Strategy Use in Mathematics Assessment: Does Spatial Reasoning Matter?

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Mathematics and spatial reasoning are inextricably linked. While this is increasingly recognised, a clearer understanding of how spatial reasoning improves your mathematics performance is needed. We sought to explore the role of spatial reasoning strategy use across a range of mathematics assessment tasks with Grade 7 and 9 students. Interviews were conducted and data from students with varying levels of spatial ability was explored to determine if spatial ability had an observable effect on strategy use and success.

Introduction

The link between spatial reasoning and mathematics is increasingly being acknowledged. As such, the spatial-mathematics link may play a key part in ensuring the success of not only an Australian, but a global push toward greater participation in Science, Technology, Engineering, and Mathematics (STEM), particularly for women (Lowrie & Jorgensen, 2017; Wai, Lubinski, & Benbow, 2009). This is partly due to evidence that spatial reasoning can be trained, and that such training can improve mathematics performance (Uttal et al., 2013). While studies investigating the link between spatial reasoning and mathematics performance have in some cases had remarkable results, few studies have attempted, and fewer still succeeded, to clearly identify the direct effects of spatial reasoning and spatial training on how children solve mathematics tasks.

Recently, Verdine, Golinkoff, Hirsh-Pasek and Newcombe (2017) found evidence for links between spatial thinking and mathematics in the early years, highlighting the importance of early spatial experiences and instruction. However, their study did not identify specific skills or processing. Uttal et al. (2013) conducted a comprehensive meta-analysis of spatial training studies. While providing a plethora of evidence for the malleability of spatial skills, Uttal et al. (2013) urged caution when interpreting findings concerning studies documenting transfer of training, suggesting that transfer is possible, but requires a considerable amount of training and experience. This raises a crucial point as evidence of mechanical underpinnings related to the spatial-math link is scarce. This is despite the long-standing suggestion that this link may be observable by examining strategy use on mathematics assessment (Lowrie, Diezmann & Logan, 2011; Lowrie, Logan & Ramful, 2016).

Strategy use has long been an important area of mathematics education research. This has also been true for researchers examining the link between spatial thinking and mathematics performance. Hegarty and Kozhevnikov (1999) identified two types of visuo-spatial representations used in mathematics problem solving, 1) schematic representations focusing on spatial relations described in a problem, and 2) pictorial representations that involve encoding the visual characteristics of described objects. Thirty-three grade 6 students were then tasked with completing mathematics and spatial tests, and then interviewed to identify self-reported strategy use. Hegarty and Kozhevnikov (1999) found that use of schematic representations was linked with higher spatial visualisation ability, providing practical evidence of the link between spatial thinking and mathematics performance on tasks encouraging the use of visual representations. Van Garderen (2006)

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 276-283. Perth: MERGA.

investigated the use of visual representations on mathematical word problems in a grade 6 student sample consisting of students with learning disabilities, average achievers, and high achievers. High achievers were found to use significantly more visual images than participants with learning disabilities. As found by Hegarty and Kozhevnikov (1999), use of schematic representations was positively correlated with performance on spatial visualisation measures, while use of pictorial representations was negatively correlated with the same measures. However, few correlations were statistically significant when broken down by assessment and participant condition, raising questions about the generalizability of findings. Diezmann and Lowrie (2012) examined the performance of grade 6 students on numeracy items involving graphics. Students were found to lack spatial skills required to succeed on mathematics tasks involving graphics, raising concerns in light of the increasing use of graphics in mathematics assessments (Diezmann & Lowrie, 2012). Authors concluded that embedding spatial skill development into the mathematics curriculum and introducing explicit instruction of spatial thinking will not just be beneficial, but essential to future success in mathematics learning, teaching, and education.

The Study

The link between spatial thinking and mathematics has the potential to inform mathematics instruction (Lowrie & Jorgensen, 2017; Lowrie, Logan, & Ramful, 2016). However, it is still unclear how spatial thinking influences performance on mathematics tasks in a practical sense. While it has been shown that children with greater spatial visualisation skills are more likely to use schematic representations on tasks requiring the use of visual imagery as a primary strategy (Hegarty & Kozhevnikov, 1999; Kozhevnikov, Kosslyn, & Shephard, 2005), the effects of spatial thinking on strategy use across mathematics problems with different task demands has not been truly investigated. We sought to explore the relationship between spatial thinking and strategy use on mathematics problems in a sample of students drawn from grades 7 and 9. Students from four secondary schools located in the ACT, Australia, completed spatial and mathematics tests. A subset of students completed in-depth follow-up interviews during which strategy use on various mathematics assessment items was discussed. We hypothesized that students who differed in their performance on spatial thinking measures would differ in their reported strategy use.

