Throw Away the Script: Examining the Introduction of a Guided Mathematical Inquiry Unit

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Explicit teaching, an instructional method promoted by Australian Education Departments over student-centred methods, entails teacher-led instruction and sequenced breakdown of student tasks. Inquiry-Based Learning is often considered an opposing, student-centred approach, which explicit teaching proponents argue lacks guidance and support. In this paper, we examine one experienced teacher's introduction of a Guided Mathematical Inquiry (GMI) unit to provide insight into key teacher practices and illustrate the approach in practice. The findings suggest that GMI establishes, guides and supports contextual and discipline knowledge in mathematics from the start of the lesson. Teacher considerations for implementation of GMI are provided.

Engelmann (1967) believed that students' learning is determined by the teacher and is independent of student characteristics, leading to the introduction of a teacher-centred approach to instruction commonly referred to as explicit instruction. Explicit instruction entails a teacher-controlled, highly scaffolded and sequenced breakdown of tasks for learners with ample opportunities for student repetition and practice (Archer & Hughes, 2011). By contrast, constructivist and socio-cultural theorists promote student-centred instruction, such as Inquiry-Based Learning (IBL), to maximise learning outcomes. Student-centred approaches emphasise the roles of students in exploring, modelling, guiding, collaborating, questioning, and carrying out activities that enhance the development and ownership of mathematical ideas. There are research-identified limitations and benefits to both teacher-centred and student-centred instructional approaches. As such, teaching through only one method is likely to maximise the limitations inherent with that method and minimise the opportunities associated with the other.

While explicit instruction is promoted in Australian schools, IBL is less so (e.g. NSW Centre for Education Statistics and Evaluation, 2020). Part of the reason for this is the challenge imposed on teachers through lack of structure and difficulties in getting started (Makar & Fielding-Wells, 2011). To counter this, we examined ways in which an experienced teacher of Guided Mathematical Inquiry (GMI) introduced an inquiry unit to develop insights into, and provide an illustration of, key components to be considered. The research question addressed was:

- What insight into GMI can be gleaned from analysing the introduction phase of a GMI unit.

Literature

Explicit Instruction

Explicit instruction refers to a teaching approach where the teacher provides clear statements of objectives, instruction, and practice to achieve the intended outcomes of a lesson. When introducing a mathematical concept, the teacher gains students' attention, presents the objectives and relevance of the lesson, and then proceeds to teach the concept by breaking tasks down into small steps and providing significant teacher guidance (Archer & Hughes, 2011). A rationale for using this approach is to effectively manage and maximise students' cognitive resources (Sweller, 2012). However, since its' inception, explicit instruction has received significant criticism. In the Australian

(2023). In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives*. *Proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 27–35). Newcastle: MERGA.

mathematics context, explicit instruction has been associated with non-participation and disengagement; passive approaches to learning; and reduced opportunities for students to "access or use the language of mathematics to express their mathematical ideas" (Ewing, 2011, p77). Classroom interactions are largely a one-way process and memorisation, rehearsal and rote learning are the outcome. An assumption that underpins this approach is that mathematical knowledge can be handed from teacher to student (Ewing, 2011), which has implications for student transfer of knowledge to new applications.

Guided Mathematical Inquiry (GMI)

GMI is an approach to IBL which maintains a student-centred focus whilst incorporating scaffolding of the learning process by the teacher—hence the term *Guided.* While the extent of guidance can vary, here it is taken to mean that the teacher provides students with the support necessary to enable them to engage with an inquiry problem while scaffolding the process and supporting student direction (Fielding & Makar, 2022).

IBL approaches to learning involve students addressing a complex problem or question, identifying or developing a method or strategy to solve it, and then evaluating the outcome and approach (e.g., Bruder & Prescott, 2013). During IBL, "students are cognitively engaged in sense making, developing evidence-based explanations, and communicating their ideas" (Hmelo-Silver et al., 2007, p. 100). Through these activities, students are encouraged to engage with mathematical concepts in depth, promoting a deep understanding of the subject matter (Carpenter et al., 2012), increased transferability of learning (Duffy & Raymer, 2010), improved motivation and perceived relevance of mathematics (Bruder & Prescott, 2013), and a critical stance (Goos, 2004). However, IBL is not without difficulties, including: extended time demands limiting curriculum content coverage; inadvertent privileging of students who have prior knowledge (Kazemi & Franke, 2004); student struggle and frustration (Sengupta-Irving & Enyedy, 2015); and, the need for teacher preparation and skill at implementation, which can be challenging for some teachers to acquire (Makar, 2007).

