Using a Contextual Pāsifika Patterning Task to Support Generalisation

Jodie Hunter
Massey University
<J.Hunter1@massey.ac.nz>

Jodie Miller
University of Queensland
<Jodie.Miller@uq.edu.au>

Pāsifika cultures have a rich background of mathematics including a strong emphasis on patterns used within craft design (Finau & Stillman, 1995). However, there have been limited studies which have investigated the use of contextual Pāsifika patterns in mathematics classrooms. The aim of this study was to explore how contextual Pāsifika patterning tasks can potentially support young children to develop their understanding of growing patterns. Ten lessons using Pāsifika and Māori patterns were undertaken with 27 Year 2 students (6-year-old). In this paper, analysis of one of the lessons is used to examine how a contextual task assisted these young students to generalise growing patterns.

Mathematical achievement of culturally diverse students is a challenge in many countries. Teaching in ways responsive to the cultures of our students is an important step in enhancing equity of access to mathematical achievement and enacting educational policy (e.g., Ministry of Education, 2012). Within New Zealand, similar to other countries, there is a changing student population that is increasingly culturally diverse. This includes a large number of Pāsifika students, a heterogeneous group from a range of Pacific Island nations and including both those born in New Zealand who identify themselves with the Pacific Islands and those who have migrated from the Pacific Islands (Coxon, Anae, Mara, Wendt-Samu, & Finau, 2002). In New Zealand schooling, Pāsifika students' results are characterised by under-achievement when compared to students of other ethnicities (University of Otago & NZCER, 2014). Deficit theorising is frequently used by educators to explain this under-achievement with Pāsifika cultures being positioned as mathematically deficient (Hunter & Hunter, 2018; Turner, Rubie-Davis, & Webber, 2015). However, Pāsifika cultures have a rich background of mathematics including a strong emphasis on patterns used within craft design (Finau & Stillman, 1995). This paper investigates the use of contextual Pāsifika patterning tasks to support young children to develop their understanding of growing patterns.

Research Literature

Over the past decades, early algebra has been the focus of both research studies and curriculum reform with calls for a greater emphasis on the teaching and learning of algebra in primary classrooms (Blanton et al., 2018; Ministry of Education (MoE), 2007). Both patterning activities and functions offer an opportunity to integrate early algebraic reasoning into the existing mathematics curriculum. Evidence from research studies highlights that young learners can engage in early algebraic reasoning and generalise from patterning tasks (Blanton et al., 2018), and this supports students development of deeper understanding of mathematical structures (Warren & Cooper, 2008). Early algebraic thinking comprises of three key components: (1) generalising mathematical relationships and structure; (2) representing generalised relationships in diverse ways; and (3) reasoning with generalised relationships (Blanton, et al., 2018; Kaput, 2008; Warren & Cooper, 2008). For the purpose of this study we will be focusing on generalising mathematical

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relationships and structures. It is argued that before students can generalise relationships, they must be able to identify the underlying mathematical structure (Warren & Cooper, 2008). However, it is often the way in which the pattern structure is represented, in conjunction with teacher instruction, which inhibits primary school students to successful access mathematical structures.

For the purpose of this study we are considering the structure of growing patterns. Linear growing patterns are characterised by the relationship between elements which increase or decrease by a constant difference. Students are often introduced to growing patterns through visual images of mathematical shapes (e.g., squares, circles) as a series of stages adding in one direction of the pattern (e.g., adding on another line of squares or a layer of circles) as evident on www.nzmaths.co.nz. Less common in curriculum documents and teaching materials are the introduction of growing patterns as instances of growing in multiple directions (e.g., in the shape of a cross growing in four directions).

The majority of studies have considered how students see the structure of growing patterns and how they form generalisations with geometric growing patterns often not embedded or connected to students' culture. The aim of this study is to explore how contextual Pāsifika patterning tasks can potentially support young children to develop their understanding of growing patterns. In particular: (1) How do young culturally diverse students see the structure of Pāsifika patterns? and, (2) How do young culturally diverse students generalise the structure of Pāsifika patterns?

Theoretical Frameworks

There are two theoretical frameworks that underpin this study; first, a framework for generalisations in early algebraic thinking as identified by Radford (2010); and, that of culturally responsive pedagogies with a particular focus on Pāsifika values and culture.