Research Design and Methods

Participants

As part of a larger study involving 136 grade 7 and 9 students (63 girls, 73 boys) (*M* age = 13.3, *SD* = 1.1) recruited from two secondary schools located in the ACT, sixteen students agreed to take part in semi-structured interviews designed to elicit explanations of students' thinking and strategy used on mathematics test items.

Materials

Mathematics Assessment Instrument

The mathematics assessment was developed as part of a previous investigation into the effects of a spatial training program on the mathematics performance of primary and secondary school students (Lowrie, Logan, Harris & Hegarty, 2018). The assessment consisted of 17 items reflecting content and typical presentation of Australian standardized mathematics assessment items (ACARA, 2017). The assessment was developed for tablet devices and computers, with some items designed to include interactive elements not possible on traditional pen-and-paper assessments.

Spatial Reasoning

Participants completed three spatial reasoning measures covering mental rotation, spatial orientation, and spatial visualisation. The Card Rotation Test (CRT; Ekstrom et al., 1976) was used to capture participants' mental rotation ability. Participants were presented with one reference shape and eight comparison shapes and instructed to select only the comparison shapes that were the same (i.e., rotated) as the reference shape. On the Spatial Orientation test (Kozhevnikov & Hegarty, 2001) participants were asked to imagine being at the position of one object in the display facing another object (defining their imagined perspective within the array) and required to indicate the direction to a third (target) object. The 10-item Paper Folding Test (PFT; Ekstrom et al., 1976) was used to capture students' spatial visualisation ability. Participants were presented with directions indicating how a piece of paper was folded before a hole was punched through it, and then tasked with choosing one of several options representing what the piece of paper may look like when unfolded. From these, an overall spatial reasoning score was derived.

Semi-structured Interviews

Sixteen students from grades 7 and 9 took part in semi-structured interviews designed to elicit explanations of students' thinking and strategy use on test items with different demands (Diezmann & Logan, 2011). Participants were presented with assessment items they had encountered during the mathematics test. They were asked to review these, and to then answer the following questions to the best of their ability:

- What are you being asked in this problem (What are the important/essential elements in this problem)?
- Which mathematical ideas/concepts/properties can/did you use?
- What do you notice about the problem and how it is presented?
- Tell me more about the presentation (graphics/image, dynamic components).

Procedure

Students completed the mathematics assessment and spatial test battery on tablet devices or computers. Students were provided working out paper but not permitted the use of calculators. All assessments were completed within one hour. Interviews were conducted at students' schools. Some completed interviews on the same day as assessments were completed, while others only participated in interviews up to seven days after initial test administration. Participants were informed that participation was voluntary, and that interviews would be recorded for further analysis. Length of interviews ranged from fifteen to forty minutes.

Analyses

Analyses were conducted to determine if performance on the mathematics test differed between low, moderate, and high spatial students. Strategy interview data was disseminated to identify differences in strategy use on mathematics assessment items between students with low and high spatial scores.

Results

Mathematics Performance and Spatial Ability by Grade level

Independent samples t-tests were conducted to determine if scores on the mathematics and spatial reasoning assessments differed across grades. It was found that students significantly differed in their mathematics and spatial reasoning scores across grades 7 and 9. See Table 1.

Table 1.
Means (Standard Deviations) and t-test results for Maths and Spatial Reasoning across grade levels

	Grade	N	*M (SD)*	Difference	*t*	*p*
Maths Assessment	7	76	26.43 (16.11)	-7.20	-2.24	>.01
	9	60	33.63 (21.31)			
Spatial Reasoning	7	76	48.48 (16.26)	-8.16	-2.53	>.01
	9	60	56.64 (21.39)			

Note. Test scores expressed as percentages

Mathematics Performance by Spatial Ability

Students were categorized as high (top 25%), moderate (middle 50%) or low (bottom 25%) spatial ability. Analyses were then conducted to determine if mathematics performance differed across grades and spatial categories. It was found that performance on the mathematics assessment significantly differed by spatial category, $F(5, 130) = 39.35$, $p = .001$, $\eta^2 = .14$ (see Figure 1).

