Investigating Mathematics Students' Motivational Beliefs and Perceptions: An Exploratory Study

Claudia Orellana
RMIT University, Australia
<claudia.orellana@rmit.edu.au>

Tasos Barkatsas
RMIT University, Australia
<tasos.barkatsas@rmit.edu.au>

The purpose of this study was to explore the factorial structure of motivation and perception items from a student survey utilised as part of the Reframing Mathematical Futures II (RMFII) Project. Data was collected in 2017 from 442 students in Years 7 to 10 from various different States across Australia. An exploratory factor analysis identified four factors which were consistent with the studies the items were adapted from: Intrinsic and Cognitive Value of Mathematics, Instrumental Value of Mathematics, Mathematics Effort, and Social Impact of School Mathematics. An analysis of variance (ANOVA) also revealed that there were statistically significant differences between Year Level and State for some of these factors.

For many years, researchers in the field of mathematics education have acknowledged the significant role that affective factors play in the teaching and learning of mathematics (Goldin 2002). Although defined in many different ways, the affective research area within this field focuses on "the interplay between cognitive and emotional aspects in mathematics education" (Di Martino & Zan, 2010, p. 1). Based on McLeod's (1992) work, the affective domain is also seen as composed of three major constructs - *beliefs, attitudes,* and *emotions* – with each representing "increased levels of affective involvement, decreased levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (p. 579).

While there are many interpretations for each construct in the literature, they are often considered difficult to define, particularly due to their overlapping nature (Di Martino & Zan, 2010). For example, early definitions of attitudes by Neale (1969) and Hart (1989) embedded beliefs about mathematics as a key element of this construct along with its usefulness to the learner. Hart (1989) also considered "one's emotional reaction to mathematics" (p. 39) in his definition of attitudes. However, in alignment with McLeod's (1992) initial interpretation, Goldin (2002) conceptualised each construct as follows:

> (1) emotions (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context), (2) attitudes (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition), (3) beliefs (internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured). (p. 61)

While the three constructs of beliefs, attitudes, and emotions have been widely studied within the mathematics domain, they do not cover the entire field of affective research (Zan, Brown, Evans, & Hannula, 2006). *Motivation* is another construct which has had significant implications for student achievement in mathematics although it has not been a prominent field of study within this context (Hannula, 2006; Middleton & Spanias, 1999). As with its other affective counterparts, motivation has also been defined in various different ways in the literature. While Middleton and Spanias (1999) proposed that "motivations are reasons individuals have for behaving in a given manner in a given situation" (p. 66), different theoretical perspectives have varying interpretations for how

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces *(Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia)* pp. 615-622. Auckland: MERGA.

these reasons may arise. For example, the behaviourist perspective sees motivation as resulting from external incentives, such as for rewards or to avoid punishment, whereas the social cognitivist view sees motivation as resulting from a sense of self and self-efficacy (Churchill et al., 2013). Each perspective has been referred to in the literature as extrinsic and intrinsic motivation respectively with the latter seen as valuable in promoting pedagogically desirable behaviours in mathematics such as persistence and risk taking (Middleton & Spanias, 1999).

Regardless of the challenges faced in defining the aforementioned affective variables, there are a number of instruments designed to measure constructs such as attitude and motivation in mathematics. However, as with their definitions, some constructs are measured as part of others. For example, Tapia and Marsh (2004) developed an instrument to explore the construct of attitudes called the Attitudes Towards Mathematics Inventory. They conceptualised attitudes as having four underlying dimensions, one of which was motivation. Additionally, the Fennema-Sherman Mathematics Attitudes Scales, one of the most popular instruments in mathematics education, also views motivation as a sub-set of attitudes, with 12 items on this construct forming one of the nine scales (Fennema-Sherman, 1976). As can be seen from the aforementioned instruments, development of a scale measuring an affective construct is no easy task. Although referring to attitudes, Taylor (1992) makes an important point in that the formation of a construct "is a complex process involving the interaction of many factors. It cannot be explained simply or completely" (p. 12).

With this in mind, the research presented in this paper will examine the motivations and perceptions items from a student survey which was utilised as part of a larger project, and will outline key findings with respect to the variables explored.

Aims

The aims of the study were to investigate:

The factorial structure of the motivations and perceptions items

The existence of statistically significant differences between the derived factors and the independent variables Year Level and State.

