Subject Matter Knowledge: It Matters!

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In mathematics education, subject matter knowledge matters! This paper reports findings from a study of relationships between teachers' understandings of content, the tasks they design and their interpretations of student thinking. A combined methods approach was used to gather multiple data sets from 64 upper primary teachers. The study found that differences in teachers' understandings of area, perimeter and volume accounted for approximately half of the variance in two aspects of pedagogical knowledge, when the content remained constant and subject matter knowledge was probed through problem solving.

There may be nothing more foundational to teaching than subject matter knowledge (Ball, Thames & Phelps, 2008). However, empirical evidence supporting the impact of teachers' subject matter knowledge on student learning remains inconclusive. A consistent finding is that students learn more when teachers focus on understanding (Hattie & Anderman, 2012). The centrality of understanding, as a dimension of teacher knowledge that impacts student achievement, is reflected in research on conceptual understanding (Kilpatrick, Swafford & Findell, 2001), profound understanding (Baumert et al., 2010; Ma, 1999) and relational understanding (Skemp, 1976). Excellence in mathematics education involves opportunities for students to solve complex problems, high expectations for communicating thinking and exposure to alternative solution approaches (Thomson, Hillman & Wernert, 2012). Equity necessitates access to quality mathematics teaching for *all*, rather than *some*, students (Gonski, 2011). Investigating how teachers' understandings of mathematics influence their knowledge for teaching is thereby central to increasing equity and excellence in education.

Designing mathematical tasks, and interpreting students' responses to them, exemplify ways in which daily teaching draws upon teachers' mathematical knowledge. The design of tasks matters. "It is through tasks, more than in any other way, that opportunities to learn are made available" (Anthony & Walshaw, 2010, p.96). To shift teaching beyond procedural exercises, teachers need a repertoire of tasks and problems through which students can explore and understand concepts (Shulman, 1986). *Noticing* refers to the ways that teachers attend to, interpret and respond to student thinking (Jacobs, Lamb & Phillips, 2010). Noticing is central to student achievement because it provides the connection between students, the task and the content (NCTM, 2014). To design tasks that stimulate learning, and "scrutinize, interpret, correct, and extend" thinking (Ball, Hill & Bass, 2005, p.17), teachers need to represent ideas in multiple ways and carry out and understand multi-step problems. Thus, effective mathematics teaching involves teachers in doing mathematics.

The research investigated how teachers' understandings of mathematics influence their knowledge for teaching it. Numerous studies have explored relationships between subject matter and pedagogical knowledge using attainment in coursework, generalised measures of subject matter knowledge, or measures emphasising Number and Algebra. In this study, area, perimeter and volume provided a specific analysis of relationships, using content that is problematic for students and teachers (Blume, Galindo & Walcott 2007; Steele, 2013).

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Data Gathering Methodologies

A combined methods approach (Gorrard & Taylor, 2004) was adopted to investigate a complex question, involving multiple aspects of teacher knowledge, and facilitate the selection of tools and methods for gathering data and testing relationships in the same study. Within this approach, correlational research offered the benefit of identifying and evaluating the strength of relationships between aspects of knowledge without the need to assign teachers to different learning conditions (Cresswell, 2003).

