Fluency with number facts – Responding to the Australian Curriculum: Mathematics

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This paper reports on results of a targeted and extended (two school terms) program for building Grade 3 and Grade 4 students’ facility with number facts and application for mental computation. As part of a larger project, this paper reports results from two schools. Results indicated strong gains in number fact recall and mental computation for both cohorts at both schools. The similar gains in outcomes at both schools suggests the power of a targeted and extended program to build basic fact fluency for mental computation.

Introduction

Teaching the Australian Curriculum: Mathematics (ACARA, 2017a) is to address the development of the four proficiencies of understanding, fluency, problem solving, and reasoning. Of most importance to this project is the term fluency. The focus of the research reported here was on building Year 3 and Year 4 students’ fluency with the basic facts of addition (and subtraction) and multiplication (and division) respectively.

Fluency is a term often misunderstood. Fluency is about skills in “choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately” (ACARA, 2017b). Fluent number fact knowledge spans far beyond conventional automatic mastery practices involving rote learning (Baroody, 2006). Students demonstrate fluency when they “calculate answers efficiently, when they recognize robust ways of answering questions, when they choose appropriate methods and approximations…” (ibid). Having the ability to automatically recall facts fluently does not mean students will have strong conceptual understanding (Hurst & Hurrell, 2016).

In the Australian Curriculum: Mathematics (AC:M), the development of number fact knowledge is an explicit feature in Year 3 and Year 4. In Year 3, the *content descriptor* is expressed as follows:

- Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (ACMNA055)
- Recall multiplication facts of two, three, five and ten and related division facts (ACMNA056)

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces *(Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia)* pp. 266-273. Auckland: MERGA.

The associated aspect of the achievement standard is stated as follows: "By the end of Year 3, students … recall addition and multiplication facts for single-digit numbers."

For Year 4, the content descriptor is as follows:

- Recall multiplication facts up to 10 x 10 and related division facts (ACMNA075)

The associated aspect of the achievement standard is stated as follows: "By the end of Year 4, students … recall multiplication facts to 10 x 10 and related division facts."

Number fact recall is a component of the AC:M, yet the pathway to attainment of these achievement standards is not prescribed. The term 'recall' may also serve to impact approaches to developing number facts. Does recall mean automatic/instant recall of a number fact, or the capacity to use strategies to achieve a correct calculation? How long does a teacher wait for a student to 'recall' a number fact before she/he determines that the student has attained the achievement standard?

Conceptual Framework

It can be argued that the purpose of number fact recall is to assist mental computation and estimation, one of the most-used mathematical skills of adults in their daily lives (Northcote & McIntosh; 1999). It has been long-established that automatic recall of basic facts is a prerequisite for mental computation facility (Sowder, 1992).

Research has highlighted the value of instructional programs that emphasise strategic and flexible thinking and provision of opportunities for students to explore, discuss and justify their strategies and solutions (e.g., Blote, Klein, & Beishuizen, 2000; Gravemeijer, Cobb, Bowers, & Whitenack, 2000). Young children bring to school sophisticated number knowledge and strategic thinking around number, and there is general consensus that rich learning environments, where students are provided with opportunities to explore number combinations and arrangements, assist students to derive their own strategies for basic fact combinations (e.g., Baroody, 1985; Fuson, 1992) and the development of number sense (Wright, 1996). However, some students do not develop efficient strategies for basic facts, and predominantly rely on inefficient counting strategies for mental computation (McIntosh & Dole, 2000; Mercer & Miller, 1992; Ruthven, 1998; Steinberg, 1985). Explicit strategy instruction, modelling, discussion, questioning, feedback and guided and independent practice has shown pleasing results, including for early years learners as well as students identified as exhibiting mathematics learning difficulties (e.g., Bryant, Bryant, Gerston, Scanmacca, & Chavez, 2008; Gerston, Jordan, & Flojo, 2005). There is also an extensive body of research that has shown that explicit teaching of strategies for particular groups of basic facts has facilitated fact recall and application in problem solving (e.g., Mercer & Miller, 1992; Rightsel & Thornton, 1985; Steinberg, 1985; Thornton & Smith, 1988).

The conceptual framework for this study was based on the premise that it is important for teachers to spend time in promoting thinking skills and strategies for basic facts with the goal of automaticity of basic facts. If learners are armed with automatic recall of basic facts as well as a wide range of thinking strategies, they are in a strong position to develop fluency in mental computation.

