is there a diagram you could draw?" Non-example: • The teacher suggests that the student use the method just taught.
Worked Example	The teacher (or student) writes out the steps involved in order to illustrate the type of solution expected to a problem or task (a model answer) with or without student involvement.	For example: • Teacher writes out the solution to a problem on the whiteboard, providing oral explanations and clarifications along the way. Non-example: • Students recording their solutions at the board.

Figure 3. A sample of the operational definitions developed of the terms in the lexicon.

For each term, composite operational definitions were generated including the following essential elements: (a) the agreed name of the term; (b) a description, (c) examples, and (d) non-examples. These operational definitions were subjected to a validation process to investigate the extent to which the community of mathematics education researchers would endorse the constituent terms of the Australian Lexicon. A selection of terms together with their operational definitions is provided in Figure 3.

Local Validation. Two groups of people were invited to participate in a local validation of the lexicon: mathematics education researchers (specialists) and education researchers. The intention for recruiting the first group was to investigate the extent to which the local community of mathematics education researchers would endorse the purpose, the structure, and the constituent terms of the Australian Lexicon. The second group was recruited to provide a check on the possible cross-disciplinary nature of the mathematics lexicon. The first group (eight participants) was, strictly speaking, the group that was "validating" the terms, descriptions, examples, and non-examples from the perspective of the discipline of mathematics education, but not from the perspective of mathematics teachers, which is being undertaken separately. The second group (11 participants) provided an understanding of how widely used and understood the terms are outside of mathematics education.

This supplementary data collection to validate our reflections confirmed that the 63 terms in the Australian lexicon all identify general pedagogical practices. Not one of the terms is unique to the classroom of mathematics. Although a practice like practising might "appear" quite different in the mathematics classroom from, say, a music classroom, the intent and description of the term might be understood by both teaching communities albeit illustrated with different examples and video material.

National Validation. 120 teachers across Australia participated in an online survey in which they indicated their familiarity with each of the terms in the Australian Lexicon, as well as commenting on the clarity and appropriateness of the descriptions and examples and non-examples provided for each term. The 63 terms, descriptions, examples, and non-examples in the Australian Lexicon were validated locally and nationally in this fashion. Whilst the online survey has fulfilled the purpose of national validation, its function as a data collection device continues, and, in a later phase of the project, teacher responses across Australia will contribute to the profiling of term familiarity and use for different sectors of the Australian mathematics education community, including differentiation by level of experience.

Structure of the Lexicon. Whilst identifying terms for inclusion in the lexicon, thought was given to the possible structure or format that would best communicate the content of the lexicon. A university class of practising teachers was invited to group the items in the lexicon. Three categories were identified across almost all the item clusters generated: Administration, Assessment, and Classroom Management. However, the categories suggested in addition to these illuminated a very interesting aspect of classroom practice (see Figure 4 for these additional categories).

Team 1	Team 2	Team 3	Team 4	Team 5
Building Self-Confidence (Intrinsic)	Differentiation Strategies	Group work	Discussion	Cognition and Metacognition
Metacognitive Skills (Learning Strategies)	Direct Instruction	Positions in the Room	Questioning	Directing
Peer Learning and Teaching	Teaching Strategies	Student Output	Showing Understanding	Feedback
Setting the Scene/ Structuring the Learning Environment	Relationships	Tone of Environment	Student Activity	Grouping
Teacher Directed throughout Lesson	Responses	Strategies of Teaching and Learning	Teacher Movement	Representation
	Student Directed		Teaching Strategies	Speech
	Students Working Together			Student Mechanics
				Support
				Teacher Mechanics

Figure 4. Additional teacher-suggested categories for the lexicon terms.

The diversity of groupings employed across five teams of teachers was initially quite surprising. However, on reflection, it is quite reasonable to suppose that individuals' associations with the mathematics classroom would reflect the diversity of their personal histories and, in addition, that teachers practise their art differently. This tolerance of idiosyncrasy within education may indeed be one of the defining characteristics of the Australian teaching community. In other words, teachers have the freedom to develop a highly personal pedagogical style, with an associated personal vocabulary, and a sense of the context in which such a term might apply that reflects the teacher's personal educational history. The research team decided on two additional categories that captured the spirit of the teachers' suggestions: Learning Strategies and Teaching Strategies.

Communicating the Lexicon. The 63 terms of the lexicon were organised into the five categories consistent with the groupings suggested by teacher practitioners: Administration (eight terms), Assessment (11 terms), Classroom Management (six terms), Learning Strategies (27 terms), and Teaching Strategies (50 terms). An illustrative selection of terms, organised by category, is given in Figure 5.

Administration	Assessment	Classroom Management	Learning Strategies	Teaching Strategies
Collecting Work	Giving Feedback	Disciplining	Answering Questions	Prompting
Handing Back Work	Monitoring	Giving praise	On Task	Wait Time
Marking the Roll	Test/Testing	Raising Hands	Practising	Whole Class Discussion

Figure 5. A sample of terms present in each of the organisational categories.

A number of terms appeared in more than one category when members of the research team agreed there was a strong association of the term with each of the categories. Indeed, 24 terms were found in both the Learning Strategies and Teaching Strategies categories, whilst three terms were associated with three categories (see Figure 6 for a selection).

Learning Strategies	Teaching Strategies
Board Work	
Group Discussion	
Questioning	
Summarising	

Assessment	Learning Strategies
Correcting	
Note-taking	

Assessment	Learning Strategies	Teaching Strategies
Reviewing		

Figure 6. A sample of terms present in more than one category.

Another interesting feature of the lexicon is that very few terms reveal a singular pedagogical intention or purpose in engaging in the particular instructional practice or activity. For example, the Worked Example might be used to introduce a new skill, review a homework task, or model an approach to a worded problem. This attribute of many of the terms of the Australian Lexicon might be seen either as inclusiveness or as lack of precision. Other lexicons employed by other communities may employ terms that are far more specific as to the purpose of a named activity. By way of contrast, some terms within the Australian Lexicon are specific either as to purpose or as to location in the instructional sequence. For example, one could argue that Reviewing and Summarising suggest that the named activity assumes the occurrence of some prior event or activity, providing a partial specification of both purpose and location in the instructional sequence. The majority of terms in the Australian Lexicon lacked even this level of specificity.

Another form of imprecision or ambiguity within the Australian Lexicon arose from the prevalence of gerunds (noun/verbs). As can be seen from the examples in Figures 5 and 6, participles were widely employed in this way (marking, questioning, monitoring). Such terms give a sort of dynamism to the Australian Lexicon that may or may not be evident in the lexicons employed by other communities.

Conclusion

A sophisticated professional language of practice would greatly advance discussion about classroom practice. Our entry point in the development of this professional lexicon has been the empirical identification of the lexicon in current use by middle school mathematics teachers. A robust and coherent lexicon, defined and illustrated, would provide a common point of reference for teachers and teacher educators alike. Then, the adequacy of this lexicon to encompass and distinguish the variety of practices and pedagogical and didactical phenomena prioritized by contemporary mathematics education could be evaluated. We could also determine if differences exist between the language that the researcher community of practice (CoP) uses to identify classroom events and actions, and the language used by the educator CoP (Lave & Wenger, 1991). Any differences might have significant implications for the translation of research findings for practitioner use.

If the general aim of an education research community is to better equip pre-service and in-service teachers, an essential starting point is engaging both groups in a study of the "terms" that feature in teachers' professional speech when conceptualising the practice of the classroom. Equipped with such a lexicon, teachers will be better able to reflect on and improve their practice. The primary intention of this research was to provide insight into the naming system employed by middle school mathematics teachers in Australia in relation to their classroom practice, by documenting and interpreting the constructs that are well-known, understood, and used in discussions with others. From this foundation, we hope to inform national and international efforts to better equip contemporary mathematics teachers with a sophisticated lexicon to shape their professional practice.

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Using Coding to Promote Mathematical Thinking with Year 2 Students: Alignment with the Australian Curriculum

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In this paper, we present data from a study exploring the use of coding to promote mathematical thinking. A teaching experiment was undertaken with 40 Year 2 students participating in six 45-minute lessons of coding (one lesson per week for six weeks). All lessons were video-recorded and analysed to determine students' mathematical thinking. Insights from the study reveal that coding contexts promoted higher levels of mathematical thinking for Year 2 students including measuring angles, orientation and perspective-taking, and deducing repeating patterns.

There is an international urgency to improve Science, Technology, Engineering and Mathematics (STEM) education in preparation for a scientifically and technologically advanced society (Office of Chief Scientist, 2014). This push is also in response to the rapid decline in secondary school students' engagement in STEM disciplines (e.g., advanced mathematics, chemistry) (Australian Academy of Science [AAS], 2016). Disengagement in STEM begins at an early age (Larkin & Jorgensen, 2016), with many students from the upper primary years onwards failing the most important STEM subject – mathematics (AAS, 2016). To address this issue, coding has recently been included in the Australian Curriculum: Technology as a way to "re-engage" students in the sciences and potentially develop mathematical thinking. One example of a recent government initiative is the National Innovation and Science Agenda, which funds ($51 million) programs specifically dedicated to coding for primary school students (Department of the Prime Minister and Cabinet, 2016). The convergence of policy and curriculum directions is heartening; however, it is also highly problematic as there is a limited evidence base to inform the implementation of STEM in classrooms (English, 2016). We were particularly concerned with: How does the use of coding in primary school classrooms support, or provide opportunity for, the learning of mathematics? Using the coding program Scratch, we will discuss the ways that working with such programs provide opportunities to develop mathematical thinking. Here, we present the first lesson in a teaching experiment, where students used Scratch to draw a square.

Literature Review

Scratch is a visual programming language developed by the Lifelong Kindergarten group at MIT Media Labs (Resnick et al., 2009). Designed for students eight years of age and older, Scratch promotes creative thinking, reasoning, and innovation for those who engage with the program (Resnick et al., 2009). The rich digital environment utilises building block command structures to manipulate graphics, audio and video functions (Calder, 2012). The building block commands are a form of simplified syntax, so students are not required to type the code themselves; rather, they drag and drop the interlocking blocks of symbolic code together to create chains of code. There are 10 categories of building block command structures action, and each is represented by a specific colour. Examples of these colour categories include: motion blocks (blue); logic/control blocks (gold); and, data blocks (orange) (Francis, Khan, & Davis, 2016). Each coding block

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within a category has text and symbolic commands to assist the user to select the appropriate code for the action that they would like to undertake. Similar to the LOGO turtle (Papert, 1980), the interface enables a cat to move on a two-dimensional screen. Figure 1 presents the Scratch interface with an example of code for drawing a hexagon.

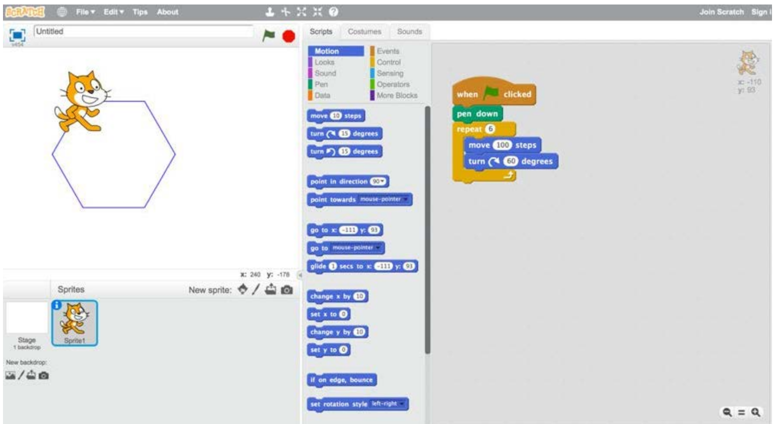


Figure 1. Scratch interface.

Studies Focusing on Coding

Research indicates that computer programming (henceforth coding) provides an opportunity for developing students' cognition and mathematical knowledge (Papert, 1980). Noss and Hoyles (1996) state that "writing a computer program provides a broad canvas on which the learner can sketch half-understood ideas, and assemble on the screen a semi-concrete image of the mathematical structures he or she is building intellectually" (p. 55). Research into the teaching and learning of mathematics through coding and programmable robots, including the use of LOGO (Clements, Battista, & Sarama, 2001) and Beebots (Highfield, 2010), have indicated that programmable robots support students in exploring problem solving, measurement, geometry and spatial concepts (Savard & Highfield, 2015). In addition, findings from quantitative studies have revealed that there is a correlation between computer coding using Scratch and mathematics test grades for Year 5 students (Lewis & Shah, 2012). With the exception of research conducted using LOGO, much of this research is in its infancy. Many studies examined the use of Scratch in middle school but few examined the use of Scratch in lower primary years. Finally, Benton, Hoyles, Kalas, and Noss (2017) stress that much of the past research into the impact of coding on students' mathematics acquisition is inconclusive; and, due to the diversity of adopted research paradigms across these studies, it is difficult to compare the results.

During the period 1970-2000, there were pockets of enthusiasm regarding the teaching of coding (e.g., LOGO and BASIC); however, there were several factors hindering its wider application: (a) many students had difficulty mastering the syntax of the program, (b) programing had little connection to young people's interests (e.g., generating lists of prime numbers), and (c) coding was introduced when there were few experts who could provide the students guidance (Resnick et al., 2009). It could be argued that the last point still resonates in the current educational climate. Many generalist primary school teachers are underprepared to teach coding and therefore will likely have difficulty in establishing links between coding and the teaching and learning of mathematics (Benton et al., 2017). The underlying mathematics that students engage in when coding can be unseen by teachers who often focus on the use of the tool (visual coding program or robots) rather than the mathematics within the tasks (Savard & Highfield, 2015). Few studies focus on the classroom implementation of coding and the curriculum (Lye & Koh, 2014). Coding appears in the Australian Curriculum areas of Mathematics and Technology (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016a, 2016b). The relationship between these documents is important to teachers who use the curriculum to plan, teach and evaluate student learning. Making the mathematics in coding apparent to teachers in curriculum documents is essential.

Linking Mathematics and Coding in the Australian Curriculum

Coding is explicitly embedded within the Digital Technologies (DT) strand of the Technology Curriculum and given that this paper concerns coding; we will limit discussion only to this component. The DT curriculum outlines that students will use "computational thinking and information systems, to define, design and implement digital solutions" (ACARA, 2016a). The DT curriculum in primary school is divided into three bands (F-2, 3-4, and 5-6) that are further subdivided into content descriptors under the sub-headings of Knowledge and Understanding and Processes and Production Skills. When examining the ways in which the DT curriculum builds opportunity for development of mathematical thinking, its alignment with the Mathematics curriculum is an important consideration. As we are solely concerned here with using Scratch with Year 2 students, the relevant Year 2 Mathematics and DT descriptors are displayed in Table 1.

Table 1
Mathematics Curriculum and Digital Technologies Curriculum

Subject	Content Description	Elaboration
Mathematics Year 2: Geometry	Describe and draw two-dimensional shapes, with and without digital technologies (ACMMG042)	Identify key features of squares, rectangles, triangles, kites, rhombuses and circles, such as straight lines or curved lines, and counting the edges and corners.
Digital Technologies Foundation – Year 2: Process and skill production	Follow, describe and represent a sequence of steps and decisions (algorithms) needed to solve simple problems (ACTDIP004)	Experimenting with very simple, step-by-step procedures to explore programmable devices, for example providing instructions to physical or virtual objects or robotic devices to move in an intended manner, such as following a path around the classroom.

The learning sequence in this study was designed to address the content descriptors in Table 1. The research questions explored in this paper are:

1. In what ways does working with coding contexts such as Scratch provide opportunity to develop mathematical thinking?
2. How do these opportunities align with the Australian Curriculum content descriptors for Mathematics and Digital Technologies?

Research Design

Participants

Six classes of Year 2 students (7-8 years old) from two schools located in Brisbane participated in the study. In total, there were 153 Year 2 students: 74 students from School A and 79 students from School B. Both schools were matched for socio-demographic characteristics. Both schools are just above the median for socio-educational advantage (School A = 1,056; School B = 1,037; ICSEA median value = 1,000).

Methodology: Teaching Experiment

A six-week coding and robotics teaching experiment was conducted with Year 2 students. The aim of the teaching experiment was to explore how students developed mathematical knowledge and thinking as they participated in coding and robotics lessons. Teaching experiments were used in this study for the primary purpose of directly experiencing students' mathematical learning and reasoning in relation to their construction of mathematical knowledge (Steffe & Thompson, 2000). One of the researchers (Author 1), in consultation with the class teacher, assumed the role of teacher in these experiments at both school sites. The teaching experiment comprised of: (a) pre-testing, (b) 6 x 45 minute lessons of either coding or robotics lessons (one lesson per week for six weeks with two groups of 10 students at each school site), and (c) post-testing.

All students participated in pre-testing measures at the commencement of the teaching experiment to identify their prior patterning knowledge and coding knowledge. As coding and mathematical patterning are related, it was decided to test the students on these two constructs. The test data are not the focus of this paper, however, do explain how students were selected for the study. This was a pen and paper test that focused on patterning (10 items) and coding from Scratch contexts (10 items). All test items were read to the students, and the test took approximately 30 minutes to complete. The items from these tests were developed from previously trialled patterning test items (Miller, 2015; Warren & Cooper, 2008) and then modified for coding contexts. Data from the pre-tests were analysed to determine a smaller, experimental group of students (n = 40) to participate in the coding and robotics lessons. Students were selected on their prior knowledge of patterning and coding (low-mid-high test scores). Four subgroups of students were identified: low patterning/low coding, low patterning/mid coding, mid patterning/low coding, and mid patterning/mid coding. No students were classified as high in either pattering or coding. Each subgroup consisted of 10 students with an even number of male and female students in each. Students not selected for the study (n = 113 spread over the six participating classes) stayed with their classroom teacher and participated in normal class lessons as planned by their teacher for that time. These teachers (n = 6) did not teach robotics and coding in their classrooms during the experiment. At the conclusion of the six weeks, post-testing (patterning and coding test) was conducted with all students (n = 153).

The teaching experiment consisted of six lessons, three with a coding focus using Scratch and three with a robotics focus using LEGO Mindstorm robots. Each lesson focused on teaching a mathematical concept using coding or robotics (e.g., drawing a square, drawing spirolaterals, moving a robot along a particular path). In this paper, we only present findings from the first lesson in the teaching experiment, where students were required to use the Scratch program to draw a square (See Figure 2). This lesson was aligned to the Year 2 Mathematics and DT Curriculums (see Table 1).

Task	Mathematical concepts	Scratch coding concepts
Scratch Cat's favourite shape is a square. Can you help Scratch Cat draw a square? 	Discussing properties of a square Testing and measuring lengths. How far is one step in Scratch? Testing and measuring Angles/turns Identifying any repeating patterns	Exploring the coding blocks and symbols used in scratch Measuring in pixels Building a code Running a code Editing a code if there are errors Using the repeat/loop function

Figure 2. Drawing a Square task using Scratch with links to mathematics and coding concepts.

Two video cameras were used to collect data during each lesson of each teaching experiment, with one camera focussed on the researcher and one on a group of students. These video-recordings were used for in-depth analysis by the authors.

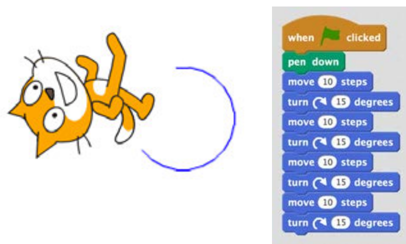
Data Analysis

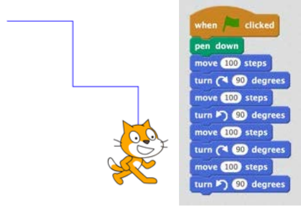
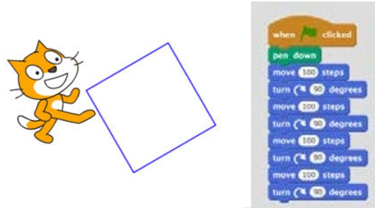
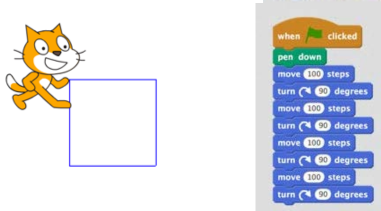
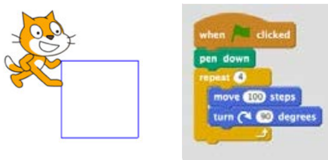
An iterative approach, using iterative refinement cycles for videotape analyses of changes in students' thinking, was adopted to analyse the data from the teaching experiment lessons (Lesh & Lehrer, 2000). Due to the unique application of mathematics, coding and robotics, this data-analysis model, utilised in prior studies (Miller, 2015), comprises two key stages. First, the lesson video-footage was transcribed to capture students' verbal responses. These transcriptions were then analysed to consider emerging mathematical thinking evident during the lessons. Second, the data were analysed to align the curriculum descriptors with student responses to the coding and robotics lessons.

Findings and Discussion

The findings are presented in two sections. Firstly, the emerging themes of the students' response to the "Draw a Square" Task are discussed. Secondly, the alignment of the task against the Australian Curriculum Mathematics and Digital Technologies descriptors is reviewed. Each of the 40 students provided a response to the task. After analysis of the student responses, it was evident that there were five common types of responses to the "Draw a Square" Task. Table 2 displays the student approaches, the frequency of the types of responses, and an example of a student's work fitting this type.

Table 2
Student Response and Explanation of Approach, Frequency of Student Responses, and Example of Students' Work

Response and Explanation	Frequency	Example
I can't draw a square but I can draw a hexagon Student attempted to draw a square but used 15 degree turns. While this does not draw a hexagon, the majority of the Year 2 students articulated they were creating a hexagon. They clicked on the code four times to make this shape.	8	

Response and Explanation	Frequency	Example
I can draw stairs: Why is the cat not turning the right way? Students did not construct a square as they alternated the turns to the right and left.	4	
Is this still a square? Students correctly coded a square but were unsure whether it was a square or not because of the orientation.	3	
I made a square. Students were able to write a code to make the Scratch cat draw a square parallel to the bottom of the screen.	20	
I can see a pattern: move, turn, move, turn, move, turn…. Students identified that they can see a repeating pattern and used the repeat coding block to draw a square.	5	

When considering the above responses, in relation to the content requirements of the Year 2 Mathematics curriculum (see Table 1), it is evident that some students were working at a higher level than required. There were three key insights from the data that demonstrated higher levels of mathematical thinking: (a) working with 90 degree turns, (b) orientation and perspective taking, and (c) deducing a repeating pattern to provide a generalised code for making a square. When drawing a two-dimensional shape, using a digital tool such as Scratch, it is conjectured that this provides the opportunity for students to engage with higher levels of mathematics. This may occur for three reasons: first, as a consequence of the visual programing language (icons) and representations; second, the perspective which performing the task requires; and third, using a coding chain that represents the structure of the mathematical shape the students have drawn.

When using Scratch to draw a square, it appears the language and representations depicted in the coding blocks require higher levels of mathematical thinking and knowledge for these young students. For example, Scratch uses measures of degrees for turns, rather than language such as 1/4 turn. This program appears to offer more opportunities to explore some aspects of mathematics (e.g., measurement of angles) than programmable robotics toys, such as the commonly used Beebots, that have an arrow indicating left or right which results in the Beebot performing a 90 degree turn. When using Scratch students have to determine the number of degrees themselves in order to perform an accurate 90 degree turn. Aligning this mathematical thinking to the Year 2 Mathematics curriculum, students are only required to identify the corners of shapes and use the language of "quarter and half turns". It is not until students are in Year 5 that they are required to estimate and measure angles using degrees (ACARA, 2016b). Our initial supposition is that the representations and language in Scratch supports the development of higher levels of mathematical thinking for these young students.

Second, the way in which students engage in the task of drawing a square using Scratch is vastly different to drawing a square on paper using a pencil and ruler. As students code the Scratch cat to draw a square, they are required to take the perspective of the cat (i.e., the square will be drawn in the same orientation as the cat is initially facing). Students who started their cat either facing up or down, or an alternative sideways orientation other than the cat facing directly left or right of the screen (student's perspective while looking at the screen), meant that the square could be drawn from different initial perspectives and thus end up looking different to the prototypical depictions of squares (parallel to bottom and sides of the screen). This was evident in the case where students were unsure if they had still drawn a square for the reasons outlined above. While they could code a square, their limited mathematical understanding of "squareness" meant they were unable to reason if their shape was a square or not.

Finally, unlike drawing a square on paper using a pencil and ruler, some students could see on the screen the "structure", that is a semi-concrete mathematical structure (Noss & Hoyles, 1996), of their drawing in the Scratch code. This led to five students, unprompted by the researcher, identifying units of repeat (e.g., move 100 steps, turn 90 degrees) and deducing that their code (repeat four times – move 100 steps, turn 90 degrees) would draw a square. While, students in Year 2 should be able to identify a repeating pattern, this moves beyond the typical patterns presented to students (e.g., ABAB). This led to students then deducing a generalisation for the perimeter (e.g., move *n* length, turn 90 degrees and repeat four times) of the square and even further to discussions about measuring the perimeter of squares using the code (e.g., If my square has a length of 10, the total perimeter will be 40). We suggest that these students were demonstrating, and engaging in, early algebraic thinking (deducing patterns) and higher levels of measurement (identifying the perimeter), beyond the current required curriculum standard, such as Year 6 – Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133) and Year 5 – Calculate perimeter and area of rectangles using familiar metric units (ACMMG109) (ACARA, 2016b).

Conclusion

The implementation of coding into the lower primary years presents a challenge for generalist primary school teachers. Commonly, past research in this area has provided insights into how students in the middle years of schooling work with coding contexts, but at times the impact on mathematics acquisition for these students are inconclusive (Benton et al., 2017). This study adds to the current literature by examining the use of Scratch in a lower primary context, and identifying the types of mathematical thinking these students engaged with while undertaking the task. Research with primary school students, when using robotics programs, have identified that young students use problem solving, measurement, geometry and spatial concepts (Savard & Highfield, 2015), in these contexts. Our early conjecture is that coding also provides an opportunity to identify and deduce patterns and therefore is a platform to engage with early algebraic thinking.

With few studies focusing on the classroom and curriculum implementation of coding (Lye & Koh, 2014), there is a need to examine the relationship between the Mathematics and Digital Technologies Curricula and coding contexts to maximise learning opportunities for primary students. As teachers use the curriculum to manage their planning, teaching and evaluating of student learning, making the mathematics in coding apparent for teachers within both curriculum documents is essential. Although this is an initial, small-scale study over a relatively short intervention, it begins to indicate the potential of coding programs such as Scratch to support students' mathematical thinking and concept development.

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Online, Anytime, Anywhere: Enacting Flipped Learning in Three Different Secondary Mathematics Classes

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Flipped learning is gaining in popularity as a teaching approach in secondary mathematics classrooms. Traditionally seen as the domain of tertiary teaching, flipped learning has a number of affordances that address the challenging demands of teaching secondary mathematics. Enacting this approach requires a reconceptualization of traditional secondary mathematics instruction in that instructional content is assigned as homework before class, providing for more targeted in-class teaching. I describe three different enactments of the flipped learning approach and report on the teachers' and students' experiences of such an approach and the affordances it offers.

Traditionally considered the domain of higher education, flipped learning is increasingly being implemented in secondary school settings. While terms such as "flipped classroom", "inverted classroom" and "flipped learning" are used interchangeably in the literature, Bergmann and Sams (2012), who are credited with conceptualising the approach, prefer the term "flipped learning", which is defined as:

> a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter. (Flipped Learning Network [FLN], 2014, para. 1)

Advocates of the approach report increases in student achievement, success, and engagement (Hamdan, McKnight, McKnight, & Arfstrom, 2013), along with benefits such as increased student-teacher interaction and differentiated teaching for a range of student abilities (e.g., Straw, Quinlan, Harland, & Walker, 2015). The flexible nature of the approach allows students to extend their knowledge "at a pace, in a place and with an educational purpose that suits them" (Ministerial Council on Education, Employment, Training and Youth Affairs, 2003, p. 4), with technology giving them greater control over how, where and when they learn (Australian Curriculum, Assessment and Reporting Authority, 2014). It provides an arguably more engaging alternative to traditional homework practices, which have been perceived by many middle school students as boring, too easy or too hard, or irrelevant (Xu & Wu, 2013). It also has the potential to enhance secondary mathematics practice which has traditionally been dominated by textbook use and externally imposed assessment measures (e.g., Muir & Chick, 2014). Students have shown motivation to engage with the approach, which is important as student disengagement in mathematics is of ongoing concern (Skilling, Bobis, & Martin, 2015). This paper adds to previous research through describing three different enactments of flipped learning in secondary mathematics classes, along with students and teachers' perceptions of the impact of the approach. Specifically, the paper addresses the following research questions: How is flipped learning enacted in three different secondary mathematics classes? How do these enactments impact upon students' uptake of the approach?

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Review of the Literature

Flipped Learning

According to Bergmann, Overmyer and Wilie (2013), flipped learning is characterised as a space where students take responsibility for their own learning, a classroom where students who are absent are not left behind, all students are engaged in their learning, class content is permanently archived for review or remediation, and students receive a personalised education. The Flipped Learning Network (FLN, 2014), established by Bergmann and Sams, distinguishes between a flipped classroom and flipped learning, and advocates that teachers must incorporate the four pillars of FLIP into their practice in order to engage in flipped learning. Table 1 provides a summary of each of the pillars.

Table 1
Overview of Four Pillars of FLIP

Pillar	Characterised by
Flexible environment	Establishment of spaces and time frames that permit students to interact and reflect on their learning as required; flexible spaces which allow students to choose when and where they learn
Learning culture	Giving students opportunities to engage in meaningful activities without the teacher being central; activities are accessible to all students; learning is personally meaningful
Intentional content	Concepts used in direct instruction are prioritised for learners to access on their own; relevant content is created or curated for students; content accessible and relevant to all students
Professional educator	Teacher available to all students for individual, small group, and class feedback in real time as required

It is important to emphasise that flipping a class can, but does not necessarily, lead to flipped learning, and that there is "no single way to flip your classroom… no specific methodology to be replicated, no checklist to follow that leads to guaranteed results" (Bergmann & Sams, 2012, p. 34). While "flipped mastery" may be the ultimate aim, Bergmann and Sams (2012) recommend that teachers gradually make the change to flipping, adapting it to their current practices and contexts.

Enactments of flipped learning and classrooms in the literature include examples of teachers sourcing existing online resources (e.g., Straw et al., 2015), creating video content for teacher-paced instruction (e.g., Muir & Chick, 2014; DeLozier & Rhodes, 2017) and flipped mastery where students set the pace for their learning (e.g., Muir, 2016). In a collective case study involving nine U.K. secondary schools, mathematics teachers were asked to use Khan Academy mathematics resources in their delivery of flipped learning for one year (Straw et al., 2015). Straw et al. (2015) reported a range of benefits including increases in students' knowledge and understanding, confidence, progress and attainment. Reported challenges included access to technology, identification of appropriate online resources, students not participating in preliminary homework, and teachers and/or students' preferences for face-to-face, as opposed to remote instruction.

Arguably the most widespread approach reported in the literature requires students to watch pre-recorded video lectures or screencasts prior to attending class and is particularly popular in tertiary settings (e.g., Abeysekera & Dawson, 2015; DeLozier & Rhodes, 2017).

The flipped learning approach may assist with student motivation through developing students' autonomy, competency, and sense of relatedness (Abeysekera & Dawson, 2012).

There are few examples in the literature of mathematics secondary classrooms that have adopted a flipped mastery approach. Muir (2016), for example, described two cases whereby senior secondary mathematics teachers provided a bank of teacher-created video resources for students to access individually and work through at their own pace. Both cases involved courses that were based upon textbooks, with one being subject to externally imposed assessment measures. Muir (2016) found that in contrast with traditional practices experienced in the past, students reported increased satisfaction with the relevancy of materials provided, and greater engagement with, and autonomy over, their learning. Other identified affordances included accessibility, assessment preparation, self-pacing, and optimisation of class time.

Methodology

The research reported in this paper was part of a larger study that employed a mixed-methods approach (Creswell, 2003) to investigate students' and teachers' experiences of flipping the classroom in 10 secondary mathematics classes. Participating students completed online surveys containing a mix of Likert-scale items and open-ended questions, interviews were conducted with teachers and students, and classroom observations were undertaken. Sequential methods (Creswell, 2003) were used to inform the interview questions, allowing more detailed exploration with a few cases or individuals.

Table 2
Overview of Participants in Each Case Study

Enactment	School[1] and context	Grade	Teacher	Students
Teacher paced curated	Keating College (independent metropolitan)	8 (mixed ability)	Mr Shepherd	22
Teacher paced created	Howard College (large independent metropolitan)	12 (Mathematics Methods)[2]	Ms Brown	15
Student paced, teacher created	Fraser College (large metropolitan secondary college)	12 (Specialist Mathematics)[3]	Mr Burns	9

For the purpose of this paper, three cases which illustrate three different enactments of the flipped classroom approach have been selected for discussion. An overview of each case's context and participants is presented in Table 2. Nineteen of the 46 students participated in focus group interviews, which were all audio-taped, fully transcribed, and took approximately 20 minutes. Qualitative data from the interviews and open-ended survey responses were analysed using reflexive iteration (Srivastava, 2009), whereby each sentence in the transcript was coded using open themes. These themes were then analysed

[1] Pseudonyms are used for schools, teachers, and students throughout this paper.

[2] Mathematics Methods is a senior secondary pre-tertiary course that covers topics such as functions, calculus, and statistics and is externally examined.

[3] Specialist Mathematics is considered the top-level mathematics course for Year 12.

to identify evidence related to the four pillars of Flipped Learning (FLN, 2014), along with affordances identified by Muir (2016).

Results

This section has been organised around the three different enactments of the flipped learning approach as experienced in the classrooms of Mr Shepherd, Ms Brown, and Mr Burns. It contains evidence from all data sources as indicated throughout.

Teacher Paced Curated

Lesson observations showed that Mr Shepherd's enactment of the flipped classroom involved an expectation that his students had watched the prescribed videos before class (The majority had). The lessons observed and video tutorials assigned were based around identifying number patterns as part of an algebra unit. Following a brief review of patterning and balancing equations, the majority of the lesson was spent on students individually completing the class allocated exercises from the prescribed textbook. Mr Shepherd's role was to monitor student on task behaviour and assist individuals as required. He occasionally stopped the class to seek feedback on progress, but there was little teacher demonstration or whole class facilitated discussion. In his interview, Mr Shepherd indicated that he generally sourced his video tutorials from 8-10 YouTube channels or regular contributors, and acknowledged that it was sometimes time-consuming to identify appropriate material: "One video might only take about 12 minutes to watch, but it could have taken me an hour or more to find".

The students who were interviewed following the lesson indicated that what was observed was typical in that "we review what we did, what we learnt from the videos and then if we have questions from the video, we ask and then we go over it … We then use the textbook to answer questions based on what we had just learnt for practice" (Chloe). Students generally described the nature of the video tutorials as "a video of someone talking while drawing up the problem and how to solve it using a diagram" (open-ended survey response). Quantitative student data showed that 86% of students had accessed the online tutorials throughout the year and 89% agreed that the tutorials helped them to understand a concept. Only 50% of students agreed that the tutorials were of the right length, and 58% agreed that they watched the tutorials from beginning to end. In terms of engagement, 50% indicated that they found the tutorials boring, yet 75% indicated that the tutorials helped them to better understand the work in class. When identifying advantages of online resources as compared to text books, opinions were mixed, with only 47% of students indicating they preferred online resources. Pragmatic reasons given for this included: "You can search what you want to know", "You can see people do it and it makes it easier to understand", and "You don't have to carry a text book home". Only one response referred to the affordance of being able to replay the video. When comparing the approach with mathematics teaching experienced in the past, interview responses showed that students compared it favourably:

> I would probably prefer to do this … you have more time in class to really understand it … to do the questions and stuff. (James)

> You don't go into the class cold, so you know what you're going to be talking about and you know what you're going to be doing. (Albert)

> It's good – I'm actually starting to kind of enjoy maths more … I especially enjoy maths when I get it, and when I don't I just hate it to be honest – but I'm understanding it better now. (Chloe)

Other interview comments revealed that for these groups of students at least, it was not important that their teacher prepared the videos, but appreciated that Mr Shepherd "looks for the best ones so they're really good and show understanding of the topic" (James). Along with relevance, students also indicated that videos which were entertaining were particularly effective, including reference to "the lady with the mammoth – it was actually quite funny … and I actually learnt a lot more then" (Chloe).

Overall this cohort of students could see some benefits of flipped learning, but "wouldn't recommend it to everyone … I think it sort of depends on the person and your ability to do it" (Martha). Reasons for this included the perception that "basically you have to teach yourself for most of it" (Arthur) and that "the harder the subject, the harder it might be teaching yourself" (Alison).

Teacher Paced Created

In this enactment, Ms Brown, like Mr Shepherd, also had an expectation that students had watched the allocated video tutorial/s before attending class, with the difference being that the video was prepared, delivered and recorded by Ms Brown. Classroom observations showed that, again like Mr Shepherd, students spent the majority of class time individually completing exercises from the prescribed textbook, which in this case involved the solving of simultaneous equations using matrices. Each lesson observed began with an eight minute "warm-up" where students worked individually from their textbooks. Ms Brown then facilitated students' oral responses to the problems, before briefly revising some of the content from the video tutorial that most students indicated they had watched. The remainder of the lesson (approximately 40 minutes) involved students working individually through allocated questions in the textbook, with Ms Brown individually assisting students who required assistance. Talk occurred between students but there was little whole class demonstration. Students indicated in the interviews that the lessons observed were typical.

At the time of the study Ms Brown had recorded approximately 20 video tutorials, all based upon topics in the textbook and all about an hour in duration. As reported in Muir (under review) Ms Brown preferred to create a video for each topic and then direct students to watch different parts of it, rather than break it up into shorter videos. She used PowerPoint with an OfficeMix add on to record her videos, which students accessed through an emailed link. This was provided to students at least three days prior to class. Student survey data showed that 100% of students had accessed the videos throughout the year and within the last month, and 100% of students agreed that the tutorials helped them to understand a concept and that the tutorials were helpful. Just over half (54%) indicated that the tutorials were of the right length, with only 38% indicating that they watched all of the tutorials from beginning to end. Interestingly, 77% indicated that they found the tutorials boring, yet 85% of students indicated that they accessed all or most of the video tutorials that were made available. In terms of comparisons with the text book, students' responses indicated that they viewed them as complementary, rather than a replacement, for either the textbook or the teacher:

> You can access the videos and information from anywhere. (open-ended response, survey)
>
> They are presented in different ways – if you don't understand the book, watching another person explain the concept can help you to gain a better grasp of the ideas and skills needed. (open-ended response, survey)
>
> For me, if the teacher said, watch this video as compared to doing 20 questions in the text book, I would do the video – it's more appealing. (Anna, interview)

When asked to compare the approach with that experienced in mathematics lessons in the past, the following comments were illustrative of students' perceptions:

> We didn't do questions like this, not all the time, like we used to sit and listen, but now she's doing more questions in class so that gives you more time with her one on one if you have questions, whereas I can remember some other topics, we would just sit and listen, and … we wouldn't do as many questions like we were doing today. (Hayley, interview)

> It's better having the video and watching it at home and being able to come and ask the teacher if I am still unclear about how to do something or a particular concept … I think it's better than last year where we would go through the book and rather than have lengthy explanation in class, it's better to have an idea before you get to class. (Anna, interview)

In their interviews, students identified a number of affordances with the approach, including reference to self-pacing, accessibility and convenience. Helen, for example, appreciated the autonomous nature of the approach, stating that: "You can always go back and view them, not like last year when you had to continuously ask for help".

Interestingly, students varied in their perceptions as to whether or not it was important that Ms Brown had prepared the videos. Abigail, for example, stated that "You understand it better when it's someone you know… and they can explain it again in a similar way in class if they have to". Anna, however, commented that:

> I don't think it's really important who does it – whether one teacher does the video or the entire maths faculty… but what's good about a teacher from school doing it as opposed to Khan Academy is that they know what the curriculum is and know what's important to focus on… The few times I did that [looked up on Google] it was extremely lengthy and only a few relevant points so it is easier having Ms Brown give us the videos – it's a lot more concise and relevant to what we want.

The above comment indicates that a sense of relevance should be considered as a motivator for students' engagement with the videos, along with competence, autonomy and relatedness (Abeysekera & Dawson, 2012).

Student Paced, Teacher Created

At the time of the study, Mr Burns had created 193 video tutorials for his students to access that covered the requirements of the Grade 12 Specialist Mathematics course. Topics in the course included conic sections, complex numbers, and differential equations. Typically students would access the videos for each topic, attend class where they would individually work through associated exercises in the textbook, then sit a test to demonstrate mastery of the topic. As with Ms Brown's approach, students were expected to access the video tutorials and complete related textbook exercises in class, but in Mr Burns' class, the students were all working at their own pace and on different topics. While some class discussions and demonstrations were observed, students tended to "opt in" according to relevance, and the majority of class time was spent on individual work, with Mr Burns providing personal assistance when required.

Student survey data indicated that 100% of Mr Burns' students had accessed his videos throughout the year, and most of the students had accessed them within the last month. Interestingly one student revealed in the interviews that he no longer accessed the videos as he felt that he already had a firm understanding of the topics covered. Other survey data showed that 86% of students agreed that the tutorials helped them to understand a concept, with 100% finding the tutorials helpful. Similar to Ms Brown's students, 29% of Mr Burns' students indicated that they found the tutorials boring, yet interview data showed they appreciated Mr Burns' dedication to producing the videos and his sense of humour.

Comparisons with the textbook revealed that, like Ms Brown's students, these students tended to view the approach as complementary. Open-ended survey responses included: "Online resources often give harder examples and more variety", and "Using online resources makes it possible to learn more information than what is possible in a textbook alone. A textbook can also be difficult to comprehend sometimes". Comparisons with past traditional mathematics instruction involved reference to affordances such as a capacity to focus and accessibility (Muir, 2016). Chris (Grade 12), for example, stated that:

> There's no interruptions [at home] whereas in class there are so many interruptions … he might be halfway through an explanation and then somebody interrupts … if you get distracted, [at home] you just pause the video and come back to it.

Mr Burns also identified similar affordances, along with the autonomous nature of the approach:

> A couple of months ago… one of the girls in the class came to me and she said… I've got to watch those videos three or four times before I understand what's going on and I thought to myself, gee I only teach it once and if I only taught it once, she wouldn't have got it.

As with the other enactments, students in Mr Burns' class were ambivalent about the importance of the teacher preparing the videos. Again students emphasised that the videos needed to be relevant and acknowledged that it was helpful having access to Mr Burns in class, having watched his videos beforehand. Mr Burns, however, was adamant about the importance of creating the videos:

> The students relate [to me] I think better than they do to somebody talking about a video that may contain 40 or 50% of what they are looking for; the videos they are looking at now contain 100% of what they are looking for so it's more important in that respect… I think it's very important… because you still need that teacher/student relationship and that works for the student and that works for the teacher. I can give them 20 or 30 videos to look at on a particular topic that are on YouTube but whether they'll get anything out of it compared to having me do the video and talking about them at their level… I know who they are and what they're doing, I think makes a big difference. I think it really is important that the teacher does the video, it really is.

Discussion and Conclusions

In terms of enacting the pillars of flipped learning as depicted in Table 1 (FLN, 2014), while all teachers demonstrated aspects of these in varying degrees, Mr Burns' students were arguably experiencing flipped learning rather than a flipped classroom. Both Mr Shepherd and Ms Brown were definitely providing students with intentional content through either selection or creation of relevant and appropriate videos, and both made themselves available as a professional educator. Aspects of the learning culture were present in that the teacher was not always central to the learning and all activities were accessible to students. Classroom observations, however, showed that students' experiences were centred on individual textbook exercises which did not seem to be especially personally meaningful. Similarly, while the homework environment may have been different, the classroom space was not a flexible environment in terms of students choosing where and when they learned. Mr Burns' students, however, were experiencing a flexible learning environment through having autonomy over their learning and self-pacing their progress through their course. Intentional content was provided through the bank of videos that students could access where and when it suited them, while still having access to a professional educator. It appears that despite the different enactments, students in all three classes were willing to take up the approach and compared it favourably with traditional forms of teaching as experienced in the past. While it is important to acknowledge that the cohort of students taught by Ms Brown and Mr Burns were enrolled by choice in their classes and arguably strongly motivated to achieve, Mr Shepherd's class arguably represented a more "typical" Grade 8 class and also reported favourably on the approach.

In conclusion, this study has contributed to the field of flipped classroom research through its focus on three different enactments of flipped learning within secondary mathematics classrooms. The results show that while the flipped classroom may be enacted in various forms and to varying degrees, student and teacher perceptions indicate that the approach has merit, particularly in terms of complementing existing practices. The study has practical implications for teachers, educational providers and students who may be teaching and learning within the constraints of a traditionally imposed curriculum and delivery method.

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Learning from our Neighbours: The Value of Knowing Their Number History

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Recent research has supported and extended earlier research on how and for how long Indigenous people of Australasia have been counting. This history values the long history of Indigenous knowledge and re-writes the limited and sometimes false history that many Australian teachers accept and teach about number systems. The current views on the spread and innovation of number systems are critiqued in terms of how oral cultures used and represented large numbers.

Forty years ago, ethnomathematics was beginning to be recognised more widely (Bishop, 1979; Lancy, 1978; Van der Waerden & Flegg, 1975; Wilder, 1974; Wolfers, 1971; Zaslavsky, 1973). Many linguists were recording the counting systems of the people whose language(s) they were learning (Panoff, 1970; Pumuye, 1975); Wurm and his colleagues (Wurm, Laycock, Voorhoeve, & Dutton, 1975) were documenting the languages in New Guinea, and others were considering Oceania (Lynch, 1977; Ross, 1988). Anthropolgists were incorporating counting into the various complex activities and beliefs of people (Strathern, 1977) while cognitive, developmental psychologists (e.g., Saxe, 1979) were considering how cultural context affected concepts. Papua New Guinea University of Technology had a Mathematics Education Centre and the University of Papua New Guinea had an Education Research Unit, both of which supported research that took account of cultural difference in mathematics learning. Some of this work was focused on difference using Piagetian studies and some on cognitive development for mastery of concepts. It would be another decade before ethnomathematics was widely discussed (Ascher, 1994; Bishop, 1988; D'Ambrosio, 1990).

Counting System Diversity and How They Developed

With written records of number, there are symbols used in the various languages for numbers. These reflect the ways that the people combined numeral words to make new number names. For example, the Romans, at least in their later history, said four was IV, that is, one before five. In one sense, place or position was important. We are also cognisant that the Arabs used and modified the original Indian ways of recording numbers to give us our current Hindu-Arabic system of base-10 with place value and a zero. But what happened before this? In the second half of the 18th century, it was considered that numerals were a hallmark of civilisation (Crawfurd, 1863), and then there was a debate about whether different civilisations invented their counting and numerals or whether there was a diffusion from either Egypt around 4,000 BCE (G. E. Smith, 1933) or the Sumerians (Raglan, 1939). The diffusionist ideas prevailed and Seidenberg (1960) suggested that counting systems diffused from the Middle East civilisations starting with two-cycle systems having one and two as frame words around 3,500 BCE "spread out over the whole earth; later, other methods of counting arose and spread over almost all, but not quite all, of the world" (p. 218). His view was based on the anthropological evidence that suggested these systems only occur on the vestiges of the world in southern Africa, southern America, and Australia. Some systems with numerals for three and perhaps four, but

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without combinations for higher numbers he claimed as systemless and an aberration of the two-cycle system. Seidenberg then suggested that paired systems developed (He called them neo-2 systems) in which 6 = 2 x 3 and 7 = 2 x 3 + 1 and so on. Then, the base-10 system developed in conjunction with these paired systems spreading around the world.

Seidenberg went on to suggest that the North American Inuits that have a (5, 20) cycle system came after the 10-cycle and there was a fusion, slurring (his term), or modification made with the (2, 10) cycle system resulting in (10, 20), or 20-cycle systems. He suggested that when the 10-cycle system developed in different places it was diffused, unlike the two-cycle system that began in the Middle East. From African systems, he suggested that the (2, 10) and (5, 20) systems came together to make the second pentad (6 to 9) as five plus one to four or another word plus one to four. Finally, Seidenberg believed that the body-part tallying was a combining of the two-cycle system with the (5, 20) system as they linked the idea of body part tallying to represent numbers when *tabu* prevented them from using counting words.

Glendon Lean began collecting the counting systems of his students from across Papua New Guinea (PNG) and Oceania in the late 1960s and located first contact records in linguistic and government records and in research journals. Through careful collation and analysis, he was able to publish these data in 1988 for each province of PNG. These data formed part of his doctoral thesis (Lean, 1992) together with counting system data for Oceania (West Papua – the name refugees use – then called Irian Jaya, Indonesia; Island Melanesia; Micronesia; and Polynesia). G. Smith (1984) had also studied the systems in Morobe Province. Both used Salzmann's (1950) method of ascertaining the frame words with which all other numerals were named, deciding the operative patterns by which the frame words were combined to make the other numerals and then deciding on the primary and superordinate cycles. For example, if a system has frame words of 1, 2, 5, and 20 by which all other numerals were named then the system was a (2, 5, 20) cycle system. In an earlier paper (Owens, 2001), much of Lean's work was summarised.

Countering the Diffusionist Theory

Seidenberg claimed the two-cycle system was once widespread, but Lean found that there were virtually no systems that were pure two-cycle systems in PNG, and they have not been found en route to South America (e.g., in North America). In PNG, as elsewhere, most two-cycle systems, in fact, went on to have a (2, 5) or (2, 5, 20) cycle system and these existed well before the 10-cycle systems introduced mainly by Austronesian (AN) Oceanic speakers. Furthermore, the paired system was a variation of the AN Oceanic speakers on the southern coast of PNG and generally not found in the non-Austronesian (NAN) systems although there is evidence of different group counting. These Oceanic languages spread from their homeland near New Britain (Owens, Lean, Paraide, & Muke, 2017).

Seidenberg thought 10 cycles preceded (5, 20) and body-tally systems, but Lean countered this by recognising that finger gestures while counting are so widespread among Indigenous people that digit-tally systems would have developed across the world and not after base-10 systems were in place. It is not commonly considered that the body tally systems were older than (5, 20) systems as they are more complex and in many cases, there is no evidence that this practice was linked to sharing body parts in rituals, as Seidenberg claimed. Furthermore, Seidenberg has not explained how the systems with 1, 2, 3 and/or 4 evolved into a system with 10. Finally, having a system that is efficient such as the base 10 system may not be the only reason for a group to take up a counting system from a neighbour. A few AN 10-cycle systems in PNG changed to a two-cycle system when the people moved inland along a major river valley, the Markham, and were building relationships with people using two-cycle systems, including variants like (2, 5) cycle systems (Holzknecht, 1989; Owens, Lean, Paraide, & Muke, 2017). There are many other reasons: friendliness, trade, length and intensity of contact, desire to be separate or similar, or for what is valued such as multilingual skills, care, confounding others, and for extravagance or prestige or just confusion (Jett, 1971).

Extending Our Understanding of How Counting Systems Develop

Not only does Lean's (1992) evidence contradict Seidenberg on a number of major issues but his analysis also found several other factors of interest in discussing the spread of counting systems. First, it was found that in West Papua, PNG, and the Solomon Islands that NAN and older languages had borrowed some words, mostly in the second pentad from Austronesian (AN) languages. Similarly, some AN languages borrowed words from NAN languages. Table 1 indicates these phenomena. However, these borrowings are not particularly common and more significant is where a whole system is adopted as mentioned above in the Markham Valley (Owens, Lean, Paraide, & Muke, 2017).

Table 1
Summary of Five-Cycle Systems in Austronesian and Non-Austronesian PNG Languages

Phylum or Cluster	(5)/(5, 20)	(5, 10)	(5, 10, 20)
Non-Austronesian	79	21	13
Austronesian	63	113	46

The majority of (5)/(5,20) cycle systems are found in the NAN Sepik-Ramu and Trans New Guinea (TNG) Phyla; (5, 10) in East Papuan Phylum; and (5, 10, 20) in West Papuan Phylum. Among the AN languages, the majority are in North New Guinea, Papuan Tip and Vanuatu; Vanuatu; and New Caledonia, respectively.

Evidence of Large Numbers and Concepts of Infinity

Non-Base-10 Systems

In order to illustrate how these Indigenous systems used large numbers, three language groups are selected: Yu Wooi (Mid-Wahgi) from the Jiwaka Province of PNG that displays a digit tally system with vestiges of a four-cycle and from some informants, a body-tally system; Iqwaye from the Eastern Highlands Province bordering Morobe that is a digit-tally system with (2, 5, 20) cycles; and six-cycle systems from a large island off the southern coast of West Papua and languages around the border of West Papua and Western Province, PNG.

Yu-Wooi. Muke (2000) obtained data from a number of people from his four tribal groups that have different dialects. The system is a typical (2, 5, 20) cycle system, e.g.

27 is *angek yem yemsi simb yem yemsi, angek yemsi, tak*
both hands both legs half hand two
40 is *hi ende simb angek*
2nd man's legs hands

However, they also count items by twos and in hundreds by grouping them in tens and then tally each ten using their fingers and toes to reach 200. Thus, as one participant said, “for six hundred pigs, they would say that they will kill pigs equal to the hands and legs of three man” (Muke, 2000, p. 134). For the larger thousands, people used the fingers and toes for groups of ten and when they had ten groups of ten, they referred to a specific body part, starting from the head towards the legs, aiding memory of giving for reciprocity (Table 2).

Table 2
Yu Wooi (Mid-Wahgi) Counting System for Large Numbers

Numeral	Yu Wooi and Explanation		
100	*elsi or angek yem yemsi peng ngond* or *peng* 10th ten or head 1st 100		
200	*komuk*	ear – 2nd 100	600 *angek daro* left hand – 6th 100
300	*gnumb*	nose – 3rd 100	700 *buk* back – 7th 100
400	*gupe*	mouth – 4th 100	800 *kumbuk* belly – 8th 100
500	*angek woiro*	right hand – 5th 100	900 *simb woiro* right leg – 9th 100
1,000	*simb daro or hi ende simb angek poro bekenj* left leg – 10th 100 or whole body parts of one person		
2,000	*hi tak* two persons		3,000 *hi takendeka* three persons

Source: Owens, Lean, and Muke (2017)

One variation of this was the use of the names of fingers for adding almost as a vestige of a body-part tally system. Furthermore, when it came to counting and having the opportunity to use hands, people often counted in twos. They would fold down two fingers at a time saying *eraksi* meaning “take two” each time: two then two on one hand followed by two and two on the other and bringing the four folded fingers of each hand together being *mam erak* followed by the two thumbs with the words *angek yem yem* “together hands” (some also folded two, two, then thumb *eraksi eraksi el* and then two and two and thumb repeated before bringing together and saying *angek yem yem*).

When deciding the number of pigs to be given by each person in a compensation claim, the leader asked people to take the number that they would give from a bunch of small banana fruit. When everyone had offered as they wished, the banana fruit were put together and tallied in groups of 10, each group matched with a digit tally part starting with the fingers.

Iqwaye. Building on a binary system of relationships in which a pair is one or where another number is linked to one (the whole), it seems that PNG and Oceania cultures have a richer understanding of number. There is order in the counting, but temporality related to this order may not be of the Western kind but pulsating back and forth (Mimica, 1988). One such group, Iqwaye, has a digit-tally system starting with the thumb of either open hand, then the index finger, etc., moving to the other hand, then counting the toes. (Most language groups seem to count by bending the fingers down starting with the little finger.) Twenty is two hands and two legs or a person. Iqwaye refer to the link between the creator and each of the five children represented by a finger as one child or one to five children so one can be one or five can be one.

Using this digit-tally counting system, by which each digit represents a counting word in order, the man standing up becomes one denoted by the thumb again. So, each digit then represents a multiple of 20. Three fingers could represent three persons or 3 x 20. The next iteration is for the digits to represent 20^2 counted off by the crouching man with fingers touching the toes. Thus, two groups of 20 men (20 x 20 = 400) is represented by the thumb and index finger so 1,000 is represented by two fingers and then 10 fingers (each finger representing 20).

> 400 is Aa' 'mnye, aa'mnye, toqwotni tepu hyelaqa kokoloule hyule hwolye hyelaqapu
>
> Person person this-me this that-all their leg hand that-all
>
> '[as] this many persons [as] me this [one] person [speaker] all their legs and hands'
>
> 1,000 is 'two persons [as] me this [one] person all their legs and hands and to another person's two hands (= ten persons) all their legs and hands' (Mimica, 1988, pp. 35-36)

Thus, the notion of infinity is generated (Mimica, 1988). This self-generating system of numbers is reminiscent of some modern Western mathematical and binary systems. He suggested that a study of the system shows an intuitive non-Western origin of number, capable of developing into a system and purpose for counting (Brouwer, 1975).

Six-Cycle Systems Near the Border of West Papua and PNG. Donohue's (2008) study of the languages of Kolopom Island has shown that languages like Kanum have developed an interesting variety of counting systems to manage their base-six system for large numbers. There are in fact three systems for small numbers, moderate numbers, and a complex system for one to large numbers, 6^5. Some number words occur in more than one system, but the complex system is well established except that Indonesian currency note 1,000 has brought confusion:

> Some younger speakers are reinterpreting ntamnao '1296' as '1000', ... ntamnao tamp is effectively ambiguous between '5000' (1000 × 5; new reading) or '6480' (1296 × 5; old reading), although only the latter is prescriptively correct. (Donahue, 2008, p. 427)

Evans (2009) has shown that languages of the Morehead-Maro language group in Western Province, PNG and further west also have large numbers for six-cycle languages. For example, Nen count to 6^5 or 6^6. The counting in these languages seems to relate to counting yams (three and three) six times, since Williams (1936) recounts that in Keraakie with two counters had a yam representing 36 yam in a daisy pattern. Interestingly, different groups represented six by different parts or gestures of the hand.

Base-10 Systems of the Region

The AN languages, and in some places neighbouring NAN languages such as Nasioi and Uisai on Bougainville, used numeral classifiers for large numbers. In most cases, the classifying prefix or suffix were for counting specific groups of objects such as single bananas, a hand of bananas, long thin objects, food items and so on (Chapter 8 of Owens, Lean, Paraide, & Muke, 2017). Fisher (2010) noted, for example, the prefix *po-* for 100,000. Bender and Beller (2006) also suggest that:

> The Samoan expression refers to just 2 coconuts whereas the corresponding article in Tongan (2) multiplies this amount by 10-score (200), thus yielding 400 coconuts. It is only when numeral and classifier change their position (as *infua-lua*) in Samoan that a numerical change occurs (from 2 to 20). (p. 396)

Rennelles, a Polynesian Outlier, in the Solomon Islands also shows multiplication for large numbers so the practice was widespread.

Evidence for the Longevity of Counting Systems in Our Region

Lean (1992) drew heavily on linguistic data and archaeological linguistics, and his thesis is supported by more recent evidence. For example, the dating of the spread of Oceanic languages is based on Proto-Oceanic (POC) community and the cultural complex associated with the Lapita-style ceramic tradition found throughout Island Melanesia and western Polynesia (Allen, 1996; Pawley & Green, 1985; Spriggs, 2011).

The deep conceptual structure of counting systems, namely their cyclic nature, rather than just loanwords suggests a long-standing existence of these counting systems. Table 3 indicates the diversity. TNG Phylum accounts for most of the mainland of New Guinea outside of the coastal AN areas has a diversity of types of systems. Eighty percent of body-part tally systems are found in one of the sub-phylum. There is also a cluster of four-cycle systems but they are quite diverse in themselves and so seem to be localised innovations. The majority are variants of two-cycle systems and mainly (5, 20) cycle systems.

Table 3
Distribution of Counting System and Tally Types Among the NAN Phyla

Types	West Papuan	East Papuan	Torricelli	Sepik-Ramu	Trans New Guinea	Minor Phyla	Total
(2)	0	0	0	3	39	0	42
(2, 5)	0	1	16	5	86	1	109
(2', 5)	0	1	3	5	17	1	27
(2'', 5)	0	0	5	3	31	1	40
(5, 20)	0	1	2	17	52	7	79
(4)	0	0	0	1	6	2	9
(6)	0	0	0	0	5	0	5
Body-Parts	0	0	0	8	58	4?	70?
(5, 10)	2	12	0	3	4	0	22
(5, 10, 20)	5	0	0	0	4	3	13
(10)	1	8	0	1	2	0	13
(10, 20)	2	0	0	0	1	0	3

Note. These are numbers from Lean's (1992) collected data, which are most languages but not all languages in PNG and Oceania. They exclude 11 West Papuan languages in North Halmahera.

The Sepik-Ramu Phylum tend to have body-part tallies as well as (2, 5, 20) systems with some variants due to the nearby AN languages. There are also body-tally systems in south-east Australia and Torres Strait Islands. The southernmost languages of the Sepik Hill Stock, especially Hewa, have been influenced by East New Guinea Highlands Stock and other TNG Stock (Wurm, 1982). Body-part tallies may have been introduced into the Sepik-Ramu Phylum languages by such influence and not been an original feature (Lean, 1992). The two branches (Kanum and Moro) of Morehead-Wasur languages have six-cycle systems with large numbers suggesting the Proto-Morehead-Wasur language contained a six-cycle system, thousands of years ago (Evans, 2009). The East Papuan and Bougainville languages tend to have classifiers and there appears to be some influence with AN.

POC contained at least terms for 100 and 1,000 with some northern areas losing these for various reasons, perhaps due to NAN influence with other languages inventing further systems. Higher powers are common in Polynesian languages sometimes as 2 x powers of ten and probably due to tributes to chiefs. According to Harrison and Jackson (1984), higher powers, at least for Micronesia, may have a later history and several innovations.

The genealogy suggested for the age of the systems is given in Figure 1.

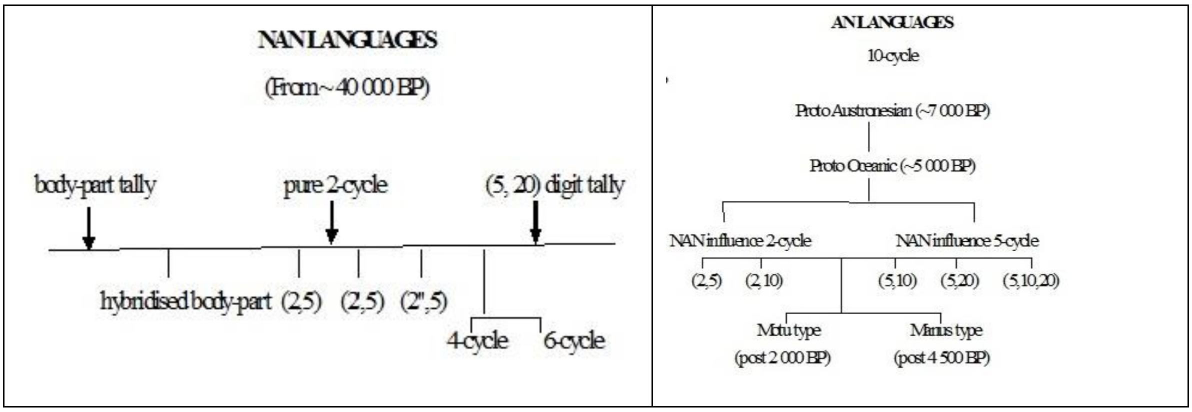

Figure 1. Genealogy of Languages of PNG and Oceania (Source: Owens, Lean, Paraide, & Muke, 2017).

Implications for Changing our Teaching on the History of Number

One of the most significant aspects of this research is that a complex connection between archaeological linguistics, an understanding of counting systems and their cycles, and an acceptance of oral understandings has resulted in new insights into how counting began and was used. Learning about these systems extends students' understanding of counting systems, respects Indigenous cultures, and provides a global mathematics perspective. It counters limited conceptions of history including the development of number and mathematics. It values archaeological linguistics of which Australasia is rich. The importance of groups in early arithmetic and higher levels of mathematics and relationships between numbers can also be enriched. For some students, the link to culture will be a critical way of engendering an interest and understanding in mathematics.

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Generalising Fraction Structures as a Means for Engaging in Algebraic Thinking

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In this paper, we report on how Year 5 and 6 students (10 to 13 years old) solve reverse fraction problems; that is, where students are required to find the quantity of an unknown whole given a known partial quantity and its equivalent fraction of the unknown whole. To what extent do students' solutions generalise fraction structures that indicate algebraic thinking? Which solution strategies to reverse fraction problems seem to promote generalisation and which appear to hold students back?

The links between fractional knowledge and readiness for algebra have been highlighted by many researchers (e.g., Empson, Levi, & Carpenter, 2011; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Siegler et al., 2012; Wu, 2001). The current study builds on the research of Lee (2012) and Lee and Hackenberg (2014), who investigated students' quantitative reasoning with fractions and algebraic reasoning in writing and solving equations. Their research showed that fractional knowledge appeared to be closely related to establishing algebra knowledge in the domains of writing and solving linear equations. They concluded: "Teaching fraction and equation writing together can create synergy in developing students' fractional knowledge and algebra ideas" (p. 9).

After analysing the data, Lee (2012) constructed models to determine the fraction schemes used by students and their reasoning about unknowns and writing equations. These two studies focused on whether students were able to execute specific algebraic performances, namely, to write and solve an algebraic equation to represent a multiplicative relationship between two unknown quantities. Our study involves students in the final two years of primary schools, and our research question focuses on identifying indicators of generalised fraction thinking as students attempt to find an unknown whole when presented with one known quantity representing a known fraction of the whole.

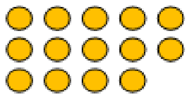
Reverse Fraction Task 1	Reverse Fraction Task 2	Reverse Fraction Task 3
This collection of 10 counters is $\frac{2}{3}$ of the number of counters I started with.  a. How many counters did I start with? b. Explain how you decided that your answer is correct.	Susie's CD collection is $\frac{4}{7}$ of her friend Kay's. Susie has 12 CDs. How many CDs does Kay have? ______ *Show all your working.*	This collection of 14 counters is $\frac{7}{6}$ of the number of counters I started with.  a. How many counters did I start with? b. Explain how you decided that your answer is correct.

Figure 1. The three reverse thinking fraction questions.

These three reverse fraction tasks (Figure 1) formed the core of our earlier study (Pearn & Stephens, 2015). Unlike the Lee and Hackenburg (2014) paper, which described an extended interview of one student, our study looks at the performances of whole-class

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 405–412). Melbourne: MERGA.

groups on these and related tasks, through a written test and selected interviews, and aims to develop criteria to identify the emergence of algebraic thinking and to evaluate students' different solution strategies. To address our research question, we employed an interview protocol using reverse fraction tasks similar to those shown in Figure 1 but with progressive levels of abstraction to capture students' ability to generalise.

The Australian Curriculum Context

According to the rationale given for the Australian Curriculum: Mathematics (ACM; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016),

> The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, reasoning, and problem-solving skills. These proficiencies enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently.

The Australian Curriculum: Mathematics (ACARA, 2016) presents fractions as an important topic across all years with emphasis in Years 5 to 8 on fraction operations. The focus at Year 6 is on finding fractional parts of a known whole and at no stage directs the attention of teachers to finding the whole when given a known fractional part. Year 7 students are expected to solve problems involving addition and subtraction but this appears to exclude multiplicative solutions to fraction problems, especially those involving an unknown whole. Table 1 implies that the link between fractions and algebra is limited to number patterns and sequences involving fractions. However, in Year 7, when students are introduced to the concept of variables, we argue that the bridge between fractional knowledge and algebraic thinking needs to be more explicit. This connects to the synergy emphasised by Lee and Hackenburg (2014) above and forms a major focus of this paper.

Table 1
Content Descriptors from Australian Curriculum: Mathematics (ACARA, 2016)

Year	Fractions and Decimals	Patterns and Algebra
5	Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (ACMNA103)	Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107)
6	Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (ACMNA127)	Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133)
7	Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)	Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)
8	Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)	Simplify algebraic expressions involving the four operations (ACMNA192)

This Study

At the end of 2016, 46 students from Years 5 and 6 from an inner-city Melbourne primary school completed two paper and pencil tests: The Fraction Screening Test (Pearn & Stephens, 2015) and The Algebraic Thinking Questionnaire (Pearn & Stephens, 2016). Three days later, 17 students were interviewed to gain further insights into the strategies they used to solve the three reverse fraction tasks. Their selection will be described later.

Results of Paper and Pencil Assessment

The students' written responses to three reverse fraction tasks from the Fraction Screening Test were analysed to determine the types of strategies that students were using. These responses were then classified according to six categories: Incomplete, Visual, Additive/Subtractive, Mixed, Multiplicative, and Advanced Multiplicative.

Table 2
Methods Used for the Three Reverse Fraction Tasks (Figure 1)

	Incomplete	Visual	Additive/ subtractive	Mixed	Multiplicative	Advanced multiplicative
Year 5 (*n* = 26)	6	3	4	10	3	0
Year 6 (*n* = 20)	6	1	5	4	2	2
Total (*n* = 46)	12	4	9	14	5	2

Incomplete refers to students who did not attempt more than one of the reverse fraction tasks in Figure 1 and who provided no explanations. Visual refers to students who showed explicit partitioning of the diagrams (Figure 1) for Task 1 and Task 3 before using additive or subtractive strategies. Some students created a diagram for Task 2 to represent the 12 CDs before solving the task. Additive refers to students who used additive or subtractive methods without explicit partitioning of the given diagram, or a new diagram, to find the whole. These students could find the number of objects needed to represent the unit fraction and then added or subtracted objects correctly to make the whole.

Mixed refers to students who used multiplicative strategies to solve at least one task while still using additive/subtractive strategies to solve at least one other task. Multiplicative refers to students who only used multiplicative reasoning to solve at least two questions successfully. Generally, these students found the quantity represented by the unit fraction and then scaled up or down to find the whole. Advanced multiplicative describes students who used either algebraic notation to find the whole, or used a one-step method to find the whole by dividing the given quantity by the known fraction. It needs to be noted that 6 of the 12 students who were unable to complete more than one of the reverse fraction tasks were in Year 6 and about to transition into secondary school.

Developing an Interview Protocol and Selection of Students

The follow-up interview was designed to respond to questions raised by Kieran (personal communication, 2016) who encouraged the researchers to vary the quantities associated with each of the given fractions. This would provide stronger evidence of generalised thinking if students were consistently able to use the same methods in similar reverse fraction questions where the quantities were changed but the fractions remained the same. This suggestion is supported by Marton, Runesson, and Tsui's (2004) research, which shows how numbers can be varied to foster a generalisable pattern. Could students treat different given fractions as "quasi-variables" (Fujii & Stephens, 2001)?

Figure 2 gives an example of one of the three extended fraction questions used in the interview based on the original tasks in Figure 1 using 2/3, 4/7, and 7/6. For each fraction, there are two changes. *Change of number 1* is a change of the given number of counters. *Change of number 2, and Unknown number* gives a new number of counters and then asks the student to find the whole if given any number of counters.

Original tasks	**Change of number 1**	**Change of number 2, and Unknown number**
1. This collection of 10 counters is $\frac{2}{3}$ of the number of counters I started with.  a. How many counters did I start with? b. Explain how you decided that your answer is correct.	**1.** Imagine that I gave you 12 counters which is $\frac{2}{3}$ of the number of counters I started with. a. How many counters did I start with? b. Explain your thinking.	**4.** a. If I gave you 18 counters, which is $\frac{2}{3}$ of the number of counters I started with, how would you find the number of counters I started with? b. If I gave you any number of counters, which is also $\frac{2}{3}$ of the number I started with, what would you need to do to find the number of counters I started with?

Figure 2. Example of an extended fraction question based on Figure 1.

If students completed the related questions for 4/7 (Questions 2 and 5) and for 7/6 (Questions 3 and 6), a final question, Question 7, was given:

> Think about the tasks you have just done.
> What if I gave you any number of counters, and they represented any fraction of the number of counters I started with, how would you work out the number of counters I started with?
> a. Can you tell me what you would do?
> b. Please write your explanation in your own words.

When the two changes shown in Figure 2 were used with different known quantities, our aim was to see if students' solution approaches replicated those they used in the written test or whether the interview questions induced them to change from additive/subtractive methods to generalisable multiplicative methods. In particular, we needed to ascertain whether students who had relied on additive or subtractive methods, with or without a diagram, were able to use multiplicative methods once diagrams were no longer provided.

Subsequent interview questions involving any number were designed to make additive and/or subtractive strategies less attractive and less easy to use and so to push students to generalising the fractional structure. Students were asked to find an unknown whole if they had any number of counters that represented a specific fraction of the whole. Finally, students were given the general question about any quantity with any fraction and asked how they would find the whole. This question was designed to provide conclusive evidence that students could generalise their solution method (e.g., by dividing the unknown quantity by the numerator and then multiplying by the denominator). This question also allowed more confident multiplicative thinkers to use algebraic notation to represent the unknown quantity and its accompanying fraction,

Students to be interviewed were chosen from the 32 students who successfully solved and explained their solutions to at least two of the three reverse fraction tasks. Initially, 19 students were interviewed but two interviews were terminated as students went "off-task". The final sample is shown in Table 3.

Table 3
The Sample of Students Interviewed

Year Level	Boys	Girls	Total
Year 5	5	4	9
Year 6	5	3	8
Total	10	7	17

Referring to Table 2, three of four students described as using visual strategies were interviewed, five of nine students who used additive strategies, 4 of 14 students who used a mix of multiplicative and additive methods, three of five students who used only multiplicative methods, and the two students who used advanced multiplicative strategies were both interviewed. The subgroups of students' methods are shown in Table 4.

Table 4
Methods used by interviewed students to solve the three Figure 1 tasks (n = 17)

Year Level	Visual	Additive	Mixed	Multiplicative	Advanced multiplicative	Total
5	2	3	2	2	0	9
6	1	2	2	1	2	8
Total	3	5	4	3	2	17

Administration of the Interview

The record of interview consisted of a three-page document that included the questions, and subsequent space for students to explain their thinking and record their answer and explanation. Each interview took approximately 15 minutes. Interviewers encouraged students to first verbalise and then record their thinking. Students could leave the interview at any point. The written records from the interviews were analysed by two researchers independently, using the following scoring framework shown in Table 5.

Table 5
The Scoring Framework for Interview Questions 1 to 7

Level	Description
0	Not able to successfully complete any questions
1	Completed some or all questions with known fractions and a given quantity (Questions 1-3, 4a, 5a, and 6a) but unable to answer any other questions.
2	Completed all questions with known fractions and a given quantity (Questions 1-3, 4a, 5a, and 6a). Relied on additive methods to solve Questions 4b, 5b, and 6b. Could not give a suitable multiplicative response to Question 7.
3	Completed all Questions 1-6 using multiplicative and/or mixed methods. Gave an appropriate multiplicative response to Question 7.
4	Completed all Questions 1-6 using consistent multiplicative methods. Used suitable algebraic notation to give a multiplicative response to Question 7.

Interview Results

No student was scored at Level 0, an unsurprising result given that all selected students had successfully answered two of the three reverse fraction tasks from the written test. Two of the three students who used visual methods in the written test were able to use additive methods for the interview with no diagram, and were deemed to be at Level 2. The third student who had used visual methods was scored at Level 1 as shown in Table 6.

Table 6
Evidence of Generalising Fraction Structures as a Result of the Interview

Reverse Fraction Tasks (Figure 1)		Interview Score (Table 5)			
Written Test Methods	Number	Level 1	Level 2	Level 3	Level 4
Visual	3	1	2	-	-
Additive	5	1	2	2	-
Mixed	4	-	-	4	-
Multiplicative	3	-	-	3	-
Advanced Multiplicative	2	-	-	-	2

Two students using additive methods in the written test were also at Level 2 in the interview, and one was at Level 1. Two students who used additive methods in the written test converted to fully multiplicative methods and were deemed to be at Level 3. Four students who used mixed methods in the written test successfully solved all other interview questions and solved Question 7 multiplicatively. These students were scored at Level 3.

All three students who used multiplicative methods in the written test were scored at Level 3 in the interview. These students continued to apply generalisable multiplicative methods in the interview. Two students who used advanced multiplicative methods for the written test answered all questions multiplicatively in the interview and responded to Question 7 using appropriate symbolic notation and were deemed to be at Level 4. The numbers highlighted in Table 6 show that 11 of the 17 students, including two who had used only additive methods on the written test, scored at either Level 3 or Level 4 when interviewed, thus demonstrating an ability to generalise a procedure that is independent of a particular fraction or quantity. At Level 3, this was typically expressed as “divide by the numerator and multiply by the denominator”.

Illustrative Examples

Students scoring at Level 2 or below were restricted to using additive methods when it was no longer appropriate or useful. This is illustrated in Figure 3 by Student G’s attempted solution to Question 7, which only appears to be generalizable since it depends on knowing how many times the quantity equivalent to the unit fraction needs to be added or subtracted.

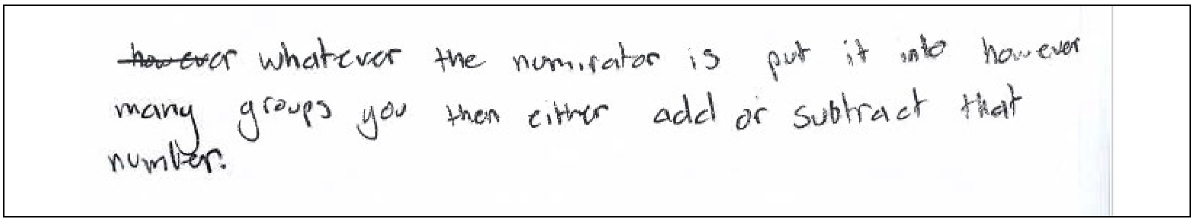


Figure 3. Student G’s additive response to Question 7b.

Student E used algebraic notation in what looks like a generalised solution, but only works for specific cases where the given fraction is one part more (or less) than the whole. This student confidently solved all preceding interview tasks, but she was scored at Level 2 because she was unable to give an appropriate multiplicative response to Question 7.

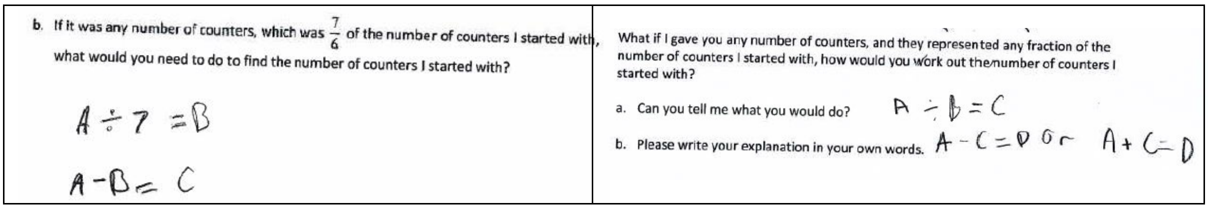


Figure 4. Student E’s responses to Question 6b and 7b.

Student K used a similar additive/subtractive method as Student E to answer Question 6b. When presented with Question 7, Student K suddenly realised that the preceding method would no longer work, and then gave a fully multiplicative answer to Question 7. Student K was scored at Level 3.

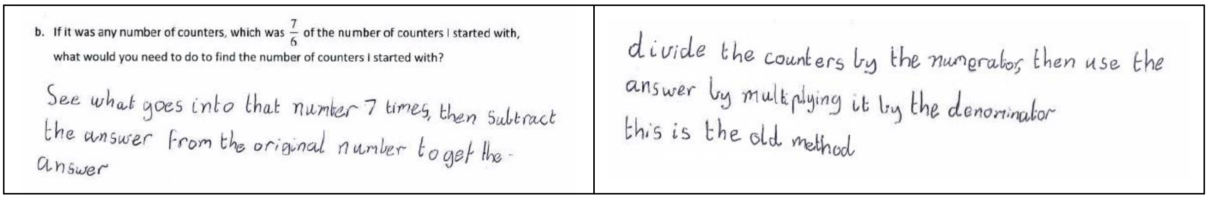


Figure 5. Student K’s additive response to Question 6b followed by Question 7b.

Student J consistently solved all previous interview questions by dividing whatever quantity was given by its equivalent fraction. In Question 7, Student J represented all quantities and fractions algebraically in a fully generalised solution, scored at Level 4.

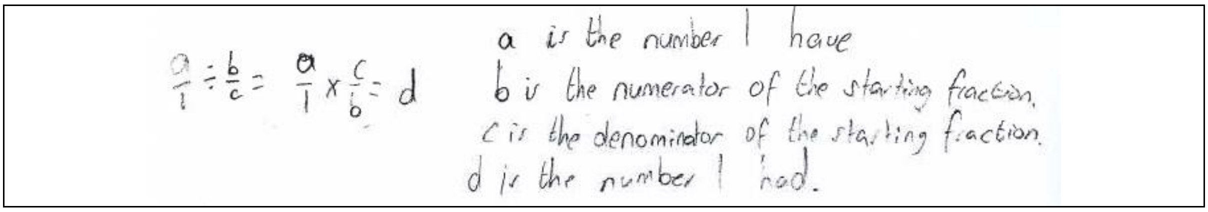


Figure 6. Student J’s symbolic multiplicative response to Question 7b.

Discussion and Conclusion

All nine students who used either mixed methods or multiplicative methods to solve the three reverse fraction tasks on the written test showed in the interview that they were able to deal with variations in both fractions and corresponding quantities and to generalise their methods. One student, having successfully solved the first three questions, explained subsequent solutions as “same as I did before”, but later gave an explicit symbolic response for Question 7.

The interview format exemplified in Figure 2 allowed these nine students to treat variations in the given fractions as “quasi-variables” (i.e., recognising that the same multiplicative operations applied regardless of the fraction). These students appear well-positioned for formal algebra as expected in Year 7 as given in the Australian Curriculum:

Mathematics (ACARA, 2016) where they will be introduced to "the concept of variables as a way of representing numbers using letters (ACMNA175)". The two students in this group who were scored at Level 4 showed even stronger evidence of being able to create and "simplify algebraic expressions involving the four operations" (ACMNA192), as recommended for Year 8 (Table 1).

The data collected from the interview demonstrated that students who relied on visual methods or additive methods experience difficulty in adopting a multiplicative approach and describing a rule as implied in ACMNA133 (Table 1). These students appear to be most at risk in subsequent years when meeting linear equations involving rational numbers also in relation to proportional reasoning.

The interview task design acted as a scaffolding mechanism for two students who had relied previously on additive methods for all three reverse fraction tasks in the written test to use multiplicative and generalizable methods to solve questions that presented either an unknown quantity or, in the case of Question 7, with both an unknown fraction and unknown quantity. This has clear implications for teaching in helping those students who are dependent on visual or additive methods to move to more generalisable approaches. Visual and additive methods have limited power because every problem has to be treated afresh. This study suggests that by using careful scaffolding more students can be helped to recognise and use repeatable multiplicative methods. Being able to generalise fraction structures is vital for all students to achieve success.

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First-Year University Students' Difficulties with Mathematical Symbols: The Lecturer/Tutor Perspective

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School and university mathematics: Do they speak the same language? Our university mathematics students come from amongst the successful school mathematics students, but what difficulties with symbols do their lecturers and tutors observe? In this paper, we report on data from interviews with 21 first-year lecturers and tutors from four universities. Key emerging themes focused on mathematical communication: the importance of comprehending mathematical symbols and of composing a mathematical narrative consisting of both explanatory words as well as symbols.

Mathematics comes with its own language, a mix of words and symbols, each of which is infused with meaning that is agreed upon by the community of practice. This sounds straightforward, but we know that, in practice, symbols that look the same may have different meanings in different contexts, or symbols that look different may, in different contexts, be used to mean the same thing. The "rules" of syntax may change as we learn to operate in different domains, and the meaning to be assigned to a symbol also depends on the domain in question. At school and university, students learn to do mathematics by studying it in increasingly complex contexts and extended domains. While the Australian curriculum (Australian Curriculum, Assessment and Reporting Authority, n.d.) promotes the broad mathematical proficiencies of understanding, fluency, problem-solving, and reasoning, the pressures of "fair" high-stakes senior secondary school assessment result in agreed use of symbols and a narrow range of questions. Concerns have been expressed that students are choosing not to proceed to higher mathematics (Chubb, Findlay, Du, Burmester, & Kusa, 2012). One hypothesis under investigation is that a change in symbols load or complexity between school and university may disturb students' confidence to proceed (Bardini & Pierce, 2015). In this paper, we consider the research questions: What do university teaching staff recall of first-year students' difficulties with symbols? What issues do they perceive that students have reading and writing the language of mathematics? We begin by revisiting some of the issues previously reported in the literature by researchers and by summarizing the framework we are using to discuss the different aspects of symbols. This is followed by the methodology of this part of our project, the findings, and finally some implications.

Background and Framework

Symbols in the Literature

Students' misconceptions with the meaning of letters in algebra have been well documented over a long period of time (Küchemann, 1981; Stacey & MacGregor, 2000) and recent research (Bardini, Vincent, Pierce, & King, 2014) suggests that some fundamental uncertainties, like feeling that a different letter must indicate a different number or relate to a different process, exist for some university level students.

Researchers over past decades have agreed that too often students have learned to respond/react to certain patterns of symbols without paying attention to the domain,

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 413–419). Melbourne: MERGA.

context, or purpose of the mathematics in focus. Twenty years ago, Barbeau (1995) articulated a common concern that some students with technical proficiency (who no doubt would be able, through practise, to score well on high-stakes examinations) may lack insight and only view variables as placeholders for numbers. Arcavi (1994, 2005) offered a way forward for the pedagogy of mathematics by drawing attention to the importance of promoting symbol sense. This, he said, includes an understanding of and an aesthetic feel for the power of symbols, an ability to manipulate and also to "read through" symbolic expressions, the ability to use symbols to represent problem situations, and the realisation that symbols can play different roles in different contexts. Pierce and Stacey (2001), motivated by the introduction of computer algebra systems (CAS) to mathematics classrooms, identified components of "algebraic insight" (i.e., the understanding required when the focus can shift from memorising and automating procedures handled more efficiently by CAS). This, they said, involved both "algebraic expectation" and "ability to link representations". They coined the term "algebraic expectation" to summarise the notions of recognition of conventions and basic properties: "identification of structure" and identification of key features". They suggested that teachers should encourage the development of their students' algebraic expectation by using this framework as guide to thoughtfully consider any algebraic expression before embarking on any processes.