Table 1

Model of Knowledge Domains in Inquiry-Based Learning (Fielding-Wells, 2016)

Knowledge Domain	Knowledge Sub domain	Code	Description
Context Knowledge:	Prior Experience	CK:P	Past involvement with the context at school or otherwise.
	Understanding	CK:U	Knowledge of the context from experience, but may also derive from teaching, reading or discussion.
	Discourse	CK:D	Underlying language and terminology required to support understanding and discussion.
	Affect	CK:A	Feelings or intuitively held beliefs about a context.
Mathematical Knowledge:	Knowledge	MK:K	Conceptual and procedural knowledge associated with the discipline and curriculum.
	Discourse	MK:D	Using the language and terminology of the discipline to express and comprehend ideas.
	Community Practices	MK:C	Mathematical practices that increasingly approximate the authentic practices of mathematicians.
	Affect	MK:A	Disposition and willingness to engage in mathematics and use the knowledge of the discipline.

To engage with IBL, students are required to draw on a number of knowledge domains, including, but not limited to, Context Knowledge and Mathematical Knowledge. These domains have been conceptualised, along with a number of sub-domains, through classroom research of IBL in practice (see Table 1, Fielding-Wells, 2016). It is important to note that context in this model usually does not have an immediately visible mathematical underpinning, for instance, a sport, a children's book or game, or a common practice such as a meal.

Researchers have identified a number of issues when supporting teachers to teach using IBL, particularly if they are accustomed to more teacher-centred approaches (e.g., Makar, 2007). Whereas teachers might be accustomed to planning a lesson in detail, teaching through IBL requires some uncertainty about the direction students may take and potentially the content that may be covered, necessitating deeper consideration of the content by the teacher (Makar, 2007). Teachers must also know how to support a classroom culture conducive to inquiry, including students' familiarity with ambiguity, propensity to risk-take, collaborative skills, and comfort with not 'being told' (Goos, 2004; Makar et al., 2013).

Method

The findings reported in this paper are taken from a larger study which involved repeated interventions into the development of argument-based mathematical inquiry with children. The larger study adopted a Design-Based Research (DBR) (Cobb et al., 2003) approach as DBR is typified by practitioner and researcher collaboration to plan successive learning episodes, reflecting on progress and making progressive adjustments in planning with the intent of improving learning approaches.

The participants reported in this paper included a Year 3 class (~8 years of age) from a high-average socio-economic suburban public school in Australia. The teacher was experienced in IBL in mathematics, having used this method of teaching mathematics for approximately 3 years. The teacher and researcher identified a goal of addressing observed student equiprobability bias by engaging students with a problem context in which outcomes would have unequal likelihood of being obtained. They worked together to develop a game of addition bingo with the inquiry question: *What is the best card for a game Addition Bingo?* In this version of Addition Bingo, each possible combination of the sum of two numbers (1 to 10) is placed in a box. Players have a card with a 5 x 5 array of self-selected numbers. As each paper is drawn (e.g., 9+6) from the box, players mark off the sum (in this case, 15) if it appears on their card. The winner is the first to mark off all of their numbers.

Data included student work samples, transcribed videotapes of each lesson, and field notes including pre-and post-lesson discussions with the teacher. The focus was an analysis of the approach the teacher took to introducing the inquiry prior to presenting the inquiry question. Accordingly, the introductory lesson was viewed in full by all researchers, transcribed in full by the first researcher, and linked to relevant representations. Thematic analysis was carried out following a process described by Braun and Clarke (2022) with initial codes derived from the Domains of Knowledge Framework (refer Table. 1, Fielding-Wells, 2016).

Two of the authors independently analysed the transcript by applying the initial codes and noting any lesson aspects that could not adequately be coded to these initial codes. They then compared their coding and discussed any discrepancies until coding was agreed, referring back to research field notes as warranted. The coding focus was on teacher 'moves' and this included both the teachers' initiation of activity or discussion as well as her responsiveness to opportunities presented by students' discourse/activity.

Findings

The introduction lasted for 100 minutes and was bounded by the commencement of the first lesson and the point at which the teacher introduced the inquiry question: *What is the best card for a game of Addition Bingo?*. Context Knowledge sub codes were most prevalent in the initial stages of the lesson before a shift to a high proportion of Mathematical Knowledge sub codes. Below, we briefly highlight key aspects of the lesson with excerpts to illustrate. Numbers in parentheses refer to line numbers in the excerpts. The excerpts are numbered sequentially for ease of discussing, however, not all interactions are reported. Ellipses (…) indicate omissions.

