Examining a teacher's use of multiple representations in the teaching of percentages: A commognitive perspective

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Learning Mathematics in Primary Schools is often mediated through the use of multiple representations. However, teachers may not pay enough attention to the way they use these representations. Given that the translations among representations may not always be smooth, it may be insightful to examine how teachers mediate learning through the use of multiple representations. In this paper, I will share key ideas in commognition before I present a case study of how Hannah, a teacher, mediate learning of percentages in her class. I will also introduce the idea of a 'Mediation Flowchart' and demonstrate how it can be used to describe and analyse a teacher's use of multiple representations.

Representation is one of the five process standards stated in the principles and standard of school mathematics (National Council of Teachers of Mathematics (NCTM), 2000). Representation is both a process and a product (NCTM, 2000). As a product, representation refer to external form of representation (Goldin, 1998) such as symbols, graphs and diagrams. As a process, it is seen as the internal thinking in the teachers and pupils' mind when working with representations. Representation can then be viewed as a useful means for communicating mathematical ideas. More specifically, pupils demonstrate their ability to connect mathematical ideas when they are able to translate among different representations of the concepts fluently, resulting in deeper and meaningful mathematical understanding (NCTM, 2000). Mathematics communication and connection are important mathematical processes under Singapore's Mathematics framework (Ministry of Education (MOE), 2012). Hence, a study on the use of mathematical representations would also improve mathematics communication and connection.

Although the use of multiple representations is an integral part of mathematics teaching and learning and teachers are also encouraged to integrate a variety of multiple representations into their teaching (Goldin 1998; NCTM 2000), several studies have raised issues on the use of multiple representations in the teaching and learning of mathematics. One issue is that teachers often use representations in isolation (Dreher & Kuntze, 2015; Goldin & Shteingold, 2001; NCTM, 2000). When representations are not connected fluently, mathematics communication will be affected and the lack of representational fluency may hinder deep and meaningful mathematics learning (Goldin 1998; NCTM 2000). In addition, the translation between representations is also often challanging (Pape & Tchoshanov, 2001), especially in topics such as fractions and percentages.

Percentages is an essential topic in the Singapore primary school mathematics syllabus (MOE, 2012). However, many pupils do not have a good understanding of percentages (Zambo, 2008). Moreover, teachers may not have a clear understanding of this topic. For instance, in a study done by Koay (1998), she found that many pre-service teachers in Singapore did not have a good understanding of the percentages topic. Her findings, and others like hers, suggest that the teaching and learning of percentages should be more closely examined. However, there are only a few studies (e.g., See Parker & Leinhardt,

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces *(Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia)* pp. 591-598. Auckland: MERGA.

1995) which focus on the teaching and learning of percentages as most studies focus on fractions and decimals instead. More importantly, there are not many studies which explore the use and interplay of representations in the teaching and learning of percentages, given that the use of representations may play a critical role in teaching the topic. In addition, there is also no recent study on the use of representations in the teaching and learning of percentages. Therefore, this study aims to shed some light on the use and interplay of representations and contribute towards a better understanding of how multiple representations can be used in the teaching of percentages.

A Commognitive Perspective of Learning and Teaching

This paper positions the interaction between the teachers and the pupils when using multiple representations within a participationist view of learning. A participationist perspective of learning reflects a shift from the acquisitionist perspective. According to Sfard (2001), an acquisitionist perspective describes learning as a mental action such as learning new concepts and forming new schemas. Some researchers challenged the acquisitionist perspective which did not consider the social cultural context which learning takes place (Sfard, 2015). On the other hand, participationist perspective views "learning is first and foremost about the development of ways in which an individual participates in well-established communal activities" (Sfard, 2001, p. 10). In other words, participationist perspective focuses on the interaction between the learner and the rest of his community. Participationism is able to complement the acquisitionist perspective in analysing pupils' learning (Sfard, 2001). In addition, an important aspect of examining the use of multiple representations is to investigate how they are used within a social context (Pape & Tchoshanov, 2001). Hence, examining the use of multiple representations using the participationist perspective will provide new insight to current research on the use of multiple representations.

Sfard (2008) introduces the commognitive perspective to analyse mathematical communication and thinking. Commognition, which is formed using the words 'communicating' and 'cognition', stems from a participationist perspective that views thinking as a form of communication. In this section, I will first introduce the key terms from the commognitive framework used in the study, as summarised in Figure 1.