In analysing some difficulties apparent in university students' mathematical work, Pierce and Bardini (2016) identified instances of what Tall (2008) calls "met-befores". He defines a met-before to be "a current mental facility based on specific prior experiences of the individual" that can be "sometimes consistent with the new situation and sometimes inconsistent" (p. 6). Pierce and Bardini note that these met-befores may be encountered at various stages along the mathematical journey, but at university, where the mathematical domain in focus may change, memorised processes, applied efficiently but unthinkingly to seemingly familiar patterns, can lead to illogical results.

Framework

In our study, we have taken the principles of the work of Serfati (2005) for our framework for analyzing mathematical symbols and discussing the implications for students' learning. This framework challenges us to consider symbols from three aspects:

- *Materiality* (i.e., what they look like): This includes whether they are a Latin or Greek letter, or an operator, and their physical attributes.
- *Syntax* (i.e., their position and the conventions associated with this): For example, an = sign must have some symbol or expression on either side of it. We expand this to consider *syntax templates* where a sequence of symbols typically forms a block that is used as a "template" for a particular mathematical action.
- *Meaning* (i.e., the meaning of the symbol within the domain of interest and the context of the problem)

Methodology

For this study, interviews were undertaken with 21 first-year mathematics and/or statistics lecturers and tutors at four Australian universities. Sampling was purposeful. This combination of universities was selected for location and student mix. They represented a mix of urban and regional locations with students selected on the basis of their tertiary entrance rank and others offering more open access. Within each university, first-year mathematics or statistics teaching staff (lecturers and tutors) were identified and asked to participate. Participation was voluntary, but coordinators did encourage their staff.

The semi structured, face-to-face interviews were audio-recorded and later transcribed. Any writing or drawing by the participant was retained and digitised. These transcripts were then subject to thematic analysis, with two researchers separately analyzing all 21 scripts and then discussing the emerging themes. Close agreement between the researchers was achieved on the first reading and further refined by discussion about how best to "name" these themes. In this paper, we report findings from the opening question that we asked of first-year students' university lecturers and tutors: "Just thinking about students' use of symbols. Can you recall any particular difficulties where they might struggle?"

Findings

The over-riding theme that emerged from the analysis of interview transcripts was communication. This theme essentially had two aspects. First and most emphasized by lecturers and tutors was the importance of students communicating – being able to compose a mathematical narrative including their mathematical understandings or statistical findings and the related, underpinning reasoning. The second aspect to which lecturers and tutors drew attention was the difficulty that students have in comprehending certain symbols or material variations of symbols they have met previously at school. We will address these in reverse order.

Comprehension of Symbolic Statements

Certain symbols were reported consistently as causing difficulty. T9, T12, T13, T15, and T17 provided the examples included below. If we analyse their observations in terms of our framework, we see that both symbols with unfamiliar materiality and symbols with familiar materiality but changed syntax or meaning can present stumbling blocks for students.

T9 reported that students have difficulty with the meaning of the square root symbol. This is an example of a symbol, introduced early in the secondary school curriculum, whose materiality remains unchanged but whose associated syntax and meaning depend on the domain of interest and the context of the question. T9 also draws attention to students' fundamental understanding of the meaning of letters in algebra, as illustrated by students' ability to solve a problem with x and y but who are bewildered by the same problem written with a different pair of letters. In this case, the materiality appears different to the student who does not recognize that the syntax and meaning have not changed.

Several tutors reported that students stumbled at the use of capital Greek sigma to signify the operation of summation. The unfamiliar materiality of this symbol may be an issue (T12, T13). At this stage of their mathematical learning, this is the only example of a letter being used to signify an operation. Perhaps both unfamiliarity with the shape of the Greek letter and an "unusual" meaning assigned to a letter present difficulties for the novice. It is also possible that, since most students are unfamiliar with Greek letters, Σ is just seen as a squiggle to be copied thus any potential mental link between Sigma, S, and summation is lost.

T15 and T17 provide examples of the use of symbols, familiar in their materiality but taking new meanings in new contexts, in this case vectors. The dot and cross product symbols "·"and "×" look familiar to operators in arithmetic and algebra but are associated with new syntax and new meanings when working with vectors. Similarly, both points and vectors may be written as ordered triples but the appropriate meaning must be inferred from the context of the question (T15).

> T9: The square root symbol is one of the most common difficulties. They do not understand what the square root means. They have difficulties going from solving equations, so things like $x^2 = 16$ they can tell me that $x = \pm 4$ but when they write down $\sqrt{16}$ they don't know what that actually means. Is this two roots? One root? … if it's got a square root … especially if the square root comes out to be the square root of a negative number and they want a complex root and then they get *i* so then this is a major, major problem.
>
> …Using a different letter upsets them, you know a different parameter… if you start with y as a function of x and you change the problem to x is a function of t, you're in trouble, and if I change to p is a function of q or something that's really, really not common, it's like they cannot do the question anymore…they know how to solve a problem with y in terms of x.
>
> T12: The one that does seem to hold them up is the sign sigma, capital sigma…I remember my stats students saying things like "What's that big E-looking thing?"
>
> T13: They don't have a concept of x representing anything…if you give them numbers they can add it up but if you say, "Okay, that's the sum of *Xi* i is from 1 to n" and they say, "What's that, it's so scary that I can't do this, I can't do this."
>
> T15: (*x, y, z*) - (*a, b, c*) If it says, "What is the vector connecting this to this?", then they're both points because you've got to read the language around it. Or if it says you've got to minus (*a, b, c*) or something like that then they have to be vectors because points can't be sort of added. So, you've got to think about what it is actually. You've got to read the context of the question to understand what the issue was… They see something written down and think they know what it is and so that sends them down the wrong path and then you go part way down there and then it's really hard to come back from that, because you've already started to build a picture.
>
> T17: Another problem is distinguishing shorthand notation for dot and cross products … we write the divergence of F is equal to del dot F. And again, for them to understand what that translates to in mathematical notation becomes difficult for a lot of students. Because this one here, they don't understand that the order is important with these two here. Like this one is not equal to F dot del, which is something quite different. $\text{div}\,\underset{\sim}{F} = \Delta \cdot \underset{\sim}{F} \neq \underset{\sim}{F} \cdot \Delta$

Composing Mathematical/Statistical Narratives

The most common theme across the interviews was that written mathematics should make sense and that it should communicate clearly to the reader. This is exemplified in the statements from T2, T3, T4, T6, T9, T12, and T13 included below. These first-year university lecturers and tutors consistently reported that students were reluctant to use words. Instead, they provided mathematical solutions as strings of symbols that might or might not make sense to the reader. Not only is poor communication due to a lack of words but also to the misuse of symbols, particularly $=; \Rightarrow;$ and $\therefore$, with these symbols being liberally scattered in students' work without apparent attention to their meaning (T2, T9) Concerningly, T9 suggested that students' lack of proficiency with the logical connectors of mathematical notation not only impacts coherent writing and explanation of mathematical workings, but it also creates a barrier to learning. In addition to valuing words, as well as symbols, to elucidate reasoning throughout a problem, statistics staff look for a concluding statement that includes an interpretation of the results of any symbolic or numeric process (T12, T13).

University teachers agreed that it is important for them to model the composing of mathematical narratives and that marking schemes for assignments, at least, should reflect this expectation in students' work. They also commonly reported that they were not aware of the expectations set at the secondary school level but guessed that externally marked high-stakes examinations would be likely to reward correct final answers rather than encouraging full logical mathematical communication.

> T6: …just thinking about symbols. I mean one of the sort of things that I find …I really want students to do is like develop mathematical reasoning and communication skills and I find that often you see what is just a blind sort of statement of symbols one after the other and one of those things that I sort of try to get students to do is to basically write less symbols, write more English. Not necessarily less symbols but write more words to connect them….

> T3: …their general layout and the way they present their mathematics on a page. You and I will write it down in a linear fashion, each using ideas, explaining a point mentioning what we're about to do next then do the calculation, then continue, so there's an interplay between English and mathematics together in a consistent story. They don't think English is appropriate; they don't put any reasoning in their line of argument …

> T4: There are so many things that they're missing, it's the communication. They don't seem to appreciate who is going to read what they've written and how that person might actually understand what was written.

> T2: They just don't write down what they're doing, they don't explain. It's just literally, they think they just need to write a page of equations with each [of] these funny little symbols joining everything together and they'll think they're done.

> T9: …I'm finding that all the problems they have with understanding the notation, the symbols, the way you use the logical connectors – this is actually affecting their ability to understand what we're doing in first-year basic calculus, and then to be able to explain it to me and write something coherent that I understand.

> T13: …when you do the actual workings through the maths part, you do really need the symbol, but the interpretation, you need the language, you need the English to interpret it.

> T12: I do say that at the end there should always be a sentence, at the end you know, summarising what you've done. Their idea of a sentence and mine don't always tally.

Discussion and Implications

Value of the Framework

First, the framework provides us with a systematic way to analyse students' difficulties with symbolic notation. Mathematics teachers at all school and university levels need to explicitly alert students to a change in the accepted meaning of a materially familiar symbol, or a change of symbol to be used with familiar syntax and meaning. Concurrently, we need students to be alert to the domain (e.g., natural numbers, integers, real numbers, complex numbers) or context of each problem (e.g., points, vectors, planes) before they engage with mathematical processes. Instead of just noting a symbol that seems to cause an issue, considering the materiality, syntax, and meaning associated with that symbol can bring in to focus the aspect of the symbol to be attended to when teaching new symbols, or when changing the use of a familiar symbol. The framework can also structure our thinking when we pose questions to students to prompt their thinking about a symbol and so help them overcome barriers to their mathematical progression.

Tension between Trying to Reduce Cognitive Load and Developing Flexible Thinking

The student difficulties noted by first-year university lecturers and tutors have implications for teaching both at the school and university level. At both levels, there is a tension between helping students to achieve fluency in algebraic manipulations and routine problem-solving by limiting what could be seen as extraneous variation (e.g., by using the same letters). This is done with good intention – to help students focus on the process. However, for some students, even some of our "best" secondary school students, this can create a barrier when they perceive that a change of letter for parameter or variable indicates a new and different problem.

Clear Communication must be Valued by Assessment

It is commonly accepted that assessment drives learning. While assigning marks for clear communication in assignments, tutors and lecturers typically told us that exam marking is not so strict. They said things like "if you read this literally, it does not make sense; we can see that the student knows what they are doing (especially if the final answer is "correct")". In fact, markers impute what they expect the student was thinking based on their own understanding of the problem and its solution. Students who arrive at a "wrong" answer also expect that they may be awarded marks for "working" while markers tend to assume that a correct final answer comes as a result of correct mathematical thinking to solve the problem. Here we see the importance of a clear narrative involving words and symbols so that a marker may indeed see if the student has understood the problem and made a slip due to the time pressure of an examination or, alternatively, been lucky to arrive at the preferred result.

Mathematical Reasoning must be Paired with Clear Mathematical Communication

University mathematics and statistics lecturers and tutors expressed the concern that students focused on symbols rather than clear communication, which requires a combination of words and symbols. The Australian mathematics curriculum sets mathematical reasoning as a key proficiency to be developed. However, students apparently too often take the narrow view that the symbolic processes are the total of the mathematics. The habit of communicating mathematical reasoning in words, symbols, and other representations as appropriate needs to be demonstrated and encouraged throughout schooling. Perhaps students need to peer review each other's work to see if they can follow the reasoning.

What Issues do First-Year University Staff Perceive that Students have Reading and Writing the Language of Mathematics?

University lecturers and tutors recalled commonly observing students having difficulties with particular symbols: sometimes because the symbol shape and syntax were unfamiliar, but often because a symbol that was materially familiar, or a syntax template that looked like one they had met before took on a new meaning or range of meanings on an extended domain or new context. Most of these staff had come to realise that this comprehension of symbols, so familiar to them, needs to be explicitly taught to novices.

Staff also expressed concern that students focused on processes rather than clear mathematical communication. It was common for students to write a series of disconnected or incorrectly connected results from applying algebraic algorithms. Students need to learn to make their reasoning explicit by composing mathematical narratives using words and symbols.

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11th Grade Students' Self-Regulated Learning in a Mathematics Problem-Based Learning (PBL) Classroom

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We proposed to study self-regulated learning (SRL) of 11th grade students in a PBL classroom (n = 36). The data were collected in November 2016 using Students' Self-Regulation Strategy Inventory (Students' SRSI), Teachers' Self-Regulation Strategy Inventory (Teachers' SRSI), students' interviews forms, students' reflections, and the teachers' notes. Descriptive statistics (percentages, means, and standard deviations) and descriptive analysis were used to analyse the data. We found that in the PBL classroom, the students demonstrated self-regulated learning in three phases – the forethought phase, the performance phase, and the self-reflection phase – at high levels.

In the 21st century, students need to develop life skills along with content knowledge, thinking skills, and social and emotional competencies to be able to survive in this era of globalization (Trilling & Fadel, 2009). One of the important life skills is Self-Regulated Learning (SRL). SRL is the process by which individuals exercise autonomy and control cognition, affect, and behaviours to achieve a defined learning goal (Kaur, 2012; Zimmerman, 2000, 2002). In this study, we used the process of Zimmerman and Campillo's SRL (2003) that involved three phases as follows: (1) the forethought phase including task analysis (goal setting and strategic planning) and self-motivation beliefs (self-efficacy, outcome expectation, intrinsic interest/value, and goal orientation), (2) the performance phase including self-control (imagery, self-instruction, attention focusing, and task strategies) and self-observation (self-recording and self-experimentation), and (3) the self-reflection phase including self-judgement (self-evaluation and causal attribution) and self-reaction (self-satisfaction/affect and adaptive/defensive). The SRL has three phases. In the first phase, forethought refers to processes and beliefs of students before the students start to do their action or performance. In the second phase, performance refers to behavioural implementation and strategies for learning by students. In the third phase, self-reflection reflects students' evaluation of their achievement and their reactions to performance goals compared to the outcomes.

In the past, Thai students in formal education became accustomed to receiving knowledge from lectures by a teacher, which provided fewer SRL opportunities such as setting goals, communicating with their teachers and peers, getting feedback, and adapting their own knowledge during learning (Suanpang & Petocz, 2006; Tsai, 2010). These days, many Thai students are still accustomed to passive learning with less emphasis on SRL (Park & Nuntrakune, 2013). As still found in many countries, most Thai students showed a lack of SRL behaviours. For example, in the forethought phase, some students lacked goal-setting behaviours. They neither read nor planned on solving mathematics problems by themselves. In the performance phase, some students did not solve mathematics problems by themselves. Instead, they would rather wait to get assistance from their friends or to get answers from their teachers. In the self-reflection phase, some students were not concerned about self-evaluation. For instance, some students could not evaluate their assignments to check if their works were correct. In order to develop students' SRL, teachers must transform the students from passive learners to active learners (McDonough & Sullivan, 2008; Zimmerman, 2002).

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 420–427). Melbourne: MERGA.

Several studies showed that SRL was fostered by Problem-Based Learning (PBL, Blumberg 2000; Sungur & Tekkaya, 2006). PBL is an active learning strategy that stimulates students to learn about a subject through real-world problems and promotes the development of critical thinking skills, problem-solving abilities, communication skills, and SRL (Duch, Groh, & Allen, 2001; Paris & Paris, 2001). Therefore, we were interested in studying 11th grade students' SRL in a mathematics PBL classroom. We adapted the PBL learning processes from Othman, Salleh, and Sulaiman's study (2013). They proposed five steps in the PBL processes: (1) an introduction to the problem, (2) self-directed learning, (3) group meeting, (4) presentation and discussion, and (5) exercises.

Method

In this mixed-method research study, we aimed to study students' SRL in a mathematics classroom by implementing a PBL approach. The participants were 36 11th grade students (eight boys and 24 girls) from a high school in Chiang Mai, Thailand. The research instruments were:

1) Eight PBL lesson plans: One of the researchers taught the PBL lesson plans for four weeks in the second semester of the academic year 2016. Each lesson took 100 minutes.
2) Students' Self-Regulation Strategy Inventory (Students' SRSI), a 48-item self-report instrument with 5-point Likert scale adapted from Cleary's study (2006). Before using the Students' SRSI, we examined the reliability of students' SRSI by testing it in a parallel classroom (n = 40) (Cronbach's alpha co-efficient, r = 0.92).
3) Students' reflections
4) Students' interview forms adapted from Callan's study (2014)
5) The teachers' notes
6) The Teachers' Self-Regulation Strategy Inventory (Teachers' SRSI) adapted from Callan and Cleary (2012)

The participants took the Students' SRSI (Pre-Test) at the beginning of the lessons. In the classroom, data were collected by one of the researchers who taught the eight lesson plans. A mentor teacher observed the students in the classroom by using the Teachers' SRSI. In the meantime, other sources of data were students' reflections and the teacher's notes (Video recordings were used to provide backup data). At the end of all the lessons, Students' SRSI was used to verify the students' SRL (post-test) (Cronbach's alpha co-efficient, r = 0.912). Moreover, we selected nine students with mixed mathematical abilities (three high, three average, and three low) to interview in order to get in-depth information on SRL. The collected data were analyzed using both quantitative and qualitative methods. The data from Students' SRSI and Teachers' SRSI were analyzed by using descriptive statistics, including percentages, means, and standard deviations. The data from students' reflections, interview forms, and the teacher's notes were analyzed by descriptive analysis.

Results

We employed a mixed-method design using multiple data sources, as described above to investigate 11th grade students' SRL in a PBL context. Our findings are presented according to Students' SRSI, Teachers' SRSI, students' interview forms, students' reflections, and teacher notes.

Regarding the students' SRSI, the mean scores of all three phases of SRL increased from the pre-test to post-test (see Table 1). The mean scores of the task analysis in the forethought phase showed the greatest improvement from pre-test to post-test. In contrast, the students' self-reaction in the self-reflection phase showed the lowest improvement. It can be noted that the mean scores of self-observation and self-reaction in the post-test were similar (Mean = 3.69).

Table 1
Means and Standard Deviations of Scores on Students' SRSI (n = 36)

SRL Phase	Pre-test		Post-test	
	Mean	S.D.	Mean	S.D.
Forethought				
Task Analysis	3.38	0.91	3.68	0.81
Self-Motivation Beliefs	3.59	0.90	3.70	0.84
Performance				
Self-Control	3.31	0.90	3.56	0.82
Self-Observation	3.43	0.81	3.69	0.79
Self-Refection				
Self-Judgement	3.45	0.92	3.54	0.78
Self-Reaction	3.60	1.01	3.69	0.90
Overall	3.46	0.91	3.64	0.83

Focusing on the analysis of teachers' SRSI based on observations in the PBL classroom, we found that the mean scores of all phases (i.e., the forethought phase, the performance phase, and the self-reflection phase) had an increasing tendency. In the forethought phase, the mean scores for task analysis were less than the mean scores for self-motivation beliefs (see Figure 1). In the performance phase, the mean scores for self-control were greater than the mean scores for self-observation in most of the PBL lessons (see Figure 2). In the self-reflection phase, the students' self-judgement mean scores were greater than the students' self-reaction in most of the PBL lessons (see Figure 3).

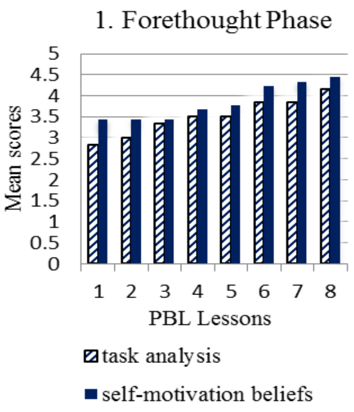

Figure 1. Mean scores of Students' SRL in Forethought Phase (*n* = 36).

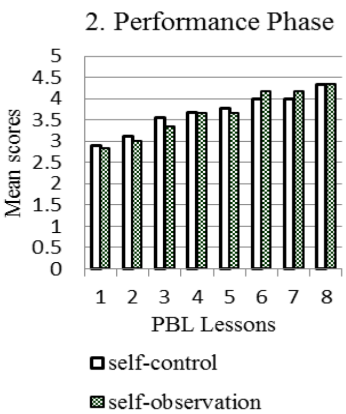

Figure 2. Mean scores of Students' SRL in Performance Phase (*n* = 36).

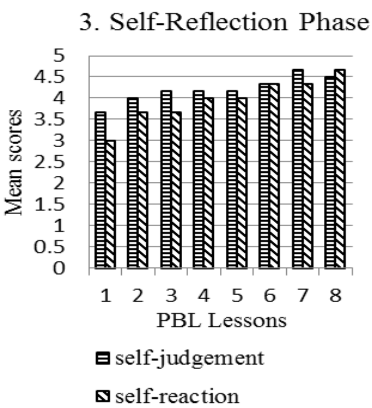

Figure 3. Mean scores of Students' SRL in Self-Reflection Phase (*n* = 36).

In addition, the data from the interviews of the nine students from different mathematics achievement levels provided more details on the students' SRL in the forethought phase, the performance phase, and the self-reflection phase, which are presented in Tables 2, 3, and 4, respectively.

Table 2
SRL in the Forethought Phase from Students' Interviews

Mathematics Achievement Level	Forethought Phase
High	1. Task Analysis: The students set very clear goals to complete the assignments (Goal setting). They had their plans to select or create the best ways to solve the problems (Strategy planning).
	2. Self-Motivation Beliefs: The students were very confident in their efficacy for solving the problem situation (Self-efficacy). They not only believed that their solutions were aligned with their goals (Outcome expectation), but also believed that mathematics was valuable to their daily lives (Intrinsic interest/value).
Average	1. Task Analysis: The students set pretty clear goals to complete the assignments (Goal setting). They had plans to select or create the best ways to solve the problems (Strategy planning).
	2. Self-Motivation Beliefs: The students were confident in their efficacy to solve the problem situations (Self-efficacy). They also believed that their solutions aligned with their goals (Outcome expectation). They believed that value of mathematics was only for doing the exams (Intrinsic interest/value).
Low	1. Task Analysis: The students set vague goals to complete the assignments (Goal setting). They were uncertain about their plans for selecting or creating the best ways to solve the problem (Strategy planning).
	2. Self-Motivation Beliefs: The students were uncertain about their efficacy to solve the problem situation (Self-efficacy). They were uncertain that their solutions were aligned with their goals (Outcome expectation). Moreover, they believed that the value of mathematics was only for doing the exams (Intrinsic interest/value).

As seen in Table 2, focusing on the students' SRL in the forethought phase, we found that all of the students explained the behaviours and belief in their task analysis (goal setting and strategies planning) and their self-motivation belief (self-efficacy, outcome expectations, and intrinsic interest/value). The students with high mathematics achievement had very clear goal setting and self-efficacy, greater than that of the other students. In addition, the students with average and low mathematics achievement believed that the value of mathematics was only to benefit them in doing the exams.

Table 3
SRL in the Performance Phase from Students' Interviews

Mathematics Achievement Levels	Performance Phase
High	1. Self-control: The students preferred self-instruction such as thinking aloud and asking themselves questions. They preferred task strategies such as underlining important information and planning before solving problems.
	2. Self-observation: The students preferred self-recording such as taking notes and checking their solutions.
Average	1. Self-control: The students preferred imagery such as drawing pictures to do mathematics. They preferred task strategies such as rereading the problems and planning before solving problems.
	2. Self-observation: The students preferred self-recording such as taking notes and checking their solution. They tried to use self-experimentation such as using different strategies in the same problem situations.
Low	1. Self-control: The students preferred attention focusing such as rechecking their answers. They preferred task strategies such as rereading the problems.
	2. Self-observation: The students preferred self-recording such as taking notes. However, they did not use taking notes to recheck their solutions.

As presented in Table 3, focusing on SRL in the performance phase, we found that the students showed different preferences for self-control (self-instruction, imagery, attention focusing, and task strategies). In the self-observation sub-process (self-recording and self-experimentation), the students with high and low mathematics achievement preferred self-recording such as taking notes, but the students with low mathematics achievement did not recheck their solutions in their notes. The students with average mathematics achievement added self-experimentation for finding the best ways to solve the problem because they wanted to confirm their answers again.

Table 4
SRL in the Self-Reflection Phase from Students' Interviews

Mathematics Achievement Levels	Self-Reflection Phase
High Average	1. Self-judgment: The students evaluated the strategies that they used in doing tasks by comparison with their goal (Self-evaluation). They could explain the causes of success or failure in an assignment (Causal attribution). 2. Self-reaction: The students were satisfied with the strategies and solution that they used (Self-satisfaction/affect). They wanted to improve by using new ones to obtain better results (Adaptive/defensive).
Low	1. Self-judgment: The students evaluated the strategies that they used in doing tasks by considering their feelings (Self-evaluation). 2. Self-reaction: The students were not satisfied with the strategies and solution that they used (Self-satisfaction/affect). They wanted to improve the mistakes in mathematical process such as calculating mathematics (Adaptive/defensive).

As presented in Table 4, focusing on the SRL in the self-reflection phase, we found that the students with high and average mathematics achievement had similar perspectives about self-judgment (self-evaluation and causal attribution) and self-reaction (self-satisfaction/affect and adaptive/defensive). In contrast, the students with low mathematics achievement used their feelings to evaluate tasks instead of using their goals. They could not give reasons about the causes of success or failure in an assignment. They had less satisfaction with their strategies and solutions than the students with high and average mathematics achievement.

Furthermore, the data analysis from students' reflection and teacher's notes showed the Students' SRL in the PBL classroom by focusing on the forethought, performance, and self-reflection phases. The Students' SRL are discussed next.

In the forethought phase, most students (80%) showed improvement in their task analysis (goal setting and strategic planning) and self-motivation beliefs (self-efficacy, outcome expectations, intrinsic interest/value, and goal orientation) in the 1st, 2nd, and 3rd steps of the PBL process: an introduction to the problem, self-directed learning, and group meeting. For example, in the 1st step of the PBL process, after the teacher introduced the problems, the students created goals of learning (goal setting). For example, "I want to learn and understand the tasks by myself so that I can use it in my daily life or use it to learn in higher education". The students believed that they must understand mathematical content in the problem situation (outcome expectation). In the 2nd and 3rd steps of the PBL process, the students planned to solve the problems by trying find the means required for the problem situation or trying to manage their time and group members' duties in solving the problem (strategies planning).

In the performance phase, most students (85%) showed improvement in their self-control (self-instruction, imagery, attention focusing, and task strategies) and self-observation (self-recording and self-experimentation) in the 2nd, 3rd, 4th, and 5th steps of the PBL process: self-directed learning, group meeting, presentation and discussion, and exercises. For example, in the 2nd and 5th steps of the PBL process, the students used many strategies to solve the problem. For example, they used imagery such as drawing pictures or using mind mapping or they underlined the important points and marked the questions to help them solve the problem situation (task strategies). They tried to find the best solution from their prior knowledge or their experience (self-experimentation). In the 4th step of the PBL, they tried to record the important data to make conclusions from their learning (self-recording).

In the self-reflection phase, most students (78%) showed improvement in their self-judgment (self-evaluation and causal attribution) and self-reaction (self-satisfaction/affect and adaptive/defensive) in the 2nd, 3rd, 4th, and 5th steps of the PBL process: self-directed learning, group meeting, presentation and discussion, and exercises. For example, in the 2nd and 5th steps of the PBL process, the students rechecked their answers with friend (self-evaluation). They evaluated their performance by level of satisfaction. They were rather satisfied of their solution. In the 4th steps of the PBL processes, they evaluated and adapted the solutions of problems presented by other students in order to identify their solution for the problem situations (see Figure 4).

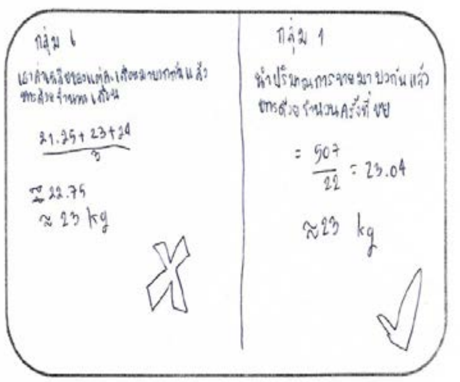	Group 6 Sum mean of Each month, and divided the summation with the number of months $\frac{21.25 + 23 + 24}{3}$ $= 22.75$ ≈ 23kg	Group 1 Sum sale quantities, and divided the summation with terms of sales $\frac{507}{22}$ $= 23.04$ ≈ 23kg

Figure 4. Example of self-evaluation the solutions of problems by presentations of other students. The left image shows a student's work, and the right shows its English translation.

Conclusion

The findings of this study showed that 11th grade students in a PBL classroom demonstrated the SRL in all three phases: the forethought, performance, and self-reflection phases, based on the study of Zimmerman and Campillo (2003). By using Students' SRSI adapted from Cleary (2006) and Teachers' SRSI adapted from Callan and Cleary (2012), we found that the students' mean scores in all phases of SRL increased. The results of the study, from interviews with students, students' reflections, and teachers' notes showed that in the 2nd step of the PBL processes (self-directed learning), the forethought and performance phase were observable. In the 4th step of the PBL processes (presentation and discussion), the students showed obvious expressions SRL in the self-reflection phase.

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Statistics Instructors' Beliefs and Misconceptions About *p*-values

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It is well known that students of inferential statistics find the hypothetical, probabilistic reasoning used in hypothesis tests difficult to understand. Consequently, they will also have difficulties in understanding *p*-values. It is not unusual for these students to hold misconceptions about *p*-values that are difficult to remove. In this study, 19 Australian tertiary statistics educators were surveyed about their beliefs about *p*-values, and it was found that some of these instructors held misconceptions about the nature of *p*-values. These findings suggest that professional learning of statistics instructors is urgently required so that instructors may have their beliefs challenged and corrected.

It is well known that the processes of inferential statistics are difficult for students to understand and, as a consequence, these students resort to blindly following procedures (Garfield & Ahlgren, 1988). One of these procedures is that of the Null Hypothesis Significance Test (NHST). Briefly, the procedure involves making a hypothesis about a parameter of a population, such as the null hypothesis. A sample is then taken from the population and the appropriate sample statistic is calculated. For example, if the hypothesis is about the population mean, then the sample mean would be determined. Then, assuming that the null hypothesis is true, the probability of obtaining the sample mean or one even more extreme would be calculated. For the situation regarding the population mean, this probability is:

$$P((|\bar{x} - \mu| \geq 0)|H_o = \mu)$$

If this resultant probability (known as the *p*-value) is greater than a preset value, then the sample data are considered to be consistent with the hypothesised population parameter and the null hypothesis is not rejected (according to some texts the null hypothesis is accepted). If this p-value is less than the preset value, the sample data are considered not to be consistent with the hypothesised population parameter and the hypothesis is rejected.

This form of hypothetical, probabilistic reasoning is not one with which students are generally familiar, and misunderstandings are common. In fact, in contrast to deterministic reasoning, it is not unusual for students to find any form of probabilistic reasoning difficult (Kahneman, & Tversky, 1982; Tversky & Kahneman, 1982a, 1982b). To complicate matters, full understanding of the NHST process also requires understanding of randomness, probability, sampling, sampling distributions, and variability, about which students may also have misconceptions (Castro-Sotos, Vanhoof, Van den Noorgate, & Onghena, 2007). As a consequence of these difficulties, students develop misunderstandings about the nature of the *p*-value. Previous research has shown that students may believe that the *p*-value is the probability that the null hypothesis is true (Reaburn, 2014; Gliner, Leech, & Morgan, 2002). Students may also believe that the same *p*-value will be obtained if the experiment were to be repeated (Cumming, 2006; Mittag & Thompson, 2000; Nickerson, 2000). Further problems occur because students may not fully understand what not rejecting or rejecting a null hypothesis actually means.

Oakes (1986, as cited in Haller & Krauss, 2002) found that there was also widespread misunderstanding about *p*-values among academic psychologists. Building on that research, Haller and Krauss (2002) surveyed 113 staff and students (including 30 statistics instructors) from psychology departments from six German universities and found that

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some of the instructors and students had misconception, such as believing that NHSTs prove or disprove hypotheses. Some of these participants also indicated that they believed that the *p*-value indicated the probability of the null hypothesis being true. They also indicated that they believed that the value of 1- *p* gives the relative frequency of the null hypothesis being rejected if the experiment were to be repeated. The aim of this study was to determine if there is a similar problem of misinterpretation of *p*-values by statistics instructors in Australian universities.

Method

In the study, I adopted an exploratory paradigm (Mackenzie & Knipe, 2006) to investigate the understanding of *p*-values of statistics instructors in Australian universities. It was a quantitative study in which I used a survey to collect data. The initial participants constituted a sample of convenience, and a snowball recruiting strategy was used (Babbie, 2014). The initial potential participants were recruited from nine universities from New South Wales, Queensland, South Australia, Tasmania, and the Northern Territory. These potential participants were selected because their email addresses and details about their teaching responsibilities were freely available on their respective university websites. Academics who were teaching statistics courses were sent emails with a link to an online survey platform with a request to complete a survey. Because the links were able to be sent to other potential participants, no identifying information was collected and this ensured that the data remained anonymous.

The survey administered included the scenario and six statements utilised by Haller and Krauss in their 2002 study, with small alterations (Figure 1).

Suppose you have a treatment that you suspect may alter the performance on a certain task. You compare the means of your control and experimental groups. You then use a 2-sample independent means *t*-test and your *p*-value is 0.01. What can you conclude? Please mark the following as "true" or "false".

1. You have disproved the null hypothesis that there is no difference between population means.
2. You have found the probability of the null hypothesis being true.
3. You have proved your experimental hypothesis. That is, there is a difference between the population means.
4. You can deduce the probability of your null hypothesis being true.
5. If you decide to reject the null hypothesis, you know the probability that you are making the wrong decision.
6. You have a reliable experimental finding in the sense, that, if, hypothetically the experiment were repeated a number of times, you would obtain a significant result on 99% of occasions.

Figure 1. Survey scenario and questions.

The Haller and Krauss scenario included the details about the *t*-statistic and degrees of freedom, and their wording of Statement 5 was slightly different to that shown in Figure 1. The statements represented common misconceptions of the meaning of a significant test result. Statements 1 and 3 addressed the belief that NHSTs can give definitive proof – the illusion of certainty (Gigerenzer, 2004). Statements 2 and 4 addressed the beliefs that probabilities can be assigned to hypotheses. Statement 5 was similar to the definition of a Type I error, but did not include the proviso that the null hypothesis must be true.

Statement 6 addressed the replication fallacy (Gigerenzer, 2004). Also included in the survey were some questions to collect data about the units into which the academics taught, the courses in which the units were embedded, and the number of years of experience the academics had in teaching statistics.

Results

Fifty emails were sent to potential participants, as described in the Method section. Of these, 19 completed surveys were received. Eleven participants indicated that they were lecturers in a statistics unit, two were tutors in a statistics unit, one was a lecturer and tutor, and one was a statistician embedded in a science faculty. The status of the others is not known. Two participants indicated that they were teaching in units that were part of a mathematics or statistics major, and 14 indicated that they were teaching in service units where the students were not expected to become professional mathematicians or statisticians. The other participants did not answer this question. The academics' years of teaching statistics ranged from two years to 40 years, with a mean of 18 years and a median of 13 years.

At least three participants agreed with each of the statements about the hypothetical two-sample *t*-tests. The number of participants who agreed with each statement is indicated in Table 1. The greatest level of agreement was with Statement 6, with the next highest level of agreement with Statement 5, followed by Statements 4 and 2. Five participants did not agree with any of the statements.

Table 1
Number of Participants Who Agreed with Each Statement

Statement about *t*-test *p*-value	Number who indicated "true" (n = 19)
You have disproved the null hypothesis that there is no difference between population means.	3
You have found the probability of the null hypothesis being true.	5
You have proved your experimental hypothesis. That is, there is a difference between the population means.	3
You can deduce the probability of your null hypothesis being true.	6
If you decide to reject the null hypothesis, you know the probability that you are making the wrong decision.	7
You have a reliable experimental finding in the sense, that, if, hypothetically the experiment were repeated a number of times, you would obtain a significant result on 99% of occasions.	8

Discussion

It has to be admitted that the sample size in this study is small. However, it is concerning to see that all of these misunderstandings about the nature of *p*-values were held by at least three of the participants, all of whom, apart from one, identified themselves as instructors in statistics. In this study, 42% of the participants agreed with Statement 6.

This is a description of the replication fallacy, the idea that 1-p represents the proportion of significant results if the experiment were to be repeated many times. However, as Cumming (2013) so engagingly demonstrates in his *Dance of the p-values*, replications of experiments can result in widely varying p-values. In the study by Haller and Krauss (2002), 37% of statistics instructors also held this misunderstanding.

Statement 5 appears to describe the definition of a Type I error; that is, it is the probability of rejecting the null hypothesis if this null hypothesis is true. A close reading, however, shows that the important proviso that the null hypothesis is actually true was not included. In this study, the proportion of participants who agreed with this statement (41%) is much less than that obtained in the study by Haller and Krauss (2002), where 73% of the instructors agreed with this statement.

Statement 4 is incorrect as it is not possible to assign a probability to a hypothesis in the NHST process (Haller & Krauss, 2002). Statements 1 and 3 are also untrue as we cannot be sure about any conclusion about a population when only a sample is taken. The NHST process is one way that the uncertainty of conclusions based on samples can be addressed; it is disturbing to see that this fundamental issue of uncertainty may be missed by some instructors of statistics. Statement 2 has been shown to be a common misunderstanding among students, and it is concerning to see that 29% of the participants also hold this misunderstanding. This compares with 17% of the instructors in the study by Haller and Krauss (2002).

These results beg the question: How do such misunderstandings arise? Reaburn (2014) noted that the misunderstanding indicated by Statement 2 was not in any of the materials supplied to the students in her study. She proposed that this misunderstanding may arise from the students' attempts to rationalise the difficult material with which they were dealing. This study, along with that of Haller and Krause (2002), suggests that students may also be gaining their misunderstandings from their instructors. It is reasonable to posit that if these students in turn become instructors, they will pass on these misunderstandings to their students. In addition, Gigerenzer (2004), Haller and Krauss (2002), and Pollard and Richardson (1987) have described examples where textbooks have given inaccurate descriptions of the interpretation of NHSTs.

It is also possible that misunderstandings arise because of the teaching methods that are chosen by the instructors. It has been suggested that despite the availability of computers, instructional methods have remained substantially unchanged (Garfield, delMas, & Zieffler, 2012). Modern computers allow much more exploration of the data than was formerly available. In particular, computers allow repeated randomization and simulation. Computers allow students "to create models, repeatedly simulate data from the model, and then use the resulting distribution of [the] computed statistic to draw statistical inferences" (Garfield et al., 2012, p. 883). As they do this, the students can see what "sort of results are typical, and what should be considered unusual" (Cobb, 2007, p. 3).

Whether or not p-values should be used at all is debated in the literature. This is partly due to their tendency to vary, as noted above. This is problematic in the scientific endeavour where replicable experiments are desired. Varying p-values may also be obtained with the same data depending on the method of analysis chosen by the researcher and on whether a one- or two-tailed test is chosen (Hubbard & Lyndsay, 2008). In addition, p-values do not indicate the effect size; a study with a large effect with a small sample may result in the same p-value as a study with a small effect size and a large sample (Hubbard & Lindsay, 2008; Wagenmakers, 2007). In contrast, confidence intervals, which give an estimate of the range within which the value of the parameter of interest may be found, give an idea of the precision of the estimate, make it easier to determine the effect size, and also have the same metric as the point estimate (Cumming 2010; Wagenmakers 2007). Their use also avoids the complicated logic used in the NHST. Whatever one's view on this debate, the NHST is still widely used in the scientific literature, and even if it is replaced by the use of confidence intervals or other methods in the future, there are many years of literature using *p*-values that need to be understood.

Whether not the use of *p*-values continues into the future, and whatever the source of misunderstandings, the findings of this study suggest that professional development for statistics instructors in our tertiary institutions is urgently required so that their misunderstandings are corrected. The findings of this study also suggest that research is needed to see if similar misunderstandings are held by school teachers who teach in pre-tertiary subjects where *p*-values are used. Examples of such subjects include Specialist Mathematics in Victoria (Victorian Curriculum and Assessment Authority, 2015) and Mathematics Applied in Tasmania (Tasmanian Qualifications Authority, 2013). If future researchers are to produce the best possible research, and interpret their findings accurately, then they must be able to understand the statistical procedures they use in their work.

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Revisiting Friedrich Froebel and his Gifts for Kindergarten: What are the Benefits for Primary Mathematics Education?

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This paper provides historical insights and educational background of Froebel's Gifts, hands-on materials developed in the early 19th century. Based on an explorative study with 54 German children (aged 5 to 10) in 2016, we first took steps to explore how these materials meet the demands for early mathematics learning of primary children. Starting with a brief introduction into the work and life of Friedrich Froebel, we outline how using Froebel's Gifts can stimulate the acquisition of mathematical knowledge and abilities, then conclude by considering future research directions within Australian schools.

Introduction

Knowledge of geometrical shapes and solids is a core element of geometry and mathematics education in primary schools. Classroom activities often focus on sorting and naming (prototypical) representatives of three-dimensional objects. However, recent international studies suggest that there is a need to shift focus from naming and sorting shapes to learning experiences that develop primary children's spatial reasoning, including "working on the composing/decomposing, classifying, comparing and mentally manipulating both two- and three-dimensional figures" (Sinclair & Bruce, 2015, p. 319).

Block play has been a popular activity in early childhood for decades. Block-building activities provide children with opportunities to classify solids, manipulate shapes, and develop spatial sense (Reinhold et al., 2013). In this paper, we describe block play activities invented by Friedrich Wilhelm August Froebel, recent studies that have explored the mathematical potential when using blocks in German schools, and conclude with suggestions for future research within Australian schools.

Historical Background – Froebel at his Time

Friedrich Wilhelm August Froebel (1782-1852)

Friedrich Wilhelm August Froebel, one of the most famous European philanthropists and reform pedagogues, was born in Germany in 1782. He lived during a historical period marked by social and political turbulences with feudalism turned to capitalism in most parts of Europe. Froebel, the son of a priest, completed a forester's apprenticeship after finishing school. This was followed by uncompleted studies in science at the University of Jena (Germany). After a short employment as a private secretary, Froebel moved to Frankfurt/Main (Germany) to apply to do architecture, but changed his plans to train as a teacher. He taught in schools in Frankfurt and Yverdon-les-Bains (Switzerland) that used the pedagogy of Johann Heinrich Pestalozzi (1746-1827). Froebel also worked as a private tutor in a nobleman's family and returned to Germany (University of Göttingen) to study languages and natural sciences. In 1816, Froebel finished his academic studies and founded

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a correctional institution in Griesheim (Germany). During this phase (1817-1830), he focused on establishing his ideas of pedagogy for early childhood and released one of his most in-depth publications: *Die Menschenerziehung* (*Education of Mankind*; Fröbel, 1826). His philosophical and ideological views were further expanded in the publication, *Die Menschenerziehung* providing insights into Froebel's understanding of educational development beginning with early childhood (Fröbel, 1832/1982).

Based on this concept, Froebel opened the *Anstalt zur Pflege des Beschäftigungstriebs für Kindheit und Jugend* (institute for the care of the children's desire for occupation) in 1837, situated in Bad Blankenburg (Germany). Thereby, he established what he called the first "kindergarten" (children's garden) – where the children were supposed to raise and bloom like flowers in the metaphorical sense. Friedrich Froebel died in 1852, aged 70.

Children's Play and Play Gifts Invented by Froebel

Froebel's influence on early childhood pedagogy (including early mathematics learning) went beyond establishing childcare institutes. In doing so, he developed standards for childcare that were compatible with a humanistic and democratic approach to education. No doubt, his approach was influenced by the philosophical discussion during the period of Enlightment in the 18th century in Europe. Froebel's opinions of how children engaged at kindergarten differed from his contemporaries' concepts that were often limited to ideas of adult supervision and instruction (Heiland, 1991).

Similar to Pestalozzi, Froebel was convinced that children gain a deeper understanding of the world around them when given opportunities to interact with the world via concrete activities, including manipulation of prepared hands-on materials. In addition, Froebel stressed the key role of children's play for their educational development (Fröbel, 1832/1982). He recommended providing children with well-chosen materials during play with the intention to stimulate their understanding of the world. Hence, Froebel's constructive approach (in the sense of discovering the world, e.g., spatial relationships or properties of geometrical solids by constructing with blocks) forms an important part of the work in "kindergarten" (Frey et al., 2006, p. 169).

Pursuing his intentions, Froebel developed special educational toys for his "kindergarten", namely the so-called *gifts* ("Spielgaben"). Two meanings are associated with this term: first, the idea of 'presenting the child a gift to play with', and second that each child possesses innate human 'gifts'. The gifts were embedded in a variety of learning settings, and kindergarten educators were encouraged to guide the children during their reconstruction of geometrical arrangements.

In total, Froebel developed six gifts, encompassing the growing complexity of the world around us: Gift 1 includes six balls made of different materials whereas Gift 2 includes a variety of solids (cube, cylinder, sphere). In contrast, Gifts 3, 4, 5, and 6 represent the idea of decomposing the cube into smaller units like small cubes, small cuboids/rectangular prisms, triangular prisms of different sizes. For example, Gift 3 consists of a threefold divided cube which (after being cut orthogonally in each direction), produces eight congruent smaller cubes that can be assembled into a variety of (new) arrangements while playing (see, Figure 2). Gift 4 (Figure 2) is made of eight smaller cuboids (relation of side lengths is 1:2:4). This gift and Gifts 5 and 6 (including prisms) invite children to explore mathematical concepts such as measuring heights and surfaces and encourage discoveries of the idea of balance or part-whole relationships (Henschen & Tescher, 2013). Furthermore, Gift 5 resembles Gift 3, however instead of eight cubes (Gift 3), Gift 5 appears to consist of 27 cubes. By cutting three of the 27 cubes into two congruent triangular prisms, and cutting another three cubes into four smaller triangular prisms Gift 5 offers another expansion of possible constructions (Hebenstreit, 2003). Here, the ratio of volume to single part is 1:2:4 (Figure 1). Gift 6, resembles the less complex Gift 4 comprising the division of 27 cuboids, making a composition of 36 elements of different shapes (cuboids of the same size as in Gift 4 and divided cuboids).

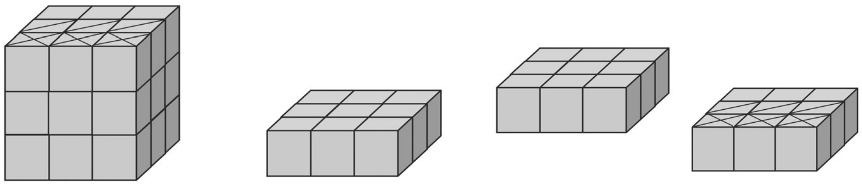

Figure 1. Froebel's Gift 5 (parts arranged as a cube on the left, decomposed into layers on the right).

An analysis of the elements of the Froebel's Gifts 3, 4, 5, and 6 reveals the growth in structural complexity. The gifts have the potential to encourage children to explore and discover geometric properties of three-dimensional objects, including spatial relations among parts of the gift pieces. In addition to arranging the pieces of a gift as a cube, Froebel suggested to arrange all parts of a gift in new arrays. While arranging the pieces in these ways, a child explores what Froebel called *forms of knowledge* (dividing the entire cube into smaller, equal pieces), *forms of life* (arrangements that resemble objects from children's environment like a sofa or staircase) and *forms of beauty* (symmetrical arrangements of smaller parts). This element of Froebel's concept aligns with Pestalozzi's philosophy, reflecting three different approaches of life and learning: *Forms of knowledge* – our head and mind; *forms of life* – what we touch with our hands; and *forms of beauty* – what touches our heart (Frey et al, 2006; Heiland, 1991).

Exploring Froebel's Gifts in Primary Mathematics – First Insights

A Traditional Hands-on Material Seen from Today's Perspective

The original setting for using the Froebel's Gifts with children was in early education of Friedrich Froebel's first kindergarten, established in the early 19th century. Having a closer look at the structural properties of the gifts, it could be argued that from today's mathematics educational perspective the gifts have the potential to foster young children's understanding of geometry, deepen their geometric knowledge and spatial skills.

Geometry and spatial thinking is viewed as children's foundation knowledge needed for learning mathematics in the early years (e.g., Horne, 2003). Guay and McDaniel (1977) found that mathematically high-achieving children in elementary school outperform mathematically low-achieving children not only in terms of their arithmetic skills and knowledge but also concerning their visualization skills. This connection is shown specifically for the ability to mentally rotate or mentally compose/decompose spatial arrangements (Linn & Petersen, 1985). In addition, activities in the first years of learning arithmetic are most often based on the use of hands-on materials (e.g., depicting an arithmetic operation with counters placed in rows of tens, each of the rows partitioned in two lots of five counters laid down with a blank space). Children have to identify the geometrical structure of these hands-on materials before developing mental representations, based on the intentional manipulation of these materials (Radatz, 1990). In line with these arguments, Clements and Sarama (2011) have indicated that geometry and spatial reasoning are important to the development of mathematics. However, they also indicated that these areas of mathematics are often neglected, or receive little attention in the early years' classrooms. This is a concern as geometry forms a fundamental dimension of the mathematics curriculum.

Meeting Australian and German Standards in Primary Education

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that spatial reasoning and geometric modelling includes creating mental images of geometric shapes, recognizing and representing shapes from different perspectives. Similarly, within the Australian Curriculum (AC; Australian Curriculum and Reporting Authority, 2017), children develop geometric reasoning skills when describing, analysing, and understanding structures in their world. However, geometric reasoning skills as suggested within the AC are introduced to children at Grade 3 (aged eight) rather than when commencing school.

Geometry has been in the German curriculum for primary education since the late 1960s (Franke & Reinhold, 2016), with a strong focus on shape recognition and drawing skills, especially in the Deutsche Demokratische Republik (DDR, communist part of Germany until 1990). During the 1990s, the mathematics education discussion on the importance of geometric activities and visualization influenced a shift from shape recognition, drawing and preparation for geometry at secondary level, to geometrical reasoning and opportunities to develop spatial abilities. Today's German national standards for mathematics in primary (KMK, 2004) outline a wide range of geometrical competencies children should develop by the end of Grade 4. These refer to three (among) five key content areas, namely *space and shape* (Raum & Form), *pattern and structure* (Muster & Struktur), and *measurement* with length and volume (Größen & Messen). Regarding *space and shape*, the standards stress the importance of visualization skills (e.g. creating mental images of geometric shapes, mentally manipulating them, orientation in space, changing perspectives). The standards also indicate that children should be able to identify, name, depict, produce and systematically investigate the geometric properties of representatives of geometrical shapes and objects.

Raising New Questions Concerning Traditional Hands-On Material

Studies with German preschool and primary school children's use of Froebel's Gifts provided evidence of geometric reasoning and strategies when solving construction problems (Reinhold, 2015; Reinhold & Wöller, 2016). In these studies, the tasks using the gifts were initially used as a tool to investigate the development of young children's geometrical concept knowledge. However, experiencing the gifts in these studies raised the question: "To what extend are the gifts suitable to pursue current objectives in primary mathematics education (Grades 1 to 4, aged 5-10)?" This broad and challenging question motivated an explorative project involving Froebel's Gifts in German schools.

In 2016, six complementary studies conducted by Master of Education students contributed to the exploration and use of Froebel's gifts in German primary classrooms (see also Friedl et al., 2017). Each master's student developed an individual research focus and question, including various methods for data collection and analysis. They collected and interpreted data with a common aim to empirically support the hypothesis that learning environments encompassing Froebel's Gifts contributes to children's development of a mathematical issue. The chosen research topics included geometry and arithmetic. The master's students observed the children working in small groups or with larger groups of children during mathematics lessons. All activities were video-recorded and mostly accompanied by one-to-one interviews with individual children after their work with the Froebel's Gifts. Altogether, 54 German children (Grades 1 to 4) in different primary schools participated in the project and complementary studies. The following examples from these explorations report insights into the results.

Discoveries in Geometry

Previous studies within German schools focused on how analyses of differences in children's block construction processes and products contribute to a deeper understanding of their conceptual knowledge of geometrical solids (e.g. Reinhold & Wöller, 2016). For younger (preschool) children, we also reported on their difficulties in (re)constructions of cube arrays for purposes of enumeration (Reinhold et al., 2013). However, there was some evidence that children's fine motor function and their general kinaesthetic competence when assembling single blocks or components of three-dimensional arrays was usually developed at the beginning of school. Therefore, we asked primary children to articulate their conceptual knowledge of geometrical solids when constructing with Froebel's Gifts. These investigations revealed that primary children (aged eight to nine) faced difficulties when demonstrating their geometrical understanding of cuboids and concepts of a cube. During their construction of cubes and cuboids in the interviews, children explained why they considered their construction was a cube by naming properties or offering other comments such as: "Because it has equally long sides", "It looks the same from all sides", or "All surfaces are the same and it's three-dimensional."

These research findings were the starting point for the design of classroom activities with Froebel's Gifts 3 and 4 for Grades 1 and 2. Wiesner (2016) included explorations of the blocks that led to a range of constructions – with some children discovering (without further explanation) that two of the rectangular prisms in Gift 4 had the same volume as two cubes in Gift 3. Further group activities included the use of several sets of Froebel's Gifts led to the (re)construction of bigger cubes or cuboids (Figure 2). These activities were accompanied by exercises that required the children to name the shapes of the blocks they used. During the discussion at the conclusion of each lesson, the teacher (a master's student) stressed the special relationship between cube and cuboids. Individual interviews before and after this intervention supported the hypothesis that the activities with the gifts had a positive influence on the development of children's conceptual geometrical knowledge of cubes and cuboids. For example, children were likely to correctly name shape properties by making connections to cubes or cuboids (see also Friedl et al., 2017).

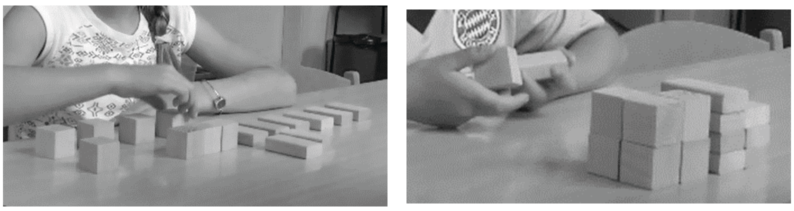

Figure 2. Exploring the properties of cubes and cuboids with Froebel's Gifts 3 and 4 (Wiesner, 2016).

Daniel (2016) conducted a study with a pre-post-test design and an intervention using the Gifts 3 and 4. He designed activities according to the concept of natural differentiation (e.g., Krauthausen & Scherer, 2013) for a group of Grade 4 students. For example, providing children with a construction plan (Figure 3) challenged them to construct various possible solutions using the pieces of Gifts 3 or 4.

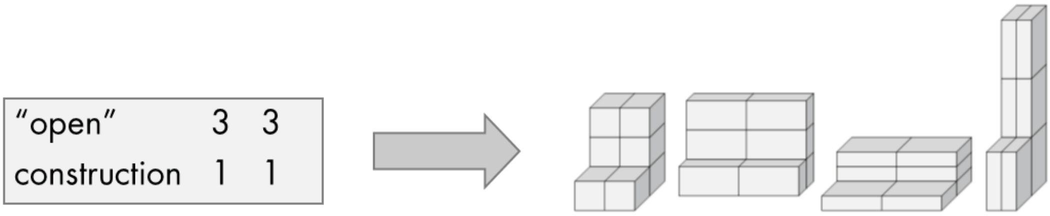

Figure 3. Construction plan and solutions using the blocks of Froebel's Gifts 3 and 4 (Daniel, 2016).

Another challenge was to analyse a complex array made from pieces of Gift 4, that required children to visualize (mentally move and compose pieces) how many and what type of different birds-eye views can occur if all pieces of Gift 4 (made of eight congruent rectangular prisms) are constructed as a cube (Figure 4). The analyses of the pre- and post-test revealed that children demonstrated a more analytic approach when distinguishing between cubes and cuboids. Terms such as "surface" or "volume" were used accurately.

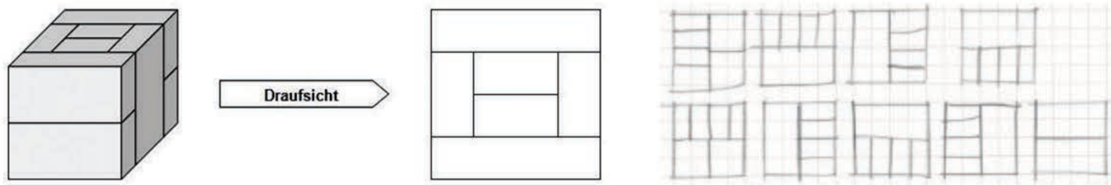

Figure 4. Task and student's drawings of (possible) top views (Daniel, 2016).

Tackling Arithmetic Issues

In addition to using Froebel's Gifts to explore geometric topics, there were investigations relating to arithmetic issues. For example, Laußmann (2016) focused on part-whole relationships and used the pieces of Gift 3 in her work with first-graders. After a lesson that provided opportunities for children to explore free constructions with Gift 3, Laußmann asked the children (in an interview) to find as many different partitions (groups) into three (four, five, or six) subsets as possible (Figure 5). All children found at least one possible solution, some found a range, while others worked systematically and tried to find *all* combinations. Sullivan (2015) described a task designed to find multiple solutions as having the potential for all students to find a solution, as 'low floor' benefits, and when students generalise having found all solutions, as 'high ceiling' benefits. This activity not only addressed the arithmetic idea of part-whole-relationships, but challenged the children's problem-solving, argumentation skills and ability to justify their argument.

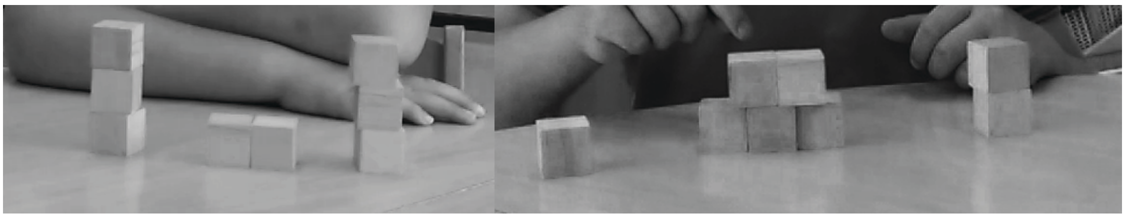

Figure 5. Dividing the number of cubes of Froebel's Gift 3 into three subsets (Laußmann, 2016).

Von Lucke (2016), explored Grade 1 children's arithmetic knowledge when using Gift 6 and focused on number concepts. Her study explored children's knowledge of sorting and classifying objects and counting representations of the same objects. In her interviews, Von Lucke asked the children to compare the number of single units in two sets, of same cardinality (e.g., six smaller and six bigger cuboids), exploring the concept of invariance, or one-to-one correspondence, respectively. As a result, activities of construction and re-construction with the pieces appeared to positively influence children's understanding of the cardinality as most children were able to demonstrate one-to-one correspondence by rearranging the pieces (Figure 6).

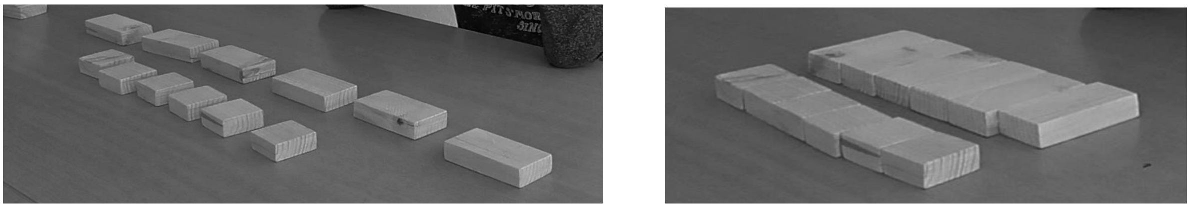

Figure 6. Invariance of the number of single units in two sets of the same cardinality (Von Lucke, 2016).

Conclusion: Research Desiderata and Ensuing Studies

The focus of our paper was to provide a historical overview of Froebel's Gifts and to present examples of the mathematical attributes of the gift sets. In previous studies on children's geometrical concept knowledge (e.g., Reinhold & Wöller, 2016), activities with the gifts were informative for exploring children's knowledge of geometrical objects. Moreover, several explorative studies by master's students suggest that using the gifts in German primary classrooms contribute to meeting today's international standards for mathematics education in Grades 1 to 4 – aligning the German and the Australian curriculum, and providing opportunities for natural differentiation in tasks with "low floor and high ceiling" (Sullivan, 2015). However, a review of literature suggests that Froebel's Gifts have yet to be used in Australian research studies. Therefore, we identify three possibilities for prospective research based on the results of these (explorative) studies with the gifts, namely using Froebel's Gifts (i) as a *tool to investigate Australian children's geometrical concept knowledge* on cubes and cuboids, (ii) in *pre- and in-service teacher education*, and (iii) *in the Australian primary classroom*. The outcomes will contribute to knowledge for improving educational opportunities and outcomes in education, with a focus on the early years of primary mathematics education, teaching and learning, and ultimately develop a theoretical framework for international comparisons.

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Perceived Changes in Teachers' Knowledge and Practice: The Impact on Classroom Teachers from Leader Participation in Whole-School Reform of Mathematics Teaching and Learning

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Thirty primary classroom teachers quantified perceived changes in their practice and knowledge attributed to their school leaders' participation in a six-day course focused on leaders designing and implementing a whole-school reform of mathematics teaching and learning, and the school-based professional learning that followed. The project and framework underpinning the reform are outlined, and the teachers' reported changes in pedagogical practices are described. Teachers identified many changes in pedagogy and growth in knowledge. Not surprisingly, changes in pedagogical practices were even greater for specialist mathematics teachers who had also participated in a separate six-day course.

Teachers' mathematical knowledge and instructional practice are keys in improving student outcomes (Artz & Armour-Thomas, 1999; Hill, Rowan, & Ball, 2005). Previous research has helped to describe the nuances of effective mathematics' teaching that facilitates learning for all students (Anthony & Walshaw, 2009; Clarke et al., 2002; Siraj, Taggert, Melhuish, Sammons, & Sylva, 2014). The instructional leadership of principals and other school leaders has also been shown to have an impact on teacher change and, ultimately, improvements in student learning (e.g., Hallinger & Leithwood, 1994; Leithwood, Louis, Anderson, & Wahlstrom, 2004; Robinson, Lloyd, & Rowe, 2008). In particular, Robinson et al. (2008) found a strong link between leaders' who promote and participate in teacher professional learning and improved student academic outcomes, and concluded that, "the more leaders focus their relationships, their work, and their learning on the core business of teaching and learning, the greater their influence on student outcomes" (p. 636). Leithwood, Harris, and Hopkins (2008) contended that "school leadership is second only to classroom teaching as an influence on pupil learning" (p. 27). However, leading and initiating school reform is complex and takes time (Desimone, 2002; Fullan & Steigelbaur, 1991).

Professional learning models take many forms, but Fullan (2001) suggested that "learning in the setting where you work, or learning in context, is the learning with the greatest payoff because it is more specific (customized to the situation) and because it is social (involves the group)" (p. 14). However, this can lead to models where key teachers in a school participate in professional learning with an expert in the field, and then attempt to replicate and apply the experience for colleagues at school. Notably, this "train-the-trainer" model has been reported to lead to variations in the quality of the professional development delivered to teachers and "shallow training experiences" (Watt, 2015, p. 87).

In this study, we aim to describe the impact on teacher and student learning associated with a professional learning program that prepared school leaders to support and initiate whole-school reform in mathematics teaching and learning. The school leaders became the professional learning providers for their teaching staff, with the benefit of teacher learning occurring in context and focused on impact for their own students. However, the quality of the professional learning for teachers was dependent on the capacity of their leaders to translate and reproduce the key messages and quality of the original professional learning.

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The Interconnected Model of Teacher Professional Growth (IMTPG, Clarke & Hollingsworth, 2002; see Figure 1) built upon previous change models by adding more detail, distinguishing the processes of reflection and enactment, and broadening changes in student outcomes to include all aspects that teachers believe to be salient in their context.

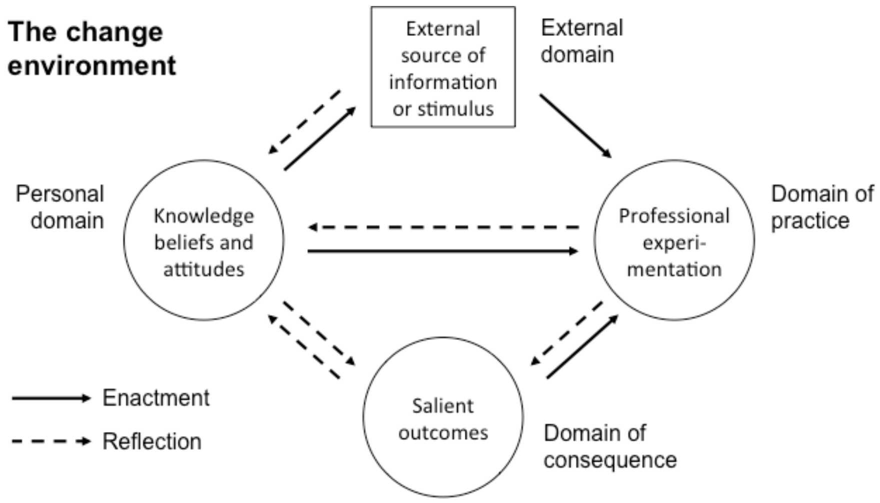

Figure 1. The Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002).

The IMTPG provided a framework for understanding the professional growth process in this study, with the external source of information or stimulus being the teachers' professional learning provided by their school leaders and the change in knowledge (and practice) being influenced by professional learning, professional experimentation, and salient outcomes. The professional experimentation was the opportunity for teachers to implement new or refined teaching strategies and assessments and the salient outcomes were the benefits they derived and valued from these influences.

Background to Leading Mathematics Learning and Teaching Course

From 2010 to 2017, all primary school leaders from four Catholic Dioceses in New South Wales have participated in a course aimed at developing their instructional leadership in mathematics so that they might implement a whole-school approach (Fullen, Hill, & Crévola, 2006) to improving mathematics learning for all. In each Diocese, this has been a system-wide and well supported initiative. The Leading Mathematics Learning and Teaching (LMLT) Course, developed by the second author, included six full days of professional learning spaced over a year, and targeted school leaders including principals and those staff with a mathematics or pedagogical leadership role.

The research cited earlier highlighted the positive impact of principals being instructional leaders, and participating in professional development alongside their staff in their own school context. These were key factors in deciding to engage leaders in the LMLT course rather than the classroom teachers. The participants' role was to consider insights from the research, scholarship, and experiences explored during the course, and then, as a leadership team, to design and implement an action plan that would initiate or extend positive changes to the teaching and learning of mathematics in their school. This required designing and delivering a program of contextualised teacher professional learning. The professional learning provided by leaders was often informed by, or replicated, aspects of the LMLT course. For example, all school leaders introduced the Mathematics Assessment

Interview (MAI) and associated growth point framework (Gervasoni et al., 2011) as an assessment tool, and designed and implemented professional learning so that all teachers could learn about and use this assessment tool. The LMLT course focused on the school leaders first learning about the MAI and growth point framework so that they could design and lead the associated teacher learning at their school. The course content also included: exploring the features of high quality learning environments; mathematics inclusion within a social justice framework; and effective pedagogical practices associated with whole number learning (e.g., Clarke et al., 2002; Siraj et al., 2014). It was anticipated that the learning activities, critique of research and scholarship, and professional discussion during the course, would inform the leaders' approaches to designing and implementing relevant professional learning for their school community and context.

All leaders introduced the MAI to their school to monitor all students' developing knowledge and strategies across four whole number domains (Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies). Analysis of these assessment data was used to inform the school leaders' school action plan and the professional learning and support they would provide for their teachers.

Ultimately, the aim of the LMLT Course was to support school leaders to improve and sustain improvement in mathematics outcomes for all primary students in their school. In conjunction with strategies to support all students, each school employed a specialist teacher who had participated in the Extending Mathematical Understanding (EMU) Specialist Teacher course to support students who were identified as vulnerable in learning mathematics. These specialists were qualified to conduct the EMU intervention program (Gervasoni, 2004) for these students, who were identified as mathematically vulnerable and who may not fully benefit from the classroom mathematics program. The EMU specialist also had a role in supporting the professional learning of staff with leaders during the change process.

In this paper, we address the research question: What changes in professional knowledge and practice do classroom teachers attribute to the professional learning and support offered by their school leaders as a result of their leaders' participating in a six-day course focused on whole-school reform of mathematics teaching and learning?

Method

After two years of implementing the whole-school approach for the teaching and learning of mathematics, classroom teachers in 26 schools across two New South Wales Catholic Dioceses were invited to complete an online survey that addressed the effectiveness of the LMLT course and its impact on a whole-school approach for enhancing mathematics learning for all students. School leaders, EMU specialist teachers, classroom teachers, parents, and children completed the surveys in late 2016. In this paper, we focus on the results of two items from the classroom teacher survey that explored the impact on classroom teachers' mathematics teaching and knowledge.

Participants

Thirty teachers responded to the two survey questions. Their school leaders had participated in the LMLT course in 2015. Teaching experience ranged from 1 to 37 years, with a mean of 14.2 years, and a mean of 10.9 years in their present school. They taught mathematics on average 4.7 days per week and 5.5 hours per week, and taught a range of year levels from the first year of school (Kindergarten) to Year 6, and some taught composite year levels (e.g., Year 1/2).

Online Survey Items

Previous surveys and discussions with teachers by researchers indicated that they felt that they had learned a great deal and their teaching of mathematics had changed. To explore this further, we posed the following two questions in an online survey:

1. We are interested in your perceptions of the *amount of change in your teaching practice* since you commenced professional learning aligned with [their whole-school] approach. Please indicate the extent to which these aspects were/are a part of *your* teaching of mathematics (*prior to* the implementation of [their whole-school reform] and now) using the scale below. [The scale was from 0 = Not at all and 10 = A great deal.]

2. We are interested in the extent to which the recent focus of mathematics in your school has contributed to a change in your knowledge about the teaching of mathematics. Please rate your understanding of this content (*prior to* the implementation of the [their whole-school reform] and *now*) using the scale below. [The scale was from 0 = No understanding and 10 = A thorough understanding.]

The statements in Tables 1 and 2 were developed as a reflection of the recommended aspects of pedagogy and knowledge that were addressed in the course with the exception of items l, m, and n, which we anticipated may have reduced as a result of the reform.

Data Analysis

Mean scores and standard deviations were calculated for each item, as was the change in means. Paired t-tests were performed to assess the statistical significance of the changes for each item. Statistical significance was set at 0.05 and all tests were two-sided.

Results

In Tables 1 and 2, we report the mean ratings from teachers of the stated practices and knowledge, before and during the reform, respectively and the associated change in means. In Table 1, p values for paired t-tests are given to indicate the statistical significance of the change. No p values are given in Table 2, as all changes were significant at the 0.05 level.

Change in Teachers' Instructional Practice

For teachers' instructional practice, the positive change of mean (see Table 1) varied from 0.6 to 2.9, indicating there was substantial variation between the amount of change for each activity across this group of teachers. Out of the 16 statements provided, the practice for which there was the greatest mean change (2.9) was (b) "I use open tasks in mathematics lessons." Other practices with considerable perceived change were (j) "I expect students to explain their thinking and reasoning" (2.0) and (i) "I allow wait time for students to think prior to answering questions" (1.9). These data may indicate that teachers initiated a greater use of discourse in mathematics lessons than was previously employed.

Table 1
Means, Standard Deviations, Changes in Means, and p values for Teacher Practice Ratings Prior to and During the Implementation of a Whole-School Reform

Teacher instructional practices (n = 30)	Mean (SD) Prior to reform	Mean (SD) Now	Change of mean	p value
a. I use assessment data to inform planning and teaching.	7.0 (2.3)	8.5 (1.5)	1.5	<0.001
b. I use open tasks in maths lessons.	5.2 (1.7)	8.1 (1.2)	2.9	<0.001
c. I use concrete materials to assist students' learning.	7.5 (1.9)	9.0 (1.2)	1.5	<0.001
d. I'm enthusiastic about teaching maths.	7.1 (2.3)	8.1 (1.7)	1.0	0.015
e. I plan collaboratively.	7.2 (2.4)	8.0 (2.3)	0.8	0.001
f. I discuss my successes and challenges about my maths teaching with others.	7.1 (1.9)	8.3 (1.3)	1.2	<0.001
g. I address individual learning needs.	7.0 (1.7)	8.3 (1.1)	1.3	<0.001
h. I do professional reading about the teaching and learning of mathematics.	5.6 (2.0)	6.9 (1.6)	1.3	<0.001
i. I allow wait time for students to think prior to them answering questions	6.5 (2.4)	8.4 (1.4)	1.9	<0.001
j. I expect students to explain their thinking and reasoning.	6.7 (2.3)	8.7 (1.5)	2.0	<0.001
k. I use questioning to support students' learning and promote thinking.	7.0 (1.9)	8.5 (1.3)	1.5	<0.001
l. I use assessment data to group students into like ability groups.	7.2 (1.9)	8.3 (1.8)	1.1	0.002
m. I provide opportunities for students to learn and practise formal written algorithms.	7.1 (1.6)	6.8 (2.5)	-0.3	0.463
n. I use worksheets in mathematics lessons.	6.1 (2.1)	4.4 (2.3)	-1.7	<0.001
o. I prepare enabling and extending prompts before the lesson begins.	5.8 (1.9)	6.4 (2.5)	0.6	0.032
p. I spend time talking to individuals about their thinking and understanding.	6.5 (1.6)	8.0 (1.6)	1.5	<0.001

Only two statements revealed perceptions of negative change. These were (n) "I use worksheets in mathematics lesson" (-1.7) and (m) "I provide opportunities for my students to learn and practise formal written algorithms" (-0.3). This may suggest that teachers now implemented less of the "drill and practice" that might be associated with more traditional mathematics teaching. However, the change in item m was not statistically significant. While nine teachers had a reduced rate for (m), 11 teachers indicated their practice had not changed. The mean score for these 11 teachers was 7.5, which might suggest learning and practising formal written algorithms was, and still is, a common practice for these teachers. Also, 10 teachers indicated an increased rate, suggesting that this practice was occurring more often. The highest mean response for an item at the time of completing the survey was for item (c) "I use concrete materials to assist students' learning" (9.0). However, this item also had the highest mean response (7.5) prior to the reform, indicating that for some teachers, this may have been a well-established practice prior to the reform.

Change in Teachers' Knowledge for Teaching Mathematics

The teachers were also asked to rate their professional knowledge in relation to certain statements both prior to the reform process, and at the time of completing the survey. While we cannot be sure of the basis of teacher judgements about the extent to which their knowledge had changed, they clearly indicated that their knowledge had increased, and in all cases these changes were statistically significant (Table 2).

Table 2

Means, Standard Deviations, and Changes in Means for Teacher and EMU Specialist Knowledge Ratings Prior to and During the Implementation of a Whole-School Reform

Teacher knowledge (T) (n = 30) EMU specialist knowledge (ES) (n = 13)	Mean (SD) Prior	Mean (SD) Now	Mean change (n = 30)
a. Knowledge of the key steps that describe children's progress in whole number learning	T 5.7 (1.7) ES 4.3 (1.7)	7.6 (1.4) 8.2 (0.7)	1.9 3.9
b. Knowledge of students' mathematical understandings/abilities	T 6.4 (1.4) ES 4.9 (1.8)	8.1 (1.2) 8.2 (0.7)	1.7 3.3
c. Knowledge of appropriate questions to extend mathematical understanding	T 5.9 (1.4) ES 4.0 (1.6)	7.7 (1.4) 8.5 (0.8)	1.8 4.5
d. Knowledge of features of high quality tasks	T 5.8 (1.6) ES 4.8 (1.9)	7.5 (1.6) 8.3 (0.6)	1.7 3.5
e. Knowledge of common misconceptions or difficulties in mathematics learning	T 5.7 (1.6) ES 4.4 (1.9)	7.0 (1.9) 8.2 (0.9)	1.3 3.8
f. Knowledge of students' mental and invented calculating strategies	T 6.4 (1.8) ES 4.5 (1.5)	7.7 (1.4) 7.8 (0.6)	1.3 3.3

We also compared their responses to those of the EMU specialists who were from the same cohort of schools as the teachers. The EMU specialists had participated in a six-day course focusing on effective teaching strategies for students who are vulnerable in learning mathematics. These specialists also conducted an intervention program for young students (aged 5-7) during the year while participating in the course, allowing them substantial opportunity to refine and implement new pedagogies and understandings.

The mean change in scores for the EMU specialists was just over double the mean change for classroom teachers' for the same set of statements. This is not surprising given their extensive professional learning opportunities when compared with the classroom teachers. The difference in reported change of knowledge between these two groups may provide some evidence that the teachers were rating the extent of their knowledge in a thoughtful way.

Conclusion

In this study, teachers' responses to survey items suggest that aspects of their practice and knowledge for teaching mathematics changed as a result of their schools' implementation of a whole-school reform for the teaching and learning of mathematics that was initiated through their school leaders participating in the LMLT Course. In some cases, this reflected an increased use of a particular practice; in others, a decrease. The amount of change varied across aspects of practice and knowledge. Classroom teachers' self-reported change of knowledge was on average less than that of the intervention specialist teachers who had experienced substantially more professional learning directly with an expert.

With respect to the IMTPG model, changes in the Personal Domain (knowledge) and the Domain of Practice (instructional practice) were initially influenced by the External Domain (the professional learning). Through enactment and reflection, it appears that changes in teachers' knowledge may have been influenced by outcomes which they regarded as salient, such as student opportunities to reveal their understanding. For example, consider those perceived changes in practice that were reported as greatest, such as the use of open tasks (item b) and expecting students to explain their thinking and reasoning (item j). Both practices enable students to demonstrate to the teacher what they knew and could do, and were reflected in the considerable change identified in their response to item b in Table 2.

Financial and organisational issues often drive the decision to provide external professional learning for a select group of teachers as compared to all school staff. Our findings suggest that the leaders were able to implement professional learning that led to important changes in teachers' practices. However, perceived change was greatest for the specialist teachers who participated in professional learning at its original source.

Overall, there are features of this professional learning approach that appear to contribute to developing and sustaining teacher capacity to enhance student learning. These include:

- Principals and other leaders participating in the professional learning (Robinson et al., 2008).
- Professional learning for classroom teachers was situated in their own school and classroom contexts (Elmore, 2004; Stoll, 1999).
- Data-based decision-making was utilised to inform the school action plan. This has been shown elsewhere to improve the quality of teacher instruction and therefore student performance (van Geel, Keuning, Visscher, & Fox, 2016).

Further analysis of the survey data and future focused case studies will enable the project team to determine the impact of the professional learning for leaders, specialist teachers, and children, and make recommendations for those creating school reforms in mathematics that includes building school leaders' capacity to support teachers' professional learning.

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Examining the Impact of Lesson Structure when Teaching with Cognitively Demanding Tasks in the Early Primary Years

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The current investigation systematically contrasted teaching with cognitively demanding (challenging) tasks using a task-first lesson structure with that of a teach-first lesson structure in a primary school setting (Year 1 and 2). The findings indicate that there is more than one way of incorporating challenge tasks into mathematics lessons to produce sizeable learning gains. Analyses of interviews with teachers and students regarding their perceptions of learning with challenging tasks suggest that each type of lesson structure has distinct strengths. It is concluded that teachers should consider varying the structure of the lesson to provide a range of learning experiences for students.

Over the past few decades, there have been calls to reform mathematics education in Australia to increase the amount of time students spend engaged in deep problem solving and cognitively demanding mathematical tasks (e.g., Hollingsworth, McCrae, & Lokan, 2003). As part of this reform process, it has been argued that traditional lesson structures (i.e., teacher explanation, followed by student practice and correction) are inherently inadequate for meeting contemporary mathematical learning objectives (Sullivan et al., 2014). Instead, reform-oriented teaching approaches have frequently employed a triadic lesson structure: Launch, Explore, Discuss (Stein, Engle, Smith, & Hughes, 2008). Considerable empirical evidence is emerging as to the efficacy of reform-oriented approaches. To summarise, classroom climates perceived by students or expert observers to be more reform-oriented appear to foster students who are more intrinsically motivated to learn mathematics (e.g., Middleton & Midgley, 2002) and perform better mathematically (e.g., Jong, Pedulla, Reagan, Salomon-Fernandez, & Cochran-Smith, 2010).

However, from the perspective of cognitive load theory, launching a lesson with a cognitively demanding activity, which is not explicitly linked to teacher instruction and prior learning, may be problematic (Kirschner, Sweller, & Clark, 2006). This argument is based on the idea that working memory has limited capacity to process novel information and is easily overloaded when required to solve an unfamiliar problem (Sweller, Kirschner, & Clark, 2007). Consequently, it may be that an alternative lesson structure which adopted the same reformist content and pedagogy, but began with a less cognitively demanding activity, would result in even larger gains in mathematical performance.

Given these contrasting evaluations, there is a need to disentangle the various elements of a reform-oriented lesson and to empirically investigate the impact that systematically varying one aspect, such as lesson structure, has on subsequent student learning outcomes and the learning experience of students. To address this issue, the current investigation contrasted teaching with cognitively demanding tasks (challenging tasks) using a task-first lesson structure (Task-First Approach) with that of a teach-first lesson structure (Teach-First Approach), through the delivery of two programs of mathematics instruction to Year 1 and 2 students. The aim was to investigate how varying lesson structure impacts teaching and learning with challenging tasks. The central question was:

What are the advantages of using cognitively demanding tasks to launch lessons and support subsequent instruction and discussion (Task-First Approach) compared with using

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cognitively demanding tasks to extend understanding, following instruction and discussion and the completion of several more routine tasks (Teach-First Approach)?

This question was explored from a variety of different perspectives. Study One considered the question from the point of view of evaluating the impact of lesson structure on student outcomes, including mathematical performance, task-based persistence and intrinsic motivation to learn mathematics. Study Two examined teachers' confidence and competence in teaching with cognitively demanding tasks, and whether lesson structure impacts teacher willingness to incorporate such tasks more comprehensively into future instruction. Study Three explored student perceptions of learning with cognitively demanding tasks and whether students prefer a given lesson structure.

Method

Participants

Participants included Year 1 and Year 2 students (n = 75), and their respective teachers (n = 3), who attended a primary school located in Melbourne. In Victoria, students typically turn seven years of age during Year 1, and eight years of age during Year 2.

Procedure

The first author was responsible for designing and teaching two units of work in number and algebra across two school terms. The first unit of work related to number patterns (Patterning Unit), and comprised 16 lessons. The second unit of work related to addition and missing addend problems (Addition Unit), and comprised 12 lessons. The three classes of students included in the study were composite classes of Year 1 and Year 2 students.

Figure 1. Contrasting the Task-First Approach with the Teach-First Approach.

Each lesson involved four aspects: work on a challenging task (15 minutes), a teacher-facilitated discussion (15 minutes), work on consolidating worksheets (15 minutes), and a teacher-led summary of the lesson (5 minutes). Whereas the Task-First Approach occurred in this order (i.e., Challenge, Discussion, Worksheets, Summary), the Teach-First Approach began with the teacher-facilitated discussion, proceeded with work on the consolidating worksheets, then shifted to work on the challenging task, with the teacher-led summary again concluding the lesson (i.e., Discussion, Worksheet, Challenge, Summary). Figure 1 summarises these two approaches.

Each of the three classes were initially randomly allocated to one of three intervention conditions: Task-First Approach, Teach-First Approach or the Alternating Approach (two lessons task-first, two lessons teach-first, two lessons task-first etc.). Across both units of work, Class C remained in the Alternating condition, whereas Class A and Class B were inverted (see Table 1).

Table 1
Structure of the Overall Research Program

Unit of Work	Task-First	Teach-First	Alternating
Patterning: Term 2	Class A	Class B	Class C
Addition: Term 3	Class B	Class A	Class C

The researcher (first author) was responsible for developing the units of work and the respective lesson plans, and for leading the teaching during the sessions. By contrast, the regular classroom teacher acted as a relatively passive co-teacher, assisting with classroom management and providing occasional support and guidance to students to assist with the smooth running of the lessons (under the assistance of the researcher).

Summary of Results and Discussion

Study 1

In Study One (see Russo & Hopkins, 2017a), a series of mixed randomized-repeated design analyses of variance (Mixed Design ANOVAs) were employed to explore the relationship between participation in the program and student outcomes, including mathematical fluency, problem-solving performance, intrinsic motivation to learn mathematics and task-based student persistence. For each analysis, the within group factor was time (i.e., pre-, post-program) and the between group factor was lesson structure (i.e., Task-First Approach, Teach-First Approach, Alternating Approach).