The research question of interest in this project was: What is the impact of an enriched program of number study targeting basic number fact recall upon mental computation fluency for children in (a) Year 3 associated with addition and subtraction; and (b) Year 4 associated with multiplication and division?

The Study

A key element of our design was to align the project goals with the needs of participating teachers. We recognized the importance of providing teachers with authentic, practice-based learning opportunities drawn from research into basic number fact development, opportunities to experience these investigations as learners themselves, and opportunities to share their ideas and experiences with colleagues, including the challenges encountered and their insights into the process. We drew on the Loucks-Horsley, Love, Stiles, Mundry and Hewson (2009) research and development framework as the methodology to our study.

As part of a larger study, this paper reports on results from two schools (A and B). School A is a large (approximate enrolment of 950) primary school (Prep-6) located in an outer-urban community of low to middle income (ICSEA 985). All six Year 3 classes (140 students) and five Year 4 classes (134 students) participated in the study. School B is a mid-range (approximate enrolment of 750) primary school (Prep-6) located in a coastal community of middle to high income (ICSEA 1073). All four Year 3 classes (87 students) and five Year 4 classes (129 students) participated in the study.

Instrument

A purpose-designed pen and paper Number Fact Quiz was developed by the researchers. It consisted of 60 items organised into three sets. Sets 1 and 2 consisted of 25 items each that targeted number facts for either addition and subtraction (Grade 3) or multiplication or division (Grade 4). Each set was organized according to perceived complexity (i.e., items in Set 1 were perceived to be less complex than items in Set 2). Set 3 consisted of 10 items requiring mental computation (that is, at least one of the numbers was greater than 20).

Procedure

At the beginning of the school year, the pre-test was administered to each class of students. To ensure a level of consistency, the lead teachers at each school implemented the test with each grade level at their respective schools. They read each item to the students and allowed 6-8 seconds for the basic number fact items (Sets 1 and 2) and 10 seconds for the mental computation items (Set 3). Project teachers attended a half-day professional development session where strategies and teaching approaches for developing number fact recall and mental computation was presented. The place of mental computation and basic fact recall within the Australian Curriculum: Mathematics was also revisited. Over a 20-week period (two school terms), classroom teachers prioritised number fact lessons for at least 15 minutes per day. Project leaders conducted school and classroom visits and undertook ad hoc classroom observations during each school term. The post-test occurred at the beginning of the last school term.

Results

The pre and post-test results for Grade 3 at both Schools A and B for each set of items on the quiz are presented in Figures 1 – 3. The pre and post-test results for Grade 4 at both Schools A and B for each set of items on the quiz are presented in Figures 4 – 6. What is most notable from all figures is the substantial gain in performance between the pre and post-test. In both schools, students' performance is 100% or very close to for particular number fact items, particularly for those items in Set 1.

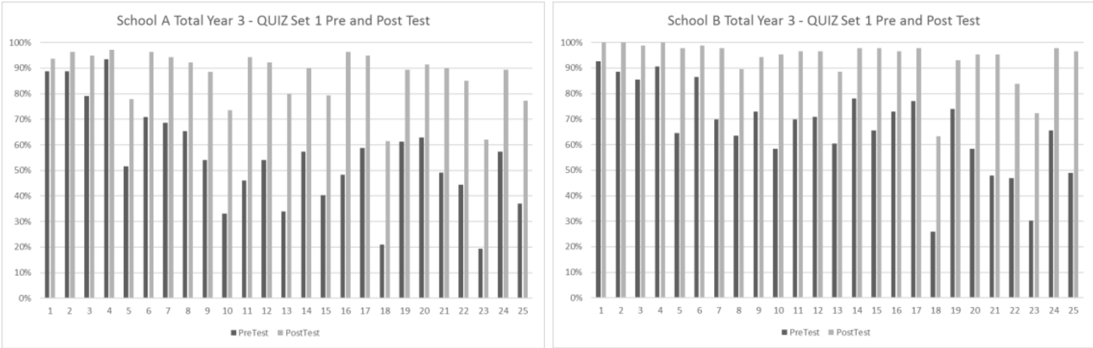

Figure 1. Set 1 pre and posttest scores for Year 3 School A (*n* = 140) and School B (*n*= 87) respectively