The teacher commenced the inquiry by ascertaining student familiarity with Bingo. Even though all students stated prior experience, she proceeded to question the students' familiarity with the game and asked it to be explained [CK-P]. She then had students play two games to ensure that they all have common contextual experience [CK-P], and as a means of developing common contextual language [CK-D].

In the excerpt below [1-3], the teacher is introducing the game to be played [CK-U]. After which the students design their own Bingo cards, as distinct from using an existing game cards. This was deliberate [field notes] to provide experiences which would focus students' attention on the need to consider the parameters of the set of possible outcomes to be called.

1	Teacher:	… before I start I'm going to tell you something. These are all the numbers from 1 to 50. … and these are going to be my numbers for calling out. What I want you to do is choose any numbers you like between 1 and 50 and write them in the first square—in the first bingo card. …So how many numbers will you have in there?
2	Students:	25
3	Teacher:	… So, I want you to choose 25 numbers between 1 and 50…and fill up that box.

The teacher moved between the students as they created their cards, observing and answering direct questions, noting errors but not correcting or providing feedback [MK-C]. A sample of the interactions below [4-5] illustrates. These reflect the teachers' desire for students to learn experientially rather than being told, and was observed frequently through the lesson.

4	Student:	Can you have the same number twice?
5	Teacher:	If you would like, you can. You decide whether you think that would be a good idea to have the same number twice.

The teacher proceeded to call the Bingo numbers, using language often associated with Bingo for authenticity (e.g., "legs eleven"). She observes students' emerging realisations of their errors but does not discuss these yet. When the first student to call Bingo does so erroneously (duplicated a number on their card), she mentions this but defers it, "Oh you had number three two times, I will get back to that Troy" [MK-C]. As the games progress, students began realising their errors and calling out (e.g. "I had 50 twice"; "oh …there's two eights"). The teacher attempts to continue; however, the students continue to comment about duplicates, so she addresses the issue [6-7], using the opportunity to draw students' attention to possible outcomes while also prompting the language of chance [MK-K, MK-D].

6	Teacher:	If I had all the numbers from 1 to 50 and then all the numbers from 1 to 50 again, so if I had two of each, how many threes could you have written down then? … would it be possible for you to get three twice?
7	Students	Yeah

The teacher continued calling numbers until a student legitimately won. She then used students' newly developed collective experience and understanding [CK-P; CK-U] [8] to begin to focus on the mathematical content of the lesson, drawing on students' prior knowledge while focussing attention on the need to consider possible outcomes [MK-K]. She elicits students conceptions about chance [MK-K][8-16] The teachers' intention for the students to learn experientially [MK-C] is further apparent in this exchange.

8	Teacher:	Ok Duncan has got Bingo. If we play this game again, using exactly the same numbers, will Duncan win again?
9	Students:	[calling out simultaneously] No; possibly; 50:50; even; yes.
10	Teacher:	50:50 [acknowledging this response]?
11	Justine:	The same numbers called can be called again or they might not.
12	Teacher	So, 50:50 means and even chance? Is there an even chance that I could call out those numbers again? Like 50:50
13	Students:	Yes
14	Justine:	Like you've got 20 numbers…25 numbers, so that's half of 50… but you've still got 25 numbers that haven't been called out so they might be the ones that are being pulled out this time.
15	Teacher:	That is a really interesting way of explaining 50:50. You think that because we've got 25 numbers that we've written down and there are 50 tickets to pull out …then 50:50. [no judging of response by T—just accepting]
16	Student:	Not 50-50, 25-25

Following on from this interaction, the teacher began to address the issues that she had previously observed by drawing on students' experiences to support them to challenge their practices/ideas [MK-C]. The first issue addressed was a reiteration of the duplication of numbers beyond what was supported by the possible outcomes [MK-K]. The second was the use of numbers that did not occur within the range of possible outcomes [MK-K][17-22]. In both instances the teacher was guiding students to developing awareness of the importance of considering the outcomes possible.

17	Teacher:	Clay, what happened with you? What did you have on your card that you said, I can't win?
18	Clay:	Zero
19	Teacher:	You had a zero?
20	Clay:	I had a zero and I had numbers twice
21	Teacher:	So, you had two things on yours which stopped you from being able to win?... So, you had a zero. Why is that a problem?
22	Clay:	Only 1 to 50

The teacher then proceeded to elicit potential (mis)conceptions related to the topic that had not yet arisen through game play, including the notion of 'luck' in respect to numbers being drawn [MK-K] [23-26]. Once again, these were elicited but not corrected [MK-C].