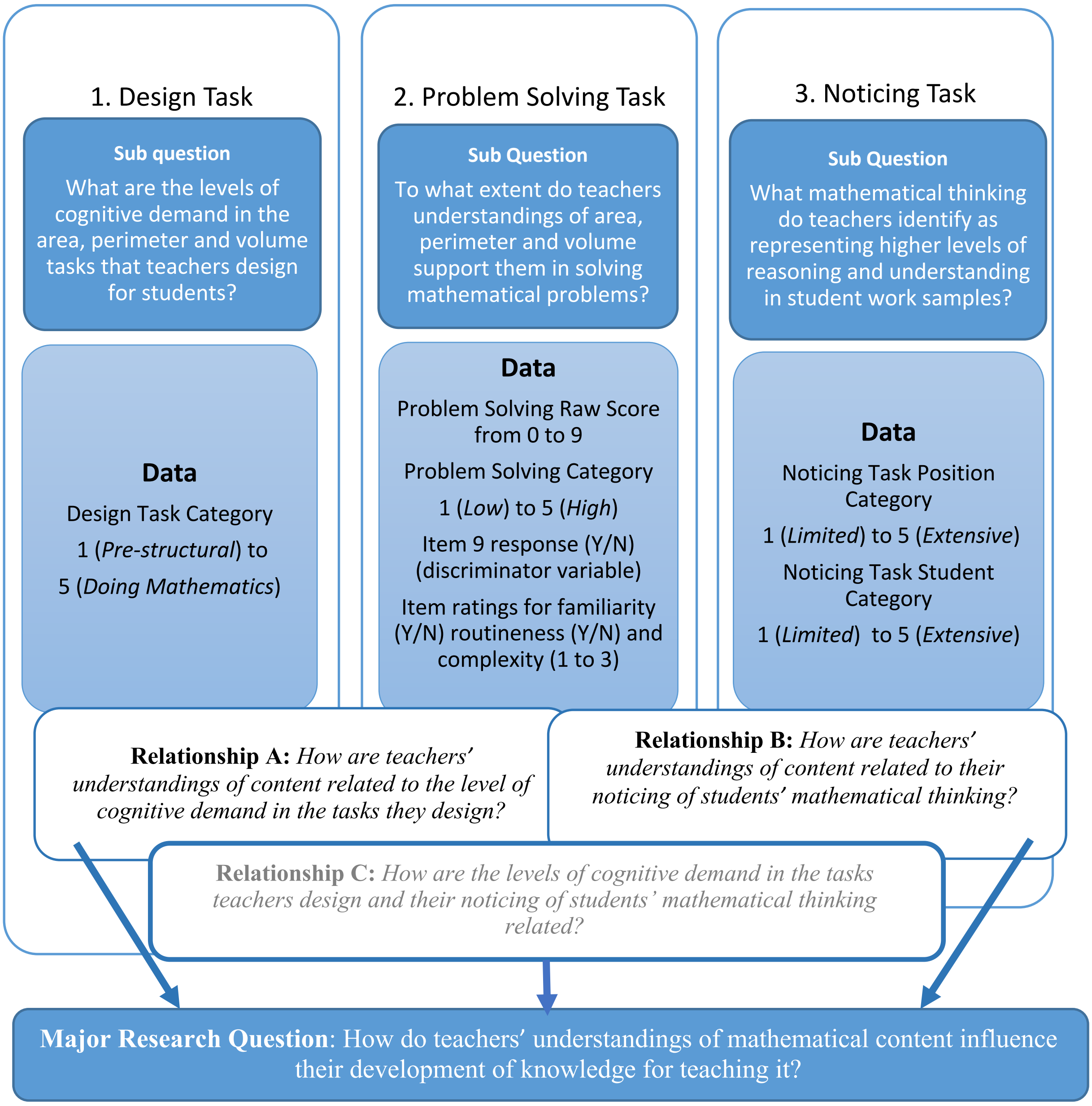


Figure 1. Summary of methods.

Figure 1 provides a summary of the research, including the research questions, data gathering, analysis and relationships studied. A cross-sectional research design created a 'snapshot' across three aspects of teacher knowledge. Multiple sets of data were gathered from 64 participants, from the same schooling system, teaching the final two years of primary school (students aged 10-12 years) in the same metropolitan area, at the same point in time, and in relation to the same content (Gorrard et al., 2004). Teachers engaged in three tasks that were replicas of the types of challenges faced in their daily teaching, to provide valid information about aspects of teacher knowledge (Goe, Bell & Little, 2008). On a single day of participation, teachers engaged in a Design Task, Problem Solving Task and Noticing Task.

