Spatial ability, skills, reasoning or thinking: What does it mean for mathematics?

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Spatial reasoning is identified as a Numeracy general capability in the Australian Curriculum, and more globally as a significant precursor to mathematics proficiency. Currently, the literature surrounding mathematical-spatial relations remains largely removed from classroom practice. This paper provides a reflection on the spatial cognition field as it relates to mathematics. The focus of the review is to find points of connection between psychological notions of spatial skills and spatial reasoning as it stands in curriculum and assessment.

Spatial reasoning as an instinctive, vital, human capability has been demonstrated throughout history (e.g., locating the source of the Cholera epidemic in London; supporting the discovery of DNA; NRC, 2006). At a global level it refers to proficiency in mentally representing and transforming objects and their relations (Mulligan, 2015). At a local level, spatial reasoning is ingrained in daily activities, such as the ability to locate our keys, the process of parking a car or packing a suitcase. Although these different skills are often taken for granted and fall under the label *spatial reasoning*, it may not be the case that being good at one type of skill ensures aptitude for another (Newcombe, 2010). Spatial reasoning as an umbrella term has been deemed so closely related to mathematical proficiency it no longer makes sense to explore whether the two are related (Mix & Cheng, 2012). Whole books (Mix & Battista, 2018) and mathematics research journal special issues (Resnick et al., 2020; Sinclair & Bruce, 2015) have been dedicated to the theoretical positing of mathematical-spatial relations. Despite the decades of analysis, the gap between cognitive theories of the mathematical-spatial relationship, and classroom promotion of spatial reasoning remains vast (Lowrie et al., 2020). This paper presents a review of some of the different spatial understandings brought about by differences in terminology, and how these link to the current state of spatial instruction in mathematics classrooms. The aim of this paper is to identify connections across the fields about mathematical-spatial relations, with a view to providing a common conceptual framework on which to build future empirical studies.

Spatial Vocabulary

Spatial terminology varies across discipline, country of origin and research intent. One reason may be that the richness of our mental imagery is poorly articulated by our linguistic capabilities (Hayward & Tarr, 1995). Consequently, a range of terms have been used to define spatial concepts with little consistency. Here I seek to define key spatial vocabulary to provide a shared conceptual framework that is currently lacking in the literature.

The term *ability* is often used to differentiate students in education and has been defined as a "salient psychological attribute" (Wai et al, 2009, p. 817), implying it is stable over time. By contrast, spatial *skills* suggest the opportunity for growth and change (Uttal et al., 2013). More generally, spatial *reasoning* invokes thoughts of non-verbal problem-solving while spatial *thinking* conjures up images of a habit of mind or more holistic spatial sense (Whiteley et al., 2015). These terms are distinct from the mental processes that occur during spatial tasks. *Visual imagery* (imagining a referent object(s); Presmeg, 1986) and *spatial relations* (relative position or movement between objects; Hegarty & Kozhevnikov, 1999)

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are often used to describe stable spatial characteristics. By contrast, spatial *manoeuvres* are the dynamic mental processes undertaken when performing tasks (Ramful et al., 2017). The accuracy and usefulness of these processes may vary depending on spatial aptitude and task demands (Hegarty & Kozhevnikov, 1999; Presmeg, 1986).

Never is the lack of consistency in terminology more evident than when searching keywords in the literature. For example, in conducting their spatial training meta-analysis Uttal et al. (2013) searched 14 different terms yet failed to include spatial reasoning or thinking. To move the field forward there needs to be consistency in the meaning and use of spatial terms. A proposed conceptual model for spatial terminology is presented in Figure 1.

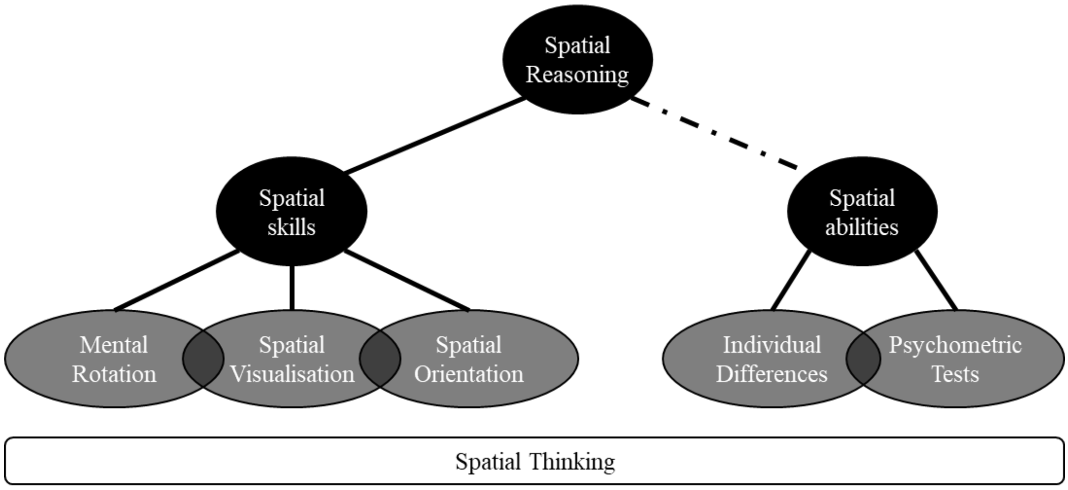

Figure 1. Conceptual model of spatial terminology.