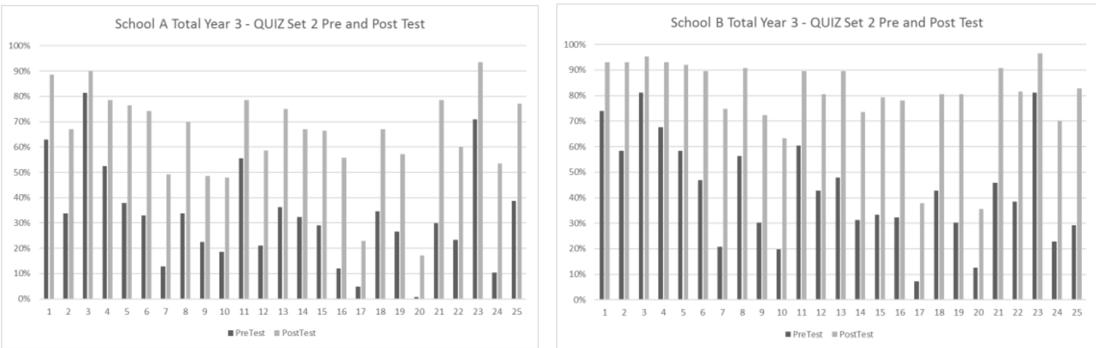

Figure 2. Set 2 pre and posttest scores for Year 3 School A (*n* = 140) and School B (*n* = 87) respectively

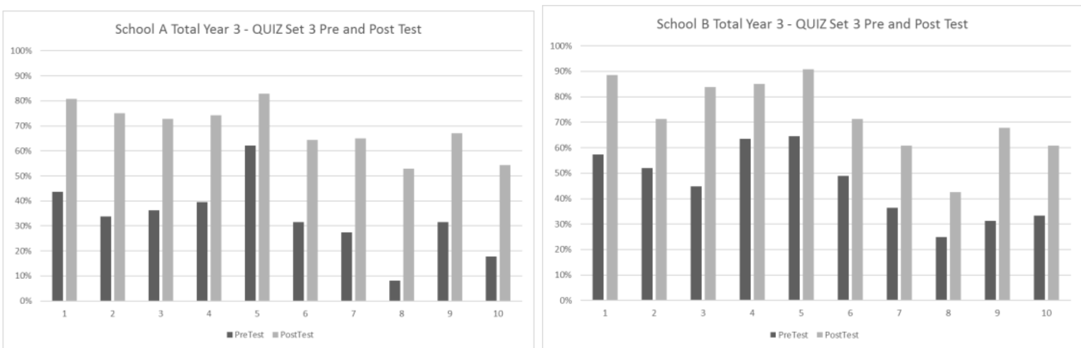

Figure 3. Set 3 pre and posttest scores for Year 3 School A (*n* = 140) and School B (*n* = 87) respectively

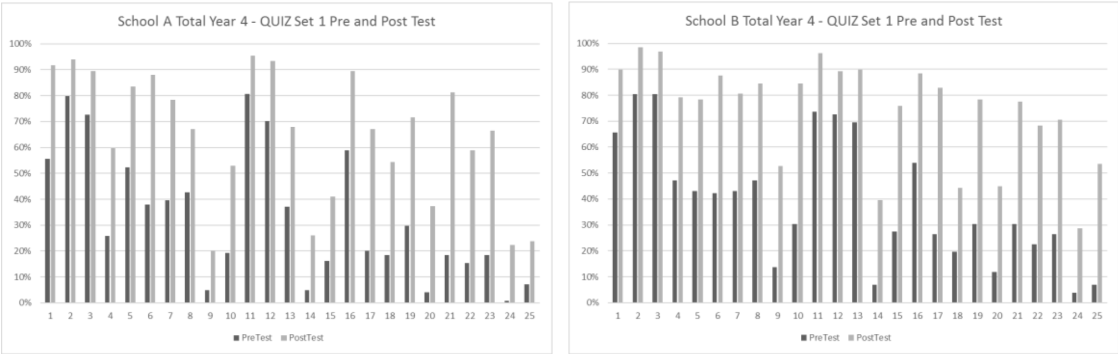

Figure 4. Set 1 pre and posttest scores for Year 4 School A (n = 134) and School B (n = 129) respectively