The Design Task

The Design Task was used to gather data regarding the level of cognitive demand in a task designed by each teacher. Teachers used the syllabus as a starting point for designing a task to assess students' understandings of area, perimeter or volume. A framework, combining levels and characteristics of the Task Analysis Guide (TAG) developed by Stein & Smith (1998) with an adaptation of the Structure of Observed Learning Outcomes (SOLO) taxonomy (Biggs & Collis, 1982), was used to analyse and categorise all tasks. The frameworks were combined to include a *Pre-structural* category describing tasks that provided no relevant opportunity to learn the selected content, whilst utilising the levels of the TAG to discriminate lower and higher level tasks. Numerous steps were taken to establish reliable levels for all tasks, including the use of multiple expert raters and clearly stated guidelines to remove potential ambiguities or biases.

The Problem Solving Task

The Problem Solving Task was used to gather data regarding teachers' understandings of area, perimeter and volume when solving mathematical problems that were applications of content they teach to students in the final two years of primary school. The task consisted of a set of nine items, adapted from the National Assessment Plan for Literacy and Numeracy (NAPLAN) Numeracy tests between 2008 and 2013. The items were selected through a trial in which 32 teachers solved and rated a larger set of 22 items assessing the same content. Data were gathered to provide information about participants' understandings of the content as well information about the items. Participants' raw scores were used to allocate responses to categories, while item analysis included the number of correct responses to each item, the content focus of each item and teachers' ratings of the familiarity, routineness and complexity of each item (Hirstein, 1981; Mevarech & Kramarski, 2014).

The Noticing Task

The Noticing Task was designed to gather data regarding teachers' interpretations of students' understanding and reasoning in written work samples. Teachers were presented with a set of five work samples in response to an area problem, representing different levels of student thinking, ranging from incorrect solutions due to predictable misconceptions to correct solutions using sophisticated reasoning (Goe et al., 2008). The work samples were based on the final item from the Problem Solving Task, which involved calculating the area of a plane shape with numerical dimensions on all sides – a situation noted as problematic for students (Hirstein, 1981). Teachers ranked the work samples from Extensive (A) to Limited (E) and recorded feedback to students to confirm their rankings. Teachers' responses were categorised according to where they ranked a work sample with sophisticated thinking (Noticing Task Position Category) and which work sample they ranked as Extensive (Noticing Task Student Category).

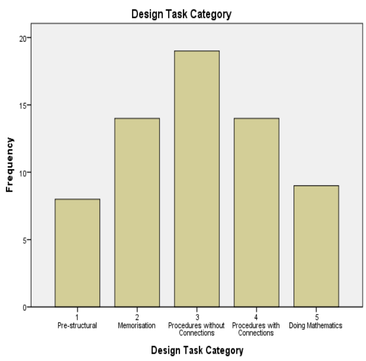
Analysis of Data

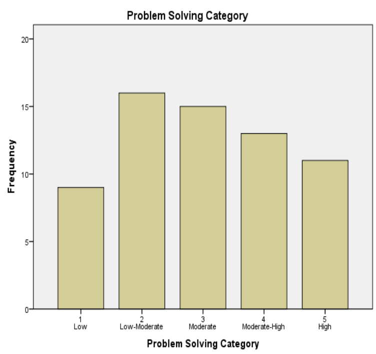
Data were analysed for distribution and central tendency [$M = 3.03$ ($SD = 1.23$, $N = 64$)]. Figure 2 illustrates the distribution of Design Task responses across all levels of cognitive demand. More teachers designed tasks in the *Procedures without Connections* category than in any other category. Approximately one-third of the teachers designed the types of higher level tasks recommended for student learning: *Procedures with Connections to Meaning* or *Doing Mathematics* tasks. The eight tasks in the *Pre-structural* category did not reflect any relevant opportunity for students to learn the selected content.

Problem Solving Task scores were used to allocate responses to Problem Solving Categories [$M = 3.02$ ($SD = 1.32$, $N = 64$)]. Figure 3 illustrates the distribution of responses across the full range of Problem Solving categories from *Low* (0 or 1 correct) to *High* (8 or 9 correct). On average teachers answered half of the problems correctly. However, 37.5% of teacher responses demonstrated stronger subject matter knowledge by solving most or all problems. Notably, teachers were far more likely to solve problems rated as familiar, non-routine and/or low in complexity.