Spatial abilities

Spatial ability was described as an intelligence distinct from verbal ability almost 140 years ago (Galton, 1883). The measurement of spatial ability was predominantly conducted with instruments developed by psychologists (Hegarty & Waller, 2005). Despite research now indicating that spatial aptitude is not fixed (Uttal et al., 2013), there are individual differences that show trends in spatial abilities. Generally, males perform better on spatial ability tests (Hegarty & Waller, 2005). However, research is emerging to suggest that gender differences may lie in strategy choices, thus calling into question some of the long-held beliefs about gender factors in spatial ability theory (Newcombe, 2020).

Piaget and Inhelder (1967) proposed that although infants show evidence of spatial coding, mature spatial reasoning does not emerge until age 9 or 10. Congdon et al. (2018) report evidence for pre-schooler's awareness of spatial properties, but it is not until a few years into formal schooling that language and conceptual understanding develop. Separate spatial abilities also seem subject to different developmental trajectories. Crescentini et al. (2014) found that the ability to perform object-based spatial tasks emerged earlier than tasks requiring awareness of one's body and environment. This may be largely due to children's exposure to activities and environments that support the development of spatial reasoning at the different scales (Newcombe, 2002; 2020). Understanding the developmental path of spatial abilities may assist educators and researchers in providing appropriate experiences for children to foster their spatial reasoning. The dotted line between spatial abilities and spatial reasoning in Figure 1 above signifies that spatial capacity (or ability) exists for everyone, however, education, experience and environmental interaction are most influential in the development of the more holistic notion of spatial thinking (Newcombe, 2002).

Spatial skills

Spatial skills are the quantifiable factors comprising spatial reasoning that are distinct, yet related (Carroll, 1993). The structure of these skills, much like the overarching theme itself, remains under some debate. Newcombe and Shipley (2015) proposed a typology of spatial tasks categorised by the characteristics of the referent object(s); whether they remained still (i.e., static) or moved (i.e., dynamic) and whether spatial relations were within (i.e., intrinsic) or between (i.e., extrinsic) objects. Such a framework could provide researchers with the foundations for linking the mental manoeuvres undertaken during spatial tasks and skills in other fields, such as mathematics. However, the typology proposed by Newcombe and Shipley is largely based on psychological tests and has yet to be supported by further research or in applications beyond lab-based studies (Mix et al., 2018).

Measuring spatial skills. The idea of assessing different spatial skills emerged in the field of aptitude testing for occupations (Hegarty & Waller, 2005). As psychometric tests measuring spatial skills continued to evolve, correlations with other skills and outcomes emerged. For example, spatial skills were the strongest predictor of Science, Technology, Engineering and Mathematics education success and career choice in a 50-year longitudinal study (Wai et al., 2009), above verbal ability and mathematics proficiency.

Spatial task performance has been related to mathematics outcomes in correlational (Gunderson et al., 2012; Mix et al., 2016) and intervention studies (Cheng & Mix, 2014), leading to categorisation of spatial skills based on test affordances. For example, object-based spatial skills are considered in a different category to egocentric skills, where one's perspective becomes the reference point (Sorby, 1999). This distinction is a consequence of test design and the intentional promotion of specific strategy use (Hegarty & Waller, 2004). This psychological approach results in cognitive theories limited by the measures used in empirical studies and may be counter-productive to the development of robust models of mathematical-spatial relations that are based on applications of spatial skills.

Ramful et al. (2017) adopted a different methodology in the development of their spatial reasoning instrument (SRI). They defined spatial constructs (as opposed to skills) by the spatial manoeuvres found in the Australian Numeracy curriculum; namely, mental rotation (i.e., imagining an intact 2D shape or 3D object in a different orientation; Cheng & Mix, 2014), spatial visualisation (i.e., performing complex, multi-step manoeuvres that change the form of the referent object; Hegarty & Waller, 2005), and spatial orientation (i.e., imagining different perspectives, navigating, or taking different orthogonal views; Newcombe & Shipley, 2015). These constructs correlate with psychological tests of spatial skills but provide opportunities to explore links with mathematical-spatial processing in a more applied way. This measure is the first of its kind but there are calls for more measures of spatial reasoning that consider real world spatial problem-solving (Mix, 2019).