There was evidence that a Teach-First Approach improved mathematical fluency more than a Task-First Approach. Specifically, growth in mathematical fluency in the addition unit was highest for the teach-first group [$F(2, 70) = 3.913$, $p < 0.05$, $\eta^2 = 0.101$], whilst a combined analysis which pooled data from both units of work suggested that participants improved their mathematical fluency more in the teach-first condition than in the task-first condition [$t(47)= -2.05$, $p <0.05$ ($d = 0.30$)]. It is worth noting that the main effect for time (i.e., pre vs. post) was notably larger than the effect of lesson structure. This suggests that participation in any form of the program when challenging tasks are used was more important than the manner in which the respective lessons were structured for improving mathematical fluency. By contrast, there was no evidence that lesson structure resulted in differential improvement in students' problem-solving performance. However, again it was apparent that participation in the program overall had a large impact on problem solving performance. Finally, both intrinsic motivation to learn mathematics and task-based student persistence appeared wholly unrelated to lesson structure, although this may have been in part a consequence of limitations regarding the specific instruments employed to measure these constructs. In particular, students in the study demonstrated very high levels of intrinsic motivation to learn mathematics prior to participation in the program, and the associated ceiling effect may have undermined the capacity of this measure to detect group differences.

It is necessary to try and explain why lesson structure was found to impact fluency but not problem-solving outcomes. Spiro and DeSchryver (2009) have argued that more explicit approaches to teaching may be more effective for learning in well-structured domains, and that inquiry-based methods more appropriate for learning in ill-structured domains. Although this position is contentious (e.g., Clark, 2009), it may explain the differential findings in the current study. Perhaps most obviously, the skills and knowledge that facilitate fluency performance are almost by definition more clearly structured than the equivalent skills and knowledge that facilitate problem solving performance. Whilst developing mathematical fluency involves acquiring and flexibly applying algorithmic-type knowledge, problem solving ability necessarily involves contexts where the individual is assumed not to know how to solve the problem a priori. Consequently, if we adopt the position of Spiro and DeSchryver (2009), it is perhaps not surprising that whilst the Teach-First Approach resulted in greater relative improvements in fluency performance, the Task-First Approach was equally effective when it came to problem solving performance.

Study 2

Study Two (see Russo & Hopkins, in press) employed interpretative phenomenological analysis to examine teacher-participant interviews following each unit of work. The findings revealed that teacher-participants perceived that students responded positively to learning with challenging tasks. Teacher-participants described students as autonomous, persistent and highly engaged. Such positive student reactions were attributed by teacher-participants to a variety of factors, including a classroom culture which embraced struggle, high teacher expectations, and consistent classroom routines. However, other previously identified barriers to teaching with challenging tasks, including time and resource constraints (e.g., Sullivan et al., 2014) and possessing the relevant mathematical knowledge (e.g., Charalambous, 2008), remained (to varying degrees) a concern for teachers. In addition, teacher-participants differed in their views of whether challenging tasks were a suitable means of differentiating instruction, with such evaluations apparently linked to how they defined student success.

With regards to lesson structure, teacher-participants perceived both the Task-First Approach and the Teach-First Approach to teaching with challenging tasks to have particular strengths. Teacher perceptions uncovered in the current study were highly consistent with the arguments and evidence contained within prior research. Specifically, it was found that the Task-First Approach was perceived by teachers as better able to (i) foster mathematical creativity as students had the opportunity to ‘discover’ idiosyncratic, and often more than one, solution methods (e.g., Leikin, 2009); (ii) promote meaningful discourse amongst students (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998); (iii) build student persistence (Sullivan et al., 2014); and (iv) effectively engage students through challenge (e.g., Sullivan, Clarke, Michaels, Mornane, & Roche, 2012). Conversely, there was also some support for the postulation that a lesson which begins with some form of explicit teaching, such as the Teach-First Approach, constitutes a more focussed, efficient approach to instruction (e.g., Kirschner, et al., 2006). Teachers also perceived that the Teach-First Approach was more supportive, particularly for lower-achieving students (Westwood, 2011) – although this latter view was not supported by student outcome data. Overall, these findings imply that framing the Task-First Approach or Teach-First Approach as an either/or proposition is perhaps overly simplistic, as both approaches were perceived by teachers to have distinct advantages in terms of student learning outcomes.

The central tension identified by teacher-participants in the current study between wanting students to discover and subsequently own their personalised solution method and teachers leading students towards the most efficient (or mathematically important) solution method has been revealed in previous research. For example, Star and Rittle-Johnson (2008) found that encouraging Year 6 students to discover their own methods for solving linear equations led to them demonstrating a broader variety of problem solving strategies, however directed teaching in how to solve such equations resulted in students incorporating more efficient strategies. This tension has been described elsewhere by Baxter and Williams (2010) as "managing the dilemma of telling", and is the central theme of their paper which observes the classroom practice of two teachers who are attempting to employ problem-based approaches to learning mathematics (Baxter & Williams, 2010, p. 7).

A corollary of the finding that the Task-First Approach and the Teach-First Approach have distinct strengths is that a particular teacher's preference for one approach over another will likely depend in part on what student learning outcomes she prioritises as a teacher. For example, a teacher who is strongly focussed on meeting the needs of the three or four students in her classroom who have severe difficulties with mathematics may be inclined to embrace the Teach-First Approach. By contrast, a teacher who views mathematics learning as being principally about struggle and discovery will likely embrace a Task-First Approach. The notion that the idiosyncratic values that teachers hold regarding what they believe should be the primary learning objective impacts on their subsequent approach to instruction has been raised in a variety of other primary education contexts, including foreign-language learning (e.g., Pichon, 2014) and the use of technology in classrooms (e.g., Warwick & Kershner, 2008).

Study 3

Study Three (see Russo & Hopkins, 2017b) was divided into two sections. The first section used the Constant Comparative Method to analyse the interview responses of 73 young students regarding the work artefacts they were most proud of creating and why. Five themes emerged which characterised student reflections: Enjoyment, Effort, Learning, Productivity and Meaningful Mathematics. Whereas Enjoyment reflected a single category, Effort encompassed the categories of having a go, conscientiousness and persistence. Learning described students either learning something new or trying something new, and disproportionately reflected the views of female participants [X^2 (1, 73) = 3.895, p <.05]. Productivity captured three interrelated categories, specifically the notion of taking pride in a work artefact because the task was completed, because a large quantity of work was produced or because the work was produced quickly. Meaningful Mathematics was the final theme discussed, and reflected students valuing work because it was presented in a rich context, or because the challenging task involved doing 'real maths', as opposed to more routine mathematical work. Overall, there was evidence that students embrace struggle and persist when engaged in mathematics lessons involving challenging tasks, and moreover that many students enjoy the process of being challenged.

The second section considered the lesson-structure preferences of a subset of participants (Class C; n = 23) when learning with challenging tasks. Most students (58%) in the study preferred the Teach-First Approach when learning mathematics in lessons involving challenging tasks. According to these students, this was primarily because the teacher-facilitated mathematical discussion and the consolidating worksheets served as cognitive activators, effectively ‘warming up their brains’ so students were ready to work through the challenging task. Lower-performing students were disproportionately inclined to indicate that they preferred the Teach-First Approach, which provides further validity for the cognitive activation explanation offered by students. These observations are consistent with Pekrun’s (2006) control-value theory of emotions in an achievement setting. When students are simultaneously given substantial autonomy over how they approach a task and provided with opportunities for cognitive activation, they are likely to experience a high level of control. When accompanied with a high level of value, this is theorised to generate academic enjoyment. By contrast, the comparative expertise of higher-achieving students implies that their sense of control is less likely to be undermined when confronted with a challenging task prior to any instruction (i.e., Task-First Approach). For these students, the activation of knowledge held in long-term memory may effectively substitute for knowledge provided from an external source (e.g., a teacher), which may explain their greater relative preference for the Task-First Approach.

Still, a considerable proportion of students (42%) indicated that the Task-First Approach was their preferred lesson structure. Several students with this preference indicated that they valued the fact that the focus of the lesson was very much on the challenging task. This appeared to partially relate to students acknowledging that they have finite mental resources available, and would rather use their available ‘energy’ to work on the challenge. However, most compellingly, three students specifically indicated that they found discussing the mathematics after exploring the challenging task particularly important, as it provided them with an opportunity to learn from other students.

The potential power of the discussion component of the lesson following work on a cognitively demanding task to build students’ mathematical understanding has been noted elsewhere (e.g., Stein et al., 2008). Indeed, ensuring that teachers possess both the pedagogical and mathematical knowledge to value (in the first instance) and facilitate (in the second instance), such a discussion has been viewed as a critical aspect of converting a task into a “worthwhile learning experience” (Sullivan, Clarke, & Clarke, 2009, p. 103). However, the notion that students as young as seven or eight years old can identify that they themselves benefit directly from participating in such discourse is noteworthy, and contrasts with teacher concerns that even much older students struggle to meaningfully engage in whole-of-class discussions around mathematics (e.g., Leikin, Levav-Waynberg, Gurevich, & Mednikov, 2006).

The other theme to emerge, Cognitive Demand, can be considered the antithesis of students preferring the Teach-First Approach because it supported Cognitive Activation. Specifically, it appears that that some students preferred the Task-First Approach precisely because the lack of discussion beforehand made it more challenging. This notion that ‘hard is good’, which exists in juxtaposition to the idiom ‘help is good’, is a reminder that one size is unlikely to fit all within the context of mathematics education (Ridlon, 2009), and suggests that teachers may contemplate varying the structure of lessons on equity grounds.

Implications

A key implication to emerge from this suite of studies is that the findings do not support the assumption that for students to learn from cognitively demanding tasks, lessons must begin with these tasks. They instead suggest there is more than one way of incorporating challenge tasks into mathematics lessons to produce sizeable learning gains. However, this does not imply that the Task-First Approach and the Teach-First Approach generate equivalent learning experiences for students. Specifically, taken together, the three studies provide distinctive, contrasting portrayals of the two approaches. The teach-first lesson structure can be described as a highly focussed, efficient approach to learning that effectively activates prior knowledge and provides opportunities for students to be successful and to feel suitably supported. On the other hand, the task-first lesson structure can be described as a highly dynamic, explorative approach to learning that effectively maintains a high level of cognitive demand and provides opportunities for students to be mathematically creative and to feel suitably challenged.

It appears that teaching with more cognitively demanding tasks in any capacity constitutes a significant departure from how mathematics is typically experienced in schools, at least for participating students and teachers in the current investigation. Moreover, teaching with more cognitively demanding tasks improves both mathematical fluency and problem-solving performance, regardless of how the corresponding lesson is structured. Consequently, teacher-educators should continue to encourage and support teachers to incorporate such tasks into their mathematics instruction, even in the early years of primary school. Part of the role of outside expertise, such as teacher-educators, would appear to be to design suitable cognitively demanding tasks, whilst perhaps initially allowing teachers to structure lessons around these tasks in a manner in which they are most comfortable. Although there seems little doubt that the task-first and teach-first lesson structures have distinctive strengths and generate different learning experiences, it is difficult to prescribe one particular structure over another based on the results of the current investigation. Such a determination likely depends on the skill, personality and knowledge of the teacher, the nature of the mathematical material to be learnt, the specific learning objectives emphasised during the particular lesson (or suite of lessons), and the preferences, personalities and mathematical ability of students. Ideally, teachers should strive to provide students with opportunities to experience both types of lesson structures when planning lessons incorporating challenging tasks.

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Pricing: Exploring the Intersection Between Values, Maths, Finance, and Entrepreneurship

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In this paper, I explore data collected from more than 300 Year 5 and 6 students in four government primary schools in urban Darwin. Students were asked to respond to real-world problem contexts involving fundraising as an example of an enterprise activity. The findings reveal that familiarity with the problem context, personal values, and language and literacy skills influenced students' decisions how to price goods for sale. It is argued that contextualised learning tasks that require students to apply mathematical, financial, and entrepreneurial thinking can provide insights into students' family backgrounds, personal values, and learning needs while guiding and informing culturally responsive teaching.

Policy and Curriculum Background

Over the past decade, policymakers have become increasingly interested in the potential for school education to prepare enterprising, financially literate graduates. This goal has been the focus of various initiatives and reports by the Organisation for Economic Cooperation and Development (OECD) and its member governments. For example, the European Commission and the OECD developed Entrepreneurship360, an online platform intended to promote an "entrepreneurial mindset" through primary and secondary education (Lakeus, 2015). In Australia, the Office of the Chief Scientist recently released a report arguing a need for educational pathways to instil commercial and financial acumen so that Science, Technology, Engineering and Mathematics (STEM) research and innovation might become a source of national competitive advantage and economic growth (Spike Innovation for the Office of the Chief Scientist, 2015). Such statements imply a necessary intersection between STEM and commerce teaching and learning through enterprise and entrepreneurship education.

Enterprise and entrepreneurship education are not new – but there is renewed enthusiasm for their importance at the policy level given that global markets are challenging to navigate. Enterprise education is typically conceptualised as being about identifying and creating new business opportunities, predominantly for self-employment (Fayolle & Gailly, 2008). Many primary and secondary schools enact this approach by involving students in planning market stalls for profit and fundraising. By contrast, entrepreneurship education is framed as developing personal attitudes and attributes that foster creativity, initiative and risk-taking while critically and sensitively attending to possible social and environmental considerations (Fayolle & Gailly, 2008). Both enterprise and entrepreneurship education are intended to cultivate a repertoire of skills, including critical and creative thinking, communication and interpersonal skills, problem-solving, digital literacy and financial literacy (Foundation for Young Australians, 2016). These learning outcomes are notable in the Australian Curriculum in the articulation of both learning area content and the seven general capabilities.

On the one hand, enterprise and entrepreneurship education would seem an ideal solution to steel school leavers for tough labour markets. Yet while STEM and entrepreneurship are referred to as educational priorities, teachers face the challenging task of reading and interpreting substantive curriculum documentation to develop meaningful

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learning programs that might engage students. There is also the tension that teachers' and students' work is evaluated by standardised assessments, the results of which are relied upon to evaluate the extent to which students are equipped to apply their learning. This suggests a need for educational research that examines student responses to real world problem contexts as well as the explanations they give for these responses. Research of this nature has the potential to guide and inform teaching and learning.

Literature Review

Perhaps one of the most important aims of school mathematics is to prepare students to apply the learned content to the real world (Verschaffel, de Corte, & Laure, 1994). The research reported in this article was critically informed by academic literature related to three particular factors that have been found to influence students' approaches to real world mathematical problem-solving tasks: the choice of problem context, including the extent to which students are familiar with it; personal values; and language and literacy skills.

The choice of problem context and how students interpret and engage with it can also influence their performance on problem-solving tasks. Whether a problem context being familiar helps or hinders student learning and assessment performance is contentious. Neuroscience has shown there is a strong relationship between processes underlying episodic memory and the ability to solve open-ended problems (Sheldon, McAndrews, & Moscovitch, 2011). So, when faced with a problem context that is familiar, a problem solver is more readily able to identify the problem space and retrieve information that is relevant and useful (Sheldon et al., 2011). This explains why problem contexts that are familiar to students can make for fun, engaging lessons – students typically contribute to their classroom learning with confidence when they have experiential knowledge to share. This is not to say that such contributions will always be productive. Problem contexts that are familiar have also been found to lead students to misinterpret, overlook or ignore the intended relevance and meaning of a task (Van den Heuvel-Panhuizen, 2005). Boaler (1994) found that girls were more likely to apply common sense as well as mathematical knowledge when faced with a fashion-related question that was considered more familiar and real to them. While the students became engaged and involved with the problem context, they underachieved on this question. Cognitive psychology offers insights why. Problem contexts provide data that are intended to activate mathematical thinking, but these data can operate in ways that cue different facts, concepts, processes, prior experiences and semantic knowledge (Tulving, 1985). Further, the salience of these cues can vary from problem-solver to problem-solver (Kaplan & Simon, 1990).

Students bring to their learning knowledge and understanding filtered through their social and cultural lenses (Vale, Atweh, Averill, & Skourdoumbis, 2016). Personal values – the convictions which one finds important (Seah, 2016) – have also been found to influence students' responses to worded mathematical problems. In previous iterations of the research reported in this article, personal values learned within the home were found to shape students' responses to real world mathematical problems involving money, both in interview and classroom settings (Sawatzki, 2015). This makes sense since values have been found to be influential in the formation and development of attitudinal and behavioural tendencies, including financial behaviour (Homer & Kahle, 1988).

A growing body of research is showing that language and literacy skills and worded mathematical problem-solving skills are interrelated, not only during the primary school years but in early adolescence too (Kyttala & Bjorn, 2014; Vilenius-Tuohimaa, Aunola, &

Nurmi, 2008). Pimperton and Nation (2010) argued that mathematics assessments that place high demands on verbal ability and linguistic comprehension serve to underestimate the underlying mathematical abilities of students who tend to experience comprehension difficulties.

To explore the influence of the above factors on students' interpretation of and responses to real world problem contexts involving enterprise and entrepreneurship, data were collected from 14 teachers and more than 300 Year 5 and 6 students in four government primary schools in urban Darwin. The findings reported in this article are based on quantitative and qualitative student data collected via online surveys and face-to-face discussion groups.

The research questions were:

- What factors seem to influence students' responses to real world problem contexts involving fundraising as an example of an enterprise activity?
- What are the insights and implications for schools and teachers?

The Research Context and Methods

The study reported in this article was part of an ongoing educational design research project. As schools servicing students from diverse and low socioeconomic backgrounds were sought to participate, the Index of Community Socio-educational Advantage (ICSEA), created by the Australian Curriculum, Assessment and Reporting Authority (ACARA), was used to understand the socioeconomic profile of potential school communities. An ICSEA value below the Australian average of 1,000 was a qualifying criterion to participate. Table 1 consists of My School data to describe each school's size and student characteristics (socioeconomic background, identifying as being Indigenous, and being from a language background other than English).

Table 1
My School Data Describing Each School Community

	Total enrolments	ICSEA value	Indigenous students	Language background other than English
School A	433	912	26%	14%
School B	397	983	9%	2%
School C	407	995	10%	14%
School D	270	935	24%	45%

These figures serve in some way to describe the diverse, often challenging communities within which the teacher participants work.

Students were asked to complete two surveys online: one before and one after they had completed a series of 10 lessons exploring "financial dilemmas" over the course of Term 2 (see Sawatzki, 2016). The pre-intervention survey was open for one week in April and consisted of three multiple choice items seeking to find out about students' learning preferences and seven financial literacy assessment items. It was completed by 331 students. The post-intervention survey was open for one week in August. While it was similar to the pre-intervention survey, one item was modified, a new financial literacy assessment item was added, and two open-ended questions about learning through challenging problem-solving tasks were included. It was completed by 302 students. The attrition from pre- to post-intervention is mostly explained by student absences and turnover across the four participating schools. The pre- and post-intervention survey data were analysed in preparation for the student focus group discussions, the intention being that preliminary findings might guide and inform the choice of issues and questions to be explored further.

Twenty-eight students (seven groups of four students drawn from each of the four participating schools) participated in 20-minute focus group discussions where they shared insights into their observations and experiences with money in their family and community life, as well as their learning through the series of 10 lessons.

This article explores insights into students' emerging capacities to apply mathematical, financial and entrepreneurial thinking to real world problem contexts involving fundraising. In the section that follows, findings related to two financial literacy assessment items that were included as part of the student surveys and a financial dilemma that was presented as part of the student focus group discussions are presented and analysed.

Findings and Discussion

The findings are presented in two parts: insights from the student survey data and insights from the student discussion group data.

Insights from the Student Survey Data

Seven financial literacy assessment items were included on the pre-intervention student survey. These were developed in the style of the National Assessment Program – Literacy and Numeracy (NAPLAN) items. Table 2 presents Item 7. This item requires students to employ a simple mathematical operation: divide the total cost ($6) by the number of items (12). Performing such a calculation is well within the expectations of the upper primary years of the Australian Curriculum: Mathematics. The options presented provide two loss-making, a break-even, and a profit response. The intention was to give the students a range within which a solution was situated. While not referred to in the problem, the cost per cupcake is otherwise known as the break-even price. This economics and business concept is typically explored in the upper primary years of the Australian Curriculum: Humanities and Social Sciences. While the survey was completed online and students were allowed to use a calculator, they were encouraged to use pen and paper to note their working.

Table 2
Item 7: It Costs $6 to Make 12 Cupcakes. What is the Cost per Cupcake?

	Pre-intervention (*n* = 331)		Post-intervention (*n* = 302)	
Option	No.	%	No.	%
30c	46	14	24	8
40c	35	10	33	11
50c	198	60	200	66
60c	46	14	41	14
No response	6	2	4	1

Pre-intervention, 325 students completed this item and 60% responded correctly. Post-intervention, 298 students completed this item and 66% responded correctly. On both occasions, there was a very low no-response rate. While there was a notable improvement in students' performance on this item, the number of students unable to achieve success is interesting when you consider that mathematical problem-solving of this nature is essential to complete simple everyday financial transactions. For example, a visit to the supermarket presents a similar scenario: Should I pay 50c per lemon, or buy a bag of five for $2?

An additional, related financial literacy assessment item was included in the post-intervention student survey. Table 3 presents Item 7b, which requires students to reason that money is able to be raised when the price per cupcake is higher than the cost per cupcake. Such reasoning, which relies on a correct response to Item 7, implies an understanding of profit – an economics and business concept that is generally explored through the upper primary years of the Australian Curriculum: Humanities and Social Sciences. The decision to link Item 7 and Item 7b in this way, while not typical, meant that one problem context was able to be leveraged in two ways, thereby limiting the language and literacy demands associated with the assessment. Again, the options presented provide two loss-making, a break-even and a profit response.

Table 3
Item 7b: If Grade 6 Wants to Make and Sell Cupcakes to Raise Money for an end of Year Party, How Much Should They Charge per Cupcake?

	Post-intervention (*n* = 302)	
Option	No.	%
30c	8	2
40c	18	6
50c	124	41
60c	108	36
No response	44	14

Post-intervention, 258 students completed this item. Most students (41%) nominated the break-even price and a little less (36%) nominated the profit-making price. Note the relatively high no-response rate – 14% of the sample did not even attempt this question. The fact that students struggled with this item is particularly interesting given that the four participating schools reported Year 5 and 6 students being involved in kitchen garden programs that included growing, preparing, and marketing fresh produce and home-made goods and/or other fundraising initiatives. In fact, the problem context – a cake sale to raise money for an end of year party – was selected since upper primary students routinely organise fundraising activities like this as they plan and budget for Year 6 graduation celebrations. At various times over the course of the study, the teachers described these practical initiatives as being rich in experiential learning related to entrepreneurship.

Students' responses to these items signalled a need to explore what influences students as they consider problem contexts related to pricing and profit. Might the high error and non-response rates be explained by miscalculation, or other factors? This was done through the student focus group discussions.

Insights from the Student Discussion Group Data

As part of the student focus group discussions, the following financial dilemma was presented:

> Year 6 would like to raise money to donate to the RSPCA. The teacher has suggested making lolly bags to sell at school. Each lolly bag will cost $2 to make. What price should Year 6 sell the lolly bags for? Justify your thinking.

Note the shift in the terminology used to pose the question. In Item 7b, the word "charge" was used and here the word "price" was used. This was intended to bring the inquiry into sharper focus.

In each focus group discussion, the researcher introduced the task by saying, "I've got a problem here and there's probably more than one answer. I'm interested in your thinking." Students were then given time to read the task and pose questions. A range of questions were raised, examples of which include:

> How much money do they need to raise?
>
> How many students are there?
>
> Is there a budget?

These early reactions signal that while some students were familiar with the problem context inquisitive as to the possibilities associated with it, they seemed to draw on experiential knowledge in ways that were not immediately relevant to the task at hand or helpful to the problem-solving process. To re-focus students on the financial dilemma and initiate quiet problem-solving time, the researcher asked, "So, how much do you think we should sell these lolly bags for?" On occasions, the researcher reframed the question, "What price should we charge?"

Transcripts of the student discussion group audio recordings were analysed and student responses assigned to one of three categories: loss-making responses, break-even responses, and profit-making responses.

Students who gave loss-making and break-even responses were price-conscious and preoccupied with providing value for money to the market. Three students from School C agreed to sell the lolly bags for $1.50, giving the following explanations:

> Because $2 is a lot to be spending on a lolly bag.
>
> I'd probably do $1.50. So, it's not so expensive.
>
> In the shops, you'll find lolly bags are normally $1.50. I reckon $2 is too much.

In these examples, students seem to draw on their observations and experiences with similar products in the market to judge a price point they believe purchasers will reasonably tolerate. They infer a need for the price point to be competitive. Similarly, two students, one at School A and one at School B, determined that the price should be $1.50. Their explanations revealed an emerging understanding of demand and supply theory. For example, one commented, "If the price is lower, more people might buy them. Then they'll make more money." A price that would enable more children to participate in purchasing lolly bags reveals sensitivity to others' financial circumstances and suggests these children value inclusion. However, with a price point below break-even, their reasoning was flawed by capitalist standards. Further, in these examples, the students seem to confuse what is meant by "cost" and what is meant by "price". There is no evidence that they understood the meaning of "profit" – a concept that is central to any enterprise activity.

Break-even responses (to price the lolly bags at $2) were justified in similar ways. As one student from School A explained, "It's not too expensive or too cheap. Plus, that's what it costs to make the lolly bag." This particular response seems to be motivated to avoid financial gain – a goal that is contrary to that specified by the problem context. Again, being conscious of what might be a fair price to ask other students in the school community to pay again reveals sensitivity to others' financial circumstances and suggests these children value inclusion.

Students who gave profit-making responses revealed more sophisticated understandings of the problem context. They applied reasoning that was purely mathematical, financial and entrepreneurial. These students competently and confidently used the term "profit" in their explanations, as shown by this interaction between the researcher and students from School B:

Gen: You can't sell it for $2 because you need to make a profit. And the profit is the money that will go to the RSPCA.

George: I reckon around $2.50 or $3.00.

Gen: Yeah.

I: Just explain to me, how do you know about this word "profit"? That's a nice word you've used there.

Gen: For a field day at our school, we were making chutney…

George: And rosella jam… And we needed to make a profit out of it so we were adding up how much it cost and deciding what the profit should be. How much profit we'd get. And then what the price should be.

Here, the students reference prior learning through their school kitchen garden program. George clearly distinguishes between cost, price and profit. It seems that these students' mathematics learning was situated within an enterprise initiative where concepts in mathematics, economics and business were meaningfully explored. Further, to the extent that students' vocabulary was added to, language and literacy learning outcomes were achieved. For these students from School B, what was learned in their school kitchen garden program was able to be transferred to the lolly bags task - a similar real-world context. By contrast, students from School A who were also regularly involved in a kitchen garden program did not mention profit or engage in conversation like this.

Conclusion and Implications

In the first instance, students were presented with two assessment items based on real world problem contexts involving fundraising as an example of an enterprise activity. While 66% of students were able to calculate the cost per cupcake, only 36% of students nominated a sale price that would enable a profit to be made.

Subsequent student focus group discussion data revealed that students' responses to a similar task were influenced by their familiarity with the problem context, personal values, and language and literacy skills. Students who gave loss-making and break-even responses to the lolly bags problem were price-conscious and preoccupied with providing value for money to the market. They seemed motivated that the price per lolly bag be affordable for the majority of students, rather than "too expensive". These considerations reveal sensitivity to others' financial circumstances and show the students value inclusion. This is likely due to social and cultural norms within their families and school communities. Interestingly, there seemed to be a gap in these students' language and literacy skills in terms of important understandings that underpin the mathematical calculations and reasoning necessitated by the problem context (i.e., "cost," "price", and "profit").

By contrast, students who gave profit-making responses applied reasoning that was purely mathematical, financial, and entrepreneurial. They were not distracted by social and cultural sensitivities. They interpreted that the price per lolly bag must be higher than the cost to make one and were more likely to use the term "profit" to explain their thinking.

There are several insights and implications for schools and teachers. These findings highlight that problem contexts of this nature are in fact values-laden and interdisciplinary. Not only did the problem contexts appeal to students' personal values, there were implicit language and literacy demands that meant students were required to apply mathematics alongside economics and business. This suggests a need for teachers to select tasks and pedagogies that meaningfully connect learning areas and general capabilities through real world problem contexts that allow for exploration, conversation and discovery. Further, the findings highlight the importance of adopting culturally responsive teaching practices that seek to align classroom tasks and pedagogies with the diverse identities, experiences, values and norms students bring to their learning (Vale et al., 2016). Inevitably, there is a need for professional learning that orients teachers to work in this way.

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Using Activity Theory to Understand a Mathematics Leader's Motivations and Use of Mathematical Knowledge for Teaching

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Despite the significance of the role, little is known about mathematics leaders in schools. Rachel, a mathematics leader, was observed leading planning meetings with junior primary teachers. Using activity theory, three of Rachel's motivations were identified: influencing teacher affect, developing shared understandings, and avoiding conflict. Observation and interview data analysis revealed the use of pedagogical content knowledge (PCK) as a tool far more than subject matter knowledge. We posit that the dominant use of knowledge types associated with PCK was due mostly to Rachel's object of avoiding conflict.

This paper is about one mathematics leader, Rachel, and her work in leading planning meetings with junior primary teachers at her school in 2015. Mathematics leaders like Rachel are regarded as having significant influence in affecting change in teachers' professional learning and the mathematics practices enacted in schools (Grootenboer, Edwards-Groves & Rönnerman, 2015; Jorgensen, 2015; Millet & Johnson, 2004). Despite the importance of this critical role in schools, little can be found about these middle leaders in the research literature (Sexton & Downton, 2014). With this paper, we aim to contribute new information about mathematics leaders by identifying the motivations of Rachel, and how she used mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) when leading planning meetings with teachers. We used activity theory (Engeström, 1987) to guide our research work in describing Rachel's motivations, and knowing more about the Ball et al. (2008) knowledge types she used when leading those meetings with her teachers.

Literature Background

Millett and Johnson (2004) claimed that mathematics leaders in schools have opportunities to lead teacher professional learning and mediate change in teachers' practices associated with mathematics teaching. This image of the mathematics leader as a professional learning provider in schools has endured since that time (Grootenboer et al., 2015; Jorgensen, 2015; Sexton & Downton, 2014). Jorgensen (2015) revealed that schools make provisions to develop this leadership role as a form of "coach" who supports teachers' learning about curriculum and planning. Mathematics leaders have identified the facilitation of planning meetings as an important aspect of their work in primary schools (Sexton & Downton, 2014).

Teachers' planning for mathematics teaching has garnered attention from the MERGA community in recent years (e.g., Davidson, 2016; Roche, Clarke, Clarke, & Sullivan, 2014; Sullivan, Clarke, Clarke, Farrell, & Gerrard, 2013). Planning is an important role in primary teachers' work responsibilities (Roche et al., 2014). Sullivan et al. (2013) reported that primary school teachers place a greater emphasis on collaborative planning with peers (compared to secondary school teachers). Many Victorian Catholic school teachers claimed that they do indeed work in teams of two or more when planning for mathematics teaching (Roche et al., 2014). Creating units of work appears to be a goal when primary school teachers meet to collaboratively plan (Davidson, 2016; Roche et al., 2014), but planning can be a complex task for teachers (Davidson, 2016). Teachers' mathematical

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knowledge is a critical issue in mathematics planning processes because that knowledge impacts the decisions that teachers make during planning opportunities (Davidson, 2016). This recognition of the importance of teacher knowledge is not new (e.g., Ball et al., 2008; Shulman, 1986; Sullivan, Clarke, & Clarke, 2009).

Building on Shulman's (1986) seminal yet theoretical work, Ball and colleagues sought to conceptualise, measure, and evaluate the knowledge that is required for mathematics teaching. Ball et al. (2008) gave rise to their framework named mathematical knowledge for teaching (MKT), which they believed captured both the subject matter knowledge (SMK) and pedagogical content knowledge (PCK) needed for the work of teaching mathematics. Ball et al. included three types of SMK and three types of PCK in their knowledge framework.

According to Ball et al. (2008), the three types of knowledge that comprise SMK are common content knowledge (CCK), specialised content knowledge (SCK), and horizon content knowledge (HCK). CCK is considered the knowledge of mathematics that is commonly used by adults in settings outside of teaching (Ball et al., 2008). It is knowledge that is required to solve mathematical tasks encountered in everyday life (Sullivan et al., 2009). Examples of CCK include recognising when answers are incorrect, calculating answers correctly, and using mathematical terms and definitions appropriately (Ball et al., 2008; Hurrell, 2013). SCK relates to mathematical knowledge that is unique to the teaching of mathematics. This knowledge is generally not required for purposes outside of the teaching profession (Ball et al., 2008). In a sense, this type of knowledge goes beyond what most adults require for doing and using mathematics. Examples of SCK include knowing if a student-invented algorithm or strategy could be generalised, finding examples and non-examples to make a mathematical point, and understanding the different interpretations of the operations (Ball et al., 2008; Hurrell, 2013; Sullivan et al., 2009). HCK is understood as the knowledge of how mathematical topics are connected with other topics, and knowledge of those mathematical ideas that students encounter in the later years of schooling (Hurrell, 2013; Sullivan et al., 2009). Examples of this knowledge type include the teacher's ability to articulate how a mathematical idea develops through curriculum documentation (Hurrell, 2013).

Pedagogical content knowledge within the Ball et al. (2008) framework includes knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of content and curriculum (KCC). KCT concerns knowledge of mathematics content and knowledge of teaching. A teacher who enacts KCT can discern the advantages and disadvantages of particular representations for ideas, identify appropriate tasks to use in lessons, and sequence them in ways that support learning and teaching (Ball et al., 2008; Sullivan et al., 2009). KCS is teaching knowledge that combines mathematical content and ways that students come to understand (or misunderstand) mathematical ideas and content (Ball et al., 2008). Examples of KCS include knowing common misconceptions often developed about mathematical topics, anticipating likely student responses to tasks, and knowing which aspects of mathematical tasks students will find easy or challenging (Ball et al., 2008; Hurrell, 2013; Sullivan et al., 2009). The final knowledge type, KCC, is knowledge of mathematical content and curriculum. With this knowledge, teachers understand how concepts are represented within the curriculum and when they are expected to be taught. KCC is also knowledge of teaching materials and resources that support the teaching of ideas in the curriculum (Hurrell, 2013).

With planning meetings acting as an important aspect of mathematics leaders' work (Sexton & Downton, 2014) and knowing that mathematical knowledge plays a critical role in those meetings (Davidson, 2016), we wanted to contribute to the literature about mathematics leaders. We wanted to gain insights into Rachel's motivations when leading planning meetings with her teachers. We also wanted to highlight the types of mathematical knowledge for teaching (Ball et al., 2008) that she used when leading those meetings. With a need to know more about Rachel's motivations and use of the Ball et al. knowledge types, activity theory was chosen as the theoretical perspective for the study.

Theoretical Perspective

The theoretical perspective for the doctoral study on which this paper is based uses an activity theory framework influenced mostly by the work of Engeström (1987). A number of important concepts are engaged when using activity theory. The first idea is the activity system (Engeström, 1987), which forms the main unit of analysis. The activity system is comprised and organised by a number of nodes (italicised in this paragraph). The activity system is concerned with the subject (which can be an individual or collective) who is working towards an object whilst enacting activity within a social context that includes the community. Within activity theory, the activity system is viewed as dynamic and ever-changing, where rules, division of labour, and mediating artefacts act as mediators of the activity under investigation (Engeström, 1987). Acting as mediators, these nodes have the potential to support or constrain the achievement of the object. Due to the confines of this paper, only the object and mediating artefacts will be discussed briefly.

Within activity theory, the object of activity has two different meanings. In one sense, the object may be a physical and/or mental product (Engeström, 1987), or the object can also be the motivation or goal held or pursued by the subject (Leontyev, 1978). The object provides reasons for the subject's behaviour(s) within the activity system. Mediating artefacts, or tools as used by Vygotsky (1978) and thus used as a term by us in the remainder of this paper, are an important feature of activity theory. The subject uses tools to mediate activity so that the object can be achieved. Vygotsky (1978) claimed that tools can be both physical and psychological in nature. Examples of physical tools include computers and pens, and psychological tools include language and signs. Psychological tools are used by humans to influence themselves or other human beings. The subject of any activity system uses a combination of both types of tool (Engeström, 1987).

Another important concept of activity theory is contradictions. Contradictions are inherent and ever-present in activity systems (Engeström, 1987). Due to the dynamism of the activity system, interrelationships exist between the different nodes of the system (Engeström, 1987). By examining these interrelationships, contradictions can be identified within the system. Contradictions are viewed as catalysts in transforming the activity system, bringing about potential change if addressed by the subject (Engeström, 1987).

In using this theoretical perspective, we mobilised activity theory concepts associated with the activity system – namely subject, object, tools, and contradictions – to know more about Rachel's leadership work. We positioned Rachel as the subject within the activity system, and then situated the Ball et al. (2008) knowledge types as the psychological tool that Rachel used to mediate the objects of her leadership when facilitating planning meetings with her teachers. We then wanted to identify the objects (motivations) of Rachel's activity system, and know which knowledge types she used to mediate those objects that she pursued in planning meetings.

Method

Participant

Rachel, the mathematics leader, works in a Victorian Catholic primary school that participated in the Contemporary Teaching and Learning of Mathematics (CTLM) project (Sexton & Downton, 2014) in 2011 and 2012. Rachel leads planning meetings with teachers who are released from teaching duties once a fortnight to attend the meetings.

Data Generation

Rachel was observed on three occasions during 2015 (from April to November) at times that she nominated. Rachel worked with her Grade 1/2 teachers during each meeting. One meeting was focused on planning a unit of work on time measurement (which included a focus on a unit of work on addition and subtraction), and two other meetings focused on 2D transformation, symmetry, and visualisation. The total amount of observation time was 165 minutes. Rachel was interviewed on five occasions during the time of the observations: once after the first observed planning meeting, and then before and after the remaining meetings. The data generated through observations and interviews were recorded and transcribed. In line with ethics protocols, only Rachel's activity (speech, actions, etc.) was recorded and analysed.

Data Analysis

Analysis of data was approached using a deductive method where *a priori* concepts, derived from theory and literature – namely object, tools, and the Ball et al. (2008) knowledge types – were used to analyse data. Specific examples of the knowledge types were retrieved from literature to support the coding of data (e.g., Ball et al., 2008; Hurrell, 2013; Sullivan et al., 2009). Interview transcriptions and observation notes were coded for Rachel's use of particular knowledge types using those examples. To illustrate our coding, we provide some examples of our work. In one planning meeting, Rachel referred to tessellation and provided a definition for this mathematical idea. We coded this as knowledge associated with common content knowledge (CCK). Another time, Rachel warned her teachers that understanding angle as rotation is challenging for younger students to understand. This was coded as a form of knowledge of content and students (KCS). We read and coded interview transcriptions and observation notes for specific examples of the MKT knowledge types as well as the concepts from activity theory. Parts of observation notes and transcriptions were then independently coded by a colleague to check coding consistency.

Results and Discussion

The findings that we share are tentative ones that have been highlighted so far in the doctoral study. Rachel's leadership of planning meetings is indeed dynamic, as she was charged with leading up to at least five teachers in each meeting. Rachel said that she saw leadership of planning meetings as part of her role (Sexton & Downton, 2014), and confirmed that planning was a critical part of teachers' work (Roche et al., 2014).

Motivations of Rachel's Work

We identified three of Rachel's motivations (objects) when coding the data: influencing teacher affect, developing shared understandings, and avoiding conflict. The first two motivations were explicitly discussed and enacted by Rachel. Therefore, we have deemed these to be conscious objects. The third object, avoiding conflict, is one that we believe is unconscious to Rachel, yet it was one that was pursued by her in her work.

Influencing teacher affect. Rachel identified developing teachers' confidence about mathematics topics as one of her motives when planning with her junior primary teachers. When asked about what she wanted the teachers to learn, Rachel would often refer to the teachers' confidence. An example of this was when Rachel spoke about the purpose of her work: "I want them [the teachers] to have more confidence in the language of the unit…I really want them to be confident in the unit". This behaviour of instilling confidence has been described by Edwards-Groves, Grootenboer, and Rönnerman (2016) as a dimension of trust which they have termed "interpersonal trust" (p. 378). This is an important aspect of the work of middle leaders in primary schools (Edwards-Groves et al., 2016). Rachel also made references to teachers feeling comfortable with what to do, say, and use during lessons that formed the units of work. In terms of activity theory, this object would be considered a motive (Leontyev, 1978) as well as a mental product (Engeström, 1987).

Developing shared understandings. The second object of Rachel's activity concerns the development of shared understandings between teachers. During interviews, Rachel often commented that she wanted consistency in teachers' understanding of teaching approaches, tasks, and language and terms that were to be used when the teachers taught. Rachel once commented, "We want to make sure there is consistency and the teachers are aware of the important steps and processes to teach it [the unit of work]". Rachel also said it was important that she supports the teachers in consistent understanding of the important mathematical ideas that were to be taught.

Rachel's focus on developing shared understandings of mathematical ideas during planning meetings is important because Sullivan et al. (2013) found that teachers often experience difficulty in articulating important ideas during planning meetings. It was observed, however, that during the meetings, Rachel rarely engaged teachers in discussions about those mathematical ideas. Instead, Rachel would often tell teachers the mathematical ideas that the topic focused on, and she would tell them statements about those ideas that she called "key understandings". These key understandings have been described by Roche et al. (2014) as a feature of planning documents used in many Victorian Catholic schools. Mathematics leaders have reported using such understandings in planning meetings during the CTLM project (Sexton & Downton, 2014).

Even though it is important that Rachel is focusing on these aspects of planning, her practice of telling teachers and not engaging in mathematical discussions about mathematical ideas could be viewed as a contradiction with her activity system (Engeström, 1987). We believe this because Rachel said that she viewed facilitated planning meetings as opportunities for collegial discussion; however, there were very few opportunities for teachers to engage in discussions where they were required to question, explain, and elaborate upon their own understanding of mathematical content that they were expected to highlight when teaching. Addressing that contradiction by including more discussions about important mathematical ideas between the teachers could be one way that Rachel uses that contradiction to develop her activity system (Engeström, 1987). By attending to this, Rachel has an opportunity to align her activity more closely with her espoused object. Rachel could enact the practice of inviting her teachers more often to discuss ideas in planning meetings.

During the second planning meeting, Rachel did do this by organising one of the teachers to bring work samples associated with one of her suggested tasks. This task was related to the conservation of volume where the students had to use 24 cubes to construct different containers that held the 24 cubes exactly. It was noted that during that experience, there were in-depth discussions among the teachers about important measurement ideas, ways of supporting students to notice the differences and similarities of the containers, and ways that teachers might use iPad photos as assessment opportunities.

Avoiding conflict. The third object identified in Rachel's activity relates to the relational dimension of her work. Rachel often discussed a sense of empathy for her teachers and their work demands. Rachel mentioned that, as a school leader, she needed to be aware of how busy teachers were in her school. She often said that she had to be conscious of the teachers' work responsibilities. Rachel acknowledged this by stating: "they are so busy especially because they're all writing reports at the moment... I've got to be very mindful of that". This empathy for teachers is another description of interpersonal trust reported by Edwards-Groves et al. (2016). Rachel's response, generated by her empathy, was to offer and seek ways to reduce teacher workload.

During each of the meetings, Rachel said that she would create particular resources (e.g., laminated flashcards), collect materials for classroom use, prepare and photocopy templates for teachers to use, or complete the planning of the units of work for the teachers when time ran out for the scheduled meetings. Rachel explained the reason for completing the units of work during an interview: "If I didn't do that, they would then have to get together tonight… That takes another whole night of planning for them. And that's a lot". In each meeting, Rachel would offer ideas about tasks that formed the basis of lessons. When asked about reasons for this, Rachel said, "They've been teaching all day and suddenly they've got to focus in on something and that's why I think, sometimes they're happy if I've got an idea." We elaborate on this further in the next section.

We have coded this as an unconscious object of Rachel using the code "avoiding conflict". We do not mean any negative connotations by this label. We use this term because the empathy that Rachel feels for her teachers provides reasons for her behaviours. This object of avoiding conflict would also be deemed a motive or goal (Leontyev, 1978) that is driven by Rachel's concern for her teachers. We are questioning, however, if this object of avoiding conflict is in fact actually contributing to Rachel's own workload as the mathematics leader, and thus another contradiction has surfaced in her activity system.

Use of Mathematical Knowledge for Teaching by Rachel

Rachel used both subject matter knowledge (SMK) and pedagogical content knowledge (PCK) when leading the three planning meetings. The most used knowledge type of SMK as a tool was common content knowledge (CCK). There were 22 observed uses of this knowledge type during the meetings. Examples of CCK used by Rachel in the meetings included rotation as a transformation type, common language used to describe rotations (e.g., quarter turn, clockwise rotation), and representing, reading, and matching times on analogue and digital clocks.

Rachel also used knowledge associated with the specialised content knowledge (SCK) during the meetings. There were 14 uses of SCK as a tool to mediate the objects of her activity. Examples of SCK used by Rachel included subitising (as means of quantifying small collections) and recognising and naming time and volume as measurement attributes.

The least used knowledge type associated with SMK was horizon content knowledge (HCK). There were only four coded uses of this knowledge type as a tool during the 165 minutes of planning meeting time. This suggests that little attention was given to how mathematical topics are connected to other topics, which could result in a fragmented picture of the mathematics curriculum and how it develops into the secondary curriculum.

The pedagogical content knowledge (PCK) category of Ball et al.'s (2008) framework and its knowledge types were used far more dominantly by Rachel. Rachel used PCK types nearly three times more often (110 uses) than the combined use of knowledge types associated with SMK. Knowledge of content and teaching (KCT) was the most used knowledge type by Rachel with it being used 57 times during the three planning meetings. This is more than the combined uses of knowledge of content and students (20 uses) and knowledge of content and curriculum (32 uses).

Knowledge of particular tasks/activities for teaching content (which is categorised as KCT) was used 18 times, and knowledge of concrete materials for teaching content (KCC, also 18 times) featured mostly as tools that mediated the objects of Rachel's activity. Rachel provided insights into reasons why she uses these particular knowledge types:

> Yeah, I found that in general, and the reason I keep doing it is because they go, "Yeah! I really like that." So, it makes me think, "Oh well, maybe the activity is a good one." But then I also think I've got more time to think about maths and be looking, be on the lookout for activities and… that I've got some of them in my head… So, part of me thinks yes, that's what I should do, but then the other part thinks, I hope I'm not taking over too much.

Rachel identified the reason why she tends to focus on the use of KCT and KCC. Rachel was very aware of teachers' work pressures, so we believe that in order to avoid conflict, Rachel used knowledge of tasks (KCT) and materials (KCC) as a psychological tool to minimise teacher workload, and thus achieve that unconscious object. The avoiding conflict object of Rachel's leadership in planning meetings influenced the knowledge types she used as a tool, which then in turn, mediated the object of her activity system. This is where we see the dynamism of Rachel's activity system at work (Engeström, 1987).

The quotation from Rachel above also suggests the presence of another object that relates to responding to teacher affirmation. The teachers make up the community node of Rachel's activity system, and this node is interacting with this new object. We will need to generate and analyse further data to substantiate this claim. Rachel did appear to question the support she provided during planning times, thus diagnosing her own contradiction.

Conclusion and Implications

We have highlighted conscious and unconscious objects of Rachel's activity system when leading three planning meetings with her teachers. These motivations are influencing teacher affect, developing shared understandings, and avoiding conflict. Without being explicitly aware of the Ball et al. (2008) framework, Rachel did indeed use all knowledge types captured in that framework. She did, however, use knowledge types associated with PCK far more than those categorised as SMK.

The knowledge types of knowledge of content and teaching (KCT), in particular knowledge of tasks/activities, and knowledge of content and curriculum (KCC), as knowledge of concrete materials and resources, were prevalently used by Rachel. We conclude that her use of these knowledge types was influenced by the motivations of her work, namely the object of avoiding conflict.

We suggest there is potential for mathematics leaders to become more familiar with knowledge types, like those of the Ball et al. (2008) framework to support their work. Mathematics leaders could use the framework as a tool to achieve, and possibly transform, objects of their leadership activities. In this way, leaders have the opportunity to plan, facilitate, and evaluate opportunities that influence teacher learning. Leaders will need specific examples of the knowledge types if this was to happen.

By using activity theory, we were afforded new ways to view the knowledge types of Ball et al. (2008). We now see the knowledge types as psychological tools that mathematics leaders might use to mediate objects of their leadership work in schools. We believe that this new view of the knowledge types has potential for further research.

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Exploring Critical Thinking in a Mathematics Problem-Based Learning Classroom

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In this study, we explored the critical thinking of 47 eleventh-grade students in a mathematics problem-based learning (PBL) classroom in November 2016. A critical thinking test was used along with classroom observations to gather the critical thinking data in five dimensions according to the Association of American Colleges & Universities (2009). The findings indicate that students' critical thinking scores in all dimensions are at an average level. The students demonstrated strength in explaining issues and analyzing influence of context and assumptions. However, students had greater difficulty in stating their positions and drawing conclusions.

Introduction

Efforts to develop thinking skills have become essential goals in mathematics classrooms (Hurst & Hurrell, 2016). Thinking in mathematics can be referred as an important "process" to foster students' mathematical problem solving (Hwa & Stephens, 2011). In particular, critical thinking is claimed to be the most important skill for problem solving, research, and discovery (Thompson, 2011) as it encourages students to think independently and solve problems in school or in the context of everyday life (Jacob, 2012). The National Council of Teachers of Mathematics (NCTM, 2000) also mentioned that the development of critical thinking generates improvement of mathematics achievement. Thus, critical thinking has become the main agenda of worldwide mathematics education.

The first question aimed toward understanding critical thinking is "What is critical thinking?" The answer is not simple since critical thinking is a complex phenomenon. Critical thinking is viewed from several distinct perspectives and thus is referred to by different definitions (Ennis, 2003; Facione, 1990; Halpern, 2006). Despite differences among these perspectives, Lai (2011) noted that definitions of critical thinking overlapped on several specific abilities, including (1) analyzing arguments, claims, or evidence, (2) making inferences using inductive or deductive reasoning, (3) making decisions or solving problems, and (4) judging or evaluating. In this study, critical thinking is defined as an ability to think objectively in order to make a decision. This definition emphasizes skills in five dimensions adapted from Association of American Colleges and Universities (AACU, 2009), consisted of (1) explanation of issues, (2) evidence, (3) conclusions and related outcomes, (4) influence of context and assumptions, and (5) student's position. The critical thinkers, therefore, should be able to comprehensively explain given issues or problems, thoughtfully select and use evidence, inquire about possible outcomes and relate them to each other in a conclusion, as well as analyze contexts, situations and others' assumptions to synthesize them to make their own positions.

Despite the importance of critical thinking, most of the teaching and learning process in school is the traditional lecture method, which is based on memorization, leading students to think less critically (Cobb, Wood, Yackel, & McNeal, 1992). The negligence of the importance of thinking skills also occurs in mathematics education in Thailand. The recent results of PISA 2015 showed that Thai students ranked 54th in mathematics from 70

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countries with a mean score of 415 points, which was significantly below the Organisation for Economic Co-operation and Development (OECD) average (2016). The PISA reports and other evidence lead to the recommendations for curriculum development in Thailand where the content, textbooks and teacher are the targets for major changes (Sunee, 2015). In these recommendations, teachers were encouraged to change their teaching behaviour from telling to questioning, to create learning activities that promotes students' participation, and to motivate students to think critically. Thus, active learning strategies such as problem-based learning were introduced.

Problem-Based Learning (PBL) is an educational approach where learning is driven by real-world problems (Othman, Salleh, & Sulaiman, 2013). Students learn by working individually and in teams to investigate, communicate, and apply essential skills to solve the problem. Therefore, the PBL environment is claimed to support students' problem-solving skills and higher-order thinking as well as their critical thinking (Roh, 2003).

In this study, students' critical thinking was explored in a mathematics PBL classroom. The learning process was adapted from Othman, Salleh, and Sulaiman's study (2013), which consists of five ladders (i.e., introduction to the problem, self-directed learning, group meeting, presentation and discussion, and exercises). The critical thinking was described in terms of the subskills adapted from AACU (2009) as mentioned before.

Methods

The participants in this study were 47 eleventh-grade students from a public high school in Chiang Mai Province, Thailand. The data were collected for four weeks during December 2016. Students' critical thinking was observed during eight PBL classes while video was recorded as supporting data. After four weeks, students were given a critical thinking test. The test consisted of two real-world open-ended problem situations and five questions to evaluate each of the critical thinking dimensions (see Table 1). The PBL lesson plans and the critical thinking test were approved by three experienced teachers for content validity.

Table 1
Critical Thinking Test Questions

Dimension	Question
1. Explanation of issues	What is the problem in this situation?
2. Evidence	What evidence do you plan to use in solving the problem? How do you solve it?
3. Conclusions and related outcomes	What are your results and conclusions?
4. Influence of context and assumptions	Do you agree with his/her idea? Why?
5. Student's position	If you can make a decision, what will you do? Why?

The data were interpreted using both qualitative and quantitative methods. The critical thinking test was analyzed using descriptive statistics including percent, mean, and standard deviation. Furthermore, the PBL classroom observation data was described by means of descriptive analysis.

Results

Students' overall critical thinking scores were assessed by the critical thinking test via AAC&U's critical thinking VALUE rubric (2009) (see Table 2). The critical thinking score varied from 1 (low level), 2 and 3 (average level) to 4 (high level).

Table 2
Critical Thinking Test Results

Dimension	Critical Thinking Score				Mean	S.D.
	1	2	3	4		
1. Explanation of issues	4 (8%)	5 (11%)	26 (55%)	12 (26%)	2.98	0.84
2. Evidence	3 (6%)	6 (13%)	34 (72%)	4 (9%)	2.83	0.66
3. Conclusions and related outcomes	3 (6%)	31 (66%)	11 (24%)	2 (4%)	2.26	0.63
4. Influence of context and assumptions	2 (4%)	14 (30%)	24 (51%)	7 (15%)	2.77	0.75
5. Student's position	5 (11%)	23 (49%)	16 (34%)	3 (6%)	2.36	0.76

Focusing on findings from the PBL classroom observation, the five critical thinking dimensions including 1) explanation of issues, 2) evidence 3) conclusions and related outcomes, 4) influence of context and assumptions and 5) student's position are discussed respectively in the subsections below.

Additional details are provided with examples of exercise problem situations. Problem Situation 1 was an exercise problem for the first three critical thinking dimensions including explanation of issues, evidence and conclusions and related outcomes, and Problem Situation 2 was an exercise problem for another two critical thinking dimensions including influence of context and assumptions and student's position.

Problem Situation 1

You are going to participate in a water rocket competition. In the competition, each team can launch a rocket three times. The winner is the team with the highest mean of their rocket flying distances. Therefore, you invented two types of water rocket, tested them and recorded the results as follows:

Water rocket	Flying distance (meter)									
	1st try	2nd try	3rd try	4th try	5th try	6th try	7th try	8th try	9th try	10th try
Type 1	150	170	165	157	166	153	155	167	152	165
Type 2	170	151	167	155	154	171	153	161	170	148

Problem Situation 2

An educational organization surveyed learning outcomes in two neighborhood schools. Students in all classrooms of both schools were given an exam. The result showed that the schools' classroom score deviations were significantly different. Therefore, the organization conducted an interview about classroom management in both schools. The interview results are shown as follows:

First school: "Our school groups students into classes according to their entrance exam scores. Therefore, students with the same ability level are assigned to the same classroom, so they can learn in a suitable learning environment"

Second school: "Our school groups students into classrooms randomly. So, the classrooms consist of students of different ability and interest. We manage classrooms this way because we believe in diversity, and students should learn to live with others."

Question 1: What is the best classroom management policy? Why?

Question 2: What does the classroom's score deviation reflect?

Figure 1. Problem situations.

Explanation of Issues

Students were able to explain their understanding of the problem situations after the introduction to the problem in the first step of the PBL process. The students could highlight the necessary information and summarize the problem in their language. Most of the students were able to summarise the problem correctly but incompletely, as the problem's significance and supporting contexts were often ignored.

Table 3 presents examples of students' explanations of Problem Situation 1. Students with high explanatory skills described the situation including necessary information. Students with average explanatory skills usually omit details of the situation and always describe the situation's goals in an insufficient manner. Students with low explanatory skills sometimes misinterpret the problem situation including the situation's goals, leading them to misuse their evidence.

Table 3
Examples of Students' Explanations of Problem Situation 1

Level	Students' Explanation
High	Find rocket with the greatest flying distance and the highest reliability for the competition by using flying distance data to make a decision.
Average	Find rocket with the highest mean of flying distance.
Low	Find rocket that flies the greatest distance using first three of the greatest distance records.

Evidence

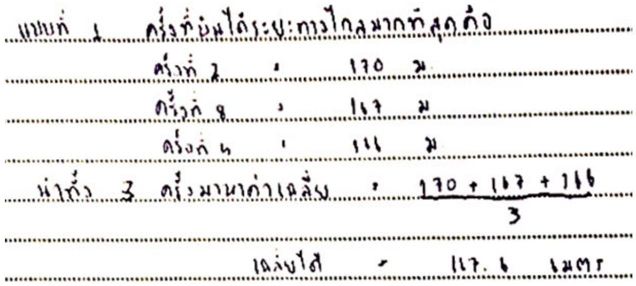
Students examined the situation, interpreted information and made their plan individually in the second step of the PBL process, self-directed learning. The students listed relevant information including given data, existing mathematics knowledge, and sometimes listed alternative hypothesis or approaches. Additionally, in the third step of

PBL, group meeting, the students shared individual work with their groups. The students discussed the problem goal and presented evidence to derive a group strategy and draw up a solution.

Table 4 presents students' selections and their use of evidence in Problem Situation 1. Students with high skill in selecting and using evidence always found multiple approaches and synthesized them to solve the problem. Students with average skill in selecting and using evidence were entirely focused on one approach, and they would give up when the approach failed. Students with low skill in selecting and using evidence could not interpret evidence in the situation, and thus they always came up with inappropriate approaches.

Table 4
Examples of Students' Selection and use of Evidence in Problem Situation 1

Level	Students' selection and use of evidence
High	Students used experimental records to calculate measures of central tendency including mean, median and mode.
Average	Students used experimental records to calculate one of the measures of central tendency and used it to make their decision.
Low	Students used experimental records in inappropriate approaches, and used them to make their decision as shown in Figure 1.



Type 1: The greatest distances are
2^{nd} try: 170 meters
8^{th} try: 167 meters
10^{th} try: 165 meters
Use the distances to calculate mean

$$= \frac{170+167+165}{3}$$

Mean is $=167.6$ meters

Figure 2. Example of inappropriate approach: Three of the greatest records were used to compute the mean of expected distance. The left image shows a student's work, and the right shows its English translation.

Conclusions and Related Outcomes

Students concluded and presented their findings to the class in the fourth step of the PBL process, group meeting. The groups' discovered evidence, working processes, outcomes and conclusion were demonstrated in the presentation and discussion while other students in the audience wrote summaries of the presented idea. After the presentation and discussion, students evaluated and concluded the ideas presented by the different groups to identify the best practices for the problem situation.

Table 5 presents students' outcomes and conclusions in Problem Situation 1. Students with high skill in drawing conclusions found possible outcomes and evaluated them to reach a conclusion. Students with average skill summarized their findings in a conclusion. They also created an alternative plan if the existing outcome could not reach the desired conclusion. Students with low skill formulated a conclusion from a single finding. Most of the conclusions were oversimplified and therefore were incomprehensible.

Table 5
Examples of Students' Outcomes and Conclusions of Problem Situation 1

Level	Students' Outcome and Conclusions
High	Means of rockets' flying distances are 160 meters for both types. Medians of rockets' flying distances are 161 and 158 meters. Modes of rockets' flying distances are 170 and 165 meters. We choose the first rocket type according to the mode because the difference between the modes (5) is greater than that between the medians (3).
Average	Because both rockets' flying distances have the same mean, we therefore need to find other method which is median. The calculated median of the first rocket is higher than that of the second type. Thus, we choose the first rocket type.
Low	We can choose either rocket because mean of 10 recorded flying distances of both rocket types are equal.

Influence of Context and Assumptions

Students examined their colleagues' thoughts and hypotheses in group meeting. The students analyzed and evaluated individual works to synthesize a group solution. Once a student proposed his/her idea, the others criticized the idea based on the problem's context, and their ideas and perspectives.

Table 6
Examples of Students' Analysis on Influence of Context and Assumptions in Problem Situation 2 on the Question "What does Classroom's Score Deviation Reflect?"

Level	Students' analysis on influence of context and assumptions
High	It reflects knowledge diversity in classrooms. From the interview, the first school manages classrooms by students' entrance scores which causes student at the same level to be together, and thus classrooms in the first school have low score variation. On the other hand, the second school mixes students in each classroom which causes diversity, and thus classrooms in the second school have high score variation.
Average	It reflects how many scores each classroom earns. The classroom which has low score variation is where the student scores are in the same interval and the classroom which has high score variation is where the student scores are greatly different.
Low	It reflects the teachers' principles and teaching methods because the teaching method of the first school is specified in each classroom, but the teaching method of all classrooms in the second school is the same.

Table 6 presents students' analysis on influence of context and assumptions in Problem Situation 2 on the question "What does the classroom's score deviation reflect?" Students with high context analytical skill considered given information and found a relation to their perspective. Students with average skill could use sufficient information to reach their answers. The answers, however, ignored some important information. Students with low skill were aware of existing information, but could not analyze it to reach an answer.

Students' Positions

Students expressed their thoughts through the group meeting. The students brainstormed their ideas to analyze problem objectives, evidence and hypotheses. The students' ideas and perspectives were usually unmatched and therefore were discussed to reach a group decision. The students also demonstrated their positions in the presentation and discussion step. The presenting group illustrated their problem solution to the class, and sometimes the others criticized and suggested alternative opinions or refinements.

Table 7
Examples of Students' Positions in Problem Situation 2 Toward the Question "What is the Best Classroom Management Policy? Why?"

Level	Students' Positions
High	I agree with the first school's principle because we can study effectively with classmates at the same level. Moreover, in my experience, putting diverse students together make the students unmotivated because the low achievers stick together, and there is no challenge for the high achievers. However, I disagree with the use of entrance score because it cannot always define students' abilities. I prefer using students' grades to arrange classrooms in every year. This method will motivate students to study with their friends.
Average	I agree with the first school's principle because it helps teachers to teach systematically and helps students with similar ability level to learn together. The second school's principle, on the other hand, makes high achievers learn less effectively while waiting for low achievers who also get depressed.
Low	I agree with the second school's principle because bringing diverse students together creates opportunities for high achievers to help the lower ones. Low achievers can develop themselves while high achievers can practice their skills and knowledge.

Table 7 presents students' positions in Problem Situation 2 toward the question "What is the best classroom management policy? Why?" Students with high skill in stating a position specify their position considering multiple viewpoints. The chosen path and unchosen paths were discussed in order to reach their decision. They mentioned the limitation of their decision and sometimes came up with alternative ideas. Students with average skill in stating a position considered both chosen and unchosen viewpoints and made their decision. Students with low skill in stating a position considered only a single viewpoint and made their decision, which thus was too simplistic and biased.

Discussion and Conclusion

This study explored students' critical thinking in five steps of a mathematics PBL classroom. The findings revealed that PBL learning processes allowed students to express their critical thinking in all of the dimensions as "students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions" (Roh, 2003, p. 1), and the PLB tasks also gave opportunities for the students to share and evaluate their thoughts and opinions in a group as "these tasks require an open exchange of ideas and engagement by all members of the group" (Hmelo-Silver, 2004, p. 241).

Additionally, the critical thinking test results indicated that the students showed the highest score in the explanation of issues dimension and the lowest score in the conclusions and related outcomes dimension. These results are compatible with AACU's current report (2017) that demonstrated explaining of issues as the strength and drawing conclusion as the weakness of students. Furthermore, the results showed that the students also had difficulty in the student's position dimension as they omitted available viewpoints and specified their position inadequately. This finding was not specifically mentioned in the AACU report. Therefore, future work should focus on confirming students' weakness and strength, and supporting students' critical thinking in the specific dimensions.

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10th Grade Students' Participation in a Mathematics Problem-Based Learning Classroom

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In this study, we investigated 10th grade students' (n = 46) participation in a mathematics problem-based learning classroom. The data were collected from 10 PBL lesson plans, students' participation observation forms, teacher's notes, students' reflections, students' participation self-surveys, and students' interview forms. The students' participation was described in six dimensions that were adapted from Abuid (2014). We found that the students performed in positive dimensions of students' participation at a very high level and students expressed in a negative dimension of students' participation at low level.

Introduction

Students' participation is considered to be a significant factor in learning (Sadker & Sadker, 1994). Much research has shown strong evidence for the importance of students' participation in classrooms (Petress, 2006). Participation can actively bring students into the learning process (Cohen, 1991). Liu, Yao, and Yao (2005) found that students who participate actively in their classrooms tend to have better academic achievement. In addition, there are various activities to support students' participation, such as questioning, discussion and explanation, that help the students to gain in-depth knowledge and understanding (Boyle & Nicol, 2003).

Many researchers provide different points of view of the definitions of students' participation. Vonderwell and Zachariah (2005) defined participation as a method where students engage and are active in learning. The definition from Selun and John (2008) showed that students' participation is a behaviour of students who act as active participants in their own learning. Therefore, this study defined students' participation as an expression of student behaviours in classroom that creates a learning experience.

In collecting evidence of students' participation, many researchers have proposed several dimensions of students' participation. Interestingly, Abuid (2014) identified eight dimensions of participation that can be flexibly applied in any classroom i.e., answering questions addressed to the class, answering questions addressed to the individual, volunteered participations, group discussion, e-learning forum, attendance and disruptive participation. In this study, the six students' participation dimensions including (1) answering questions addressed to the class, (2) answering questions addressed to the individual, (3) long in-class written answers, (4) volunteered participation, (5) group discussion, and (6) disruptive responses were adapted from Abuid (2014). In fact, Abuid's work (2014) has another two dimensions, including e-learning forum and attendance dimensions. However, in this study, the dimensions of students' participation in e-learning forum and through attendance were excluded in order to suit the context of the target classroom.

In the 21st century, learning approaches emphasize student-centred activities. This pedagogical approach for mathematics education shifts the educational paradigm away from traditional approaches (Schmude, Serow, & Tobias, 2011). The student-centred model moved attention from whole-class instruction to small-group work and individual inquiry, which brings about active learning and extends students' participation, motivation

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and achievement (Cannon & Newble, 2000). The study of McManus (2001) showed that passive learners did not receive the content of knowledge along with understanding. On the other hand, students who were expected to participate in active classrooms constructed and applied their new knowledge with understanding. Othman, Salleh, and Sulaiman (2013) recommended problem-based learning (PBL) as one of the most powerful student-centred approaches in the 21st century.

PBL is an instructional process where problems are used in the beginning of the instruction to introduce and provide the topics of learning (Chagas, Mourato, & Sousa, 2007). Students work in groups to solve a problem; the learning is enhanced by solving an ill-structured real-world situation. Then, students learn to assume a role as owner of the situation (Torp & Sage, 2002). Barrow (1996) identified the processes in a PBL classroom where learning occurred in small student groups while students learn together. Moreover, PBL motivates students to curtail disruptive behaviour and engages students to participate in learning (Achilles & Hoover, 1996). Therefore, the aim of this study was to investigate students' participation in a mathematics classroom using problem-based learning. The PBL can be defined by five steps of learning that are adapted from Othman, Salleh, and Sulaiman's study (2013): (1) introduction to the problem, (2) self-directed learning, (3) group meeting, (4) presentation and discussion, and (5) exercises.

Method

The study adopted a mixed research methodology using both quantitative and qualitative data collection. In order to validate and crosscheck the findings, we used different data sources (Patton, 1990). The research instruments included ten PBL lesson plans, students' participation observation forms, teacher's notes, and students' reflections, participation self-surveys, and interview forms. The participants were 46 tenth grade students from a high school in Chiang Mai Province, Thailand. The data were collected for four weeks from mid-December 2016 to mid-January 2017.

One of the researchers taught the students using the PBL lesson plans for 100 minutes per lesson. Additionally, in each lesson, the mentoring teacher used the students' participation observation form to collect the students' participation data. The teacher's notes, students' reflections and video tape recordings (to provide backup data) were used to reflect on teaching and students' participation. At the end of each week, students' participation self-surveys were used to evaluate the students' participation in all dimensions. At the end of the four-week PBL lesson plans, the teacher interviewed six students selected according to their mathematics abilities (two high, two average, and two low) in order to provide in-depth information.

In the data analysis, quantitative data that was collected from students' participation observation forms, students' reflections and students' participation self-surveys were analysed by using descriptive statistics including percentage, mean, and standard deviation. In addition, qualitative data that was collected from teacher's note, students' reflections and students' interview form were analysed by means of descriptive analysis.

Results

After using two PBL lesson plans with 46 tenth grade students, the results are reported in two parts. Part 1 describes data collected in the PBL classroom and Part 2 describes data collected from students' participation self-survey and students' interview.

Part 1 Description of Data Collected in the PBL Classroom

Step 1: Introduction to the problem. In this step, the teacher introduced real-world problems to the class. Data collected from teacher's notes and students' reflections showed that most of the students (80%) were interested in the topic that was introduced. Many students (70%) participated in answering questions. A few students (30%) shared their own ideas that were involved with the problem situation. Photographs of the students' participation in this step are shown in Figure 1.

Figure 1. Students' participation in the first step of PBL classroom: Introduction to the problem.

Interestingly, in the first step of the PBL lesson, students showed four out of six dimensions of students' participation (i.e., answering questions addressed to the class, answering questions addressed to the individual, volunteered participation, and disruptive responses). The researchers found that participation along the first two dimensions occurred very often. Meanwhile, the dimension of volunteered participation was at a high level, while the dimension of disruptive responses was at low level as seen in Table 1.

Table 1
The Occurrences of Students' Participation in the First Step of PBL From Students' Participation Observation Forms

Dimension	Mean	SD	Level
Answering questions addressed to the class	3.80	0.42	Very high
Answering questions addressed to the individual	3.60	0.52	Very high
Volunteered participation	2.90	0.88	High
Disruptive responses	1.80	0.63	Low

Step 2: Self-directed learning. In this step, students began to solve a problem by themselves. They attempted to do their individual work. Data collected from teacher's notes and students' reflections showed that most of the students (90%) attempted to write down their own ideas and tried to solve the problem. When the students were uncertain, they usually asked for help from the teacher or their peers. Photographs of the students' participation in this step are shown in Figure 2.

Figure 2. Students' participation in the second step of PBL classroom: Self-directed learning.

The second step of PBL lesson, the students were involved with two dimensions of participation (i.e., long in-class written answers and disruptive responses). We found that the first dimension was at a very high level and the other one was at low level as seen in Table 2.

Table 2
The Occurrences of Students' Participation in the Second Step of PBL From Students' Participation Observation Forms

Dimension	Mean	SD	Level
Long in-class written answers	3.80	0.42	Very high
Disruptive responses	2.10	0.74	Low

Step 3: Group meeting. In this step, students were divided into eight small groups of five to six students to participate in group meetings. The students worked together with their peers to find the solution as a group. Then, the students wrote down the ideas on worksheets and prepared for the presentation. Data collected from teacher's notes and students' reflections showed that many students (70%) were interested in sharing their own ideas. Furthermore, some students (60%) often wrote down new ideas to solve the problem before sharing again. The students asked for help from the teacher after they had discussed the problem in their group. In addition, the teacher asked some questions of particular students to help them and check their understanding. Many students (80%) participated in answering the questions. Photographs of the students' participation in this step are shown in Figure 3.

Figure 3. Students' participation in the third step of PBL classroom: Group meeting.

In the third step of the PBL lesson, the students were involved with five dimensions of participation (i.e., answering questions addressed to the individual, long in-class written answers, volunteered participation, group discussion, and disruptive responses). We found that the first four dimensions were at a very high level and the disruptive responses was at low level as seen in Table 3.

Table 3
The Occurrences of Students' Participation in the Third Step of PBL From Students' Participation Observation Forms

Dimension	Mean	SD	Level
Answering questions addressed to the individual	3.70	0.48	Very high
Long in-class written answers	3.80	0.42	Very high
Volunteered participation	3.30	0.67	Very high
Group discussion	3.60	0.52	Very high
Disruptive responses	2.10	0.74	Low

Step 4: Presentation and discussion. In this step, the teacher asked for volunteers to present their group work. Then, a whole class discussion brought the students to the conclusion of the topic being studied. Many students (80%) paid attention to the presentations of their peers. A few students (30%) wrote down ideas from the presenting groups. After that, the teacher asked some questions to clarify each idea. Many students (70%) answered the questions that were addressed to the class. In addition, many students (70%) who were called upon by the teacher always answered the questions. Some students (60%) volunteered to ask other groups and give some counterexamples. Photographs of the students' participation in this step are shown in Figure 4.

Figure 4. Students' participation in the fourth step of PBL classroom: Presentation and Discussion

Obviously, in the fourth step of PBL classroom, the students were involved with all six dimensions of participation i.e., answering questions addressed to the class, answering questions addressed to the individual, long in-class written answers, volunteered participation, group discussion, and disruptive responses. We found that four out of six dimensions (answering questions addressed to the class, answering questions addressed to the individual, volunteered participation, and group discussion) were at a very high level whereas another two dimensions (long in-class written answers and disruptive responses) were at a high level as seen in Table 4.

Table 4
The Occurrences of Students' Participation in the Fourth Step of PBL From Students' Participation Observation Forms

Dimension	Mean	SD	Level
Answering questions addressed to the class	3.50	0.53	Very high
Answering questions addressed to the individual	3.60	0.52	Very high
Long in-class written answers	3.00	0.82	High
Volunteered participation	3.50	0.71	Very high
Group discussion	3.30	0.67	Very high
Disruptive responses	2.60	0.52	High

Step 5: Exercises. In this step, the teacher promoted students' learning by allowing them to do exercises. Data collected from teacher's notes and students' reflections showed that many students (80%) attended to do the exercises. When the students didn't understand, they asked for help from their peers. Photographs of the students working during the exercise step are shown in Figure 5.

Figure 5. Students' participation in the fifth step of PBL classroom: Exercises

In the fifth step of the PBL approach, the students demonstrated two dimensions of students' participation (i.e., long in-class written answers and disruptive responses). The researcher found that the first dimension was at a very high level and the other one was at low level as seen in Table 5.

Table 5
The Occurrences of Students' Participation in the Fifth Step of PBL From Students' Participation Observation Forms

Dimensions	Mean	SD	Level
Long in-class written answers	3.60	0.52	Very high
Disruptive responses	1.80	0.92	Low

Part 2: Description of Data Collected from Students' Participation Self-Survey and Interview

The data were described by using means and the standard deviations of students' participation self-surveys based on six dimensions of students' participation adapted from Abuid (2014). The students usually participated in the dimensions of long in-class written answers, often participated in answering questions addressed to the class and group discussion dimensions, sometimes participated in the dimensions of answering questions addressed to the individual and volunteered participation, and seldom showed disruptive responses dimension in the PBL classroom as seen in Table 6.

Table 6
Means and Standard Deviations from Students' Participation Self-Surveys (n = 46)

Dimension	Mean	SD
1. Answering questions addressed to the class	3.50	0.81
2. Answering questions addressed to the individual	2.89	0.90
3. Long in-class written answers	4.67	0.60
4. Volunteered participation	2.91	0.99
5. Group discussion	3.54	1.13
6. Disruptive responses	1.86	0.53

From the interview data of six selected students with mixed mathematics ability (two high, two average, and two low), the researchers found that student's participation in five out of six dimensions (answering questions addressed to the class, answering questions addressed to the individual, long in-class written answers, volunteered participation and group discussion) were usually at a high level. Students at an average level usually participated in three out of six dimensions (answering questions addressed to the class, long in-class written answers and group discussion). Students at a low level usually participated in three out of six dimensions (long in-class written answers, volunteered participation and group discussion), as seen in Table 7.

Table 7
Comparing Characteristics of Behavioural Participation from Students' Interviews

Level of students' achievement	Students' expression
High	Students usually answered whole class questions. They usually answered particular questions that were addressed to the individual. They usually answered questions and expressed their opinions by writing and drawing. They usually volunteered as the first speaker in group discussion to share their opinion and give examples. They usually concluded with various ways to solve a problem in group discussion. After presentations, the students occasionally gave a counterexample to the group. In addition, they sometimes expressed disruptive responses such as playing on mobile phones when they already finished their individual work.
Average	Students usually tried to answer when teacher addressed questions to the class. They sometimes participated in answering particular questions that were addressed to the individual. They usually tried to solve problem by themselves. Before starting discussion in a group, they wrote down their opinion. In addition, they did exercises by themselves. They sometimes volunteered to help their peers in the group meeting. They usually expressed their comprehension by discussing in a group and shared their own opinion with peers in the presentation and discussion step. Finally, they seldom expressed disruptive responses, such as falling sleep and playing mobile phones when their peers were giving a presentation.
Low	Students sometimes answered whole class questions. They seldom answered particular questions that were addressed to the individual. They usually participated in writing down their own answers. The students can answer easy exercise questions. They usually volunteered to prepare presentation tools. They seldom discussed in their groups. In addition, they usually disrupted peer learning while the teacher was giving a problem. The students talked about unrelated topics and used mobile phones when their peers were giving a presentation.

Discussion and Conclusion

This research investigated students' participation in a mathematics problem-based learning classroom. From the results in the classroom, students' participation was described in six dimensions (Abuid, 2014): (1) answering questions addressed to the class, (2) answering questions addressed to the individual, (3) long in-class written answers, (4) volunteered participation, (5) group discussion, and (6) disruptive responses. The overall level of the first five dimensions, which are positive behaviour, were at a very high level and the other one, which is negative behaviour, was at a low level. These findings aligned with the results of Achilles and Hoover's study (1996) that showed PBL enhanced students' participation and decreased disruptive behaviour in classroom.

The results from students' participation self-survey showed that all students usually participated in the dimensions of long in-class written answers. According to the results from interviews, students at all levels of mathematics ability usually participated in the dimensions of long in-class written answers. In addition, students at average and low levels sometimes participated in the dimensions of answering questions addressed to the individual, and students at high and low levels seldom participated in the dimensions of disruptive responses. The results revealed that all positive dimensions of students' participation in all steps of the PBL process were observable. This may be caused by the processes of PBL classrooms that support students and give them a chance to participate in different kinds of activities such as individual work and group work (Torp & Sage, 2002).

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Mathematics Identities: From Motivations to Turning Points in Mathematics Identity Construction

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In this paper, I examine mathematics identity construction of university mathematics lecturers and higher secondary mathematics teachers. Past mathematical learning experiences are elicited through narrative accounts of personal stories. Some of the lecturers and teachers described their positive mathematics identities. Others experienced transitions in their mathematics identity construction. The mathematics identity transitions are elaborated using turning points, and their stories are interpreted in the wider societal contexts. Findings from this study suggest that motivational experiences from the teachers, the school, and family had pivotal roles in developing a positive mathematics identity.

Developing a strong disciplinary bond with mathematics is important for mathematics teaching. Teachers' mathematics identities are essential in developing mathematical knowledge and in shaping mathematics teaching practices (Hodgen & Askew, 2007). Mathematics identity construction is an important phase in mathematics learners' life. Teachers' mathematics perceptions influence students' mathematics identity performances (Darragh, 2013) and consequently their mathematics learning. This is important especially when mathematics anxiety and mathematical phobia among learners are contributing towards a lesser number of students opting to take mathematics for higher studies (Boaler & Greeno, 2000; Boaler, Wiliam, & Brown, 2000).

Identity is an important construct to understand the interaction between the affective and cognitive factors (Grootenboer, Smith, & Lowrie, 2006) in mathematics teaching and learning. The literature on mathematics identity has examined students' mathematics identity (Boaler, 1997), pre-service teachers' mathematics identity (Bibby, 1999; Bjuland, Cestari & Borgersen, 2012; Walshaw, 2004) and in-service teachers' mathematics identity (Walshaw, 2010). Drake, Spillane, and Hufferd-Ackles (2001) studied teachers' mathematics identities from the perspectives of teachers of mathematics and learners of mathematics. The construct of identity in mathematics is examined in the context of mathematics subject and pedagogical knowledge (Bibby, 1999; Bjuland et al., 2012; Hodgen, 2011; Hodgen & Askew, 2007; Smith, 2007; Stein, Silver, & Smith, 1998). Positive mathematics identities are connected to knowing mathematics and in developing professional identities (Hodgen, 2011). Identity is defined by different theoretical positions. One aspect of understanding lecturers' mathematics identity is through "identity-in-discourse" (Weedon, 1997). In this paper, identity is understood as a discursive construct and is defined as "our understanding of who we are and who we think other people are" (Danielewicz, 2001, p. 10). "Identity-in-discourse" recognises identity as being represented discursively, that is, "what the lecturer or the teacher tells about their understanding of who they are as mathematical learners in the past".

It is important to extend the notion of identity construction to university mathematics lecturers specifically to understand their present positive mathematics identities, as this was not a focus of the past research. According to Boaler (1997), negative mathematics identities are one of the reasons for delineation from the subject and not choosing mathematics for higher studies. Following this, for the purpose of this study, I hypothesised that university mathematics lecturers and school teachers had positive

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mathematics identities to choose the profession of mathematics teaching. It thus becomes important to understand how university lecturers and mathematics teachers constructed their positive mathematics identities. Understanding the contributing sources for motivations in their identity transitions will be useful in better understanding their role in developing positive mathematics identity in learners. It will provide insights into the lecturers' and teachers' present positive mathematics identities. This paper addresses the research questions: How have university lecturers and mathematics teachers constructed their present positive mathematics identities? In particular, did they experience any transition in their mathematics identity construction?

Theoretical Background

I take a sociocultural perspective on identity (Wenger, 1998), where identity is seen as a constructive process between the individual and the society. According to Wenger (1998),

> The concept of identity serves as a pivot between the social and the individual, so that, each can be talked about in terms of the other... It does justice to the lived experience of identity while recognizing its social character - it is the social, the cultural, the historical with a human face. (p. 145)

That is, identity is a discursive construct in which individual identity construction is shaped and influenced by the culture and the society to which one belongs. Identity is used as a theoretical construct to understand how, when, and why teachers change their teaching practice (Hodges & Hodge, 2017). In a similar sense, I argue that by examining mathematics lecturers' and teachers' mathematics identities, we can understand how, when, and why they were motivated to construct their present positive mathematics identities.

Weedon (1997) considers identity as a discursive construct where language and identity are considered complementary. Sfard and Prusak (2005) accept the identity as "discursive counterparts of one's lived experiences" and maintain identity construction as a communicational practice. Narrative accounts of stories are used to understand teachers' mathematics identities (Drake et al. 2001; Sfard & Prusak, 2005; Stein et al., 1998). In the context of this paper, identity is understood as a discursive construct and uses the narrations or storied identities of participants to explore mathematics identities and "turning points". Turning points are key episodes in life through which one identifies substantial changes (Drake et al., 2001). According to McAdams (1993),

> A turning point is an episode wherein you underwent a significant change in your understanding of yourself. It is not necessary that you comprehended the turning point as a turning point when it in fact happened. What is important is that now, in retrospect, you see the event as a turning point, or at minimum as symbolising a significant change in your life. (p. 258)

In this paper, a turning point refers to a transition in the process of lecturers' and teachers' mathematics identity construction in the past. The turning point is their comprehension of transition from a negative mathematics identity into a positive mathematics identity.

The focus of this paper is participants who experienced a turning point in constructing their mathematics identity. It is noted that all participants who experienced the transition in their mathematics identities are university lecturers. They narrated stories of lived experiences to the researcher. These turning points are stimulated by motivations from different sources, which participants have identified. These sources are from members of the immediate school environment and family. They collectively form members and a part of the society. Thus, implications of these contributing sources on developing positive mathematical identities of learners are discussed from the wider societal point of view.

Hodgen and Askew (2007) maintained that teachers' knowledge of mathematics has intellectual and emotional connections. They suggest that teacher identity and teacher knowledge are related to the extent where a change in teacher identity corresponds to a change in teacher's knowledge. This also implies that change in mathematics teacher identity from a negative to a positive corresponds to a positive change or an increase in teachers' mathematical knowledge (Hodgen, 2011; Smith, 2007; Stein et al., 1998).

Research Design

Data reported here are taken from an exploratory study undertaken in India. The nine participants were six university mathematics lecturers (L1, L2, L3, L4, L5, and L6) and three higher secondary mathematics teachers (T1, T2, and T3). The teachers and the lecturers volunteered to participate in this study. The lecturers were from the mathematics department of the same university and the teachers were from different schools in the educational district. Different groups were included in the sample to understand if there existed any difference between school teachers and university lecturers in their mathematics identity construction. The teachers were teaching Grade 12 mathematics (17- and 18-year-olds) and the lecturers were teaching university mathematics. Most of the lecturers had more than 15 years of teaching experience and the teachers had between 8-12 years of experience. All the teachers and lecturers had a MSc degree in Mathematics and a teaching qualification. Additionally, lecturers L1, L2, L4, and L5 had a research degree in mathematics (MPhil or PhD).

Stories are considered rich sources for examining identities and such studies need more attention (Sfard & Prusak, 2005). Drake et al. (2001) used stories elicited through interviews from teachers to make sense of their learner as well as teacher identity in the subject matter context: "Stories as lived and told by teachers, serve as the lens through which they understand themselves personally and professionally" (p. 2). Individual identities have been understood through stories in which individuals make sense of themselves and their lives (McAdams, 1993).

Narrative data are in the form of interview transcripts and observations (Connelly & Clandinin, 1990). This study made use of a combination of semi-structured interviews and observations of classroom teaching. The interviews lasted for 30 minutes to one hour and were audio-recorded and transcribed. During the interview, I asked questions that focussed on their mathematical learning experiences during school years, such as: Did you like learning mathematics during school years? Can you explain your experiences of learning mathematics? I also checked with the participants if there was further change in their mathematics identity in any stage of their university studies or professional life by asking: Have you always liked learning mathematics? Did you experience any change in your liking for mathematics in later stages of your study or profession? In replying to these questions, participants recalled and narrated storied identities of their understanding of who they are as mathematics learners, revealing turning points in mathematics identity construction. Thus, identities are shaped as discursive constructs through individually told stories (Sfard & Prusak, 2005).