Methods

Data Source and Sample

An online survey was undertaken as part of the Reframing Mathematical Futures (RMFII) Project, which aims to find ways to improve the teaching and learning of mathematics for students in Year 7 to 10. The purpose of the survey was to examine students' views regarding their learning experiences in mathematics. The participants came from Australian State and Catholic schools involved in the RMFII project across various Australian States. A total of 442 Year 7 to 10 students from eleven schools across Victoria, New South Wales, Queensland, Northern Territory, South Australia, and Tasmania responded to the survey.

Instrument

The survey consisted of 95 items and was designed by adapting items from instruments developed in prior studies (Dweck, Chiu, & Hong, 1995; Frenzel, Goetz, Pekrun, & Watt, 2010; PISA, 2006; Watt 2004; 2010; Wyn, Turnbull, & Grimshaw, 2014; You, Ritchey, Furlong, Shochet, & Boman, 2011). The survey examined the following constructs: *Mathematics Learning Climate, Friends Perceptions of Mathematics, Perceptions of NAPLAN, Homework, Mathematics Motivations and Perceptions, Gender Perceptions of Mathematics, Personal Goals in Mathematics, Mindset, Perceptions of School, Perceptions of Mathematics Teaching, and Mathematics Career.*

For the purposes of this paper only the 2017 Mathematics Motivations and Perceptions item responses will be examined. There are a total of 21 items adapted from Watt (2004; 2010) and PISA (2006) examining factors that influence students' perceptions of mathematics and their beliefs about themselves as mathematics learners.

Data Collection

A link to the online survey was provided to participating students by their teachers from February 2017 and it was completed either in the students' own time at home or during class time. The survey was anonymous and students and their respective parents were made aware of the purpose of the survey.

Results

An initial data screening was carried out to test for univariate normality, multivariate outliers (Mahalanobis' distance criterion), homogeneity of variance-covariance matrices (using Box's M tests), and multicollinearity and singularity (tested in the ANOVA analysis). Descriptive statistics normality tests (normal probability plot, detrended normal plot, Kolmogorov-Smirnov statistic with a Lilliefors significance level, Shapiro-Wilks statistic, skewness and kurtosis) showed that assumptions of univariate normality were not violated. Mahalanobis' distance was calculated and a new variable was added to the data file. There were fewer than twenty outlying cases, which is acceptable in a sample of 442 students. These outliers were therefore retained in the data set. Box's M Test of homogeneity of the variance-covariance matrices was not significant at the 0.001 alpha level and we therefore concluded that we have homogeneity of variance. The questionnaire items were subjected to an Exploratory Factor Analysis (EFA) by using SPSSwin. Reliability tests were also conducted. An Analysis of Variance (ANOVA) statistical test was used to investigate statistically significant differences by Year Level and by State.

Exploratory Factor Analysis (EFA)

Given the exploratory nature of the study and that the structure could vary, three factor analyses – one for each of the possible combinations between the three Year Levels (7, 8, and 9) categories (Year 10 was not used because of the relatively small number of students in that category) with sufficient student numbers - were performed in order to investigate possible differences between Year Levels. Since no differences were observed in the three initial analyses, a final factor analysis using data from 438 complete students' responses to the 21 items forming the questionnaire, indicates that the data satisfy the underlying assumptions of the factor analysis and that together four factors (each with eigenvalues greater than 1) explain 72.4% of the variance, with 44.5% attributed to the first factor – *Intrinsic and Cognitive Value of Mathematics* (see Table 1).

Further, according to Coakes and Steed (1999), if the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy is greater than 0.6 and the Bartlett's test of sphericity (BTS) is significant then factorability of the correlation matrix is assumed. A matrix that is factorable should include several sizable correlations. For this reason (Tabachnick &

Fidell, 1996) it is helpful to examine matrices for partial correlations where pairwise correlations are adjusted for effects of all other variables. The Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy in this study is greater is 0.92 and the Bartlett's test of sphericity (BTS) is significant at 0.001 level, so factorability of the correlation matrix has been assumed.

Reliability analysis yield satisfactory Cronbach's alpha values for each factor: Factor 1, 0.93; Factor 2, 0.90; Factor 3, 0.85 and Factor, 0.80. This indicates a strong degree of internal consistency in each factor.

