Senior Secondary Probability Curriculum: What has changed?

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In this paper, the probability content in senior secondary mathematics Victorian curriculum between 1978 and 2016 was classified and compared, by content, context, procedural complexity, the SOLO thinking frameworks and use of technology. While probability continues to form an important and increasing component of the curriculum, it has moved from the popular general mathematics to the specialised subjects. The senior secondary probability curriculum as described by the textbooks, provided students with a variety of levels of complexity and thinking in both 1978 and 2016, although low complexity questions were more common in 2016.

The mathematical knowledge and skills of Victorian students has faced scrutiny because of reported declining performance in the international studies TIMSS and PISA (Mullis & Martin, 2015; Thomson, De Bortoli, & Underwood, 2016). The number of students studying advanced mathematics is decreasing (Kennedy, Lyons, & Quinn, 2014). Much debate has occurred around why this has transpired. As part of this discussion, this paper describes the changes to the senior secondary mathematics curriculum, in particular the content and complexity of the probability component of the curriculum.

As an experienced teacher of senior secondary mathematics, I am interested in the recent changes to the curriculum and spectacle if this is related to the perceived decline of mathematical knowledge of Australian students in popular discourse. As a contribution to investigating this issue, this paper provides a comparison of the changes to one element of the curriculum. The topic of probability was chosen as it has recently gained a greater focus within the senior secondary mathematics curriculum (VCAA, 2015). Yet probability is often overlooked and is not even mentioned in the TIMSS Advanced study (Mullis & Martin, 2015).

In this paper, the 1978 curriculum was compared to the 2016 newly modifed Austalian curriculum. The comparison year of 1978 was chosen due to the availability of the ACER report, *Changes in Secondary School Mathematics in Australia, 1964-1978* (Rosier, 1980). The second comparison year of 2016, was when the Australian Mathematics curriculum was updated in an attempt to align Australian curriculum across all states and territories, and incorporate recommendations from international research (ACARA, 2012). The changes to the senior secondary mathematics curriculum included an increase in the focus on the topic of probability, as described by the formal written curriculum (VCAA, 2015). The textbooks were used as an indicator of this intended curriculum, following the example of TIMSS (Lokan & Greenwood, 2001).

Research questions

The research questions considered in this paper are:

1. How has the topic of probability within the Victorian senior secondary mathematics curriculum changed over the years 1978-2016?
2. Which methods have been found to be most appropriate for classifying mathematical questions?

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Background

This section describes the structure of the mathematical subjects in the comparison years of 1978 and 2016. Mathematics was non-compulsory in senior secondary schools in both years.

Each mathematics subject in both years was described in its own textbook, which many teachers used as a pseudo- curriculum (Lokan & Greenwood, 2001).

In 1978, senior mathematics subjects in Victoria were General Mathematics, Pure Mathematics and Applied Mathematics. These subjects contributed to the HSC (High School Certificate) (Rosier, 1980). General Mathematics was the most popular mathematics and was taken by any student wanting to continue to university. General Mathematics and Pure Mathematics could have been taken alone, while Pure Mathematics was also a co-requisite for the specialised Applied Mathematics. Pure and Applied mathematics lead to tertiary studies where mathematics was an integral part of the discipline. Four figure mathematical *tables* (Kaye & Laby, 1977) were supplied for all examinations, which included probability distribution *tables* and miscellaneous formulae.

In 2016, the senior mathematics subjects in Victoria were Further Mathematics, Mathematical Methods and Specialist Mathematics, which contribute to the VCE (Victorian Certificate of Education) (VCAA, 2015). Further Mathematics was designed for general preparation for employment or further study and was the most popular subject. Mathematical Methods could be taken alone, with Further Mathematics or was a co-requisite for Specialist Mathematics, which was designed for further study in engineering or pure mathematics. CAS (Computer Algebra System) calculators and bound reference books were assumed for some examinations, but not all.

Methodology

Mathematics curriculum has been investigated from various perspectives, including content, context, procedural complexity, levels of thinking and use of technology. These perspectives were supported by the frameworks used in PISA, TIMSS, and the SOLO taxonomy (Structure of Observed Learning Outcomes) (Biggs & Collis, 1991).