Figure 4 captures the distribution of Noticing Task Position responses [$M = 3.20$ ($SD = 1.33$, $N = 64$)] across all categories. Most teachers did not interpret the correct, sophisticated mathematical thinking in a work sample as evidence of Extensive achievement. Teachers were equally likely to rank this work sample as Thorough or Sound as they were to rank it as Extensive. Approximately one-third of responses ranked this work sample lower than at least one work sample with an incorrect solution. Notably, teachers who correctly solved the item that work samples were based on (indicated by darker shading) were far more likely to notice higher levels of student thinking.

Figure 2. Design task.





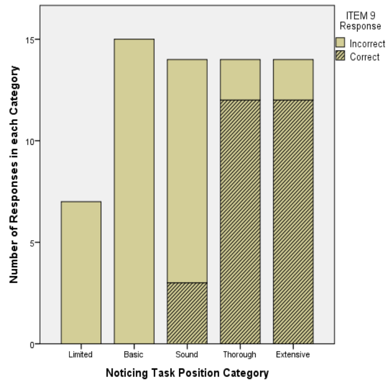


Figure 3. Problem solving task. *Figure 4.* Noticing task.

Data from the three teacher tasks were used to identify relationships between aspects of teacher knowledge. Figure 5 illustrates the relationship between Problem Solving and Design Task data. The Pearson correlation coefficient [$r = 0.759$, $n = 64$, $p = 0.01$] revealed teachers' understandings of content as highly, significantly predictive of the level of cognitive demand in the tasks they designed using the same content. A significant regression ($F(1,62) = 84.485$, $p<.001$) was found, with an R^2 value of 0.577.

Visual inspection of the scatterplot in Figure 6 shows the way in which Problem Solving was predictive of Noticing Task Position Categories [$r = 0.674$, $n = 64$, $p = 0.01$]. A significant regression equation ($F(1,62) = 51.550$, $p<0.001$) was found, with an R^2 value of 0.454. Fisher's exact test identified solving the problem that student work was based on as highly, significantly predictive of noticing student thinking [<0.00001, $p<0.05$].

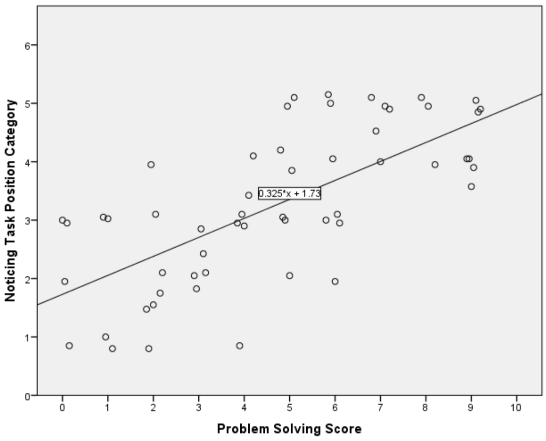


Figure 5. Relationship A.

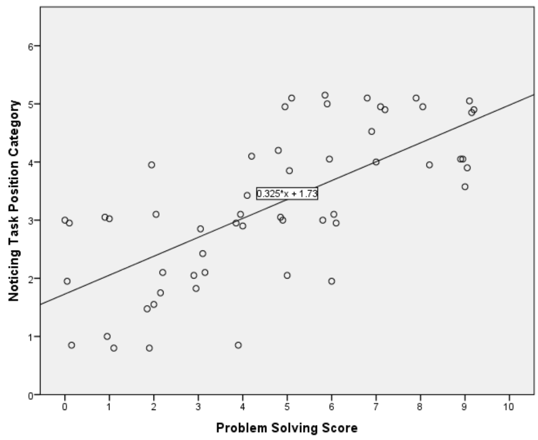


Figure 6. Relationship B.