Spatial Reasoning

Spatial reasoning, as a foundational component of Numeracy, requires an awareness of space, the ability to imagine objects and relationships, and use this information to reason and problem-solve (ACARA, n.d; NRC, 2006). Spatial reasoning manifests differently across situations (such as the constructs identified by Ramful et al., 2017). For example, mental rotation involves imagining an object's position within its direct environment, while spatial visualisation exists in isolation, where the environment is less important than the relations within and the ability to visualise and transform the object's form. Spatial orientation requires imagining dynamic interaction with an environment on a larger scale.

Spatial reasoning is not easy to quantify, and researchers look to spatial tests (Mix, 2019) or to the most explicitly spatial aspects of curricula (i.e., geometry; Battista et al., 2018) to make inferences about its underlying structure. Spatial reasoning in education goes beyond success on spatial tests. Educators need to be equipped with the tools to recognise and foster students' awareness of space in the mathematics classroom, and to encourage them to notice spatial relations in their interaction with the world.

Spatial Thinking

Spatial thinking is less well-defined by literature, except where used interchangeably with spatial reasoning. Newcombe (2010) used the term spatial thinking to describe Albert Einstein's unique way of seeing the world, that is, in pictures and relations. In this paper, I propose a distinction between spatial reasoning, the application of spatial skills during problem-solving, and spatial thinking, the tendency to visualise non-verbal aspects of objects and relations, separate to mathematical thinking (Newcombe, 2010; Whiteley et al., 2015).

In the National Research Council (2006) report, spatial thinking was defined as an "amalgam of three elements: concepts of space, tools of representation, and processes of reasoning" (p. 3). Figure 1 shows spatial thinking as underpinning all forms of spatial representation and assessment discussed above. This model positions spatial thinking as a habit of mind that guides communication, reasoning and problem-solving. Therefore, promoting spatial thinking across education, provides students with strategies when faced with new or complex materials (Uttal & Cohen, 2012).

Visualisation

Much like the absence of spatial terms in Uttal et al.'s (2013) literature search, Figure 1 did not capture all aspects of spatial vocabulary. One missing element is visualisation, which is critical for spatial thought (Battista et al., 2018). Visualisation occurs differently for those with varying spatial skill levels. Strong spatial thinkers tend to generate mental images that facilitate problem-solving and concept development, poor spatial thinkers tend to produce mental images that, while detailed, offer little in their affordances for problem-solving (Hegarty & Kozhevnikov, 1999; Presmeg, 1986).

Mathematics and Spatial Reasoning

A complete review of the mathematics-spatial literature is beyond the scope of this paper and well captured in Mix and Battista's (2018) edited book. Here, I focus on the connection between cognitive theories of mathematical-spatial relations and spatial reasoning in mathematics curricula and assessment based on Ramful et al.'s (2017) three constructs.

Mental Rotation

Mental rotation is one of the most extensively studied spatial skills in the mathematics literature. In fact, 3D mental rotation training by Cheng and Mix (2014) was found to lead to improvements on missing term addition and subtraction tasks. Furthermore, mental rotation is thought to support geometric reasoning by providing the mental models on which to examine geometric properties (Battista et al., 2018). For example, to perform mathematical rotation tasks on a coordinate grid, one must first be able to correctly visualise the rotation of the referent object. The disconnect between the psychological and educational notions of mental rotation are evident in these two lines of thought. While one is focused on repetitive, comparison tasks that rely on speed to force rotation (psychology; Hegarty & Waller, 2005), the other advocates for mental rotation processes in building conceptual knowledge for geometric understanding (mathematics education; Battista et al., 2018).

Apart from small-scale studies (e.g., Bruce & Hawes, 2015; Cheng & Mix, 2014), the development of mental rotation in mathematics classrooms remains largely incidental as a result of engagement with geometry material (Lowrie & Logan, 2018). Lowrie et al. (2018) provided a pedagogical model for developing mental rotation beyond curriculum learning through a classroom-based spatial intervention. However, the unique contribution of mental rotation to mathematics improvement was not addressed.

Spatial Visualisation

The complex, multi-step manoeuvres that constitute spatial visualisation are evident within mathematics curricula in geometric concepts of symmetry and net to solid conversions. Furthermore, psychological measures of spatial visualisation such as paper folding have been found to relate to multiplicative and algebraic thinking by reflecting students' ability to map folds to parts (Empson & Turner, 2006). Lowrie et al. (2019) trained spatial visualisation skills, which led to improvements in geometry and word problems. They concluded that the impact on geometry tasks was reflective of students' increased ability to manipulate spatial properties, while the word problems were evidence for improvements in representing information spatially during problem-solving (Hegarty & Kozhevnikov, 1999).