Generalising mathematical concepts must go beyond just the act of noticing (Radford, 2010). For all elements of a pattern sequence, students must develop the capacity to see the underlying structure and articulate this algebraically (Radford, 2010). Underpinning this assertion is Radford's (2010) three 'layers of generality': factual, contextual and symbolic generalisations. Factual generality is an elementary level of generalisation where students engage heavily in gestures, words and perceptual activities often attending to particular instances of the pattern rather than general elements across the pattern. In developing a contextual generalisation, students will often refer to the "the next figure which supposes a privileged viewpoint from where the sequence is supposedly seen" (Radford, 2010, p.52). Finally, the symbolic level requires students to replace words with symbols such as letters to express the generality of the rule. The majority of studies in this area utilise tasks from a Western context with few drawing on students' cultural backgrounds.

Despite mathematics being positioned as a value and culture free subject area (Presmeg, 2007), researchers (e.g., Bishop, 1991; D'Ambrosio, 1985, Tate, 1995) have shown that mathematics is a cultural product. The perspective taken within this paper is that the teaching and learning of mathematics cannot be decontextualised from the learner as this is wholly cultural and closely tied to the cultural identity of the learner. Similar to Tate's (1995) argument related to African American students in the USA, we contend that failing to provide Pāsifika students with tasks and learning experiences that are centred on their traditions, experiences, and culture is the major reason for inequity in mathematics education in New Zealand. To develop culturally responsive mathematics classrooms, educators need to ensure that tasks are set within the known and lived, social and cultural reality of the students. Acknowledging that students bring their own cultural ontology (ways of being, knowing and doing) and discourse to the classroom provides an opportunity for students of diverse cultural backgrounds to make more meaningful connections to mathematics (Miller, Warren, & Armour, 2018). Many Pāsifika learners have a rich environment of patterns from cultural activities and artefacts. For example, cultural activities such as the Samoan sasa (slap-dance) draw strongly on patterns as does Cook Island drumming and drum dances. An emphasis on patterns is also evident in cultural artefacts ranging from Tongan and Samoan ngatu/siapo/tapa (a form of bark-cloth) to Cook Island tivaevae (quilts). We contend that these contexts provide a means of drawing upon the mathematics already evident in Pāsifika culture to develop young learners' early algebraic reasoning.

Research Design

This research reports on one aspect of a larger study which focused on the use of authentic patterns from Pāsifika and Māori culture to develop young culturally diverse students' understanding of functional patterns. It was conducted with one classroom of Year Two students in a low socio-economic, high poverty, urban school in New Zealand. Twenty-nine students (aged 6 years old) participated in the study including 17 male and 12 female students. The students were predominantly of Pāsifika descent (n = 24), with three students from an indigenous New Zealand Māori background, and two students from South East Asia. The teacher in this classroom was an experienced teacher who had been involved in an ongoing professional development and research project entitled *Developing Mathematical Inquiry Communities* (for more information see Hunter & Hunter, 2018).

Drawing on the design of a classroom teaching experiment (Steffe & Thompson, 2000), students participated in ten 30-minute lessons exploring and developing their understandings of functional growing pattern generalisation. The students were taught in small groups with between 12 – 14 students involved in each lesson. Each lesson involved a similar structure with the launch of the task, paired work, a large group discussion and a teacher facilitated connection to a generalised rule. Students in this classroom had previously engaged with tasks involving repeating patterns but growing patterns were unfamiliar as this is not a curriculum expectation until Year Four (MoE, 2007). The focus of this research paper is two lessons focused on a pattern from a Cook Island tivaevae.

The video footage of the lessons was wholly transcribed and analysed to identify themes. To manage these documents a coding system was utilised to determine how to examine, cluster, and integrate the emerging themes (Creswell, 2008). Researchers coded the data at each phase with respect to early algebraic thinking, teacher actions, Pāsifika values, and student actions and met to discuss their themes and recode any data.

Findings and Discussion

The findings draw on the analysis from one lesson to provide an exemplar of how an authentic cultural pattern can be used to develop early algebraic reasoning. This includes an examination of the task structure and launch, teacher actions and student responses.

Task Structure and Launch: Engaging in Pāsifika Culture

Tivaevae is a traditional form of Cook Island quilting which involves groups of women designing, cutting, and embroidering these quilts. These are usually only given as gifts on special occasions such as weddings or significant birthdays. Designs are often based on plants and flowers and frequently incorporate forms of growing patterns. The task was designed in collaboration with the teacher using a photograph of part of a pattern of a tivaevae. The focus was on the number of leaves on the pattern.

Figure 1. Cook Island Tivaevae task.