Participants' narratives from interview transcripts were analysed using inductive thematic analysis (Braun & Clark, 2006). First, the analysis was done at an individual level, as each storytelling is different from one another depending on participants' personal experiences. After analysing the narratives of participants independently of one another, they were analysed together to find connecting themes across and within the stories. The emerging themes were searched again for commonalities and contrasts, which were grouped. The main themes emerged were "positive mathematics identities", "turning points" characterising feelings of failure in mathematics in transition to positive mathematics identities, and "motivations" underlying these turning points.

Results and Discussion

The analysis of interviews with nine participants resulted in three important themes, which are "positive mathematics identities", "turning points", and "motivations". The theme of positive mathematics identities emerged from five participants who always held positive mathematics learner identities. Of these five, three were higher secondary mathematics teachers and two were university mathematics lecturers.

Positive Mathematics Identities

Of the nine participants, five (T1, T2, T3, L1, and L5) talked about their liking for mathematics and provided their points of view.

> T1: I always like mathematics and mathematics is an interesting subject to me.

T2 expressed positive mathematics identity highlighting his "genuine interest in mathematics as a subject". Another pointer to T2's positive disposition is:

> T2: I have a good mathematics library at home.

Having a mathematics library at home connected T2 with mathematics. His positive identity was nurtured from home in combination with his interest in mathematics. This encouraged T2 seeing the positive influence of mathematics in daily life.

> T2: I always observe its [mathematics'] influence in daily life situations.

T3 presented with a positive mathematics identity as he always likes mathematics. T3 shared another positive view:

> T3: I like mathematics for its domination over other subjects.

L1 had positive mathematics identity, as he said:

> L1: I always like teaching and learning mathematics.

Similarly, L5 presented a positive mathematics identity, saying that he always liked mathematics. Teachers' and lecturers' positive mathematics identities give us an understanding of how their present mathematics identities are formulated over the years from mathematics learner to mathematics teacher/lecturer. Their positive dispositions of mathematics reveal that our thinking about the subject partly forms our mathematics identity. If a mathematics learner could perceive mathematical influence in daily life and can observe its practical utilities, this may act as a motivation to study mathematics. Further, early childhood habits of interacting with mathematics assist in developing positive thoughts about learning mathematics, as T2 relate the experience of reading and working with mathematics books at home. How we perceive mathematics shapes our thoughts about mathematics which influences mathematics identity construction. Thus, developing a positive outlook towards mathematics and maintaining it are useful in developing positive mathematics identities.

Turning Points

Four participants (L2, L3, L4, and L6) experienced a transition in the process of identity construction. In other words, the theme of turning point came from these four participants' narrated "stories of lived experiences" (Sfard & Prusak, 2005). Interestingly, all these four participants were university lecturers.

L2 recalled negative mathematics identity using expressions of fear and anxiety

> L2: A fear of the subject was unknowingly built into my psyche during my childhood.

L2 was talking about the mathematical phobia and mathematical anxiety experienced during student life, but continued to narrate how he was able to develop a positive mathematics identity from his pre-university year onwards, which helped him "master the subject". L2 referred to his uncle motivating him to pursue mathematics for higher studies.

> L2: But a question from my uncle...changed the whole scenario, which helped me ignite a new mathematical mind.

When asked about that question, L2 said his uncle knew he had good marks in other subjects and lower marks in mathematics.

> L2: The question was, "Why should you choose mathematics for higher studies when you are more comfortable learning other subjects?" Then, I thought why I cannot do mathematics. I took the rebellious and challenging decision of becoming a master of this subject, which I feel, to a certain extent, I have been able to fulfil.

The question from his uncle challenged L2 and motivated him. L2 decided to opt for mathematics and learn it for advanced studies. It is impressive that L2 is now a research mathematician. L3 presented with a negative mathematics identity when she talked about her dislike for learning mathematics in the early years of school:

> L3: I did not like mathematics until I was in Year 3.

Then, L3 talked about how it had evolved into a positive mathematics identity when she started elementary mathematics.

> L3: My tuition teacher created interest in learning mathematics. I liked her teaching method. She taught me mathematics using short-cuts. I used to get confused when learning mathematical problems with a lot of procedural steps involved. She taught me mathematics taking straight to the point.

According to L3, it is "the thrill and originality of mathematics" that fascinated her on knowing more mathematics. L4 narrated how an unexpected reward from his head teacher at the senior school acted as a turning point. Though he did not like or dislike mathematics until that time, this incident motivated him to consider studying mathematics seriously.

> L4: I used to read books from mathematics library in the senior school. One day, when I was in the library, my head teacher happened to notice the mathematics book I was reading (which was related to advanced level mathematics) and he praised me for reading that book at that age. Following this, my head teacher spoke about this incident at the school assembly and rewarded me. After this incident, I started seriously studying mathematics.

L4 recollects this incident after many years (similar to L2 and L3), which means that the motivation he received from the head teacher for reading an advanced mathematics book encouraged him to learn and continue studying mathematics for higher degrees. He remembered that although he had no particular interest in mathematics until then, this incident made him love mathematics and assisted in developing a positive mathematics identity. Another example of a turning point participant is L6.

> L6: I did not like mathematics until Standard 7 [Year 7]. My mathematics teacher motivated me and created interest in studying mathematics.

L6 narrated how his teacher motivated him to study mathematics:

> L6: I used to fail in mathematics exams, but my teacher knew I can do mathematics. He called me individually to his office and talked about how to learn mathematics. It was about learning basic steps in doing mathematics, the patience and regularity needed to work with mathematics. Exactly after that incident, I got the motivation to learn mathematics. Then, I started working on mathematics and slowly build up my maths. I started liking mathematics, passed mathematics exams and gradually got good marks.

L6 identified his mathematics teacher encouraging his transition from a negative to a positive mathematics identity. Four lecturers in the interview narrated how, when, and why they had the transition from a negative to a positive identity. Notably, many years after these turning points, the participants could recall the events and strongly relate their mathematical learning experiences leading to the turning point in their mathematics identity construction. The negative and fearsome experience of engaging with mathematics turned into a fascination for mathematics, leading to joyful learning. Transitions were substantial in that the university lecturers embraced learning higher mathematics and teaching it for the rest of their lives. This evidences the effect of these turning points in continuing mathematics learning and in choosing a career in teaching mathematics. Stein et al. (1998) observed teachers' identity change with a corresponding change in teacher learning. In this study, the negative to the positive mathematics identity change was correlated with an affective change in their mathematics learning in relation to their liking for mathematics. For example, L2's "igniting a new mathematical mind", L3's experiencing "the thrill and originality of mathematics", L4's "seriously studying mathematics", and L6's "created interest in studying mathematics" and "liking mathematics" show their deep engagement with mathematics and its learning. The "turning point" lecturers identified school (the mathematics teacher and the head teacher), family (uncle), and the tuition teacher as sources for developing positive mathematics identity. These sources form the theme of motivations as discussed below.

Motivations

The theme of motivations has the potential for showing the lecturers' mathematics learner identities. Motivation to do mathematics and to teach are both individual and social (Hodgen, 2011). The stories narrated by turning point participants had one striking similarity – They identified a potential source of motivation that contributed positively to their mathematics identity construction. L2 identified his uncle as a motivation to choose mathematics for university study. This led him to continue learning higher mathematics and teaching university mathematics. For L3, the source of motivation for positive mathematics identity was her mathematics tuition teacher. The head teacher of the school acted as the source of motivation for L4's positive mathematics identity and the mathematics teacher at school motivated L6.

Motivations are useful in changing from a negative to a positive mathematics identity. It provided the opportunity for the mathematics lecturers in learning higher mathematics and in choosing a career in mathematics teaching. The potentials of motivations are utilised in positive mathematics identity construction, in overcoming mathematical phobia and mathematical anxiety, in developing positive mathematics learner identity, and, finally, in mastering advanced mathematics and in teaching.

For the turning point participants, the sources of their motivation came from members of society, such as the mathematics teacher, the head teacher of the school, mathematics tuition teacher, and a family member. Collectively taken, the lecturers' self-identified sources of motivation are the individual members of the community. This reinforces wider societal role in constructing and developing positive mathematics learner identities. It thus becomes necessary to look at the role and potential of motivations in constructing positive mathematics learner identities from a wider societal point of view.

Mathematical Knowledge and Positive Mathematics Identity

In this study, I hypothesised that university mathematics lecturers and the mathematics teachers have positive mathematics identities. However, I found that not all university lecturers had positive identities in the beginning; some had turning points or transitions in constructing their present positive mathematics identities. Teacher learning is connected to identity change (Stein et al., 1998). The positive identities enabled university lecturers in learning higher mathematics and in developing an affective connection with mathematics. In this way, positive mathematics identity and gaining mathematical knowledge are connected (Hodgen, 2011). For example, in this study, the turning point participants L2 and L4 had research degrees in mathematics. The other turning point participants L3 and L6 expressed the desire to study for a PhD in mathematics. Thus, gaining mathematical knowledge is central to lecturers' and teachers' mathematics identities (Smith, 2007).

Conclusion

This study used sociocultural conceptualisation of identity to understand participants' experiences. It defined identity as a discursive construct. Sfard and Prusak (2005) discuss the weakness of using motivational concepts in analysing data as they lack operational definitions. Defining identity as a discursive construct in this study thus gives greater insights into analysing and understanding participants' experiences. The mathematics teachers and lecturers whom I interviewed narrated their past mathematics learning experiences. Out of the nine participants, five of them always held positive mathematics identities. For them, positive dispositions generated towards mathematics from living contexts assisted in developing positive mathematics identities. The remaining four participants have compelling stories about their motivations to study higher mathematics. That is, not all university mathematics lecturers held positive mathematics identities as learners of mathematics; instead, they experienced a transition in formulating present positive mathematics identities. The lecturers who had turning points in the process of mathematics identity construction acknowledged different sources of motivation to study mathematics. The motivation to study mathematics primarily arose from different members of society. This study suggests that the sources of motivations from wider societal contexts have societal implications. The role of the society in constructing and developing positive mathematics learner identity is crucial. Motivations turned mathematics anxiety and mathematical phobia into joyful mathematics learning, enabling learning and teaching higher mathematics. Developing a positive mathematics identity and gaining mathematical knowledge is connected as positive identity change and linked to participants' desire to engage with and learn higher mathematics. I conclude that mathematics identity construction is an important phase in mathematics learning that has the potential to facilitate one's ability to do mathematics and choice to study higher-level mathematics.

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Using Drawings in Solving Mathematics Word Problems

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Widely used in mathematics teaching, word problems are often more difficult for students than comparable number only problems. This study investigated whether Year 4 and 5 student success with mathematics word problems could be improved using a drawing-based intervention. In this mixed method study, five students at risk of low mathematics achievement participated in three 30-minute intervention sessions. Data included pre- and post-tests, verbal student reflections, and student drawings. Findings include that creating representations of word problems may be important for student understanding and that teaching students to create drawings of word problems can help promote achievement.

Realistic contexts are known to enhance the effectiveness of teaching and learning. With curricula that emphasise the use of real world examples in mathematics, the wide use of word problems in teaching, resources, and assessment, it is crucial that students have strategies to be successful word problem solvers. However, mathematical word problems (mathematics problems presented in words rather than solely using numbers) can be difficult for students (Hegarty, Mayer, & Monk, 1995; Verschaffel & De Corte, 1993), with many performing worse on word problems than comparable number only problems (Cummins, Kintsch, Reusser, & Weimer, 1988).

Word based problems place greater cognitive demands on students in relation to reading comprehension skills, unknown vocabulary, unfamiliar contexts, and common words used in unfamiliar ways than number-only problems (Benjamin, 2011; Chinn, 2004; De Corte & Verschaffel, 1991; Whelan Ariza, 2006). Students often attempt to solve word problems using only the numbers and an operation with little thought to the context or meaning of the problems (Kajimes, Vauras, & Kinnunen, 2010), an approach named “direct translation” (Hegarty et al., 1995, p.18). Direct translation can be successful for simple word problems. However, students using direct translation tend to struggle when confronted by complex problems and those with irregular wording.

Representations (e.g., graphs, diagrams, images) are useful tools for mathematical learning (Anthony & Walshaw, 2007). In their early years, New Zealand students are encouraged to use a variety of representations to learn and show mathematical concepts. As they move through schooling, there is increased emphasis on using number properties and using symbols, rules, and formulae. This changing emphasis can inhibit students from creating new knowledge or linking it to previously constructed knowledge and representations (Hiebert &Wearne, 1985).

Language-based difficulties that students face when solving mathematics word problems include: linguistic demands (Abedi & Lord, 2010; Jordan, 2007); wording, length, grammatical complexity (Fuchs & Fuchs, 2007); different usage of a word in a mathematics setting as opposed to everyday life (e.g., table, of, difference) (Sousa, 2011); having multiple words with the same meaning (e.g., add, join, plus) (Sousa, 2011); semantic structure and complex problem layout (Thevenot, 2010; Verschaffel & De Corte, 1993); and comprehension of the problem (Fuchs & Fuchs, 2007; Jordan, 2007).

To succeed, students need to be able to understand the context and nature of the word problem as well as be able to carry out the mathematics. English Language Learners (ELL), students with special educational needs, and students with low and average

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mathematics levels perform worse when language is involved in mathematics learning, but can benefit when the language demands are simplified (Abedi & Lord, 2010). Given the challenges students face with solving word problems and the prevalence of word problems in many assessments, helping students solve word based problems is a major challenge for teachers (Verschaffel & De Corte, 1993).

Helping Students to Solve Word Problems

Ways that word problems can be made more accessible for students include:

- simplifying the language demands (Abedi & Lord, 2010);
- having the problems written in the student's first language (Sousa, 2011);
- using specific computer programmes (Glenberg, Willford, Goldberg, & Zhu, 2012; Kajimes et al., 2010); and
- teaching representation and key word identification skills (Xin & Jitendra, 1999; Zhang & Xin, 2012).

Many of the studies above focused on changing the word problems or providing ways for teachers to help students to access word problems during learning. These modifications cannot happen during formal assessments, so it is also important that students have strategies for dealing with word problems as written. Using representations and visualisation of word problems are known to help readers connect text with their prior knowledge and experiences (De Koning & van der Schoot, 2013; Ministry of Education, 2006), and investigation into instruction in the use of drawing as a strategy has been called for (Van Meter & Garner, 2005). In this study, we sought to build on what is known about helping students solve word problems by investigating the extent to which learner-generated drawings and visualisation can help students to solve word problems.

Drawings can Assist Comprehension of Text

Drawings have been found to increase comprehension and learning in many contexts (e.g., Leopold & Leutner 2012; Schwamborn, Mayer, Hubertina, Leopold, & Leutner, 2010) and for specific groups (e.g., dyslexic readers, see Wang, Yang, Tasi, & Chan, 2013). Many believe that learners generating an illustration for themselves is more effective for comprehending text and solving word problems than viewing an illustration prepared by someone else (Dewolf, Van Dooren, Cimen, & Verschaffel, 2013). Students can be taught how drawings can assist them (Csikos, Szitanyi, & Kelemen, 2012), with schematic representations found to be more effective than pictorial representations (Edens & Potter, 2010). However, many students appear reluctant to use representations to solve mathematics problems unless specifically directed to, despite their teachers using them in their teaching (Uesaka, Manalo, & Ichikawa, 2007).

Working Memory and Cognitive Load

Working memory, important for performance in mathematical tasks, is "the retention of a small amount of information in a readily accessible form, which facilitates planning, comprehension, reasoning and problem solving" (Cowan, 2014, p. 217). Working memory can assist learning when not overloaded (Cowan, 2014; Farrington, 2011), but is affected by trying to hold too much information in mind, distractions, and doing more than one activity simultaneously (Alloway & Alloway, 2012). De Koning and van der Schoot (2013) suggest that using representations aids comprehension through lessening the demands on working memory, enabling a focus on information in the text.

Cognitive Load Theory (Sweller, 1994, 2016) is concerned with minimising the cognitive load placed upon students to allow learning to occur. Sweller identifies three types of cognitive load – intrinsic (related to the demands of learning the content), extraneous (the way in which new learning is presented), and germane (the way learners create schema and make new learning automatic and permanent). Sweller contends that when intrinsic and extraneous loads are too high, the working memory is overloaded and learning is compromised. Word-based mathematics problems place higher extraneous cognitive load demand on students than non-word problems, as language processing, reading comprehension, selecting a suitable operation, and then processing the numbers is more demanding that working solely with numbers.

In summary, text comprehension and cognitive load can cause difficulties with word problems for many students. Generating pictures helps with learning new information and increasing comprehension. Word problems are prevalent in mathematics assessment in New Zealand and elsewhere. The use of drawings has not been investigated in a New Zealand setting in relation to mathematics learning and assessment. Hence, this study investigated whether learner-generated drawings can be effective in helping New Zealand Year 4 and 5 students improve their achievement when solving mathematics word problems.

The Study

Within a social constructivist framing, a mixed-methods approach (Johnson & Christensen, 2008) including an experimental research design was used in this study as the research questions involved exploring links between an intervention and scores (causation), students' feelings and attitudes about mathematics word problems, and their thought processes when solving them. Convenience and purposeful methods of sampling (Creswell, 2012) were used resulting in five Year 4 and 5 student participants from an inner city, mid-socio-economic school, identified by their teachers as finding word problems challenging and likely to benefit from learning a new strategy. The intervention was designed to be practical for teachers to use in everyday mathematics lessons. Full ethical approval was gained for the study and full and informed consent gained from students and their parents.

Number only and word problem test questions included addition, subtraction, multiplication and division questions such as 6 + 7 = ___, and change unknown questions such as 7 + __= 12 (Carpenter, Fennema, Franke, Levi, & Elpson, 1999). Three 30-minute intervention sessions in which students were taught how to draw representations of word problems were held over one week. One pre-test was carried out before the interventions and three post tests were used, the last held three weeks after the final intervention session as a measure of retention. Tests comprised 10 word problems and 10 number problems with matching mathematical difficulty. Data collected from the tests included a raw score out of 10 for number-only and for word problems, the times taken to complete each set, and the number of times students selected the correct operation for a word problem but made a calculation error. Scores were compared by group and individuals. Each question type (addition, subtraction, multiplication, division, change unknown) was analysed to examine whether effectiveness varied by type.

In each intervention session, the researcher modelled generating a schematic drawing to represent the word problem by reading the question, explaining his visualisation of the problem using a think-aloud technique (Ericsson & Simon, 1980, 1993; McGuiness & Ross, 2003), then drawing a picture while talking through the thinking behind the drawing.

Students had opportunities to make their own schematic drawings, share solutions and thinking, with feedback from the researcher. The students then completed their own examples for three other word problems. Each student completed a drawing they felt would help them solve the problem and then attempted to solve the problem, writing down any working and answers. The students were not taught how to solve the problems. After each problem two or three students were asked to explain their drawing, every student explaining a drawing in every session at least once. The researcher asked questions for elaboration or to explore reasoning. At the end of each session, students completed a written reflection and several were interviewed. There were two think-aloud sessions, pre- and post-intervention, in which students were individually video-recorded solving word problems and explaining their thinking and any representations they had produced.

Results

At the start of the first intervention session, students were shown a word problem and a matching number only question. Every child identified the word problem as being the more difficult to solve with responses relating to knowing which operation or numbers to use, (e.g., “It doesn’t really tell you if its times or what”), the semantics of word problems (e.g., “When it’s less, I don’t know if it’s a takeaway or a plus”), and greater cognitive demands:

> It’s easier there (pointing to number only question) because you see what it is and just write it down but that (pointing to word problem) you have to read it, see what its telling you because it could be times, divided by…”

In the final video-recorded reflection session, eight of the nine students stated they felt they were better at solving word problems following the interventions (e.g., “I can finally understand them”) and enjoyed them more (e.g., “It seems a lot easier now. Even though it was really hard it was fun.”).

Students’ achievement with word problems was better after the drawing intervention than before, with test data showing significant improvement for the number of number problems solved correctly and the number of word problems solved correctly, and a trend for the number of operations chosen correctly (Table 1).

Table 1

Paired t-test Comparison between Test 4 and Test 1

	Number	Mean Difference (Test 4 & Test 1)	Std. Dev.	t	P
Number problems	5	2.6	0.894	6.5	0.003
Word problems	5	4.4	0.812	5.416	0.006
Correct operations	5	2.6	0.98	2.654	0.057

Improvements were also found in students’ achievement with number-only problems and the number of operations chosen correctly, whether or not the word problem could be solved post-intervention. A paired t-test was used to compare the number of correct number problems with the number of correct word problems to see of the gap between the two was reduced by the intervention, with a statistical trend towards improvement found ($t(4) = 2.250$, $p = 0.088$). Including numbers in the drawing appeared to be important in helping students identify suitable equations for the word problems.

There was no statistical difference found between time the students took to solve the problems before and after the intervention, showing that while the students used extra steps in their problem-solving process, they spent no longer solving the problem overall. This is an important result given the common use of timed tests and concerns regarding the "crowded" curriculum.

Using Representations

Comparisons of the think-aloud data from before and after the interventions suggest that asking students to use internal or external representations can help them to solve mathematics word problems. Before the interventions, only one of the five students made any reference to the setting of the word problem when explaining their thinking with the others identifying numbers and making equations without any reference to the setting. After the intervention four students described the setting of the problem.

Drawings used by students were categorised into pictorial, schematic, notational or a combination of these. There was a shift in the content and style of the drawings produced during the think-alouds before and after the intervention sessions. Purely pictorial drawings used by one student in the initial think-aloud, were not present in the final think-alouds and only three were present in the post-tests. In post-intervention testing, every student produced schematic representations with most including the numbers from the problem (Table 2). Rather than producing a purely literal representation of the setting or just selecting the numbers described in the word problem the schematic diagrams showed that students have been able to integrate the context and mathematics needed to solve the problem. However, even detailed verbal descriptions or accurately drawn pictures of the problem did not help students find solutions if the equation of the problem could not be found.

Table 2
Representations Produced by Students, Test 4 and Test 1

	Notational				Problems where an equation or working out was present	Pictorial	Schematic	Number of problems solved correctly
	Answer only	Equation and answer	Pictorial/Notational	Schematic/Notational				
Test 1	32 (12)	13 (3)	0	4 (2)	17 (6)	0	1(1)	18
Test 4	3 (2)	13 (10)	14(11)	16 (14)	40 (33)	3 (3)	0	40

Note: Numbers in brackets indicate successful solutions to word problems.

The ability to accurately include or extract the mathematical information and operation in the problems was important for solving the problem, regardless of the neatness, format, or realism of the drawing (e.g., Figure 1). Examples that did not include the relevant information described in the problems were often not solved successfully.

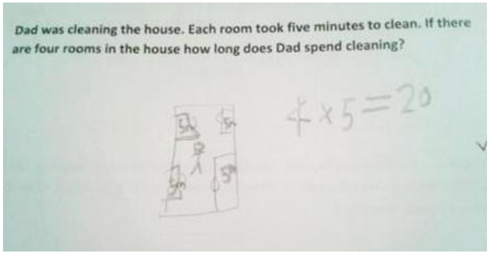

Figure 1. Representation that led to an equation and successful solution.

Students' written reflections, collected at the end of each intervention session, showed students felt drawings could help them when they perceived word problems to be hard (e.g., "I prefer not to [draw a picture] but if it's hard it's sort of easier to draw a picture."), creating a drawing helped them to understand the problem ("[drawing] sort of helps me a lot 'cause then you get a better picture in your head about what to do…because it helps you figure out the maths and solve the problem as well.") and drawings helped then to be accurate with their working.

Consistent with Raghubar, Barnes, and Hecht's (2010) review that pen and paper can lessen the load on working memory, student comments suggested that they felt their drawings enabled them to move some information from their head to paper and using drawings lessened the demand on their brain:

> You can actually see it there (points to the desk indicating on paper) and you don't have to, um… because you might lose count in your head, or otherwise when its right in front of you down there, you can actually see what's going on.
>
> It [solving word problems] pushes your brain to the limit…because really if there's no picture you kind of have to visualise it really hard, which pushes your brain to the limit, which basically means you won't be able to think straight, so when it right in the middle you won't have to think as much.

The student responses support Sweller's (1994, 2016) view that by reducing the cognitive load through the use of effective strategies students can be more accurate and achieve more learning.

Discussion and Conclusion

Study limitations include that think-alouds may have been new to students which may have affected data differently across the study as they became more familiar with the method. In addition, the study was carried out with a small number of students and further development of the study with larger groups would be useful.

Encouraging students to see the word problem as a real event, through visualising, may lead to the problems making more sense to students. The drawing strategy worked across all types of word problems with students' accuracy increasing for all types of wordings and operations. This is a powerful result in contrast to strategies which involve knowing an appropriate format for different types of question, as it involves students knowing only one strategy (making a schematic representation) and being able to apply it to all types of word problems. Having just one strategy lessens the demands on student thinking as they know they have a strategy which can help with word problems without spending thought or time deciding which strategy or format is needed for a new problem.

As the group was removed from their classroom for the intervention sessions, it is unclear whether the approaches are manageable and effective as a classroom teaching strategy, warranting further exploration. Further investigation is also needed to explore the use of visualisation and drawing for word problems in areas of mathematics other than numeracy and for other levels of the school.

The results from this study provide evidence of the power of reading strategies commonly used in many schools to assist students with solving mathematics word problems. Given the reading requirements involved in mathematics learning and assessment, it is important for pedagogy to include reading comprehension strategies. This study has shown that teaching visualisation and how to make schematic learner-generated drawings can enhance achievement in number-based problems. Further implications include ensuring visualisation and drawing are included as a word problem solving strategies promoted in teacher and student resource materials and in teacher education.

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Examining Non-Traditional Pathway Preservice Teachers' Attitudes Towards Mathematics

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In this study, we examined non-traditional pathway preservice teachers' attitudes towards mathematics to inform decision-making on designing the course and the units related to mathematics and how to support the teachers in transitioning smoothly into higher education programs. We adopted a mixed design approach to document teachers' attitudes towards mathematics. Results show that the teachers revealed positive attitudes towards their mathematical self-ability of the subject, the enjoyment of mathematics, and the utility of the subject.

As a way of increasing the quality of teacher education courses, the Australian Institute for Teaching and School Leadership (AITSL, 2014) has recently called for only the top 30% of school applicants to be admitted. For students who do not meet the Australian Tertiary Admission Rank (ATAR) score requirements, their chances of entering the teaching profession are consequently limited. Nevertheless, Australian researchers/educators have argued that the success of non-traditional students at university is related as much to how the institution engages, supports and challenges the students in constructive relationships as to student pre-entry scores (Funston, Gil, & Gilmore, 2014). To address this, Victoria University offers a one-year intensive program of supported study developed in a unique collaboration between Arts and Education in 2011. The Diploma in Education Studies (EDES) was designed for students who do not meet the entry requirements for selection for the Bachelor of Education (BEd P-12) through the Victorian Tertiary Admission Centre (VTAC) selection process, and provided a pathway that offered extensive and supported study to build capacity in literacy and numeracy appropriate for a career in teaching. These students are able to transition into the second year of the BEd program via an internal course transfer process conditioned on completing requirements of the EDES. In order to prepare students for successful transfer into the second year of the BEd, the EDES curriculum included a significant focus on increasing the literacy and numeracy of students, with a quarter of the course credit points (24 out of a total of 96) devoted to developing these skills. The length of individual units is also unusually long (16 weeks as opposed to 12), with an explicit focus on learning support and retention.

Attitudes towards mathematics appear to be an important variable in teaching and learning, and in predicting students' mathematical performance (e.g., Sakiz, Pape, & Hoy, 2012). Positive attitudes towards mathematics help reduce anxiety (Akin & Kurbanoglu, 2011), and students with more positive attitudes towards mathematics have higher perceptions of the utility of mathematics that motivate them intrinsically towards their study (Perry, 2011). The students have a better mathematical self-concept (Hidalgo, Maroto, & Palacios, 2005), and are more confident that they can learn mathematics (McLeod, 1992). As these students, hereafter called non-traditional pathway preservice teachers, are perceived to accrue low academic ability (low ATAR), gauging their affective factors is critical to help address the challenges the students might have, and determine

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how to move the students towards the academic standards of higher education. In the current study, we address the research question: What are the attitudes of non-traditional pathway preservice teachers towards mathematics in the first year of study?

Literature

Increasing attention is being paid to the role of affective factors in mathematics education as “students who have positive emotions towards mathematics are in a position to learn mathematics better than students who feel anxiety towards that subject.” (Organisation for Economic Co-Operation and Development, 2013, p. 42). In a recent review of research into the affective domain in mathematics education, Attard, Ingram, Forgasz, Leder, and Grootenboer (2016) summarise six prominent constructs, including identify, beliefs, attitudes, engagement, motivation, and anxiety. McLeod (1992) argued that beliefs, attitudes, emotions (anxiety could be grouped here), differ in stability and intensity. He stated that, “beliefs are largely cognitive in nature, and are developed over a relatively long period of time. Emotions, on the other hand, may involve little cognitive appraisal and may appear or disappear rather quickly” (p. 579). Importantly for this study, significant attention from the research community is recently focusing on attitudes (Palacios et al., 2014).

In regard to research about attitudes, consistent findings show that negative attitudes towards mathematics persist across primary and secondary students (Larkin & Jorgensen, 2014), and preservice teachers (e.g., Afamasaga-Fuata’i & Sooaemalelagi, 2014). Winheller, Hattie, and Brown (2013) found that for secondary students, self-efficacy in mathematics was primarily related to their outcomes, and confidence in and liking mathematics influence their perception of quality of learning. However, research shows that their attitudes towards mathematics could be changed. For example, primary pre-service teachers’ positive mathematical attitudes increase during their initial teacher education program (Bailey, 2014) through the use of open-ended investigations in their program. The scarcity of research related to non-traditional pathways pre-service teachers’ attitudes towards mathematics necessitate the current study. It is essential to know where these teachers come from and what beliefs they bring with them into the course that might help or hinder their learning. Affective factors could inform educators in designing learning opportunities to meet their need as well as to help them transition smoothly into higher education programs.

Methodology

We adopted a mixed design approach to document teachers’ attitudes towards mathematics. Data include digital reflective journals collected in the first semester and a survey about attitudes towards mathematics administered in the second semester.

Participants

This paper focused on the 2016 cohort of 163 full-time students, of whom two are international students. The students ranged in age from 18 years to 32 years. Most were young adults (median age = 18), with nearly 67% of the cohort was 19 years old or younger, and 25% of the cohort aged between 20 and24. Most were female (65.6%). All did not have an ATAR score. For 77.6% of the cohort, English was the only language spoken at home. The demographic information indicates that the course was meeting the institutional goals of widening participation and enabling new cohorts of students to enter this course.

Data Collection

Data were collected in two units related to mathematics (numeracy), EDC1002, Semester 1 2016 and EDC1003 Semester 2 2016 when students were in their first year of study. In the first semester, as part of reflective journals, students were asked to write a response to the following prompt: "Describe your self reflection as a mathematics learner and teacher. What are your goals for this unit?" Data from 70 students' responses were used for data analysis. In the second semester, towards the end of the Unit EDC1003, students were asked to fill out a paper-and-pencil survey including 32 items to measure attitudes towards mathematics (Palacios, Arias, & Arias, 2014). This scale categorises attitudes into four factors: *perception about mathematical incompetence* (12 items), *enjoyment of mathematics* (12 items), *perception about utility* (4 items), and *mathematical self-concept* (four items). This is a five-point Likert scale ranging from strongly-disagree to strongly-agree. 58 students filled out the survey and 12 of those did not leave their identity in their response. This scale, Attitudes towards mathematics, was validated and checked for reliability using a sample of 4,807 students at the school levels building on previous efforts to measure attitudes (e.g., Aiken, 1979; Fennema & Sherman, 1976; Tapia & Marsh, 2004). Across the two data points collected, only 26 students finished both. As the data were collected from a voluntary sample, caution is also needed when making generalisations from the data to the whole cohort.

Data Analysis

The digital reflective journals were coded through two phases: open coding and a priori coding. For the first phase, an emergent coding was applied to describe what the students alluded to in their reflection, such as their perception of self-ability in mathematics, enjoyment/dis-enjoyment of mathematics, and their attitudes towards improving their mathematical knowledge. After this, the second phase built on the emergent coding, using an a priori coding scheme related to attitudes towards mathematics adopted from Palacios et al. (2014). The scheme includes four factors: perception of mathematical incompetence, enjoyment of mathematics, perception of utility, and mathematical self-concept. Students' responses were coded for the four factors if evident. When it was evident, the direction of the factors was also noted as positive or negative. For example, in responding to the prompt, Malinda indicated: "My reflection from my past high school and college mathematics experience has given me some passion and opportunity to enjoy and achieve well with results. I'm very interested and willing to learn more about primary mathematics." This was coded as *enjoyment of mathematics* (positive) and *mathematical self-conception* (positive). It was also coded for *looking forward* in addition to the four factors specified. Then, counting and percentages were processed to quantify reflection of the group. For the survey, students' responses were entered into a spreadsheet as a numerical scale from 0-4. Questions with negative wording were recorded to reflect the scale. Then the items in the survey were regrouped based on the four factors and summary statistics (mean and standard deviation) of each of the factors were developed. Percentages were also reported for each of the items.

Results

Reflective Journals

Among 70 responses, 35 commented on their perception about their mathematical ability. Of the 35 responses, 23 teachers perceived that they felt incompetence about mathematics whereas 12 teachers perceived that they were able to learn mathematics. These 12 teachers saw themselves as mathematically capable and felt self-confident about the subject and that they could do well if they were trying hard enough. For examples, Kat responded “I have always struggled with mathematics, believing it to be a rigid and formulaic discipline that my artistic brain is not suited for. This is untrue though, as I have the potential to study maths further and to explore how I can use my other learning strengths to overcome the challenge.” This teacher perceived herself to have an artistic brain, which was not appropriate for learning mathematics. Teachers with responses of incompetence in mathematics described it as either stressful, difficult, a weak subject, fixed mindset (giving up easily when frustrated), insecure, or having gaps in knowledge. Interestingly, one student saw herself better in mathematics than in English.

Only seven teachers mentioned about enjoyment/dis-enjoyment of the subject. Three teachers said they enjoyed mathematics, such as Tom, “Personally in early years of high school, I loved maths and I loved getting tests back and seeing what you answered right and wrong but as I got older I got lazy when I probably needed to work my hardest and my results reflected that.” One teacher, Sam said she did not like it, “I had ‘fixed’ (Dweck, 2014) ideas about mathematics some of which being; thinking I didn’t like anything about math." Among the teachers, three commented on the love-hate relationship, depending on their performance on and understanding of the subject, Mel said: “I as a mathematics learner have a love hate relationship with maths as I hate it when I do not understand it and absolutely love it when I understand it.”

Only one teacher commented on the utility of mathematics, and the comment was positive:

> In mathematics, there are many challenges one faces as a student, but without these challenges in the subject, would the world be where it is today? With all the addition, subtraction, multiplication, division and even the long equations that may take hours to complete are all worth it.

The vast majority of the teachers (65) aimed to improve their knowledge during the unit; 55 of those aimed to improve their mathematics content knowledge, and 24 focused on developing their knowledge for teaching mathematics including how to teach, teach in a fun way, and understand their students. For example, Tayla responded:

> A personal goal for me in this unit is to get a better understanding of maths in a different perspective, throughout our lives we have been told to do the work but not actually teach it. So, I would like to learn how to actually teach maths than actually doing it. I would also just like to make sure I understand all the content that maths has to offer.

Thirty-two teachers explicitly set the goals to improve their attitudes towards mathematics, to develop grow mindset, to perceive them as capable if trying hard, and to put more efforts into improving their knowledge. For example, Sarah after realising her own mindset stated that: “I’m now going to make a conscious effort to turn my fixed mindset into a growth mindset by learning to appreciate challenge and grow from my mistakes.”

Survey of Attitudes Towards Mathematics

The teachers tended to disagree or feel neutral about perception of mathematical incompetence, and feel neutral or agree that they had a positive mathematical self-concept. Likewise, these teachers tended to feel neutral or agree that mathematics was enjoyable, and especially agree or strongly agree that mathematics was useful (Table 1). These results are elaborated more next.

Table 1
A Summary of Mean and Standard Deviation Obtained for Each Factor

Factors	*N*	Mean	*SD*
Perception of Mathematical Incompetence	58	1.61	1.26
Enjoyment of Mathematics	58	2.21	1.26
Perception of Utility	58	3.03	1.00
Mathematical self-concept	58	2.61	0.98

More than 50% of the teachers disagreed with the notion of their mathematical incompetence as stated in six of the 12 items related to perception of mathematical incompetence (see Figure 1). For example, in response to Item 11 - "Except for a few cases, no matter how much effort I put, I cannot understand mathematics", 34% of the students strongly disagreed and 29% disagreed.

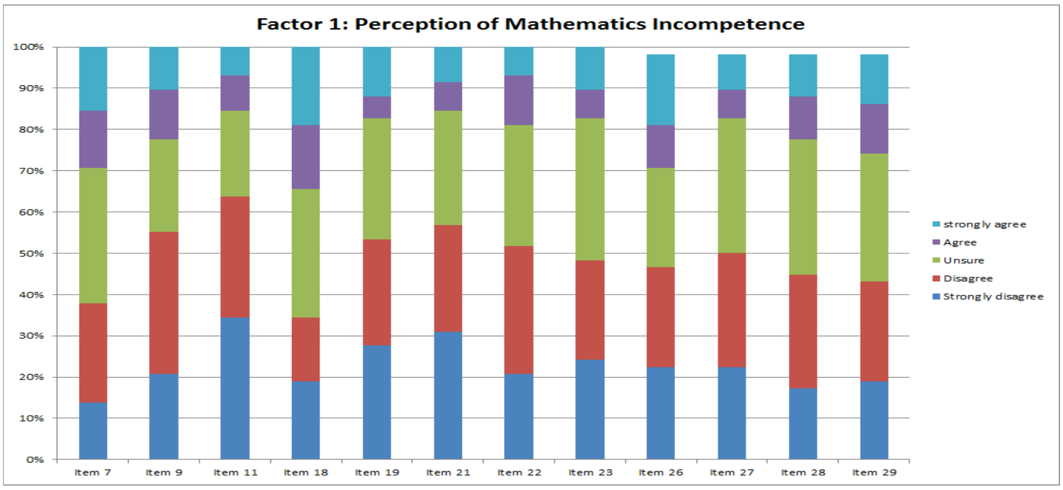

Figure 1. Teachers' responses about perception of mathematics incompetence.

Note that for Items 26 to 29, one student did not put any response. Four items (8, 14, 15, and 25) in this factor included negative statements, such as Item 8: "It's time for maths, how awful". Respectively, 53%, 60%, 50%, and 52% respondents disagreed or strongly disagreed with these negative statements. However, for positive statements such as in response to Item 1: "I like mathematics" and Item 2: " I feel comfortable doing maths problems", 58% and 64% respondents recorded a strongly agree or agree response. Overall, more than 50% of the teachers agreed that mathematics was enjoyable in 6 of the 12 items.

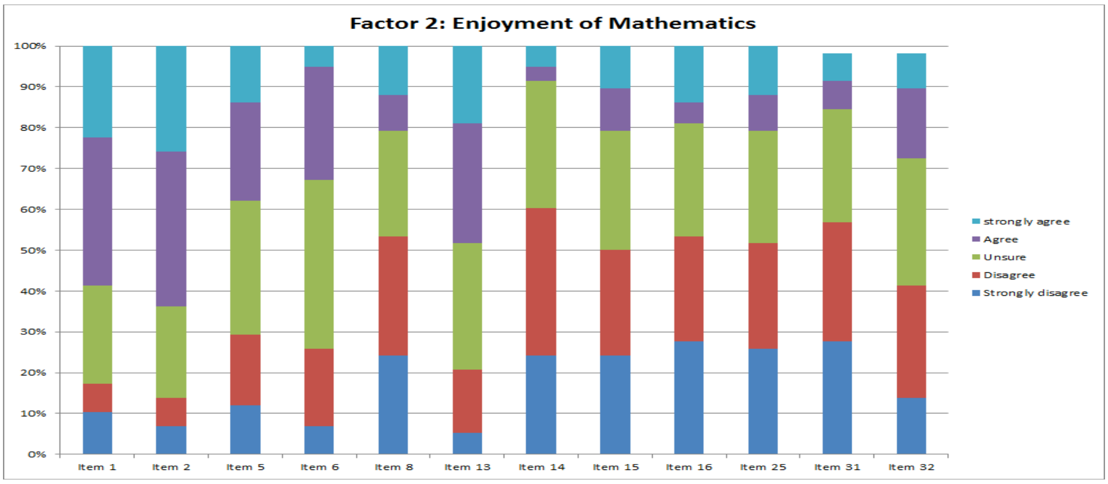

Figure 2. Teachers' responses about enjoyment of mathematics.

Note that for Items 31 and 32 one student did not put any response. Two items (3 and 4) in this factor also included negative statements. For example, Item 3: "Mathematics is useless" and Item 4: "mathematics should be present only in science careers", respectively 83% and 81% students disagreed or strongly disagreed with the statements. Similarly, for Item 17: "Learning maths is a matter for only few" scored a 53% strongly disagree or disagree response. (Figure 3, left graph)

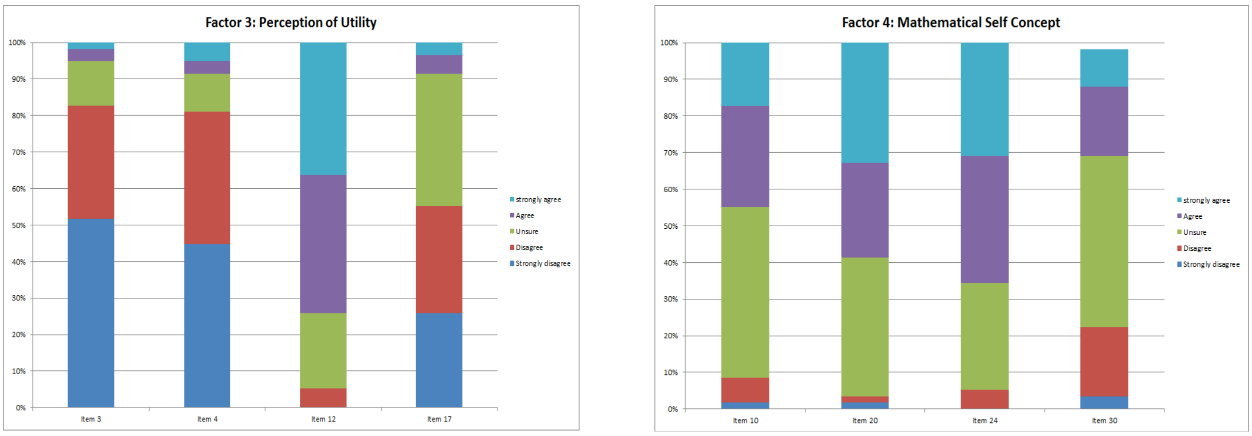

Figure 3. Teachers' responses about perception of utility and self-concept.

For Item 20 and Item 24 students showed 59% and 65% agree or strongly agree response. These items included statements, “For my math teachers and lecturers I am a good student” and “I can become a good student of mathematics” respectively. (Figure 3, right)

Discussion

This study examined non-traditional pathway preservice teachers’ attitudes towards mathematics to inform decision-making on designing the course and the units related to mathematics and how to support the teachers in transitioning smoothly into higher education programs. In the first semester, only 23 of 70 teachers expressed they felt incompetence in mathematics whereas 12 students self-perceived their capability in doing mathematics. The vast majority of students were looking forward to improving their knowledge and attitudes towards the subject, thinking they could do mathematics if they try hard. Likewise, in the second semester, there were positive attitudes towards the subject (mean for perception of mathematical incompetence was less than 1.7 and mean for mathematical self-concept was more than 2.6). The results do not corroborate previous finding that pre-service teachers tend to show negative attitudes towards mathematics (e.g., Afamasaga-Fuata’i & Sooaemalelagi, 2014). This could be for several reasons. First, the reflection journal was finished after Week 3 of the semester when students had been challenged about their own attitudes towards mathematics and encouraged to develop a growth mindset by watching a video about growth-fixed mindset. It seemed that this brief intervention had a positive impact in shifting the teachers’ thinking. Furthermore, the supporting structures put into the units, such as regular and frequent feedback to students, and their ability to do well in their first semester maths unit, might help explain the result. Likewise, the scale was administered towards the end of the second unit after they had experienced support in developing growth mindset during the first year of study.

Overall, the experience in the first year was helpful in changing teachers’ perception about the subject, which apparently encouraged them to keep challenging their self-beliefs and attitudes and improve their academic standards. However, how such experience in their first year could be followed in other years of the teacher education program still remains unclear. Furthermore, in what ways these experiences serve as a trigger for the teachers to develop their academic knowledge, and to reach the top 30% as required to registered as a teacher, is yet to be investigated. Future study could further track the students and use appropriate measure to link between cognitive and affective domains for the students. It is challenging to help teachers create a balance between perceiving the good in their ability, and continuing to improve their academic standards. Preliminary findings suggest that attending to helping teachers’ change of their attitudes seems to be a first step.

The philosophy of this sub-degree is reflected in the overall university commitment to enable entry to university degrees via alternative entry courses, increase the diversity in teacher education, and widen participation of non-traditional groups in higher education. In general, the course appeals to a recent call by Sahlberg to rethink government approaches to selecting teacher educators on the basis of elite performance at school but rather to select wider socio-economic and diverse cohorts who represent school populations (Shaw, 2015). The staff in the course sought to “find ways to enable non-traditional students to draw on the rich cultural resources, alternative knowledge and ways of knowing that brought them to the course” (Hallpike, 2014, p. 107). Non-pathway students would not traditionally be offered the opportunity to enter initial teacher education programs, and it is argued they need significant support to become qualified teachers.

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Indigenous Teacher Education: When Cultural Enquiry Meets Statistical Enquiry

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For Indigenous students in minority education contexts, it is important that teachers have strategies to combine both cultural knowledge and mathematical knowledge in appropriate ways. This paper presents the results from analysing preservice teachers´ statistical enquiry assignment linked to a cultural context, in a Māori-medium teacher education programme. The results indicate that there are many tensions in trying to honour both the learning of cultural and statistical understandings. The findings provide insights to teacher educators about what may be needed to reduce some of these tensions and the implications for teachers working in Māori-medium schools.

How teacher education prepares Indigenous teachers to teach in Indigenous schools is an under-researched area in mathematics education research. Consequently, in this paper, we investigate how Indigenous preservice students in a Māori-medium teacher education programme responded to an assignment, in which they were asked to combine cultural understandings with statistical enquiry. In setting up the project, it was anticipated that cultural understandings could be gained with a range of tribal and national literature and from elders' oral sources. The statistical enquiry used the statistical enquiry model.

Initial teacher institutes (ITE) have existed since 1862 in Aotearoa/NZ (Openshaw & Ball, 2006), but were English-medium only for over 110 years, reflecting assimilationist policies of the European colonisers. The impact of research in the 1970s (i.e., Benton, 1979) that showed te reo Māori, the Indigenous language of the Māori people, was in a precarious state and the subsequent demands by communities and activists to revitalize the language, saw a rapid growth in students learning the language (Walker, 1984). However, it was not until 1974 that there was a response to the lack of te reo Māori teachers by providers of ITE through programmes, such as Te Atakura, which fast-tracked native speakers into a teaching qualification (Shaw, 2006). However, these programmes focused on meeting the demand for language teachers of te reo Māori for secondary schools and not the chronic shortage of teachers of Māori-medium schooling, apparent in the 1980s.

By the early 1990s, various ITEs, under pressure from the schooling sector, responded by developing bilingual-type programmes. While based on good intentions, in these programmes Māori culture was acknowledged and given some emphasis, but the programs did not develop the te reo Māori proficiency of the teachers needed to teach in Māori-medium schools (Stewart, Trinick, & Dale, 2016). However, by 2008, 12 programs defined themselves as Māori-medium (Murphy et al., 2008), including the programme in this study. The programme is a three-year Bachelor of Education (Teaching) programme, located in the Faculty of Education at the University of Auckland. It is unique in Aotearoa/NZ, as it is the only degree in the University delivered in the medium of te reo Māori. The programme aims to produce teachers who can actively engage in te Ao Māori (the Māori world) in terms of language, knowledge, commitment, pedagogy, understanding of tikanga (cultural customs), and how tikanga plays out in a range of contexts. The programme is comprised of courses in each curriculum area such as mathematics

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(pāngarau) and professional knowledge It also includes a school placement each year in Māori-medium kura (schools).

From its inception, the programme has sought to address the complex challenges of language revitalization (Dale, McCaffery, & McMurchy-Pilkington, 1997) and the need to develop new linguistic resources for discussing and disseminating conceptual material at high levels of abstraction (Trinick, 2015). Where the programme has not done so well is the revitalisation of cultural knowledge. This programme, like many others in ITE, have focused on revitalising and elaborating the language to enable subjects, such as mathematics, to be taught at the tertiary level (Trinick, Meaney, & Fairhall, 2014). This is also reflected in the schooling sector where graduates of this programme and others have struggled to revive mathematical ideas or to locate mathematics in contexts that can be considered authentically Māori (see Trinick et al., 2016).

In order for Māori-medium learners to achieve success and Māori-medium schooling to survive and flourish as an indispensable component of te reo Māori revitalization, a continued supply of teachers with the necessary competencies, skills and disposition is required (Hohepa, Hawera, Tamatea, & Heaton 2014). While there is ongoing debate about what this range of skills and dispositions ought to be, we argue that teachers require two skills—the ability to discuss and disseminate conceptual material at high levels of abstraction in te reo Māori and an understanding of relevant mātauranga Māori (Māori knowledge). As Murphy, McKinley, and Bright (2008) note, Māori teachers who struggle to put “language into a cultural context for students” will contribute to “students learning language without any of the richness and experience associated with culture” (p. 36). Although we highlight only two dimensions of being an effective teacher, this definition has significant implications for the teaching and learning of school subjects and for Māori language and knowledge revitalization. In this paper, we describe the results of a project with preservice teachers that set out to merge Western mathematical ideas, from the statistics investigation cycle, with questions about the early Māori migratory voyages.

Early Canoe Migrations

Almost all the students in this BEd (Teach) programme are descendants of families who were involved in the rapid and extensive urban migration after the Second World War, where Māori families shifted from socially isolated Māori-speaking communities into English-language-dominated urban areas, and into English-language-only schooling systems and workplaces (Spolsky, 2005). Consequently, many of the younger generation, including students in this programme, struggle to reconnect to this knowledge. Early canoe migrations could provide a useful context for developing relevant cultural knowledge.

The various canoe migrations are important to the identity of Māori in Aotearoa/NZ both in historical and contemporary times (Orbell, 1975). The canoe (waka in Māori) traditions describe the arrival of Māori ancestors from a place, most often called Hawaiki. The exact location of this Hawaiki has been lost in the mist of time and has been the cause of much speculation. With the advent of technology such as DNA mapping, it is now well established that most of the Māori migratory canoes came from different points in East Polynesia, more specifically, Raiatea, Tahaa, Porapora, Tahiti and some of the islands of the Cook Group (Underhill, et al. 2001) (see Figure 1). The migration stories refer to the construction of canoes, conflicts before departure, voyaging at sea, landing, inland and coastal exploration, and the establishment of settlements in new regions (Orbell, 1975). Genealogical links (whakapapa) back to the crew of founding canoes have established the origins of tribes and define relationships with other tribes. For example, several tribes trace their origin to the Tainui canoe, while others, such as Te Arawa, take their name from their ancestral canoe. When identifying themselves on a marae (meeting house) outside their tribal area, people refer first and foremost to their waka. Alongside tribal dialects, a person's waka is a significant identity marker for Māori. Canoe traditions explain origins and also express authority and identity, and define tribal boundaries and relationships. They "merge poetry and politics, history and myth, fact and legend" (Taonui, 2006).

Figure 1. Canoe journey in the Pacific (from Royal, n.d.).

While the precise date of the canoe migrations is a matter of debate, there is a large amount of material, both traditional (oral stories and artefacts) and contemporary (modern voyages using traditional methods, DNA testing) that provide information about these earlier voyages. However, as can be expected considering the antiquity, there are many unanswered questions surrounding aspects of the voyages, for example, how long did it take the waka to get to Aotearoa? Questions such as these have implications for the validity and reliability of traditional wayfinding knowledge. This is one of the questions, the preservice teachers in this research were asked to evaluate, drawing on the various data sources that were available to them using the statistics enquiry cycle.

The Statistics Enquiry Cycle

The separate and distinct nature of statistics is well-established in New Zealand and recognised as having different ways of thinking and solving problems from mathematics. Moore (1998) argued that, "Statistical thinking is a general, fundamental, and independent mode of reasoning about data, variation, and chance" (p. 1254). With technology to do the "number crunching", it is more valuable to focus on the statistical process.

With the publication of the English and Māori-medium versions of the New Zealand curriculum in 2007 and 2008 respectively (Ministry of Education, 2006; Te Tāhuhu o te Mātauranga, 2008), the subject "mathematics" became "mathematics and statistics", with the difference being connected to two related, but different, ways of thinking.

> Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data. These two disciplines are

> related, but they use different ways of thinking and solving problems. Both equip students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live. (Ministry of Education, 2006, p. 26)

Statistical investigations involve information gathering to seek meaning from and to learn more about phenomena as well as to inform decisions and actions. Wild and Pfannkuch (1999) argue that the contextual nature of the statistics problem is an essential element and how models are linked to this context is where statistical thinking occurs. Thus, one of the goals of the preservice teachers' task was to learn more about a real-world situation, canoe migrations. While based in antiquity, canoe migrations and the resurrection of wayfinding skills is a contemporary topic of interest (Trinick et al., 2016).

In the Aotearoa/NZ schooling system and curriculum, including Māori-medium, it is recommended that statistical investigations are conducted using an enquiry cycle. The cycle defines the way one acts and what one thinks about during the course of a statistical investigation (Wild & Pfannkuch, 1999). The cycle as proposed by Wild and Pfannkuch (1999) consists of five stages: Problem, Plan, Data, Analysis, and Conclusion.

- The **problem** section is about understanding and defining a problem and formulating a statistical question.
- The **planning stage** is about how the data will be gathered and measured, and the design of the study. This also includes the sources of data: a primary source, i.e., data collected by students, or secondary sources, i.e., data already collected by someone else.
- The **data** stage is about how the data is managed and organised and cleaned and
- The **analysis** stage is about sorting the data, constructing appropriate data displays and numerical summaries, looking for patterns and reasoning with the data.
- The final stage of the cycle involves interpreting, generating **conclusions**, new ideas and communicating findings.

One of the advantages of a cyclic approach according to Wild and Pfannkuch (1999) is that this model provides more structure in the learning process, particularly for those who are generalist teachers, as opposed to specialist statistics teachers.

Data Collection and Analysis

Data were collected from 18 preservice teachers in the final (3rd) year of a Māori-medium initial teacher education programme. The preservice teachers were required to complete two assignments, one was a statistical investigation on their ancestral waka, the other was to develop a unit of work showing how they might teach the statistics enquiry cycle to students. The data that were analysed were the written assignments completed for the statistical investigation. Permission was asked of the students to use the assignments for the data analysis, in the semester following their completion of the course.

The statistical investigation included the following guiding questions:

- Name your ancestral waka, and where do you think it came from and why?
- How many people were on board?
- What do you think was the gender, age group mix?
- What sort of foods did they bring?
- How long did they take to get to Aotearoa?
- If all of the above is not available you will need to predict the answers using the variables (information) that is known.

Wild and Pfannkuch (1999) argue that thinking about the variables involved in a statistical investigation is an important consideration particularly when linking the context to the statistics model used. However, it is known that inferential thinking is difficult for preservice teachers to understand (Leavy, 2010). Statistical thinking also occurs when statistical processes within the cycle are questioned; for example, what type of data is needed to answer the questions posed? Which graph is best for this type of data? What are the statistics, graph or table inferring about the waka data? What are the statistics, graph or table inferring about the waka population?

Findings and Discussion

When the statistical assignments were original graded, it was noted that the preservice teachers faced issues that had been unexpected when the project had been set up. Thus, it was decided to investigate these issues further. To do this, the issues were faced at different points in the statistical enquiry model were identified and described below.

Understanding the Problem

The first challenge for the students was coming to terms with investigating a topic which was traditionally considered a "taonga tuku iho"—a treasure handed down from the ancestors, thus not generally considered open to question and enquiry. This is an issue which has sometimes impinged upon the revitalisation of mātauranga Māori (Māori knowledge) in schooling and this issue was connected to choices about relevant data that the students drew on to answer the questions.

The Planning Stage

The choice of where to gain data from seemed to be affected by how much the preservice teachers felt that the knowledge was open to questioning. As a consequence, some students drew on only one source of data, such as an elderly relative, while others drew on a range of both written and oral sources. The source and authenticity of the data created a real tension throughout the process. As noted, the migration stories contain both fact (they did occur) and legend (the prowess of the captain of the canoe) (Orbell, 1975).

The Data Stage

It was difficult for the preservice teachers to completely discount evidence from elders when managing and organising the data. However, one of the unintended consequences of the students' investigations was that they also used the investigation to survey the knowledge of their own extended families including their relatives by marriage from other tribal areas. A number sent out an electronic survey via Facebook and/or used Survey Monkey. The answers tended to vary wildly from relative to relative. However, some students did note it was because mātauranga Māori (Māori knowledge) is not standardised and it was important is to acknowledge the difference. For example, one student noted:

> Each tribe or hapu have their own pūrakau (stories), hitori (history) and kōrero (talk) pertaining to Mahuhu-ki-te-rangi. Who am I to question it. I can't tell the people of Te Roroa that Rongomai is the captain because he is most likely according to the data, therefore their history is koretake (useless).

Thus, the assignment also turned into an assessment of how much their relatives knew. It may be that school statistical projects which often survey family members (see for example, Makar & Rubin, 2009) influenced the preservice teachers' perceptions that such a survey was useful for gaining relevant knowledge. Nevertheless, several were able to navigate between a range evidence by deferring to a reasonably logical option:

> According to the data I collected, the journey ranged between 5 days to 76 days, so I chose somewhere in the middle.

The Analysis Stage

In the analysis stage, the preservice teachers responded to the questions, some of which had a large degree of uncertainty, for example, the number of people on board the canoe. While there are formal statistical methods to reduce uncertainty, one of the goals of this assessment was to encourage students to draw on mathematical ideas, such as measurement estimation to support the interpretation of the varied data.

While there were one to two outliers, where South America was identified as a possible source of origin for the waka, most preservice teachers identified East Polynesia as the place of origin of their ancestral canoe. However, the exact island in East Polynesia differed even when the preservice teachers identified with the same canoe.

> Through extensive research the closest I have come to identifying the island of Hawaiki which our ancestors claim to have come from is the modern-day island of Tahiti…. Looking back on our history, these are also similar traits to Māori people. Tupaia was on board Captain Cook's ship when it arrived in Aotearoa and even though he was from a completely different island far away, Captain Cook´s records showed that he [Tupaia] had great success communicating with Māori.

The next question, which asked, "How many people on board?" required a synthesis of the various variables. To calculate the estimated length of the journey requires rational reckoning and logic to determine position, distance, size of canoe and so on. While this can never be answered exactly it can be estimated by the time it had taken modern replicas of traditional Māori sailing canoes (Taonui, 2005), which have taken between 24 and 18 days to sail to and from Rarotonga (Taonui, 2005) Most of the students used this information, particularly the accounts of the voyage Te Aurere throughout the 1990s up to recent times

> I estimated the time for the voyage of my ancestral waka at 36.7 days from Rai'atea to Aotearoa. This was based on contemporary waka voyagers by Hekenukumai Busby which took him 30 days to travel 3233.73 km from Aotearoa to Rarotonga, traveling around 146.98 km per day

These results differed to the information gained from whānau (family) informants who predicated the voyages as being very quick (seven days) or very long (four months). One preservice teacher working it out using distance and time calculations, stated:

> In relation to the question on the estimated number of people on board and the gender mix, estimates varied from a low of 15 to a high of 250. There was some doubts expressed about this later number, I decided there was between 20 to 30 because it was the most common estimate. This was based on the length of the voyage, the amount of food needed to be carried and so on.

Conclusion Stage

As noted in the previous section, conclusions were drawn from the analysis stage, but the reflections needed to query the responses' appropriateness were not always apparent. In regard to the gender mix, while most preservice teachers agreed that the people would comprise a gender mix, they almost always estimated more men than women.

> It was very clear to see that the majority (21) participants of this survey believed that there were more men on board than women, and 2 participants believed there were equal number of men and women aboard the ……. waka for the following reasons:

Men were stronger at paddling

Women were there to help look after men who were paddling

Men needed women to reproduce offspring

It is also difficult to know just how much preservice teachers' knowledge of ancient canoe migration is influenced by their observations of major cultural events. Traditional ocean-going waka used for settlement were sailed, rather than paddled. Paddling is the norm for the various ceremonial waka that preservice teachers see on special cultural occasions. However, what is important for understanding the tension between cultural and statistical knowledge is that the preservice teachers did not always see the need to query what their relatives told them with other data.

Conclusion

Teaching subjects like mathematics in Māori is a relatively recent phenomenon and the issue of where Māori knowledge fits is an ongoing debate. The preservice teachers in this study accessed the literary information in the public domain. However, some knowledge still resides in the private, localised domain. How to evaluate the value of these two forms of knowledge is a significant tension. Western knowledge gains prestige by being in the public domain and contested, while Indigenous knowledge gains prestige in other ways.

One reason for choosing waka is that the knowledge about the canoe voyages can be considered something to which all the preservice teachers could relate, but relevant background knowledge was not always known to the preservice teachers, due to the loss of connection through urbanisation. In future, preservice teachers will be expected to create their own investigative questions about their waka of origin as the ability to construct questions as part of the investigation cycle is important (Wild & Pfannkuch, 1999).

Most of the students acknowledged the variation in the data between published and family knowledge and knowledge held by the tribe itself. None of these sources should be considered as having more validity, but only some of the preservice teachers were able to mediate between the variation in the data sources. For example, the assertion by some of the informants of particular waka that no women were on board was rationalised on the grounds that perhaps that the ancestral canoe was exploring new lands rather than seeking re-settlement, which would have had a different composition in those on board. However, most preservice teachers were sensitive to their elder's cultural knowledge. This is partly because Māori have collective ownership of the knowledge—hence why relatives are considered appropriate informants. Some discussion is needed in the future about how to respectfully integrate different knowledge sources.

In statistical enquiry, scepticism is considered as a disposition of statistical thinkers, but it is not necessarily so within the Māori family in regard to cultural knowledge. Knowledge is certainly contested between tribes and vigorously so, when it comes to resources. The challenge for the mathematics teacher, both in teacher education and in schools, is to utilise understandings about contesting knowledge from these discussions about resources in other situations where knowledge sources provide different information.

Upon reflection, we considered that there needed to be more discussion on "What does enquiry mean?" at the start of the project. We now considered that it was important to raise the question of how can the cultural knowledge be respected while simultaneously questioning it. This is because scepticism is identified as a disposition of statistical thinkers (Wild & Pfannkuch, 1999). One of the strategies would be to examine more culturally sensitive questions including discussing with students such questions as when does cultural understandings have precedence over statistical knowledge? It is also important to guide students to develop their own questions. Chance (2002) and others argue instruction should encourage students to view the statistical process in its entirety.

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Assessing the Creation of Value in a Community of Practice Linking Pre-Service and In-Service Mathematics Teachers

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As part of the IMSITE Program at the University of Sydney, a community of practice was formed linking three interconnected networks of mathematics pre-service and in-service teachers with the aim of building connections through shared experiences. This paper describes this community, its networks and the process of assessing their value from the in-service teachers' perspective using Wenger, Trayner, and deLaat's (2011) conceptual framework. Interviews conducted with twelve in-service teachers who had participated in at least two of the three networks revealed that resource sharing, collaborative connections, meaningful mentoring and professional affirmation positively impacted teacher experience.

In Australia, qualified mathematics teachers are in high demand and the lack of skilled teachers may in part explain the decrease in the number of secondary students studying mathematics at the highest curriculum levels (Marginson, Tytler, Freeman, & Roberts, 2013). Over the last several years, the Australian government has committed significant resources to increase student success in STEM subjects in K-12 education (Office of the Chief Scientist, 2016). Aligning with this national agenda, the IMSITE (Inspiring Mathematics and Science in Teacher Education) Program was established in 2014 at six universities in three Australian states to develop and disseminate new approaches for mathematics and science teacher education programs. One of the main aims of the IMSITE project is to institutionalise new ways of integrating the content and pedagogical expertise of education and science academics to enhance recruitment and retention into teaching careers, and continued professional learning following graduation (Goos, 2015).

The focus of this paper describes the efforts at the University of Sydney to create a community of practice joining together pre-service and in-service mathematics teachers with the goal of promoting these aims, particularly as it relates to the retention and continued professional development of in-service mathematics teachers. Importantly, these efforts were conceived of, and implemented by, a mathematics educator (Anderson), and a mathematician (Poladian) with support from Tully. We describe three unique, yet interconnected networks in which in-service mathematics teachers participated, and a framework for assessing the value of the community of practice that emerged.

Literature Review and Background

A recent review of research on early career teachers (Buchanan et al., 2013) confirms that between 20% and 40% of teachers have contemplated leaving the profession at some stage in their first five years. Real and perceived isolation and the level of collegial support were amongst the main contributing factors. However, collaborative learning cultures in which teachers learn from each other facilitates not only teacher learning but also reduces the felt experience of isolation (Le Cornu & Ewing, 2008). As teaching is indeed a social

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enterprise, our conceptual framework draws on Wenger's (1998) social theory of learning and the notion of communities of practice:

> a learning partnership among people who find it useful to learn from and with each other about a particular domain. They use each other's experience of practice as a learning resource. And they join forces in making sense of and addressing challenges they face individually or collectively (Wenger, Trayner, & deLaat, 2011, p. 9)

Such communities exhibit mutual engagement of participants and coordinate their complementary expertise as they help develop a shared repertoire of language and concepts. The opportunity to share experiences and engage in challenging but supportive conversations about teaching are an important feature of successful learning communities (Grossman, Wineburg, & Woolworth, 2001) but such communities are difficult and time-consuming to build. Fostering participation in such communities builds familiarity and confidence for both pre-service and in-service teachers as they engage in critical dialog and reflection for ongoing professional learning. Within the heart of these communities is the value of co-learning and co-mentoring that takes place within and among teacher participants (Le Cornu & Ewing, 2008). The perceived benefits for in-service teachers include the ability to critically assess their teaching practice through reflective discussions with pre-service and in-service teachers (Hudson, 2013), the potential to reinvigorate a passion for the teaching profession (Inzer & Crawford, 2005) and the ability to extend their professional networks through a community of practice (Wenger, 1998).

Nested within the concept of communities is the notion of networks. Wengeret al. (2011) define a network as a "set of relationships, personal interactions, and connections among participants who have personal reasons to connect" (p. 10). At the University of Sydney, and under the umbrella of the IMSITE project, we instituted three networks that joined together pre-service and in-service mathematics teachers within specific social spaces for learning to form a broader community of practice focused on teacher development: Sydney University Secondary Mathematics Alumni Conference (SUSMAC), the Teaching in Practice (TIP) Day, and Mentoring Mosaics. Informed by research, these opportunities for connected learning encouraged the sharing of resources, perspectives and stories between and amongst pre-service and in-service mathematics teachers (Graven, 2004; Mason, 2013; McGraw, Lynch, Koc, Budak, & Brown, 2007). After each network was implemented, feedback from participants informed the development of the next networking opportunity.

Sydney University Secondary Mathematics Alumni Conference (SUSMAC)

The initial approach to connecting pre-service and in-service teachers involved the creation of an annual one-day alumni conference beginning in 2014. Building on the networks that already existed between groups of alumni who had studied together while at university, the aim of SUSMAC was to provide an opportunity for them to reconnect each year by sharing experiences and for them to engage with the new pre-service teachers to help initiate them into the profession with practical advice about teaching and learning (Mason, 2013). With a focus on a current issue in mathematics education (e.g., catering for diversity, motivation and engagement, inquiry-based learning), each year we invited a keynote speaker to begin the day's deliberations. The remainder of the day was devoted to panels of experts (leaders in schools, mathematicians, teacher educators) discussing aspects of school life, and two or seven minute short presentations called *TeachMeets* from both pre-service and in-service teachers providing practical examples of useful teaching ideas, new technologies, and creative ways to engage and assess mathematics learners.

We always provide a space in the program where we pair an in-service teacher with a pre-service teacher to encourage exchange of ideas and sharing of experiences. These pairs typically connect with other pairs over lunch with teachers frequently offering professional learning placements to their pre-service colleagues. With between 120 and 150 participants each year, the energy and enthusiasm for mathematics teaching is palpable. Evaluations are overwhelmingly positive, with particularly favourable comments about the short TeachMeet style presentations. As noted by an in-service teacher, “The TeachMeet presentations were absolutely amazing, so many new and innovative ideas”.

While community building may be challenging given the size of the event, the feedback from participants indicates they feel connected with the University, connected with staff members who attend the day (mathematicians, mathematics tutors, and mathematics education lecturers), and connected with peers from their University graduating class. One teacher indicated he felt “valued”, particularly when invited to be on a panel of experienced teachers, and that he appreciated the opportunity to “give back” to the profession. Evaluations suggested that there was interest from alumni teachers to continue their engagement with the University by volunteering to attend further professional networking opportunities.

Teaching in Practice (TIP) Day

Beginning in 2015, we designed the next network, Teaching in Practice (TIP) Days – a one day forum held early in Semester 2 each year before pre-service teachers begin their first professional experience placement. Facilitated by three academics (from both mathematics and education) the purpose of this smaller community experience is for in-service and pre-service mathematics teachers to work together on classroom issues and curriculum design, thus providing a platform for in-service teachers to contribute their experience and inspire the pre-service teachers, while the pre-service teachers raise and discuss issues of concern. Typically, there are 12 volunteer in-service teachers (from the previous SUSMAC) and 24 pre-service teachers placed in mixed groups of three.

Prior to the TIP day, we asked participants to suggest or contribute issues and questions which we reframed into scenarios to use as discussion triggers throughout the day. The program also included sessions dedicated to identifying aspects of quality mathematics lessons, group lesson planning, strategies to combine topics from the syllabus documents, working with mixed-ability classes, designing rich tasks from newspaper articles, and classroom management. During morning tea and lunch breaks participants continue informal conversations and build upon prior connections from SUSMAC. Similar to findings from Murray, Mitchell, and Dobbins (1998), data from participant evaluations indicated in-service teachers particularly valued the shared conversations with pre-service teachers, and the strategies learned from other teacher participants.

Mentoring Mosaics

The final network was also based on feedback from the SUSMAC evaluations, with most pre-service and in-service mathematics teachers indicating a desire and willingness to participate in some form of mentoring relationship. Considering this level of demonstrated interest, a pilot mentoring program was instituted for second semester 2016 linking pre-service mathematics teachers who were preparing for their first professional experience placement, with in-service teachers. After exploring a variety of mentoring models that could be adopted for this pilot program, we organised “mentoring mosaic” groups of two to three pre-service teachers with two to three in-service teachers. This arrangement allowed for a diverse range of sharing experiences as well as an efficient use of the mentor's time. Also, mosaic groupings permitted a more informal style of mentoring hinged on the relational component of shared trust and the potential of peer mentoring between in-service teachers in each of the groups (Ambrosetti, 2014; Inzer & Crawford, 2005). To facilitate open and honest sharing within groups, no in-service teachers served in a supervisory role for the pre-service teachers during their professional experience.

Overall, 10 pre-service and 10 in-service teachers participated in this mentoring pilot program. The program "kick-off" started with a two-hour workshop where participants met each other, engaged in discussions on meaningful mentoring and designed their online platform which served as the medium for ongoing "mentor-type" discussions. Before the completion of the workshop, each group's online platform was established and participants sent multiple exchanges to test their platform. Within each mentoring mosaic group, the minimum requirements for each participant were to post one original comment/question a week and also to respond to at least one other post during the week. Each group designated a "point-person" who would encourage on-going participation of group members. At the conclusion of the semester pilot program, we hosted a "wrap-up" dinner in which participants shared their stories of their mentoring experiences. While the expectation of commitment to this community was one semester, one of the mosaic groups decided to remain together due to the perceived benefit to all participants. Although only a small number of teachers participated in this pilot program, the responses from their evaluations were consistently positive. Using a Likert-type scale of 1-6 (1 = strongly disagree to 6 = strongly agree) mean responses are presented in Table 1.

Table 1.
Mentoring Mosaic Evaluation Prompts: Responses of In-Service Mathematics Teachers (n=7)

After participating in the mentoring program…	Response
I feel more connected to others in the teaching profession	5.7
I feel more valued as a teacher	5.3
I feel more of a commitment to the teaching profession	5.4
Overall, I found the mentoring experience encouraging	6
Overall, I found the mentoring experience worthwhile	6

Research Question and Methodology

The flexibility of the shared learning spaces provided though SUSMAC, TIP Days, and Mentoring Mosaic groups allowed in-service mathematics teachers to engage across these interconnected networks while forming a learning partnership within a community of practice focused on teacher growth. Considering the literature explored in the context of these described networks, our research question emerges as: How has a community of practice linking pre-service and in-service-mathematics teachers created professional value for in-service teachers?

To assess the value created through this community of practice, we adopted Wenger et al. (2011) conceptual framework. Adapted from Kirkpartick's (1994) work on program evaluation, this framework is built on five cycles of value:

Cycle 1: Immediate value-activities and interactions,

Cycle 2: Potential value-knowledge capital,

Cycle 3: Applied value-changes in practice,
Cycle 4: Realised value-performance improvement, and
Cycle 5: Reframing value-redefining success.

This framework does not assume that one cycle will lead linearly to another, nor does it assume that a community is only successful if it has reached the fifth cycle. However, it is presumed that some causal relationships may exist between the different cycles, such as gaining knowledge (Cycle 2) and applying knowledge (Cycle 3). These cycles provide a means to assist us in appraising the value of the IMSITE community of practice in which University of Sydney trained in-service mathematics teachers participated.

We conducted semi-structured interviews with 12 in-service mathematics teachers who were involved with at least two of the three networks discussed earlier. During the interviews, we asked teachers to discuss their experiences with professional communities and networks, as well as their overall impressions, and personal observation and experience of SUSMAC, the TIP Day and their Mentoring Mosaic group. During the interviews, we also asked teachers if these specific networks, connected to the IMSITE community of practice, may have contributed to their ongoing professional development, and if so how. Interviews lasted from 20-60 minutes, and were audio-recorded and transcribed. Our intention was to gather a collective narrative of community participants. It is in the "interplay between personal and collective narratives" (Wenger et al., 2011, p. 16) that we begin to assess the value created through a community of practice.

In analysing the interview data, we independently used the five cycles outlined in the framework to facilitate the coding and categorisation process of responses, assigning various teacher reflections to the different cycles. After this iteration was complete, we then shared the categorisation to collectively determine subthemes within each cycle. This analytic process yielded a matrix that offered a multi-dimensional summary encapsulating the value of our community of practice as ascribed through the lens of the in-service mathematics teacher participants (Miles & Huberman, 1994).

Towards an Understanding of Value Creation of a Mathematics Teacher Community of Practice

In this next section, we use the data from the teacher participants to present evidence for each cycle by presenting the in-service teachers' reflections and stories, weaving together themes through the five cycles of Wenger et al.'s (2011) framework.

Cycle 1: What immediate value did in-service teachers place on their participation in these networks and in this community? One of the main benefits noted by teachers was the ability to link with other teachers, to strengthen prior ties, and to feel connected to a larger network of other alumni.

> For me, I have gained a lot from that community, and the Alumni conference has kept me connected, has kept me looped back into that. Just personally having those connections and just knowing someone to ask a question to, I feel is incredibly powerful. (T1)

> It was a great way to network; it was a great way to link. I've linked with other people who I wouldn't have. (T10)

Another aspect of immediate value expressed by the in-service teachers was the ability to be in a space of learning again.

> I think part of having that alumni day or the mentoring program helps because there is a lot of teacher talk and exchanging of anecdotes and advice and stuff and I think that is quite helpful in itself. (T6)

Aside from the connections, sharing, and value placed on learning, in-service teachers expressed how the communities inspired and re-invigorated them towards their profession.

> I love the ideas, but also the energy. Still it's quite inspiring. I just love to have that and get the reinvigoration. (T11)

> Gosh, I found it really energetic and inspiring and I thought as a new person, fantastic. (T8)

> Yes, I can't wait to get this out there and get into the classroom. (T5)

Cycle 2: What resources were produced through participation in this community? In-service teachers placed a high value on the variety of resources they acquired through their participation in these various networks. In particular, teachers spoke favourably of the generation of new ideas, and immediate ideas to implement in their classrooms.

> Everyone was talking about these great ideas, I think that's fantastic. I think as teachers there is no time to come up with all this information for yourself. (T7)

> Seeing an idea and I can implement it the next day when I walk into my classroom. (T1)

In-service teachers, though, expressed an even greater value on the people resources that were now at their disposal, someone to not only share teaching resources, but teaching stories as well.

> We're sharing stories which is nice and that kind of support is really nice to have for one another, just to share teaching ideas, "I did this. It was really great", "That's awesome. It's really awesome. I did this this week". I'm just being really encouraging. I think just having that positivity and that recognition is something we kind of don't get as much on a day-to-day basis. So just having that online [mentoring mosaic group] with a group is quite nice. (T5)

Cycle 3: How have in-service teachers applied what they have learned through this community? Several teachers shared how they have not only acquired fresh resources but have been able to apply these new teaching ideas in their classrooms.

> I thought that was really good [alumni conference]. I was astounded by how good those students [pre-service teachers] were presenting when they had the TeachMeets. That was so good. Some of that stuff I was going, "That's awesome". One pre-service teacher presented on Kahoot and I was like, "Well, yes. That's really good", so I've used that in the classroom. (T7)

Also, building on the theme of people resources, one of the interviewed teachers forged a working relationship with another teacher from a different school to work on designing assessment tasks together.

Cycle 4: What has been achieved through in-service teacher community participation? While the authors' intention of this cycle may relate more to concrete indicators of performance or achieved success, we took the liberty to shift this interpretation to focus more on indicators within the affective domain. Considering that many teachers may leave the profession due to a lack of collegial support (Buchanan et al., 2013), in this cycle, we highlight teachers' perceptions of personal value and sense of self-worth as it relates to their participation in these communities of practice. These data also confirm anecdotal comments received from the various network event evaluations.

> One of the things that has engaged me in what Sydney Uni is doing, is feeling like I matter. I think that could go some way [reaching out to alumni teachers] to make teachers feel more valued in a field where they quite often don't get very much of that. (T4)

> The sense of community that came from the set up [mentoring mosaics]. I am part of a few different teacher Facebook groups, but they are more about sharing news that affects teachers on a whole, to be part of this where the purpose was to share your own experience and feelings was really cathartic. (T5)

> It's certainly inspired me to feel that I am valued. (T7)

Several teachers also noted that participating in this community promoted opportunities to reflect on their personal growth as teachers.

> It's also nice for me to come into an environment and just realise how much I know about teaching that I didn't know when I was an undergraduate. It's like a marker of how far I have come. (T4)

Cycle 5: How has participation in this community changed teachers' view of what matters? Within this cycle, it was difficult to identify any consensus amongst teachers. However, there were individual teachers who expressed a change in some personal metric of what mattered to them as a teacher. One teacher was motivated by discussions on the future of STEM teaching that occurred during the community gatherings, and has thus enrolled in a PhD program. Another teacher, after recognising the immediate value that in-service teachers offered to pre-service teachers within this community, believed that more effort should be given to formalising this arrangement within the university curriculum.

> I know I personally would love the opportunity to come in, perhaps during school holidays, to assist in some of the tutorials in a similar way to how I did at the Teaching in Practice Day. If expert teachers were paid to contribute their time, perspective and insights to the students, I think it would contribute a huge amount to developing the kind of community of practice that we're seeking. (T1)

Concluding Comments

In our study, we sought to learn how a community of practice linking pre-service and in-service-mathematics teachers created professional value from the viewpoint of in-service teachers. Schlager and Fusco (2003) assert that the means by which communities of practice in education support teachers' professional growth is often under-researched and not well understood. Through applying Wenger et al.'s (2011) framework of "cycles of value" we were able to determine specific ways in which a community of practice elevated the professional experience of in-service mathematics teachers.

The community discussed in this paper appears to have not only provided access to practical teaching ideas and resources, but has also inspired emotional fortitude, professional encouragement and invigoration towards the profession for in-service teachers through shared collaborative connections, meaningful mentoring and critical reflection (Cordingley, Bell, Thomason, & Firth, 2005; Hudson, 2013; Loughran, 2002). Additionally, in-service teachers expressed feeling affirmed and valued, an often necessary component for continued commitment to the profession (Morgan, Ludlow, Kitching, O'Leary, & Clarke, 2010). Contributing to the learning experience of pre-service teachers provided in-service teachers with recognition as valued members within the profession. Additionally, the personal invitation extended to in-service teachers by the university to participate in these professional growth endeavours reinforced this affirmation.

In their definition of a community of practice, Wenger et al. (2011) apply descriptors such as "a learning partnership", "use each other's experience of practice" and "join together in making sense" (p. 10); each of these terms aptly describes the community discussed in this paper. Applying a framework to assess the value creation of an interconnected community of practice allows us to move forward to further support the development of this community from the perspective of the in-service teacher participants towards enhancing professional learning and retention in the profession. While this paper focused mainly on the experiences of the in-service mathematics teachers, this community also contained pre-service teachers, and mathematicians and mathematics educators.

Further evaluative work can be done to assess the value communities such as these offer pre-service mathematics teachers, particularly as it relates to their retention and identity in the profession of mathematics teaching.

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Supporting Teachers in Planning for Interactions with Students' Ideas

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We explore how instructional sequences, grounded in a conjectured learning trajectory, can support teachers' preparation for classroom interactions with students' ideas. Using two examples from design experiments, we illustrate that teachers in transition (a) develop a need to select and design classroom activities in which students would come to problematise some of their reasoning, and (b) require substantial support in planning for productive classroom interactions. We argue that instructional resources can and should be designed to provide some of this support.

Supporting teachers' development of classroom practices that aim at ambitious goals for students' mathematical learning is a complex undertaking. These practices place student mathematical reasoning at the centre of a teacher's decision-making and foreground classroom interactions in formats such as project-based, inquiry-based, or problem-based learning. These practices are currently not typical, and teacher learning involved is substantial (Maaß & Artigue, 2013).

Material resources such as online and physical textbooks, and instructional sequences are necessarily only a piece in a puzzle of facilitating teachers' transition (e.g., Remillard, 2012). Nevertheless, the question of how material resources can and should support such transition has captured interest of both researchers and professional development (PD) practitioners. The term *educative curriculum materials* was coined by Davis and Krajcik (2005), who clarified that such materials should ground teacher learning "in specific instances of instructional decision making" (p. 3) and indicated that materials should also help teachers "develop more general knowledge that they can apply flexibly in new situations" (p. 3). These goals are broadly compatible with approaches to supporting teacher learning where teachers are oriented to make visible, interrogate, and refine over time, the rationales that underpin their instructional decisions, and where mature rationales bring students' reasoning to the fore (Cobb & McClain, 2001).

One of the design heuristics proposed by Davis and Krajcik (2005) explicitly targets supporting teachers in anticipating, understanding, and responding to their students' ideas. We explore some of the functions that resources can fulfil in supporting the changing classroom interactions of teachers in transition. We propose that, with appropriate support, the demanding in-the-moment decisions about guiding classroom interactions can become more manageable for teachers, making ambitious teaching centred in students' reasoning a realistic possibility.

We draw on PD collaborations with teachers, in which instructional sequences developed in classroom design experiments were used and revised (Gravemeijer, 2004). We illustrate that when teachers transition towards ambitious instructional practice, their views of useful resources shift relatively early and additional needs emerge in their planning with respect to classroom interactions and working with students' mathematical ideas. In the absence of suitable resources, the transition itself may be jeopardised. We document teachers' emerging expectations and needs and speculate whether and how these could be meaningfully addressed in material resources.

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Background to Design Experiment Illustrations

Illustrations for our discussion come from two rather different PD studies, which nevertheless share the specific design research methodology (Cobb, Zhao, & Visnovska, 2008), and conceptualisation of both goals for and means of supporting teacher learning (Visnovska, Cobb, & Dean, 2012).

The data for our first illustration come from a collaboration with Irene, an experienced Mexican teacher who agreed to conduct a classroom design experiment to test a *fractions sequence* (Cortina, Visnovska, & Zuniga, 2014) with her fourth-grade students. Prior to this collaboration, Irene's teaching could be characterised as traditional. In a series of six one-hour meetings with the second author, Irene first became acquainted with the instructional sequence, including its rationale, and how this sequence was developed and used in prior classroom design experiments. She then worked with the instructional sequence during 14 weekly classroom sessions. After each session, she met with the second author to analyse the classroom events, and collaboratively plan for the upcoming session.

Data for the second illustration come from a final, fifth year of a PD design experiment with a group of 12 middle school mathematics teachers, conducted in the south-eastern US. One of the primary goals was to support the teachers' development of instructional practices in which they would induct their students into the ways of reasoning of the discipline by building systematically on their current mathematical activity. A *statistics sequence* designed in prior classroom design experiments (Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001) was a primary means of supporting teachers' learning. The PD group met for six full-day sessions dispersed through school year and three-day summer workshop every year. In Years 1 and 2, the aim of PD activities had been to foster the teacher community, and to deepen teachers' understanding of central statistical ideas. In years 3 and 4, PD activities focused increasingly on the teachers analysing their own and others' pedagogical practices with particular attention to student learning opportunities. We approached the last year of PD program as a performance assessment, the goal of which was to understand what the teachers had learned and the role that the PD had played in supporting that learning.