The content of the senior mathematical subjects was described, focusing on the probability topic and highlighting the recent curriculum changes.

The context surrounding the mathematical questions were classified as these influence engagement according to the international study of PISA (Thomson et al., 2016). A focus on personal contexts which students could relate to, such as food or costs of phone plans, were considered more engaging than abstract contexts.

Procedural complexity as described in TIMSS (Hiebert, 2003) was based on the number of decisions: with low complexity problems described as involving less than four decisions; moderate complexity involving four or more decisions and possibly containing one sub-problem; and high complexity requiring more than four decisions and two or more sub-problems.

Levels of thinking is one method frequently used for classifying student responses. The formal curriculum guidelines for both comparison years refer to thinking and reasoning as being significant (Rosier, 1980; VCAA, 2015). When considering levels of thinking, the SOLO taxonomy has been found to be a valuable method for classifying student responses, especially in probabilistic thinking (Mooney, Langrall, & Hertel, 2014). SOLO has also been used to design assessment questions (e.g. Collis & Romberg, 1992). The aim of the current study was to classify mathematical questions using the SOLO framework and to describe the anticipated level of thinking expected for students' responses to these questions.

SOLO classifies responses and questions with unistructural/multistructural/relational (UMR) cycles within a neo-Piaget's framework (Biggs & Collis, 1991). Classification of questions according to the SOLO level of thinking required was determined through the following considerations. It is anticipated that Year 12 students would be at Piaget's formal mode of thinking. A question was classified as requiring unistructural thinking if it needed one formula or concept, including definitions. A question was deemed as requiring multistructural thinking if it involved one formula but used in different ways; for example, using the formula backwards, or if it involved two concepts. Relational thinking involved using several formula or concepts and relating them together.

The current use of technology including CAS calculators has been well documented (e.g. Geiger, Forgasz, Tan, Calder, & Hill, 2012) so will not be discussed in detail here, however calculator use has certainly changed over the years and therefore warrants a mention.

Method

This paper concentrates on the intended mathematical curriculum as described by the most popular textbooks of 1978 and 2016. Textbooks have been used as the pseudo-intended curriculum at the senior secondary level (Lokan & Greenwood, 2001; Son & Diletti, 2017). Each question in the review section of the textbooks was categorised by:

- Content: probability topics
- Context: abstract, occupational, personal
- Procedural complexity: low, moderate, high
- Expected SOLO level required to solve the question: prestructural, unistructural, multistructural, relational, and extended abstract
- Technology: CAS calculator or *tables* required.

Example of analysis method.

The method of analysis used in this study can be described though the use of two examples. The first example is from the multiple-choice section of the 2016 Specialist Mathematics exam 2, where use of a CAS calculator and reference book is assumed.

> A random sample of 100 bananas from a given area has a mean mass of 210 grams and a standard deviation of 16 grams. Assuming the standard deviation obtained from the sample is a sufficiently accurate estimate of the population standard deviation, an approximate 95% confident interval for the mean mass of bananas produced in the locality is given by **B.** (206.9, 213.1) (VCAA, 2016, p. 9)

This question involved using the formula from the formula sheet supplied, with little

$$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$$

interpretation, with the fact that z = 1.96 needed for a 95% confidence interval being the only information to remember or obtain from the reference book. $\bar{x} = 210, s = 16, n = 100$ are taken from the question. This was a question with low procedural complexity as only one formula was used, and less than four decisions were involved. Students needed only one concept of theory to solve this question so would only demonstrate thinking at the SOLO level of unistructural. This question used material which was new to the subject in 2016 and was not in any subject in 1978. Although a calculator was needed, a CAS calculator would have been unnecessary. This question had a personal context of food, as did several of the Specialist Mathematics questions.

The second question to demonstrate and clarify the analysis method is from the 1978 Applied Mathematics textbook review section.

> If X is a normally distributed variate with mean 4, and the probability of X exceeding the value 8.34 is 0.015, calculate (i) the standard deviation of X, (ii) Pr (X > 1 | X < 4). (Fitzpatrick & Galbraith, 1974, p. 103)

The solution was found by using the cumulative normal distribution *tables* to find the solution to Pr (Z > z) = (1 - 0.015), where z = 2.17. The formula $z = \frac{8.34 - 4}{\sigma}$ was then used to find standard deviation, $\sigma = 2$.