According to Sfard (2008), mathematical discourses are categorised using four characteristics: keywords, visual mediation, narratives and routines. Keywords are important in mathematical discourses because they help to convey meaning to the participants. Visual mediators are visible objects used in the communication such as symbols or iconic representations. Next, narratives involve a set of spoken and written utterances which describes mathematical objects and the relationships among them. The narratives are subject to endorsement, or rejection based on their substantiation procedure. Endorsed narratives, for example, theorems and proofs, are labelled as true. Endorsed narratives are created when there are elaborated realizing procedures between the signifiers and their realisations. Signifiers are words, symbols or other form of representations used in utterances by the participants and its realisations are objects that are operated upon their signifiers to produce narratives. Realisation can be visual or vocal (Spoken Words). Visual realisations may be represented using symbols, concrete objects, icons, gestures or written words. The last characteristic is the use of routines. Routines are sets of metarules that describe repetitive discursive action.

Routines can be further categorized into explorations, deeds and rituals (Sfard, 2008). The goal of the use of explorations is the production of endorsed narratives. Exploration can also be divided into three different types: construction, substantiation and recall (Sfard, 2008). Construction of narratives will result in the construction of new endorsable narratives. Substantiation are actions which determine whether the narratives should be endorsed, and recalling act is the process of recalling previously endorsed narratives. Deeds are defined as a set of rules that produce or change the physical object involved in discourse. Ritual is a routine which primary goal is to create and sustain relationship with others.

For example, in the teaching of addition of unlike fractions, 2/3 + 1/4, the class may be involved in the use of different types of routines. The pupils may need to recall previously endorsed narratives such as definition of like and unlike fractions (Recalling). Instead of only stating the algorithm to be performed, teachers may be involved in substantiation of narratives such as explaining the importance of converting unlike fraction to like fraction (Substantiation). Eventually, the use of recalling act and substantiation will lead to the creation of new endorsed narratives 2/3 + 1/4 = 11/12 (Construction). The use of deed may include the conversion of fractions to their equivalent forms (Deed). Inevitably, ritual such as the use of teachers' questioning will be used during the interaction between the teacher and the pupils (Ritual).

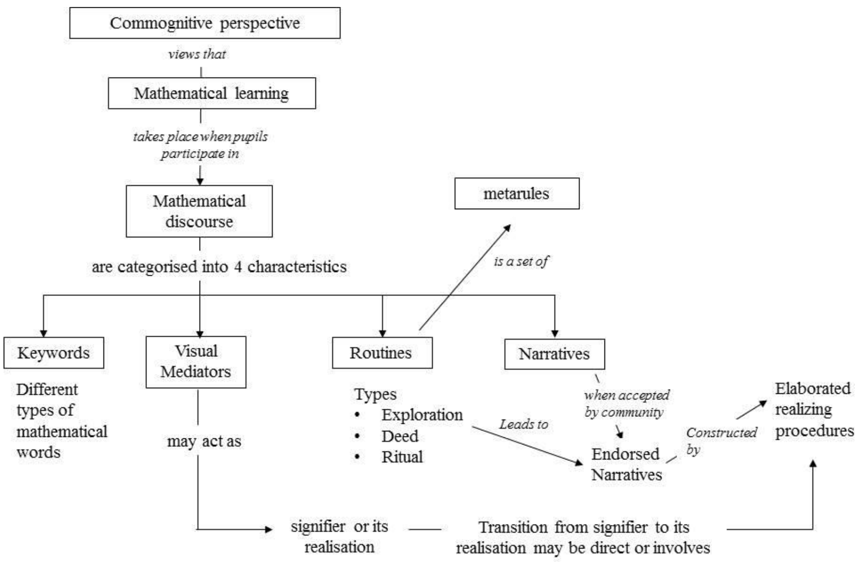

Figure 1. Summary of commognitive terms used in the study.
Adapted with permission from Choy (2015, p. 28)

Method

This study explores how the commognition framework can be used to analyse the transitions among multiple representations in the teaching of Mathematics. The participants of the study included an experienced teacher, Mrs Hannah (pseudonym) and her class of seven pupils at Primary 5 level from a Singapore public primary school. At the time of this study, Mrs Hannah had 12 years of teaching experience in primary school. She received teacher training at the Institute of Education (Singapore) and graduated with a Postgraduate Diploma in Education. During her teaching years, she had taught different profiles of pupils. The seven students in this study were identified based on their results and their behavioural needs at the end of their Primary Four academic year. These pupils have outlier scores (lowest) across all subjects and were grouped to form a small class so that they would be able to receive more attention and assistance from the teacher.