Instructional sequences with which we engage the teachers in our studies instantiate how learning in a specific mathematical domain can develop in a classroom setting, and how the outlined developments can be supported. The rationale for these sequences is grounded in analyses of how this process unfolded in actual classrooms, and is expressed in the form of a conjectured learning trajectory (Simon, 1995). In it, the designers specify the key shifts in classroom mathematical practices and how each of those shifts might be supported at classroom level. The supports include establishing productive norms of classroom interactions, engaging students in specific instructional activities, making particular issues the focus of whole class discussions, and the use of particular tools and inscriptions. These kinds of instructional sequences are different from collections of instructional tasks that address a particular mathematical construct, but do not specify (a) the mathematical insights students are expected to develop by engaging in the tasks, (b) how might those insights be related to each other in a learning process, and, important to our present discussion, (c) how to instructionally support the emergence of a new insight from the prior emergence of another.

Our goal as we worked with the teachers was that they would examine issues of teaching and learning fractions and statistics, respectively, as they adapted, tested, and modified the sequences in their classrooms. We did not focus on specific teacher moves, but instead pressed the teachers to justify the moves and actions that they chose by considering opportunities for student learning. Many of these PD discussions specifically attended to classroom interactions, as the nature of classroom discourse and organisation of classroom activities that includes whole-class discussions are among the key means of support within instructional sequences.

Methodology

The data in the first study included video-recordings and field notes of all classroom sessions and copies of students' work, as well as a log from research debriefs between the teacher and the second author. The log was the primary source of data on the teacher planning discussed in this paper. Similarly, the data from the five-year PD design experiment included video-recordings and field notes of all PD sessions, copies of the teachers' work, and a debriefing and planning research log.

We analysed the data using an adaptation of constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically. As new episodes are analysed, they are compared with conjectured themes or categories, resulting in a set of the theoretical assertions that remain grounded in the data. The thorough analysis of the second study is available in the author's dissertation (Visnovska, 2009) and the analysis of the first study is ongoing.

Resources and Teacher Learning about Student Interactions

In the following sections, we both illustrate and discuss the kinds of supports the teachers in transition drew on and came to expect from their instructional resources as they envisioned and prepared for classroom interactions.

Illustration 1. Classroom Design Experiment: Learning as Co-Participation

The notion of *fraction as measure* was at the centre of the *fractions sequence*. The instructional goal was to cultivate, from early on, an image of a unit fraction as a number that (a) accounts for the size of an attribute that satisfies an iterative condition with respect to the reference unit, and (b) is separate from the reference unit (Cortina, Visnovska, & Zuniga, 2015).

The sequence is set within a narrative about the ways in which a group of ancient Maya (the Acajay) measured first with body parts, then with a traditional standardised measure called *stick*, and subsequently with *smalls* introduced by Acajay elders to address the problem of accurately measuring the lengths that the stick does not cover exactly. Students then used plastic straws and scissors to make smalls so that they fulfil a specific iterative condition with respect to the stick: Small of two (3, 4, etc.) has such a size that two (3, 4, etc.) iterations of its length exactly cover the length of the stick (i.e., lengths of smalls represent 1/2, 1/3, 1/4, etc. of the length of the stick). The first mathematical practice that was supported in the classrooms where the sequence had been developed involved students reasoning in ways consistent with the inverse order relation among unit fractions (Tzur, 2007). The second mathematical practice involved students reasoning about the relative size of proper and improper fractions (Cortina & Visnovska, 2015).

Following the instructional sequence, Irene first engaged her 4th grade students in activities that entailed measuring different lengths with their body parts. Although her students engaged enthusiastically in these experiences, she recognised that there was a complication since, when questioned, none of them recognised measuring with body parts to be problematic.

Irene knew that within the conjectured learning trajectory, students were to start noticing limitations of measuring with body parts (i.e., inconsistent measures produced for the same length) before a standard unit of measure was introduced. She was thus not sure how to proceed in her classroom and considered disregarding the lack of students' concerns and introducing a standard unit of measure anyway.

When debriefing the initial lessons with the second author, she agreed, instead, to co-design and trial instructional activities in which the limitations of measuring with body parts could become readily noticeable to at least some of her students and, then, the focus of a whole class discussion. The activities involved noticing the difference in the number of hand-spans that would fit in the length of a table when different people measured it, and discussing when this difference would matter. Standard unit of measure was then introduced as a meaningful innovation.

In described episode, Irene was aware of her next mathematical goal within the fractions sequence (introducing a standard unit of measure). She was also aware of a heuristic that new mathematical ideas are productively introduced when they provide a response to students' existing questions and concerns. She established that the students in her classroom had no such concerns. She needed support in understanding *that* she could proactively help students to start questioning issues that were previously established as unproblematic, and develop some images of *how* she could go about it. In this instance, she could co-plan with the researcher, and without this support she would likely have resorted to the 'covering the content' strategy.

Irene encountered similar situations later in her teaching. One occurred when her students proposed a way to notate their work, which Irene knew would be inefficient in the long run (e.g., see Fig 1a). Instead of delegitimising the use of proposed notations from the outset, she incorporated them in classroom discussions. She then developed follow up tasks that helped students see how their way of notating quickly turned cumbersome with larger numbers (e.g., 17 smalls of 4), so they became eager to find a better alternative.

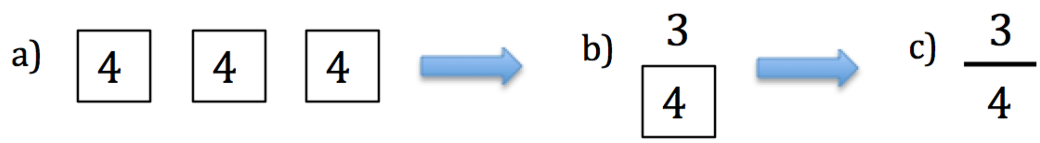

Figure 1. Evolution of notation for ¾: (a) initial student notation for *three smalls of four*, (b) more efficient notation, suitable for *17 smalls of 4*, and (c) standard fraction notation.

To take this course of action, it was important that Irene had a clear sense of the conjectured learning trajectory against which she judged the potential of specific student contributions during in-the-moment classroom interactions (understanding that she would eventually need to introduce standard fraction notation). She already knew that if she saw limitations of students' proposals, her role—with the help of the instructional sequence—was to develop follow up activities in which these limitations would become apparent to her students.

A useful material resource would need to help Irene understand that she had a role to play in supporting students to notice that their methods might at times become problematic.

Illustrations of how a teacher can provide such support, and suggestions of activities designed to problematise established forms of reasoning at multiple key points in the learning trajectory would also appear useful.

Illustration 2. PD Design Experiment: Material Resource

The focus of the *statistics sequence* was on supporting students to reason about univariate and bivariate distributions of data, often in activities that involved making recommendations based on comparing two or more sets of data. The intent of instructional activities was that students would conduct genuine data analyses in order to address problems that they considered significant. Three computer tools provided the students with a variety of options for organising data sets (for descriptions and analyses of these tools see e.g., Bakker & Gravemeijer, 2003). In the classroom design experiments in which the instructional sequences had been developed, students compared their recommendations in classroom discussions and justified them by explaining how they had analysed data.

The activities in which the teachers engaged during year 5 required the teachers to develop statistics instructional units that could be used in their school district. The teachers first reviewed and critiqued two sets of instructional units in statistics. They then selected tasks from these units, modified them for their needs, and organised them into an instructional sequence. One set of materials was an inquiry-oriented textbook series that the district had adopted. The second set of materials comprised three units that had been designed by a group of teachers at a second research site in a different US state. The units included sequences of tasks accompanied by teacher notes and were based on the teachers' work with the statistics sequence. We continued to support the teachers throughout Year 5 but we did not press them to develop instructional units that aligned with our view of effective design.

The ways in which the teachers came to participate in PD activities during Year 5, consistently indicated that they made ample use of the conjectured learning trajectory when planning instruction. They anticipated the different ways in which students could reason when engaging in specific instructional activities, and assessed the appropriateness of these activities in light of the issues that needed to be problematised, in order to make progress in the trajectory. They highlighted aspects of the tasks, and proposed changes aimed at guiding students to question the appropriateness of some of the forms of reasoning that had been regarded by the class as legitimate and mathematically adequate in earlier activities. For instance, they proposed to modify data in one task, where students were to compare two univariate datasets, displayed as dot plots, using a computer applet tool (see Figure 2). The teachers maintained that after the data were smoothed to look more like hills, the two distributions have to lead the students who compare them additively to reach a different conclusion than those who compare them proportionally. Specifically, data on T-cell counts of AIDS patients on two different treatment protocols is examined to advise a hospital on which of these treatments to adopt. Although a greater *number* of patients in the traditional treatment have T-cell counts improved above 550 than in the experimental treatment, greater *proportion* of patients in the experimental treatment for AIDS improved. This characteristic of data would make it possible to orchestrate a whole-class discussion in which the problematic nature of using an additive approach, in some situations, could be reasonably addressed.

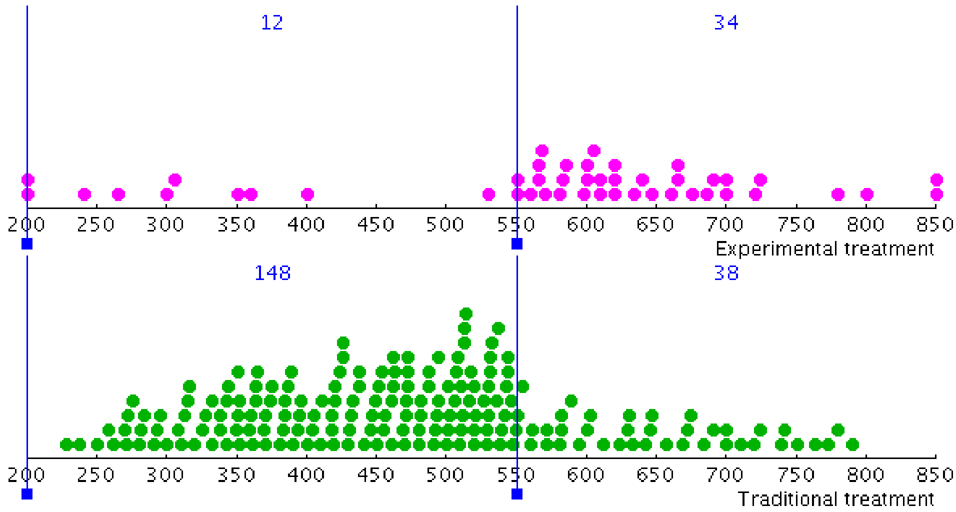

Figure 2. Comparison of two datasets. An additive comparison leads to a different treatment recommendation for AIDS than comparing proportions of patients who improved on two treatment conditions.

During Sessions 5 and 6, and the summer workshop of Year 5, the teachers proposed changes to the datasets of seven different tasks. As another example, they proposed using either more skewed or bimodal datasets to problematise solutions in which students attempted to gauge the location of the main cluster in the data by averaging only two values, the highest and the lowest. The teachers justified each of their proposals in terms of the types of students' reasoning that the change would encourage, and the ideas that it would make possible to problematise.

In PD sessions where teachers selected tasks and organised them into a unit they were to use, a researcher took collective notes that were projected for all to see, check, and modify. Researchers then used the notes to create a material resource binder where the selected tasks were edited to address the points raised in PD discussions. For purposes of this paper, we illustrate how the resource binder addressed some of the supports that were of interest in Irene illustration.

The resource, which the teachers reviewed and critiqued, merged the task descriptions with brief explanation of mathematical goals for the activity. In AIDS task, the goals described reasoning strategies that would no longer be correct (comparisons based on number of data values, Figure 2) and those that would be usable (reasoning proportionally with several options on computer tool). It provided possible student solutions and included tips for foci of classroom discussion of these.

The teachers in our PD group were adamant that in their resource, they needed to have a separate section for the mathematical goals of the task, and outline these clearly in relation to the conjectured learning trajectory. Goals for most of the tasks were written to briefly specify

(1) Solution methods students are likely to use by this point ("In this problem, students continue to reason about the shape of the data set. By this time, they might try to use most of the [options on computer tool] to describe that shape. [Two examples given]"),

(2) Ways of reasoning the design of dataset aims to overcome (e.g., "The fact that each dataset has large numbers of data points makes it more difficult for students to look at individual values (e.g., just the maximum in each group) and pushes them to find alternative methods of data description"),

(3) Where the learning is heading and which methods will soon be needed in discussions (e.g., "Building of students' solutions, you might want to support their interpretations of representations that will become crucial once they move to problems with unequal numbers of data points. For example, [a method], while not necessary for the current activity, will support their transition into the next activity").

In addition to noting which methods might become problematic for students and why, the resource binder regularly directs teachers to be proactive. For instance, it suggests to "play 'devil's advocate'" if a specific solution that students might be able to reject at the time does not come up in the classroom discussions. For each task, it also describes in detail several arguments that students could develop, and how these could be discussed in the classroom.

Overall, the resource binder suggested that, in teachers' views, a useful resource needed to outline (a) mathematical goals for each activity in relation to the conjectured learning trajectory, (b) the diversity of possible student contributions at different points along the trajectory and how these were related, and (c) suggestions for how a teacher could proactively support, with a specific task, the emergence of new mathematical ideas in the classroom. It is important to remember that the teachers created the resource as a reification of their learning in PD sessions that continued for 5 years and contributed to development of a variety of personal and institutional resources (Visnovska, 2009). We are not suggesting that the developed binder would be unproblematic, or necessarily educational, for a lone teacher with traditional teaching practice. But it indeed has characteristics of an educational curriculum material (Davis & Krajcik, 2005) and we can envision how it could be used to aid teachers' transition to ambitious instructional practice if suitable supports for teacher learning are available.

Conclusions

Teachers have a need to know where are they going with their teaching: What is it that they are trying to achieve? Often, they solve this issue by focusing on what they need to teach, and base their instructional decisions on the content that needs to be covered, the activities that need to be implemented, and the work that students need to produce. Transition to practices where students' reasoning is central entails a huge change for teachers. This goes beyond having to engage in a new kind of teaching, where problem solving plays a central role, and where students work collaboratively and share their ideas. Teachers also need an alternative way to keep track of progress and guide their teaching, in issues such as "What comes next?" and "How much time should I spend on this?" In our work, we investigate whether and how designed instructional sequences with clearly outlined conjectured learning trajectories could provide this guidance.

We discussed two examples of teachers in transition who came to value planning for quite specific classroom interactions, informed by a conjectured learning trajectory. In both examples, teachers expressed a need to problematise some of the forms of reasoning established in the classroom in order to advance their instructional agenda. In both examples, the supports involved for the teachers to design instructional activities that addressed their need were substantial (co-planning with the researcher or highly experienced peers). We took first steps in analysing and illustrating resources that were co-designed by teachers to aid in their planning for classroom interactions, and, specifically, guide teachers' design of activities in which students would come to see some of their reasoning as problematic in new situations.

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Students' Development of Statistical Literacy in the Upper Primary Years

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In a three-year longitudinal intervention study developing an understanding of statistical literacy with a cohort of students from Years 4 to 6, teachers delivered lessons from provided materials, including scripts and prompts. Besides completing seven major investigations, workbooks, and several in-class assessments, the students undertook four surveys, from which their overall progress was determined, as well as individual learning progressions. Findings indicated that despite experiencing the same lessons delivered in similar ways, students had very different learning progressions. The implications of these findings are explored.

Interest in research about school students' ability to take up concepts related to statistics has grown steadily since the National Council of Teachers of Mathematics (NCTM, 1989) published its *Curriculum and Evaluation Standards for School Mathematics*. This document laid a firm foundation from early childhood for Data and Chance, and was followed soon by documents in Australia (Australian Education Council, 1991) and New Zealand (Ministry of Education, 1992). Research on student understanding has evolved from surveys (e.g., Green, 1986) and interviews (e.g., Mokros & Russell, 1995) to interventions with pairs of students exploring data (e.g., Ben-Zvi, 2000), to interviews including cognitive conflict presented from other students (Watson, 2002), and classroom lessons led by project teachers on statistical topics with pre- and post-tests (e.g., Watson & Kelly, 2002). In the classroom, explorations of teacher-led lessons have usually been focussed on a single class (e.g., Makar & McPhee, 2009).

Following the NCTM *Standards* (1989) and its *Principles and Standards for School Mathematics* (2000), which laid firm foundations for statistical investigations, the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* (Franklin et al., 2007), set down the components of "statistical problem solving" explicitly:

- Formulate Questions - Anticipating Variability;
- Collect Data - Acknowledging Variability;
- Analyze Data - Accounting of Variability;
- Interpret Results - Allowing for Variability. (p. 11)

The acknowledgement of variation in *GAISE* components is based in a fundamental aspect of any statistical investigation (e.g., Moore, 1990) and means that variation needs to be an explicit feature of classroom investigations from the earliest exposure (e.g., Petrosino, Lehrer, & Schauble, 2003)

Besides the importance of experiencing statistical investigations first hand, there is also the goal in the curriculum that students are able to assess critically the statistical claims made by others. This is currently seen in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015) in Year 6 and in the General Capabilities (ACARA, 2013) under the description of Numeracy. The ability to assess claims critically does not necessarily imply the need to carry out an

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investigation, but requires the following components of statistical literacy: (i) understanding of the tools and terminology used, (ii) understanding of these within the context where they occur, and (iii) the ability to critique and question when necessary claims that are made (Watson, 2006). Built on previous survey research and data from nearly 4,000 students, Watson and Callingham (2003) provided a hierarchy of development of statistical literacy at school across Years 3 to 10. In a later three-year longitudinal study Callingham and Watson (in press) used 7000 responses to follow the change in statistical literacy across the middle school years in relation to an intervention project (StatSmart) (Callingham & Watson, 2007).

In this study, the interest was less on identifying global progression as students move through school, but focused on identifying individual students' observed learning progressions (e.g., Corcoran, Mosher, & Rogat, 2009). In this study, a learning progression is defined as the progress that students make as they develop skills and understanding in statistical literacy through a series of activities that address the practice of statistics as exemplified in *GAISE* in increasingly sophisticated ways.

The question arising from the previous research is whether experiences in the classroom with the process of statistical problem solving (e.g., *GAISE*) enhance statistical literacy skills in the learner beyond the particular activities. Callingham and Watson (in press) found that in the StatSmart project levels of statistical literacy improved from Years 6 to 9 but levelled off in Year 10. Across that project over 40 teachers in 15 schools in 3 states devised their own classroom activities based on their professional learning during the project. The current study uses a similar method of assessing the development of statistical literacy for students in a more focussed three-year longitudinal project for students in Years 4 to 6. The project included active involvement by the research team with the teachers in building statistical understanding through hands-on data-based investigations for the students. The general research question in the context of the current study is: What are students' learning progressions in statistical literacy across Years 4 to 6?

Methodology

Background and School Context

The government school in the project had an ICSEA value of 1,197 and 84% of students were in the top quarter of Australian students. In the first year of the study, there were five classes, one of which was a combined Year 4/5 class, but only data from students in Year 4 at the start of the study are included. In the following two years, there were four classes, consisting only of students who had been in Year 4 at the start of the study, around 90-100 students each year. Across the three years, 12 teachers were involved in the project, with only one teacher teaching across two years; this was the teacher with the combined Year 4/5 class at the beginning of the study. Professional learning sessions took place with each group of teachers at the beginning of each year and before every activity. They were provided with printed background notes and detailed lesson plans interspersed with the pages of the student workbooks for every activity. The detailed nature of the lesson plans reflected the experiences of teachers with their state curriculum (cf. education.qld.gov.au/c2c). Although the teachers followed the lesson plans closely, data collected by students during the activities within each class were, expectedly, different, prompting diverse class discussions. There were debriefing sessions with teachers at the end of the first two years. The allocation of teachers and students in the study was beyond the control of the research team and varied across years.

Classroom Activities

In the first year, when students were in Year 4, the first activity involved students posing multiple-choice questions for their classmates to answer, in order to make recommendations for improvements to the school playground (English & Watson, 2015b). The second activity focussed on measurement and variation by comparing many measurements of a single student's armspan with single measurements of the armspans of all members of the class (English & Watson, 2015a). Year 4 finished with a probability activity focussed on modelling the outcomes of the tossing of two coins and using simulations to test the model (English & Watson, 2016). The fourth activity at the beginning of Year 5 turned to implementing a complete investigation in relation to Year 5 students in Australia being environmentally friendly (Watson & English, 2015). Next, students considered two ways of collecting data to determine Year 5 students' typical reaction times, either by dropping a ruler or by using the Australian Bureau of Statistics (ABS) *CensusAtSchool* on-line reaction timer (Watson & English, in press). The sixth activity at the beginning of Year 6 extended the previous activity motivated by a media article claiming that people with brown eyes had faster reaction times than those with other-coloured eyes. Finally, the seventh activity had two parts, initially posing and refining questions in the context of the claim of athletes improving over time, and then choosing and making predictions for a 2016 Olympic swimming team given recent results for potential swimmers. The overall structure of the project was design-based following Cobb, Confrey, diSessa, Lehrer, and Schauble (2003).

Students collected their own data for every activity except the last, with the *GAISE* model for statistical problem solving the focus from Year 5 and discussion of variation prominent in all. *TinkerPlots: Dynamic Data Exploration* (Konold & Miller, 2011) was used in all activities from the second and critically analysing claims of others was a feature of the fourth, sixth, and seventh activities, building statistical literacy skills.

Participants

Over the three years of the project 65 students completed all of the four surveys and had parental permission for their data to be included in the study. Surveys took place at the beginning Year 4 (April) (Test A), end Year 4 (Test B), end Year 5 (Test C), and end Year 6 (Test D). Of the 65 students, 46 (71%) were present for all seven activities but 12 had incomplete responses in at least one workbook. Sixteen students (25%) were present for six activities with three having one incomplete workbook. One student was present for five activities and two students for only four. The incomplete workbooks were usually because the student was late arriving at school or removed for another school activity. Students were reassigned to classes in Year 5 and a few changes were made in Year 6. The range of ages at the beginning of Year 4 was eight years, nine months, to 10 years, zero months, with a mean of nine years, four months; 45% were classified as non-English speaking background (NESB).

Instruments

The longitudinal surveys were created using items from Watson and Callingham (2003) with some additional items added after the first survey, directly related to the activities in the project. Link items were used across the surveys in order to employ Rasch analysis (Bond & Fox, 2015). The Cronbach alpha values for the four tests were Test A, 0.83; Test B, 0.87; Test C, 0.81; and Test D, 0.74.

Analysis

The surveys (Test A to Test D) were analysed using Winsteps 3.92.0.0 (Linacre, 2016) using the partial credit Rasch model (Masters, 1982) to allow for different numbers of categories across items. Anchor files were produced for each of the link items (that is the ones from Test A) so that each subsequent analysis could be estimated using the same scale. Person ability measures were obtained for all students on each of the four surveys which, because of the item anchoring, could be directly compared across time. Only analyses from the 65 students who completed all surveys are reported here. In addition, “kidmaps” were obtained for every student for each survey, providing an individual profile for each student across time. These maps showed items that students had unexpectedly got correct or incorrect in each survey, by comparing their expected performance with their actual response code.

Results

Overall Results

Overall students’ performance on the tests improved significantly over the period of the study from Test A to Test D (t = 9.24, df = 64, $p < 0.0001$). This overall performance, however, masked considerable variation in survey outcomes over time as seen in Figure 1. Figure 1 shows overall learning progressions based on mean ability measures for the whole group of 65 students broken down by gender ($N_m = 29$; $N_F = 36$) and by English speaking background ($N_{Non\ NESB} = 36$; $N_{NESB} = 29$). Error bars have been left off for clarity. There were no significant differences between males and females, or between students designated NESB or not, at any point in time, although males did appear to outperform females.

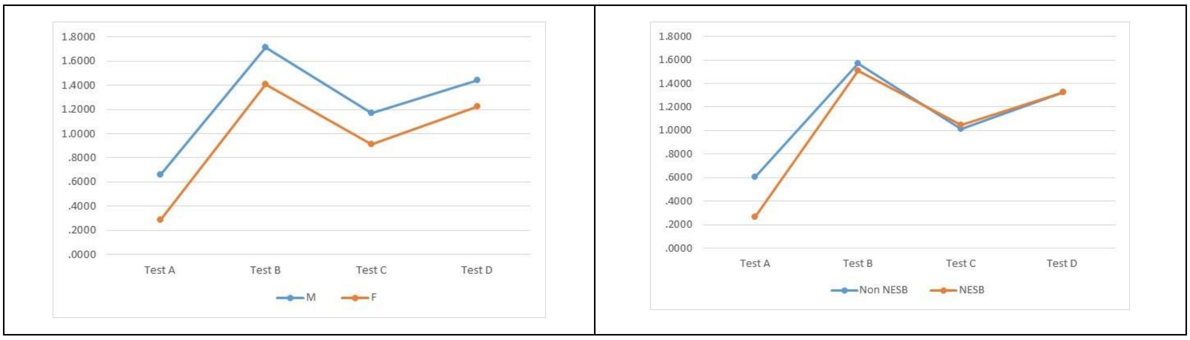

Figure 1. Results across time broken down by gender (left) and NESB (right).

Test C appeared somewhat anomalous with students apparently going backwards from Test B to Test C. This finding is addressed in the Discussion.

These overall outcomes, however, masked some considerable variations among the students. Figure 2 shows the results across the four surveys for six selected students, each of which shows a different progression. All of these students except S6 had completed every activity in the project. S6 missed one whole activity in each year. Student S5, a male from an English-speaking background was one of the highest performers in Test A. His performance, however, only marginally improved to Test B, and dropped considerably at Test C. His final measure was a little lower than at Test A. Student S3, also a male from an English-speaking background, was one of the lowest achieving students at Test A. He made a dramatic gain from Test A to Test B, dropped considerably at Test C, but gained sufficiently in Test D to be around the average performance. The majority of students had similar progressions with patterns rising at Test B, falling at Test C, with recovery nearly to the Test B value in Test D.

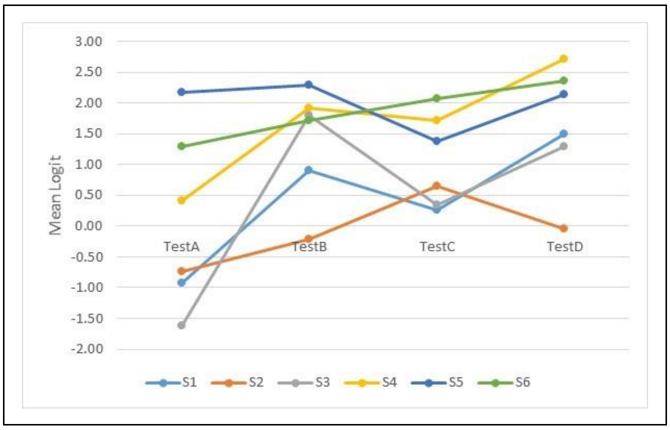

Figure 2. Individual learning progressions across time.

Student S6, a male from an NESB background, had a different progression again. He was one of the few students who gained slightly across every survey. His performance was above the mean in all tests. Four other students had continuous upward progressions.

Students S1, a female from an NESB background, and S4, a male from an English-speaking background, both showed progressions very similar to that of the group overall but had their highest values on Test D, as did six other students.

Student S2, a female from an English-speaking background, gained steadily to Test C but dropped considerably in Test D. Four other students had similar learning progressions.

To illustrate the ways in which responses to items differed across time, three items are considered, all of which expect an understanding of what a sample is. The third also focusses on the critical thinking aspect of statistical literacy. All items were in every survey. The items are shown in Table 1.

Table 1.
Items Having a Focus on Sample

SMP1	If you were given a sample, what would you have?	Code 0-3
MVE1	A class wanted to raise money for their school trip. They could raise money by selling raffle tickets for a Wii Game system. Before they decided to have a raffle they wanted to estimate how many students in the whole school would buy a ticket. They decided to do a survey to find out first…The school has 600 students in grades 1–6 with 100 students in each grade. How many students would you survey? How would you choose them? Explain your answers.	Code 0-3
MVE3	Shannon got the names of all 600 students in the school and put them in a hat. Then she pulled out 60 names. What do you think of Shannon's survey? Explain your answer.	Code 0-3

In Test A, students S2 and S3 scored unexpectedly well on MVE1, whereas student S6 unexpectedly scored low. At Test B, all students except S4 scored above expectation on SMP, students S2 and S6 scored unexpectedly well on MVE1, but students S4, S5, and S6 all scored unexpectedly low on MVE3. By Test C, students S1, S3, S4, and S5 scored unexpectedly well on MVE1, but students S4 and S5 scored lower than expected on SMP.

Students S2 and S3 scored better than expected on MVE3, but students S4 and S5 were still unexpectedly low on MVE3. Students S4, S5, and S6 again scored lower than expected on MVE3 in Test D, and students S3 and S4 also scored low on SMP. These results are summarised in Figure 3. The arrows show unexpectedly high (↑) or low (↓) performance; where there is no arrow the student performed as expected on that item, according to their measured capacity.

	SMP				MVE1				MVE3			
TEST	A	B	C	D	A	B	C	D	A	B	C	D
S1		↑					↑	↑				
S2		↑			↑	↑	↑				↑	
S3		↑			↑		↑	↑			↑	
S4	↓		↓	↓			↑			↓	↓	↓
S5		↑	↓				↑			↓	↓	
S6			↑					↓				↓

Figure 3. Individual students' performances across four tests on three items addressing the concept of "sample".

Discussion

There are several discussion points that arise from these findings. Obviously in any classroom learning context, the aim is for continual improvement in understanding of the procedures and concepts introduced. In this study, the measurement of the students' improvement in relation to statistical literacy showed overall progress at the end of the three years of the study. At the end of the second year, however, there was a decline in measured performance. Potential contributing factors are considered.

The content was more sophisticated in the second year of the study, with concepts such as categorical and numerical variables being introduced. The survey performances from Test A to Test B to Test C, however, can be directly compared and the drop was not because the surveys were more difficult. It is possible, however, that the new content to some extent interfered with students' previous understanding. In Figure 3, for example, students S4 and S5, who performed at or above expectation on the item SMP in Test B, performed unexpectedly low on the same item in Test C. In contrast, MVE1 showed that five of the six students performed above expectation, and it is possible that the classroom activities in year two of the project, which involved the complete implementation of a *GAISE* (Franklin et al., 2007) investigation, more directly addressed the understanding required to answer MVE1 at a high level.

The density of contact with project activities may also have affected the learning progression in the first two years. In the first year, Year 4, there were three activities between April and November, whereas in the second year, Year 5, there were only two major activities for the entire school year.

There were also issues in administering the survey at the end of the second year of the project, Year 5. At the debriefing session, the teachers reported timing issues, including some students not having finished when the surveys were collected because they had been working on the survey off-and-on over several days due to end-of-year school activities.

The classroom activities were taught by teachers using well described lessons and "scripts" so that delivery of the content was as standardised as possible. The teachers were familiar with this approach through their state curriculum. Nevertheless, both the overall progressions, and the individual students' performances indicated considerable variation. This finding suggests that attempts to improve teaching through the provision of detailed lesson plans and identical activities is unlikely to necessarily translate into improved performance for all students. Context and individual student differences clearly impact on students' learning outcomes. Learning is not necessarily linear (Maloney & Confrey, 2010), although it is possible to identify over-arching progressions in understanding. In the short term, there is likely to be considerable variation in students' learning progressions, and these differences should inform a teacher's next steps.

For meaningful learning in statistics, students need to have ownership of their data and hence the experiences students have in relation to data collection and class discussion are likely to be varied across classes or groups within classes, even when lessons are the same. As well, in the end-of-year debriefing sessions it became apparent that the teachers recognised that the students needed more reinforcement between the major activities.

Conclusion

Although the students' overall development of statistical literacy progressed across the years of the project, it is clear that this progression was neither linear nor uniform. This project was undertaken at a single school using uniform activities across classes, employing activities building concepts such as variation, and engaging in complete statistical investigations. The aim was to build a foundation for critical statistical literacy as the students progressed to high school. The results hence provide tentative benchmarks for others who study the potential relationship of students carrying out statistical investigations and developing statistical literacy competence.

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Why Teachers of Foundation Phase Mathematics Have Yet to "Take Up" Progressive Roles

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This paper emerged out of the first author's PhD study, which identified factors giving rise to teachers' identities and the expression thereof in teaching FP mathematics. The study analysed the mechanisms conditioning the expression of teachers' identities. We show how changes in post-apartheid systemic roles of teachers create contradictory tensions for them as these bring their mathematical learning and teaching experiences under apartheid into contradiction with the new roles they are expected to enact. We illuminate how this "situational logic of contradiction" (Williams, 2012) is managed as the complementarity and continuity between teachers' beliefs and old systemic roles remain.

The assertion that learner performance in mathematics in South African (SA) schools is in crisis may be clichéd, but a significant body of literature attests to this. Learner performance in international, regional and national mathematics benchmarking tests all paint the same picture (e.g. Reddy et al., 2015) that: SA learners are underperforming compared to their international counterparts; and the trajectory of underperformance starts in the Foundation Phase (FP; Grades R-3).

Explanations given for underperformance in SA tend to focus on systemic issues. These explanations foreground a discourse that assumes teachers and learners are products of the social (which includes the political and economic), schooling, and teacher education systems. Even research that offers explanations rooted in concerns about teachers' inadequate content knowledge and poor pedagogical practices revert to systemic explanation as teachers are viewed as products of inefficient schooling and teacher education systems. The research from which this paper emerges sought to answer the questions: What are the conditions that enable or constrain the emergence and expression of teachers' identities in the teaching of FP mathematics? Drawing on Archer's (2000) social realist work, teacher identity was considered as the way teachers express their role in teaching FP mathematics. We reflect the roles of teachers historically into the present. We contrast this with how teachers of FP mathematics express their roles. Drawing on the beliefs teachers hold, we consider why teachers tend to hold onto roles rooted in the past.

Theoretical Framework

This research is underlaboured by critical realism that posits the existence of reality independent of our knowledge of it, while simultaneously recognising that our knowledge of the world is fallible and thus relative (Bhaskar, 1978). We draw on the critical realists' conception that reality is differentiated and stratified; consisting of the levels of the empirical (consists of persons' subjective experiences and perceptions of what happens in the world), actual (events in the world, whether we experience them or not) and real (our experiences, the events and the structures and mechanisms that exist in the world). Despite these generative mechanisms, at the level of the real, not always being observable, they exert influence when activated (Bhaskar, 1978). For example, the capacity to teach mathematics with individual (agential) and extra-individual (structural/cultural) features from which it derives, occurs at the level of the real, but the exercising of the capacity to

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teach mathematics is at the level of the actual. Our experiences and perceptions of the mathematics teaching then, are at the level of the empirical (adapted from Sayer, 2000).

Drawing on Archer's (2000) delineation between structural (material) and cultural (ideational) mechanisms at the level of the real we argue here that the roles of teachers (as articulated through various educational policies) are part of the social system, while the beliefs, theories, ideologies underpinning these roles are part of the cultural system. These systems work together to condition the way teachers express their roles as teachers of FP mathematics. This assumes that there are structural and cultural mechanisms that pre-date the actions of agents, and that structural and cultural morpho-genesis or -stasis post-dates the actions of agents. In other words, when a teacher teaches mathematics, there are structural and cultural conditions at the level of the real that enable and/or constrain the way she expresses her role as a teacher of FP mathematics. The expression of these roles either reproduces or changes the structural and cultural mechanisms conditioning her practices. In this paper, we focus particularly on the interplay between the roles of teachers of FP mathematics (social system) and the beliefs they hold about mathematics and pedagogy (cultural system). For critical realists, beliefs are real in the sense that they have effects (e.g., the belief that mathematics ability is innate has effects on children's success in mathematics at school). We further delineate between systemic and social roles of teachers and show how the systemic roles of teachers are part of the real, and condition the actions of teachers. Social roles refer to the way teachers express their teacher identity. Teachers' roles are conditioned by the systemic roles, but teachers, being agents, can "act back" on these systemic roles resulting in morpho-stasis or -genesis of the systemic roles.

The Systemic Roles of Teachers of FP Mathematics

1994 was a significant year in the history of SA, as the first democratically elected government came into power. As such it marked the end of Apartheid and an opportunity to redress inequalities of the past. In 1997, the first post-democracy curriculum was implemented in all SA schools. While there have been three post-apartheid curriculum iterations, the systemic roles for teachers have remained unchanged (Department of Education [DoE], 2000).

Prior to 1994, schooling was based on the view that children were ill-disciplined and ignorant and that the teachers should discipline both the body and the mind (Woods, 1996). Teachers were expected to transmit "objective mathematical knowledge" to obedient children. Correct answers, obtained from procedures taught to the whole class, were valued and seen as evidence of learning. Teachers were viewed as implementers of a prescribed curriculum that was non-negotiable and based on rigid timeframes. In many respects, teachers were positioned as passive transmitters, expected to comply with education polices and all forms of authority. All four teachers in the broader research from which this paper emerges were subjected to this context as learners and pre-service teachers.

Post-1994 saw a marked shift in curriculum and pedagogy in South Africa. Two key curriculum policies were the first post-Apartheid Curriculum 2005 (C2005; Department of Basic Education [DBE], 1997) and the current Curriculum and Assessment Policy Statement (CAPS; DBE, 2011). C2005 was explicitly political, giving expression to the perceived opportunity for SA schooling to right past injustices (Graven, 2002). School mathematics was represented as a product with epistemological and social justice aims.

Graven (2002) identifies in C2005 four different orientations school mathematics should promote: critical democratic citizenship, have utilitarian value, induct children into the work of mathematicians, and develop the "conventions, skills and algorithms" (p. 6) that children are required to learn. New roles of teachers were thus required to enable these new orientations to school mathematics to materialise. Thenceforth, teachers were expected to prepare children through mathematics for democratic citizenship requiring a shift in teacher identities to include that of social agent (Naidoo & Parker, 2005)

Teachers were required to develop learning programmes that related to children's everyday lives, encouraged problem-solving and were integrated across and within learning areas, thus enabling learners to see the interconnectedness of knowledge in the world (Pausigere & Graven, 2013). In addition, teachers had to select content which they would sequence and pace in accordance with the diverse needs of the learners (DBE, 2000). Instead of simply adhering to a prescribed curriculum, teachers were required to make decisions about *what, when,* and *how to teach* (DoE, 2000).

Persistently poor learner performance in mathematics led to replacing C2005 with the Revised National Curriculum Statement (RNCS) and later CAPS. CAPS "deliberately attempts to define curriculum in terms of a specialisation of 'the what' of knowledge, and the organisation and structuring thereof, removing the past emphasis on knowers and knowing" (Hoadley, 2011, p. 153). While the emphasis on teaching specified content marked a significant shift from C2005 to CAPS, the pedagogy promoted however remained largely consistent. CAPS thus continues to promote learner-centeredness and social constructivism but with minimal integration across learning areas. Small group learning is encouraged and children are expected to demonstrate, record and share their thinking (DBE, 2011). Teachers are viewed as mediators of learning, who facilitate rather than transmit knowledge to children. In addition, they are expected to mediate learning "in a manner which is sensitive to the diverse needs of learners"; teaching should be contextually relevant, motivating, and effectively communicated (DBE, 2000).

As indicated above the pre-1994 systemic roles of teachers differ substantially from those post-1994 roles. This radical change in systemic roles has created a "*situational logic of contradiction*" (Williams, 2012, p. 306) for the four teachers who participated in the broader research. In other words, the pre- and post-1994 systemic roles have placed the teachers in a situation where the two 'sets' of systemic roles seem incompatible (Archer, 1995). We illuminate this situational logic of contradiction through analysing selected extracts from lesson observations and interviews of the aforementioned teachers.

Methodology

As indicated above this paper emerges from the PhD study of the first author who spent four to five weeks in the classrooms of four Eastern Cape Grade 3 teachers. The schools were purposefully selected as representative of the majority of schools in the province. These were no fee-paying schools where all the children were black South Africans and the language of learning and teaching was isiXhosa. All four teachers were black, female isiXhosa South Africans, between the ages of 39 and 55. This is representative of the primary school teaching force in terms of age and gender. During this time, the first author researched their respective life histories, mathematics histories, and mathematics teaching. All interviews were audio-recorded and three mathematics lessons were video-recorded for each teacher. Field notes were taken for the other lesson observations. All interviews were conducted in English and then transcribed. The video-recorded lessons were in isiXhosa and were transcribed and translated into English.

Three modes of inference informed the data analysis. Inductive reasoning was used to develop codes and categories emerging from the data. Inductive reasoning however could not identify possible generative mechanisms giving rise to the way teachers express their identities in teaching FP mathematics. Thus, the critical realist thought processes of abduction and retroduction were used. Abduction enabled movement "from a conception of something to a different, possibly more developed or deeper conception of it … [by] interpreting original ideas about a phenomenon in the frame of a new set of ideas" (Danermark et al., 2002, p. 93). The new frame was social realism. Retroductive reasoning assisted in moving beyond the empirical to identify structural, cultural and agential mechanism (Danermark et al., 2002). To do this, the study drew on transfactual questioning and argumentation to identify the basic conditions without which the way teachers expressed their identities would not exist. Transfactual questions require looking back (e.g., What is it about teachers' identities and their expression in teaching mathematics that makes them such?). Judgemental rationality was used to provide validity to the analysis. This involved evaluating findings in relation to existing findings, while simultaneously being aware that descriptions are always fallible.

Results and Discussion

The principles of active learning and sense making, and inclusion are key in the role of learning mediator in curriculum documents. Teachers are required to ensure that learners participate in a variety of activities that enable them to make sense of mathematics. Teachers are expected to mediate learning in a way that is respectful of difference and which demonstrates sensitivity to the needs of all learners. Drawing on a limited selection of extracts from each of the four participating teachers' broader data corpus, we illuminate the way in which the two key beliefs teachers hold militate against their expression of the systemic role of learning mediator in teaching FP mathematics. These are that mathematics learning is "difficult"/"not for everyone" and that it is about "listening and following clear explanations". While we focus here on only these two key beliefs (to adhere to length restrictions), Westaway (2017) presents an extended account of more beliefs and mechanisms that influence teacher expressions of identities as FP mathematics teachers.

"Not Everyone Can Do Maths"

All four teachers spoke of mathematics as difficult in relation both to their own experiences as learners and the extent to which their learners could grasp mathematics concepts. Nomsa explained

> Basically, I think maths is a very challenging subject, especially to learners, as you can see, look at their faces when they do maths (refers to the learners). They look so lost and they know nothing. I don't know what it is. (Nomsa, MHI, t.114)

Beauty similarly pronounced that her learners have challenges particularly with the foundational concept of place value.

> Yes, if my learners have got that (referring to counting and number operations), I'm sure they will pass. Other things like place value, expanded notation, it's a lot, but I must make sure they can add. These kids it's so difficult for them. (Beauty, MHI, t.94)

Her fear that *these* children will be overwhelmed by the difficulty of learning mathematics, along with other structural and cultural conditions, led to a restricted mathematics curriculum with low cognitive demand. In a lesson on addition of two three-digit numbers, Beauty explicates:

> Have you noticed that I just did addition, I didn't do the carrying? I'm afraid, they don't know how to. It's few that can [carry]. I've done that with the two-digit numbers and we haven't done it with the three-digit. (MHI, t.96)

Her comment that "few can carry" (t.96) is underpinned by a belief that mathematics is not a subject that all learners can grasp. One of the many criticisms of SA mathematics teachers is that they have low expectations of the children and thus expect little cognitive engagement from them (e.g., Hoadley, 2012). By contrast, Nomsa's belief that mathematics is difficult leads to her presentation of a mathematics curriculum that is abstract and devoid of sense-making. The vignette below is taken from a measurement lesson. Nomsa and her class were discussing the mass of various household products that the children had brought to class. Nomsa holds up an empty 2,5kg packet of sugar. She asks what this is and the children reply that it is a 2,5kg packet of sugar. She suggests they add two packets of sugar and asks what the answer will be. One of the children says '5kg'. She repeats the question and the children respond in unison '5kg'. She explained that she wanted to check if they are "really going to get 5kg" (t.98). She writes '2,5kg + 2,5kg' using the vertical format (shown in Figure 1) and asks the class what they must do. There is no response. Eventually a child from the 'first group' (Nomsa's term for learners deemed to be mathematically competent) comes up to the board and stares at the sum for a few seconds. Nomsa writes '10' next to the sum and asks what we should do with the '0'. The child does not respond, so Nomsa says "we are going to scratch that zero and put it where?" (t.100) to which another child, from her 'first group' replies, "there by the five" (t.101). The child comes to the board and places the '0' under the '5's' (see Figure 1). Nomsa asks the children if the '1' (in her written 10 on the board) represents a 'ten' or a 'unit'. The children respond: 'tens'. Pointing to the '2', she asks the children "what are the tens here?" (t.108) to which the children reply '2'. She puts the '1' next to the '2' and asks the children to add. They eventually say '5'. Nomsa writes in the '5,' in front of the '0'.

^{1}2,5kg	
<u>2,5kg</u>	10
5,0	

Figure 1. From Nomsa, FN, pp.10-11.

In this lesson children are required to add the decimals 2,5kg to 2,5kg. The children seemingly manage this without written calculation but struggle when expected to use the standard vertical algorithm. In trying to assist the children, Nomsa and the children confuse tenths with units and units with tens. The result is an example of mathematics that provides little opportunity for sense-making. Throughout this section of the lesson, Nomsa's body faces the 'first group' in her class. When asked about this she responded, "it's because I know the answer is going to obviously come from them" (Nomsa, PI2, t.88). The children's perceived inability to understand the mathematics entrenches Nomsa's view that mathematics is not for everyone. Thus, participation in the lesson appears directed towards those deemed mathematically competent. Both Nomsa and Beauty stated that there were some children in their classes who would never be able to succeed in mathematics. In both Beauty and Nomsa's classes, children were grouped according to ability. The children perceived to be the 'weakest' were seated at the back of the class. When asked why this was the case Beauty expressed that "these children are not struggling like that won't know it tomorrow, they won't know it forever" (PI 2, tt.196). Nomsa similarly expressed,

> Eish, I don't know, but like in my first group, if a learner did something wrong, then sometimes I explain and ask that learner to do corrections, you see but with that last group, I will know that if he does not know it, he will never know it (Nomsa, PI2, t.114).

When children make a mistake, Nomsa re-explains to those children who she believes can do maths, but not to those who she believes cannot do maths. Children deemed not capable of learning mathematics are thus excluded from the lesson even though this goes against the systemic role of meeting the needs of diverse learners. While Beauty's solution to the belief that mathematics is difficult is to offer a restricted curriculum, Nomsa promotes a proceduralised view of mathematics. Like Nomsa, Veliswa also stressed a proceduralised view of mathematics. Despite her children demonstrating that they could work out the difference between 35 and 45 in a variety of ways, she wanted them to use the standard vertical algorithm. While the children solved the sum quickly and easily using their own methods they were confused by the formal procedure and were not able to tell her what was 'five take away five'. Veliswa drew tallies on the board to show them. Nokhaya however introduces her own rules into procedures to avoid common learner errors. That is, she required the children to insert "00" into the calculation 108 – 66 = (100) + (00 - 60) + (8 - 6) in order to signify the 'tens'. The double zero was an important feature in her class when children decomposed numbers. Nokhaya altered the way the children decomposed a number where there are "0 tens" (e.g., 309). Nokhaya insisted that the children write "300 + 00 + 9" and when asked why the children should write "00" for the 'tens' she stated that they need to know that tens consist of two digits. She was concerned that they would get confused if they did not write the "00" (Nokhaya, FN, p. 12). Yet this intention to bring attention to the place value of the tens through writing "00" created problems further down the line. For example, in a later lesson observation children were required to write three digit numbers in a "house" that Nokhaya had drawn on the board. Nokhaya asked one of the children to write "401" in the house as shown below.

Nokhaya's diagram

K	S	M

The child's work

K	S	M
4	00	1

Figure 2. Nokhaya, FN, p.11.

The examples above demonstrate how the beliefs that mathematics is difficult and that not all children can do mathematics are expressed by these four teachers in the process of teaching FP mathematics. The teachers tended to offer a restricted curriculum, presented mathematics procedurally, and attempted to make calculating "error proof" through stating "rules" for children to follow. In the latter two the mathematics taught interfered with rather than supported the development of conceptual understanding while the former worked against providing equitable learning opportunities for learners with diverse needs.

Teachers Must Explain Mathematics Clearly and Children Must Listen Attentively

The word "clear" was used by all four teachers to distinguish between teachers deemed to be good at teaching mathematics and those who weren't. All emphasised the importance of explaining mathematics clearly. Nokhaya noted that when she was in school:

> [In primary school] [o]ur teacher was good at explaining because maths is about explaining what to do. So, he was good … [whereas in high school] I didn't like it much. There was a lot to be done and our teacher was not quite clear about maths because he was not a maths teacher, but he was told to teach maths so I didn't get it the way I wanted (Nokhaya, MHI, t.20).

Here clarity of explanations is noted as fundamental to Nokhaya's learning. Veliswa too complained about the poor-quality explanations given by one of her high school mathematics teachers, coupled with the fact that the class feared him because he was a "very strict man". She said, "It's just that the person who taught us that maths, didn't explain thoroughly how it was done and because I was scared, I didn't understand it clearly" (Veliswa, MHI, t.112). Reflecting on her class of Grade 3 learners, she emphasised that she wanted them to understand the mathematics they were learning "It's just that I'd like the learners to pass and understand. Not to pass by doing rote learning, but to understand methods that they must use" (Veliswa, MHI, t.140)

Both Beauty and Nomsa contrasted their high school mathematics experiences with that of their experiences in their teacher training colleges. Both were critical of their high school teachers' inability to explain mathematics thoroughly or clearly, whereas they held their college lecturers in high regard. Beauty explained "[t]he teacher that was teaching us in Grade 12, mmm, Mr, I forgot who he was, he was not clear when he was teaching us" (Beauty, MHI, t.22); however, at the college "It (mathematics content) was very good. It was clear. I never failed that" (Beauty, MHI, t.50). Nomsa was similarly very impressed with her mathematics lecturer at college: "She was teaching us very clearly and if you don't understand, she was willing to assist" (Nomsa, MHI, t.44). In contrast, she reflected that at high school the teachers were "so negative sometimes in maths. Like, like they would shout at you if you don't know something and not be able to explain to you" (Nomsa, MHI, t.68). The emphasis on teachers giving clear explanations in mathematics suggests a belief that the teacher's role is to transmit mathematical knowledge through clear explanations to learners. The learners in this context have the responsibility to listen and learn. In this respect Nokhaya, Veliswa and Nomsa emphasised the importance of learning by listening. Nokhaya and Veliswa both reflected on their success at school, particularly at primary school, and attributed this to their being good listeners. Nokhaya comparing her primary and high school experiences commented:

> Yho, it was very different, because in primary school, I used to listen from the teacher, now in high school I had to study. I didn't like studying; I preferred listening and then go to the book. But most of the teachers there, they referred us to the book. You must read chapter what, chapter what and chapter what, then the following day, you must come and discuss. I didn't like that. (Nokhaya, LHI1, t.130)

Veliswa attributed her achievement in primary school to her being a good listener:

> It's just that I was a good listener. I listened and if, apart from the fractions that I had to learn by rote, I listened and kept what I was learning. I tried to keep it and remember it when I was to write an exam or something. (Veliswa, MHI, t.14)

Nomsa too reflected on the importance of listening in her classroom. During the maths history interview she reflected on why her children struggle with mathematics.

> I think another problem, when you are doing examples on the board; they are playing, they are not listening. They are not listening at all, especially those ones who are not clever in class. They are the ones who are not listening while you are teaching… When it comes the time to answer questions, they know nothing. (Nomsa, MHI, t.116)

Three of the four teachers emphasised the importance of listening in the mathematics classroom, seemingly equating learning with listening.

The belief that the teachers' role is to explain mathematics clearly is consistent with the policy expectations and pre-1994 systemic roles. Teachers were regarded as the all-knowing authority in the classroom expected to transmit objective mathematical knowledge to passive recipients of knowledge. Rather than deal with the situational logic of contradiction brought about by the radical pendulum swing (Graven, 2002) from the pre- to post-1994 systemic roles for teachers, these teachers continue to hold onto their pre-1994 beliefs. These beliefs generate a *situational logic of protection,* enabling teachers to reproduce systemic roles of the past, resulting in morphostasis of pre-1994 roles.

Concluding Remarks

The systemic roles of teachers as expressed through the post-1994 Norms and Standards for Educators (DoE, 2000) and CAPS (DBE, 2011) are at variance with the systemic roles that the four teachers experienced as learners, during their initial teacher education and as young teachers. This paper has highlighted how newer systemic roles have created a situational logic of contradiction for teachers and how teacher responses to this are to 'hold onto' the systemic roles that they were taught mathematics under (both schooling and tertiary). These thus stand in a logical relation of complementarity with the pre-1994 systemic roles of teachers and in contradiction with the post-1994 systemic roles. The above teacher data illuminates the ways in which these systemic roles (i.e., the structural mechanisms) and beliefs (i.e., cultural mechanisms) have conditioned the teachers' identities, into the present. As such, the paper has explained how the roles of the past exist in necessary and logical relation with the beliefs of that period, generating a situational logic of protection of pre-1994 roles.

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Relating Emotions to Motivational Processes using Middle-School Students' Expressed Aspirations for Learning

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Understanding more about how students' motivation is influenced by intrinsic and extrinsic factors can support teachers in their approaches, choice of tasks, and strategies for helping students engage and achieve in their mathematics learning. As a complementary approach to normative surveys with pre-defined constructs and Likert-scale items, this study used a free-format prompt that sought students' voice expressed in their own words. Data were collected on over 3,500 middle-school (Years 5 to 8) students' aspirations for their learning as a way to explore motivational processes. In this paper, I focus on a sample of responses in which the students explicitly related their aspirations to the emotional dimension.

Although there are various, sometimes conflicting and overlapping, operational definitions of affective and motivational variables in the literature (Grootenboer & Marshman, 2016; Pintrich & Schunk, 2002) – such as values, attitudes, identity, self-efficacy, beliefs, emotions, engagement, and effort – there is general consensus that along with mathematical knowledge, motivational factors directly influence students' academic achievement. Motivation, in this study, was defined as "a potential to direct behaviour through the mechanisms that control emotion" that is "structured through needs and goals" (Hannula, 2006, p. 175). It can be viewed as relating both to intrinsic factors inherent in a student's disposition and extrinsic factors, such as the school environment and mathematics learning activities provided to students (Eccles & Wigfield, 2002; Middleton & Spanias, 1999; Skinner, Kindermann, Connell, & Wellborn, 2009).

This perspective on motivation posits an influential role for the extrinsic environment in which students learn and their emotional appraisal of their experiences. It suggests that the choices that teachers make in their teaching practice and how they manage the social environment of the classroom affect students' goal setting, emotional reactions, interest in learning, and ongoing motivational outcomes. This study provided the opportunity to examine middle-school (Years 5 to 8) students' spontaneous descriptions of their wishes for learning mathematics and explore how students describe their motivation relative to extrinsic or intrinsic aspects. Its intent was to consider how these aspects might relate to theoretical perspectives on motivational processes and school-based influences: evidence of particular mindsets and goal orientations, achievement aims, emotional responses, and various intrinsic and extrinsic motivational factors. This paper focuses on one subset of the overall data – students' responses that explicitly mentioned emotional aspects in describing their aspirations.

Theoretical Perspectives on Motivation

In asking students about their aspirations for learning mathematics, it was thought that their responses might provide evidence of intrinsic motivational aspects, such as their mindset or goal orientation, or might relate to extrinsic aspects, such as certain types of experiences or environmental features in their classes. A number of theoretical perspectives on each of these themes were used in the overall study for data analysis (Wilkie, 2016). Here, I briefly overview relevant theoretical perspectives on motivation.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 554–561). Melbourne: MERGA.

In a large review of studies on motivation, Middleton and Spanias (1999) sought to explain student effort and achievement using context-specific mathematics studies. One finding was that students' evaluation of their likely success in mathematics influences the extent to which they engage, and that believing their effort will result in success can increase their motivation. Students' definitions of success relate to their beliefs about the nature of mathematics as a domain and about the nature of the process of learning mathematics. In a three-year longitudinal qualitative study, Hannula (2006) found three aspects that affected students' regulation of their motivation in mathematics: their derivation of goals from their needs, their evaluation of the accessibility of their goals, and automated regulation based on emotional reactions to previous experiences that act as an inertia against change. Motivation in mathematics can involve automated regulation based on emotional reactions to previous experiences. Hannula (2006) conceptualised motivation as "a potential to direct behaviour through the mechanisms that control emotion" that is "structured through needs and goals" (p. 175). In this paper, the term disaffection is used to describe students' expressions of negative emotions that suggest a loss of satisfaction, interest, affection, or engagement in their experience of learning mathematics. Examining those students' self-reported aspirations that describe emotional reactions can provide evidence of their self-perceived needs and goals, and thus give potential insight into motivational processes involved in their mathematics learning.

A recently proposed theoretical model of motivation variables was derived from structural equations modelling using a large data set of 24,000 U.S. middle-school mathematics and science students (Middleton, 2013). It also informed the data analysis of the study in this paper, with a particular focus on *interest* as a central variable, which influences effort, self-efficacy, utility, and achievement. Space limit prohibits inclusion of the diagram of the model here, but it will be shared in the presentation. Interest can be related to both intrinsic (goal orientation, curiosity) and extrinsic characteristics (novelty, challenge, social interaction). Interest can therefore ebb and flow in different contexts (Schweinle, Meyer, & Turner, 2006). The results from Middleton's (2013) research also indicated that extrinsic factors are likely to be present, which influence interest and effort to then influence achievement. Many were suggested, including teacher attitude, classroom environment, learning strategies and resources, and family environment. Grootenboer and Marshman (2016) reviewed studies on affective development, concluding that particularly for New Zealand middle-school students, their relationship with their teacher seems to be a key factor. Teacher actions, such as the choice of mathematical tasks, use of tools, and operation of the classroom environment can influence students' effort and achievement. Sullivan, Tobias, and McDonough (2006) demonstrated that the social climate and interactions with peers in the mathematics classroom also appear to play a role in influencing students' motivation. Over 50 Australian middle-school students were given the scenario of a hypothetical student who is "good at mathematics but does not try" and nearly half of them suggested that the lack of effort related to a desire to be "popular or cool" or to avoid getting "picked on or bullied or teased" (p. 91).

Confirming Hannula's (2006) conceptualisation of motivation as structured through needs and goals and directed by mechanisms that control emotions, Martínez-Sierra and García-González (2016) found that older mathematics students also appraised various situations (in their undergraduate course) that triggered emotions, in terms of their own goals. Additionally, the students' emotional experience appeared to be a contextual rather than individual phenomenon. Even though individuals experienced emotions, their appraisals seemed to be social and contextual in origin: socially organised and historically constituted. This suggests that extrinsic contextual features of students' environments (including social and historical aspects) play a role in their appraisal of experienced emotions during their learning activities. Grootenboer and Marshman (2016) highlighted the pervasive poor image of mathematics among adults in society, despite its being considered important as a school subject. It is likely that the influence of these societal views over many years also plays a role in mathematics learners' appraisals of emotions that are triggered by previous experiences (Hannula, 2006), particularly negative emotions.

Research Design

The issue of "voice" in research is discussed extensively in the literature: how to give participants greater voice about issues that directly and indirectly affect them, especially those whose voices are not often heard (Flutter & Rudduck, 2004). There seems to be an increased valuing of consulting learners about their views, thoughts, feelings, and experiences, to make teaching and learning more effective (Flutter & Rudduck, 2004; Robinson & Taylor, 2007). Much of the research literature describes attempts to define and to measure different aspects of the affective dimensions of students' learning – their emotional responses, attitudes, beliefs, and other motivational variables – to theorise causal relationships with behaviour and achievement. Structured surveys and Likert-scale items are typically used in large-scale studies, which aim to measure pre-defined aspects of motivation (Hannula, 2006) and approach the relationships from a normative view (di Martino & Zan, 2010).

As an alternative and complementary methodological approach, this study used an open-response survey item enabling qualitative and interpretive content analysis (Lieblich, Tuval-Mashiach, & Zilber, 1998; Wolcott, 2009) for exploring the different facets of students' aspirations as expressed by the students themselves, without any *a priori* motivation constructs. The aim was not to infer causal relationships but to understand more about what students themselves focus on when articulating their aspirations and how they relate to different aspects of their emotional responses, needs, goals, and motivation in learning. The students were asked about their aspirations with the following question: "If you had one wish for your mathematics learning, what would it be?" The word "wish" was deliberately chosen as conveying the most familiar and open-ended meaning out of the group of related synonyms, understandable to young people (e.g., "Make a wish for your birthday"). It was considered useful for capturing the intended sense of a longing or desire or hope that may or may not be attainable, a felt need or lack ("Wish", n.d.), and related well to Hannula's (2006) definition for motivation.

The study was conducted as part of a larger project[1] exploring tasks and strategies for promoting student effort. The prompt was included in a pre-test at the beginning of the students' participation, which included achievement and attitudinal items. Although seeking qualitative data, the free-format prompt generated a large amount of data from 3,562 students in Years 5 to 8 (93% response rate) across 31 government and independent schools representing a diverse range of school types, sizes, and socioeconomic status across three Australian states: Victoria, New South Wales, and Tasmania. The responses were coded line-by-line (Creswell, 2007) using the *NVivo 10* qualitative analysis program (Bazeley, 2007). Responses ranged from a few words to long paragraphs. Since the responses were anonymous, the students had the freedom to choose what they wanted to

[1] Encouraging Persistence Maintaining Challenge (EPMC): Australian Research Council Discovery Project (DP110101027) involving Monash University and Australian Catholic University.

focus on, without being obliged to say what they thought was expected of them (Robinson & Taylor, 2007). They could describe critical, negative, or nonsensical ideas or not respond at all. The coding process was cyclical and involved repeated check coding with other researchers and refining of the categories (For further details, please see Wilkie, 2016). This paper focuses only on those responses (*n* = 153) in which the students referred specifically to emotions and discusses them in light of theoretical perspectives on motivational mechanisms when learning mathematics.

Discussion of Results and Implications

Within the students' responses that related to emotional aspects, the types of feelings that were described included positive ones such as like, love, and enjoyment, and negative ones such as dislike, hatred, embarrassment, confusion, struggle, frustration, and boredom. Some students described transferring from "like" to "dislike" or vice versa, and several students expressed wishing to become "more confident". Table 1 presents the categories of the responses along with illustrative examples. Note that some responses needed to be coded in more than one category.

Table 1
Students' Responses that Included Reference to Emotions (n = 153)

Category		Illustrative examples
About struggle or confusion	41%	"To not struggle with more difficult maths problems." (Year 5/6 girl #425) "To try a lot harder and focus more. I seem to doze off a bit when I get really confused." (Year 7/8 boy #606)
About wanting more confidence	25%	"If I could have one wish granted in my maths it would be to be confident in my work and to be very smart and fast with all of the questions and to understand very complex and hard maths." (Year 5/6 boy #653) "My wish would be that I can complete my maths work with confidence and that most of the times my answers are right." (Year 7/8 girl #303)
About wanting to be interested or challenged	20%	"I wish that we could do harder things in class because everything in math is so easy. Maths isn't fun anymore because it's so easy. I want a challenge." (Year 5/6 boy #641) "I wish that maths lessons were not as boring, because we do the same thing every lesson." (Year 7/8 girl #157)
About perceptions of others	12%	"To be able to work with a group of people I am comfortable with and that I am able to ask questions without feeling embarrassed." (Year 7/8 girl #562) "My wish is that I could become smarter because every time my class do math I feel left out because I don't know the question and that means I can't contribute to the lesson." (Year 5/6 boy #922)
Positive emotion	7%	"Being good at every maths topic. I'm a little bit good at maths, it's also my one of my favourite school subjects." (Year 7/8 girl #58) "My wish would be that there was a lot of mathematical questions in the world, because I love maths." (Year 5/6 boy #787)
Negative emotion or desire for positive emotion	7%	"To not be annoyed." (Year 5/6 boy #578) "To love Maths and to not think Maths is hard." (Year 5/6 girl #212)

In the following sub-sections, three emergent themes are discussed, with illustrative responses highlighting how students' emotional appraisals relate to their descriptions of motivational aspects of their learning. They are not intended to represent responses proportionally from the whole dataset but rather to focus on responses that can provide theoretical insights into and practical implications for different aspects of these students' experiences in mathematics.

Students Relate Struggle and a Desire for Understanding

Many students used the word "struggle" to describe their experience of learning mathematics; others described being confused or stressed or frustrated. Some students explained their reasoning – They did not understand the mathematics they were studying.

> My one wish for my maths learning would be to always understand what we are doing and not get stuck too often. (Year 5/6 girl #109)

> "To help me understand more because I struggle at maths and need help most of the time. I would like for the teacher, Mrs * to help me more to understand fractions and other things in maths." (Year 5/6 girl #271)

A few students related their struggle with particular areas of mathematics:

> I want to be better at maths like division, multiplication times table so for once I could get something right in maths subject. (Year 5/6 girl #164)

> I would think that to get better at my times tables because I struggle with them and it affects when I'm learning because I have to stop and think about it. (Year 7/8 girl #182)

Other students appeared to relate their difficulties with mathematics itself:

> To love Maths and to not think Maths is hard. (Year 5/6 girl #212)

> To have it make more sense and learn it better because it is hard! (Year 5/6 girl #343)

There were some responses where students related their struggles to an extrinsic feature of their experience, such as how they were being taught:

> For the teacher to explain better and to not confuse us. Also, if ever I need help with my maths, a teacher can always be there to help me with my maths problems and to correct my mistakes. (Year 7/8 girl #98)

> For the teacher to explain more so I could understand the question better and get my work done instead of struggling. (Year 5/6 girl #444)

> That my teachers can help me more so I don't struggle. (Year 5/6 girl #486)

These students' responses also include a suggestion for how teachers might address their difficulties and support their goal for understanding the mathematics. The first two examples highlight the students' wishes for more and better explanations.

Other students seemed to focus on an intrinsic aspect:

> If I had one wish for maths would to be smarter at it because I am really dumb at it so I wish that I could get better at it. (Year 7/8 girl #219)

> To understand the teacher and others. I find it hard to understand others when they explain math questions to me. (Year 5/6 boy #569)

> That I wouldn't immediately judge my intelligence off of how well I do in maths. (Year 5/6 girl #523)

Some of the students connected low self-efficacy with a dislike of mathematics. Yet interestingly, a few Year 5/6 girls described a *positive* affect along with a low self-efficacy:

> To do better in times and understand it more and times. I would also wish that I could get better at maths because I'm not that good at maths and I really like maths. (Year 5/6 girl #594)

> To learn all of my x's tables and be the best I can at all of the maths work we do here in year 5 and at school. I want to learn this because I usually love maths so much when I was in year 4 but now I don't seem to like it anymore. I WANT TO LEARN THESE THINGS :) :) :) ;) :) :) ;) :) :) :) [sic]. (Year 5/6 girl #711)

> To be really good at maths because I love maths and I'm not really good at it and I want to be good at maths. (Year 5/6 girl #719)

Thus, not all students who believe that they are not good at mathematics also dislike it.

These examples highlight that some students may evaluate their likely success in mathematics learning as low and relate it not to their own level of effort, but to an aspect outside of their locus of control, such as the mathematics itself, how they are taught, or their own intelligence. Middleton and Spanias (1999) found that such evaluations of likely success influences the extent to which students engage. Experiencing success, particularly in understanding the mathematics they are studying, seem important in sustaining these students' ongoing motivation. As with Hannula's (2006) conceptualisation of motivation as structured through needs and goals, these students' responses highlight their own expressed goal to understand the mathematics, which affects their evaluation of success.

> My one wish is to understand math much more and to get it a lot more. My only weakness in learning is math. My wish is to stop getting frustrated when I do it all the time. If I understand math more than I would enjoy math a lot more. It would be my dream to get math and to understand it more. I have difficulty a lot in math and like I said I would really really REALLY LIKE MATH A LOT MORE if I UNDERSTAND! (Year 5/6 boy #892)

Students Relate Disaffection and a Desire for Interest

Another type of response seemed to relate to a sense of boredom when learning mathematics. Some of the students explicitly linked their emotional appraisal to the type of work they were given in class:

> My wish would be to make things more interesting in maths. It can get very boring sometimes and you don't really feel motivated to work. So maybe try and mix it up and try using the computers more or having breaks and letting us talk a bit more. (Year 7/8 girl #312)

> If I could be granted one wish for my math learning it would be to have more advanced questions that gets your brain thinking more and so that you can have fun while doing the tasks and so that you're not bored doing the same thing every task. (Year 5/6 girl #430)

These types of responses can be related to Middleton's (2013) central motivational variable, specifically situational (extrinsic) interest from tackling a novel task or interesting work. In the first quotation above, the student explicitly referred to motivation as being affected by their level of interest. The second quotation described a desire for advanced questions that require thinking, suggesting that some students value challenge in learning mathematics.

Some students communicated the sense that they did not want to be unmotivated or disaffected:

> Not having a bad attitude!!! (Year 7/8 girl #321)

> That I have more of a passion for it and the teachers are more understanding (Year 7/8 girl #375)

> To learn more about it and to love it (Year 7/8 girl #916)

It seems that some students not only value opportunities to experience success with their learning but also interesting tasks and challenge at a level that is engaging, not disheartening. This might also benefit students who attribute their disaffection to an intrinsic feature of themselves, by enabling them to experience positive emotional reactions to succeeding with their learning that over time might disrupt their inertia against change (Hannula, 2006).

Students Relate Embarrassment and a Desire for Social Competence

The following responses highlight students' perception of embarrassment in class through having to ask for help or not knowing the answer to a question:

> To know everything in maths since I had such a hatred for maths in my previous years and now I am growing to like it. I feel embarrassed when the teacher asks me something and I don't know and I need Mrs *'s assistance. (Year 5/6 boy #306)

> I would choose to put my hand up more in class and believe in myself more then what I do. Also not think that my answer is wrong and not ask anyone for help. (Year 5/6 girl #330)

> My wish for learning my maths would be to not have as much students that will distract me and my learning and to not have people that think they're better than everyone and when we need help from other students they always say "oh that's easy" and when I ask for help from other students I feel embarrassed. (Year 5/6 girl #275)

One student drew attention to the possibility of embarrassment through being socially perceived as too smart:

> To be the best in maths but not so smart that people start teasing me. (Year 5/6 boy #176)

This response resonates with Sullivan and colleagues' (2006) finding that some students connect a choice to disengage in class with the need to avoid peer censure.

These examples highlight that for some students, their aspirations for mathematics learning relate to the avoidance of experiencing embarrassment. Martínez-Sierra and García-González (2016) found that undergraduate students appeared to appraise their emotions using contextual and historical features of their learning environment in terms of their own goals. The younger students in this study also explained their emotional appraisal (embarrassment) in terms of their perception of the negative social connotation of asking for help or being unable to answer a question in class. It seems that normal activities for learning, such as seeking further assistance, can trigger emotions in some students based on their appraisal of prior experiences, and lead to a disinclination for those activities. To teachers, this can appear as disaffection in the classroom, and can be difficult to respond to with warmth and involvement (Skinner et al., 2009). If motivation is directed by mechanisms that control emotions (Hannula, 2006), then it is likely that there may be underlying social and historical appraisals of emotions involved in a student's apparent disengagement in class.

Conclusion

There is much to learn about students' engagement and motivation in learning mathematics. This study provided an opportunity to investigate the issue of apparent pervasive disaffection among middle-school students from a different angle, by asking them to describe their wishes for mathematics learning in their own words. Some of the students attributed their difficulties to intrinsic factors, such as their intelligence or lack of confidence. Others referred to extrinsic aspects, such tasks they are given or their teacher's actions. Yet overall, the students' expressed aspirations to understand the mathematics and be challenged and develop competence suggest that many students in this cohort may not necessarily perceive their learning environments as supportive of their goals, they *wish* for them, and link them to their experience of struggle, or boredom, or embarrassment. Their willingness to share their emotional responses and give specific suggestions for how their motivation might be improved suggest that overall, students do not want to be disaffected in mathematics and would value teachers' efforts to support their learning needs in particular ways. It appears that such a methodological approach, giving students the opportunity to share their own voice using an open format, can provide helpful insights into the affective and motivational dimensions of their engagement and achievement in mathematics. It is also an approach that teachers could try with their own class as a way to understand their students more, and respond to their individual aspirations strategically.

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Maths Anxiety: The Nature and Consequences of Shame in Mathematics Classrooms

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This paper presents an analysis of pre-service teachers' reflections on the consequences of their perceived public humiliation in school mathematics classrooms, based on Torres and Bergner's (2010) model of the stages of humiliation. It analyses two examples of pre-service teachers' critical incident reflections from studies at two Australian universities. This research contributes to the frameworks through which primary pre-service teachers' mathematics anxiety, and its implications for their identity development, might be understood.

Introduction and Context

Humiliation has a potentially devastating effect on people. Torres and Bergner (2010) promote the need to identify its actual or potential occurrence, as they argue that "humiliation tends to be under-recognised, trivialized or insufficiently confronted in many kinds of settings" (p. 203). They make the distinction between embarrassment, which they regard as a minor violation of social decorum or conduct, and humiliation, which leads to loss of status, and subsequent outcomes such as feelings of hopelessness and helplessness, anxiety, powerless rage and possibly even revenge.

The paper investigates humiliation in the context of the development of mathematics anxiety (maths anxiety). Maths anxiety is recognised as important issue in mathematics education nationally and internationally (Wilson, 2012, 2015). Hembree's (1990) definition as "a general fear of contact with mathematics, including classes, homework, and tests" (p. 34) focuses on academic situations and doesn't include the aspect of ordinary life or the stress caused by being required to use mathematics in public (Tobias, 1981), both of which are important to the understanding of maths anxiety as important in terms of lifelong learning.

The theoretical framework gives an analysis of humiliation and the components/elements that make events humiliating for individuals. It presents the model of Torres and Bergner (2010).

The results section presents two examples of pre-service primary teachers (PST) who experienced humiliating events as students in mathematics classrooms.

The discussion details the effects on them as students and the way which humiliation can be so devastating for them, using the model of Torres and Bergner (2010) to examine and elucidate the impact they describe.

Theoretical Framework

Humiliation has been described in civil and legal contexts, by Torres and Bergner (2010). Adshead (2010), from her point of view as a forensic psychiatrist, followed up their article by emphasising the importance of the social context in which such humiliation occurs, and the issue of the normalisation of the behaviours involved. This is important to consider when relating these ideas to the classroom setting. Adshead (2010) views

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humiliating and bullying behaviour as a fundamental attack on narrative identity (see Sfard & Prusak, 2005). She explicates, referring to loss of face, that one's psychological face is the narrative that makes up our social identity, and therefore "events that disrupt our narrative identity cause us to lose face in front of others, and it is this disruption that is the cause of shame and humiliation" (p. 207).

One of the areas where humiliation needs to be recognised and addressed is in educational settings, and specifically in mathematics classrooms. In a previous study of narratives of events in mathematics learning that evoked emotions, Ingleton and O'Regan (2002) identified four themes that arose: "the uses of power in the classroom, the threat of exposure from being publicly shamed, the impact on the teacher's judgement on developing identity as a mathematics learner, and the role of emotions in making self-judgements and decisions" (p. 98).

In a consideration of ethical issues, Torres and Bergner (2010) suggest that it can be interpreted as a human right not to be subjected to public humiliation. In educational contests, there is a corresponding need for debate raising awareness of humiliation as an inadvertent consequence of teachers' teaching or assessment strategies in the classroom. The Convention on the Rights of the Child (Unicef, 1990) Article 28(2) holds that discipline in schools should respect children's human dignity and be administered in a manner consistent with the Convention. This has implications for the responsibility for respectful treatment of students in classrooms.

According to Torres and Bergner (2010), "humiliation has been subjected to relatively little conceptual or empirical scrutiny" (p. 195). This paper takes the formulation of humiliation that they have designed based on civil actions and law enforcement cases, and applies it to the analysis of mathematics classroom experiences recalled by PST.

The model of humiliation proposed by Torres and Bergner (2010) comprises four components or elements:

Element 1: The Status Claim

The individual attempts to claim a certain status, meaning that they presented themselves having a certain social position or as a certain kind of person.

Element 2: The Public Failure of the Status Claim

The status claim fails, and individuals failed to gain the status to which they aspired or lost their existing or perceived status. When this happens privately, the result "may be painful self-realization" (Torres & Bergner, 2010, p. 197), but in a public situation, the scenario becomes humiliating.

Element 3: The Status of the Degrader to Degrade

The person who rejects the claim has the status to do so. Their position is legitimated or valid. They may be an expert, respected or held in esteem, or in a position of power. However, in some cases, they may not have any special standing, but people's ordinary presumption is to regard statements by others as genuine until shown otherwise.

Element 4: Rejection of the Status to Claim a Status

The bid or claim for status is rejected. Humiliation results from the manner with which this is done. When not only the bid is rejected, but also the legitimate status of the person to be able to even make the bid, the result is humiliation. The humiliation is even deeper because in rejecting the person's status to make the bid, the person is held up as a pretender and denied the potential to counter claim or make subsequent claims. If their reading of reality is dismissed or invalidated the person has no voice to make any counter claims. They are powerless to claim what is important to them. This leads to the perpetration of a profound sense of powerlessness to make any claims. The denunciation without the person having the recourse to defend means that the person is deprived of their status of being one entitled to make a counterclaim

Torres and Bergner (2010) suggest that there are people that are prone to feeling humiliated. This may be a consideration in peer-to-peer interactions in educational settings. However, in extending this to the educational context, it is important to consider the power differential between children and adolescents and adults, and between students and teachers. The research question is: How might applying the Torres and Bergner (2010) model of humiliation to the classroom experiences recalled by pre-service primary teachers inform our understanding of the development of maths anxiety?

Methodology

The study is based on the understanding that people create and associate their own meanings of their interactions with the world. The research study participants were pre-service teachers from the Bachelor of Education (Primary) course at two Australian universities.

Methods

Ethics approval, based on accepted informed consent procedures, was received from the university's ethics committee. The researcher used Critical Incident Technique (Flanagan, 1954; Wilson, 2015) to investigate how PST felt about themselves as learners and future teachers of mathematics, by asking them to recall a critical incident from their school mathematics classrooms that impacted on these feelings. The two responses below were from PSTs studying initial teacher education courses at two Australian universities.

Findings

The findings from the critical experience reflections from two participants are presented below.

Cathy

Cathy was a mature-age female. She attended a traditional country primary school. She had enjoyed good experiences with her previous teachers and her reports praised her quiet and polite behaviour. In Cathy's classroom, the teacher's desk was placed diagonally in one corner of the room. The floors were wooden, noises from the corridor were easily transmitted into the room and the room was visible from outside. Cathy was very impressed by her new teacher, who was new to the school, young and attractive, and dressed fashionably. In the class, there was a focus on rote learning of times tables which Cathy was never good at, as there was never a clear explanation that she understood. The teacher commented on her tendency to daydream and lose concentration.

This new teacher liked to set a competition before letting the students go out to lunch. The class had to line up between the front groups of desks and the board. The space was not wide enough for only one line so there were usually two lines one behind the other, facing the front. Cathy often would be in the front line, which she found very uncomfortable as the students behind would be standing very close and she was very close to the teacher. Then the teacher would give times-tables questions, which the students had to answer correctly before they could go to lunch. If students answered wrongly or took too long to respond they would have to wait and be given a second or third question. This was repeated until the question was answered correctly. The students who answered their questions correctly would be very happy, and could leave the room. On one occasion, Cathy was not able to respond fast enough or not at all because she relied heavily on counting with her fingers. After she had failed to give the right answer a number of times the teacher's facial expression changed and Cathy found it unbearable to look at her.

Cathy felt the students who were leaving as they answered correctly, were looking at her and thinking she was stupid. Students outside in the playground could see those students who were still lined up, through the window. Cathy "put up a wall", avoiding the teacher's looks, but still tried to get the right answer in the right time. The teacher eventually became cross and impatient, with negative facial expressions, body language and tone of voice. Cathy shut down. She did not make eye contact. She did not react. She could feel her wall sliding up and closing her mind so that she could not respond. In this way, she tried to protect herself. Her confidence and belief in herself were shattered, and she felt like a failure from that day

Maths came to mean confusion and failure to Cathy. The incident remained with Cathy and coloured her attitude to maths. She accepted that she couldn't do maths, and assumed that she was part of the natural attrition in the mathematics classroom

John

John was a male pre-service teacher who associated maths with "being smart and successful and having advantages."

In his critical incident reflection, he wrote: "I found maths a difficult subject. When I was in year 8 I was told I had to sit permanently at the back of the classroom. I was very shy and mild mannered, I had trouble seeing the white board and understanding every single maths lesson. The room was very bare, no colourful pictures or posters on the wall. The room had a whiteboard at the front and the teacher's desk was located in front of the whiteboard. The teacher would have been in his late 40's and I believe it was his only job and he had been teaching for a very long time and I think he may have only ever taught at the one school."

The teacher often explained a problem on the whiteboard and then instructed the class to work on various activities set from a textbook. The teacher often sat at his desk after he ran through the problem on the whiteboard, on vary rare occasions would he ever walk around the room to check on work. The class was generally well behaved as the teacher was quite strict and did not allow talking. From memory, there may have been between 25 students all males. The desks were set up in a U shape with two rows in the middle of the U shape.

I had absolutely no idea what I was doing and did work on the activities however, completely had no understanding of what I was doing. On one occasion, the teacher made me complete problem in front of the entire class on the whiteboard. I had absolutely no idea what I was doing and yet the teacher still made me complete the task. I tried to attempt the problem and it made me a joke in front of all the other students. It was a humiliating and degrading experience."

As a result of this experience, when John was trying to do a maths problem during his initial teacher education course, he said that he became anxious because he thought "everyone else is finished and I should be able to do it faster", and stated, "I wouldn't be like this in another subject".

Discussion

In studying pre-service teachers' views of themselves as learners of mathematics and the critical incidents in mathematics classrooms that they recall as having an impact on these views, the author identified humiliation as an important element in the situations that these two PSTs described. The impact of this shame is evidenced not only by the language that Cathy used but also by the minute detail of her recount. The emotive language used by John emphasises the continuing impact of the incident he described.

As students, these PSTs described their sense that others were judging and deriding them and this had a negative impact on them. Hence, as students, these PST experienced what Bibby (2002) describes as a strong sense of shame. According to Munroe (2009), "a shamed child might feel that he or she is the cause of the teacher's negative attitude" (p. 65). There is no intention to claim that this is a deliberate intention on the part of the teacher. It may be that the student interprets a possibly inadvertent and accidental dismissal on the part of the teacher as criticism and rejection and is therefore humiliated. However, despite the intentions of the teacher, this may set up an association in students' minds between mathematics and public embarrassment, leading to humiliation and to a future block or choke (Beilock, 2011) and further humiliation. In turn, this can lead to a cycle of fear, failure and avoidance, as previously discussed in Wilson (2015).

According to Giddens (1991), "shame bears directly on self-identity because it is essentially anxiety about the adequacy of the narrative by means of which the individual sustains a coherent biography" (p. 65). For the individual PST who experienced situations above the result can be a sense of worthlessness, a sense of inadequacy, feel of being an imposter and not entitled to make a contribution to the discourse of the mathematics classroom. This led to emotions of anger, sadness or fear and feelings of frustration and incompetence, which were recalled vividly in their descriptions.

Table 1 shows the application of the Torres and Bergner (2010) model to studies of classroom experiences

Table 1
Applying the Torres and Bergner Model to the Classroom

Torres and Bergner (2010) Humiliation element	Related to pre-service teachers' critical incidents
Person claims status	As students in the classroom, the pre-service teachers presented themselves as a person able to do mathematics (part of their identity as a mathematics learner).
Public failure	The PST recalled that as the students they were held up in public as having experienced failure

Status of the degrader to denounce	As students, they did not question their power relationship with the teacher, and the teacher's right to make a judgement of their mathematics ability or potential
Bid for status rejected	The students were rejected as a person able to do mathematics successfully. They were emotionally involved in their experience and began or continued to reject their self-identity as a person who could do mathematics
Rejection of the individual to claim status	The teacher's interpretation or evaluation was accepted and the student has little recourse to make a counterclaim against the teacher's academic judgement and what this could mean for themselves as learners of maths

The PSTs both described situations in which they were called upon to demonstrate competency in mathematics in the classroom. In the terms of the model, this was a public demonstration, both of their claim and of their subsequent failure.

Cathy felt the students who were leaving as they answered correctly, were looking at her and thinking she was stupid, and for John, the experience "made me a joke in front of all the other students". According to Torres and Bergner (2010), "branding" such as reading out marks in public or publicly setting a student up in a remedial group, exposes the student to judgements of their peers, without any recourse. In the model, this lack of recourse is explained by the status of the degrader, and the power differential. Furthermore, the effect of the rejection of the individual's attempt on their self-identity as a learner of mathematics, in the model, makes sense of their subsequent experiences of maths anxiety As well as the public humiliation, there is the private effects as the students degrade themselves. The story relates to an inner dialogue (Sfard & Prusak, 2005) that each low grade or confusion confirms they cannot do and should avoid mathematics.

Conclusions

This paper applies Torres and Bergner's (2010) model of the stages of humiliation, to the analysis of experiences that PSTs identified as humiliating situations that they recalled in their school mathematics classrooms. This structure of humiliation, presents the factors that make a situation shameful and humiliating for a student and some of the long-term consequences of these in terms of the development or exacerbation of their maths anxiety.