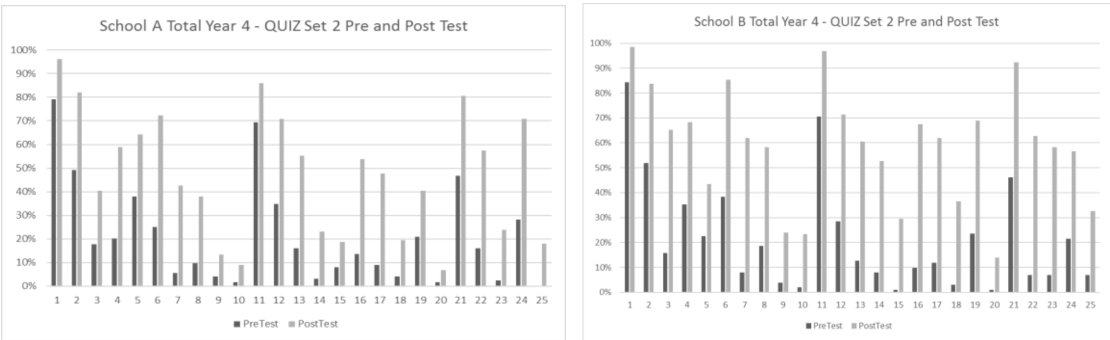

Figure 5. Set 2 pre and posttest scores for Year 4 School A (n = 134) and School B (n = 129) respectively

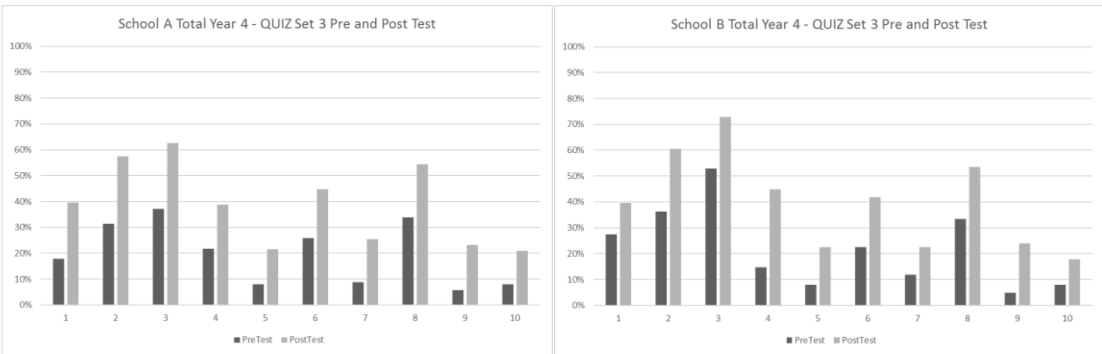

Figure 6. Set 3 pre and posttest scores for Year 4 School A (n = 134) and School B (n = 129) respectively

The overall mean score for each section of the quiz for School A and School B by year level is presented in Table 1. From Table 1, it can be seen that each Year level in both schools had considerable gains in results at the pre-test mark compared to the post-test. The year level cohorts from both schools started at similar but slightly different baselines and the overall performance on each set of quiz items was greater for School B than School

A. However, the percentage increase gains for School A and School B in each year level cohort is similar.

Table 1
School A and B pre and post-test mean scores for each section of the quiz

	School A		School B	
	Pre-test	Post-tests	Pre-test	Post-test
Grade 3				
Set 1 (/25)	14 (56%)	22 (88%)	17 (68%)	24 (96%)
Set 2 (/25)	8 (32%)	16 (64%)	11 (44%)	20 (80%)
Set 3 (/10)	3 (30%)	8 (80%)	5 (50%)	7 (70%)
Grade 4				
Set 1 (/25)	8 (32%)	16 (64%)	10 (40%)	19 (76%)
Set 2 (/25)	5 (20%)	12 (48%)	6 (24%)	15 (60%)
Set 3 (/10)	2 (20%)	4 (40%)	2 (20%)	4 (40%)

Discussion

The results presented in Table 1 show the overall gains from the pre-test to the post-test for both cohorts in both schools. The data indicate that the Grade 3 students returned much stronger results on the pre-test that their Grade 4 counterparts. The Grade 3 test items all focused on addition and subtraction whilst the Grade 4 test was only multiplication and division. What is noticeable are the strong gains on the post-test for the Grade 4 students indicating the value of a targeted focus on multiplication and division number facts. Also noticeable is the similarity of gains for each school particularly in relation to Set 3.