Rittle-Johnson et al. (2019) found strong relationships between patterning, mathematics and spatial visualisation. They found that spatial visualisation at the beginning of the pre-school year was a significant predictor of later numeracy performance (a subset of the mathematics assessment) in that same year. They also found that initial patterning skills were a significant predictor of later mathematics, over and above prior mathematics knowledge and a composite spatial measure. Their findings shed light on the complex relationship between spatial skills, patterning and mathematics. It is possible that spatial visualisation is helpful when developing mathematical understanding but is less influential long term when content knowledge increases.

Spatial Orientation

Few psychological studies have explored the direct role of spatial orientation in mathematics (Newcombe, 2010) but mapping tasks and orthogonal perspectives are explicit elements of the Measurement and Geometry strands of the Australian Curriculum (ACARA, n.d.). Two longitudinal studies have examined the unique role of spatial orientation in mathematics performance. Frick (2019) found that spatial orientation skills measured in kindergarten predicted performance in quantity, magnitude and geometry tasks, but not arithmetic in year 2. Mix et al. (2016) found significant contributions of spatial orientation to a general mathematics measure in years 3 and 6. Spatial orientation skills such as understanding scale and magnitude are critical for performance on mapping tasks as well as development of number line knowledge (Gunderson et al., 2012) and proportional reasoning (Möhring et al., 2016). Given these preliminary findings, I propose that this is a spatial construct that should be included in empirical studies of mathematics-spatial relations.

Spatial Intervention

Many studies to date have analysed correlational data, providing valuable insight into areas where spatial intervention may support student learning (Mix, 2019). Spatial skills have been found to be malleable and responsive to training (Uttal et al., 2013). In their meta-analysis of spatial training studies Uttal et al. (2013) found no difference in effect sizes as a result of the form of the training (i.e., instructional courses, video games or spatial skills training) on spatial outcomes. However, when examining potential transfer to mathematics the range of outcomes has produced variable results (Stieff & Uttal, 2015). Stieff and Uttal acknowledge the difficulty in conducting classroom-based studies on a large-scale but when done successfully, they have the greatest potential for effecting change.

To progress the field and transfer theoretical understandings to practical, student benefits we need to shift the focus from performance on cognitive tests to how spatial reasoning manifests in mathematics. Lowrie and colleagues have demonstrated reliable transfer to mathematics achievement (Lowrie et al., 2017; 2019; in press) in ways that others in the spatial cognition field have not (Cheng & Mix, 2014; Hawes et al., 2017). They achieved this through the integration of spatial skills in a pedagogical framework, delivered by classroom teachers (Lowrie et al., 2018). In their intervention studies, ranging in length from 3 to 10 weeks and focusing on mental rotation, spatial visualisation and spatial orientation (Lowrie et al., 2017; in press) or spatial visualisation alone (Lowrie et al., 2019), Lowrie and colleagues consistently demonstrated mathematics improvements with effect sizes ranging from .38 – .40 (Cohen's d) compared to business as usual control groups.

There remains a gap in the literature. To date there have been no systematic studies of spatial skill interventions to determine their contribution to mathematics. Studies remain either isolated (e.g., 3D mental rotation; Cheng & Mix, 2014; spatial visualisation; Lowrie et al., 2019) or combined (e.g., Lowrie et al., 2017; in press), making it difficult to identify the unique contributions of spatial skill development to mathematics. Similarly, the effect on mathematics has been too broad, leaving the field still speculating about the mechanisms that result in improvements in mathematics based on spatial training (Stieff & Uttal, 2015).

Limitation and Conclusion

The focus of this review has been to highlight some of the connections between cognitive theories of spatial skills, emerging from lab-based studies, and applied spatial reasoning, in education. This review could not be exhaustive and there remains a considerable absence of spatial terminology as well as spatial concepts such as transformation, and representation. These were excluded based on the goal of seeking common ground across psychological and educational domains, as these terms often have different meaning in the two fields. For example, in mathematics transformations are functional in problem-solving (Battista et al., 2018), while in psychology transformations refer to mental manoeuvres (Frick, 2019).

To progress the field in practical and constructive ways the focus on spatial reasoning in mathematics needs to be situated within real world applications (Lowrie et al., 2020). Spatial instruction needs to be explicit, not merely fostered through the more spatial content within the curriculum. To bridge the disconnect between cognitive theories of mathematical-spatial relations and classroom practice, there needs to be shared meaning and studies need to be conducted at scale with teachers instrumental in the process. Finally, experimental design needs to allow for conclusions to be drawn about the mechanisms that connect spatial thinking, reasoning and skills with mathematics understandings to ensure sustainable and positive outcomes for students.

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