This paper raises the issue of the ongoing perceptions of students of their position in the classroom. In the light of Adshead's (2010) questioning of the ethos or organisations that tacitly support bullying behaviour by "indirectly blaming the victim for being sensitive and weak" (p. 206), and excluding them, there is potential for using the Torres and Bergner's (2010) model for further research examining school and classroom cultures.

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Graphic-Rich Items within High-Stakes Tests: Indonesia National Exam (UN), PISA, and TIMSS

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In this paper, we undertake a content analysis of mathematics assessment tasks to understand how often graphical representations are embedded within high-stakes national and international tests. A total of 274 items were analysed, consisting of 160 Grade 9 UN items, 88 Grade 8 TIMSS items, and 26 PISA items. Analysis showed that all items in the PISA test were embedded with graphics, with far fewer graphical items in the TIMMS and national UN tests (47% and 33% respectively). We also found that graphical items in UN tests are distinct from PISA and TIMSS, suggesting a misalignment between what is represented in UN tests and international instruments.

Graphics-based representations are a powerful tool to communicate information (Cleveland & McGill, 1985), and a number of studies provide strong evidence for the importance of graphic representation in mathematics assessment (Kulm, Dager Wilson, & Kitchen, 2005; Lowrie & Diezmann, 2009; Lowrie, Diezmann, & Logan, 2012). The emphasis on the importance of graphical representation within mathematics and teaching has encouraged many parties to place concern on principles and standards for graphical representation within school mathematics, including assessments. Such parties include the U.S. National Council of Teachers of Mathematics (NCTM), Department of Education U.K., and Australian Association of Mathematics Teachers (AAMT). Since curriculum development is influenced by assessment practices, we investigated the characteristics of graphical items within three high-stakes tests: Indonesia National Exam (UN), Trends in International Mathematics and Science Study (TIMSS), and Programme for International Student Assessment (PISA).

Although high-stakes testing is viewed as problematic by some (Abrams, Pedulla, & Madaus, 2003), observing assessment trends has its merits. These large-scale tests are used to make important decisions that affect students, teachers, administrators, communities, schools, and districts. Graphics-based representations in international tests are of particular interest, as Lowrie and Diezmann (2009) noted that graphics-rich tasks have become increasingly used in national tests over the past decade. It is important to understand the characteristics of graphical items within high-stakes tests, as test results are used for ranking and categorising schools, teachers, and children. Results are also reported to the public as part of the accountability movement (Au, 2007).

Graphical and Non-Graphical Mathematics Items

Graphics are defined as representations used to store, understand, and communicate essential information in a visual form (Bertin, 1983). Graphics include number lines, scales, maps, charts, and Venn diagrams (Logan, Lowrie, & Diezmann, 2009). Diezmann and Lowrie (2008) divided the roles of graphics in mathematics into two categories: context and information. In the present study, we extend this classification beyond the dichotomous categories of information and contextual graphics to include items that contain both of these attributes. The new category "combination graphic" is used to signify items that have both a contextual and information graphic embedded within the item. In

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 569–576). Melbourne: MERGA.

this study, we categorise graphical items into three types: contextual, information, and combination graphics. Non-graphics items are described as items that only contain texts and/or symbols, and are categorised into two types: word problems and symbolic. Explanations and examples of each category are presented in Table 1.

Table 1
Definitions of Types of Graphics and Non-Graphics Test Items

Type		Definition	Examples in Appendix A
Graphics	Information	Conveys mathematical information that is required to solve the task (Diezmann & Lowrie, 2008)	Item 1 and 3
	Contextual	Represents objects, people or locations for illustrative purposes only, with no mathematical information related to the task (Diezmann & Lowrie, 2008)	Item 2
	Combination	Represents information that contain both the attributes of information and contextual graphics	Item 4
Non-Graphics	Word problem	Includes a text passage without a picture or graphic associated with it	Item 5
	Symbolic	Consists of numbers or symbols with short instructions, such as ‘do the following’ and ‘solve the following’	Item 6

The Nature of National and International Mathematics Assessment: UN, TIMSS, and PISA

A test is identified as a high-stakes test when its results are utilised to make critical decisions that affect test participants such as students, teachers, and administrators. It is often considered part of a policy design, and results are reported to the public. For instance, tests are administered to decide grade promotion or categorise school performances (Au, 2007; McNeil, 2000). In this study, we explored three high-stakes mathematics tests: the Indonesian national assessment (UN) and international tests TIMSS and PISA.

The UN is administered every year in Indonesia. It targets students in Grade 6 (11- to 13-year-olds), Grade 9 (14- to 16-year-olds), and Grade 12 (17- to 19-year-olds). It is intended to ensure that Indonesian education providers assess their students’ achievement against national education standards. It has been used to decide whether students can progress to the next level of schooling (Kemendikbud, 2007). Thus, UN significantly impacts classroom instruction.

TIMSS is administered every four years and targets students in Grade 4 (9- to 10-year-olds) and Grade 8 (13- to 14-year-olds). It is administered by International Association for the Evaluation of Educational Achievement (IEA), and is well known for providing international comparative assessments of educational achievement.

PISA is administered every three years, and assesses Grade 10 (15- to 16-year-olds) to determine the extent to which they have acquired key knowledge and skills that are fundamental for full participation in society. PISA is designed to assess whether students can reproduce what they have learned. It also examines how well students can extrapolate from what they have learned and apply that knowledge in unfamiliar settings, both inside and outside of school. In other words, it assesses what students can do with what they know (Organisation for Economic Co-Operation and Development [OECD], 2012).

Both PISA and TIMSS are widely accepted as performance benchmarks by participating countries (Mullis, Martin, Foy, & Arora, 2012; OECD, 2012). The key difference is that TIMSS aims to assess the coverage of mathematics curriculum of participating countries, while PISA aims to assess mathematical literacy that is considered critical for a student's life (Stacey, 2014).

Items in these three high-stakes tests have different formats. UN includes only multiple choice questions, while TIMSS includes multiple choice and short response questions. PISA items are group-based with a passage of text setting out a real-life situation (OECD, 2012). Students are required to provide their own answers. Each PISA group item can have three to four questions, and each question refers to the passage. Although TIMSS and PISA characterise item test differently, items can be categorised into four common strands: numbers, algebra, geometry, and statistics (Gronmo, Lindquist, Arora, & Mullis, 2015; Kemdikbud, 2016; OECD, 2012).

Method

This study is a part of an ongoing PhD project investigating the correlation between Indonesian students' spatial ability and mathematics performances. Mathematics performance is often measured through high-stakes tests, and the type of questions in the tests determines how much spatial reasoning is needed in solving them. For example, graphical items often require more spatial reasoning than non-graphical items (Lowrie & Diezmann, 2007). Students with better spatial ability can decode graphics relatively easy compared to those with lower spatial ability (e.g., Hegarty & Mayer, 2002; Vekiri, 2002). This led us to address the following questions:

1. What proportion of graphic and non-graphic items appears in high-stakes tests (UN, TIMSS, and PISA)?
2. What is the nature of graphical items in four different mathematical strands in these high-stakes tests?

Instruments and Procedure

To select sample tests for analysis, we mainly considered two aspects: the students' age (i.e., 14 to 16 years of age) and the time frame of the tests. UN is administered every year, TIMSS every four years (Mullis et al., 2012), and PISA every three years (OECD, 2012). Therefore ,we analysed test items from UN 2011–2014 inclusive, TIMSS 2011, and PISA 2012. As a result, the data set included 160 items from UN, 88 released items from TIMSS 2001, and 26 released main survey items from PISA 2012.

UN items were downloaded from various websites created by teachers in Indonesia. TIMSS items were retrieved from the National Center for Education Statistics website (IEA, 2013). PISA items were downloaded from the Organisation for Economic Co-operation and Development website (OECD, 2013).

Data Analysis

Content analysis was employed to classify test items. Two researchers independently coded all the items with the following procedure: First, each item test was coded as either a graphical (G) or a non-graphical (NG) item. Second, all graphical items were further assigned with one of the three codes: information graphic (IG), contextual graphic (CG), or combination graphic (MIX). Third, each of the non-graphical items were coded as word problems (WP) or symbolic (SM). An example of each type of graphic item is provided in Appendix A. Fourth, each item was classified into four strands: number, algebra, geometry, and statistics (IEA, 2013). Coded items were recorded in an Excel spreadsheet for descriptive data analysis. Coding reliability was high, with 95% agreement between the two researchers. The remaining 5% of the codes were agreed upon after discussion.

Results

Results from this study are presented in two parts, according to the research question they address. Part 1 presents the proportion of graphics and non-graphical items within three high-stakes tests. Part 2 reports the nature of graphical items in mathematical strands.

Proportion of Graphic and Non-Graphic Items by Instrument

Analysis of the data revealed a wide diversity in the proportion of graphic and non-graphic items across the three tests. Table 2 shows that graphical items accounted for approximately 33% in UN, and 47% in TIMSS, while PISA items are completely (100%) graphics-based.

The next stage of analysis revealed that UN and TIMSS graphical items were mostly information graphics, with a small proportion of combination graphics and no contextual graphics. Most PISA items were coded as combination, but there was also a reasonable proportion of information and contextual graphics. Data analysis also revealed that non-graphic items were mostly in the form of word problems. Results are displayed in Table 2.

Table 2
Item Representation across the Three High-Stakes Tests

Type of test	Graphics (%)				Non-Graphics (%)		
	IG	CG	MIX	Total	WP	SM	Total
UN (160)	32	0	1	33	53	14	67
TIMSS (88)	38	0	9	47	41	13	54
PISA (26)	27	35	38	100	0	0	0

Note: Percentage is rounded to the nearest whole number.

Graphic Items by Strand

Another level of analysis involved identifying the proportion of graphic items in each of the four mathematical strands: numbers, algebra, geometry, and statistics. As shown in Figure 1, graphical items appeared in all of the four strands.

In each instrument, geometry items were the most prevalent among the high-stakes tests, with the UN having the highest proportion of geometry items. This indicates that the Indonesian graphic items are most commonly aligned to geometry content. The proportion of graphic items within the number strand varies across the tests, with UN having the least and PISA having the most. PISA more than doubled the proportion of graphics in number items, compared to TIMSS. In the algebra strand, UN had the lower proportion compared to TIMSS and PISA, which have the same proportion. In the statistics strand, TIMMS has the highest proportion of graphics elements, followed by PISA and UN. Generally, PISA has a more balanced proportion across strands, compared to UN and TIMSS.

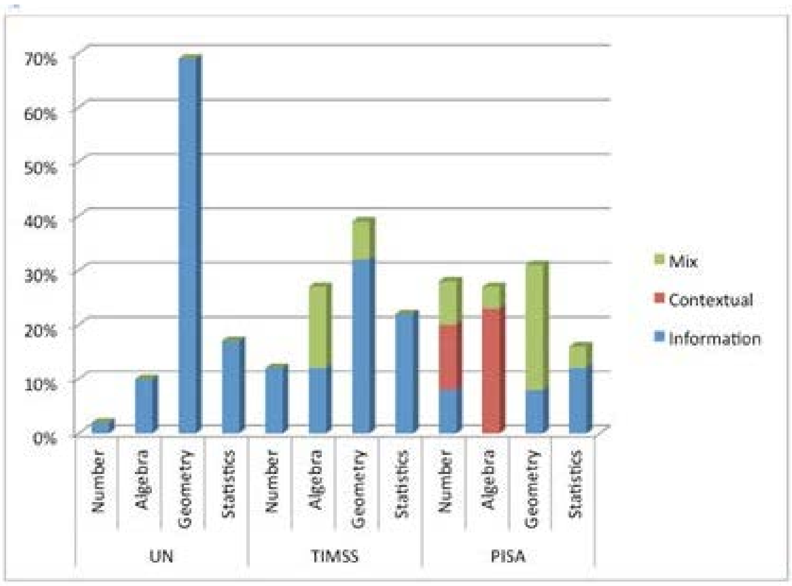

Figure 1. Proportion of graphical items in three high-stakes tests by strands and types.

Discussion and Conclusion

Our analysis of items in high-stakes tests identifies three discussion points. First, this study highlights the importance of graphics-based representations in mathematics assessments today. Content analysis showed that graphics-based items were used frequently in high-stakes tests. Overall, approximately 65% of all mathematical items in the three high-stakes tests contained a graphic. Since high-stakes tests often inform education policies (Au, 2007; Madaus, 1988), this can impact students' lives. In the case of UN tests, results can determine whether a student progresses to the next grade or graduates, so it is critical for students to learn graphical mathematics tasks (Logan et al., 2014). A rigorous design for graphical tasks in high-stakes tests should be applied, to ensure that students are not unfairly disadvantaged (Lowrie, Diezmann, & Logan, 2012).

Second, the findings suggest that Indonesian students are exposed to more word problems than graphics, despite Indonesian society using a vast array of information graphics outside of school, such as graphs, diagrams, tables, and maps. The Indonesian National Exams (UN) use the fewest graphical items, compared to TIMSS and PISA. UN items are predominantly word problems (55%) across each of the strands (number, algebra, geometry, and statistics). It is somewhat the opposite with the other two high-stakes tests, which have a lower proportion of word problems. In TIMSS, for instance, word problem items took up on average 45% of the total items, while PISA does not have any questions categorised as word problems.

Third, this study found that graphics-based items mostly appeared in geometry strands for all three high-stakes tests. This is not surprising, as school geometry is highly linked to interpreting shapes and other graphics. However, number and algebra questions in UN tests hardly contained any graphical items, compared to TIMSS and PISA. This indicates that number and algebra items in international high-stakes test often include context, while UN items tend to measure fluency (See examples in Appendix A: Item 3, Item 5, and Item 6). This pattern has been recognised by other researchers (e.g., Edo, Ilma, & Hartono, 2014), and Indonesian researchers recently advocated for the use of context with assessment (e.g., Kohar, Zulkardi, & Darmawijoyo, 2014).

This investigation highlights differences in the types of graphics used in UN tests compared to other international tests. Since UN is the national exam in Indonesia, it represents the type of content to which Indonesian students are exposed at school. The item structure of the UN test is quite different from that of international comparison tests, and Indonesian students may find it difficult to decode representations that frame mathematics thinking within contexts. As other researchers (e.g., Greenlees , 2015; Logan & Greenlees, 2008) maintained, contextual information embedded within graphics can dramatically influence sense-making. Though further research is required in this area, our findings provide some understanding of why Indonesian students perform poorly on TIMSS and PISA assessments.

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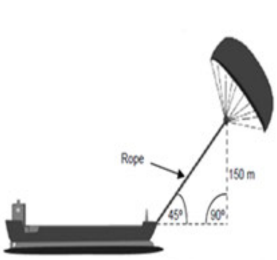
Appendix A: Examples of Graphical Type of Items

Question 3: SAILING SHIPS PM923Q03

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?

A 173 m
B 212 m
C 285 m
D 300 m



Note: Drawing not to scale.
© by skysails

SAILING SHIPS SCORING 3

QUESTION INTENT:

Description: Use Pythagorean Theorem within a real geometric context
Mathematical content area: Space and shape
Context: Scientific
Process: Employ

Item 1. PISA: Information graphics (Geometry strand).

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

Question 1: CLIMBING MOUNT FUJI

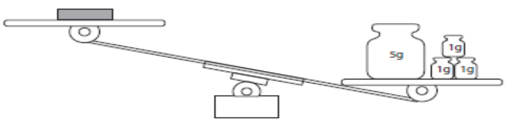
Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

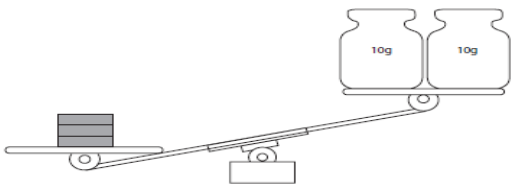
A 340
B 710
C 3400
D 7100
E 7400

Item 2. PISA: Contextual graphics (Number strand).

Jo has three metal blocks. The weight of each block is the same. When she weighed one block against 8 grams, this is what happened.



When she weighed all three blocks against 20 grams, this is what happened.



Which of the following could be the weight of one metal block?

A. 5 g
B. 6 g
C. 7 g
D. 8 g

Item 3. TIMSS: Information graphics (Algebra strand).

WHICH CAR?

Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.

Model:	Alpha	Bolte	Castel	Dezal
Year	2003	2000	2001	1999
Advertised price (zeds)	4800	4450	4250	3990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783

Question 1: WHICH CAR?

Chris wants a car that meets **all** of these conditions:

- The distance travelled is **not** higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is **not** higher than 4500 zeds.

Which car meets Chris's conditions?

A Alpha
B Bolte
C Castel
D Dezal

Item 4. TIMSS: Combination graphics (Statistics strand).

A workman cut off $\frac{1}{5}$ of a pipe. The piece he cut off was 3 meters long.

How many meters long was the original pipe?

A. 8 m
B. 12 m
C. 15 m
D. 18 m

Item 5. TIMSS: Non-graphical word problems (Number strand).

Hasil dari $\sqrt{3} \times \sqrt{8}$ adalah ....

A. $2\sqrt{6}$
B. $3\sqrt{6}$
C. $4\sqrt{3}$
D. $4\sqrt{6}$

Item 6. UN: Non-graphical symbolic (Number strand).

Pre-Service Teachers' and Tutors' Perceptions about the Value of Talk Moves

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Talk moves simulations were used in tutorials for a mathematics education unit. Pre-service teachers (PSTs) and tutors were surveyed about their perceptions of the purposes, benefits, and drawbacks of the simulations. There was strong support from both groups for the benefits of talk moves in developing PSTs' ability to manage discussions, ask good questions, and understand students' thinking. Tutors were more inclined than PSTs to note improvements to PSTs' mathematical knowledge. Challenges to implementation were authentic engagement in the simulations, PSTs' lack of experience with children, the cognitive load associated with managing discussions, and limited mathematical knowledge.

Providers of Initial Teacher Education (ITE) in Australia are under continuous scrutiny (Louden, 2008), exemplified by recent public demands that the institutions prepare "classroom ready" graduates (Teacher Education Ministerial Advisory Group, 2014). This challenge is particularly difficult in primary mathematics education, as a significant proportion of students enter courses with modest levels of achievement, which in turn impacts their attitudes and beliefs about mathematics and their personal confidence for teaching the discipline (Hine, 2015; White, Way, Perry, & Southwell, 2006). It is also recognised that subject knowledge alone is insufficient for classroom readiness. The pedagogical content knowledge required for teaching mathematics is complex and connected (Beswick & Goos, 2012; Depaepe, Verschaffel, & Kelchtermans, 2013; Hurrell, 2013). Furthermore, graduate teachers of mathematics face a plethora of demands including participation in communities of practice (Alvalos, 2011), demonstration of adaptive expertise (Anthony, Hunter, & Hunter, 2015), and iterative inquiry into their own praxis (Kazemi & Hubbard, 2008). Therefore, educators in ITE need to provide learning opportunities for their students (i.e., PSTs) that meet these multiple demands. In this paper, I report on PSTs' and tutors' perceptions about the value of talk moves simulations within tutorials of an ITE mathematics education unit.

Background Literature

In his recent meta-analysis of research into discourse in mathematics education, Ryve (2011) stated that discourse is underpinned by three principles: (a) Language constitutes and builds ideas, (b) Discourses construct versions of reality reflective of social objectives, and (c) Meaning is co-constructed with others though talk. Consistent with these principles, participation in discussions that promote mathematical understanding involves students both explaining their ideas and actively engaging with the ideas of others. Explanation and justification of their own ideas helps students to reflect on, monitor, and refine their ideas while analysis of others' ideas prompts students to broaden their ideas and develop their identities as participants in a mathematical community (Hiebert & Grouws, 2007). Productive talk is essential to providing effective opportunities to learn mathematics (Walshaw & Anthony, 2008).

The management of classroom discourse creates pedagogical tensions, mostly between the achievement of social outcomes and mathematical outcomes (Sherin, 2002). Two

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 577–584). Melbourne: MERGA.

approaches used by researchers to support teachers are to establish rules for participation and to provide tools for orchestrating discussion (Franke et al., 2015). Talk moves (Chapin, O'Connor, & Anderson, 2009; Michaels & O'Connor, 2013) are a set of actions that a teacher may use in managing discussion, though the authors also address norms for participation in their later work. The "moves" have both function and form, in that they are observable actions with goals for creating "academically productive talk". In Table 1, I summarise the five original talk moves (Chapin et al., 2009) that were used in this study and the corresponding goal for using each move.

Table 1
Five Original Talk Moves (Chapin et al., 2009)

Talk Move	Example	Goal
Revoicing	"You said you did/thought X. Is that right?"	A student clarifies her/his own thinking
Repeating	"Can you repeat what M said?"	A student expresses her/his interpretation of another student's thinking
Eliciting	"Do you agree or disagree with what M said? Why?"	A student analyses the validity of another student's thinking
Adding One	"Would you like to add on to what M said?"	A student expands on the thinking of another student
Wait Time	Teachers waits for an extended period before expecting a response from students	Students get space to reflect on their own thinking and that of other students

Anthony et al. (2015) used talk moves successfully to develop the adaptive expertise of PSTs in New Zealand, while Michaels and O'Connor (2013) used their framework in the professional development of teachers in mathematics and science. In this small-scale study based in Australia, PSTs' and tutors' perspectives about the value of talk moves simulations within tutorials were examined.

Methodology

The opportunity to use talk moves arose within an undergraduate mathematics education unit taught to third-year primary Bachelor of Education PSTs. The 195 PSTs who enrolled in the unit attended a large city campus in Melbourne, Australia. Prior to the unit, the PSTs studied three units in mathematics education, two designed to develop their personal knowledge of mathematics, and the other aimed at developing their capacity and confidence for teaching mathematics. Hence, the PSTs in this research were studying their final unit in a suite of four units. This unit focused on the learning and teaching of challenging concepts such as rational number, decimals, proportional reasoning, probability, and algebraic thinking at the primary school level. While in their second semester of their third year, these PSTs had limited experience on placement in schools, having spent 10 days as "observer/helpers" as first-year students, and only 15 days on placement in their second year.

Talk moves formed an integral part of nine concept-based tutorials, out of 12 tutorials for the whole unit. For approximately 45 minutes, the PSTs attempted two different simulations in each tutorial. One PST acted as the teacher while four other PSTs role-played the part of primary-aged children. In the first three simulations, a fifth student monitored the teacher PST's frequency of using different talk moves using a checklist. An indicative example of a simulation task based on proportional reasoning is given in Figure 1. In a simulation, the teacher PST set up any materials s/he needed while the student PSTs read and rehearsed their strategies. A role-play then occurred in which the teacher PST used talk moves to manage the discourse, eliciting and responding to the strategies of the students. The role of the student PSTs was to stay true to the way of thinking of a student who used the strategy that each participant had received on an allocated card. Tutors used variable ways to support the simulations such as modelling themselves "fish bowling", an interesting scenario within one group, and inviting PSTs to record their reflections of the activity.

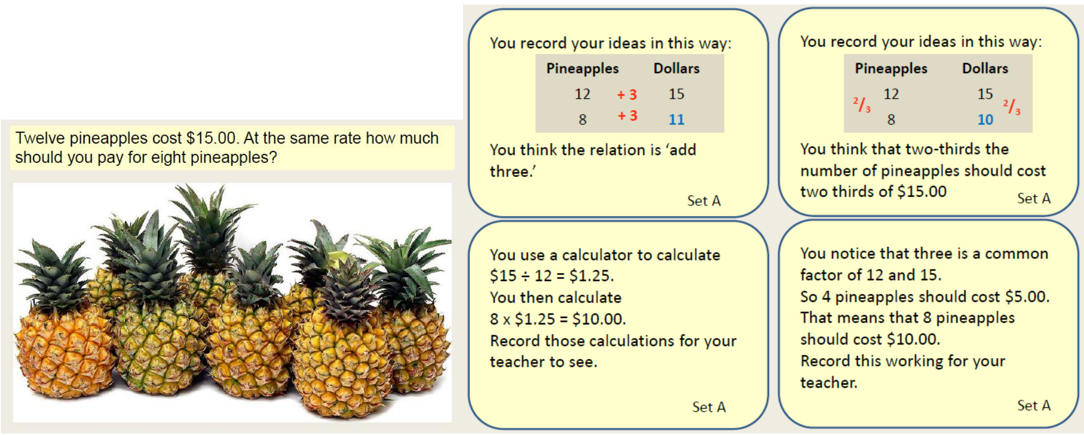

Figure 1. Example of talk moves simulation task.

The research question for this study was "What are PSTs' and tutors' perceptions about the purposes, benefits, and negatives of using talk moves as a tutorial activity?" At the end of the final lecture, all PSTs in attendance were invited to complete a survey. Participation was voluntary and anonymous. The survey consisted of four statements with five-point Likert-scale response options (strongly agree, agree, uncertain, disagree, and strongly disagree). The four statements were:

1. Talk moves helped me to engage students in productive mathematical discussion.
2. Talk moves made me aware of the mathematical thinking that primary-aged students might use.
3. Talk moves improved my questioning skills.
4. Talk moves improved my own mathematical understanding.

In addition, PSTs were provided an open prompt to which to respond: "Use this space to make any comment you want about the value of talk moves to your development as a teacher of mathematics."

Six tutors, five of whom were sessional academics, taught in the unit. Three of these tutors were in their first year of university teaching, two of them had three or four years of experience while the Lecturer in Charge was a very experienced academic. Tutors were asked to complete a survey at the end of semester consisting of open responses to four questions.

1. What is the purpose of using talk moves as an instructional tool with PSTs?
2. What do you consider to be the main benefits and negatives of using talk moves?
3. How do you anticipate that PSTs will evaluate the usefulness of talk moves?
4. If you were to modify talk moves, as it was used this semester, what would you change and why?

The rationale for Question 3 was to investigate if the tutors' expectations of the PSTs' perceptions of talk moves were consistent with what PSTs actually reported.

Results

Seventy-five of the 195 (38%) PSTs who completed the unit provided survey responses. High proportions of respondents agreed or strongly agreed that the use of talk moves in tutorials had helped them engage their students in discussion (92%), made them more aware of students' thinking (88%), improved their questioning skills (87%), and improved their own mathematical understanding (72%). Though the proportions in the complementary categories were small (8%, 12%, 13%, and 28%, respectively for the four statements), a lower proportion of the participants agreed that talk moves had improved their personal mathematical knowledge.

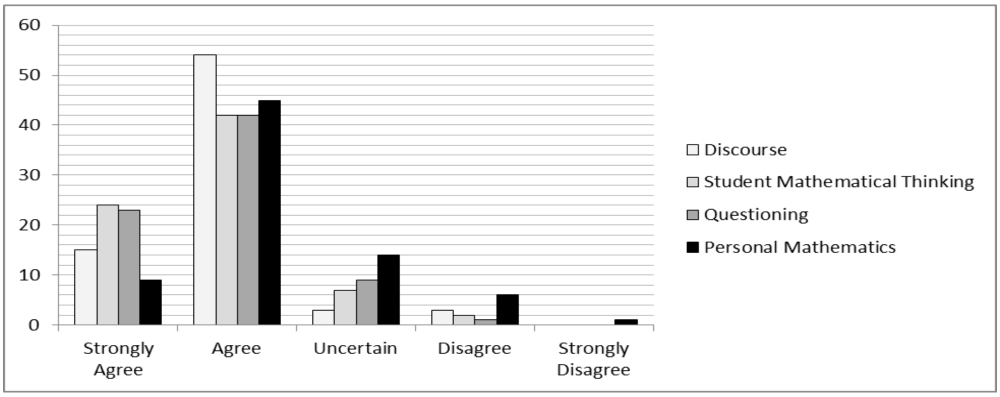

Figure 2. PSTs' responses to Likert-scale questions.

Fifty-one of the 75 respondents (68%) provided a comment in the open section of the survey. The comments aligned with two categories: references to the development of personal abilities and advice about talk moves as an activity within tutorials. Table 1 contains the frequency of comment types by these categories. The tenor of responses to the Likert-scale questions was also reflected in open comments about the development of personal abilities. Understanding children's thinking, developing questioning and other scaffolding strategies, orchestrating discussion, working with small groups, and modelling for students were reported, as highlighted by the below sample responses.

> Talk moves gave me great insights into students' possible thinking and how to work at possible misconceptions and getting children to help each other.

> Talk moves allowed me to use effective questioning to gain insight into student thinking.

> Helpful in making mathematics lessons more of a social experience, and creating authentic conversation that helps build and develop ideas.

Few PSTs reported development of their personal knowledge of mathematics, or of their ability to support their students with the mathematical processes (proficiencies).

The "advice" category highlighted some of the issues related to enacting talk moves in the artificial setting of a university tutorial. PSTs commented that the value of a simulation was dependent on the authentic engagement of their peers, particularly in acting out the roles of primary-aged children, and that this role-playing was difficult, given their lack of experience with children on school placements. These ideas are discussed in the responses provided by the PSTs:

> Wonderful to be able to practise in tutorials each week. Of course it is dependent on students engaging with the activities appropriately – In my case it went really well!

> It is hard to practice [*sic*] talk moves when there is [*sic*] no students, rather peers.

> The talk moves activities only worked well when peers took the tasks seriously. Often there were times when others couldn't be bothered and so rather than actually practising the talk moves they would just get each student to read out what their 'student' did.

Some PSTs felt the need for tutor modelling on enacting talk moves early in the first tutorials, and they provided suggestions regarding some simulation tasks and spending less time overall on the activity.

Table 2
PSTs' Open Comments

Personal Abilities	Frequency	Advice	Frequency
Children's thinking	13	Challenging at first	3
Questioning/Scaffolding	12	Dependent on positive engagement by peers	3
Importance for classroom readiness	8	Difficult without "real" students	3
Discussion	4	Some simulations better than others	3
Personal learning of mathematics	3	Too much time/Too many tutorials	2
Modelling	4	Need for tutor modelling first	3
Small group	3	Needs to be applied in practice	1
Mathematical processes	1		
Total	40	Total	18

All six tutors who taught the unit agreed to complete the survey. Tutors' beliefs about the purpose of using talk moves generally matched the improvements to personal abilities reported by the students. Purposes discussed by the tutors were the development of understanding of children's thinking (n = 6), and classroom discourse (n = 5), and PSTs' mathematical knowledge (n = 2). A sample of responses is provided next.

> It gives them a framework to explore concepts and develops their own understanding.

> The scenarios given to PSTs, with pre-empted misconceptions and achievements, are highly beneficial as PSTs at this stage of their degree often lack the classroom experience to consider the wide variances that might occur.

Purposes discussed by only one of the tutors each were questioning, improving achievement, facilitating co-operative learning, representation of concepts, and making learning "visible". Tutors were also asked about the positives and negatives of using talk moves in tutorials (see Table 3). The development of PSTs' personal mathematical knowledge was given as a strong positive by four tutors, although only two had listed it as a purpose. The positive impact on students' ability to understand students' thinking, to question, and to effectively orchestrate classroom discourse was aligned with the benefits also reported by the PSTs. Tutors' beliefs about the negative aspects of the simulation did support those of a few students about the need for authentic and equitable participation by PSTs in role-playing within the simulation. One tutor reported occasions when the tasks failed:

> Really only with PSTs who did not embrace it fully. Maybe were embarrassed? But some PSTs found it difficult to engage in the student's method/thinking. May not have been used as effectively as it could have.

However, tutors noted other issues related to implementation such as the challenge talk moves presented to PSTs' cognitive load, understanding of given strategies, and time to explore the simulations adequately. One tutor questioned her own preparedness to model talk moves and another suggested other possible frameworks for classroom interaction.

Table 3
Tutors' Beliefs about the Positives and Negatives of Using Talk Moves

Positives	Frequency	Negatives	Frequency
PSTs' mathematical knowledge	4	Difficulties assuming roles (authenticity)	4
Understanding students' ways of thinking	3	Cognitive load of teaching situation	2
Developing questioning	2	PSTs not understanding strategies	1
Developing discourse	2	Equitable opportunities to play different roles	1
Encouraging active participation in tutorials	2	Inadequate time	1
Illustrating specific mathematical concept	2	Not all possible student responses covered	1
Simulating real classroom	1	Ineffective tutor introduction	1
		Other discourse frameworks exist	1
Total	16	Total	12

Tutors were asked to anticipate PSTs' evaluation of the talk moves simulations. The tutors' comments aligned well with the focus on discourse, questioning, and thinking given by the PSTs. However, four tutors expected that PSTs would acknowledge that the real benefits of talk moves would be most visible in the next school placement. Three tutors anticipated acknowledgement of how the simulations built up PSTs' ability for informal assessment of specific mathematical concepts. Neither expectation was evident in PSTs' open comments.

Asked how they would change talk moves-based tutorials in the following year, tutors provided a range of practical suggestions, including tutor modelling of teacher-student interaction in the early tutorials ($n = 2$), flexible balancing of PST groups ($n = 2$), linking to WALTs (outcomes; $n = 1$), showcasing skills as well as strategies ($n = 1$), providing talk moves charts ($n = 1$), setting norms for participation ($n = 1$), and providing videos of real primary-aged students ($n = 1$). One tutor felt that the talk moves simulations should be left "as is". In general, the tutors' suggestions amounted to fine-tuning the talk moves simulations.

Discussion

There was strong agreement between the perceptions of PSTs and tutors about the positive contribution of the talk moves simulations to PSTs' abilities to manage classroom discourse and to engage with students' thinking. The main differences in perception were that PSTs put more emphasis on their improvements of questioning skills while tutors noted the observed effects on PSTs' personal mathematical knowledge. Gains in conceptual knowledge are not discussed in the literature about talk moves but emerged as a significant benefit.

Both PSTs and tutors provided useful suggestions regarding implementation. Both groups mentioned the importance of authentic engagement from the PSTs involved, and tutors suggested that they would manipulate groupings of PSTs to improve group dynamics in future. The challenges of talk moves simulations were attributed differently by the two groups. PSTs viewed their ability to assume the roles of children as their major challenge, presumably due to a lack of classroom experience. In contrast, tutors attributed the challenge to the cognitive load associated with the PSTs "teachers" managing interactions among their "students" and to gaps in mathematical knowledge prohibiting PSTs making sense of strategies. Both groups promoted the need for tutors to model talk moves in early tutorials, and one tutor commented on her personal lack of confidence to do so. Tutors expected PSTs to comment that the benefits of the simulations would be more visible on placement but such comments were not provided. PSTs did not share their tutors' perception of the value of talk moves in terms of preparing them for informal assessment.

Talk moves simulations were generally perceived as beneficial, though the artificial setting of university tutorials, coupled with the lack of classroom experience of PSTs, raised some barriers to implementation. The lack of placement experience provided to PSTs in this pre-service program appeared to work directly against attempts by tutors to make the tutorial offerings relevant to their students' needs and to prepare "classroom ready" graduates. Talk moves simulations may result in more benefits to practicing teachers who have the experience to take up role-playing with greater authenticity.

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SYMPOSIA

STEM Professional Learning: Evaluating Secondary School Teachers' and Students' Experiences

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The four papers presented in this symposium report on the evaluation strategies and feedback from teachers as they embarked on designing and implementing STEM approaches to learning in secondary school contexts. The STEM professional learning program was designed to provide time and expert support so that cross-disciplinary school teams of up to six teachers from science, mathematics and technology/engineering could develop new school-based initiatives. A range of evaluation strategies were used in the first two STEM Academy programs to identify factors and approaches that supported teachers' and students' needs, and to further enhance the STEM Academy program. This symposium addresses ways in which the progressive evaluations have informed this change process.

Paper 1: Judy Anderson, The University of Sydney. *The STEM Teacher Enrichment Academy Approach*

Paper 2: Kathryn Holmes, Western Sydney University. *Evaluation of the First STEM Teacher Enrichment Academy*

Paper 3: Gaye Williams, Deakin University. *The Second STEM Teacher Enrichment Academy Evaluation: Teachers' and Students' Perspectives*

Paper 4: Debbie Tully, The University of Sydney. *Developing an Evaluation Framework for Future STEM Academies*

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The STEM Teacher Enrichment Academy Approach

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The STEM Teacher Enrichment Academy was developed to promote the STEM subjects in schools so that more students engage with, and consider pursuing a STEM-based career. Since teachers are key to student engagement and interest, a professional learning program for multi-disciplinary school teams was developed to support teachers in identifying and designing the most appropriate STEM approach for their students. Offered for the first time in 2014, the Academy has been implemented four times, reaching 260 secondary teachers from 47 schools. Feedback using a range of data collection tools has enabled the evolution of the program to better address school, teacher and student needs.

With a global decline in students enrolling in mathematics and science subjects at the senior secondary and tertiary levels (Kennedy, Lyons, & Quinn, 2014), and predictions that we will need many more mathematicians and scientists to meet workplace demands of STEM (Science, Technology, Engineering, and Mathematics) related professionals into the future (Office of the Chief Scientist, 2016), school systems and other stakeholders have embarked on developing new approaches to promoting STEM. To build on, and coordinate the range of reforms, a STEM Education Forum was held in Sydney in 2015 to develop a National STEM School Education Strategy, 2016-2026 (National Council, 2015). Driven by the two key goals of wanting all students to finish school with strong foundational STEM skills and capabilities, and ensuring all students want to embark on more challenging STEM subjects, the Strategy identified five key areas for national action:

1. increasing student STEM ability, engagement, participation and aspiration;
2. increasing teacher capacity and STEM teaching quality;
3. supporting STEM education opportunities within school systems;
4. facilitating effective partnerships with tertiary education providers, business and industry; and
5. building a strong evidence base.

It is noted in the Strategy that these key actions relate to both the individual STEM subjects as well as to any integrated approaches to STEM education.

While many factors influence student participation as measured through subject choice and subject engagement in secondary schooling, McPhan, Morony, Pegg, Cooksey and Lynch (2008) determined the lower participation of students in senior mathematics was particularly influenced by poor pedagogical practices, perceived level of difficulty, and irrelevance. Traditional approaches to teaching mathematics and science do not capture the multi-disciplinary nature of contemporary mathematics and science practices (Tytler, Symington, & Smith, 2011) and their connections to the systems thinking, design thinking, or computational thinking of engineering and technology (Bybee, 2013; English, 2016). Proponents of integrated STEM curriculum argue for its potential to increase student motivation and engagement (Beane, 1993); to enable students to transfer knowledge, make connections and see the relevance of the STEM subjects (English, 2016); to develop students' "STEM literacy" and their understanding of local and global challenges (Bybee, 2013); and to provide an impetus towards the further study of STEM subjects in senior schooling and STEM degrees at university (Freeman, Marginson, & Tytler, 2015).

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To date, there has been little research conducted into the efficacy of STEM subject integration in secondary classrooms (Bruder & Prescott, 2013), but there is some evidence to suggest that STEM integration is successful in increasing student engagement within mathematics classrooms (Venville, Wallace, Rennie, & Malone, 1998). Based on the assumption that students benefit from opportunities to connect knowledge across the curriculum, a professional learning approach was developed to support teachers in planning and implementing connected approaches in secondary schools. The design of the Academy program was informed by research into effective professional learning practices but with little research available about the best approaches to integrated STEM learning more generally, the program has evolved based on feedback from teachers' experiences.

The Initial Academy Design

In 2014, the Faculty of Education and Social Work collaborated with the Faculties of Science, and Engineering and Information Technology, to develop the initial program of teacher enrichment and professional development. The multi-day on campus program for up to 75 teachers (from 12 schools) of Year 7-10 mathematics, science and technology/engineering was designed to be foundational in enhancing teachers' knowledge of content and pedagogy, inspiring them to reinvigorate their classroom practice and improve student engagement in STEM subjects. The overall Academy aims were to:

- introduce and support exciting and effective approaches to learning, enhance teachers' knowledge of content and approaches to teaching mathematics, science and digital technologies in Years 7-10;
- develop a community of practice for participating STEM teachers, with ongoing support and engagement through mentoring, online forums, newsletters, seminars and events; and
- develop teachers' knowledge of STEM-related research and industry as well as knowledge of STEM programs at university and career pathways.

Modelled on commonly agreed core features, the Academy professional learning approach was developed to incorporate a content focus, active learning, coherence, duration and collective participation (Desimone, 2009). With a focus on examining content and processes from the STEM subjects, Academy sessions were facilitated by the University's academic specialists and STEM leaders, as well as teacher/peer-led sessions. The program involved a three-day on campus program at the University followed by up to two full school terms working on developing, planning and implementing STEM strategies in school-based teams. Teachers then returned for a further two-day program at the University to share their experiences, present evidence of teacher and student learning, discuss issues and challenges, and consider future initiatives. Each cross-disciplinary school team of two mathematics, two science and two technology teachers worked together to develop inquiry-based learning approaches to teaching both within their subject discipline as well as across the subject disciplines (Maaß & Artigue, 2013). Initially focusing on the individual STEM subjects was adopted because mathematics and science teachers made limited use of inquiry-based learning approaches in lessons that is recommended in curriculum documents and in research into meaningful learning (Sullivan, 2011; Tytler, 2007).

The First and Second Academies

For the first Academy, 64 teachers from 13 schools visited the University in November 2014 and returned in March 2015 (see Table 1 for sector representation) – schools were invited to participate based on engagement with the University. While most schools are Sydney based, four are clustered near Mudgee in the central West of NSW. This small country hub of schools enabled greater opportunity for collegiality, an essential ingredient given the small size of these schools with some teachers reporting feeling isolated and with limited access to quality professional learning. Like the first Academy, the second involved 70 teachers from 12 schools with a country hub of two larger schools from Wagga Wagga (see Table 1) and took place in November 2015 with a subsequent return to the University in May 2016. When selecting each group of schools, we sought diversity in school systems, socio-economic status, gender composition, and size to further expose teachers to the range of issues involved in curriculum redesign and promote community engagement.

Table 1
School Sector Representation for the First Two STEM Academies Including School Gender Composition

	Department of Education	Catholic	Independent	Total
2014/15	8 (1 all girls)	1	4 (2 all boys, 2 all girls)	13
2015/16	7 (1 all boys)	2 (1 all girls)	3 (1 all boys, 1 all girls)	12

While overall the feedback from teachers has been positive, the key challenges to be addressed based on the first two academies included implementing inquiry-based learning approaches in regular classrooms, understanding the connections between the separate STEM subjects, working effectively in school teams, designing a STEM strategy most suitable for school contexts, and building the community of practice. Further detail about the evaluations of each of these first two academies is presented in the second and third papers in this symposium.

Our experiences from both academies revealed some schools move more quickly to developing integrated STEM approaches because of experiences prior to academy participation of writing integrated units of work, and/or working together as a team. This highlighted the diversity of teachers' knowledge and experiences of integrated STEM before coming to the Academy and the influence this had on their progress within the Academy. Some teams were cohesive while others were dominated by one or two teachers who already had a plan which would be implemented regardless, while others had never worked together on creative programming and curriculum design. It became clear that we needed to conduct school audits of their STEM work as well as to consider teachers' experiences of working together before they arrived to participate in the program.

Team building and effective whole school planning have now become critical components of the Academy and these begin with each school before they attend the first session at the University. On site, preliminary planning meetings include the school principal and other school leaders who need to play a key role in supporting the development of STEM initiatives which frequently have implications for timetabling, teacher allocation to classes, alignment of STEM subjects on timetable lines, and resourcing. Schools have adopted a wide variety of approaches to implementing STEM education – frequently these decisions have been based on available personnel, teacher interest and resources but school structures can act as impediments to innovative practices.

Because the schools were so diverse, particularly in relation to teachers from different subjects working together, the approaches they initially adopted were equally disparate. From embedding more cross-curriculum applications within regular lessons to conducting cross-disciplinary investigations in several STEM subject lessons, schools adapted and designed their approaches around perceived student needs sometimes finding lateral ways to overcome constraints from school structures and resources. Our purposeful tolerance for such diversity acknowledges that schools need to consider the needs of their students, the competence and interest of teachers, the overwhelming influence of siloed assessment in many schools, and that real change takes time.

Building the community of practice has been a challenge. While on campus at the university, teachers willingly discussed ideas with teachers from other schools, and engaged in worthwhile sharing of ideas but the busyness of school life frequently meant little ongoing sharing in the online community. In some schools, finding time to meet as a school team was enough of a challenge and proved to be an inhibiting factor in moving plans forward. To alleviate some of these challenges, for schools to become STEM Academy participants, we had requested principals provide time for teachers to work on their projects. Unfortunately, this was not always achieved and some academy teachers have admitted this is as much they themselves not wanting to take time away from something else. Teachers being provided with school time to work on their projects, and accepting to do so remains another challenge to be addressed.

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Evaluation of the First STEM Teacher Enrichment Academy

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The initial STEM Teacher Enrichment Academy was held in 2014/2015 involving 64 teachers from 13 schools. The teachers attended two on-campus sessions which bookended their STEM work in schools supported by Academy mentors and online interactions using Edmodo. In survey responses, the teachers reported that the Academy had extended their pedagogical knowledge for engaging students in STEM and to a lesser extent their STEM content knowledge, however they all valued the networking opportunities afforded by the Academy. Despite their enthusiasm for implementing new STEM activities in their schools, teachers were significantly challenged by a lack of time to plan adequately.

The initial STEM Teacher Enrichment Academy (2014/2015) was designed to improve STEM teachers' capacity to plan and implement engaging STEM lessons for Years 7 to 10, provide opportunities for within and cross-school collaboration and to improve teachers' knowledge of STEM industries and tertiary STEM study options. The Academy pre-empted the National STEM School Education Strategy, 2016-2026 (National Council, 2015) but its goals were closely aligned with the five areas for action identified later in the Strategy. In this sense, the Academy has acted as a forerunner in providing leadership and direction for STEM teacher development in recent years.

Currently teachers and school systems are grappling with forecasts for new student skill sets requiring integrated models of curriculum development (ITL Research, 2011), particularly in the STEM areas where skill shortages are predicted. The National STEM School Education Strategy identifies the need for students to develop scientific, mathematical and technological literacy along with 21st century skills such as problem solving, critical analysis and creative thinking (National Council, 2015). These 'softer skills' are also embedded in the cross-curriculum capabilities identified in the Australian Curriculum (ACARA, 2015). The initial STEM Academy was designed with these advances in mind, aiming to prepare teachers to foster inquiry approaches to the teaching of STEM subjects so that students' interest in STEM and development of '21st century skills' could flourish.

The Initial STEM Academy Evaluation Design

The evaluation of the initial STEM Academy was designed as a mixed-method, survey and interview study. All teachers (n = 64) completed surveys during the on-campus sessions and a sample of volunteer teachers (n = 22) also participated in telephone interviews after the second on-campus session. The three Academy mentors also took part in a telephone interview at the end of the Academy.

The survey instrument was designed to provide a measure of teachers' existing pedagogical practices prior and subsequent to participating in the Academy. The survey items were derived from the Innovative Teaching and Learning survey instrument (ITL Research, 2011) which provides measures of teachers' propensity to use pedagogies focussed on real world problem solving and student collaboration. The teachers also responded to open-ended questions about how best to engage students in STEM. The mathematics teachers also completed an additional survey focussed on their beliefs about

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the discipline of mathematics and pedagogical approaches. After each on-campus session the teachers completed additional surveys asking them to rate the various components of the session and to reflect on the impact of the Academy in their schools

Evaluation Results

The Academy was attended by 64 teachers from 13 schools (24 mathematics, 23 science, and 17 technology). Approximately 60% of the teachers were female and the mean number of years of teaching experience was 17.9 years (*SD* = 9.4). The schools represented a range of school systems (eight government, four non-government, and one Catholic), primarily from the Sydney metropolitan area with only three provincial schools.

A key aim of the Academy was to increase teachers' capacity to engage students in STEM subjects in school. At the beginning of the first on campus session, teachers (n = 64) were asked to respond to the following open-ended question: What are the best ways for teachers to promote student engagement? Their responses emphasised the importance of focussing on real-life examples that are relevant to students' lives (n = 20), using hands-on activities and ICT where appropriate (n = 13), building a respectful, positive classroom environment (n = 9), facilitating an inquiry based approach to learning (n = 7), and student collaboration and group work (n = 7). Although the teachers emphasised the value of using real-life examples, few of the teachers indicated that they regularly planned for real-world connections in the classroom, such as allowing students to consult with experts outside of the school setting, involve parents or community members in school activities, listen to guest speakers or produce something for use outside of the classroom. Therefore, prior to the Academy, despite many teachers acknowledging the value of real-world problem solving as a means of engaging students, it seemed that few teachers actively planned for real-word interactions in their classrooms. Teachers were more likely to plan for student collaboration in the classroom, however, the mathematics teachers were less likely to do so in comparison to the science and technology teachers.

When teachers were asked about the value of the various components of the Academy program they consistently responded that they felt that they had extended their knowledge of pedagogical approaches for teaching STEM and were excited and enthusiastic about trialling their new teaching approaches with their students back in their schools. They were less positive about the degree to which their knowledge of STEM content had increased. They greatly valued the networking opportunities with other teachers and tended to value the collaborative on-campus sessions more highly than plenary/guest speaker sessions.

> I think from talking to the teachers at other schools and getting ideas and sharing ideas and just that was just really the best thing. There were things I came away with or things I could help people with that you know, it would take a lot of time of your own to be able to achieve. (Science Teacher, Non-government school)

While the teachers were in their schools in between the two on-campus sessions they were visited by an Academy mentor and invited to participate in an online community via the Edmodo platform. The mathematics (7.6/10) and technology teachers (7.4/10) valued their interactions with their mentors more highly than the science teachers (5.6/10), perhaps reflecting the degree to which a successful mentor/mentee relationship depends on the personnel involved. The teachers appreciated the outsider expertise, a sounding board, confirmation, insight and advice, although there were a noteworthy number indicating that they didn't really need a mentor, all from schools fitting a higher SES school category. In contrast, there was a substantial number who indicated that they would have appreciated more visits or contact with their mentor. Many of these schools were from the lower SES spectrum of schools. When interviewed the mentors emphasised the importance of timely visits to schools and the importance of school support for teachers as they planned and implemented their STEM activities.

> So the fact that you do have mentors is a massive plus in the program. I'm just wondering as we go forward, do we actually need more mentors so that perhaps a contact could be a little bit more consistent or a little bit more often, and whether or not it's the actual teachers who've been involved in this first round –could see how that would be a great advantage if that could happen down the track." (STEM Teacher, Catholic School)

An Edmodo site was established to facilitate communication and resource sharing amongst STEM Academy participants. Forty teachers replied in the survey that they had used the site, but of those teachers only 10 replied that they had used the site frequently. Thirteen teachers reported that they did not use the Edmodo site at all. Teachers who used the site regularly did so because of the great resources being shared. Those that didn't use the site, or who used it infrequently, said in the survey that a lack of time was the main reason for not doing so and some experienced problems navigating to the group pages within Edmodo.

> That really made me realise how important it is to keep in contact with other maths teachers and the Edmodo page that the STEM Academy came up with is great and I've actually joined other Edmodo pages for maths teachers as a way of – I guess being isolated, that's one way that I can keep in contact with other maths teachers and share ideas and things like that (Mathematics Teacher, Government school).

After the second on campus session the teachers were asked for their reflections on the Academy in terms of the impact within their schools. On the positive side, the teachers said that there was improved student engagement, more use of interdisciplinary projects (although not in mathematics), and increased enjoyment of mathematics, particularly for girls, possibly due to the use of more challenging problems. When asked about the challenges they encountered, overwhelmingly the teachers cited the lack of time that they had for planning, the difficulty in involving other staff in their schools, lack of resourcing for new equipment and in some cases, a perceived lack of support from senior staff in their schools.

> The students were engaged before, but I think that they are engaged in a slightly different way because I'm asking the questions, the students are wanting to find the answers. Instead of just wanting to be successful, they want to find an answer, so I think there is a slight difference there in just that whole thing. I think that enquiry process is starting to take hold, if you know what I mean (Science Teacher, Non-government school).

Conclusion

Overall, the first STEM Teacher Enrichment Academy was positively received by the participant teachers. However, based on the evaluation conducted several recommendations were made:

1. to plan for more time in interdisciplinary groups working on real world problem tasks;
2. to include more planning time within the residential schedule so that teachers would be more prepared to implement STEM activities in their schools;
3. to reconsider the use of Edmodo to encourage teachers to use the site more frequently and to view the site as a rich source of resources for sharing;
4. to expand the use of mentors within the Academy to ensure that teachers can access this expertise in a timely manner;

5. to provide teachers with strategies for including and enthusing other staff in their schools to use the STEM activities; and
6. to target disadvantaged and/or isolated schools, where possible, as these teachers appeared to gain the most benefit from the Academy.

On the basis of the recommendations from the evaluation, several changes were implemented to better support teachers learning and to enhance the learning experiences of students. The Expression of Interest template for the second Academy required schools to nominate a STEM Leader who would manage team meetings, encourage teachers to use Edmodo for sharing experiences, support teachers to consider using the Academy experience for accreditation requirements, and to coordinate the STEM school team's presentation and final report to the Academy. In addition, after schools were selected, each school site was visited by a member of the Academy team to meet members of the STEM team as well as school Executive members to ensure all were clear about Academy requirements before the program began. They were encouraged to pre-plan before coming to the first on-campus session by examining each subject area's scope and sequence, identifying common content and processes, comparing assessment requirements, and considering when and how they would develop a cross-disciplinary approach to STEM teaching and learning.

During the on-campus session, schools were provided with more time to work in school teams to develop cross-disciplinary programs, mentors played an active role in working with the school teams and then visiting each school during the two on-campus sessions. Some schools from the first Academy were also invited to attend the second Academy to share their experiences and provide feedback on early plans. This approach extended the network of STEM schools and helped to build the community of practice. One of the challenges remaining in the Academy program is to consider ways to sustain the early STEM work in each school and to scale the approach to more teachers in each of the subject areas.

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The Second STEM Teacher Enrichment Academy Evaluation: Teachers' and Students' Perspectives

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This paper briefly describes the aims and research design for the 2015-2016 STEM Teacher Enrichment Academy Evaluation and gives detail about the research design for the case study reported herein. A subset of findings from the case study school, developed through analysis of student and teacher interviews, is reported to highlight STEM program strengths and how the Academy was perceived as contributing to what was achieved. Student interviews showed the development or strengthening of STEM students' career aspirations.

The underlying intentions of STEM education are to engage students in STEM subjects, so they develop deep understandings, and draw flexibly on what they know when exploring unfamiliar situations (Freeman, Marginson, & Tytler, 2015). The Melbourne Declaration of Educational Goals for Young Australians (Ministerial Council on Education, Employment, Training and Youth Affairs, 2008) shares these goals: "Successful Learners… are creative, innovative and resourceful, and are able to solve problems in ways that draw upon a range of learning areas and disciplines" (p. 8). As Anderson (Paper 1 in this symposium) states, we need to know more about effects of interdisciplinary STEM education on outcomes for students. The findings reported herein from the STEM Teacher Enrichment Academy Evaluation 2015-2016 (undertaken by Tytler and Williams, Deakin University) add to the body of knowledge on how interdisciplinary STEM Education might influence the potential for STEM career aspirations of students.

Foci of the Evaluation

The second STEM Academy Evaluation was designed to find out more about how the STEM Academy experience: (a) led to changes in pedagogical practices, (b) supported the design and implementation of interdisciplinary projects, (c) encouraged collaboration between teachers in STEM disciplines within and beyond the school, and (d) influenced student engagement with STEM subjects. This paper focuses within that evaluation on: "What influences does the model of interdisciplinary STEM education, as developed and implemented in the case study reported, have on student engagement in STEM subjects?"

Research Design Elements

Design of the evaluation. Data for the broader study included document analyses, survey and questionnaire responses, field notes including photos, and interviews with various stakeholders. Both the Component Mapping teacher survey and the Change in Teacher Practices Questionnaire (administered after each of the two workshops) informed probing in interviews and foci of attention for STEM class observation. The survey (Tytler et al., 2004) and the questionnaire (Williams, 2000) were refined from research tools previously developed by evaluation team members. The survey captures where teachers perceive they lay along various evidence based spectra of high quality pedagogical practices. The Change in Teacher Practices Questionnaire was adapted from a

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questionnaire on student learning change during problem solving employed previously by Williams (e.g., Williams, 2000)). Teachers were asked to identify what they had learnt from Academy participation, and what had influenced that learning. This questionnaire and interviews with teachers and students provided opportunities for them to identify what they attended to in STEM Ed in the school rather than only respond to researcher identified elements.

Case study design. The case study design as implemented included observations of STEM Ed activity (in STEM-Tech and STEM-Maths classes), and interviews with various school community members: a) school leaders directly involved in supporting STEM initiatives; b) Academy teachers (which included school leaders); and c) six students from STEM classes (three boys, three girls) who were identified by the teachers as demonstrating engagement with STEM subjects. Engagement was also identified through teacher and student interviews, and observed body language in interviews and classroom observations through focus of attention, body direction, lack of awareness of activity external to their focus, connecting to the ideas of others, and exclamations (EyDUPLEx Framework: Williams, 2003). The interviews included probes about what had influenced new learning that occurred. This helped identify links between engagement and creative STEM activity.

STEM-ED

The school's Year 7/8 STEM program (STEM Ed) commenced two years prior to Academy participation. It was funded by the supportive principal who had faith in the primary STEM-ED instigator (now Vice Principal: VP), who developed the program in conjunction with the heads of technology and science. The team perceived benefits from joining the Academy: feelings of increased obligation (thus motivation) to succeed because they were given this opportunity, increased confidence that the innovation was worthwhile because the Academy selected them, access to a network of schools with similar interests and challenges through the Academy, raised STEM education profile at school (which encouraged more teachers to commit to STEM), valuable PD for staff, and additional time for team planning.

Given timetable and resource constraints in this small school, STEM-ED was located in the classes of each discipline, renamed STEM-Tech, STEM-Science, and STEM-Maths. Activity undertaken in these classes was interdisciplinary. For example, during data collection for the case study, the Year 7/8 STEM-Ed group were undertaking a multi-disciplinary project on Space, and Rockets. Each of STEM-Tech, STEM-Science, and STEM-Maths focused within this topic in ways that interconnected the STEM subjects.

STEM-Tech groups of three or four built rockets to launch to test their capacity. The teacher posed questions rather than gave answers when students struggled to find ways to proceed. Student 1's (S1's) eyes lit up and his voice became animated (demonstrating engagement with STEM-Ed) as he described this program: "It's more open [than other subjects] ... you got to think out ways to do it for yourself rather than be taught a certain way by the teacher". A rocket design sketch (with design features justified), and rocket construction are shown (Figure 1, left and right, respectively). Learning included: "centre of mass and ... thrust need to align so rocket does not tip ... wings need to be down low as stabilisers" (S1).

STEM-Science students tested materials to help make decisions about what materials to use for their rocket: "We were testing how fire-resistant leather was – put leather over a Bunsen burner and it shrivelled up still intact" [S3]. They also studied the Solar System.

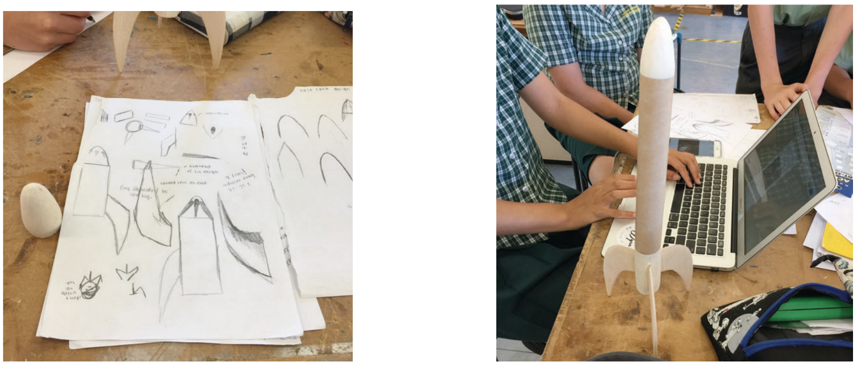

Figure 1. STEM-Tech design of one student group and rocket assembly of another group.

STEM-Maths students in groups of four, allocated group roles. They made a calendar for Mars using conversions from Earth to Mars time. The teacher stimulated class discussion by raising questions from these discussions, for students to discuss and resolve.

> The number of months were completely up for grabs for a start- we talked about that and well okay Mars has how many moons ... [and] does Mars have seasons and what are seasons and do they occur on every planet and we found that Mars does have solstices and equinoxes but they do not quarter the year as ours do ... it was a class conversation I ran and there were times where I spotted the questions to ask and they discussed and ran with that. [Teacher of Class, Vice Principal]

Group roles included a "think big" group member who selected a focus to explore beyond the core requirements of the project. S1 decided to explore Mars leap years.

Design Thinking Embedded in STEM-ED

All six students drew attention to autonomy enabled in STEM-Ed and the focus on learning for a purpose rather than just learning for assessment. Learning in STEM-ED was described in various ways: "Compared to learning in other classrooms this [STEM-ED] tries to use your brain more, it challenges you more" [S2]. 'Using what is learnt rather than just learning it' was a common comment made:

> I just like the whole STEM program because you don't just get to learn stuff in the classroom and not do anything with it- in STEM you learn information but then you get to put it into a practical use in Tech [S6]

The STEM-ED team devoted time and energy to familiarising other teachers with Design Thinking. Student comments showed Design Thinking was embedded in STEM-ED. They either made explicit references to it or described the process in activity reported:

> liked the immersion ... how you do things ... the design where you get told everything then do two designs on paper if possible a scale and then pick one of them and that usually takes most of the time ... then testing and readjusting (S4: female student)

S6 captures the cyclical nature of the process: trying to solve a problem within certain constraints, testing products to work out how to proceed in environmentally friendly ways:

> we are given like strict materials that we can use ... it helps us to think about 'how to use this in a productive way like an *effective* way so that we don't *waste* any materials but ... if it doesn't work we try not to use more materials but if we have to use more materials then we will but we are trying to make it as effective as we can

The design process appeared crucial to changed aspirations of students not previously considering STEM careers.

STEM Career Aspirations

This different way of learning shifted the career aspirations of the three girls. The boys and one girl (S6) were already interested in at least one STEM discipline at the start of secondary school. S6 was interested in Science, but her greater interest was English at the start of secondary school. The boys displayed their interest through their voices becoming more intense and their faces more animated as they discussed various STEM-ED activities. STEM-ED changed the career aspirations of all students interviewed with each of them shifting more towards pursuing STEM careers, For example, S4:

> "[In primary school, I preferred] Art [as] more creative but ... now with science not so right and wrong ... more like creativity ... I like Art still ... but Sci and Tech I like ... now and careers in that area."

S6 who wanted to be a teacher or missionary nurse included engineering as an aspiration after the excursion to the university drew her attention to engineers helping others:

> I wasn't really thinking about being an engineer in primary school but then when we had the STEM-ED camp ... visit[ed] the university and we got to see ... different engineers and what projects they do ... it really like *inspired* me because it ... showed me that we don't just learn this stuff in school- and then not use it in real life- you can ... use the information you have learnt ... to make a *difference*

Concluding Comments

This study shows that this interdisciplinary STEM education model that physically located classes in each discipline, while employing interdisciplinary projects that emphasised design thinking, achieved student outcomes consistent with the broader STEM agenda. The processes employed increased/strengthened STEM career aspirations for both boys and for girls. Creative opportunities involving design thinking (S4, S5) and raised awareness that engineering can involve helping others (S6) influenced these changes. The University of Sydney STEM Teacher Enrichment Academy contributed to team opportunities to achieve these changes by affirming the directions the team were taking, raising the STEM school profile, and resourcing the project. Preliminary outcomes for students interviewed indicate that selection by the Academy and the way teachers have used this opportunity has raised student awareness of STEM and stimulated interest in it. The team look forward to finding which senior secondary subjects STEM-ED students will select.

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Developing an Evaluation Framework for Future STEM Academies

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Building upon prior evaluations, a comprehensive evaluation plan for the STEM Teacher Enrichment Academy at the University of Sydney is currently being devised and will consider the multiple perspectives of teachers, students and school leaders, and the interplay between these key stakeholders as it affects program outcomes. Additionally, effects of parents and industry partners will also be examined. The evaluation will follow a mixed-methods protocol with the additional collection of pertinent school-level data of all participating schools. Evaluation results will not only prove beneficial in shaping future academies but will also add to the literature in this growing field of academic research.

The evaluations conducted as part of the first and second STEM Teacher Enrichment Academies offered key formative assessment data that proved useful in improving the structure and content of the academy as seen through the lens of the academy participants. As we move forward in developing an evaluation framework for future academies, our focus will become more outward with the aim of evaluating not only the initial objectives of the academy but also keys goals as outlined by the National STEM School Education Strategy (National Council, 2015).

The professional development offered through the STEM Teacher Enrichment Academy endeavours to increase teachers' pedagogical content expertise through guiding teams of teachers in their development and delivery of integrated STEM units of study within each of their schools. While it is anticipated that this approach would inspire teachers to expand their own personal interests in STEM, for some teachers this team-styled approach towards creating and disseminating integrated STEM content in their classrooms may be a novel experience affecting their personal beliefs towards teaching, as well as their understanding and knowledge towards their subject area. The literature is replete with examples of how teacher beliefs and self-efficacy significantly impact student learning and achievement, and we assume likewise in our evaluation (Tschannen-Moran & Barr, 2004).

As the need for an expanded STEM workforce grows, the role that teachers play towards encouraging students to pursue STEM fields of study appears critical. In building an evaluation model, emphasis will be given not only to teachers' perceptions of their own capabilities and beliefs both individually and collectively, but also towards their capacity in affecting students' interests, motivation, 21st century learning and future career choices in STEM (Betz & Hackett, 1986; Dick & Rallis, 1991; DuFour & DuFour, 2010).

One of the major outcomes envisioned of the STEM Teacher Enrichment Academy is an eventual interest and increase in student participation in STEM occupations. Accordingly, an assumed pathway towards that goal is increased student engagement with senior school mathematics, science and technology subjects. The attitudes students possess towards STEM are a significant factor in not only influencing future STEM subject choice but also in students' pursuit of STEM related careers (Maltese & Tai, 2011), and as such become an important area for examination in our evaluation plan.

Additionally, school principals are appropriately considered as drivers who may influence the effectiveness of professional development programs aimed at student success and teacher growth in STEM education (Prinseley & Johnston, 2015). Without the advocacy, vision and leadership of principals in STEM curricular efforts, the impact of the

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STEM Teacher Enrichment Academy's professional development program in any one school may be short-lived. The academy aims to provide teachers with the tools, resources and knowledge to affect positive change in their classroom teaching through encouraging collaborative hands-on STEM learning in a "real-world" context. Yet, the leadership, condition and culture within each particular school context may also influence the eventual outcomes of this STEM focused professional development program. Therefore, when evaluating the program effectiveness of the STEM Teacher Enrichment Academy it seems essential to consider and understand the multiple perspectives of teachers, students and school leaders, and the interplay between these key stakeholders as it affects program outcomes.

Designing an Evaluation Framework for the STEM Teacher Enrichment Academy

Integrated STEM teaching and learning is a new endeavour for many Australian schools, and as such their effects are under-researched. While the quality and scope of STEM education evaluation schemas varies widely (Brody, 2006), the conclusions drawn by Bryk and his fellow researchers (2010) offer a concise reflection on the multiple factors considered as pivotal in successful science and mathematics educational programs: (a) school leadership as an impetus for change, (b) professional capacity of faculty through engagement with professional development, changes in values and beliefs, and the ability for collaboration amongst faculty, (c) outreach that strengthens the ties between parents, community, academic institutions and industry, (d) student-centred learning environments, and (e) instructional guidance that advances learning. In addition to these attributes, two specific STEM evaluation models with both theoretical and systems-based approaches, offer beneficial insight as we devise our own evaluative strategies (Arshvansky et al., 2014; Saxton et al., 2013).

Our specific evaluation plan for the STEM Teacher Enrichment Academy is designed as a mixed-methods protocol, with both survey and interview components, and will measure outcomes for principals/school leaders, teachers and students based on key program objectives. Additionally, effects of parents and industry partners will also be examined (see Figure 1).

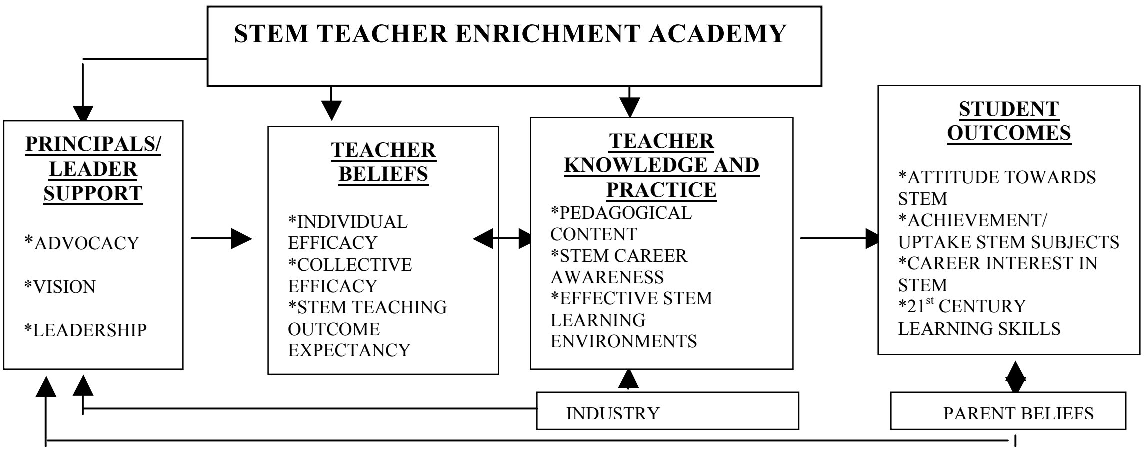

Figure 1. STEM Teacher Enrichment Academy comprehensive evaluation model.

Principals/School Leaders. The survey items designed to measure the advocacy, vision and leadership of school principals are derived from the P-STEM survey instrument which specifically measures the leadership offered by school principals towards STEM education within their specific school contexts (Friday Institute for Educational Innovation, 2014). Using a 5-point Likert scale, principals indicate their response to items such as, "Regarding the STEM work at my school, I… enable collaboration of teachers across content areas…. ensure technical support/other resources are available for STEM teaching…maintain strategic partnerships with STEM industries". Reliability testing has produced a Cronbach's alpha above 0.90 for this instrument. The 37-item survey will be administered to principals prior to their school's enrolment in the Academy and one year after their school's participation, and will also serve as a formative reflection for principals. Additionally, principals will be interviewed to capture a more nuanced understanding of the specific adaptive STEM culture within their schools, and their partnerships with STEM specific industries.

Teachers. The teacher questionnaire will consist of items from the T-STEM survey instrument (Friday Institute for Educational Innovation, 2012b) and the Collective Teacher Efficacy Measure (Goddard, Hoy, & Hoy, 2000). The T-STEM survey assesses teachers' personal STEM teaching efficacy beliefs, STEM teaching outcome expectancy beliefs, reflection on STEM instruction, 21st century learning attitudes, teacher leadership attitudes and STEM career awareness. Reliability testing on each of these scales has produced Cronbach's alphas ranging from 0.814 to 0.948. The Collective Teacher Efficacy Measure is comprised of four subscales that assess group competence and task analysis. The survey will also contain four reflective open-ended prompts. The teacher questionnaire will be administered during the on-campus components of the academy and one year post-academy participation. Additionally, teachers will be interviewed at the conclusion of their school's involvement with the academy to probe the STEM initiatives in their school, their perceptions of the impact of the STEM Teacher Enrichment Academy, adjustments made to their school's curriculum to accommodate STEM teaching and learning, their involvement with communities of practice, partnerships with industry, and efforts to sustain STEM initiatives in their schools.

Students. A student questionnaire will be devised of items from the S-STEM survey (Friday Institute for Educational Innovation, 2012a) and the STEM Semantics survey (Tyler-Wood, Knezek, & Christensen, 2010). The S-STEM survey measures student attitudes in mathematics, science and technology, future career interest, self-assessment of current achievement across STEM subjects, future projected STEM subject uptake and 21st century skills. CFA Goodness of Fit Indices and Cronbach's alphas indicate a high level of validity and reliability of this instrument. The STEM Semantics survey assesses student perceptions and attitudes towards each of the separate STEM disciplines in which students indicate on a scale of 1 to 7, for example, their opinion of science as fascinating (1) to mundane (7). Surveys will be administered to students prior to and after their engagement with integrated STEM teaching and learning.

Parents. Parents are certainly influential in their children's career selection, particularly for students choosing an engineering or science pathway (Dick & Rallis, 1991). The STEM-CAT survey (White, 2015) will assess parents' beliefs about STEM education, their values towards STEM education and their perception of the resources that their child's school offers in STEM education.

School Level Data. As the numbers of schools and educators who participate in the STEM Teacher Enrichment Academy grows, so does the need to create a database in order to track the long-term impact of student and teacher engagement with integrated STEM teaching and learning. Data to be collected will include school level data such as demographics of student body, school affiliation, offerings and enrolments in STEM subjects, and faculty level data.

The data collected as part of the STEM Teacher Enrichment Academy evaluation plan will allow us to assess both short and long-term effects of the academy. Principal, teacher and student surveys will be administered both pre-test and post-test allowing for comparison through repeated measures statistical testing. Further inferential testing may reveal connections across the participant data. Interviews will further elucidate quantitative findings. The data gathered through the STEM Teacher Enrichment Academy evaluation will not only prove beneficial in shaping future academies but will also add to the literature in this growing field of academic research.

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Transitions in Mathematics Education

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Introduction to the theme: The Merriam-Webster dictionary defines *transition* as: (a) the passage from one state, stage, subject, or place to another: change; or (b) a movement, development, or evolution from one form, stage, or style to another. These imply that the word transition can refer to an active shift of the person in space and time or status, for example; it can also refer to developments taking place within the person. Transitions may be anticipated by those involved, and hence planned for, or they may result from unexpected crises in people's lives and are likely to require innovative and sensitive solutions. In this symposium, we will be considering transitions in mathematics education affecting both students and teachers, and these can occur at various points throughout a person's educational trajectory. Here, we include student development across primary, secondary, and tertiary sectors; also studies of transitions made between cultural groups and the teaching/learning of high functioning students with ASD.

In *Transitions in Language Use in Primary School Online Mathematical Problem Solving*, Duncan Symons and Robyn Pierce adopt a Bakhtinian lens to examine upper primary school students' use of informal and formal language registers in CSCL mathematical problem-solving. They argue that online discussion assists in development of mathematical language as demonstrated by students' use of a *transitional mathematical register* combining new mathematical words with their own natural language.

In *Mathematical Writing and Writing Mathematics: The Transition from Secondary to University Mathematics*, Caroline Bardini and Robyn Pierce present a framework based on their research on students' use and understanding of mathematical symbols, recognised as crucial in students' successful transition from school to university mathematics. In particular, the framework supports a fine-grained analysis allowing better appreciation and understanding of the subtle differences in students' experiences with symbolic expressions.

In *The Valuing of Deep Learning Strategies in Mathematics by Immigrant, First-generation, and Australia-born Students: Transitions between Cultural Worlds*, Abi Brooker, Marian Mahat, and Wee Tiong Seah take an intercultural approach to propose a research framework that facilitates investigation of the mathematics learning experiences of the many students who move from one culture to another. They investigate how students' family and school cultural environments contribute to their engagement with and performance in mathematics, and the influence of this transition on their experiences.

In *Supporting Mathematics Students with Autism Spectrum Disorders through the Lens of Teacher and Student Values: A Research Framework for Teacher Transformation*, Monica Carr and Wee Tiong Seah note that most intervention research relating to Autism Spectrum Disorder (ASD) has been conducted with lower functioning students with skill deficits. By examining what high functioning ASD students value in mathematics teaching and learning, their study aims to enable teachers and students to transition beyond notions of associating ASD with deficits, and also contribute to improvements in mathematics teaching and learning outcomes for this growing population of learners.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (p. 603). Melbourne: MERGA.

Transitions in Language Use in Primary School Online Mathematical Problem Solving

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Online text discussion highlights “teachable moments”. A Bakhtinian lens is used to examine upper primary school students’ use of informal and formal language registers in mathematical problem solving in a Computer Supported Collaborative Learning (CSCL) environment. The authors argue that online discussion assists in the development of mathematical language as the data shows evidence of students’ use of a ‘transitional mathematical register’ combining new words discussed in class with their own words that the “community of practice” would not think of as part of mathematical vocabulary.

Transitional Mathematical Register

Students in the upper primary years of schooling are in a state of flux or transition (Attard, 2010; Downs, 2003). At this age (11 to 12 years), students are preparing for the transition from primary school to secondary school as well as the social and physiological transition from childhood to adolescence. Both Attard (2010) and Downs (2003) note that the transition to secondary school requires students to negotiate new social, organisational and academic structures. Academic expectations also change at this point. In mathematics, this includes the use of mathematical language.

Barwell (2012) describes this as a transition from an informal to a formal mathematical register where “register” is understood in the sense that it is used by Halliday (1978, p. 195) as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings.” For students working mathematically at this level, the informal register encompasses everyday words such as “going” or “pointy,” while “multiply”, “equation,” and “median” lie in the formal register.

In this paper, we report on an investigation into the notion that a *Transitional Mathematical Register* (TMR) exists between the informal and formal registers. We demonstrate the interplay, and movement between registers in students’ use of language by analysing two short excerpts of online discussion between students tasked with solving problems by using mathematics.

Theoretical Framework

This study draws on Barwell’s (2012) application of the Bakhtinian (1981) dialogic perspective as a means to expose the tensions that exist between informal and formal mathematical language. He demonstrates that the formal mathematical register is privileged throughout international curricula by pointing to a tendency of these documents to require simple, informal mathematical language to describe mathematical ideas in the earlier years of schooling, whilst working towards usage of the “formal mathematical register.” He argues that informal and formal registers are always required and are always in tension.

Barwell (2012) suggests that privileging the formal mathematical register within the curriculum is not ideal, because it places greater importance on the *correct* use of mathematical language at the potential expense of meaning making. Bakhtin’s (1981) view

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 604–607). Melbourne: MERGA.

of language was that it is situated, dynamic, and dialogic. He sees languages as being either unified (unitary) or, to use his term, in a state of *heteroglossia*. The theoretically complete formal mathematical register can be seen as a unified language. The tensions and centripetal forces that exist within curricula, our schools and educational institutions, and society, expect a single, agreed upon language and register for the teaching and learning of mathematics. Valuing only this unitary language may inhibit students' experimentation and trialling of new and unfamiliar language. As a theoretical construct, there is a place for the formal mathematical register, however in our lived reality it is unlikely to truly exist.

Context of the Research

Students in the present study worked in a Computer Supported Collaborative Learning (CSCL) environment. The work reported here took place in a larger project conducted in an Australian primary school. Thirty-eight percent of students attending the school are from a language background other than English. This has implications for working in a CSCL environment which places demands on students' abilities in the area of literacy. Participants were 54 Year 5 students (10 to 12 years old) allocated to ten online, mixed ability groups. Over the ten weeks in which the unit was delivered, students collaboratively solved and/or investigated nine mathematical problems incorporating an aspect of each content strand of the Australian Curriculum: Number & Algebra, Statistics & Probability, and Measurement & Geometry (ACARA, 2014).

Students were expected to engage in iterative asynchronous online discourse where they would build on each other's ideas. No online adult facilitator took part in the CSCL. This decision was taken in order to avoid discussion between students being heavily influenced by an "expert" other. Each week for the first seven weeks, prior to students commencing work online, an hour of standard classroom discussion was facilitated by the first author of this paper. This time was spent with the class performing three tasks. First discussing expectations of behaviour, and appropriate approaches to collaboration within the online space. Second reviewing the previous week's solutions and discussing students' perceived challenges and successes then finally reading through and discussing the following week's problem.

The first piece of data presented (on *Wallpaper Symmetry*) was facilitated using this approach, however the second piece of discussion data (on geese "V patterns"), was from the ninth week of the intervention. At this stage, instead of providing the students with classroom support, they were expected to use this hour to work in the online space. They received no specific advice from the facilitator. They were only allowed to communicate in the online environment (they were not permitted to speak to each other). We were interested in gaining an understanding about whether the students would be able to apply the language and problem-solving strategies without any teacher support.

Findings

Attempts at conveying mathematical ideas or concepts have been bolded, highlighting attempted use of formal or informal mathematical registers. In *Wallpaper Symmetry* students were asked to represent line/ mirror symmetry, rotational symmetry and translational symmetry using Microsoft Word and making use of the shapes provided. They uploaded their file and then described how symmetries were used in their wallpaper.

Olivia: i think to **work this out** we would need to **choose a shape** with the **pointy sides** (dont really know how to say it) so it would be easier with for us to do it does anyone agree with me?
Chris: What **shape** is everyone deciding on. i was thinking of a **hecsigon**
Olivia: i **changed it** i have **done a triangle** i **created something like a fan** so when it **spins** you could see **the pattern** and also **it would never changes** i have uploaded mine to edmodo.
Zander: **Mirror/Line Symmetry- line symmetry** means when you have a **shape** or anything, and you **cut it in half, it looks exactly the same size and lining on every single thing as the other side.**
Rotation Symmetry- rotational symetry means, depending on **how many pointy sides they have**, say for example, i had a **plus sign +,** it has **4 pointy sides**. So then, after you **move it 4 times**, it **goes back to the same spot**.
Reflection Symmetry- reflection symmetry means if you have a **picture of your face, you keep drawing that, making look the same height, the same lengh**, and etc.

Figure 1. Discussion of Wallpaper Symmetry Problem.

In Figure 1 we see examples of students struggling to express their mathematical thinking. Olivia's opening statement suggests a tension caused by her desire to use the formal mathematical register while lacking the words to do so. Her language is disjointed but when read carefully we see that Olivia is developing an understanding that rotational symmetry means a shape will look identical when rotated on its axis the number of degrees corresponding to its order of rotation. She does not have the words to describe this accurately but is able to make her meaning known through her use of the TMR.

It is interesting to note that Olivia's choice of words to represent mathematical thinking evolves and becomes a shared language through dialogue with other students in her group. Zander adopts her TMR phrase to progress the discussion. This shared language to create shared meaning exhibits Barwell's (2012) interpretation of Bakhtinian dialogism.

In Figure 2 (Modelling Middle School Mathematics, 2014) students explored the *V pattern* made by flocks of geese as they fly. The discussion below, between Indigo and Maddie was typical of discussion that occurred during this investigation.

Indigo: The **rule** is it is **going up by twos** as an **odd number** so instead of the **simple 2 4 6 8** it is **1 3 5 7 9** etc

Indigo: and my **formula** I am still **working out**

Indigo: OK I have **found** a **formula**! What I did was (say the **square** was **b2** and the **number** in it was **three**) I did **=b2+2** because **3 plus 2 is five** which is the **next equation** in the **pattern**. It **goes up by two every time** so that would be the **formula**.

Maddie: Do you know how to **drag down the numbers** so you can **go to 100**?

Maddie: That is very good Indigo. I liked how you explained the **formation**.

Indigo: thank you Maddie

Figure 2. Discussion of Geese Problem.

In Figure 2, we see Indigo experimenting with the identification of growing patterns. She is able to establish that each successive term changes by the same amount as the preceding term. Her mathematical vocabulary is not yet developed enough to express herself using formal vocabulary. However, she is able to establish meaning through her utterances. In the study, we saw many examples of students using the phrase *going up by*, which, while not part of the formal mathematical register, is helpful in the meaning making process. A tension exists between Indigo's desire to communicate her ideas and her lack of approved formal words. Indigo's emergent language provides another example of a TMR.

Discussion and Implications

Upper primary school is one bridging point in student mathematical language development. No longer are students only required to use the language of basic place value and four operations, they must begin to develop language for more sophisticated concepts; such as algebraic and relational thinking. While the goal is their use of formal mathematical language, students make sense of these new concepts through appropriating familiar language in combination with the new formal vocabulary. This hybrid language, or *TMR*, allows students to reason and communicate their emerging understandings.

The dialogic nature of language is also evident in this data. Students who have been exposed to new terms in the classroom use a variation of this formal mathematical vocabulary within the CSCL environment. The language is used with various degrees of precision. This data supports Barwell's (2012) contention that insisting on use of the formal mathematical register rather than acknowledging this transition phase could be counter-productive. We see students' use of the TMR as evidence of "teachable moments" when students' correct ideas should be validated but formal language modelled without any suggestion that the student is in some way 'wrong' so that the dialogic cycle may continue to have impact.

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Mathematical Writing and Writing Mathematics: The Transition from Secondary to University Mathematics

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This paper reports findings from research on students' use and understanding of mathematical symbols, which has been recognised as playing a major role in students' success in mathematics. One goal is to identify potential trouble spots for the usage of symbols as students travel the path between school and university mathematics. We present a framework that supports a fine-grained analysis allowing us to better apprehend the subtle differences in students' experiences with symbolic expressions. This is illustrated by considering some first-year mathematics students' written responses to questions and insights from interviews with senior secondary teachers and university lecturers and tutors.

Students' use and understanding of mathematical symbols, what we call 'symbolic literacy' (Bardini, Pierce& Vincent, 2015) plays a major role in students' success in mathematics. Gaining fluency with symbols is especially important at university, when not only does mathematics become much more symbolic, but its writing is more subtle and requires increased 'flexibility' from the reader. This paper reports findings from an on-going three-year project on students' symbolic literacy. One of the project's goals is to identify and investigate potential trouble spots for the usage of symbols as students travel the path between school mathematics and mathematical sciences subjects at university. We present a theoretical framework that has been shown to support a fine-grained analysis allowing us to better apprehend the subtle differences in students' experiences with symbolic expressions between their previous encounter in school and their new journey at university. This is illustrated with three examples and finally some implications for teaching are suggested.