The teacher began the lesson by acknowledging the cultural knowledge of her young students. After showing the class a photo of a tivaevae, she positioned a child of Cook Island heritage as an expert to share her knowledge of tivaevae with the other students:

> Mereana (excitedly): My Mama, she makes that, she makes heaps, my whole family does.
>
> Teacher: Does she? Wow, you can help me then. I am so pleased that you came and sat over here Mereana because I knew that you would know about this.
>
> Mereana: It's a Cook Island… (pause) tivaevae
>
> Teacher: Yes it's Cook Island, and they are beautiful, aren't they?
>
> Mereana: We use it for weddings and birthdays
>
> Teacher: Listen to Mereana, Mereana is going to tell us, who makes them?
>
> Mereana: My Mama and the girls in our family.

In this example, the teachers' actions supported the students to begin making a meaningful connection to mathematics in relation to their cultural context.

The teacher then began to orient the students to the structure of the pattern: *let's just get really clear where the leaves are, Sima, so we are all going to do it together because otherwise we will all be talking about different designs*. In doing so, she consistently drew student attention to the constant four in the middle of the pattern.

Student Approaches to the Pattern: Seeing the Pattern in Multiple Ways

Initially, the students attempted to draw or count to find the number of leaves for the pattern positions. For example, Sebastian and Cruz began by counting the twelve leaves for position one (four leaves in the centre (4) and two leaves up each of the four stems (8)) and for position two counted another eight leaves up to 20 leaves. After the third position, they noted the regularity of the increase by eight. They used this to continue to count by visualisation for the successive positions. For example, when they came to position four which was not pictured, Sebastian and Cruz then counted from 28 to 36.

At this point, noticing that many of the students were either drawing or counting, the teacher stepped in to press the students beyond counting:

> Teacher: What I noticed is that lots of people were busy drawing that picture and you were doing lots and lots of counting. But sometimes when you count really big numbers and draw lots and lots of leaves, what happens to our counting?
>
> Tiare: You lose the count.

This teaching moment was key to shift student attention to more explicitly noticing the structure of the pattern rather than using a count all or count on technique. The teacher finished the first lesson by selecting a pair of students to share the pattern that they noticed:

> Mereana: (indicating on the picture to outside circle of leaves) Whenever you add the leaves you add eight.
>
> Teacher: So what did you notice? Say it again
>
> Mereana: You can, if the number gets higher, you just add eight leaves
>
> Teacher: So who understands what she is talking about? What does she mean, when it gets higher we keep adding eight?

This is the first instance of a generalisation articulated by a student. Mereana is seeing the structure of the pattern as adding eight leaves as the numbers get higher. This demonstrates that she is coordinating two variables in the pattern: (i) the structure of the eight leaves; and, (ii) the position of the pattern (if the number gets higher). The teacher then pressed Mereana to link her explanation back to the structure of the pattern. When this occurs Mereana begins to talk about the pattern as individual instances.

> Teacher: Can you show on that picture behind you?
>
> Mereana: (Draws circle with finger around first eight leaves outside) There is eight on the first one, and then eight on the second one (draws imaginary circle around the next layer of eight).

In this instance, Mereana has formed a factual generalisation (Radford, 2010). She is articulating the pattern as instances and using gesture to support her reasoning. This aligns with past research that indicates that the use of gesture for young students as they articulate generalisations appear to be a key stage in their development of algebraic thinking (Miller, 2015; Radford, 2010). It is unclear whether she sees the pattern more generally or as only series of instances, however it is clear that she has been able to begin to notice and articulate the two variables of the pattern with only a few explicit teaching moments which facilitated the students to attend to the variables. Research highlights that young students often refer to only one variable of the pattern and do not attend to the co-variational relationship of the two variables. In addition, it appears that there is more success if the variables are embedded in the one structure so the students cannot ignore it (e.g., kangaroo tails and ears) rather than separated (e.g., geometric shapes and words or number cards under the pattern indicating the position) (Miller, 2015). In this case it appears the tivaevae pattern provided an opportunity for these young students to begin to see co-variation.

Lesson Two: Three alternative approaches

The structure of the tivaevae pattern allowed the students to see it growing in multiple ways. At the beginning of the next lesson, the teacher first facilitated student awareness that the pattern could be seen in multiple ways: *different people see it growing in different ways, have a little think first.* She then provided the students with time to talk in pairs to further develop their ideas of how the pattern of leaves was growing.

The teacher noticed that students viewed the pattern as growing in three distinct ways. She carefully selected specific students to share the alternative ways that they saw the pattern growing. The teacher began by reminding students of the constant in the middle: *just before we say about the outside can someone remind us how many leaves there are on the inside? What about in that middle bit?* Following this, she asked a pair to share:

> Asher: The pattern is, they’re putting two each on the outside.
>
> Teacher: Two what? Asher? Two what?