Table 1
Rotated Factor Matrix (Varimax Rotation)

	Factor			
	1	2	3	4
Factor 1: Intrinsic and Cognitive Value of Mathematics				
Q46 I find mathematics enjoyable	.866			
Q45 I find mathematics interesting	.856			
Q48 I would like to find out more about some of the things we deal with in our mathematics class	.820			
Q51 Being good at mathematics is an important part of who I am	.807			
Q44 I like mathematics more than other subjects	.806			
Q49 I want to know all about mathematics	.803			
Q47 After a mathematics class, I look forward to what we are going to do in the next lesson	.784			
Q52 It is important for me to be someone who is good at solving mathematics problems	.738			
Q50 Being someone who is good at mathematics is important to me	.709			
Factor 2: Instrumental Value of Mathematics				
Q33 What I learn in mathematics is important for me because I need this for what I want to study later on		.854		
Q35 Studying mathematics is worthwhile for me because what I learn will improve my career prospects		.849		
Q32 Making an effort in mathematics is worth it because this will help me with what I want to do		.835		
Q34 I study mathematics because I know it is useful for me		.795		
Q36 I will learn many things in mathematics that will help me get a job		.787		
Factor 3: Mathematics Effort				
Q40 It worries me that mathematics courses are harder than other courses			.856	
Q41 I am concerned that I won't be able to handle the stress that goes along with studying mathematics			.827	

Q39 Achieving in mathematics sounds like it really requires more effort than I'm willing to put in	.735	
Q37 When I think about the hard work needed to get through in mathematics, I am not sure that it is going to be worth it in the end	.696	
Q38 Considering what I want to do with my life, studying mathematics is just not worth the effort	.535	
Factor 4: Social Impact of School Mathematics		
Q42 I'm concerned that working hard in mathematics classes might mean I lose some of my close friends		.903
Q43 I worry about losing some valuable friendships if I'm studying mathematics and my friends are not		.899

The naming of the *four factors* was guided by the relevant literature and the nature of the questionnaire items associated with each factor. This resulted in the following four factors (F1-F4) described below:

F1: Intrinsic and Cognitive Value of Mathematics. The first component consists of nine items, which examine the intrinsic and cognitive value of mathematics. Three of these items were adapted from Watt (2004) and examine students' intrinsic value of mathematics (i.e., how likeable or enjoyable students find the subject). Three items were adapted from Watt's (2010) STEP study which examine the attainment value of mathematics (i.e., how important it is to do well in mathematics). The final three items were adapted from Frenzel, Goetz, Pekrun, and Watt (2010) and examine students' interest in mathematics. All nine items explore mathematical value in terms of personal enjoyment, importance, or interest hence the construct has been labelled to encompass these factors (intrinsic and cognitive value).

F2: Instrumental Value of Mathematics. The second component consists of five items that examine the instrumental value of mathematics (i.e., that the learning of mathematics is valuable for students' futures). The five items have all been adapted from the PISA (2006) questionnaire and specifically examined students' instrumental motivation to learn science subject(s) – the term science subject(s) was replaced with mathematics. These items were the only items taken from the PISA (2006) questionnaire and have loaded to develop a construct consistent with the original study.

F3: Mathematics Effort. The third component consists of five items and examines students' perceptions of the effort required in mathematics. The items were adapted from Watt's (2010) STEP study and examine the "costs" associated with mathematics. Three items examine the Effort Costs and two items examine the Psychological Costs associated with mathematics. As the latter two items can be related to the greater effort expended in mathematics (harder and more stressful) the five items have been grouped together under the overall construct of Mathematics Effort.

F4: Social Impact of School Mathematics. The fourth component consists of two items examining the social impact of school mathematics. These items were adapted from the instrument used in Watt's (2010) STEP study, which specifically examined the Social Cost perceived by students as a result of studying or working hard in mathematics. Consistent with this study, the two items have loaded to form the construct labelled here.