Relationships between aspects of teacher knowledge were then studied simultaneously. Figure 7 presents a three-dimensional view of teacher knowledge created to synthesise findings regarding how teachers' understandings of content influence their knowledge for teaching. In this graphic, each circle represents one participant. Circle size indicates the strength of subject matter knowledge, while position conveys the strength of two aspects of pedagogical knowledge. The dotted lines indicate the median score for aspects of pedagogical knowledge and divide the graphic into four quadrants of teacher knowledge.

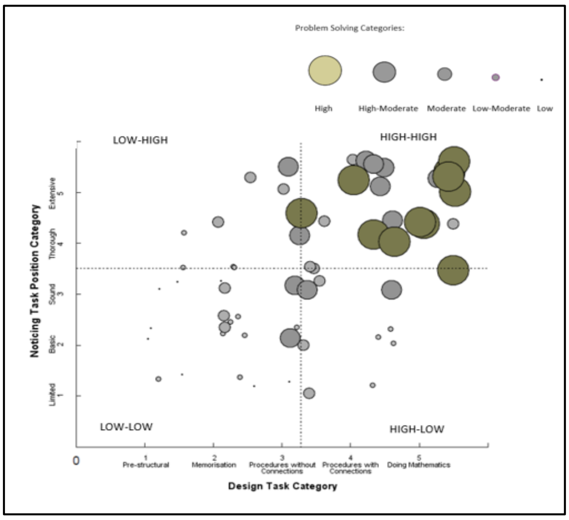


Figure 7. Relationships between three variables of teacher knowledge.

Discussion

The strength of relationships identified between aspects of teacher knowledge highlighted the important role that subject matter knowledge plays in supporting pedagogical knowledge. The results reflected the fundamental belief of pedagogical content knowledge: neither subject matter knowledge, nor knowledge of teaching alone, are sufficient for effective teaching (Shulman, 1986). However, in this study, evidence of pedagogical knowledge not supported by proficiency with the subject matter, was scarce. Subject matter knowledge matters. It impacts on teachers' interpretations of the intended curriculum, the learning opportunities they design to implement the curriculum, and their interpretations of how well students have attained the curriculum (Mullis & Martin, 2011).

The variability of teacher knowledge and an emphasis on procedural thinking were prominent throughout the results. All levels of cognitive demand were evident in the tasks, teachers' understandings of content supported them in solving anywhere from none to all of the problems presented, and teachers' interpretations of the same work sample ranged across five reporting descriptors. In the correlational analysis, teachers' problem solving scores explained 57.7% of the variation in the level of cognitive demand in tasks, and 45.4% of the variation in their noticing of higher levels of student thinking. An emphasis on procedural thinking was also evident throughout the results. More teachers designed tasks that focused on *Procedures without Connections*, most teachers solved only familiar, routine problems and almost half interpreted procedural thinking as evidence of *Extensive* achievement - even though the thinking was inefficient. Notably, teachers who solved only problems rated as familiar and routine designed lower level tasks and misinterpreted higher level reasoning in a work sample using a novel solution approach. The variability of subject matter knowledge, its association with variations in pedagogical knowledge, and a reliance on procedural thinking, present challenges for increasing the quality of mathematics education.

Teachers' understandings of content are central to student learning. The ratio of higher to lower level tasks, proportion of teachers solving only familiar, routine problems, and valuing of procedural thinking over reasoning, indicate that conceptual (Kilpatrick et al., 2001), profound (Baumert et al., 2010; Ma, 1999) and relational (Skemp, 1976) understandings of mathematics are not prevalent. If Australian students need increased opportunities to solve more complex problems, high expectations for reasoning and exposure to alternative solution approaches (Thomson et al., 2012), improving the quality of tasks, and teachers' noticing of student thinking in response to them, are fundamental to achieving standards of excellence. As equity necessitates access to quality mathematics teaching for *all* students (Gonski, 2011), the variability of teachers' subject matter knowledge, and its influence on aspects of their pedagogical knowledge, requires further investigation and investment. Ball and colleagues (2008) observed that the learning gains of students in the classes of teachers with higher levels of mathematical knowledge for teaching were equal to the effects of an additional two to three weeks of instruction per year. Hence, deepening teachers' understandings of the mathematics they teach may provide a means for overcoming educational inequity.