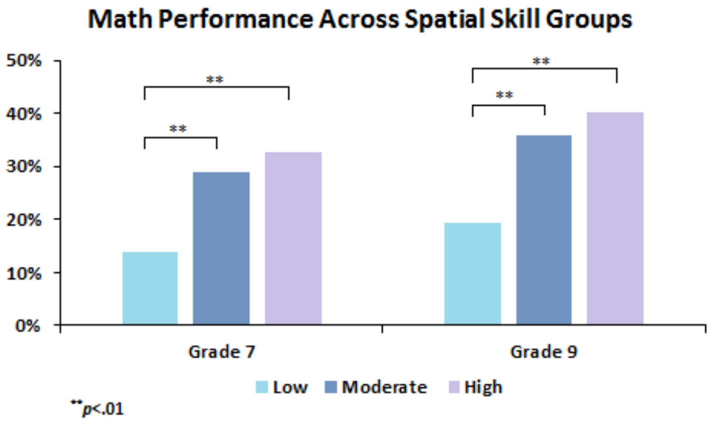


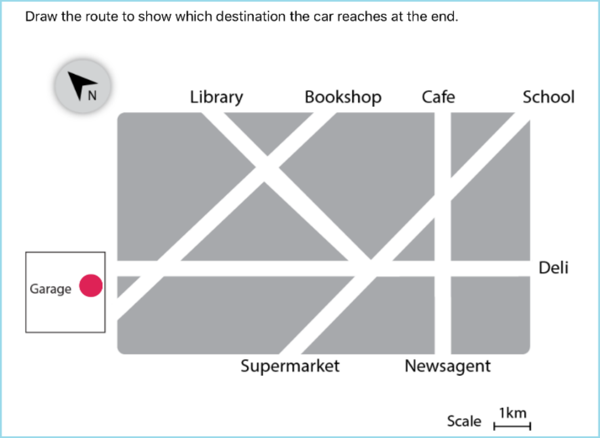
Figure 1. Math Performance Across Spatial Skill Groups

Strategy Interviews

Sixteen participants from grades 7 and 9 consented to participate in the one-on-one interviews. Of these, four (two from each grade level) were selected based on their overall spatial score in relation to scores of other interview participants, with two selected for their low (bottom 25%) spatial score, and two selected for their high (top 25%) spatial score. They were interviewed on a selection of the mathematics tasks, three of which are described here.

Road Map Item

On the road map item, participants were tasked with identifying the path taken by a vehicle in accordance with a set of instructions (see Figure 2).



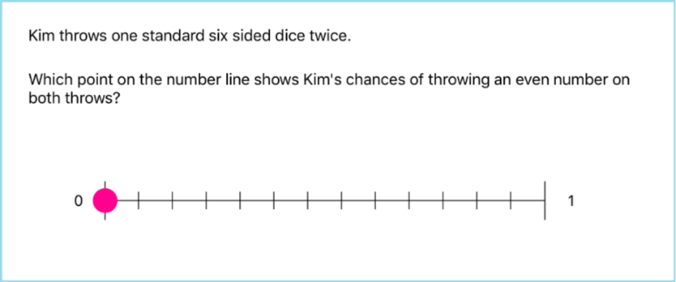
Draw the route to show which destination the car reaches at the end.

Figure 2. Road Map Item

There were clear differences between high and low spatial interviewees. Those with low spatial scores reported not noticing the compass, and completing the problem step by step, switching between instructions and the roadmap, allowing students to check their working. In contrast, both grade 7 and grade 9 interviewees with high spatial scores reported first visualizing the solution. One stated that he "*-was just drawing it in my head*" while the other reported that she "*-closed my eyes and put myself in the situation, and then noticed where it told me to go and just went there*".

Number Line Item

On the number line item, participants were required to estimate the probability of rolling an even number on two dice and locate the value on a number line (see Figure 3).



Kim throws one standard six sided dice twice.