Analysis of Variance (ANOVA) by Year Level

The existence of statistically significant differences on each of the four derived factors by Year Level was investigated by conducting an Analysis of Variance ANOVA statistical test. The dependent variables (DVs) were the four factors derived from the EFA and the independent variables (IVs) were Year Level (Levels 7-9) and State. Year 10 students' responses have not been used in this analysis because of the relatively small number of students. Effect sizes were calculated using eta squared ($\eta 2$). In our interpretation of effect sizes we have been guided by Cohen, Manion and Morrison's (2018) proposal that 0.1 represents a small effect size, 0.3 represents a medium effect size, and 0.5 represents a large effect size. We have significant univariate main effects for the following variables:

Factor 3: Mathematics Effort [$F(2, 392) = 6.85$, $p < 0.001$, $\eta 2 = 0.1$]. Effect sizes were calculated using eta squared ($\eta 2$). The effect size was 0.1 (small effect). A Games-Howell post hoc multiple comparisons test was performed. The purpose of the post hoc tests is to determine which Year Levels are statistical significant different from each other. The Games-Howell test has been used because the Year Level sizes differ. It was found that Year 8 and Year 9 students' scores had significantly different mean values ($p < 0.001$) for Factor 3: Mathematics Effort. It was also found that Grade 7 and Grade 9 students' scores were statistically significantly different ($p < 0.001$) for Factor 3. The mean scores indicate that Year 9 students had a higher mean than Year 7 and Year 8 students. Also, Year 8 students had a marginally higher mean than Year 7 students.

ANOVA by State

The existence of statistically significant differences on each of the four derived factors by State was investigated by conducting an Analysis of Variance (ANOVA) statistical test. The dependent variables (DVs) were the four factors derived from the EFA and the independent variable (IVs) *State.* We have significant univariate main effects for the following variables:

Factor 1: Intrinsic and Cognitive Value of Mathematics [$F(5, 392) = 4.99$, $p < 0.001$, $\eta 2 = .06$]. Effect sizes were calculated using eta squared (η^2). The effect size was .06 (small effect). A Games-Howell post hoc multiple comparisons test was performed in order to explore the differences for each factor. It was found that the New South Wales and the Queensland students' scores had significantly different mean values ($p < 0.01$) for Factor 1. Also, Queensland students had a higher mean than New South Wales students.

Factor 2: Instrumental Value of Mathematics [$F(5, 392) = 4.69$, $p < 0.001$, $\eta 2 = .02$]. Effect sizes were calculated using eta squared (η^2). The effect size was .02 (small effect). A Games-Howell post hoc multiple comparisons test was performed in order to explore the differences for each factor. It was found that the New South Wales and the Queensland students' scores had significantly different mean values ($p < 0.01$) for Factor 2. Also, Queensland students had a higher mean than NSW students.

Factor 3: Mathematics Effort [$F(5, 392) = 2.38$, $p < 0.05$, $\eta 2 = .01$]. Effect sizes were calculated using eta squared (η^2). The effect size was .01 (small effect). A Games-Howell post hoc multiple comparisons test was performed in order to explore the differences for each factor. It was found that the New South Wales and the Queensland students' scores had significantly different mean values ($p < 0.01$) for Factor 3. Also, New South Wales students had a higher mean than Queensland students.

Discussion and Conclusion

Examination of the survey items using an exploratory factor analysis identified four factors, each with eigenvalues > 1 that together explained 74.6% of the variance. The 21 items analysed within this paper were also found to load on factors consistent with those of the studies they were sourced from (Frenzel et al., 2010; PISA, 2006; Watt 2004; 2010). Each factor explored a different aspect of students' motivations and perceptions regarding mathematics and their beliefs about themselves as mathematics learners. The factors were labelled *Intrinsic and Cognitive Value of Mathematics, Instrumental Value of Mathematics, Mathematics Effort,* and *Social Impact of School Mathematics.*

In addition to exploring the factorial structure of the survey, this study also aimed to examine if there were any statistically significant differences between Year Levels and States on the identified factors. Using an Analysis of Variance (ANOVA), the results revealed that there were statistically significant differences for the factor *Mathematics Effort* between Year Levels, and for the factors, *Intrinsic and Cognitive Value of Mathematics, Instrumental Value of Mathematics,* and *Mathematics Effort* between States.

Further examination using post hoc tests for the Year Level variable showed that Year 9 students had significantly higher mean scores for *Mathematics Effort* when compared to Year 8 students and Year 7 students. These findings are not surprising considering that mathematics becomes more complex as students move into higher year levels and students' may perceive that studying mathematics requires more effort as a result. Comparing the Australian Curriculum Year 9 mathematics content descriptors with those of Year 8 and Year 7, there are many new concepts learned at this higher year level that have not been previously introduced in the prior years (e.g., trigonometry, Pythagoras theorem, non-linear relations), whereas Year 8 students build upon and explore similar concepts to students in Year 7 (ACARA, 2010 to present). The results are also consistent with an Australian study conducted by Watt (2004) who found that, from the end of Grade 7 through to Grade 10, students perceived mathematics as requiring slightly more effort.