This study consists of four main phases: Pre-data collection, data collection, data condensation and data analysis. During pre-data collection, the necessary ethics clearance were made. Next, for data collection, six consecutive lessons on Mrs Hannah's teaching of percentages were recorded. The average duration of each lesson is 40 minutes. Due to the huge amount of data collected, I went through a process of data condensation. I watched the six videos and identified the relevant teaching moments which may be relevant to the study. I wrote brief comments about the teaching moments (Example: Mrs Hannah connect 1% to 1 building block to 1 base ten cube.) More examples of brief comments can be found in Appendix E in Chia (2017). I categorised the brief comments into five categories. The five categories are connecting different representations, focus on percentage symbols and the use of base 100, choice of example, using pupils' common mistake and using pupils' existing knowledge of decimals and fractions. The recordings of the selected teaching moments for the first two categories were transcribed. I analysed the transcripts from a commognitive perspective and selected an episode from Mrs Hannah's fourth lesson which reflects a rich use of representations to illustrate how Mrs Hannah's use of multiple representation can be analysed with the use of a mediation flowchart. The mediation flowchart is my extension of a figure displaying the different types of signifiers' realisation in mathematical discourse (Sfard, 2008, p. 155). In the next section, I describe a pedagogically significant moment, which happened in the fourth lesson, and illustrate how the mediation flowchart can highlight the interplay between the different representations used by Mrs Hannah.

Results and Discussion

The episode described in this paper is selected from Mrs Hannah's fourth lesson. Prior to the fourth lesson, Mrs Hannah had introduced pupils to associate percentages with 100 squares. Pupils had experience learning using unit blocks and 10 × 10 square grids. She taught pupils the procedure for converting percentages to fractions and vice versa by converting the denominator to 100. For example, 25% = 25/100 =1/4. In the episode, Mrs Hannah began the discourse by revising the conversion from fraction to percentage by changing the denominator of the fraction to 100.

1.	Mrs Hannah:	Question 1, you have 1/25. Remember, let's recall what we have learnt about percentage. What do you know about percentage? Percentage is how many squares?
2.	Josh:	Hundred square
3.	Mrs Hannah:	Thank you. Josh. We learnt that percentage is equal to 100 squares. In your mind, you should picture these 100 squares. Out of 100, how many squares must you colour? So that is percentage. So 1 out of 25, can I make it into 100?
4.	Josh:	Yes, times 4
5.	Mrs Hannah:	Woah, Josh is so fast. Very good. Do you just multiply by 4 this way? [*Mrs Hannah wrote 1/25 × 4.*]
6.	Kate:	No
7.	Mrs Hannah:	What should I do? Thank you Kate. She says you must multiply the factor 4 to both the numerator and denominator. So that's one method going about doing it. You get 4/100. [*Mrs Hannah completes the working 1/25 = 4/100 as she talks.*] Ruth do you think you can help us along to change this to percentage, or perhaps you change it to decimal first? This is something which we do last week. If you can remember. Or anyone? Ruth looks so nervous. Is there anyone else who can help

		her?
8.	Mitch:	4%
		[*Mrs Hannah completes the working 1/25 = 4/100 = 4% as she talks.*]
9.	Mrs Hannah:	How do you get 4%? In your mind, how do you read this?
10.	Mitch:	4 out of 100
11.	Mrs Hannah:	[*Mrs Hannah circled 4/100 and extended an arrow out of the circle and wrote 4 out of 100.*] That's right. You must be able to read this as 4 out of 100. 4 squares out of 100 squares. 4 squares out of 100 squares will be 4 percent. Remember what I say about percentage. Percentage is about 100 squares. So it is 4 out of 100 which is 4%. Very good, Mitch.

As can be seen from the transcript, Mrs Hannah first elicited responses from the pupils that percentage is associated with 100 squares. Next, Mrs Hannah explained that 100 squares can be represented by the denominator 100, which the pupils had learnt previously. She highlighted to the pupils that they should convert the denominator to 100 when converting fractions to percentages. As seen from the above example of 1/25, the class first converted 1/25 to 4/100. Next, by replacing the denominator '/100' with the '%' sign, the class converted 4/100 to 4%. I will now provide a fine-grained analysis of how Mrs Hannah mediated the use of different representations through the lens of commognition—keywords, visual realisations, endorsed narratives, and routines.