The second part of the question requires the conditional probability formula, $\Pr(X > 1 | X < 4) = \frac{\Pr(1 < X < 4)}{\Pr(X < 4)}$ which was found on the supplied formula sheet. All values were transformed into the standard normal curve and the cumulative distribution *table* was used again. This question involved more than four decisions and two sub-problems and consequently was seen as a high procedural complexity. Using the SOLO structure, this question would require students to be at a relational level of understanding, as the student would need to use several concepts and relate them together. This question would have been easier to complete with a CAS calculator, to save converting to the standard normal curve. This question was taken from the textbook, selected from a past exam, and involved an abstract context.

Results

Probability questions from the popular senior secondary mathematics textbooks used in Victoria in 1978 and 2016 were initially classified by content and subsequently classified by context, procedural complexity, level of thinking and use of technology.

Description of the probability content

The percentage of mathematics students who studied the different mathematics subjects is interesting. The percentage of mathematics students who studied General Mathematics in 1978 and Further Mathematics in 2016 was very similar at 64-66%. The percentage of mathematics students who studied Pure Mathematics in 1978 and Mathematical Methods in 2016 are also similar at 36-34%. The percentage of mathematics students who studied Applied Mathematics in 1978 at 33% was much higher than the 9% who studied Specialist Mathematics in 2016. This decrease in the number of students studying advanced mathematics has been an ongoing area of concern (Kennedy et al., 2014).

In 1978 most students completed either General Mathematics or Pure Mathematics, with the majority of Pure Mathematics students also studying Applied Mathematics. The topic of probability made up a quarter of the core content in General Mathematics, but was not part of Pure Mathematics. Probability was included in Applied Mathematics, but this topic was internally assessed and not in the external examinations. The probability content in General Mathematics and Applied Mathematics overlaps, with discrete and continuous probability components in both subjects, including probability using calculus integration. Both subjects focus on four distribution types, where the main decisions involved the use of formula from the formula booklet or *tables*. This structure insured nearly all students who studied mathematic, studied topics of probability.

In 2016 most students studied Further Mathematics and/or Mathematics Methods. Some of the Mathematical Methods students also studied Specialist Mathematics. Probability content is not included in the Further Mathematics subject except for a brief reference to the normal distribution. Mathematical Methods includes discrete and continuous probability including calculus integration. Two probability distribution types were studied. In 2016 probability was increased in the Mathematical Methods and Specialist Mathematics subjects, to include statistical inference, sampling, and confidence intervals. This was to bring the subjects in line with the Australian Curriculum and was also recommended by international research (ACARA, 2012).

The topic of probability forms a large component of several mathematics subjects in both 1978 and 2016. The probability topic has moved from the popular General Mathematics subject to the less popular specialised subjects. The study of particular probability distributions in 1978 was decreased and the link between calculus integration and probability increased. The interpretation of probability problems increased by including inference and confidence levels in 2016.

Analysis of the probability questions

Each of the probability review questions from the popular textbooks in 1978 and 2016 were solved and classified according to context, procedural complexity, level of thinking and use of technology. A summary of the results is provided in Table 1. The 1978 review questions comprised short answer questions and extended response while the 2016 questions also included multiple-choice questions. The number of review questions in the Mathematical Methods textbook in 2016 was much higher than the other subjects, due to the inclusion of a large number of multiple-choice questions. The 1978 Pure Mathematics and 2016 Further Mathematics subjects were not included in Table 1 due to the lack of probability content.

Context remained a mixture of abstract, occupational, and personal. There were a large number of abstract questions without a real-life context, especially in the multiple-choice Mathematical Methods questions. Questions about broken machines, phones, and laptops were popular.

Procedural complexity was fairly consistent over the four subjects, except for the large number of low procedurally complex questions in the 2016 Mathematical Methods multiple-choice questions. The highest number of high procedurally complex questions is in the 2016 Specialist Mathematics subject. Questions usually explicitly stated which distributions to use, which decreased the decisions required and hence the complexity.