Post hoc tests for the State variable showed that the significant differences for *Intrinsic and Cognitive Value of Mathematics*, *Instrumental Value of Mathematics*, and *Mathematics Effort* were between students from Queensland and students from New South Wales. Students from Queensland scored significantly higher mean values for the first two factors compared to their New South Wales counterparts, but scored significantly lower mean values for *Mathematics Effort*. Thus, Queensland students see mathematics as more interesting and enjoyable, useful for their future careers, and requiring less effort than New South Wales students. Although Yates (2011) commented that different Australian States have approached the curriculum differently depending on what is valued, it is difficult to explain the results between Queensland and New South Wales based on this alone. There may be many other contextual factors than can play a role in developing students' perceptions and beliefs regarding mathematics. For example, one key finding from a review by Middleton and Spanias (1999) highlighted that "motivations towards mathematics are developed early . . . and are influenced greatly by teacher actions and attitudes" (p. 80). Fredricks and Eccles (2002) also suggested that decreases in mathematics task values over time in their study could be explained by the increased competitiveness and evaluation methods used in classrooms as students progress into higher year levels.

In summary, the results from the study have confirmed that the survey items continue to be valid and reliable in the mathematics context as the factors developed were consistent with the studies they were adapted from. The findings also highlight the need for further investigations to examine how students' motivations and perceptions of mathematics develop and differ across the different States in Australia. Having a more representative sample of students from each state across a variety of different Year Levels could provide greater insights into how students' perceptions of mathematics change in different contexts over time.

References

ACARA (2010 to present). *Mathematics*. Retrieved from https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

Churchill, R., Ferguson, P., Godinho, S., Johnson, N. F., Keddie, A., & Letts, W. (2013). *Teaching: Making a difference* (2nd ed.). Milton, QLD: Wiley.

Coakes, S. J., & Steed, L. G. (1999). SPSS: *Analysis without anguish*. Australia: John Wiley and Sons Ltd.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education.* NY: Routledge.

Di Martino, P., & Zan, R. (2010). 'Me and maths': towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education, 13 (1),* 27-48.

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: instruments designed to measure attitudes towards the learning of mathematics by females and males. *Journal for Research in Mathematics Education, 7(5),* 324-326.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: Growth trajectories in two male-sex-typed domains. *Developmental Psychology, 38(4),* 519-533.

Frenzel, A. C., Goetz, T., Pekrun, R., & Watt, H. M. G. (2010). Development of mathematics interest in adolescence: Influences of gender, family and school context. *Journal of Research on Adolescence, 20*(2), 507-537.

Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 59-72). Netherlands: Kluwer Academic Publishers.

Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics, 63(2),* 165-178.

Hart, L. E. (1989). Describing the affective domain: Saying what we mean. In D. B. McLeod, & V. M. Adams (Eds.), *Affect and mathematics problem solving: A new perspective* (pp. 37- 45). New York: Springer-Verlag.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws, *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575-596). New York: Macmillan Publishing Company.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education, 30(1),* 65-88.

Neale, D. C. (1969). The role of attitudes in learning mathematics. *The Arithmetic Teacher, 16(8),* 631-640.

PISA (2006). *Student questionnaire for PISA 2006.* Retrieved from http://www.oecd.org/pisa/pisaproducts/database-pisa2006.htm

Tabachnick, B. G. & Fidell, L. S. (1996). *Using multivariate statistics* (3rd ed.). New York: Harper Collins.

Tapia, M., & Marsh, G. E. (2004). An instrument to measure mathematics attitudes. *Academic Exchange Quarterly, 8(2),* 16-21.

Watt, H. M. G. (2004). Development of adolescents' self-perceptions, values, and task perceptions according to gender and domain in 7th- through 11th- grade Australian students. *Child Development, 75(5),* 1556-1574.

Watt, H. M. G. (2010). *STEPS: Study of transitions and education pathway.* Retrieved from http://www.stepsstudy.org/

Yates, L. (2011) Curriculum as a public policy enterprise: Australian state differences and the past forty years of curriculum reforms. [contribution to symposium on: Curriculum policy: cases of translation], BERA Annual Conference, London.

Zan, R., Brown, L, Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63(2),* 113-121.