Mediation using keywords. Keywords used in this segment can be categorised into three different categories: 'mathematical terms (percentage)', 'everyday words' and 'other mathematical terms' (Shuard & Rothery, 1984). Examples of 'mathematical terms (percentage)' are 'percentage' and 'out of 25'. 'Everyday words' are example such as 'equal to' and 'other mathematical terms' refers to words like 'numerator' and 'denominator'. In this segment, keywords, in both spoken and written forms are used to mediate between symbolic representations and algebraic representations. In turn 3, the use of mathematical terms such as percentage, 100 squares and out of 100, are used to mediate between symbolic representations, '1/25' and its iconic representation which is 10 × 10 square grids. In the case of 1/25, pupils may not be able to visualise the fraction as 100 squares directly. Using everyday words, pupils would realise that they need to 'make it' into 100 squares. After converting 1/25 to 4/100, similarly, the use of spoken and written forms of '4 out of 100' would be used to mediate between the two symbolic representations of 1/25, 4/100 and 4%. Figure 2. on the next page provides a visual flow chart of the direction of mediation in the episode.

Visual realisations of 1/25. The flowchart shows the different realisations of 1/25 in the form of concrete objects, iconic representations, spoken and written words and algebraic symbols. Base 10 blocks and 10 × 10 square grids were used in the previous lessons. Hence, pupils may make reference to these representations to make meaningful connections to the new representations used in this lesson. In this segment, the use of the phrase '____ out of ____' is frequently used by Mrs Hannah both in written and spoken form. Algebraic symbols includes 4/100, an equivalent fraction of 1/25, and 4%. The sequence of the appearance of the different realisations is presented from top to bottom with the full arrows showing the direction of mediation. These arrows also connect the signifier-realisation pairs through the mediation process which was mainly through the use of written and spoken words. The numbers and directions of the arrows reflect that the realising procedure is a non-straightforward, complicated one. There are mainly four signifier-realisation pairs as summarised in the Figure 3 below.

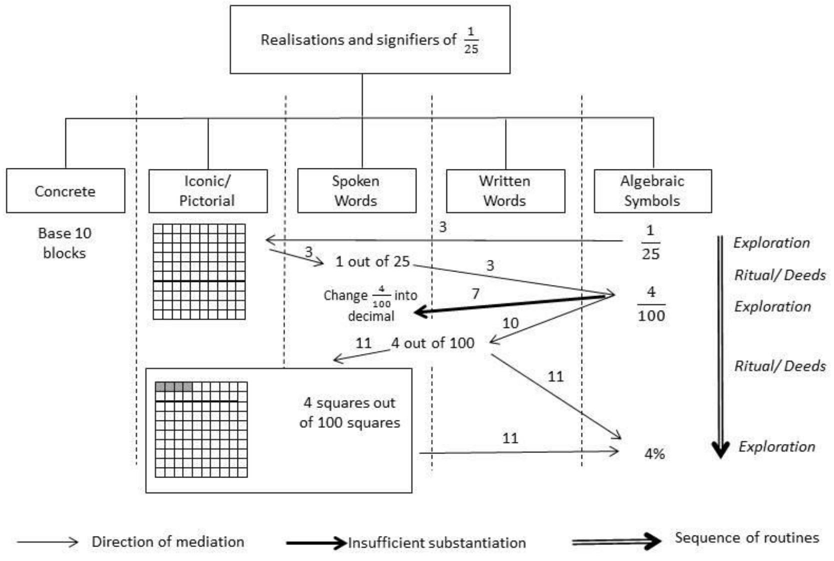

Figure 2. Mediation of algebraic-symbolic and iconic representations using written and spoken words in the episode.

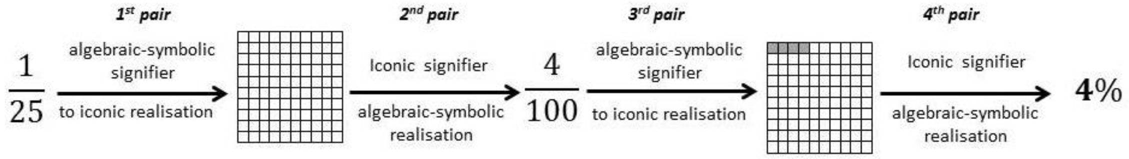

Figure 3. Signifier-realisation pairs in this episode.

Endorsed narratives. The realising procedures which translate the signifiers to their realisations lead to the creation of endorsed narratives. The first two signifier-realisation pairs lead to the endorsed narratives, 1/25 = 4/100. Using the third signifier-realisation pair, the class was able to conclude that 4/100 can be expressed using the iconic representation of 100 squares with only four squares being shaded. Lastly, in the fourth signifier-realisation pairs, we can equate 4 squares out of 100 squares to 4%. Combining the endorsed narratives found in this segment, we can express the realisation as 1/25 = 4/100 = 4%. Mathematical communication is fluent when there is coherence between the use of the keywords and narratives by the participants. In this segment, the interplay of keywords and narrative suggests that the realisation of the signifiers in the four signifiers-realisation pairs are examples of fluent communication which lead to the production of endorsed narratives. However, in turn 7, teacher asked the pupils to convert 4/100 into decimal. There is an absent of realisation of 4/100 in its decimal form in the lesson as reflected in Figure 2. This is a non-example of fluent mathematical communication.