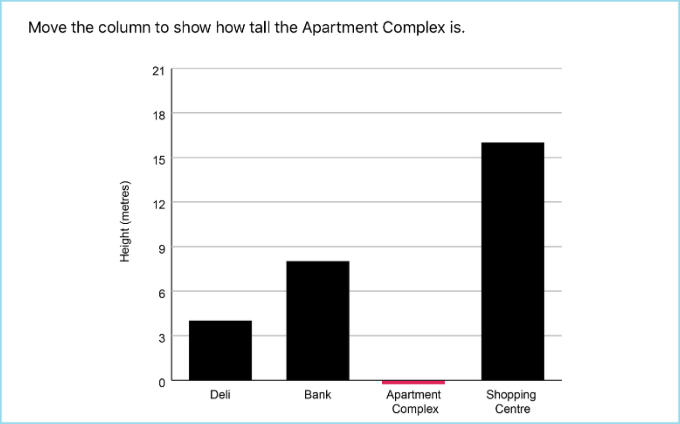
Which point on the number line shows Kim's chances of throwing an even number on both throws?

Figure 3. Number line Item

Spatial ability did not appear to factor into success on this item. Both grade 7 interviewees provided the wrong answer, while both grade 9 students provided the correct answer. Although number line estimation has been found to strongly correlate with spatial ability (Tam, Wong, & Chan, 2019), interviewees did not employ spatial strategies, instead basing their solutions on their intuitive understanding of probability. This was consistent with statements from other interview respondents.

Building Height Item

On the building height item, participants were required to find the height of a building with reference to written and visual supplementary information (see Figure 4). Although all interviewees reported using mathematics, the low spatial grade 9 interviewee stated that “-*the diagram wasn’t really of any use –I am sure it could be if you did it more visually*”. In contrast, the grade 7 high spatial ability participant remarked that he *“-looked at the height in metres, guessed 8 metres*” and that he *“used maths but completed the steps in his head”*, while the grade 9 high spatial interviewee used a creative solution, dividing the complex “-*up into apartments and figured it was 1m per floor so that got me 20 metres*”, and that, in addition to math, she employed “-*a bit of deductive reasoning to find out that four on each floor was a metre*”.



Move the column to show how tall the Apartment Complex is.

Figure 4. Building Height Item

General Observations

High spatial interview participants were clearer in communicating their strategies than their low spatial ability counterparts. Low spatial interview participants made remarks such as “*I know probability, I just knew what to do*” and “*I just don’t understand….I get it now. I just didn’t understand it but it’s a bit…I guessed a lot*”. High spatial interview participants made statements such as “-*I was more worried about the little details you left in it like where North was*” and “*I did use the scale to give me the number, but I visualised moving it up in blocks of five for each floor*”.

Discussion

Spatial ability and mathematics performance were found to be positively correlated. Participants who performed better on spatial tests also performed better on the mathematics assessment. Strategy interviews offered a number of insights. Firstly, high spatial interview participants were noticeably better at explaining their working on mathematics problems they encountered during the interview. While there was too little interview data to draw any concrete conclusions from this, researchers have previously suggested a link between spatial reasoning and verbal ability (Syzmanowicz & Furnham, 2011). More importantly, however, were differences in strategies used.

On the roadmap and building height items, high spatial participants reported using visualisation strategies, including picturing themselves outside and within tasks and graphics. In contrast, low spatial participants remarked not having used provided graphics, instead relying purely on mathematical information they could extract from text and images. As high spatial students nonetheless also reported using mathematics, their ability to supplement this with visualisation strategies led to better outcomes, including on problems where visualising was not strictly required.

Interview data on the number line item suggested that content knowledge was a more important variable on the subject of probability, and that the addition of a number line did not prompt a change in strategy use. Furthermore, none of the interview participants reported using mathematics, instead entirely drawing on their prior understanding of probability.

Limitations and Conclusions

Collecting more comprehensive interview data with an even greater focus on eliciting strategy explanations may provide a more rigorous insight into how and why students of differing spatial ability perform differently on a range of mathematics tasks.

Despite this, findings provide support for hypotheses concerning how the link between spatial ability and mathematics performance manifests. Statistical findings provided further evidence for the spatial-maths link. Interviews reinforced the view that spatial ability influences strategy use, while demonstrating that success on mathematics tasks is also linked to factors such as prerequisite content knowledge, task demands, and task representation. With this in mind, greater spatial ability appears to provide students with the capacity to approach or interpret challenging tasks in ways not available to students with lower spatial ability.

Although the move to digital assessment will provide invaluable scaffolds in the classroom, as demonstrated by the interactive road map task used in this study, developing, and capturing the development of students' spatial reasoning ability may drastically improve critical thinking in and outside of the mathematics classroom and equip future leaders with the problem solving tools and strategies they will need to overcome inevitable local and global challenges that we will encounter in the decades to come.

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