Literature Review

The transition from school to tertiary mathematics has been studied from various perspectives. Thomas (2008) summaries some of these in the introduction to a special edition of the Mathematics Education Research Journal. He notes that learning in mathematics progresses along a spiral path where students revisit concepts from new perspectives. This seems to be challenging for many students: de Guzmán et al. (1998) observed that to make the transition from school to tertiary mathematics students need to organise their knowledge to allow a global perspective supporting making connections, modifying views and adapting to new domains. Reporting on a recent study comparing mathematics in the secondary and tertiary contexts, Corriveau (2017) found that "even if the same concepts are used at both levels e.g. concept of domain, they are conceptualised differently" (p. 156). This study seeks to identify some of the obstacles students encounter in revising or expanding their thinking.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 608–611). Melbourne: MERGA.

This Study: Methodology and Theoretical Framework

Methodology

In this study, data were collected from 279 first-year mathematics students at three Victorian universities. All students were enrolled in a subject with the prerequisite of Year 12 Mathematics Methods or equivalent. Participants responded to fortnightly surveys, during tutorials, posing a probing mathematical question designed to gauge their "symbolic literacy" according to the topic they were studying. Students' responses have been categorized and responses analysed for likely links to students' past mathematical experience.

We also interviewed experienced senior secondary school teachers, university lecturers and tutors (from four Victorian universities) asking them about their students' difficulties writing and understanding symbolic mathematics. Transcripts of these interviews have been analysed by the research team in order to identify themes in their responses.

Framework

The framework that we have applied to our analysis of the symbols of concern to students and their teachers is based on the work of Serfati (2005), who provides us with an epistemological approach to mathematical notations that takes into account both the syntactical properties of a symbol and the mathematical concept(s) conveyed.

Serfati's work advocates that we consider three distinguishing features of any symbolic expression. In our simplified version of his approach we describe these components as:

- the *materiality*. The materiality of a symbol focuses on its 'physical' attributes (what it looks like). A classic example is the = sign. Materiality includes the category the symbol belongs to (letter, numeral, specific shape, conjunction etc.).
- the *syntax*. The syntax of a symbol relates to the rules it must obey in symbolic writing. This includes the number of operands for symbols standing for operators but also the appropriateness of placing certain symbols adjacent to one another.
- the *meaning*. The meaning of the symbol is the concept being conveyed, for example the representation of an unknown or of a given operation. Meaning for Serfati is that commonly agreed by the community of mathematicians and it does not refer to a person's individual understanding.

Results and Discussion

First, we saw evidence of students strongly attached to fixed materiality so that if the symbol, letter, used is changed then the student does not recognise the syntax or the meaning of the expression. In statistics staff commented for example that students do not recognise linear functions expressed using letters or the arrangement other than y=mx+c for example $y = \hat{a} + \hat{b}x$ or not using y and x.

Tutor MB001: …straight line-all the schools say now that's x, y… they say y=mx+c… but in the university level, or the mathematical convention is y=a+bx. We put the constant first…that's the same. A lot of students couldn't see that's equivalent.

In the example shown in Figure 1 students were required to “realise” the denominator. While students concerned knew the technique of multiplying by the complex conjugate, a new skill, they did not recognise the difference of two squares formed in the denominator.

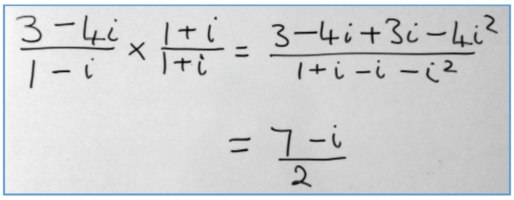

Figure 1. Student solution not recognising difference of two squares.

In later discussion, the tutor commented that many students only recognised $(a\text{-}b)(a+b) = a^2 – b^2$. The change in materiality impeded recognition of a helpful pattern.

Second, we saw evidence of students unthinkingly applying syntax templates (Bardini et al., 2015) that they have met before but without consideration of the context or the domain in which they are working. Asked to explain the meaning of -1 for in $\sin^{-1}(x)$ students responded in the variety of ways shown in Table 1.

Table 1

Students’ Meanings for -1 in $\sin^{-1}x$ *(n = 204)*

Student response	n	%
Inverse	78	
Inverse (arcsin)	15	54.4%
arcsin	17	
Flipped in y = x axis	1	
1/sin(x)	50	
Reciprocal of sin(x)	15	37.7%
cosec(x)	12	
1/sin(x) or arcsin	3	
cosec(x) as it is the inverse	3	
If capital S, arcsin (x) otherwise cosec(x)	1	7.9%
Inverse of sin. However, could also be written as 1/sin(x)	1	
Other responses	8	

Finally, we see evidence of students working without consideration of whether the symbols they have chosen create expressions that would make sense for another person reading this work. University staff consistently commented (see examples below) on their concern that students write series of symbols that do not make sense. They also comment that students often seem hesitant to use a mix of words and symbols.

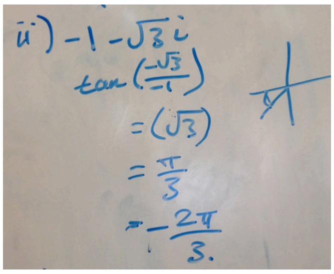

Figure 2. Student solution.

Tutor (CB0304a) …please consider the logical progression…these lines don't follow from each other

Tutor (CB304b) I think that they can understand the answer but they're failing is to express them step by step, so I say that they are lacking communication.

Teachers spoke of marking in Year 12 examinations where multiple choice and 1 or 2 mark questions reward the final answer rather than meaningful communication.

Conclusions and Implications

Analysis of the student data and staff interviews suggests that the weaknesses in the bridge between school mathematics and university mathematics began to develop in the junior secondary years where students learned to recognise patterns of symbols relying on the cue of fixed symbols. Relying on unthinking recognition continues to cause difficulties as students encounter familiar syntax templates in new contexts and domains (vectors, complex numbers). These weaknesses appear to be papered over rather than remedied at the senior secondary level as students focus on the skills required to maximise their marks on examination questions. An emphasis on speed and correct final answers values pattern recognition over mathematical thinking and communication. University staff speak of the need for words and symbols for meaningful communication in mathematical solutions.

In order to overcome these potential weaknesses and prepare school students to use mathematics in life and at university we need to explicitly model and value variety in the letters and domains for our examples and model and value clear, correct mathematical communication. In doing this it may be helpful to ask students questions or give instructions such as: Read this maths out aloud please. How else could we write this? What does this mathematical sentence mean? Could someone else, who had not been in this class, follow your working and use it to solve a new problem? Show me another way to represent and solve this problem.

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The Valuing of Deep Learning Strategies in Mathematics by Immigrant, First-Generation, and Australia-Born Students: Transitions Between Cultural Worlds

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Although foreign-born and first-generation students are in constant transitions between their home and host pedagogical cultures, they have performed better than their locally born peers in Australia. In this paper, we draw on Urie Bronfenbrenner's ecological systems approach to child development, focusing on the child's perspective to understand how cultural transitions (e.g., Gorgorió, Planas, & Vilella, 2002) interact with learning. This framework not only allows us to understand how these daily transitions facilitate different ways in which the various deep learning competencies are valued, but also assist to identify implications for even more effective mathematics learning by all students.

Introduction

The latest TIMSS 2015 (Thomson, Wernert, O'Grady, & Rodrigues, 2016) and PISA 2015 (Thomson, de Bortoli, & Underwood, 2016) results show that Australian students' performance in assessments for mathematical knowledge, its understanding, and its applications have not improved over the last 20 years or so. At the same time, mathematics education reforms in many other countries have led to noticeable improvements in their students' mathematics ability, resulting in these students surpassing their Australian peers in assessable mathematical knowledge. In fact, the latest PISA assessment results indicate that whilst Australia's ranking has dropped relative to other countries, its ranking would have been worse if it had not been supported by the performance of students from immigrant families. In PISA 2015, mathematical literacy scores of students from Australian-born families were significantly lower than those attained by first-generation and foreign-born students, as noted by Thomson et al. (2016):

> 10% of Australian-born students were high performers compared to 14% of first-generation students and 14% of foreign-born students. At the lower end of the mathematical literacy proficiency scale, the proportions of low performers for Australian-born and foreign-born students were similar (22%), while the proportion of first-generation students was 18%. (p. 65)

The mathematics learning experiences of these foreign-born and first-generation students are transition processes (Gorgorió, Planas, & Vilella, 2002). These students attend the same school as their Australian-born peers, have the same teachers, have the same in-class opportunities, and are generally treated equally in the (mathematics) education systems. Yet, there seems to be a cultural pattern in which students who have grown up moving between the Australian culture (in the mainstream classroom) and their respective home cultures (in the family) outperform those who do not make such transitions (Australian-born).

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We are interested in investigating why this pattern has emerged, and in what educators can do to improve the mathematics performance of students from Australian-born families. Drawing on developmental science (e.g., ecological models of child development), educational science (e.g., deep learning competencies and curriculum) and multiculturalism studies, we suggest that students' approaches to mathematics education are at least partially informed by their family cultural environments; and that encouraging similar approaches to learning in other students might help to promote a positive shift in Australian students' mathematics abilities.

An Ecological Systems Model of Students' Learning Experiences

The ecological systems model of child development (Bronfenbrenner, 1992) has been found useful to describe the learning experiences of immigrant children (e.g. Jensen, 2007; Paat, 2013). It describes children's development as nested within a series of interrelated systems – *microsystems* are systems with which the child interacts directly, such as family and school; *mesosystems* are the relationships between those microsystems; *exosystems* are elements of the society that affect the child's life but are typically out of the child's reach, such as mass media, industry developments, market structures, and local government; *macrosystems* are the broader ideologies and values of the culture; and *chronosystems* refer to the specific point in history in which all of these systems exist.

Although all of these systems are likely to shape students' mathematics education to some degree, the relationship between two microsystems (school and family) is particularly pertinent for this paper. We take an alternative position to this model by considering it from the perspective of the active child. As the child interacts with each microsystem, s/he takes on the values, meets relevant expectations, and masters the resources of the system.

How well the child navigates between these systems has strong implications for his or her development. On the one hand, stronger mesosystems (connections between family and school) can have positive consequences for a child's learning, as parental involvement in school-based activities reinforces important messages about the value of education for the child (Bishop, 2006; Lee & Bowen, 2006). Internationally, research consistently demonstrates this relationship between parents' and children's attitudes and behaviours towards education (e.g., Chiu, Pong, Mori, & Chow, 2012; Gibbs, Shar, Downey, & Jarvis, 2016; Li, 2016; Wang & Eccles, 2013). On the other hand, there is increasing evidence of an "immigrant paradox", in which children who successfully navigate their multiple cultural worlds are able to draw strengths from these cultures and experience more positive outcomes (in academic, mental health, and social domains) than their peers (e.g., Marks, Ejesi, & Garcia Coll, 2014). In other words, for the first-generation and foreign-born students, both their home and the Australian cultures are represented in the different micro-, meso-, exo-, macro- and chronosystems. As they are often required to practice cultural switching, they develop stronger deep learning skills than their mono-cultural or mono-linguistic peers. In a school context where deep learning is valued, this should result in higher academic performance.

Australian Culture: Shaping Children's Learning Experiences and Valuing Deep Learning Processes

The *school* context in Australia, however, is largely mono-linguistic, where the medium of instruction in almost all schools is the English language. This is despite the fact that Australia is a culturally diverse nation, in which 27% of its population were born overseas and a further 20% are first-generation (born in Australia with at least one immigrant parent; *Australian Bureau of Statistics*, 2013). This is also despite the fact that migrant and indigenous groups speak 500 other birth languages. The mono-linguistic culture of Australian classroom thus positions cultural (and linguistic) diversity as a challenge, despite its usefulness as a resource and learning opportunity for students (Scarino, 2014).

We are concerned with how deep learning strategies are represented and taught to students (of mathematics). In particular, we ask: how much do foreign-born and first-generation students value deep learning strategies? And are some deep learning strategies more useful for these students than others?

We acknowledge that Australian schools consider deep learning as a useful study approach within formal education. Australian educators' interest in deep learning is not unique: there is currently a strong international interest in the ways in which school curriculum promotes deep learning (e.g., Fullan & Langworthy, 2013). The *New Pedagogies for Deep Learning* global partnership (between Australia, Canada, Finland, the Netherlands, New Zealand, Uruguay, and USA), identifies six competencies that reflect deep learning processes essential for learning: character, citizenship, collaboration, communication, creativity, and critical thinking (DEECD, 2015). Many of these are embodied in the Australian Curriculum as capabilities that students are expected to develop. In particular, the four proficiencies of understanding, fluency, problem solving, and reasoning are reflected in the Victorian Curriculum for Mathematics. The *New Pedagogies* website (Victoria Department of Education and Training, 2014) hosts a range of case studies that demonstrate the varied ways in which students learn these competencies, including school culture (modelling good practice, school rules, and school meetings) and structured learning activities (e.g., projects, class discussions, competitions). Yet, the actions that reflect the valuing of particular attributes need not be the same in different cultures (Seah & Andersson, 2015). We further ask: If the six competencies identified by DEECD (2015) constitute deep learning strategies, what do they look like for the foreign-born and first-generation students as they move in and out of different cultures daily across systems in their mathematics learning? How similar or different are these students' deep learning strategies compared to their Australia-born peers in the same class?

What Does This Mean for Students in the Australian Context?

In our attempt to understand why foreign-born and first-generation immigrant students perform better in mathematics as they transition to Australian schools, we have found Bronfenbrenner's (1992) ecological systems model of child development useful. It has reminded us that as we assess the mathematics learning experiences of foreign-born and first-generation immigrant students, the different cultural forces and influences they experience on a daily basis has the potential to enrich the ways in which they internalise the deep learning competencies. To the extent that the home and Australian cultures do not clash, the immigrant students appear to have a richer repertoire of cultural knowledge to draw upon, with which to value attributes of deep learning of mathematics (and other subjects).

In the meantime, new questions are raised. These include: how much do foreign-born and first-generation students value deep learning strategies in mathematics? What do these strategies look like for the immigrant students? Are some deep learning strategies more useful for these students than others? We would recommend that these questions be subjected to future inquiries. Indeed, the values of students with better academic outcomes might offer insights into areas of needed support for those who are struggling. At the same time, foreign-born and first-generation immigrant students' approaches to learning are strongly interconnected with their transitions between cultures, and offer useful insights for improving all students' engagement and performance within the Australian mathematics education system.

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Supporting Mathematics Students with Autism Spectrum Disorders Through the Lens of Teacher and Student Values: A Research Framework for Teacher Transformation

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This study seeks to examine what high functioning students with Autism Spectrum Disorder (ASD) value in mathematics teaching and learning. The researchers aim to promote transition beyond common societal notions of associating ASD with deficits, and to improve mathematics teaching and learning outcomes for this growing population of learners.

Currently estimated to occur in one in every 68 births (CDC MMWR, 2014), Autism Spectrum Disorder (ASD) represents the fastest growing disability group (USDOE, 2014). A literature review confirmed that mathematics education of students on the autism spectrum is under-researched (Carr, Seah, & Moore, 2016). Most intervention research has been conducted with lower functioning students with skill deficits, typically included in special education settings. However, little is known about teaching mathematics to higher functioning students.

This paper reports part of a research study which aims to examine what high functioning ASD students value in mathematics teaching and learning. The project seeks to develop a research framework focusing on how values and valuing support students' mathematical skill development (cognitive development), as well as students' dispositions towards learning mathematics (affective growth) with a view to optimizing student outcomes when best evidence practices from Applied Behaviour Analysis (ABA) are used. Skinner (1974) asserted that his original intent of ABA was based on principles of positive reinforcement of target behaviour. Echoing this sentiment, the study intends to communicate the main components of ABA to the broader teaching community, to better support mathematics students with ASD.

Given rising prevalence estimates of ASD, increased numbers of young children receiving early intervention treatment, and a focus on least restrictive classroom placement, a growth in numbers of students on the autism spectrum attending mainstream education in many developed countries is anticipated. While there is an accumulating body of best evidence derived from the ABA paradigm demonstrating effective interventions for students on the autism spectrum, many teachers in both mainstream and special education sectors are unaware of the main intervention components used by ABA practitioners.

It has been noted in the literature that the education system should strive to create self-reliant and independent individuals, suggesting that students should develop both performance skills and principles of self-management (Lovitt, 1973). Lovitt (1973) reasoned that when effort is directed towards developing self-management skills, in addition to the acquisition of academic skills, the student will be optimally capable of using the academic skills they have learned.

The National Autism Centre has classified self-management as an established treatment for students (National Standards Report, 2009). Carr, Moore, and Anderson (2014) reported that self-management was highly effective for students with ASD for a variety of academic behaviours, including writing, task engagement, file organization,

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appropriate question-asking skills during tutorials, and academic production. One study included in the meta-analysis, reported improvements in accuracy and attention to task using self-monitoring and verbal prompting during mathematics class (Holifield, Goodman, Hazelkorn, & Heflin, 2010).

Through exploration of what stakeholders value with regards to mathematics pedagogy (see Seah & Andersson, 2015) for students with ASD, this study aims to promote interdisciplinarity of mathematics educators by communicating across research paradigms. The promotion of teacher professional development, with the view to better prepare teachers to effectively teach mathematics to this population of diverse ability learners, is also of central importance.

As a first step, a research framework focusing on how values / valuing support students' mathematical skill development and students' dispositions towards learning mathematics will be developed. By comparing and contrasting the underpinnings of a values approach and the ABA paradigm, the research framework will synthesise key components to optimize student outcomes when best evidence practices are used. The theoretical framework will be reviewed in light of relevant strengths and limitations for application to teaching students on the autism spectrum.

Thus, the research study on which this paper is based represents a transition for all participants. For high functioning students with ASD, school experience is often described as overwhelming and frustrating both inside the classroom and during less structured periods. Ideally, this study will assist these students to "move" towards greater confidence in their academic ability, improved independent functioning, and a positive affect towards mathematics achievement. Teachers describe frustration in not understanding the seemingly erratic behaviour of students on the autism spectrum, and that the additional time required to manage these students in the classroom is challenging. Ideally, these teachers will also "move" towards improved outcomes in classroom management and student achievement. Arguably, student peers may benefit by developing a better understanding of the unique challenges their classmates on the spectrum experience, leading to a greater awareness and acceptance of differences amongst members of society.

Values and Valuing in Mathematics Education

According to Seah and Andersson (2015), values in mathematics and in mathematics education reflect

> the convictions which an individual has internalised as being the things of importance and worth. What an individual values defines for her/him a window through which s/he views the world around her/him. Valuing provides the individual with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics. They regulate the ways in which a learner's/teacher's cognitive skills and emotional dispositions are aligned to learning/teaching in any given educational context. (p. 169)

This driving force nature of values and valuing might be illustrated with an example. A student might, say, value *problem-solving*. This valuing would thus mean that the student would develop a positive attitude to problem-solving situations, either at home or outside. She would feel engaged to any task in real-life which requires problem-solving. On the cognitive side of things, the valuing of *problem-solving* would drive her to learn about problem-solving strategies, or to take part in discussions.

Transformational Teaching as a Research Framework

Transformational teaching has been defined as "the expressed or unexpressed goal to increase students' mastery of key course concepts while transforming their learning-related attitudes, values, beliefs, and skills" (Slavich & Zimbardo, 2012, p. 5). It has emerged from the constructivist notion that learning occurs best when students are actively engaged in a process of discovery (Piaget, 1926), along with the social constructivist notion (Vygotsky, 1978) that social interactive educational exercises yield greater impact (Slavich & Zimbardo, 2012).

Transformational teaching may be best understood as a process that promotes student learning and personal growth through the creation of dynamic relationships between teachers, students, and the shared body of knowledge (Slavich & Zimbardo, 2012). In this sense, transformational teaching emphasizes inquiry, critical thinking, and communication using a learner-centered approach that provides positive feedback to the student.

Slavich and Zimbardo (2012) reported that transformational leadership has been associated with improvements in positive student attitudes and beliefs, self-determined motivation, classroom enjoyment, self-efficacy, and intrinsic motivation. These authors have suggested that transformational teaching can maximize students' potential for success and significantly enhance students' attitudes, values, beliefs and skills.

The framework described by Slavich and Zimbardo (2012) also includes "transformative learning theory", in which instructors can promote change in attitudes and beliefs by having students work in interdependent discovery learning. These authors suggested facilitation of change through group projects, role play, case studies, and simulations.

Slavich and Zimbardo (2012) noted that intentional change theory has been incorporated into transformational teaching. The necessity for students to establish an ideal self and a personal vision for the future, based upon developing an image of a desired future, hope to achieve their goals and identifying personal strengths upon which the vision can be realized was described.

Implications of the Transformational Teaching Framework for Students with ASD

Rosebrough and Leverett (2011) argued that education should be more about inspiration than information. Specifically, these authors suggested that teachers should also equip students with the skills and attitudes necessary to overcome challenges in life.

Two important issues are apparent within the traditional transformational teaching framework that may benefit from a modification to better address the unique learning styles of students with ASD. First, for these students, it is important to provide structured teaching in independent, realistically attainable goal-setting. Explicit teaching of (self) goals-setting, progress recording, and feedback was explored in the literature, with special consideration to learners on the autism spectrum (Carr, Moore, & Anderson, 2014). Carr et al. reported that the potential to develop this skill appeared promising, even though it is developed over time. The research framework may benefit by including a step in which students with ASD are taught to independently set, measure, and attain realistic relevant goals.

Second, the traditional transformational teaching framework often involves group work. As a result of delays in social development, the ability to function well in group settings is often problematic to students with ASD. This can cause confusion to peers and teachers working with students with high functioning autism or Asperger's syndrome, who may otherwise be cognitively very capable. Carr et al. (2014) identified four self-management interventions that reported on the inclusion of peers in various facilitating or reinforcing roles. Using the Percentage of Non-overlapping Data (PND) calculation to rate the studies as effective, Carr et al. suggested that ongoing research with peers appears highly warranted. Elsewhere, Miller, Vernon, Wu, and Russo (2014) emphasised the importance of friendship in mental health, noting that core and comorbid symptoms of ASD may be reduced. The research framework may benefit by modification from group work to peer-dyads, to better support the social delay often present in individuals with ASD.

Ultimately, this research project aims to contribute to the field by advancing a deeper and more correct understanding of the psychology and other aspects of teaching and learning mathematics to increasing numbers of diverse ability learners on the autism spectrum. By examining the valuing of students with ASD learning mathematics when best evidence practices are applied in the classroom, this study aims to enable teachers and students to transition beyond current classroom challenges, and contribute to the facilitation of improvements in mathematics teaching, and learning outcomes for this growing population of learners.

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STEM Practices: A Reconceptualization of STEM in the Early Years

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The symposium provides an overview of the Early Years STEM Australia (ELSA) program. The conceptual underpinnings of the program are framed within STEM practices, rather than traditional thinking concerning the integration of discipline content knowledge. We will argue that our focus on practices is more aligned with the play-based and intentional teaching objectives of the Early Years Learning Framework (EYLF). The symposium describes the approach we have undertaken, the extent to which some of the practices align well to mathematics thinking, and the pedagogical framework used to stimulate play and create activities for the six learning apps that form part of the program.

Paper 1: Tracy Logan, Tom Lowrie, & Claudette Bateup

Early Learning STEM Australia (ELSA): Developing a learning program to inspire curiosity and engagement in STEM concepts in preschool children

Understanding the skills, knowledge and dispositions needed for work of the future is a sustained focus of the Australian Government, with initiatives such as the National Innovations and Science Agenda at the core of Policy. The Early Learning STEM Australia (ELSA) project seeks to develop and pilot an innovative digital-based STEM learning program to be delivered within Australian preschools. This paper reports on the theoretical underpinnings of the ELSA program and describes the design framework.

Paper 2: Tom Lowrie, Tracy Logan, & Kevin Larkin

The "math" in STEM practices: The role of spatial reasoning in the early years

The paper describes the conceptual framework of the Early Years STEM Australia (ELSA) program. The conceptual framework is developed from STEM practices, rather than from play-based investigations derived from traditional understandings of content within the four STEM disciplines. One of these core STEM practices is spatial reasoning—a practice that not only has strong associations with mathematics but is also the best predictor of an individual choosing a STEM-related profession beyond schooling.

Paper 3: Kevin Larkin and Caroline Kinny-Lewis

ELPSA and Spatial Reasoning: A design based approach to develop a "mapping" app

ELSA apps will extend beyond the screen to encourage active play that supports STEM practices, such as exploring location, patterns and problem solving. Here, we present current research into spatial reasoning apps for mathematics, suggest ELPSA as a pedagogical framework to underpin mathematics learning using "mapping" as an exemplar, and then discuss the use of a design based approach to create mathematics apps.

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Early Learning STEM Australia (ELSA): Developing a Learning Program to Inspire Curiosity and Engagement in STEM Concepts in Preschool Children

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Understanding the skills, knowledge and dispositions needed for work of the future is a sustained focus of the Australian Government, with initiatives such as the National Innovation and Science Agenda at the core of policy. The Early Learning STEM Australia (ELSA) project seeks to develop and pilot an innovative digital-based STEM learning program to be delivered within preschool programs. This paper reports on the theoretical underpinnings of the ELSA program and describes the design framework.

Early Learning STEM Australia (ELSA) is a play-based digital learning program for children in preschool to explore Science, Technology, Engineering and Mathematics (STEM). The initiative is framed within the Australian Government's National Innovation and Science Agenda. The central aims of the program are to: (1) develop a learning program to inspire curiosity and engagement in STEM concepts in preschool children, (2) provide a comprehensive and holistic set of resource materials and learning apps that enhance play-based experiences for children as part of their STEM learning, and (3) establish a community of practice network for early years learning that heightens the sustainability of the initiative beyond the scope of the three-year project timeline.

Consequently, our vision for the ELSA pilot program is to design, develop and deliver quality play-based apps and integrated resources that allow children, educators and families to explore and experience powerful STEM ideas and practices in ways that are connected to the children and their environments. The intent of the ELSA program is to:

- Embed a play-based use of technology in the early years through the presentation of STEM-focused learning experiences in an online, play-based environment;
- Provide meaningful opportunities for preschool children to explore a variety of online, play-based learning environments that are rich in STEM practices;
- Engage effectively with the early learning sector to raise awareness of the importance of STEM; and
- Support early childhood educators to understand the multiple points of connection between the STEM practices and how they connect to the EYLF and Australian Curriculum.

Our Theoretical Approach to Early Learning

Building on the Early Years Learning Framework (EYLF; DEEWR, 2009) and other successful early learning programs, we acknowledge and prescribe to play- and investigation-based pedagogical approaches to early learning. Through such play and investigation, the intention is to develop positive dispositions and a range of skills and knowledge in STEM literacy for children, educators, and families. These dispositions

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include curiosity, cooperation, confidence, creativity, commitment, enthusiasm, imagination, and persistence. The skills and knowledge include the powerful STEM practices at the core of our program design, namely; spatial reasoning and problem solving through inquiry. For young children three to five years of age), there are a number of appropriate pedagogies, including play-based learning and intentional teaching, which support the construction of purposeful and thoughtful learning environments (DEEWR, 2009).

Play-Based Learning Experiences

Within early childhood, the notion of play-based learning has been a constant element. Dockett and Perry (2010) indicated that the important features of play "include the exercise of choice, non-literal approaches, multiple possible outcomes and acknowledgement of the competence of players" (p. 716). The ELSA pilot program will utilise such features within the learning apps. The overall ELSA learning program, and embedded learning apps within the program, will provide children with opportunities to develop their cognitive skills through an emphasis on reasoning and thinking skills, with core STEM practices as the foundation. The children will experience success through supported challenges, thereby developing persistence and resilience as they undertake the activities stimulated by the apps. Many of the activities will promote movement through exploration of the children's environments as well as developing the fine motor skills that will be needed in order to interact with the technology incorporated in the apps.

Interaction is central to young children's learning—through ideas, challenges, stimulating materials, peers, and learned others. Importantly, ELSA learning apps will extend beyond the screen to encourage active play that supports STEM practices, within the learning centre and home environments. The apps will support learning through play, and act as a springboard for children to explore the natural world through investigation and inquiry. According to the American Association for the Advancement of Science (1993), "students should be actively involved in exploring phenomena that interest them. These investigations should be fun and open the door to…more things to explore" (p. 10).

The learning apps will enable children to engage with the concepts multiple times, over an extended period of time. The activities within the apps will draw on themes that reflect everyday life and familiar objects. This approach will enable a richer understanding and playful exploration of learning concepts and will transfer to off screen activities in actual physical environments. This is important since effective learning in the early years requires repeated exposure to materials and concepts to acquire knowledge. The activities will encourage interaction among the preschool children and with their educators and family members in recognition of the importance of such interactions for the children's learning. The apps can be used individually, in groups and via educator led discussions to encourage social interactions. Such discussions will enable "sustained shared thinking" to solve problems and clarify concepts (Siraj-Blatchford, 2010, p.155). However, research has found that play on its own may not be enough to consolidate learning; as the educators' ability to notice learning opportunities is critical to facilitate children's learning (Dockett & Perry, 2010; van Oers, 1996).

Intentional Teaching

As indicated in the EYLF, intentional teaching is purposeful, thoughtful and deliberate teaching (DEEWR, 2009). An intentional educator thinks carefully about their actions and the potential effects of those actions (Epstein, 2007). According to Gronlund and Stewart (2011), excellent educators "are intentional in all they do with and for children. They do not assume that children's development will happen without support, encouragement, and scaffolding or without presenting appropriate challenges for the children" (p. 28). Educators deliberately plan the types of materials and equipment and carefully consider where to place them so children will discover them and use them. They use a range of teaching techniques including demonstrating, describing, modelling, co-constructing, problem solving, documenting and scaffolding (MacNaughton & Williams, 2008). However, the focus of learning remains child-centred, with the educator designing environments around a learning goal to spark children's curiosity and exploration and providing learning materials to play with. The child is an active participant in constructing their knowledge rather than a passive recipient. All concepts can be developed into inquiry projects in the centre that allow for intentional teaching and open-ended play opportunities. Thus, the ELSA learning apps will use a play-based approach that affords intentional teaching opportunities. The activities available in the apps will allow children to discover and learn concepts as well as provide opportunities for educators to plan learning experiences that reinforce and consolidate new knowledge.

The Design Framework

Our framework involves four interrelated elements, namely: program design, service delivery, the pilot study, and ongoing communication (see Figure 1). The *program design* is situated within STEM literacy and the STEM practices that make up the core learning program. The theoretical and practical knowledge of the ELSA team have ensured that early years STEM practices in the program are closely aligned to curriculum expectations. The EYLF will provide the overarching connectivity for this initiative and is designed around the engagement elements of *Being, Becoming and Belonging*. All learning app production, resource materials and professional development opportunities will make explicit reference to the EYLF to increase the likelihood of sustained use of the ELSA pilot program. The content and discipline knowledge of STEM understandings will recognise the respective school STEM disciplines to enhance smooth transitions from preschool to school. The resources developed for the educator and families apps will have appropriate levels of theoretical rigour and practical applicability to promote knowledge of STEM literacy and STEM practices. The *service delivery* element will utilise user centric and agile approaches to design that will provide the team with feedback from the target audience throughout development. Users will inform the design of every feature. Such a design affords opportunities for productive refinement and adaptation of the apps and related resource materials through conceptually-meaningful and technically-sound feedback loops. This aspect of the framework is a feature of the ELSA design and exemplifies the important relationship between quality design principles, specific STEM practices, and authentic applications in early years contexts. The important interplay between the focus groups, educator PD, and family engagement will be situated within a research design. *The pilot study* design will direct the formation of supplementary materials and resources that can maximise the holistic STEM experiences of all stakeholders—since the apps themselves are only one aspect of the project's sustainable success. This *communication* strategy will complement and enhance the other elements of the framework—through a range of physical artefacts and virtual interactions. We will create a community of practice network (CPN), which will comprise the research and design team; the learning centre educators, interested families and other stakeholders. The

CPN will afford opportunities for all stakeholders to collectively solve educational problems they have in common, especially those related to STEM.

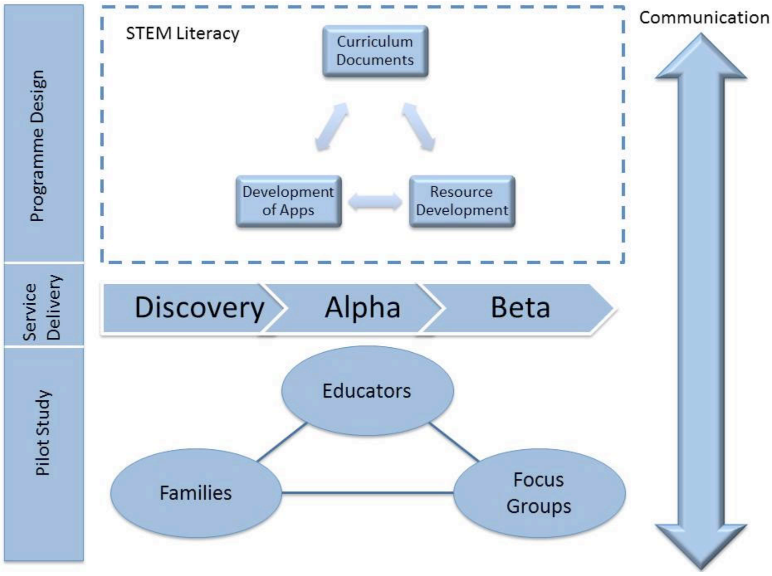

Figure 1. A representation of our overarching framework.

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The "Math" in STEM Practices: The Role of Spatial Reasoning in the Early Years

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This paper describes the conceptual framework of the Early Years STEM Australia (ELSA) program. The conceptual framework is developed from STEM practices, rather than from play-based investigations derived from traditional understandings of content within the four STEM disciplines. One of these core STEM practices is spatial reasoning—a practice that not only has strong associations with mathematics but is also the best predictor of an individual choosing a STEM-related profession beyond schooling.

Most advocates of Science, Technology, Engineering and Mathematics (STEM) education highlight the need to purposely integrate the various disciplines in situations that promote real-world problem solving. To some degree, the integration involves using a combination of the four disciplines as a cohesive entity (Breiner, Harkness, Johnson, & Koehler, 2012), although there has been a recent push by specialists outside these four disciplines to consider other domains as part of the STEM movement (e.g., The Arts and even Medicine). In early years settings, and in most real-world situations outside of school, it is unhelpful to consider learning within disciplines—whether integrated or not. Our main tenant in implementing the Early Learning STEM Australia (ELSA) pilot program was to focus on *practices* that enhanced STEM ideas and engagement rather than developing integrated content-based learning experiences derived from the respective disciplines. As Sanders (2009) argued, *practices* are more likely to alter the status quo of traditional content-based learning that has monopolized education for more than a century. Moreover, the focus on practices supports the notion that learning and understanding is best supported in contexts where values described in curricula and policy, the methods employed by educators to support learning, and the ideas developed in children's play are aligned. Such alignment is akin to Kemmis' (2008) conceptualisation of practice architectures.

The Notion of Practices in Early Years STEM (and Beyond)

We have defined a practice as an application or use of an idea, value or method. The focus on practices helped ensure that understandings are related to the real-world contexts that are enacted through participation and engagement (Kemmis, 2008). An analysis of the Early Years Learning Framework (EYLF; DEEWR, 2009) and the Foundation year of the Australian Curriculum for both mathematics and science (Australian Curriculum, Assessment and Reporting Authority, 2016) helped us to select a number of powerful ideas related to STEM learning that could be framed into STEM practices. These practices included: (a) spatial reasoning, (b) location and arrangement, (c) patterns and relationships, (d) problem solving and inquiry, (e) design and making, and (f) understanding change. These practices were classified in terms of core ideas and methods used to enact these ideas (see Table 1). Given the strong association between spatial reasoning performance and participation in STEM-based occupations (Uttal et al., 2013), it was essential that we identified spatial reasoning as a core STEM practice. The development of learning activities associated with (1) locations and arrangement and (2) patterns and relationships provide opportunities for spatial understandings to be enhanced through intentional

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teaching and play. The second core STEM practice, identified as fundamental in STEM literacy, was problem solving and inquiry. Problem solving for young children involves identifying the problem and thinking clearly about it, determining and applying strategies, checking if they were successful and, if not initially successful, persisting in the process until a successful solution is discovered. Problem solving can be either an individual or group activity. Both types require persistence, focused attention, and creativity. After establishing our second core practice, we then identified learning activities associated with (1) design and making and (2) understanding change, which would provide authentic opportunities for young learners to inquire and problem solve.

Table 1
STEM Practices within the ELSA Framework

Spatial Reasoning	
Location and Arrangement	Patterns and Relationships
Problem Solving and Inquiry	
Design and Making	Understanding Change

Given page-length restrictions, for the remainder of this paper, and across the third paper in the symposium, we focus on the core practice of spatial reasoning.

Spatial Reasoning as a Core STEM Practice

Spatial reasoning is considered critical for everyday tasks and helps us to understand, appreciate and interpret our geometric world (National Council of Teachers of Mathematics [NCTM], 2000). Spatial reasoning is identified as a core component in the Numeracy general capability within the Australian Curriculum and thus is to be incorporated into all subjects across the curriculum. In particular, spatial reasoning is a major focus of mathematics, science understanding, and digital technologies knowledge and understanding. Spatial reasoning can be defined as the process of recognising and manipulating spatial properties of objects and the spatial relations among those objects (Mulligan, 2015). The National Research Council (2006) reported that thinking spatially is an essential aspect of learning and distinguished ways in which we think: *in* space (navigating through buildings, playing sports or organising shelving); *about* space (investigating the structure, function and motion of things in the world, designing a tool, a building or a dam); and *with* space (decoding and encoding diagrams and maps). The capacity to locate, orientate, and visualise objects; navigate paths; decode information graphics; and use and draw diagrams are identified as critical to success in STEM problem solving. Spatial thinking can be improved in primary school-aged children through exposure to explicit spatial reasoning activities and intentional teaching (Lowrie, Logan, & Ramful, 2017). Providing such spatial activities to young children in play-based environments will similarly develop their spatial reasoning.

Spatial Reasoning as a Practice in the Early Years

Our decision to endorse spatial reasoning as one of our essential STEM practices was not based on the rich literature base alone. Play-based investigations were at the core of our learning design, and such contexts are well suited to the development of spatial reasoning.

Spatial concepts tend to develop through engagement in our inherently spatial world or through activities that promote particular spatial skills or understandings. The interactions with our environment are associated with thinking *about* space, thinking *in* space, and thinking *with* space (NRC, 2005). Research has indicated that infants as young as 16 months of age are able to hierarchically code location by combining distance information with information associated with a category (Huttenlocher, Newcombe & Sandberg, 1994). Although spatial understandings develop through experiences outside-of-school contexts, there is an increasing body of evidence that demonstrates that spatial reasoning also improves as a consequence of intentional teaching (Newcombe, 2013), and this development change can be rapid (Huttenlocher et al., 1994).

Newcombe and Frick (2010) argued that two simultaneous approaches should be taken to promote spatial reasoning in the early years. The first approach is to bring spatial thinking into the learning centre, by providing opportunities for children to engage with spatial ideas through everyday experiences and embodied knowledge. Of note for ELSA, they maintained that digital media could enhance these experiences. The second approach is to engage with spatial skills at home and in play. Both these approaches align well with the principles of the EYLF (Sumsion & Wong, 2011), since they capture the authors' idea of cartography within the EYLF's motif of "belonging, being and becoming". They argue that "belonging", in particular, should consider the conceptual mapping of landscapes through approaches that provide opportunities for rich and culturally diverse ways of participating in learning. That is, belonging includes an understanding of who and where you are, and what surrounds you. Moreover, our conception of the ELSA apps extends beyond the device to encourage active play that supports STEM practices. The twin pillars underpinning spatial reasoning in the ELSA apps are: (1) Location and Arrangement and (2) Patterns and Relationships

Location and Arrangement

Practices associated with location and arrangement have strong links to the EYLF since children explore the world around them through physical play; moving and orientating themselves in space such as dance and shared play; learning through and with different text types including diagrams and drawings; thinking and learning about symbols and patterns as spatial representations; and learning and communicating about numeracy understandings. Location and arrangement is clearly related to spatial reasoning across a range of STEM disciplines. Location and arrangement activities include investigations regarding position, movement and direction and then communicating this information to others using spatial language. For example, children engage in play using actions and language associated with: space (e.g., behind, on, in front of, below); movement (e.g., over, through, between, along); direction (e.g., sideways, forwards, backwards, turn, up); sequencing movement along pathways; and visualising plans for movement around locations. These practices underpin design and construction (engineering); wayfinding and navigation (mathematics); and describing the properties and behaviours of familiar objects (science).

Patterns and Relationships

This STEM idea incorporates understandings about sequences, patterns, and relationships that children encounter in their everyday lives. As was the case with location and arrangement, the practices associated with patterns and relationships are also closely linked to the EYLF. As part of a play-based learning approach, children: explore their environment; manipulate objects and experiment with cause and effect, trial and error, and motion; make predictions and generalisations about their daily activities; and contribute constructively to discussions and argument using mathematical language. These practices are developed explicitly in our apps, and in activities that springboard from the apps, as children use actions and language when: sorting, classifying and matching objects and materials; making patterns using themselves, objects and materials; recognising predictable sequences that are part of an event; and beginning to understand notions of time in relation to their own lives. Play-based investigations utilising experiences in patterns and relationships provide core opportunities for children to *mathematise* concepts, that is, notice the mathematical ideas presented in the play. They also provide opportunities for scientific thinking as they engage in discussions about observations and ideas.

Conclusion

We acknowledge the importance of discipline knowledge in each of the four STEM areas; however, our argument is that it is developmentally inappropriate for knowledge to be the organising principle for young learners. Instead we focus on STEM practices, applicable to all STEM disciplines, but more philosophically attuned to the core principles of the EYLF. Whilst a range of practices were identified, spatial reasoning and problem solving are critical to later success in STEM.

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ELPSA and Spatial Reasoning: A Design-Based Approach to Develop a "Mapping" App

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Early Learning STEM Australia (ELSA) is a play-based digital learning program for children in preschool to explore the practices that underpin Science, Technology, Engineering and Mathematics (STEM). ELSA apps will go beyond the screen to encourage active play that supports STEM practices, such as exploring location, patterns and problem solving. Here, we outline current research into spatial reasoning apps for mathematics, suggest a pedagogical framework to underpin mathematics learning using mapping as an exemplar, and then explain how we will use a design based approach to create apps that support learning in Mathematics.

Introduction

Early Learning STEM Australia (ELSA) is a play-based digital learning program for children in preschool to explore the practices that underpin Science, Technology, Engineering, and Mathematics (STEM). ELSA apps will go beyond the screen to encourage active play that supports STEM practices, such as exploring location, patterns and problem solving. These apps will support learning through play, and act as a springboard for children to explore the natural world. In this symposium paper, we outline current research into spatial reasoning apps for mathematics, suggest a pedagogical framework to underpin mathematics learning using mapping as an exemplar, and then explain how we will use a design based approach to create apps that support learning in mathematics.

Spatial Reasoning, Apps, and Early Years Learners

Research suggests that student knowledge and understanding of spatial reasoning is vital as it enables students to understand and interpret their environment. Importantly, it relates to, and predicts, future performance in mathematics (Lowrie, Logan, & Ramful 2017) and in related STEM disciplines (Jirout & Newcombe, 2015; Uttal et al., 2013). Consequently, spatial reasoning is now viewed as a core component in promoting higher-level thinking skills in mathematics and beyond. It is generally accepted that quality digital experiences support mathematical learning (see Larkin, 2016), particularly in spatial reasoning (Lowrie, 2015). Despite the widespread use of apps in primary schools, their use in early years settings is relatively unexplored.

Focussing on developing practices rather than skills, it is necessary to locate the use of the apps in the broader ecology of early childhood. We see ecology as the interplay between the physical, social and cultural elements of the environment (see Arnott, 2016) and that when children use apps we must consider how these contribute "to their social interactions and experiences during digital play" (p. 277). This notion sits comfortably within the philosophical underpinnings of the Early Years Learning Framework (EYLF) where intentional teaching within play-based contexts is promoted. Thus, we see the role of apps as two-fold. First, they contribute towards children's "epistemic" learning – in other words, what does the app do? Second, and perhaps more importantly, apps must

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contribute towards young children's "ludic" learning i.e. what can I do with the app? (Bird & Edwards, 2014). Based on research by the lead author, we suggest that this is where many apps fail, as there is either no scope, or very limited scope, for children to extend their interactions with the app. So, they learn what the app does, often teaching just a skill, but cannot then use the app in any meaningful way in their own context. For example, in many geometry apps the children can only interact with pre-loaded shapes; whereas in one of the better apps (Area of Figures), the children have the additional capability of creating and measuring shapes they have created.

Using ELPSA as a Pedagogical Frame for the Apps

In designing the apps, we are guided by the ELPSA pedagogical Framework (Lowrie & Patahuddin, 2015), which recognises that learning is constructivist and social in nature. This framework thus recognises that young children develop understanding, and ways of knowing, both individually (intra-personal) and in social interactions with others (inter-personal) (Vygotsky, 1978). ELPSA is an acronym for Experience, Language, Pictorial Representations, Symbolic Representations, and Applications. The following examples of the five stages are adapted from Lowrie and Patahuddin (2015). Experience considers how students have used mathematics and which particular concepts they know both inside and outside of classrooms. Language is used to promote understanding and can be generic (i.e., associated with literacy) or specific (i.e., associated specifically with mathematical terms). Pictorial representations are used to represent mathematical concepts and are a critical aspect of mathematics. Symbols are then used to represent mathematical ideas and are the most common aspect of many mathematics classrooms. The final component in the learning design illustrates how pictorial and symbolic understandings can be applied in novel situations. Although the process appears linear in nature, learning is complex and unpredictable and does not occur in a linear sequence, and thus the elements of the model should be thought of as interrelated and overlapping. It is a tool that early childhood educators can use to make planned and informed interventions in the largely play-based experiences of children.

ELPSA in Action: Building a "Mapping" App

As a brief exemplar, we have outlined below the process of early ideation for the location and arrangement "mapping" app. In essence, children's progression in spatial understanding commences with their own experience and then progressively become more complex. We suggest that initially children understand themselves in space (Where am I now?) and then understand their movement in space (Where am I going?). Next, they understand that there are others in space with them (Where are you in space?) and finally they begin naïve understandings of space that they cannot see (What is around the corner? What can my teddy see?). Early childhood educators can scaffold play-based experiences to support spatial development using the ELPSA framework. The following are our initial conceptualisations of how the various elements might be supported by an app.

- **Experience**: What can we see in our childcare centre? What can we see when we are at home?
- **Language**: Using directional and positional language to describe – What does it look like from here? What would it look like from over there? Language might include Left, right, forward, backward. Kindergarten terms include next term, near and far, above and below, in front and behind.

- **Pictorial**: Represent what my teddy might see while held in my arms looking forward or what she might see sitting on the swings outside. This involves seeing from a different perspective. Movement can be represented, for example by ant trails.
- **Symbolic:** Creating a map of what I see "inside space". Creating a map of what others might see "outside space in the app". Using symbols that represent things – real or imagined.
- **Application:** Noticing new locations at home and different environments. Noticing on the route. Noticing where I am in relation to others, in relation to me, and in relation to things I can't see. Creating a map of how I find my way to a significant place in my environment – home, shopping centre etc.

Our Design-Based Process

Based on the work of Katz (2010), we suggest that young children should frequently have the following experiences: be intellectually challenged, engaged and absorbed; use language to communicate their experiences and assist others; participate in sustained investigations of their world; take initiative and develop self-confidence; and apply their developing basic literacy and numeracy skills in purposeful ways. In order to offer young children these experiences, we utilise a four-stage design approach: Discovery, Alpha, Beta, and Live (Figure 1). This approach outlines a clear process to develop the apps; from initially understanding the needs of the young children, their educators and their families, through to the delivery of the finished apps. We envisage the process of app design in the following way:

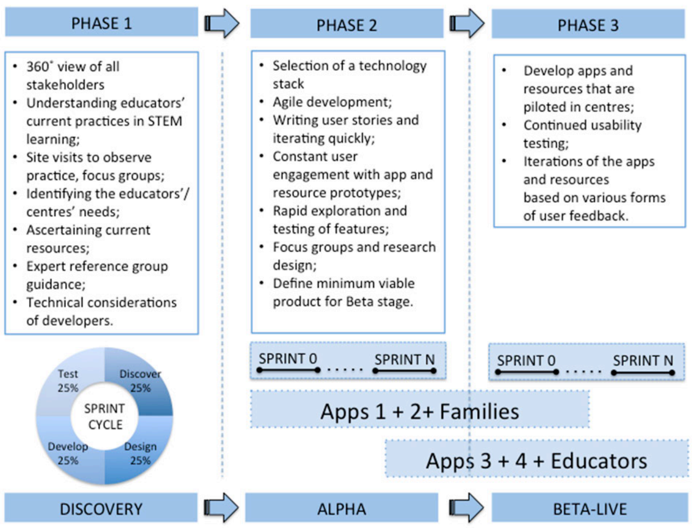

Figure 1. An overview of the app design process.

Due to their attention span, their motivation to please adults and their ability to adjust to new people and experiences, user testing with young children requires careful planning (Hanna, Risden, & Alexander 1997). Children three to five years of age will be encouraged to explore the apps according to their own interests and pacing as opposed to undertaking a series of directed tasks. They are often happy to show you what they know and what they can do independently within the app. As children at this age can find expressing their likes and dislikes in words a challenge, we will utilise observations of children's behaviours (e.g., smiling, sighing, looking confused or frustrated) as key indicators of their level of engagement with the apps during the pilot programme (Hanna et al., 1997). Two key parts of the design process are user stories and prototype testing.

User stories. The requirements of young children will be expressed via user stories, which will be developed as features that we want to test during prototyping (e.g., a preschool child indicates their favourite activities in the block corner). In essence, each user story will describe the child's primary objective in interaction with an element in the app.

Prototype testing. We envisage our apps as toys and thus they will not contain overt teacher instruction. We will initially develop "paper prototypes" (use of paper with a combination of other real-life objects) to represent what will appear in the apps. This design approach, based on Engineering principles, requires minimal investment and often provides a better indicator of engagement because the user is less likely to be distracted by technology and multitasking during play testing. Early learning mathematics concepts (e.g., mapping) can therefore be thoroughly investigated to develop our understanding of how children will engage with the app.

Conclusion

In this paper, we argued that spatial reasoning is a critical component in developing concepts that underpin later understanding in mathematics and more broadly across STEM disciplines. We suggest that thoughtfully-designed apps become a part of the ecology of young children and help them to explore their world. Finally, via the example of the design of a "mapping" app, we noted that the ELPSA framework, which supports educator intervention in play-based learning, is an important tool for supporting early learners.

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Research Engagement and Impact in Mathematics Education

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While measures of research quality are widely accepted in the education research community, there may be less agreement on what constitutes evidence of impact and on where to look for it. The aims of this symposium are to consider some key issues in undertaking the Australian government's national assessment of research engagement and impact, and to propose some approaches to evidencing engagement and impact in the context of mathematics education research. Each of the four symposium papers draws on our Numeracy Across the Curriculum (NAC) research program in order to ground our discussion in specific cases of research that have been reported at previous MERGA conferences.

In the first paper, *Evidencing research engagement and impact*, Merrilyn Goos establishes the theoretical and policy context for the symposium in terms of the apparent lack of connection between educational research and practice. She analyses aspects of the NAC research program to trace rich connections between her own teaching, research and service roles that led to beneficial knowledge exchanges (engagement), and intricate links between research activities, outputs and outcomes across multiple projects (impact). Such an analysis can suggest "where to look" for evidence of engagement and impact.

In the second paper, *The convoluted nature of a research impact pathway*, Vince Geiger develops a case study of an aspect of his own research within the NAC program to illustrate the complexity of the journey from research origin through to potential impact. The documentation of this research progress allows for reflection on how future impact can be "read" while research is taking place.

In the third paper, *Engagement and impact through research participation and resource development*, Anne Bennison and Shelley Dole illustrate how knowledge exchange and uptake of resources developed through research can provide evidence of research engagement and impact, respectively. The analysis suggests ways in which collaborative research (an ARC Linkage Project on proportional reasoning and numeracy) and contract research (funded by the Queensland College of Teachers) can be translated for economic and social benefits.

In the fourth paper, "*Numeracy for learners and teachers": Impact on MTeach students*, Helen Forgasz evaluates the impact of a compulsory unit taken by all primary and secondary pre-service teachers in the Monash University Master of Teaching. The unit design incorporates elements of the Numeracy Across the Curriculum model to address AITSL standards for knowledge and understanding of literacy and numeracy teaching strategies, and interpreting student data. The evaluation reveals substantial impact on students' understanding of numeracy and confidence in incorporating numeracy in their teaching, thus highlighting the contribution of research to improving teacher education.

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Evidencing Research Engagement and Impact

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This paper outlines current developments in the Australian government's plan to introduce a national assessment of research engagement and impact and considers implications for mathematics education. A well-established research program seeking to embed numeracy across the school curriculum is used to illustrate forms of research engagement and impact. The analysis of this program demonstrates rich connections between research, policy and practice, and suggests "where to look" for evidence of engagement and impact.

In December 2015, as part of its *National Innovation and Science Agenda*, the Australian government announced the development of a national assessment of research engagement and impact. It is envisaged that the assessment will be implemented in parallel with the national evaluation of research quality – Excellence in Research for Australia (ERA). The Australian Research Council (ARC) and the Department of Education and Training released an *Engagement and Impact Assessment Consultation Paper* in May 2016 to seek feedback from stakeholders on how this assessment should be undertaken (ARC and DET, 2016).

While measures of research quality are widely accepted in the education research community, there may be less agreement on what constitutes evidence of impact and on where to look for this evidence. The aims of this paper are to consider some of the key issues in undertaking a national assessment of research engagement and impact that were raised in the *Consultation Paper*, and to propose some approaches to evidencing engagement and impact in the context of mathematics education research.

Theoretical Background: The Gap between Research, Policy, and Practice

Education research is often criticised for its lack of impact on classroom practice. Explanations for the apparent research-practice gap sometimes highlight the different processes used by researchers and teachers to improve educational practice. For example, Richardson (1994) suggested that, whereas formal research aims to contribute to an established and general knowledge base, the practical inquiry of teachers is focused on solving immediate day-to-day problems. Writing from an educational leadership and policy perspective, Levin (2010) invoked the idea of knowledge mobilisation to examine connections between the production, communication, and use of research. He argued that not only do researchers have a responsibility communicate their findings beyond academia, but policy-makers and practitioners also need to be willing to find, share, and use good research in their work.

The apparent lack of connection between education research, policy and practice seems to be particularly relevant to mathematics education. For example, national and international assessments of mathematics achievement, such as the National Assessment Program – Literacy and Numeracy (NAPLAN), the OECD's Programme for International Student Assessment (PISA), and the IEA's Trends in International Mathematics and Science Study (TIMSS) allow governments to compare performances within and between countries and can create pressure to change mathematics curricula and teaching practices. Nevertheless, it is well documented that classroom practice remains resistant to the reform approaches promoted by mathematics education researchers (e.g., Gill & Boote, 2012). In

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light of discussions in the literature and the move within Australia towards a national assessment of research engagement and impact, it is especially timely for mathematics educators to consider how to evidence the uptake and benefit of their research.

Defining and Evidencing Research Engagement and Impact

For the purpose of illustration, I refer to the Numeracy Across the Curriculum (NAC) research program to which the presenters of this symposium have contributed in different combinations and in different ways. The program builds on sixteen years of productive engagement with teachers, teacher educators, policy-makers, school systems, and the Australian and international research community. The research was motivated by a desire to challenge narrow "basic skills" interpretations of numeracy that prepare low-achieving students to do no more than "survive" in the world beyond school. As a result, the research team developed a rich interpretation of numeracy that connects the mathematics learned at school with out-of-school situations that additionally require problem solving, critical judgment, and making sense of the non-mathematical context. This approach necessarily positions numeracy as an across-the-curriculum commitment that extends beyond the mathematics classroom. The most significant outcome of the program is a model of numeracy for the 21st century that recognises the intellectual, affective, and contextual demands of becoming a numerate person.

According to this model, numeracy development requires attention to real-life *contexts*, the application of *mathematical knowledge*, the use of representational, physical and digital *tools*, and positive *dispositions* towards the use of mathematics. A further important and overarching element of the model is a *critical orientation* to the use of mathematics (see Goos, Geiger, & Dole, 2014).

Research Engagement

The ARC *Consultation Paper* draws on the definition used by the Academy of Technological Sciences and Engineering (ATSE) to develop metrics for Australian universities' research engagement. Engagement was defined as:

> the interaction between researchers and research organisations and their larger communities/ industries for the mutually beneficial exchange of knowledge, understanding and resources in a context of partnership and reciprocity. (ATSE, 2015)

However, the *Consultation Paper* notes that metrics, which are largely based on research commercialisation income and patents, may not capture the complexity of some forms of research engagement. As a qualitative alternative, Figure 1 maps the interactions between my own academic teaching, research, and service roles that led to beneficial knowledge exchanges in teacher education, professional development, and consultancy settings, involving practitioners, school leaders, education systems, professional associations, and teacher registration authorities as part of my contribution to the NAC research program. Also noticeable in this diagram is the absence of a neat linear progression from research contexts towards contexts of application. Instead, knowledge exchange has built reciprocal relationships across the boundaries between research, policy, and practice.

Research Impact

The ARC (2012) defines research impact as "the demonstrable contribution that research makes to the economy, society, culture, national security, public policy or services, health, the environment, or quality of life, beyond contributions to academia".

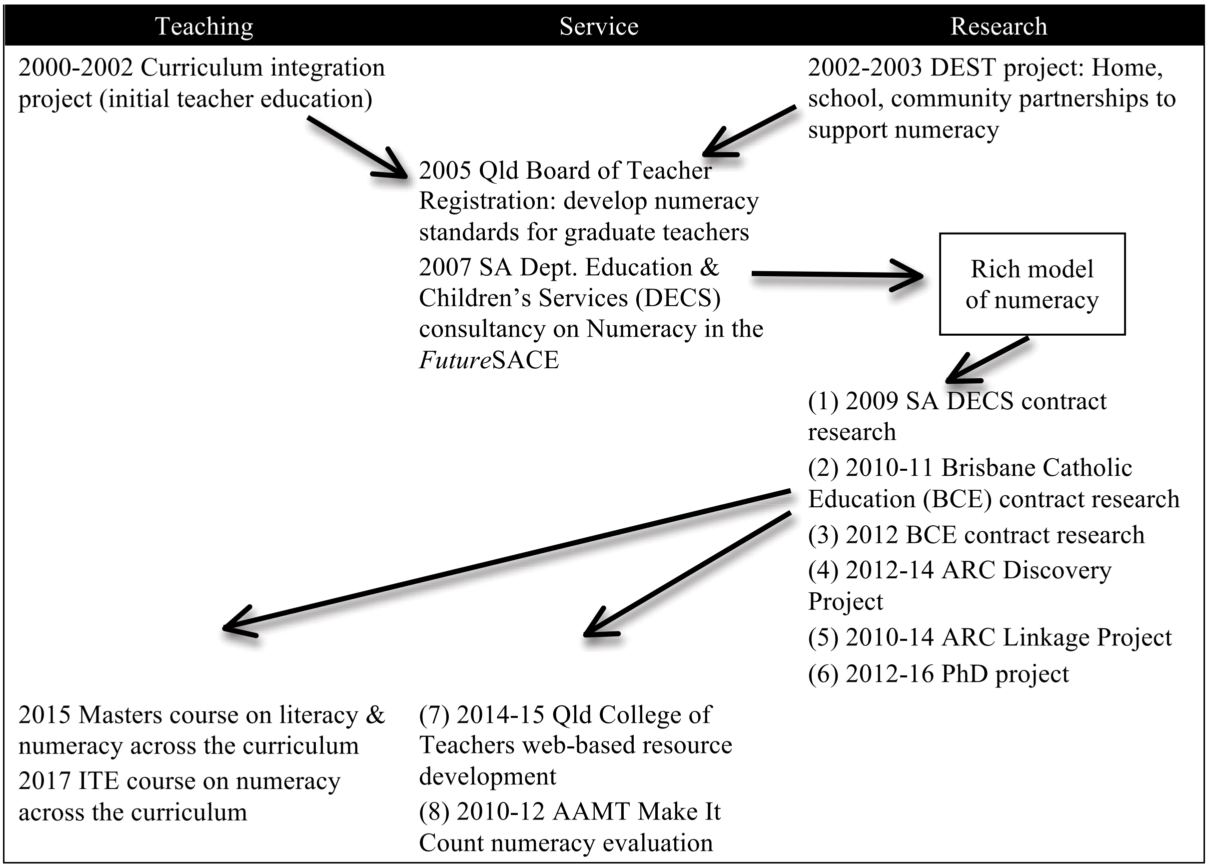

Figure 1. Personal engagement map: Numeracy across the curriculum research program

While noting that there are no clearly defined indicators for research impact, the Consultation Paper refers to peer reviewed case studies – conducted as part of the recent UK REF exercise – as being an appropriate means of assessment. Nevertheless, case studies are expensive to produce. The ARC has also developed a Research Impact Pathway table (http://www.arc.gov.au/research-impact-principles-and-framework#table) to assist grant applicants to identify the potential benefits of their proposed research. The table's column headings are listed below, together with education-relevant examples:

1. Inputs: budget, research assistants, infrastructure;
2. Activities: research project, undergraduate teaching, professional development;
3. Outputs: publications, PhD graduates, resources developed;
4. Outcomes: uptake of resources, research incorporated into teacher education and support materials, changes in policy based on research evidence;
5. Benefits: improved outcomes for learners, improved teaching practice.

Figure 2 provides a mapping of some of the outputs and outcomes of the NAC research program, using the same eight research activities identified in Figure 1. Evidencing direct benefits to students and teachers remains a pressing challenge for much education research.

Sample Research Outputs and Outcomes	Research Activities							
	#1	#2	#3	#4	#5	#6	#7	#8
Curriculum audit methodology	*			*				
Principles for task design and curriculum planning	*	*		*	*		*	
Professional development approach	*	*	*	*	*		*	*
Methods for monitoring teacher learning trajectories	*			*		*		
Whole school approaches to numeracy leadership			*		*		*	*
Resources for teachers and teacher educators	*			*	*	*	*	

Figure 2. Partial impact map: Numeracy across the curriculum research program.

Issues for Consideration

Two of the key issues identified in the *Consultation Paper* are worth considering here. The first involves accounting for the variable time lags between undertaking research and achieving identifiable benefits for end-users. As Figure 1 shows, it can take more than ten years for education research to make a demonstrable contribution to society. The second issue refers to the difficulties in determining the attribution of research engagement and impact, for example, if an impact can be traced back to more than one project, as is the case in the NAC research program (see Figure 2). Not only was impact derived across multiple research activities, but these activities also spanned the multiple universities in which the research team members worked. It remains to be seen how a national assessment of impact could be undertaken if the unit of analysis is the individual university.

Beyond the immediacy of an impending national assessment of research engagement and impact, there is surely value for mathematics educators in retrospectively analysing our own research to illuminate the opportunities taken, decisions made, and relationships built in pursuing research that makes a difference. Such an analysis might help us not only to learn "where to look" for evidence of past impact, but also to plan future research projects with an eye to demonstrating potential benefits for educational policy and practice.

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The Convoluted Nature of a Research Impact Pathway

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How research inputs and activities translate into outputs, outcomes and benefits is an increasingly important question within Australian mathematics education research. The pathway from the development of new ideas that drive educational projects through to innovations that have broad influence at local, national and international levels, however, is often convoluted and notoriously difficult to strategise. In this paper, I develop a case study of an aspect of my own research to illustrate the complexity of the journey from research origin through to impact. The documentation of this research progress allows for reflection on how future impact can be "read" while research is taking place.

The generation of innovative ideas and procurement of funds for testing new approaches in the field is at the heart of research in education. However, the "impact" of such research, educational outcomes and benefits to society, is no longer seen as a matter of potential but rather an expectation by increasingly numbers of funding bodies. While measures of research quality have been a topic of discussion for more than a decade within the Australian research context, the Engagement and Impact pilot currently being conducted by the Australian Research Council (ARC) provides evidence that a sharper focus will be drawn on this issue in future funding rounds. The proposed Engagement and Impact Assessment, as part of the National Innovation and Science Agenda (Australian Government, 2017) will consider: research interactions with a broad range of stakeholders including: industry; Government, non-governmental organisations; and research contributions to the economy, society and environment. This assessment will be conducted as a companion exercise to the Excellence in Research for Australia and is anticipated to be a significant consideration in future ARC application assessments. Such an exercise poses the challenge of how researchers will identify and then document the impact of their work? And, perhaps more importantly, dares researchers to think if it is possible to consider or strategise impact – to "read" impact – as a component of the research enterprise before and/or during the conduct of an investigation and not just in hindsight.

In this paper, I present a case study based on my contribution to the Numeracy Across the Curriculum (NAC) research program to illustrate the complexity of the journey from research origin through to impact. The journey began with a small research project and was sustained via a series of both self-generated and serendipitous opportunities. Analysis of this journey will focus on how perceptions of impact can be constructed through hindsight. A brief reflection on implications for "reading" impact in order to inform decisions about future individual, team or institution research behaviour will conclude the paper.

Research Impact Pathway

As a feature of the advice provided by the ARC on the nature of research impact a Research Impact Pathway (RIP) table (Australian Research Council, 2017) was developed. The Pathway depicts impact as a progression through five junctures – Inputs, Activities, Outputs, Outcomes and Benefits. Forms of evidence for each juncture are exemplified within the table, for example, possible Inputs include research income, staff, background IP, infrastructure and collections, while the exemplars of Outcomes are listed as

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 638–641). Melbourne: MERGA.

commercial products, licences and revenue, new companies – spin offs, start ups or joint ventures, job creation, implementation of programs and policy, citations and integration into policy. Progression towards impact is presented in a sequential, linear fashion as if one step leads naturally to the next. What happens in practice however is likely to be more convoluted. The convoluted nature of my impact progression within the NAC program is presented in the remainder of this paper.

Method

Figure 1, complemented by Tables 1-3, is a representation of evidence within impact junctures, defined by the RIP table, against time. The evidence included here relates to aspects in which I have been directly involved within the NAC research program, for example, authorship or co-authorship of publications, as named investigator or co-investigator of a project. Inputs are in the form of research income associated with projects (Table 1) supported via a number of funding sources. Only Activities (Table 2) that have direct connection to the NAC research program have been included. Outputs consist of publications and teaching resources. Outcomes have been identified as results of research that have receive attention across education systems or internationally. To date, I don't believe the program can have serious claim to Benefits as exemplified in the RIP table. Thus, Benefits is represented as a blank rectangle in Figure 1.

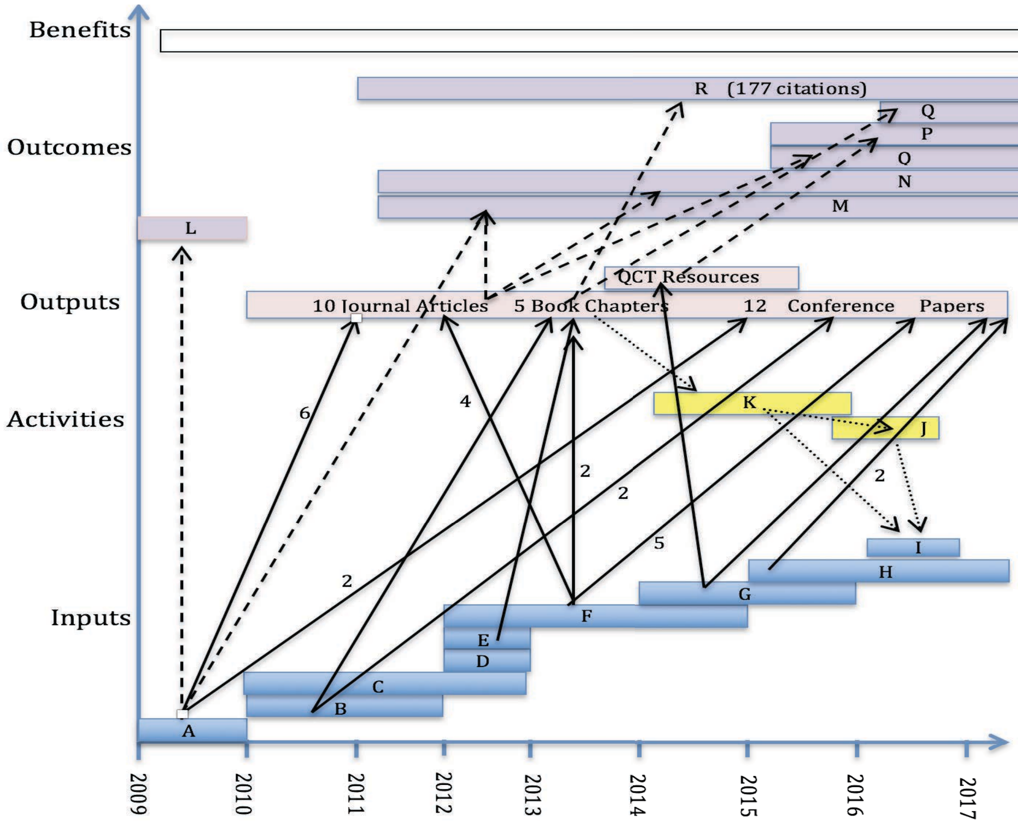

Figure 1. Representation of Geiger's RIP through the NAC research program.

Solid black arrows have been used to indicate publications or resources that were a direct Output from research projects (Inputs). The number of Outputs connected to an Input is recorded next to an arrow when this exceeds one.

Table 1
Inputs: Research Income

Project	Time	Project name
A	2009	Numeracy in the learning areas (middle years)
B	2010-2011	Leading numeracy learning
C	2010-2012	Make it count: Numeracy, mathematics and Indigenous learners
D	2012	Sustaining numeracy curriculum leadership: A whole school approach
E	2012	Models of leading curriculum reform in numeracy
F	2012-2014	Enhancing numeracy learning and teaching across the curriculum
G	2014-2015	Numeracy teaching across the curriculum in Queensland: Resources for teachers
H	2015-2017	Designing and implementing cross-curricular numeracy tasks for effective teaching and learning
I	2016	Review of the PIAAC numeracy assessment framework

Table 2
Activities

Activity	Time	Activity name
J	2016	ICME Topic Study Group Plenary
K	2014-2015	Guest Editor ZDM (Special Issue – Numeracy)

Table 3
Outcomes

Activity	Time	Activity name
L	2009	Organising structure for *Numeracy in the Middle Years* Curriculum – Department of Education and Children's Services, South Australia
M	2011-	An instructional planning tool adopted by Brisbane Catholic Education
N	2015	*Numeracy skills framework* – Department of Education NSW
O	2011-	Numeracy teaching resource package on Education Queensland's website
P	2015-	Numeracy across the curriculum resource package on QCT Website
Q	2016	One of Springer's most downloaded chapters in the last two years and was made freely available as a part of their World Teacher's Day promotion.
R	2011-	Citations (177)

Dashed arrows have been used to indicate outcomes from any of the preceding junctures – Inputs, Activities and Outputs. For example, the Outcome, *Numeracy skills framework*, developed by the Department of Education NSW draws directly on the NAC team's research to provide system wide advice to teachers about the integration of numeracy across the curriculum. In contrast, the dotted arrows flow in the opposite direction of the assumed RIP illustrating how research Outputs can also feedback into Activities that eventually new projects (Inputs). The specific example identified in Figure 1 relates to how publication Outputs helped build a case for a special issue of ZDM – Mathematics Education on Numeracy (2015) (Activities). This issue, resulted in a personal invitation for me to be part of the organising committee for the Topic Study Group (TSG) on Mathematical Literacy at ICME 2016 (Activity) and an additional invitation to deliver a plenary with two NAC team members (Goos and Forgasz) within the TSG (Activities/Outputs). An invitation from the OECD for me to contribute to Review of the PIAAC numeracy assessment framework managed by ACER (Input) followed from this series of events. This new Input will be the foundation of additional outputs and activities.

Conclusions

The general, direction of flow from Inputs to Outputs to Outcomes in Figure 1 is consistent with that of the RIP table, shown with solid and dashed arrows. The role of Activities, however, seems to be absent in the actualised pathway of my contribution to the NAC program – giving the appearance that Activities play no role in moving from Inputs to Outcomes. Perhaps this is because the exemplars provided in the RIP table are too limited (research work and training, workshop/conference organising, facility use, membership of learned societies and academies, and community and stakeholder engagement), restricting my selection of Activities and so further consideration is needed for what happenings can be considered Activities. Journal articles, for example, are not a direct output from research income (Inputs) as first the research itself must be conducted, data gathered and analysed, the article written (and usually revised); all of which are more akin to Activities than Inputs. Additionally, some influences may be too subtle to be captured in a representation such as Figure 1, such as promoting the need for research on a particular issue through a Learned Society – which leads to funds from concerned parties becoming available for research; a circuitous but still productive pathway.

The pathway indicated by the dotted arrows, however, flows in the opposite direction of that of the RIP table and demonstrates that RIP junctures can be bi-directionally influential; in this case moving from Outputs back to Inputs. Activities, in this pathway (Table 2), were significantly influential in highlighting the quality of the work in the NAC research program, leading to the inclusion of a NAC team member in a new project of international standing – a Review of the PIAAC numeracy assessment framework (Inputs).

The preceding analysis shows that my contribution to the NAC research program can be mapped from inputs to outcomes via the RIP sequence but also demonstrates that Outputs, at least, can be backward mapped to new research endeavours (Inputs). The trends identified here leads to questions about individual and institutional behaviours related to impact. How would an analysis of impact differ when considering an individual, research team or institution and what strategic decisions would result at each level? Is it possible to utilise trends identified via RIP sequences using hindsight to make strategic decisions about the type Activities, Outputs or Outcomes an individual, research team or institution should pursue in order to make greatest impact and/or lead to further research? What measures can be employed in order to shorten the timeframe of the RIP sequence – adding substance to claims of successful research investment? These are questions that will need to be addressed as we move into the new era of Engagement and Impact.

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Engagement and Impact through Research Participation and Resource Development

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This paper utilises two projects that are part of a well-established research program seeking to embed numeracy across the curriculum to illustrate how knowledge exchange and uptake of resources can be used to provide evidence of research engagement and impact. The aim of the first project was to build teachers' pedagogy around the promotion of proportional reasoning as a cross-curricular concept and a key component of numeracy; whilst that of the second was to develop resources to assist teachers to embed numeracy across the curriculum. Participation of stakeholders and the resources produced provide evidence of engagement and impact of the two projects.

A national assessment of research engagement and impact that requires universities to provide evidence of how research is translated into economic, social, and other benefits is part of the Australian Government's *National Innovation and Science Agenda*. An *Engagement and Impact Assessment Consultation Paper*, released in May 2016, sought feedback from stakeholders on how this assessment should be undertaken (ARC and DET, 2016). Several key issues associated with measuring engagement and assessing impact were raised in the *Consultation Paper*. The aim of this paper is to illustrate how knowledge exchange and the uptake of resources by teachers and education systems could be used to evidence research engagement and impact.

Research Engagement and Impact

Research engagement has been defined as the exchange of knowledge, understanding and resources that result from interactions between researchers and their wider communities (ATSE, 2015). The emphasis is on research benefit. In a recent review of trends and strategies for commercialising public research, the OECD (2013) noted that there are multiple ways in which research can be translated for economic and social benefits:

> Knowledge transfer and commercialisation of public research refer in a broader sense to the multiple ways in which knowledge from universities and public research institutions (PRIs) can be exploited by firms and researchers themselves so as to generate economic and social value and industrial development. (p. 18)

Among the various forms of research engagement considered of high significance for industry, and seen therefore as more directly transferable (and hence more impactful) are *collaborative research* and *contract research* (ARC and DET, 2016). Both these types of research were undertaken as part of the Numeracy Across the Curriculum (NAC) research program undertaken by the presenters of this symposium.

Impact is more difficult to define and assess. The ARC's Research Impact Pathway table (http://www.arc.gov.au/research-impact-principles-and-framework#table) provides one way of identifying potential benefits of proposed research. The table includes examples of impact at five stages over the life of a research project and beyond its formal conclusion under the headings of Inputs, Activities, Outputs, Outcomes, and Benefits.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 642–645). Melbourne: MERGA.

In this paper, we draw upon two specific projects from the NAC research program in an attempt to illustrate an approach to evidencing engagement and impact. The projects (an ARC Linkage and an Industry funded project) fit the categories of collaborative and contract research respectively, thus by their nature evidence research engagement. We analyse the impacts of these two projects in relation to inputs, activities, outputs, outcomes, and potential benefits to further evidence engagement and highlight the impact of each. Both these projects drew on the model of numeracy for the 21st century developed earlier in the NAC research program. According to this model, numeracy development encompasses five dimensions: mathematical knowledge, context, tools (representational, physical and digital), and positive dispositions toward the use of mathematics, which are embedded in a critical orientation (Goos, Geiger, & Dole, 2014).

ARC Linkage Project

The *Enhancing Proportional Reasoning* project was an ARC Linkage project conducted in 2010-2014 and included collaborative partner funding of 30%. The study found that improving teachers' understanding of the elements of the numeracy model broadened their teaching focus beyond the teaching of mathematical knowledge. Teachers became more aware of the need to incorporate tools, including digital tools to enhance students' numeracy ability. They more directly included a focus on student dispositions (e.g., confidence, resilience and risk taking) as a key element of promoting personal numeracy. Teachers also reported a greater awareness of and ability to identify "numeracy moments" in cross-curricular circumstances, thus broadening students' numeracy development opportunities and making this acquisition more "real life" (see Dole, Hilton, G., & Hilton, A., 2015). Teachers indicated that regular identification of proportional reasoning teaching and learning opportunities in cross-curricular contexts led to students' improved ability to identify and work with proportional situations as well as improving their meta-language, allowing them to communicate their ideas about proportional situations more precisely and concisely (see Hilton, A., Hilton, G., Dole, & Goos, 2016). This project has received international recognition for its research/practitioner focus (Hilton, A., Hilton, G., Dole, & Goos, 2013).

Industry Project

Numeracy teaching across the curriculum in Queensland: Resources for teachers was an Industry project conducted in 2014-2015 in response to a call from the Queensland College of Teachers (QCT). The project addressed a particular need of the QCT: to enhance the teaching of numeracy across the curriculum through web-based resources that could be made readily available to teachers via the QCT website. The project included a literature review of national and international good practice, an audit of existing material and consultation with stakeholders (e.g., employing authorities and teacher professional associations) to identify gaps and areas where teachers would benefit from new resources, developing video vignettes of examples of good practice in Queensland schools, and providing a brief report to the QCT (Goos, Geiger, Bennison, & Roberts, 2015). The theoretical framework that informed resource development encompassed the Board of Teacher Registration, Queensland (2005) Numeracy Standards and model of numeracy developed by the NAC research program.

The audit of existing materials and interviews with stakeholders revealed that there are very few resources available to support teachers' understanding and enactment of numeracy across the curriculum. The findings highlighted important gaps including that almost none of the existing materials addressed the need for teachers to develop the capacity to recognise and take advantage of the numeracy learning demands and opportunities within the subjects they teach. In response to these findings, six video vignettes were produced: an interview with a numeracy expert that explains some of the evidence base for the examples of good practice, a set of four classroom vignettes illustrating good practice in teaching numeracy across the curriculum at different year levels and in different subjects, and an interview with a school numeracy team that provides an example of how a whole school approach to numeracy can be developed.

Discussion and Concluding Remarks

It is possible to identify evidence of engagement and impact in the two projects presented here. There was knowledge exchange between researchers and stakeholders in the consultation process, publications, and availability of resources produced. The impact of the projects, using the column headings in the ARC's Research Impact Pathway table, are summarised in Figure 1.

ARC Research Impact Pathway	ARC Linkage	Industry Project
Inputs	Co-funding from ARC and industry partners (Education Authorities in two states)	Funding from the QCT
Activities	5 regional school clusters in two states; 60 classroom teachers; 10 school leaders; PD package for teachers (10 modules); Resource development; State conference on Proportional Reasoning in both states (105 and 130 attendees respectively);	Collaborative stakeholder engagement (teacher registration and employing authorities, teacher professional associations); Resource development
Outputs	6 refereed journal articles; 5 refereed conference papers; 6 conference presentations; 1 international research award; Book proposal	Brief report to QCT; 1 refereed conference paper; 6 video vignettes
Outcomes	11 invited keynote addresses, national and international; 20 teacher workshop presentations; Citations; Integration into school policy	Video vignettes made available on the QCT website (https://www.filmpond.com/#/ponds/qct-the-university-of-queensland)
Benefits	Potential for improved teaching practice and improved outcomes for learners	Potential for improved teaching practice and improved outcomes for learners.

Figure 1. Mapping of project impact against ARC's Research Impact Pathway.

Knowledge transfer between researchers and stakeholders, along with resources that have been taken up by stakeholders, provide evidence of engagement and impact of the ARC Linkage Project and the Industry Project. The outcomes and outputs of the two projects are summarised in Figure 2 and illustrate how each project contributes to the engagement and impact of the NAC research program which has been conducted over a 16-year period by researchers in multiple universities, with outputs and outcomes being built upon in successive projects.

Sample Research Outputs and Outcomes	Research Activities	
	2010-2014 ARC Linkage Project	2014-2015 Industry Project
Principles for task design and curriculum planning	*	*
Professional development approach	*	*
Whole school approaches to numeracy leadership	*	*
Development of resources for teachers	*	*
Assessment of numeracy capability	*	

Figure 2. Impact map for ARC Linkage Project and Industry Project.

This analysis also illustrates two issues identified in the *Consultation Paper*; that is, the time lag between research and benefits for end-users and the difficulty in attributing impact to a single project or university.

Acknowledgements

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"Numeracy for Learners and Teachers": Impact on MTeach Students[1]

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"Numeracy for learners and teachers" is a compulsory unit for all primary and secondary pre-service teachers in the Monash University Master of Teaching course. In the first two years that the unit was taught (2015 and 2016), research was conducted to evaluate the unit's impact on students' understanding of the construct, numeracy, and on their confidence to incorporate numeracy in their teaching across the curriculum. In both years, surveys were administered before commencement and on completion of the unit; a small number of students were also interviewed. The major findings from the two-year study are presented in this paper.

Introduction

There were two main drivers for the development of the unit, *Numeracy for Learners and Teachers* (EDF5017), as a compulsory study in the Monash University Master of Teaching (MTeach) course. All MTeach students, except those focusing only on becoming teachers in the early years, must complete this unit. The two drivers were:

1. Numeracy as one of seven general capabilities in the Australian Curriculum (AC; Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016), the basis of the curriculum in each state/territory of Australia. "Teachers are expected to teach and assess general capabilities to the extent that they are incorporated within learning area content" (ACARA, n.d.-a) In the AC, numeracy is defined as encompassing "the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully" (ACARA, n.d.-b).
2. The graduate standards developed by the Australian Institute for Teaching and School Leadership (AITSL). These standards must also be met as part of the accreditation process for providers of teacher education. The specific AITSL (2014) graduate standards underpinning the development of EDF5017 were:
   - Standard 2.5 Literacy and numeracy strategies: "Know and understand literacy and numeracy teaching strategies and their application in teaching areas".
   - Standard 5.4 Interpret student data: "Demonstrate the capacity to interpret student assessment data to evaluate student learning and modify teaching practice".

The definition of numeracy embraced in EDF5017 is that adopted in the AC and described above. The elements of the 21st Century Model of Numeracy (Goos, Geiger, & Dole, 2014) scaffolded the curriculum design for the unit. Focusing on the identification of numeracy demands and opportunities across all AC curricular domains at all grade levels, as well as developing the personal numeracy skills needed by practicing teachers were among the outcome goals of the unit.

[1] I acknowledge the contributions of my colleague, Jennifer Hall, who conducted this research with me.

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 646–649). Melbourne: MERGA.

EDF5017 was first offered in 2015. In that year, the cohorts enrolled were MTeach (Secondary) and MTeach (Primary/Secondary) students; in 2016, the cohorts were MTeach (Primary) and MTeach (Early Years/Primary) students. Due to a revision of the timing of some MTeach offerings, from 2017, the four cohorts of students will be enrolled in the unit simultaneously.

In this paper, I present some of the findings from a two-year study conducted with the EDF5017 students that was aimed at evaluating the unit's overall impact. The research focus reported here is on students' understanding of the construct, *numeracy*, and on the students' confidence to incorporate numeracy in their teaching across the curriculum. The MTeach cohort split in 2015 and 2016 enabled the data to be examined for any differences among secondary pre-service teachers (2015) and primary pre-service teachers (2016).

Methods and Analyses

A mixed methods approach was adopted to gather data to evaluate the success of the unit. Pre- and post-surveys were developed so that changes in views in response to studying EDF5017 could be gauged. The items focused on in this paper include those designed to identify changes in the pre-service teachers' understandings of the construct numeracy, and in their confidence to incorporate numeracy in teaching. Interviews were conducted with volunteers a short time after the post-survey had been completed, the main aim being to gather views on the content and structure of the unit.

Results and Discussion

The Samples

The pre- and post-survey samples in 2015 and 2016 are shown in Table 1.

Table 1
2015 and 2016 Pre- and Post-Survey Samples

	2015		2016	
	Pre-survey	Post-survey	Pre-survey	Post-survey
Participants	53 began; 40 finished	35 began; 20 finished	46 began; 22 finished	21 began; 13 finished
Gender	81% female	74% female	90% female	81% female
Age	77% aged 25-34	74% aged 25-34	80% aged 25-34	86% aged 25-34
MTeach stream	Secondary only (74%)	Secondary only (80%)	Primary only (79%)	Primary only (90%)

As can be seen in Table 1, most respondents were female, and most were aged 25-34. Of the 2015 cohort, more were enrolled in MTeach (Sec) than in MTeach (Prim/Sec); for the 2016 cohort, more were enrolled in MTeach (Prim) than in MTeach (EY/Prim).