Set 1 and Set 2 items on each test for each cohort indicate a different level of difficulty, as student performance was relatively better for Set 1 items than Set 2 items. This was purposely built into the design of the instrument, and results suggest that some types of number facts are easier for students than others. For the Grade 3 test, Set 1 included counting-on and doubles facts and Set 2 was associated with the ‘make a 10’ strategy. For the Grade 4 test, Set 1 included multiplication and division with zero, one, doubles (multiply by 2), double doubles (multiply by 4) and multiply by 3. Set 2 included items associated with multiplying by 10, 5, 9 and square numbers. Set 3 items were not presented as number facts and required students to apply their number fact knowledge to items requiring mental computation. Grade 3 students performed better at applying their addition and subtraction number fact knowledge than Grade 4 students in applying their multiplication and division number fact knowledge. It is argued that the Grade 3 students in this study demonstrated much greater fluency in addition and subtraction mental computation than their Grade 4 counterparts did with multiplication and division mental computation.

Teachers reported that the grouping of facts by strategy provided them with valuable diagnostic data upon which they could focus their teaching. As can be seen from Figures 1-6, there are marked dips in performance associated with particular items. For example, the majority of students in both Grade 3 cohorts experienced difficulty with item 13 (□ +7 = 9), item 18 (□ - 2 = 4), and item 23 (complete the triad: 8, ?, 2). All of these number facts were classified in the ‘counting on 2’ category and were in Set 1 of the quiz. Teachers reported their surprise that such ‘easy’ facts caused difficulty to their students, but realised that the symbolic representation of the items on the test would have most likely contributed to student performance. Varying the symbolic representation of number facts became a teaching point in the classrooms. From the Grade 4 data, noticeable difficult items were items 9, 14 and 24 in Set 2, all of which are associated with multiplication by 4. Items 20 and 25 (also in Set 2) also caused difficulty, and these items are both associated with multiplication by 3. Teachers reported that analysing the data and identifying patterns of difficulty provided them with greater focus for their teaching. The results of this study suggest the value of targeting groups of basic facts to facilitate recall and application in problem solving, as suggested in the literature (e.g., Mercer & Miller, 1992; Rightsel & Thornton, 1985; Steinberg, 1985; Thornton & Smith, 1988).

During this project, the teachers committed 10-15 minutes per day to number facts. There was no prescribed approach, but guidelines were provided. There were three common classroom practices employed that included focused teaching, practice and whole class number talks. Focused teaching comprised visual and hands-on activities that exposed students to multiple representations of facts/number combinations to develop understanding of the connections within fact families. Practice Time (3 x 10 minutes per week) included warm ups, games, partner quizzes. Students engaged in paired and group card and dice games and other activities designed by their teachers as well as students. Whole class Number Talks occurred once a week (approximately 15 minutes) with the teacher providing a question and requiring students to share their thinking with the class. Such questions might include discussing strategies for mentally computing 98+45, for example. Results from this study, in all sets of the post-test for both cohorts in both schools, indicate the value of providing opportunities to explore, discuss and justify their strategies and solutions, as suggested in the literature (e.g., Blote, Klein, & Beishuizen, 2000; Gravemeijer, Cobb, Bowers, & Whitenack, 2000).

Conclusion and Implications

This study aimed to explore the potential of an enriched program of number study on mental computation fluency in Grade 3 and Grade 4. The results of this study indicate the value of a targeted focus on number facts throughout the year. Results also indicate the developing flexibility of students in carrying out procedures for mental computation given the strong gains in performance on the pre and post-tests. The data provides some measure of students’ capacity to apply basic fact knowledge for mental computation and arguably some measure of their fluency. Further research, with a more refined instrument, would assist in exploring students’ fluency with number facts and application in problem solving. The results do indicate the capacity of Grade 3 with addition and subtraction but indicate that that Grade 4 students have further distance to travel to consolidate multiplication and division facts. These results suggest a need for a continued and targeted focus on multiplication and division facts beyond Grade 4.

Of importance to the teachers in this project was how they provided students with opportunity to learn the stated content descriptors within the AC:M associated with recall of addition and subtraction facts (ACMNA055 and ACMNA056) and recall of multiplication and division facts (ACMNA075) and to report on students’ attainment of aspects of the associated achievement standard for Grade 3 and Grade 4 respectively. On the pre and post-test quiz, students were provided with 6-8 seconds to respond to the number facts items, which means that students had time to apply strategies for number facts if they could not recall them instantaneously. Whilst this project did not specifically seek students' and teachers' interpretation of the meaning of the term 'recall', it is reasonable to suggest that the emphasis on the provision of an enriched program of number fact study emphasized the importance of building students' capacity to use strategies to achieve a correct calculation rather than emphasizing automatic/instant recall of a number fact. Further research will ensure that a more meaningful interpretation of the term 'recall' in the AC:M will be explored.

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