An underlying issue in improving student achievement is the need for opportunities that stimulate thinking beyond what students can already do (Thomson et al., 2012). Given the extent to which subject matter knowledge influenced the design of tasks and teachers' noticing of student thinking, increasing subject matter knowledge provides a powerful means for activating and increasing pedagogical knowledge. Teachers' understandings of content influenced the clarity of learning goals, expectations for student learning, opportunities to develop deeper conceptual understandings of content and noticing higher levels of student thinking. As no significant differences were observed between the Problem Solving scores of teachers in the two higher categories for the Design Task or the Noticing Task, increases in subject matter knowledge beyond a certain threshold might not be associated with higher levels of teacher effectiveness (Ball et al., 2005).

The findings support the notion that it is not how much, but how, teachers know mathematics that matters (Ma, 1999). Teachers with responses in lower Problem Solving categories may not need to learn more mathematics, but rather, to develop the Profound Understandings of Fundamental Mathematics (PUFM) described by Ma. Teachers with PUFM are more able to highlight and connect mathematical ideas and display multiple solution approaches. They place greater emphasis on justifying mathematical arguments, are more likely to approach topics in multiple ways and offer a greater variety of examples to students. By comparison, teachers without PUFM tend to focus on knowledge of how to complete procedures, rather than how procedures are connected to important principles of mathematics. Differences between teachers with and without PUFM may explain differences between teachers in the High-High and Low-Low quadrants of teacher knowledge depicted in Figure 7. As noted by Ma (1999), differences in teachers' understandings of basic mathematical principles influence their development of networks of conceptual ideas that play an important role in supporting effective teaching.

Correlations between teachers' understandings of content and their knowledge for teaching it may be due to an inextricable link between teaching and problem solving. Both problem solving and teaching require teachers to solve problems "for which the solutions may not be readily apparent" (Chick & Stacey, 2013, p.2). Teachers who were proficient with the content to the extent that they solved unfamiliar, non-routine, complex problems, were far more likely to design higher level tasks and notice higher levels of student thinking. Higher levels of subject matter knowledge did not guarantee high levels of pedagogical knowledge, but they predicted and explained variations in it. All teachers with responses in the High category for Problem Solving gave responses at or above the median score for both aspects of pedagogical knowledge. Considered simultaneously, the aspects of teacher knowledge suggest that mathematical proficiency is a critical component of teacher knowledge (Kilpatrick et al., 2001). While subject matter knowledge alone is insufficient for proficient teaching, teachers with low levels of proficiency with the content did not design higher level tasks and notice higher levels of thinking (Baumert et al., 2010; Hill et al., 2008).

Variations in, and relationships among, teachers' subject matter knowledge and two aspects of pedagogical knowledge, have implications for educational policy and the allocation of resources to support teacher education (Hill et al., 2008). Almost two-thirds of the 24 teachers demonstrating stronger subject matter knowledge also designed a higher level task and noticed higher levels of student thinking. The findings reinforce the importance of teachers possessing connected, coherent, structured understandings of mathematics (Ma, 1999) as a foundation for pedagogical knowledge. By contrast, just over two-thirds of the 40 teachers exhibiting weaker subject matter knowledge also designed tasks with lower levels of cognitive demand and misinterpreted higher levels of student thinking. However, the results also highlight that teacher knowledge is finely grained. Further research, involving larger numbers of teachers, and applying different content to the study of these relationships, is needed. To provide an excellent mathematics education for every student, we need to identify the mathematics that teachers need to know, how they need to know it and what this means for the opportunities we provide for teachers to learn and understand the mathematics they teach (Kilpatrick et al., 2001).

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