Types of routines. The endorsed narratives, 1/25 = 4/100 = 4% is the product of the interchange among the act of different routines – explorations, deeds and rituals. Although explorations are the only types of routines which lead to endorsed narrative, the use of deeds and rituals are also important in developing act of explorations (Sfard, 2008). In this segment, Mrs Hannah had used different types of routines to improve the fluency when connecting the different signifiers and realisations of 1/25 (Goldin & Shteingold, 2001; NCTM 2000). Table 1 provides a summary of the sequence and explanation of the type of routines used by Mrs Hannah in Segment 1.

Table 1
Summary of Types of Routines Used in Segment 1

Turn	Types of routines	Task
1-3	Exploring through Recalling	Pupils recalled that percentage is associated with 100 squares.
3	Deed	Act of colouring squares in 10 × 10 square grids to represent 1/25.
3-4	Ritual	Pupils responded to Mrs Hannah by explaining how they converted 1/25 to 4/100.
7	Deed	Mrs Hannah explained the conversion of 1/25 to 4/100.
7	Exploration through construction	Mrs Hannah explained the endorsed narratives 1/25 =4/100.
9	Ritual	Mrs Hannah provided scaffolding by asking them to read 4% as 4 out of 100.
11	Exploration through substantiation	Mrs Hannah explained the endorsed narratives 4/100 = 4%.

Mrs Hannah began the lesson using endorsed narratives from the previous lessons that associate percentages with 100 squares. The pupils were involved in exploration through recalling that percentages is associated with 100 squares. After that, the pupils carried out the deed of picturing the number of coloured squares to represent 1/25. Through the use of ritual, Mrs Hannah also prompted the pupils to convert 1/25 into denominator 100 and carried out the deed of multiplying both the numerator and denominator by 4 to convert 1/25 into denominator 100. The use of deeds and rituals had led to the extension of the previous endorsed narratives that percentage is associated with 100 squares and led to exploration through construction that 1/25= 4/100. Using the new endorsed narrative, 1/25=4/100, Mrs Hannah continued to teach her pupils to convert 1/25 into percentages. As Mitch had answered 4% in turn 8, Mrs Hannah substantiated Mitch's constructed narratives through the use of ritual. She questioned Mitch how he had read 4/100. After Mitch replied '4 out of 100', she substantiated his constructed narratives by explaining that 4 out of 100 is the same as 4 squares out of 100 squares which is 4%. From Segment 1, Mrs Hannah had used rituals and deeds to lead to exploration which produces endorsed narratives. The various modes of routines used is also an evident of Mrs Hannah's numeracy fluency (Sfard, 2008; Thomas, 2008).

There are several key findings from the analysis of Mrs Hannah's discourse in this episode. First, key words can be used to mediate between different representations. In turn 3, Mrs Hannah used mathematical terms and everyday words to connect different representations of 1/25. When the realising procedure that translates a signifier to its realisation is elaborated, the translation between the representations will be fluent. This can be seen in the creation of the four signifier-realisation pairs in the episode (See Figure 2). In turn 7, the absence of an elaborated realising procedure between 4/100 and its decimal form provides an example of a non-fluent transition between representations. As demonstrated in this episode, the use of different types of routines in a mathematical discourse helps to improve the fluency in the translation of different representations. Lastly, the use of a mediation flowchart serves as a tool to make the representations visible for analysis to take place. In particular, the use of arrows in the flowchart helps to identify and connect signifier-realisation pairs. Any missing or incomplete realising procedures are represented using bolded arrows. Two more episodes of analysis can be found in Chia (2017).

Concluding Remarks

Notwithstanding the limitations of a single case study, this study has demonstrated how classroom discourse can be analysed from a commognitive perspective. The use of a commognitive perspective increases teachers' awareness when using multiple representations. The coherence among the different characteristics of mathematics discourse affects the fluent use of representations. With the aim of improving teaching and learning, both researchers and teachers can better reflect on their use of representations and language during teaching by making their use of multiple representations more visible using the mediation flowchart. Through visual representations of teachers' thinking, teachers can identify gaps in their use of multiple representations and suggest alternative teaching strategies. Although it remains to be seen whether such commognitive analysis can lead to teaching and learning, this study has shed some important insights into the complexity of mathematical communication through multiple representation.

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