Asher: Ah, leaves.

Teacher: Good what are they putting on the outside each time Aurora?

Aurora: Two leaves.

Teacher: Two leaves, just two or two on every?

Aurora: Side.

Teacher: Two on every side.

As the students shared, the teacher introduced simplified representations of how the students saw the structure of the pattern.

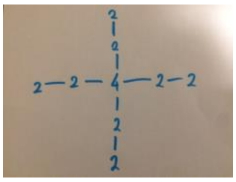

Figure 2: Simplified representation of pattern structure.

In this moment, not only is the teacher having students attend to the structure but she is also displaying the visual contractions of the pattern that students are articulating. In this instance, there has been a transfer from a cultural context into a mathematical context. Following this the teacher then asked student to share an alternative solution, similar to the example from the previous lesson:

Teacher: Ok so now Sima and Seini, and I think you guys all did something else similar I want you to talk about what you did, what did you notice?

Sima: Every time you add on leaves you add eight.

Teacher: So can I just check how many were in the middle on your one?

Seini: Four.

Teacher: It's still four in the middle, you said, every time we add eight. Turn and tell your buddy why they're saying they'll always add eight.

While the students did not articulate the constant variable in the pattern of four (the centre leaves) as an additional variable in the pattern, the teacher was drawing their attention to it as a way for them to consider their generalisations. Following this, the teacher again introduced a simplified representation to show how the students saw the structure of the pattern: *So, the first one's around it, can you see that? So it's like this, the patterns you've got your four in the middle and then it goes eight and then eight, and then eight, can you see like that, it's growing like that.* (draws diagram – see Figure 3)

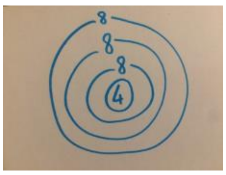

Figure 3: Simplified representation of pattern structure.

In contrast to the above figure, other students saw the pattern growing as the pattern number eight times around the stalk. That is, they saw the leaves growing up one stalk rather than in circles around the centre of the pattern.

Ngaire: There were three leaves on each stalk.

Teacher: On each stalk or on each side of each stalk?

Ngaire: Each side.

Teacher: So, there were three leaves over here, and how many leaves on this side?

Ngaire: Three.

Teacher: Have a think, have a look at that one and think about how that one works? Cos Ngaire has seen it a different way. What would come down on this stalk?

Ngaire: Three.

Teacher: Three where?

Ngaire: Three on each.

Teacher: Three here and three here (writes a three on each side of the stalk for each stalk).

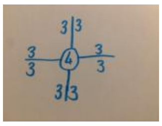

Figure 4: Simplified representation of pattern structure.

Importantly, following the sharing of the solution strategies, the teacher provided an opportunity for other students to access the ways in which their peers visualised the pattern growth. For example, using the description generated by Asher and Aurora, the teacher asked the students to describe what the third position would look like and what the seventh position would look like:

Tiare: (referring to one branch) Two times three.

Teacher: We could use our two times tables couldn't we. So there's two and two and two. So you could keep going out, how many twos would you go out for the seven?

Sebastian: Seven.

It was at this point that the students shifted their thinking from seeing the pattern as additive to multiplicative. For young students' it is often challenging to see the multiplicative structure of a growing pattern without teachers making this explicit for students (Warren & Cooper, 2008; Miller, 2015). In addition, Tiare has provided a factual generalisation (Radford, 2010) where she is referring to one instance in the pattern, however the key point here is that students are now being to see the multiplicative structures of the pattern.

Conclusion

This study begins to add to new knowledge about the use of culturally relevant tasks being used to develop early algebraic thinking for students. Acknowledging that the students brought their own cultural knowledge to the classroom provided an opportunity for these culturally diverse students to make more meaningful connections to the mathematics presented in the lesson. It is evident that contextual Pāsifika patterning tasks, such as a tivaevae pattern, can support young children to develop their understanding of growing patterns and begin to articulate generalisations. There were opportunities for: (i) students identifying multiple structures of the pattern, which were both additive and multiplicative; (ii) being to identify and articulate covariational relationships; and, (iii) form both factual and contextual generalisations. There were clear shifts in both student thinking and teaching actions across the two lessons. Students moved from count all strategies, to identifying additive thinking to then multiplicative thinking through considering the structure of the pattern. The teacher actions of mirroring student thinking and using mathematical diagrams supported students as they made generalisations and further contractions of the pattern structure.

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