Findings

Differences between numeracy and mathematics. Responses to the item "Are there differences between mathematics and numeracy?" (Yes/No/Unsure) to the 2015 and 2016 pre- and post-surveys are shown in Table 2.

Table 2
Are there Differences between Mathematics and Numeracy?

	2015		2016	
	Pre-survey (n = 45)	Post-survey (n = 21)	Pre-survey (n = 29)	Post-survey (n = 13)
Yes	76%	95%	90%	92%
No	4%	0%	0%	8%
Unsure	20%	5%	10%	0%

As can be seen in Table 2, for the 2015 cohort, there was a noteworthy increase in the proportion of secondary pre-service teachers who answered "Yes" after completing studies in EDF5017. While starting from a very high base (90% of students), among the 2016 primary pre-service teacher cohort, there is no noticeable difference in the proportions saying "Yes" in the pre- and post-surveys. One possible explanation for the 2016 cohort being aware that there is a difference is that this cohort had already completed units in the teaching of primary mathematics and the issue had already been discussed in those units.

Participants were also asked to explain their answers to the question. Typical answers are shown below:

> I'd never really given it much thought before now. Both scare me!!!
>
> One is the subject, the other is the application of the subject in real life situations.
>
> Mathematics is to numeracy what language is to literacy - only part of the whole.
>
> I think that numeracy is a broader concept than mathematics, because otherwise we wouldn't have pure maths.

Confidence incorporating numeracy into the teaching of their subject area(s). On both the pre- and post-surveys, respondents were asked to indicate on a 5-point response format (very lacking in confidence to very confident) how confident they felt about incorporating numeracy into the teaching in their subject area/s. The pre- and post-survey responses for the 2015 and 2016 samples are shown in Figures 1 and 2 respectively.

It can be seen in both Figures 1 and 2, that there were noteworthy changes in confidence from pre- to post-survey. That is, in both samples of pre-service teachers, more were somewhat or very confident after studying EDF5017 than before.

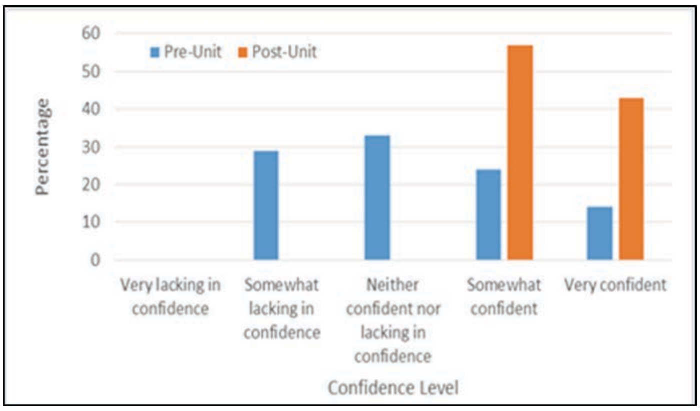

Figure 1. Pre- and post-survey responses from 2015 participants.

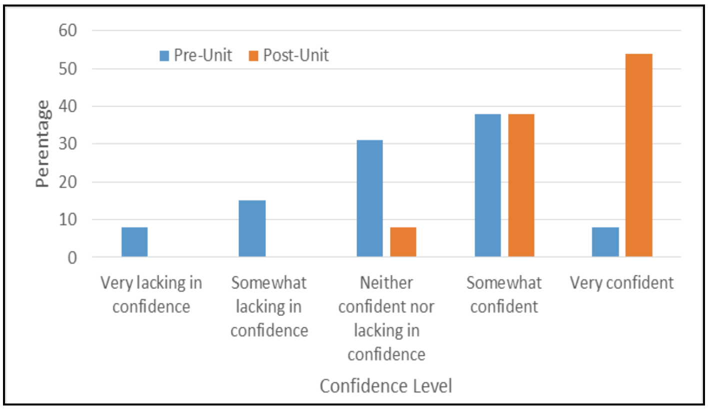

Figure 2. Pre- and post-survey responses from 2016 participants.

The following comment from one of the 2015 (secondary) post-survey respondents encapsulates the sentiments of many of the students:

> I have a clearer understanding of what numeracy entails, have been provided examples with how it would work in my method curriculum areas, and feel confident that I have adequate mathematical reasoning and numeracy skills to be able to handle this in my teaching.

In the post-survey only, students were asked if the unit had impacted their views of numeracy. The majority responded, "Yes" (86% in 2015, 85% in 2016). Some typical explanations for their positive responses included:

> I did not know the word before this unit.
>
> I understand it is my responsibility to teach this [numeracy] – AITSL and curriculum require it.
>
> I now know the difference between mathematics and numeracy.

Final Words

Clearly, completing EDF5017 resulted in a substantial and important impact on students' confidence in incorporating numeracy in their teaching, and in having a better appreciation of what numeracy is and how it differs from mathematics. Units such as EDF5017 are now expected for accreditation of teacher education programs. Based on the findings reported here, it is anticipated that the benefits to the school population and the future citizenry of Australia are likely.

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Reframing Mathematical Futures: Using Learning Progressions to Support Mathematical Thinking in the Middle Years

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The Australian Curriculum: Mathematics calls for the concurrent development of mathematical skills and mathematical reasoning. What are the big ideas of mathematical reasoning and is it possible to map their learning trajectories? Using rich assessment tasks designed for middle-years students of mathematics, this symposium reports on the preliminary phase of a large national study designed to move beyond the hypothetical and to provide an evidence-based foundation for learning progressions in mathematical reasoning in three key areas of the curriculum: Algebraic Reasoning, Geometrical and Spatial Reasoning, and Statistical Reasoning.

Paper 1: Dianne Siemon. *Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years – Introducing the Reframing Mathematical Futures II Project*

This paper presents an overview of the project and discusses the importance of mathematical reasoning.

Paper 2: Lorraine Day, Max Stephens, & Marj Horne. *Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years – Algebraic Reasoning*

The results of the initial trialling of a set of items designed to identify algebraic reasoning, and the big ideas of algebra will be discussed.

Paper 3: Marj Horne & Rebecca Seah. *Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years* – Geometric Reasoning

Little recent research addresses geometrical and spatial reasoning. This paper reports on a hypothesised learning hierarchy and the results from the trial process.

Paper 4: Jane Watson & Rosemary Callingham: *Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years – Statistical Reasoning*

Using an existing research base, and the outcomes from trial tests, this paper describes a learning hierarchy of statistical reasoning.

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Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years: Introducing the Reframing Mathematical Futures II Project

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The Australian Curriculum: Mathematics calls for the concurrent development of mathematical skills and mathematical reasoning. What are the big ideas of mathematical reasoning and is it possible to map their learning trajectories? Using rich assessment tasks designed for middle-years students of mathematics, this paper reports on the preliminary phase of a large national study designed to move beyond the hypothetical and to provide an evidence-based foundation for learning progressions in mathematical reasoning in three key areas of the curriculum.

Why Mathematical Reasoning?

The Programme for International Student Assessment (PISA) results for 2012 and 2015 indicate a significant decline in mathematical literacy rates among Australian 15-year-olds since 2003 (Thomson, De Bortoli, & Buckley, 2013; Thomson, De Bortoli, & Underwood, 2016). In particular, the results reported in 2013 suggest that

> interpreting, applying and evaluating mathematical outcomes … is an area of relative strength for Australian students, while formulating situations mathematically and employing mathematical concepts, facts, procedures and reasoning are seemingly processes of relative weakness (p. x).

This is consistent with the Middle Years Numeracy Research Project (MYNRP), which found that many students in Years 5 to 9 experience considerable difficulty interpreting problem situations, applying what they know to solve unfamiliar situations, explaining their thinking, and communicating mathematically (Siemon, Virgona, & Corneille, 2001). It is also consistent with data from the Trends in International Mathematics and Science Study 1999 Video Study that led Stacey (2003) to call for an increased focus on mathematical reasoning. Although these capacities are recognised and valued in the Australian Curriculum: Mathematics (ACM) in the form of the four proficiencies, that is, conceptual understanding, procedural fluency, mathematical problem solving, and mathematical reasoning, these are often not reflected in school mathematics at this level where:

- mathematics is typically represented as a set of disconnected topics and skills to be demonstrated and practiced rather than explored, discussed and connected (Shields & Dole, 2013; Siemon, Bleckly, & Neal, 2012)
- the vast majority of textbook problems at Year 8 tend to be relatively low-level, skill-based repetitious exercises (Vincent & Stacey, 2008)

A focus on mathematical reasoning is needed to equip teachers with the knowledge, confidence and disposition to go beyond narrow skill-based approaches to teaching mathematics in the middle years. Defined broadly in the ACM as a "capacity for logical thought and actions", mathematical reasoning has a lot in common with mathematical problem solving, but it also relates to students' capacity to see beyond the particular to generalize and represent structural relationships, which is a key aspect of further study in mathematics and a key underpinning of Science Technology Engineering and Mathematics (STEM)-related studies (Wai, Lubinski, & Benbow, 2009).

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 651–654). Melbourne: MERGA.

Why Learning Progressions?

Australian teachers of mathematics are familiar with scope and sequence charts and curriculum documents that imply broad developmental progressions in mathematics learning from the early to the post compulsory years of schooling. While the implied sequences in Number and Algebra are generally supported by research in the early years (e.g., Clarke et al., 2002; Mulligan & Mitchelmore, 2009), the evidence for the implied learning sequences beyond the early years and in other domains is less conclusive. One of the reasons for this is that although there has been considerable research on particular aspects of these domains in the middle years of schooling, much of this is "fragmented due to the variations in research questions and methods" (Confrey & Malone, 2014, p. xiv).

A more pervasive issue is the fact that curriculum content descriptors are generally expressed in a form that allows observation and measurement with little/no indication of their relative importance or how they connect to the 'big ideas' in mathematics needed to ensure students make progress. This situation inevitably privileges skills over concepts and de-emphasises the processes of mathematical problem solving and reasoning. Research is needed to identify big ideas and developmental pathways that underpin mathematical reasoning in the middle years of schooling to give "teachers, textbook authors and curriculum writers a sense of what type of reasoning they can expect and encourage at each level and in what directions students' reasoning should be developed" (Stacey, 2010, p. 19).

It is only relatively recently that learning progressions/trajectories per se have become the focus of systematic research efforts (e.g., Clements, 2002; Confrey, 2008; Daro, Mosher, & Corcoran, 2011; Siemon, Izard, Breed, & Virgona, 2006). Prompted by Simon's (1995) introduction of the notion of Hypothetical Learning Trajectories, there is debate about the meaning and use of learning progressions/trajectories in mathematics education (e.g., see the special edition of *Mathematics Teaching and Learning, 6*(2) in 2004). However, a common element in the different interpretations and use of the terms is the notion that learning takes place over time and that teaching involves recognising where learners are in their learning journey and providing challenging but achievable learning experiences that support learners' progress to the next step in their particular journey.

Outline of the RMF II Project

Reframing Mathematical Futures II (RMFII) is a three-year project funded by the Australian Government Department of Education and Training under the auspices of the Australian Mathematics and Science Partnership Programme (AMSPP). The project is working with industry partners and practitioners in each State and Territory and the Australian Association of Mathematics Teachers (AAMT) to build a sustainable, evidence-based, integrated learning and teaching resource to support the development of mathematical reasoning in Years 7 to 10 comprising:

- evidence-based learning progressions in algebraic, statistical, and spatial reasoning that can be used to inform teaching decisions and the choice of mathematics learning activities and resources by teachers and students;
- a range of validated, rich assessment tasks and scoring rubrics that can be used to identify what students know and understand in terms of the learning progressions, inform starting points for teaching and show learning over time (i.e., as pre- and post-tests);

- detailed teaching advice linked to the learning progressions that establish and consolidate learning at the level identified and introduce and develop the ideas and strategies needed to progress learning to the next level of the framework;
- indicative resources to support the implementation of a targeted teaching approach in mixed ability classrooms.

Methodology of the RMF II Project

The RMFII project has been designed in terms of three distinct but overlapping phases. Phase 1 focussed on the identification of hypothetical learning progressions from the research literature to inform task design, the provision of professional learning to support a targeted teaching approach (Siemon, 2017) to mathematics in the middle years and the development and trial of rich tasks to assess algebraic, statistical and spatial reasoning at this level. Rasch modelling (Bond & Fox, 2015) was used to analyse data collected from the trial tasks and the findings used together with the hypothetical learning progressions to formulate Draft Learning Progressions in each area of interest.

Phase 2 is focussed on the preparation and use of multiple assessment forms for mathematical reasoning, the analysis of student and teacher surveys, and the development of teaching advice and professional learning modules to support a targeted teaching approach. The final phase of the project will focus on the development and publication of project outcomes and reports. Project partners in all Australian States and Territories identified between four to six secondary schools in their jurisdiction that met the funding requirements (i.e., located in lower socio-economic regions with diverse populations). A specialist teacher was identified from each school and is being supported by the research team to work with up to 6 other teachers in their school to trial assessment tasks and implement a targeted teaching approach to mathematical reasoning. A total of 32 secondary schools, approximately 80 teachers, and 3,500 students in Years 7 to 10 are involved in the project.

This symposium will consider preliminary findings from the first phase of the RMFII project, which was focussed on the development of evidence-based Draft Learning Progressions in algebraic, statistical and spatial reasoning. This phase was designed to address the following research questions.

- To what extent can we develop rich tasks to accurately identify key points in the development of mathematical reasoning in the junior secondary years?
- To what extent can we gather evidence about each student’s achievements with respect to these key points to inform the development of a coherent learning and assessment framework for mathematical reasoning?

The first step in this process involved the derivation of hypothetical learning progressions in each domain from a review of the literature by specialist members of the research team. A range of assessment tasks and scoring rubrics were then devised to assess key elements of these progressions. These tasks were arranged in 24 different but overlapping forms and trialled with 3,075 students from 18 trial schools and coded by a team at RMIT University. The resulting data were analysed using the Rasch partial credit model (Masters, 1982) using Winsteps 3.92.0 (Linacre, 2016).

Rasch analysis allows both students’ performances and item difficulties to be measured using the same log-odds unit (the logit), and placed on an interval scale (Bond & Fox, 2015). Items that did not fit the model were examined and refined. A small number of items was removed as not useful or too complex for students to understand. A refined set of overlapping forms was constructed and used with 3,366 students from participating research schools. This allowed the further refinement of the Draft Learning Progressions and it is these and detailed processes involved that are highlighted in this symposium.

The three related papers consider the derivation of the Draft Learning Progressions for algebraic reasoning, statistical reasoning, and geometric reasoning respectively. In each case, eight incremental Zones were identified on the basis of the hierarchy of items created by the Rasch analysis. Descriptors of student behaviour were derived from a consideration of the cognitive demands of items within each Zone. Where there are insufficient items in a Zone to address a particular ‘big idea’ or generate descriptors, additional items will be developed, trialled and used to further inform the Draft Learning Progressions.

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Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years: Algebraic Reasoning

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As part of the Reframing Mathematical Futures II Project on Mathematical Reasoning, algebraic reasoning was identified as one of the three areas to be investigated. This involved developing a hypothetical learning progression for algebra to inform the design of assessment tasks to test the progression. The assessment forms were then sent to trial schools and the data was analysed using Rasch Analysis. This paper reports on the analysis of the preliminary data received and outlines some implications for teaching.

The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016) has combined Number and Algebra in a single strand to allow both to be developed together. Developing both numerical and algebraic reasoning together provides students with the opportunity to notice structure and powerful schemes for thinking about number patterns and relationships (Carpenter, Franke, & Levi, 2003). This implies that classroom practices need to adapt to build a more robust understanding of mathematics as a process of generalisation and formalisation, or as Kaput (1998) expressed it, 'algebrafying' the process. This transformation could be viewed as moving classroom practice from one of following rules and memorisation to one of sense-making (Flewelling, Kepner, & Ewing, 2007; Schoenfeld, 2008).

In order to identify a hypothetical learning progression for algebraic reasoning a review of the literature was conducted to identify the big ideas of algebra. Although the focus was to be on algebraic reasoning, it was considered appropriate to identify algebraic content, as students, at different levels, need content about which to reason. Underpinning this content focus was the understanding that in order to reason algebraically at the highest level involves visualisation, being able to move fluidly between multiple representations and having the language and discourse to reason mathematically.

Initially, hypothetical learning progressions were developed for five big ideas in algebra identified as: Pattern and Sequence, Generalisation, Function, Equivalence, and Equation Solving (Blanton, & Kaput, 2011; Blanton et al., 2015; Carraher, Schliemann, Brizuela, & Earnest, 2006; Fujii & Stephens, 2001; Mason, Stephens, & Watson, 2009; Panorkou, Maloney, & Confrey, 2013; Perso, 2003; Stephens & Armanto, 2010; Watson, 2009). However, as there was considerable overlap in the descriptors at this stage, it was decided to re-organise these in terms of: Pattern and Function, Equivalence, and Generalisation. An example of the hypothetical learning progression developed for Generalisation is shown in Table 1.

Table 1
The Hypothetical Learning Progression for Generalisation

Zone	Descriptor
1	Explain a generalisation of a simple physical situation.

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2	Explore and conjecture about patterns in the structure of number, identifying numbers that change and numbers that can vary.
3	Explain generalisations by telling stories in words, with materials and using symbols.
4	Explain generalisations using symbols and explore relationships using technology.
5	Follow, compare and explain rules for linking successive terms in a sequence or pair quantities using one or two operations.
6	Use and interpret basic algebraic conventions for representing situations involving a variable quantity.
7	Use and interpret algebraic conventions for representing generality and relationships between variables and establish equivalence using the distributive property and inverses of addition and multiplication.
8	Combine facility with symbolic representation and understanding of algebraic concepts to represent and explain mathematical situations.

Once the hypothetical learning progressions were identified on the basis of prior research, assessment tasks containing one or more items were compiled into forms that were designed to evaluate the three big ideas across Zones. Some tasks/items addressed a particular big idea while others assessed several of the big ideas in a single task. For instance, the seven-item Relational Thinking task was designed to evaluate key aspects of the hypothetical learning progressions for the two big ideas of Equivalence and Generalisation (see Table 2).

Table 2
The Relational Thinking Items and Rubrics

Item No.	Item		Rubric
1	What numbers would replace the ? to make a true number sentence (the numbers may be different). Explain your reasoning ? + 521 = 527 + ?	0	No response or irrelevant response
		1	Incorrect response but with correct reasoning based on the relationship between 521 and 527
		2	Two correct numbers given but little/no reasoning
		3	Two correct numbers given where the number on the left is 6 more than the number on the right with reasoning that reflects relationship between 521 and 527
2	Find a different pair of numbers that would make the number sentence above true	0	No response or irrelevant response
		1	A different and correct pair
3	Describe how you could find all possible pairs of numbers that would make this a true sentence.	0	No response or irrelevant response
		1	Incorrect attempt at describing based on previous answers
		2	Statement regarding difference of 6 or expression showing difference
4	What numbers would replace the ? to make a true number sentence (the numbers may be different)? ? – 521 = ? - 527	0	No response or irrelevant response
		1	Incorrect response but with correct reasoning based on the relationship between 521 and 527
		2	Two correct numbers given but little/no reasoning, may include some calculations
		3	Two correct numbers given where the number on the right is 6 more than the number on the left, with reasoning that

			reflects the relationship between 521 and 527
5	Find another set of numbers that would make the number sentence in 4 true.	0	No response or irrelevant response
		1	A different and correct pair
6	Describe how you could find all possible pairs of numbers that would make this a true number sentence	0	No response or irrelevant response
		1	Incorrect attempt at describing based on previous answers
		2	Statement regarding difference of 6 or expression showing the difference
7	What can you say about the relationship between c and d in this equation? $c \times 2 = d \times 14$	0	No response or irrelevant response
			Specific solution provided ($c = 7$ and $d = 1$) or a general statement (c is 7 times the number d)
			Statement correctly describes the relationship (c is 7 times the number d)

Results

Rasch analysis was used to rank student responses to the algebraic reasoning tasks and create a Draft Learning Progression for Algebra. From this it was possible to identify where different student responses to each of the Relational Thinking items were located on the progression. For instance, a score of 2 on RT1 (indicated by RT1.2 in Table 3 below) was located in Zone 3 while a score of 3 on RT1 (RT1.3) was located in Zone 6. Table 2 shows a range of responses to the RT items and their relationship to the big ideas of Equivalence (Equiv) or Generalisation (Gen).

Table 3
Results of Rasch Analysis on the Relational Thinking Items

RT1.2	RT1.3	RT2.1	RT3.1	RT3.2	RT4.2	RT4.3	RT5.1	RT6.2	RT7.1	RT7.2
Zone 3	Zone 6	Zone 4	Zone 5	Zone 6	Zone 5	Zone 7	Zone 5	Zone 7	Zone 4	Zone 6
Equiv	Gen	Equiv	Equiv	Gen	Equiv	Gen	Equiv	Gen	Equiv	Gen

The different student responses indicated by the scores for each item in Table 2 range from Zone 3 to Zone 7. Those that relate to Equivalence range from Zone 3 to Zone 5. Finding a correct pair of numbers to make a correct number sentence (RT1.2) was the easiest at Zone 3. Finding another correct pair of numbers to the same question (RT2.1) was at Zone 4. Whereas, finding two correct pairs of numbers that satisfied the subtraction number sentence (RT4.4) was scaled higher at Zone 5. Components that required students to give a general explanation of a relationship were scaled at Zone 6 or Zone 7. Generalisation items were typically more difficult than Equivalence items; and among Generalisation items, as Table 2 shows, explanations involving subtraction or difference tended to be more difficult than those involving addition relationships. This confirms research findings by Stephens and Armanto (2010), Mason et al. (2009), and Carpenter et al. (2003).

In most cases incorrect responses to items in the Relational Thinking task were located in the lower Zones of the progression. For example, giving an incorrect response to the missing numbers in item 1 was scaled at Zone 1. However, an incorrect attempt at describing the relationship between the two missing numbers based on previous answers for item 2 was at Zone 4; and an incorrect attempt at describing the relationship based on previous answers for item 6 was scaled at Zone 5. These latter two results which embody incorrect or incomplete generalisations show that, for our upper primary and junior secondary students, generalisation and explanation of algebraic thinking remains quite difficult. As the research of Kaput et al. (1998), Carraher et al. (1996), and Blanton et al. (2015) demonstrated, helping students to articulate and refine their algebraic thinking, especially their algebraic reasoning and justification, are complex and challenging tasks even for capable teachers. These abilities require constant and supportive cultivation if they are to be achieved by most students. The preliminary data presented above show that they have been achieved by some students. Expanding the range of achievement, especially with respect to the development of reasoning, remains our challenge as this project moves into its next phase.

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Learning Progressions to Support Mathematical Reasoning in the Middle Years: Geometric Reasoning

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Geometric reasoning is an important construct in excelling in STEM related disciplines. Yet this is a topic that is most neglected and least understood by teachers and students alike. As part of the Reframing Mathematical Futures II Project, this paper reports the development of a learning progression that provides explicit validated mapping of students' growth in geometric thinking. Thirty-six items collated into two assessment forms were administered and analysed using Rasch modelling. Eight learning Zones were identified.

The Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2016) made a distinction between 'spatial reasoning', as one of the six interrelated elements in the numeracy learning continuum, and 'geometric reasoning' as one of the content descriptors to be taught from Year 3. Beyond this, there are scant details in clarifying the differences between these and how to help children develop both reasoning abilities. Spatial reasoning is the ability to see, inspect and reflect on spatial objects, images, relationships and transformations (Battista, 2007). Geometric reasoning is the use of critical thinking, and spatial reasoning to logically deduce and find new geometric relationships in problem situations. Its success is dependent upon the use of geometric knowledge and spatial reasoning to identify and formulate axiomatic relationships. Research affirmed that individuals progress through distinct stages of geometric thinking: visualise, describe, analyse, infer and deduce geometric relationships, and formal proof (Battista, 2007; van Hiele, 1986). These five stages are seen as interconnected and developed progressively with various degrees of emphasis and importance depending on the task demand. Proficiency in one domain is supported by good command of stages developed earlier. How to move students through these stages of thinking is the issue in question, hence the development of a hypothetical learning progression (HLP).

Learning progressions are a set of empirically grounded and testable hypotheses about how students' understanding of, and ability to use, specific discipline knowledge within a subject domain in increasingly sophisticated ways develops through appropriate instruction (Corcoran, Mosher, & Rogat, 2009). The purpose is to provide explicit validated mapping of students' growth in thinking and instructional advice on how to promote thinking through the stages. We take the position that the development of geometric thought is underpinned by the degree of connectedness between representation, visualisation and mathematical discourse. Geometric representations in the form of lines, shapes and diagrams are schematic, bound by their "formal concept definitions" (Tall & Vinner, 1981, p. 152) and developed through the process of visualising, a form of cognitive process in which objects are interpreted within the person's existing network of beliefs, experiences, and understanding (Phillips, Norris, & Macnab, 2010). How accurately individuals interpret the image is dependent upon how well they communicate what they see. Students literally talk themselves into understanding geometric properties. Evidence suggests that quality and targeted teaching is crucial to help shift thinking to the next level (Corcoran et al., 2009).

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Method

A survey of literature shows that inability to recognise geometrical shapes in non-standard orientation, perceive class inclusions of shapes, visualise geometrical solids in 2D format, and solve measurement problems that require spatial reasoning are well documented as problems students face when learning geometry (Battista, 2004; Burger & Shaughnessy, 1986; Elia & Gagatsis, 2003; Levenson, 2012). To ensure that sufficient data were collected to inform the design of the HLP, a bank of assessment questions was first designed and administered to test their reliability. The questions were designed to assess what we expect middle school students to be able to do, and focused on reasoning rather than procedural skills. They are grouped into three domains: (1) properties and hierarchy, (2) transformation of relationships, and (3) geometric measurement. Within each domain is a set of mathematical concepts vital for the development of geometric knowledge.

The first trial consisting of 62 items was administered to 390 students to determine its reliability and validity. These were marked by two markers and validated by a team of expert consultants to ensure the accuracy of the marking rubric and data entry. The questions were reduced to 36 items, collated into two forms and sent out to Year 7 to 9 students in participating schools. The test was administered to 755 students with 742 valid responses.

Results

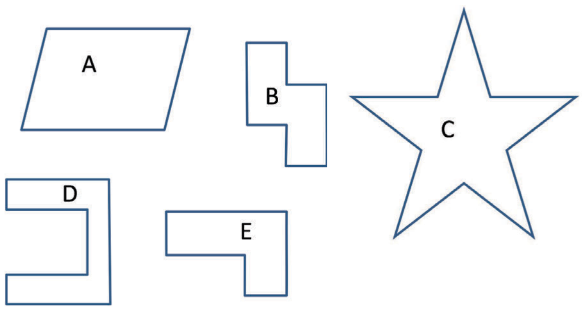
Rasch analysis of the responses resulted in identification of eight distinct thinking Zones (see Table 1). To facilitate better understanding of the formation of HLP, consider the concept of symmetry (Figure 1). It is an important aspect for developing spatial reasoning and understanding geometric properties as it promotes visualisation and geometric discourse as students learn to identify and reason about space and patterns. The code for each question, GSYM, indicates Geometry Symmetry.

5 Symmetry

Look at the shapes below

a [GSYM1] On each of these shapes draw all lines of symmetry.

b [GSYM2] For each of these shapes in part a, decide whether there is any reflectional or rotational symmetry and write the letters in the appropriate space in the table below.



	Has rotational symmetry	Does not have rotational symmetry
Has reflectional symmetry		
Does not have reflectional symmetry		

c [GSYM3] How do you know if a shape has rotational symmetry?

Figure 1. Assessment questions on concept of symmetry.

Score	Description for GSYM1	Description for GSYM2
0	No response or irrelevant response	No response or irrelevant response
1	No shapes having all lines drawn correctly	Only one shape (letter) correctly placed
2	All symmetry line(s) drawn correctly on one shape (others may be incorrect)	At least 2 correctly placed
3	Correct lines drawn on C and D but incorrect lines drawn on at least one other shape	At least 4 correctly placed
4	D: one line correctly drawn horizontally through centre C: five lines drawn from each point to opposite reflex angled corner No lines drawn on A, B or E	C has both rotational and reflectional symmetry A, B have rotational symmetry but no reflectional symmetry D has reflectional symmetry but no rotational symmetry E has no symmetry
	Description for GSYM3	
0	No response or irrelevant response	
1	An attempt at an explanation but lacking clarity	
2	Some explanation about turning shape part way around circle and it looking the same – perhaps around a pin	

Figure 2. Marking rubric for the symmetry question.

The eight Zones that were identified by the Rasch analysis are shown in Table 1. The responses to the symmetry question ranged from Zone 2 to Zone 8. Completing GSYM1 and GSYM2 with a score of 4 was in Zone 8 demonstrating understanding of reflectional and rotational symmetry. Giving a clear explanation of rotational symmetry in GSYM3 was in Zone 6. For GSYM2, students correctly placing one of the shapes in the correct position (score 1) demonstrated that they could visualise the 2D shape from a different perspective, rotating it and reflecting it, hence Zone 4, whereas identifying the reflectional symmetry of one shape (GSYM1, score 1) was in Zone 2.

Table 1
Hypothetical Learning Progression for Geometric Reasoning

Zone	Indicators
1	Recognises shapes by appearance and common orientation, shows emerging recognition of objects from different perspective, symmetry of objects and shapes and coordinate system.
2	Recognises reflection symmetry, nets of simple solids and simple shapes, shows emerging understanding of measurement concepts.
3	Able to visualise some objects from different perspective and use coordinates, uses one or two properties or attributes (insufficient) to explain their reasoning about shapes and measurement.
4	Able to visualise objects from different perspective, incomplete reasoning in geometric and measurement situations, performs measurement calculations but attends to only one attribute, gives directions on a map from personal rather than other viewer's perspective.

5	Able to visualise and represent 3D objects using 2D platform (Nets), uses either properties or orientations to reason in geometric situation, uses landmarks but retains personal orientation when providing directions, demonstrates knowledge of dilution and coordinate systems, provides partial solutions and explanations when calculating measurement situations.
6	Able to make deductions about angle situations and use properties accurately when reasoning about spatial situations but explanations are limited and lack knowledge of geometry hierarchy, provides accurate directions from a map, geometric and measurement arguments rely on examples/counter examples, omits one step when calculating multi-step measurement problems.
7	Beginning to recognise necessary and sufficient conditions, uses sound reasoning in argument/explanations, explanations often are procedurally based.
8	Constructs arguments based on multiple properties of 2D shapes and 3D objects, using the necessary and sufficient conditions to reason about geometric and measurement situations, conjectures and propositions (and theorems); demonstrates understanding of both reflectional and rotational symmetry.

These Zones reflect current students' geometric reasoning. Australian students compare poorly with students from other countries in geometry (Thompson, 2010). These Zones are not as advanced as we would desire for our students in years 7-10. Further research on improving teaching and learning practices will help to refine the HLP (Briggs & Peck, 2015). Indeed, the challenge now is to use this information to assist teachers to improve geometric reasoning in their classrooms and this assessment provides a tool to assist teachers to focus on targeting their teaching to the key ideas necessary in the development of understanding and reasoning in geometry.

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Developing Learning Progressions to Support Mathematical Reasoning in the Middle Years: Statistical Reasoning

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As part of the Reframing Mathematical Futures II (RMFII) Project, Statistical Reasoning was identified as one of three areas of Mathematical Reasoning to be investigated. A hypothetical learning progression for statistics was developed based on previous research. Assessment tasks designed to address different Zones of the progression were sent to trial schools and the data were analysed using Rasch Analysis. This paper reports on the analysis of the preliminary data received and gives some implications for teaching.

Statistical reasoning has been a more recent addition to the mathematics curriculum than algebraic or geometric reasoning. Following the National Council of Teachers of Mathematics (NCTM, 1989) in the United States, *A National Statement on Mathematics for Australian Schools* was published by the Australian Education Council (AEC) in 1991. 'Chance and Data' was one of five content areas covered in four Bands (A to D) over the years of schooling. The expectations of the *National Statement* were the foundation for much of the research in statistics education at the school level in Australia in the 1990s, conducted using surveys and interviews with students from Year 3 to Year 10. Basing analysis of student responses on the SOLO Model of Biggs and Collis (1982), hierarchical development was identified in relation, for example, to beginning inference (Watson & Moritz, 1999), to sampling (Watson & Moritz, 2000a), to average (Watson & Moritz, 2000b), and to probabilistic beliefs (Watson & Moritz, 2003). This work was consolidated for survey data by Watson and Callingham (2003), in a suggested six-level uni-dimensional construct for statistical literacy. Further analysis by Callingham and Watson (2005) identified three subgroups of items that were related to the Chance and Data content of the curriculum. These were named Average/Chance (AC), Sample/Inference (SI), and Graphing/Variation (GV).

In the light of the more recent release of the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority, 2010-2016), these three subgroups were considered against the five statistics and probability big ideas proposed as a foundation for the mathematics curriculum implementation (Watson, Fitzallen, & Carter, 2013): Variation, Expectation, Distribution, Randomness, and Informal Inference. Variation is the fundamental concept underlying the others, Expectation underpins chance and calculations of averages, Informal Inference and Randomness cover appreciation of sampling, and Distribution includes graphing as well as other representations. Recognising the fundamental influence of Variation, the three reasoning big ideas for the Reframing Mathematical Futures II (RMFII) project were hypothesised as Variation in Expectation (AC), Variation in Distribution (GV), and Variation in Inference (SI). These big ideas can be considered separately at the beginning of a learning progression but, as learning progresses, they interact with each other to provide more sophisticated reasoning. Figure 1 shows the hypothetical learning progression for statistical reasoning, including eight Zones to be consistent with the previous results related to multiplicative thinking (Siemon, Virgona, & Corneille, 2001).

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Big Idea / Zone	Expectation (AC)	Distribution (GV)	Inference (SI)
Zone 1	**Idiosyncratic response or single procedural focus**		
	Uncertainty expressed as 50%	Reads single value on graph	Ignores context
Zone 2	**Considers aggregated information but without recognising value**		
	Anything can happen	Describes isolated features of a graph	One characteristic of a sample
Zone 3	**Emerging statistical appreciation but without explanation**		
	Claims for average without justification	Elaborated physical description of graphs	Choose "all" for sample
Zone 4	**Recognises influence of variation but interprets inappropriately**		
	Rejects "luck"; suggests unlikely	Does not distinguish scale in graph reading	Recognises sample but not its bias
Zone 5	**Straightforward explanation and simple numerical justification**		
	Orders chance phrases correctly	Appropriate attention to graph details	Partial recognition of sample requirements
Zone 6	**Informal appreciation of uncertainty and variation in choices**		
	Recognises outlier	Recognises correct variation in graphs	Suggests random sampling
Zone 7	**Makes inferences across ideas using proportional reasoning**		
	Creates appropriate probability distribution	Creates hypothesis based on data	Criticises sample size and bias
Zone 8	**Integrates proportional, statistical, and contextual reasoning**		
	Correct association in 2-way tables	Conclusion with both positives & negatives	Includes human/ psychological component

Figure 1. Hypothetical learning progression for statistical reasoning with selected examples.

Based on the hypothetical learning progression for statistical reasoning in Figure 1 and items used in previous research, assessment forms were devised for the middle school students in the RMFII project consisting of statistical reasoning tasks each of which had one or more items. Three forms included only statistics tasks, whereas six others contained a mix of statistics tasks with other tasks from algebra and geometry. Common tasks and items linked the forms. The rubrics for individual items suggested scores of between two and five to distinguish increasingly sophisticated responses. The Rasch analysis allocated the rubric scores across eight Zones of the construct of statistical reasoning, mapping students' overall performances on a logit scale in relation to the difficulty of the items.

Results and Discussion

No single task addressed every Zone of the hypothetical learning progression for statistical reasoning, but when considered together some tasks with their associated items related to the same context within a big idea and provided results across several Zones. An example shown in Figure 2 employs a task related to the tossing of a fair coin with four items. Items were labelled using the convention S for statistical reasoning with a 3 letter identifier that described the context (CON for coin toss) with a number/letter identifier for the specific item. The analysis of item difficulty for the items in Figure 2 illustrates each of the Zones in the hypothetical learning progression for statistical reasoning. At Zone 1, SCON1A and SCON1B show idiosyncratic reasoning, a response being "I think 2 tails…because 4÷2=2 so the average is 2" for SCON1B. Zone 2 is the highest response for SCON1A but more sophistication is possible for SCON1B with Zone 2 responses including "it's a 50% chance" or "you can't really tell." The lowest level of SCON2 is

Zone 3, with responses indicating equally likely proportions (e.g., "20, 20, 20, 20, 20") or apparently random proportions (e.g., "10, 30, 40, 1, 19").

SCON1A: Imagine you are playing game where you throw a coin 4 times. How many tails do you think might come up?

Score	Zone	Rubric Description for SCON1A
0		No response or irrelevant response
1	1	Any other number, "you don't know, could be any of them".
2	2	2 tails or 50%

SCON1B: Explain why.

Score	Zone	Rubric Description for SCON1B
0		No response or irrelevant response
1	1	Idiosyncratic reasoning or possible misinterpretation.
2	2	2 because there is a 50% chance, 50-50 of throwing a head or tail, probability of a tail 1 in 2.
		Recognises variation by stating "You can't really tell how many tails might come up".
3	8	2 but also recognising variation and/or attempts to quantify the highest likelihood.

SCON2: Imagine you are playing a game where you throw a coin 4 times. Imagine that 100 people played the game. In the table below, fill in how many people you think will get each number of tails.

Number of tails	Number of people getting the number of tails
0	
1	
2	
3	
4	
Total	100

Score	Zone	Rubric Description for SCON2
0		No response or irrelevant response, or does not add to 100.
1	3	Assumes equality for all options.
		Seemingly random prediction; gets proportion in the wrong spot.
2	6	Too narrow or no variation – extreme probabilistic outcome.
		Primitive understanding of proportion – 50% chance for 2 tails.
3	7	Appropriate variability displayed incorporating probability and distribution.

SCON3: Explain why you think these numbers are reasonable.

Score	Zone	Rubric Description for SCON2
0		No response or irrelevant response.
1	4	Idiosyncratic reasoning and personal beliefs.
2	5	Reasoning reflecting an even or equal chance for all numbers..
		Anything can happen, chance and luck.
3	6	Implicit understanding of chance and probability, sometimes mentioning 50%, or ½ chance (or answer reflecting a '4' but not as clear).
4	8	Reasoning reflecting aspects of chance and probability including some kind of variability.

Figure 2. Example of a task having four items with scores across the hypothetical learning progression for statistical reasoning.

Score 1 in Zone 4 for SCON3, an explanation, reverts to personal beliefs in explaining the choices, but within a much more complex context. Whereas choice of equality of outcomes is Zone 3 for SCON2, for SCON3 it is Zone 5 reflecting the greater complexity, e.g., "because it adds up to 100." Zone 6 shows the first primitive use of proportion, e.g., in choosing "5, 20, 50, 20, 5" for SCON2, and explaining the values for 100 tosses of a coin four times for SCON3, such as "it is more likely people will get 2 out of 4." For SCON2, a Zone 7 response incudes variability within reasonable limits for the appropriate proportions. Finally explicit recognition of variation characterises responses at Zone 8. In explaining the original likelihood of two tails in four tosses, a response for SCON1B might be "2, because it has a 37.5% chance but the others could happen, they are just less chance." For top responses to SCON3, again explicit mention of variability or likelihood is included in the explanation. The progression of difficulty between the first two items (SCON1A, SCON1B) and second two items (SCON2, SCON3) illustrates the contribution of context as the items move from single outcomes to multiple outcomes.

Several years will pass as students develop the understanding, first to justify the actual probability for obtaining two heads when tossing a coin four times and appreciate the variation associated with experimentally checking the value, and second to move to imagining 100 such repetitions of the four tosses and experiencing variation on completing trials. After completing hands-on activities, this problem presents an excellent opportunity to introduce computer simulations in the classroom, comparing outcomes among students, and increasing the number of trials beyond what can be done by hand.

The aim of the RMFII project is to provide teachers with ways of identifying and acting upon their students' demonstrated reasoning, in this instance in statistical contexts. The initial findings indicate that it is possible to identify a progression and to match tasks and items to Zones of this progression. The next phase is to develop materials to lead students to being better able to explain their thinking and reasoning as they problem solve.

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RESEARCH PRESENTATION ABSTRACTS

Challenging Teacher Perceptions: “Those Children will Struggle No Matter What You Do to Them”

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Teachers’ perceptions of students’ capabilities are particularly important in efforts to support instructional reforms. In this presentation, we explore the efforts of one teacher to resolve conflicts and tensions as she engaged with new practices associated with ambitious mathematics teaching. We look in particular at the influence of her diagnostic framing of students’ mathematical capabilities and her beliefs about knowledge acquisition on her introduction of collaborative mixed-ability group work.

Students’ Reflections on Portfolio Assessment in Mathematics

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We discuss findings from a study that utilized students’ portfolio entries to provide initial insights into students’ views about portfolio assessment. Two Fijian Year 9 mathematics teachers implemented portfolios as a means of assessing student learning in measurement. While students in Jenny’s class noted advantages in terms of learning new content and skills, their difficulties with portfolio assessment were often bounded by the various mathematical content. Students in Gavin’s class provided insight into students’ perspectives on value of portfolio assessment. Apart from discussing the various areas of content, students in Gavin’s class pointed out many other benefits of portfolio assessment.

Tracing Student Teachers' Mathematical Modelling Motivation and Competencies over Time: A Design-Based Inquiry

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We report on the tracking of student teachers' mathematical and modelling competencies through a design-based research strategy over two iterations. Mathematical modelling was incorporated into the South African Grade 10 to 12 mathematics curriculum in 2011, and teachers were immediately expected to manage its teaching, learning, and assessment. Literature highlights the lack of preparation of mathematics teachers to teach modelling and their lack of modelling competencies. Hence, this project was introduced to more optimally prepare student teachers for this challenge. Findings reveal the development of their competencies and a pertinent improvement in their motivation.

Pedagogical Architecture for Supporting Effective Numeracy Learning

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I report on an aspect of a three-year longitudinal study that aims to generate new understandings about how teachers design and implement effective numeracy tasks. I will present a set of principles of task design and implementation, which include pedagogical approaches adopted by teachers with highly developed practices for implementing numeracy tasks. Managing the complexity associated with numeracy task implementation is illustrated though a classroom vignette in which a pedagogical architecture consisting of structural elements (e.g., initial setup), flexible pedagogical repertoire, and the ability to adapt tasks in situ is employed. Opportunities for further research will be identified.

Perceived Impact of In Situ Professional Learning on Teachers' Mathematics Knowledge Relating to Multiplicative Thinking and Their Classroom Practice

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Having an understanding of the key ideas underpinning multiplicative thinking is critical for learning beyond the primary school years. The shift to multiplicative thinking can be challenging for both students and teachers due to its multifaceted nature. We report on a pilot study of professional learning in schools that focused on multiplicative thinking, an area of concern. We sought to explore in situ professional learning in 14 primary schools across a six-month period. Pre/post survey data were collected and analysed using grounded theory. Our findings suggest that in situ professional learning had a positive impact on teachers' mathematical content knowledge and pedagogical content knowledge.

An Examination of High and Low Achievers on ACER's Numeracy Test for Initial Teacher Education Students

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In this paper, we provide an analysis of data from the Literacy and Numeracy Test for Initial Teacher Education Students, which is administered by the Australian Council for Educational Research. Specifically, we examine numeracy test data from 20 students from a prestigious Australian university who completed this test in 2016: those who achieved the 10 highest and the 10 lowest overall numeracy scores at this university on their first attempt of the test. Our analysis of these high- and low-achieving students shows that these groups clearly have particular characteristics that contributed to their success or failure on the numeracy test. We provide suggestions to support such students going forward.

Changing Pre-Service Teachers' Perceptions of Teaching Mathematics

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Pedagogical approaches to improve students' attitudes to mathematics and knowledge of mathematics are needed to enhance students' post-school opportunities and address forecasted skills shortages. We track pre-service teachers' perceptions about teaching secondary school mathematics from starting secondary mathematics curriculum units to completion of the two units. We focus on changes in learning priorities by analysing paired responses to one item in pre/post questionnaires. Statistically significant shifts in perceptions of learning priorities concerning disposition and interest were identified. We discuss explanations for these shifts and implications for mathematics teacher education.

Resources Promoting Statistical Threshold Concepts and Addressing Statistical Anxiety and Apathy

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We provide an account of a new initiative, involving resources developed for a national statistics project designed to increase access and support within higher education. The resources comprised short animated videos, interactive exercises, and extension documents developing statistical threshold concepts, plus tools to assist teachers and students to engage with statistics via a national schools poster competition. These resources enable students and teachers to understand the interdisciplinary and pervasive nature and value of statistics, and make the field of statistics accessible. The exponentially increasing annual numbers of students participating and positive feedback received are very promising.

Algorithms… Alcatraz: Are Children Prisoners of Process?

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Multiplicative thinking is a critical component of mathematics that largely determines the degree of development of mathematical understanding beyond middle primary years. A major issue is the extent to which algorithms are taught without explicit connections to key mathematical ideas. We explore primary students' understanding of the underpinning ideas and their relationship to the traditional written algorithm for multiplication. We examine the extent to which the students use the algorithm as a preferred choice of method. Indeed, we suggest that most students are "prisoners to procedures and processes", whether or not they understand the mathematics behind the algorithms.

The Valuing of Communication in Primary Mathematics Classrooms in Australia

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Communication forms an integral part of mathematics learning. The valuing of communication is reflected through the meaningful and constructive classroom interactions of teachers and students. A study involving 625 grade 5/6 Victorian students was conducted to determine the extent to which communication is valued in the primary mathematics classroom. This article also explores the various forms of communication in mathematics learning and the importance of developing effective teaching and learning approaches to promote communication in the mathematics classroom.

Metacognition of Pre-Service Mathematics Teachers

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In this study, I explored pre-service mathematics teachers' metacognition and examined whether their metacognition differed in terms of gender and grade level. A total of 67 pre-service mathematics teachers participated in the study. A personal information form, the Metacognition Questionnaire, which was developed and adapted into Thai from Schraw and Dennison (1994) and Schraw et al. (2006), and open-ended questions related to metacognition were used in collecting the data. The findings showed that preservice mathematics teachers' metacognition was generally high. A significant effect for gender difference was only found in the evaluation sub-component ($p < 0.05$).

Investigating Robust Algebraic Understanding of 9th Grade Students in a Problem-Based Learning Classroom

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In this study, we explored the robustness of 9th grade students' (n = 40) algebraic understanding in a problem-based learning (PBL) classroom. Data were collected from eight PBL lessons, algebraic tasks, a robust algebraic understanding test (RAUT), observations, and video recordings. Descriptive statistics were used to evaluate the data from RAUT that scored rubrics based on the robustness criteria by Schoenfeld, Floden, and the Algebra Teaching Study and Mathematics Assessment Project (2014). Descriptive analysis was applied to identify students' robust algebraic understanding. Interestingly, the scores of RAUT in each dimension ranged from average to high level.

Student Experiences of Remedial Mathematics Tuition Delivered via Personal Videoconferencing

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In this paper the experiences of two school student participants (Years 5 and 7) in a larger research study investigating remedial mathematics tuition delivered via personal videoconferencing (PVC) are described in case study format. Structured interviews were conducted to determine what could be learnt about student experiences of tuition delivered by PVC. Responses to the interview questions allowed some insight into the students' overall perceptions of the PVC tuition, which aspects they did and didn't enjoy and whether participation in the remedial tuition changed the way they felt about classroom mathematics.

What is a Pedagogy that Supports Teaching Mathematics for Understanding in Primary Schools – Could Teaching for Mathematising be a Solution?

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This paper is a position paper in which we argue for Teaching for Mathematising may be a pedagogy that supports teaching mathematics for understanding in primary school. Mathematics education in Australia includes an emphasis on teaching children with mathematical understanding. This shift redirects children's learning from merely memorising computation procedures to helping children construct knowledge of the mathematics that informs the *concept and processes*. However, the shift to teaching for understanding is not visible in students' item responses in different international achievement studies and NAPLAN. This paper provides one example to support a learning trajectory for teaching for understanding that builds on the framework of 'Teaching for Mathematising'.

Using a Reflective Inquiry Approach to Build "At-Risk" Learners' Confidence and Responsibility

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This paper reports on a mathematics intervention, *Prepare 2 Learn*, designed taking into account research literature and elements of other successful programs. The program prepares "at-risk" students for their upcoming mainstream lessons using a reflective inquiry approach, as well as making them aware of the impact their learning attitudes and actions have on their success. The intervention resulted in the students reaching expected achievement standard or beyond and positively impacted how they approached their mathematics learning. Using data related to one student's experiences, this paper highlights aspects of the program that help at-risk students become confident, responsible learners.

Mathematics Pre-Service Teachers' Pedagogical Content Knowledge in Planning a Lesson

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We discuss part of a larger study aiming to develop pre-service teachers' pedagogical content knowledge (PCK) through lesson study in a teaching practicum. PCK is often explored and discussed within the act of teaching. Since teaching cannot be separated from planning, the success of a lesson results from planning. We discuss pre-service teachers' PCK in the planning of a lesson study using the knowledge quartet framework. Data were collected through video and field notes from a lesson study group in a secondary school in Indonesia. Our data analysis shows that foundation, transformation, and connection dimensions of the knowledge quartet are interconnected and evident in the planning.

Understanding a Young Child's Critical Mathematical Thinking Capabilities

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Critical mathematical thinking (CMT) is the ability to reason and make judgments when solving mathematical problems. In this presentation, we specify characteristics of CMT and their alignment with eight open-ended tasks. One child's responses to these tasks indicate that CMT capabilities can be evident in young children. The tool used to gain insight could be used for assessment and teaching.

The Function of Signs and Attention in Teaching-Learning of Mathematics

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The purpose of this study is to capture and explain the roles that signs and attention play in the fraction learning process, through a previous study that employed Deleuze's perspective on sign and the role of attention. From this case study of elementary school students, we found that signs are a prerequisite for learning and that learning takes place as different forms of attention shift. The various types of semiotic resources used by teachers and students have been found to play an important role in coordinating collective attention between teachers and students.

Melody of Functions and Graphs: Improving Senior Secondary Mathematics Students' Understanding of the Function Concept by Active Integration of Mathematics and Music

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We describe a research study involving an active integration of mathematics and music applied in the process of senior secondary students' understanding of the function concept. The study was framed by the post-positivist paradigm, quasi-experimental research design, multiple research methods, experimental- and control-groups, pre-tests, and post-tests. We considered eight analogies between mathematics and music as tools to improve students' understanding of the function concept as defined in the Australian Curriculum. The findings suggest that active integration of mathematics and music makes a significant difference in students' understanding of the function concept.

Indonesian Mathematics Teachers' and Educators' Perspectives on Their Choice of Facebook Groups

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We examine participants' perspectives on Facebook Group (FG) usage. We identify their most liked FG and the reasons for their choices. Data were collected from 440 mathematics teachers (86%) and educators (14%) through an online survey across Indonesia. The findings suggest that FGs are used for professional engagement. Identification of the top five most liked FGs indicates that the participants are looking at discipline-specific (i.e., mathematics) FGs for professional learning/supports. This implies the importance for professional developers to shape the FG environment to enhance teachers' competencies and their students' performance.

An Evaluation of Online Resources Designed to Teach Mathematics for Equal Opportunity

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We report on the testing phase of a project focused on creating a set of research-informed mathematics digital resources. The resources, which comprised videos, interactive exercises, and extension documents, were informed by Polya's problem-solving approach and specifically designed for students from low socio-economic backgrounds entering science and engineering degrees. The evaluation indicates that the resources achieved the intended goals of usefulness, visual appeal, and suitability, and confirmed that pedagogically-appropriate digital resources can utilise and benefit from a mathematically-rich problem-solving approach rather than solely developing procedural skills.

Spatial Orientation Ability of 11–13 Year-old Students: Some Empirical Findings

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This study explored the spatial orientation ability of 11-13 year olds in a sample of 428 students via the formulation of newly-designed items across different contexts. The performance of the students as well the multiple-choice responses they chose were analysed on an item-by-item basis. The quantitative data analysis showed that as the students matured from 11 to 13 years, their spatial orientation ability increased correspondingly. There were significant performance differences between the 11 year olds and the 12 and 13 year olds. The answer choices revealed that the major challenge revolved around reorienting the body-centred coordinates in an opposite direction. We explain the challenge in terms of the difference between the actual and imagined frame of reference, characteristic of egocentricity. This study contributes in providing empirical data to portray the developmental change in spatial orientation ability across the age range 11-13 years.

Articulating Teacher Learning: The Power of Self-Study

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This self-study formalises reflective practices to demonstrate enhanced teacher understanding not just of the practice of teaching but of oneself as an educator. In this presentation, I highlight the growth in my teaching practice over an 18-month period through comparison of two lessons and discuss understandings that were generated, showing self-study as a viable research tool not just for researchers but for practising teachers.

Comprehending or Creating? On Sense-Making and Meaning-Making

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Previous approaches to mathematics knowing and learning have attempted to account for the complexity of students' individual conceptions of mathematical concepts, and primarily focused on students' conceptual development when a mathematical concept comes into being. However, some students give meaning not only to states/objects that have a being but also to states/objects that are yet to become. In those cases, conceptual development is not meant to reflect an actual concept (conception-to-concept fit), but rather to create a concept (concept-to-conception fit). I argue that the process of generating a concept-to-conception fit might be better referred to meaning-making than sense-making.

Assessment for Learning Techniques in the Pacific Island Context: What are Teachers' Views?

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In this study, we investigated teachers' views, all from a Pacific Island context, of assessment practices for mathematics learning in early childhood, primary, and secondary settings. Based on the analysis of these views, collected via a written survey (n = 25) from invited workshop participants in Nauru, a series of four mathematics assessment workshops was designed where the participants created authentic assessment tasks linked to the teaching/learning cycle. Post-surveys and in-depth interviews with six teachers explored their views about using authentic assessment in Nauru. The teachers' views concern the purpose of assessment and their current assessment practices.

On Translating Research in Mathematics Education

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The paper "The Challenge of Publication for English Non-Dominant-Language Authors in Mathematics Education" sensitised us to the problems and learning potential of translating mathematics education research. A translation of the paper from English into French illustrates two challenges: (1) variations between different communities of research practice using different languages as their dominant language and (2) the use of theoretical terms drawn from different theoretical frameworks that are culturally situated. We argue for tolerance of language usage, drawing on Bakhtin's (1981) notion of centrifugal forces in understanding international influences that promote both standardisation and diversity.

Junior Secondary Mathematics Teachers' Perspectives on the Transition of Year 7 into Secondary Schooling in Queensland

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We explore the experiences of Junior Secondary teachers in two Queensland schools following the 2015 transition of Year 7 students from primary school to secondary school. There is a well-documented shortage of suitably qualified mathematics teachers in secondary schools in Queensland. A significant concern is that this potential shortage will be worsened with the new cohort of Year 7 students entering secondary schooling, as the demand for qualified mathematics teachers will increase. We use a Teacher Identify Framework to identify whether Junior Secondary mathematics teachers felt their self-reported level of mathematical knowledge was sufficient for this task.

Teachers' Anticipation of the Potential of Specific Suggestions for Mathematical Learning Experience

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We report on a teacher professional learning initiative that involved specific suggestions of innovative mathematics learning experiences. The focus was on tasks and learning experiences that were intended to challenge student thinking. Teachers responded to prompts about their interpretation of the tasks prior to teaching them. The responses indicate that teachers' anticipation of the potential of the tasks matches the intentions of the task designers.

Are Students' Perceptions about Mathematics Different Amongst Those Taking Different Senior Secondary Mathematics Subjects?

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I will discuss preliminary findings of a large-scale survey. Participants, recruited from a Facebook advertisement, answered questions about their mathematics subject choices and perceptions about mathematics. Demographic data and perceptions were compared between students who took or intended to take no mathematics, elementary, intermediate, and advanced mathematics. There were no significant differences in school type and location between students taking different mathematics subjects, but gender differences and single-sex/co-education school differences were found, as well as significant differences in students' perceptions about mathematics amongst different groups.

He Puawaitanga Harakeke: Using Technology to Accelerate Learning in Māori-medium Learning Programmes

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"He Puawaitanga Harakeke" is a pilot intervention program designed to accelerate the learning of Year 1 to 8 students in Māori-medium mathematics (pāngarau) who have been identified by their schools as performing well below expectations. The program utilized a range of ICT applications that provided students with the opportunity to communicate mathematically, learn the mathematics content and share their learning with their teachers, their peers and their whānau. Most students made progress whereby they were close to, or reached, the expected stage of the number framework for their age group. This paper provides insight into the use of ICT in Māori-medium settings.

Towards a Positive Approach to Teaching for Productive Disposition in Mathematics

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The Australian Curriculum: Mathematics defines four proficiency strands. The work from which they are drawn includes a fifth proficiency (productive disposition) that relates to students' propensity to persevere and to perceive mathematics as worthwhile. We argue for the importance of productive disposition as reflective of the importance of affect in mathematics learning. We link it with work in positive education, particularly around character strengths, to suggest ways in which mathematics teachers might develop productive disposition in their students and thereby improve achievement.

ROUND TABLE ABSTRACTS

Rethinking Mathematical Tasks

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Mathematics teachers invariably use a multitude of tasks in their day-to-day practice. Indeed, “mathematical tasks provide tools for promoting learning of particular mathematical concepts and are, therefore, a key part of the instructional process” (Simon & Tzur, 2004, p. 93) Further, the National Council of Teachers of Mathematics (NCTM, 1991) postulated that tasks “convey messages about what mathematics is and what doing mathematics entails” (p. 24). Over the years, several terms have been used to describe mathematical tasks, such as worthwhile mathematical tasks (NCTM, 1991), challenging tasks (Sullivan et al., 2014), high-level tasks (Henningsen & Stein, 1997), open-ended tasks (Zaslavsky, 1995), and rich mathematical tasks (Grootenboer, 2009). While acknowledging the benefits of using such tasks, research has also surfaced some shortcomings. Stein, Grover, and Henningsen (1996) cautioned that “When employing the construct of mathematical task, however, one needs to be constantly vigilant about the possibility that the tasks with which students actually engage may or may not be the same task that the teacher announced at the outset” (p. 462). In this round table presentation, we will discuss the affordances that mathematical tasks such as those stated above offer to teachers, as well as other alternatives that are available to teachers for enhancing students’ learning of mathematics. We will provide some examples from one of our on-going projects for further discussion.

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Scaling Up and Sustaining Successful Interventions in Mathematics Teaching

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Education research journals regularly report on small-scale studies that have been successful in changing mathematics teachers' classroom practices. However, it is rare to find large-scale transfer of research knowledge into practice in mathematics education (Begg, Davis, & Bramald, 2003). In this round table discussion, we will share some early findings from research into an established, large-scale professional development project initiated and sustained by a state education system at a regional level and involving a large number of schools and teachers. In this project, we have developed a cluster model for bringing primary and secondary school teachers and principals together to analyse student performance data, create diagnostic tasks that reveal students' current mathematical understanding, and promote teaching practices that address students' learning difficulties in mathematics. The effectiveness of this approach is evidenced by reported improvements in teacher confidence and knowledge and in student achievement and enjoyment of mathematics, changes to mathematics teaching and assessment practices, and an ever-increasing number of schools volunteering to join the project and commit professional development funding. In our research, we seek to identify critical factors that support these mathematics teachers in instructional improvement on a large scale.

The round table will begin with an overview of the cluster model and then we will present some insights from interviews that we have conducted with teachers and principals. We invite MERGA members to join us and share their own experiences and ideas in response to the following research questions that are guiding our study (based on Cobb & Jackson, 2011):

1. What practices are effective in establishing a coherent instructional system supporting mathematics teachers' development of ambitious teaching practices?
2. To what extent do teacher networks and mathematics coaching of teachers support changes in mathematics teaching practice?
3. What features of school and district or regional leadership contribute to the scalability and sustainability of a cluster-based professional development model?

We also invite colleagues to suggest new lines of inquiry that could contribute to a theoretical rationale for sustained, scalable professional development.

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Exploring Emotional Aspects of Pre-Service Mathematics Learning Environments

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Preparing primary teachers for mathematics teaching typically includes attention to their existing beliefs and attitudes towards mathematics as a discipline. The potential of the pre-service learning environment to enhance emotional engagement with mathematics learning and teaching is a developing field of research. In the session, I will provide an opportunity to discuss issues around initial teacher education learning environments in terms of introducing structures to promote positive experiences in learning to teach mathematics. I will examine emerging theories of emotions from a sociological perspective that are useful for analyzing the emotional aspect of learning environments. In the round table session, I will draw on survey data about emotional dispositions and beliefs in mathematics as well as emotions associated with teaching mathematics. I will also draw on case studies of individual students who were excited and enthusiastic about teaching mathematics despite having had negative learning experiences themselves. The session will provide an opportunity for participants to discuss (a) increased awareness of emotional reactions to classroom events, (b) the connection between innovative teaching approaches and mathematics teaching and learning, and (c) the potential of games to impact the emotional aspects of learning environments.

Mathematics Leadership in Primary Schools

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In response to the recent Teacher Education Ministerial Advisory Group (TEMAG) report (2014), teacher education providers are developing new units of study and pathways of study within existing programs to cater for pre-service teachers (PSTs) who elect to undertake a specialization in mathematics. Entry requirements for students electing such a pathway are not specified; however, it is expected that when they graduate, they will have demonstrated competence in mathematics and perhaps taken one or more additional units in mathematics pedagogy.

Teacher education providers currently know little about the background experiences, aspirations, and expectations of primary school PSTs who might elect and be accepted into a primary mathematics specialization pathway. This gap in our knowledge is partly due to prior research foci on primary PSTs who lack the mathematical content knowledge required as a basis for teaching mathematics well (Callingham & Beswick, 2011).

Teacher education providers also know little about what potential employers are expecting of PSTs who will graduate with a specialization in mathematics. This knowledge is necessary for teacher education providers to be able to select appropriate candidates for a mathematics specialization pathway and plan units of study for them.

In the round table, we will begin by presenting the aim of our research, some preliminary data concerning the PSTs who have elected to undertake a new primary mathematics specialisation at The University of Sydney, and the skills and characteristics that some potential employers have identified as being essential or desirable for mathematics leadership in primary schools. These data will provide a stimulus for discussion amongst participants.

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SHORT COMMUNICATION ABSTRACTS

Use of Social Media in Preservice Mathematics Education Courses

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Facebook was used in preservice primary and secondary mathematics education units to provide a forum for students. The innovation was readily received by most of the students, but not all. Facebook was mainly used to share resources, but there was some discussion suggestive of a community of enquiry. In this presentation, I explain how to set up and run a closed Facebook group and explore the advantages and disadvantages of the system. These include the immediacy, access, and continuity Facebook provides, and the problems that inappropriate interactions generate. The research reported does not support the idea that Facebook constitutes a distraction for tertiary students.

Exploring Primary Teachers' Conceptions of Mathematical Fluency: Are We Speaking the Same Language?

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Fluency in mathematics is defined in various forms, such as computational fluency, procedural fluency, and mathematical fluency. However, terms like "procedural" and "computational" often leave teachers interpreting fluency as simply being able to follow a set formula or to compute quickly. In this study, I explore practising primary teachers' conceptions of mathematical fluency, including how they define mathematical fluency, what features they associate with the term, and what relationship, if any, understanding plays within mathematical fluency. In this presentation, I will report some initial findings from the questionnaire used in Phase 1 of the data collection for this research project.

Looking Inside the Black Box of Mathematics Teacher Noticing

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Research in mathematics teacher noticing, an important component of teaching expertise, has gained traction in recent years (Hunter, Hunter, Jorgensen, & Choy, 2016). Despite the advances in our understanding of teacher noticing as a high leverage practice (Sherin, Jacobs, & Philipp, 2011) and its application across a wide variety of contexts (Amador, 2016; Choy, 2016; Seto & Loh, 2015; Simpson & Haltiwanger, 2016; Wager, 2014), the "complex interactions of cognitive and perceptual processes and activities in dynamic situations (such as classrooms) have never been fully described in research on teacher noticing" (Scheiner, 2016, p. 234). Many of these processes remain hidden in the "black box" of noticing (Scheiner, 2016). This begs the question: How do we look inside the black box of teacher noticing? Scheiner (2016) suggests that researchers should draw on the perceptual cycle model (Neisser, 1976) and blend insights from cognitive sciences and human factors studies. In this short communication, we will present our initial idea of using wearable eye trackers to investigate teacher noticing. More importantly, we will invite feedback from the participants to explore how different video technologies could be used with the FOCUS Framework (Choy, 2015), developed for characterising productive noticing, to build a more comprehensive model of teacher noticing.

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Improving Mathematics Curriculum Support for Indigenous Language Speaking Students

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A resource to support teaching the Australian Curriculum: Mathematics to students who speak English as an additional language or dialect (Australian Curriculum, Assessment and Reporting Authority, 2014), including Indigenous language speaking (ILS) students, provides language and cultural considerations and suggested teaching strategies linked to content descriptions. Critical analysis of this resource shows much scope for improvement. There is inconsistency between the language expectations of the resource and curriculum and the English language learning progressions of ILS students (Northern Territory Department of Education and Training, 2009). Few suggestions are specific for ILS students. I provide recommendations to improve the resource for teachers of ILS students, drawing on substantial prior research.

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The Use of Contextual Patterning Tasks with Young Pāsifika and Maori Students in New Zealand Mathematics Classrooms

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Mathematical achievement of culturally diverse students is a challenge in many countries. Teaching in ways responsive to the cultures of our students is vital towards enhancing equity of access to mathematics achievement and putting educational policy (e.g., Ministry of Education, 2011) into practice. Similar to other countries, New Zealand has a changing student population that is increasingly culturally diverse. This population includes a large number of Pāsifika and Maori students whose educational results are characterised by unenviable statistics in which a large percentage are under-achieving compared to their peers. Educators frequently attribute this under-achievement to the learners themselves and position Pāsifika and Maori cultures as being mathematically deficient (Hunter et al., 2016). However, both Pāsifika and Maori cultures have a rich background of mathematics, including a strong emphasis on patterns used within craft design (Finau & Stillman, 1995). In this presentation, we report on the preliminary findings of a study in which we are investigating how contextual Pāsifika and Maori patterning tasks can potentially support young children to develop their understanding of growing patterns.

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Unidoodle

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Audience responses systems, commonly referred to as "clickers", are common in many university classes. Typically the clicker allows a student to enter an answer to a multiple-choice question. The teacher then displays the responses before (usually) either leading students in a discussion of the merits of each answer choice or asking students to discuss the question in pairs or small groups. The benefits of using clickers are well documented in the research literature and include improved classroom interaction, motivation and attendance, and improved student understanding. Unidoodle takes clickers one-step further as it enables students to submit freehand drawing and sketch-style answers. Students can write equations, draw graphs, or show their working. This allows teachers to receive much richer feedback from their students.

In this short communication I will briefly present the Unidoodle system and the way in which I have used it in two first-year mathematics classes. I will then pose some discussion questions on the use of clickers.

Factors Influencing Student Selection of Senior Secondary School Mathematics Subjects

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Declining numbers of Advanced Mathematics (AM) students at secondary school are seen as a major issue for the future of STEM in Australia and internationally (Noyes, Wake, & Drake, 2011; Office of the Chief Scientist, 2014). Few large-scale research studies have investigated why students choose particular mathematics subjects. The aim of this empirical study was to identify the reasons why students choose or do not choose AM in the last two years of secondary school. Quantitative data were collected via surveys from secondary school mathematics students and teachers, and university mathematics lecturers. The surveys contained 20 statements on reasons for choosing/not choosing AM, covering intrinsic and extrinsic motivational factors.

Practitioner Inquiry: Developing Capabilities in Mathematics Teachers

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The 21st century has seen a definite shift from teacher training towards teacher education in initial teacher education programmes. As a tertiary educator in a postgraduate initial teacher education program, I have seen the dichotomous thinking in which some of my education students embrace the idea of an inquiry-based mathematics approach with multiple solutions, while others, for whom mathematics has always been about finding a solution through learned steps of reasoning, face insurmountable challenges. While mathematics educators continue to advocate for a constructivist approach to learning, current practice has not overwhelmingly shifted from the symbol manipulation procedural approach. Could this be attributed to the notion that how we teach mathematics may be an internally learned habit from the way we were taught and that change is difficult because it requires that habit to be broken? Through the lived experiences of first-year teachers as inquiring practitioners, I explore the concept of practitioner inquiry and its implication for mathematics practitioners, and I consider how practitioner inquiry could be a catalyst for transforming mathematics education practices.

Teachers Choosing Mathematics

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The basis for teacher confidence, or the lack of it, in teaching mathematics varies across the teaching cohort, with many reporting a range of degrees of comfort with their own mathematical abilities. Through the CHOOSEMATHS Project, we have found some interesting connections in the preliminary data and suggest some explanations for it. CHOOSEMATHS is a national project aimed at getting more girls and young women into mathematics through targeted teacher professional development, career awareness, and mentoring and support across mathematics.

Student Engagement in Mathematics

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Student engagement in mathematics and mathematics learning has been a concern for educators for many years (Attard, 2011). There have been many studies and reports that have suggested ways to improve student engagement in mathematics lessons, and these seem to offer some useful approaches. In this presentation, we discuss some of the key factors related to student engagement in primary mathematics learning as have been identified in the literature reviewed. In particular, pedagogical approaches will be explored, including the use of textbooks and investigations (Langer-Osuna, 2015), and classroom factors including teacher rapport (Attard, 2011) and peer interactions (Way, Reese, Bobis, Anderson, & Martin, 2016). Finally, we will discuss issues of student engagement vis-à-vis other variables including gender, previous achievement, and socio-economic status. The initial findings presented here will underpin some empirical work that is to be subsequently undertaken.

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Testing Inquiry-Based Mathematics Competencies

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In a Danish development and research project in mathematics with focus on inquiry, a primary ICT test has to be developed, which will be able to determine development in students' mathematical competences. In order to build on earlier research, a systematic literature review was performed. The output from 1,280 articles (only 55 included) was that several foci were important: the connection between problem-posing and problem-solving, the use of visual representations, the advantages and disadvantages of ICT tests, frameworks and conditions regarding emotions, measurement of metacognitive aspects, and mathematical reasoning and thinking processes. I will discuss 10 principles in designing an inquiry-based mathematics competencies test.

Students' Espoused and Enacted Theories in an Inquiry Mathematics Classroom

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Although inquiry classrooms as learning environments for mathematics have existed for many years, only recently has literature emerged on what it means from the perspective of students to learn mathematics in these contexts. Many researchers (e.g., Attard, 2011; Fraivillig, Murphy, & Fuson, 1999; Grootenboer & Marshman, 2016; Hunter & Anthony, 2011; McDonough & Sullivan, 2014) argue the importance of listening to students' views about their experiences in inquiry environments so that mathematics educators can better meet students' learning needs. At the same time, there is a need for recognition that what students say is important about learning mathematics may not connect to what they do while learning mathematics. Previous research studies either focus on students' perspectives about classroom practices in mathematics lessons (e.g., Cobb, Gresalfi, & Hodge, 2009; Hodge, 2008; Hunter, 2006) or their theory-in-use, through classroom observations, as they engage in mathematical activity (e.g., McCrone, 2005; Perger, 2007; Pratt, 2006). In contrast, in this presentation, I report on a group of students' (aged 9-10 years old) views and attitudes towards learning mathematics and their actions within the classroom while an inquiry mathematics community was being developed.

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Task Modification to Facilitate Creativity by Korean Prospective Mathematics Teachers

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Mathematical tasks play a critical role in the teaching and learning of mathematics. Tasks with different natures can provide different opportunities to promote students' mathematical thinking and understanding (Henningsen & Stein, 1997). Teachers can read and evaluate curriculum material focusing on tasks in order to modify them based on current reforms in mathematics education. Creativity is the essence of mathematics (Mann, 2006) and has been one of the main emphases of the Korean mathematics curriculum for decades. Various attempts in the curriculum to enhance student creativity in mathematics have been made.

Attempts to promote creativity education have been made by leading mathematics teachers, and their approaches have been shared for several years in Korea (Lee, 2015). Some of these approaches to promote creativity have included promoting communication among students, practicing activity-based instruction, having students grasp fundamental ideas and knowledge in advance and focus on discussions in class, and planning and implementing storytelling lessons. Textbooks have also been redeveloped to include opportunities for creativity cultivation.

When looking back over the past few years of creativity education in Korean mathematics classrooms, one of the biggest concerns has been that we have not paid much attention to the development of tasks that are appropriate for creativity development. The majority of the tasks for cultivating creativity used in textbooks and classes have not been suitable for cultivating creativity. This is because textbook authors and teachers have attempted to design tasks without fully understanding what to consider in developing a task suitable for creativity education.

In this paper, I aim to determine what points need to be considered in the design of tasks for nurturing creativity. The definition of creativity varies, but I will discuss common and meaningful pursuits for creativity in school mathematics by analyzing some empirical data from the prospective teacher education course I ran in 2016. In order to maintain consistency with the curriculum, task design was aimed at increasing opportunities for creativity while retaining the learning objectives and content of existing textbooks. I will report the patterns that Korean prospective mathematics teachers tend to follow when they modify mathematical tasks in textbooks to facilitate creativity.

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A Five Question Approach to the Teaching of Mathematics

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According to Clements (2003), Dinham (2012), and Sullivan (2013), there is an urgent need to change the way that mathematics is taught in Australian schools. The Five Question Approach (FQA) to teaching mathematics, developed during my 30 years of secondary mathematics teaching, occurs at the commencement of every lesson. It is the subject of my doctoral research, in which I am investigating if the FQA results in an increase in students' academic achievement, perceived and/or actual, and engagement. I will discuss the final data analysis and completion stage and some of the results and implications in this session.

Numeracy of Undergraduate Business School Students

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There is growing concern of a mismatch between the numeracy of students upon entry to university and the expectations of mathematical competence by university teachers (Marr & Grove, 2010). Poor numeracy has been identified as an issue for undergraduate students across a range of disciplines (Galligan & Hobohm, 2014; Hodgen, McAlinden, & Tomei, 2014; Linsell & Anakin, 2012). In this study, we investigated student numeracy in a compulsory 100-level business school statistics paper and found that 25% of students had numeracy levels below that expected of a competent Year 9 student (13-year-old). There was a highly significant relationship between low numeracy and failing quantitative papers. Students with low numeracy were not necessarily low ability students but lacked specific skills needed for quantitative work at university level.

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Numeracy in Action in Family Shopping Experiences: A View from the Trolley

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In this short communication, we will introduce a current pilot study in which we are documenting the ways in which children and their families engage in everyday numeracy as they participate in shopping experiences. The study is underpinned by the notion that mathematical learning opportunities exist in the everyday activities of families, outside of the home, childcare centre, or school. Six families, with children ranging from 18 months to 10 years, were recorded whilst undertaking their shopping using a trolley mounted with a custom-built Go-Pro© camera rig (nicknamed "trolley-cam"). In this presentation, we will share selected "trolley-cam" data and vignettes of family numeracy engagement in action.

A Developing Framework for Identifying Young Children's Engagement with the Spatial Features of Play Spaces

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In this presentation, we report on our initial analysis of preschool children's engagement with spatial features of play spaces. The analysis focusses on noticing an awareness of mathematical pattern and structure (AMPS) evident in their play. The notion of spatial structure in play contexts will distinguish features of dynamic action such as children's movement through play spaces and the comparison, transformation, and navigation of 3D objects. The pattern and structure of mathematical concepts identified in this analysis will be compared with those evident in the Pattern and Structural Awareness Program (PASMAP, Mulligan & Mitchelmore, 2016). Future areas for research will be discussed.

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Influential Factors for Effective Problem Solving Practice in Primary Mathematics Teachers

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Problem solving is described in Australian Curriculum as one of four proficiency strands (understanding, fluency, problem solving, and reasoning), and an integral component of mathematics teaching and learning. In this presentation, I discuss the preliminary stages of a study in which I am exploring teacher knowledge and dispositions of primary educators who effectively integrate problem solving to improve student mathematics learning, as well as identify any constraints and opportunities including the Australian Curriculum and professional learning experiences. Using a mixed methods approach, in this study, I am examining past survey data from the Encouraging Persistence Maintaining Challenge (EPMC) project to identify participants for subsequent case studies.

Using Peer-Reflection to Develop Self-Regulated Learning Strategies in Year 10 Mathematics

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Recent reforms to the Australian Curriculum and the Victorian Curriculum provide a framework for developing students' awareness of metacognition and self-regulated learning strategies. In this study, I use educational design research to develop and implement a class-based intervention that aims to improve students' self-regulated learning strategies. This educational intervention structures an approach to critical peer-reflection as part of Year 10 mathematics lessons whereby students reflect, discuss, observe, and model learning strategies. In this presentation, I will explore preliminary data that have informed the development of the intervention.

Exploring Mathematics Pedagogy in Collaborative Teaching Environments

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There is an increasing number of collaborative teaching environments in primary schools. A collaborative teaching environment is a teaching situation in which two or more teachers are responsible for the learning outcomes of a number of students commensurate with the ratio of the number of teachers. What might traditionally have been two single classrooms now have two teachers who are responsible for all students in a linked physical environment. We share initial results of a study in which we explore the ways that mathematics is taught in five collaborative environments, and compare these with best practice (Anthony & Walshaw, 2007).

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The Road to Transformative Healing of Mathematics Anxiety: A Case Study in Progress

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Significant proportions of pre-service teachers in Australia are having their enthusiasm to become capable and successful primary school teachers significantly dampened by their inability to work confidently and capably with mathematical content. Wilson and Gurney (2011) defined mathematics anxiety as “a learned emotional response characterised by a feeling that mathematics cannot make sense, of helplessness, tension and lack of control over one’s learning” (p. 805). In an ongoing study into the mathematics anxiety experienced by pre-service primary teachers at an Australian university, one significant case study is explored in depth to highlight the mathematical journey of one participant in the study who has had a transformational experience through mentoring.

References

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Linguistic Obstacles to Second Language Learners’ Access to Mathematical Talk for Individualised Sense-Making

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Despite willingness, South African mathematics teachers teaching through a second language (L2) often struggle to get their learners engaging in exploratory talk. Using transcripts of talk in one South African teacher’s Grade 4 mathematics lessons, plus interview data, we will examine and share some effects that children’s diminished access to their strongest source of linguistic capital, mother tongue (L1), appears to have on their epistemological access to mathematical sense-making. Our findings suggest that use of L2 exacerbates existing inequalities in mathematics achievement across South Africa’s socio-economic sectors (sectors which, due to the legacy of apartheid, generally coincide with “race”).

Interbreeding Paradigms in Research on Mathematics Knowing and Learning

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Often, structuralism and constructivism are framed as competing paradigms in mathematics education from which one seems to have to choose. Here, we present emerging theoretical insights that recognize, rather than deny, individuals in the creation of the meaning of a mathematical concept, acknowledging the complex interaction between individual and subject matter: An individual’s knowledge system is shaped by the meaning of a mathematical concept, but the knowledge system also shapes the meaning of a mathematical concept. These recent advances in research on mathematics knowing and learning allow interbreeding of seemingly contradicting paradigms.

Fitness for Purpose of Tertiary Algebra Textbooks: An Arabic Case Study

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I will outline my PhD research into fitness for purpose of tertiary algebra textbooks used in Iraq in the education of pre-service teachers. I will consider (a) broad discourses and the use of introductions, examples, and explanations in light of cross-cultural studies such as the Japanese-U.S.A. comparison by Mayer et al., (b) pedagogies and assumptions about knowledge that can be inferred from the presentation style, referencing Magolda's theory linking forms to assessment to underlying theories of knowledge, (c) types of proof used in light of the theories of Harel and Sowder, and Stacey and Vincent, and (d) multilingual issues, given that some texts are translations and others are written in Arabic.

Evaluating Learning Analytics of an Online System to Improve Teacher Education Students' Numeracy Skills Development

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Since 2015, all teacher education students in Australia are required to pass the Literacy and Numeracy Test for Initial Teacher Education (LANTITE) in order to meet accreditation requirements. The purpose of the test is to ensure that graduate teachers meet a satisfactory level of personal literacy and numeracy, roughly equivalent to the top 30% of the adult population. To support and help students prepare for this test, we created an online Literacy and Numeracy Practice System through the university's learning management system, Blackboard. In this short communication, we will report our initial findings from evaluating the learning analytics available with this system and discuss its impact on students' numeracy skills development.

South African Vocational Engineering Students' Conceptual Understandings of Area, Surface Area, Volume, and Flow Rate Measurement: A Case Study

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Measurement is particularly important for vocational Engineering students, for whom this is a key skill required in the workplace. It is also a skill that many students find extremely challenging. In this research, I explored, through task-based interviews, 35 South African vocational Engineering students' measurement conceptualisations of area, surface area, volume, and flow rate, in order to identify the specific learning needs of these students. The amount, degree, and type of mediation required were used to map the structure of the students' measurement conceptualisations. In this presentation, I reflect on these students' conceptual understandings and the implications that these hold for mathematics educators and researchers.

Student Errors in a Mathematical Literacy Examination and the Correlated English Language Features

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Much research has established that when students have a poor command of the language of learning, teaching, and assessment, they experience a complex and deep learning disadvantage. This is the case for the majority of students in South African vocational colleges. In this study, I analysed the errors made by English language learners when writing a mathematical literacy examination in English, to determine whether the linguistic complexity of items influenced their responses. A statistically significant correlation was found between the linguistic complexity of items and certain errors, and it was possible to isolate which features of the language contributed to these errors.

Impact of Culture in Parental Control and Mathematics Achievement of their Children

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The presentation is based on a study guided by a conceptual framework developed on attributes of parental perceptions such as attitudes, beliefs, expectations, aspirations, values, and standards; parental involvement; and academic achievement of children. The participants were students (n = 128) and their parents (n = 85) from three secondary schools in Melbourne, Australia. The data were gathered by means of questionnaires and semi-structured interviews. The nature of research questions was both quantitative and qualitative, requiring a mixed-methods approach. It was found that there were significant differences in parental control between parents from European-Australian and Asian-Australian backgrounds.

High-Potential Mathematics Students and Their Mathematics-Related Activities Outside School

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Two decades ago, Csikszentmihalyi, Rathunde, and Whalen (1997) investigated what motivates young people to devote themselves to developing their potential. Studying mathematics offers long-term extrinsic rewards, but school mathematics may not be enjoyable or challenging. Hersh and John-Steiner (2011) suggested that supportive families can make a substantial difference to mathematics talent development. During a recent Australia-wide online survey of parents of school-aged children with high mathematical potential, data were gathered on the outside-of-school mathematics activities engaged in by the children. The data were examined by age and gender. The survey findings will be discussed in this presentation.

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RESEARCH INTEREST AREA DISCUSSION GROUPS

Research Interest Area Discussion Groups

Research Interest Area (RIA) discussion groups were introduced at the MERGA40 conference. The groups met for 85 minutes on 4th July 2017. Each RIA discussion group provided an opportunity for researchers to engage in discussions about the research explored in the chapters of *Research in Mathematics Education in Australasia 2012–2015* (Makar et al., 2016), and to consider future directions of research in these domains. The 15 RIAs and conveners are listed below.

Research Interest Area	*Convener(s)*
1. A philosophical gaze on Australasian mathematics education research	Steve Thornton, Virginia Kinnear
2. Researching curriculum, policy and leadership in mathematics education	Jennifer Way
3. Mathematics education and the affective domain	Catherine Attard, Naomi Ingram, Peter Grootenboer, Helen Forgasz
4. Equity, social justice and ethics in mathematics education	Colleen Vale, Robin Averill
5. Inclusive practices in mathematics education	Rhonda Faragher, Barbara Clarke
6. Distribution, recognition and representation: Mathematics education and Indigenous students	Tony Trinick, Cris Edmonds-Walthen
7. Mathematics education in the early years	Amy MacDonald, Wendy Goff, Bob Perry
8. Tertiary mathematics education	Mary Coupland, Linda Galligan, Greg Oates
9. Innovative and powerful pedagogical practices in mathematics education	Jodie Hunter
10. Assessment of mathematics learning: What are we doing?	Penelope Serow, Rosemary Callingham
11. Transformations of teaching and learning through digital technologies	Jodie Miller, Kevin Larkin, Hazel Tan, Esther Loong, Nick Calder
12. Research into mathematical applications and modelling	Gloria Stillman, Peter Galbraith, Jill Brown
13. Challenges, reforms, and learning in initial teacher education	Glenda Anthony, Audrey Cooke, Tracey Muir
14. The education and development of practising teachers	Kim Beswick, Judy Anderson, Chris Hurst
15. Advancing mathematics education research within a STEM environment	Lyn English, Noleine Fitzallen

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WORKSHOP SESSIONS

Workshop Sessions

Workshop sessions were held for 40 minutes on 3rd July 2017. The nine workshops and presenters are listed below.

Title	*Presenter(s)*
1. Using Qualtrics to design an online survey	Hazel Tan
2. Using Facebook for recruiting research participants	Simone Zmood, Gilah Leder, Helen Forgasz
3. Building an online presence: Sharing resources, exchanging ideas	Catherine Attard
4. Thesis examination: Similarities and differences from journal article and conference paper reviewing	Merrilyn Goos
5. Beyond scholarly journals: Why inform the profession and the general public	Kevin Larkin
6. Engaging teachers of mathematics in professional growth	Doug Clarke, Barbara Clarke
7. Reviewing for MERGA conference papers	Tom Lowrie
8. International Mathematical Modelling Challenge: An Australia perspective (2017)	Jill Brown, Peter Galbraith, Trevor Redmond, Gloria Stillman, Luke Bohni
9. Building a track record in readiness for major grant writing	Jane Watson

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (p. 711). Melbourne: MERGA.