In 1978 a book of *tables* was used instead of the CAS calculators of 2016. Ten of the 1978 review questions would have been considerably easier if a CAS calculator had been available, for calculations using distributions or integrals. The number of questions requiring either *tables* or CAS calculators more than doubled in 2016. All the 2016 probability questions could have been solved with *tables* or a scientific calculator.

Table 1.
Analysis of Probability Review in Senior Secondary Mathematics, 1978 and 2016

Criteria		1978 review General Mathematics (Fitzpatrick, 1974)	1978 review Applied Mathematics (Fitzpatrick & Galbraith, 1974)	2016 review Mathematical Methods (Evans, Lipson, Jones, & Greenwood, 2016)	2016 review Specialist Mathematics (Evans, Cracknell, Astruc, & Jones, 2016)
Number of questions		28	31	71	30
Context	Abstract	10	5	40	17
	Occupation	12	15	26	9
	Personal	6	11	5	5
Procedural complexity	Low	11	13	52	8
	Moderate	7	9	14	11
	High	10	9	5	11
SOLO	Prestructural	0	2	0	0
	Unistructural	10	11	41	10
	Multistructural	11	6	21	8
	Relational	7	12	9	12
CAS Calculator or *tables*		5	7	22	10

Discussion and Conclusion

The data reported above provide some insights into the intended curriculum in 1978 and 2016 as indicated by the review sections of the most popular textbooks. The topic of probability has moved from the popular 1978 General Mathematics to the more specialised 2016 Mathematical Methods. This was disappointing due to the need for an increase in the probabilistic thinking needed in the general population (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016).

There was a large increase in the number of review questions in the 2016 textbook, and there continued to be many probability questions which did not require *tables* or CAS calculators. There was a mixture of abstract, occupational and personal contexts for the questions, with many more questions of an abstract nature in the 2016. In 1978, there were 15 abstract questions, while in 2016 there were 57 abstract questions. Abstract questions were considered to be less engaging than real-life contexts (Vincent & Stacey, 2008).

While there continued to be a mixture of questions of varying complexity in all the mathematics subjects, the 2016 Mathematical Methods textbook included considerably more questions of low level complexity. This disparity could be exaggerated by the structure of the textbooks. In 1978 the review chapters were divided into *Sets* of questions of mixed complexity. In 2016 the review sections of the textbooks started with the low complexity questions and finished with questions involving high-level complexity. This could result in teachers and students not reaching the high-level questions at the end of the review chapter. The review chapters themselves may not be a good indication of the textbooks as a whole. The complete textbooks need scrutiny, including the demonstration questions.

Low complexity questions tended to be unistructural (using one concept), while highly complex questions tended to be relational (relating several concepts together).
Multiple choice questions tend to be of lower complexity and unistructural, although this was inconsistent. In 1978 there were no multi-choice questions and 24 low procedural level complexity question while in 2016 there were 60 low level questions, most of them of the multiple-choice style. A few infrequent multiple-choice questions did involve the understanding of many concepts and contained complex worded problems.

Classifying the questions became more difficult in the extended response questions, as they were scaffolded with many sub-questions, making them of a high procedural complexity. Each part of the question might be a single concept or might be related to each other. Both years had similar numbers of high level complexity questions, with 19 in 1978 and 16 in 2016.

The SOLO taxonomy, the structure of observed learning outcomes, was designed to classify student responses (Biggs & Collis, 1991). It has been used here to classify the questions as demonstrated by Collis and Romberg (1992). This assumed the students' previous experiences were known, as a question solved for the first time requires a higher level of thinking than a well-practiced question (Chick, 1998). Some of the highly complex questions in the review section of the textbook were very similar to the examples from the textbook, which would make them less difficult for the students. Classifying questions by complexity or thinking is also difficult as mathematical questions can be answered in different ways.

Analysis of the intended curriculum or textbooks is scarce in senior secondary mathematics and even more infrequent in the area of probability (Son & Diletti, 2017). Further investigation is needed in how the teachers and students use the textbooks and other curriculum resources. Students are not expected to learn from textbooks on their own in Australia (Lokan & Greenwood, 2001). The teacher's implementation of the curriculum is influential. The wider curriculum including teachers and students' perceptions will be encompassed in a larger study of the senior secondary mathematics curriculum, where the intended, implemented and attained probability curriculum will be investigated through thinking frameworks.

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