23	Teacher:	Is there a number that you think is a lucky number that comes up all the time? We don't know yet. I suppose we've only had one game.
24	Jess:	I think 11
25	Teacher:	Why do you like 11?
26	Jess:	Well I like 11 because one time like I played a game with one of my cousins and one time they called out 11 and I had 11 on my bingo card

The teacher had students design a second Bingo card, with the same requirements, reminding them to focus on what they had learned in the first round. During this second round, she took multiple opportunities to check students' understanding of the likelihood of occurrences and their use of the language of probability (e.g., likely, unlikely, highly likely, highly unlikely, uncertain, certain, possible, impossible), through having students identify and justify their number selection [MK-K; MKC; MK-D]. Once satisfied with students' use of terminology, she encouraged them to think about probability in numerical terms [27-36].

27	Teacher:	What is the chance. What is the likelihood of my next number being a one-digit number?
28	Paul:	Possible or unlikely
29	Teacher:	Possible or unlikely. Why do you say that?
30	Paul:	Because 3, 4, no 5 of the one-digits have been called out
31	Teacher:	So how many are left to be called out?
32	Paul:	4
33	Teacher:	So, there are 4 left to be called out. So, what's the likelihood of me pulling out a single-digit number.
34	Leah:	Likely
35	Teacher:	We have pulled out 26 numbers all together. There are only 4 single digit numbers left, so what is the likelihood of me pulling out a single digit number? …Can you base it on any maths?
36	Student:	Fifty-fifty

In the above exchange, the teacher tested the students comfort with representing or thinking about probability as an informal numerical representation (i.e. four out of 26) [31-36][MK-K]; however, the students were not ready to move into this practice and the lesson ended.

In the following lesson, the teacher repeated the Bingo game with a 3 x 3 Bingo card and a number set from 1-20 inclusive. After giving the instruction to the children regarding the new size and number set, she instructed them to create a new Bingo card. The new arrangement allowed her to see if the students were transferring their understanding of outcomes to an altered context [MK-

K]. After the cards were made, the teacher again drew whole class attention to taking a numerical approach to chance [37-40].

37	Teacher:	So, if we pick one number out, what is the chance of it being your number?
38	Paul:	1 out of 20. No. It would be 9 out of 20.
39	Teacher:	OK. So, the first time I pull out a number, you have a one in 20 chance of it being your number?
40	Paul:	No. 9 out of 20 still because you've got 9 numbers still.

Despite this discussion involving the whole class, the remainder of the class continued with the language of probability rather than numerical representations. About 20 minutes into the lesson, Paul again mentioned a probability in numeric terms [44]. This time, other students began to use fractional language in their comments [MK-K][MK-D] [45] and the teacher capitalised on the opportunity. However, she also validated the use of non-numerical terms [47] to support students who were not yet ready to make the shift.

41	Teacher:	There are three numbers left, what is Bruce's chance of getting his number. Do you know another way of saying it without using the language of likelihood?
42	Student:	Probably
43	Teacher:	That is the language of likelihood. What is another way we could say it?
44	Paul:	9 out of 10
45	Bethany:	I think about 7 out of 10
46	Teacher:	[Many hands up] Write down what you think, you are all itching to tell me.
47	Teacher	If you've got something written down, that's good. If you've got nothing written down you can use the language of likelihood to tell me.

The teacher continued on to use the students' responses to demonstrate a fraction, commencing with the 1/3 identified by many students, naming its parts and reminding the students of the names and meanings of numerator, denominator and vinculum. She then moved on to introduce the inquiry question to the students.

Discussion & Conclusion

In this paper, we examined how an experienced GMI teacher introduced her students to an inquiry problem. We did so in order to illustrate how a teacher might introduce a GMI question. Proponents of explicit instruction argue the need to provide a clear focus on what is to be learnt, through clear identification of the mathematics to be focussed upon and the necessary procedures, which are practiced (Sweller, 2012). Conversely, in GMI, the teacher provides and supports involvement with tasks and discussions so that the student is able to notice and identify what is important in the task experientially and through real-life or life-like contexts, thereby demonstrating the relevance and application of mathematics.

We do not dispute that explicit instruction could have reached the objectives of learning more quickly. Students could have been told the importance of considering the elements of a set, and instructed not to use numbers outside of the set or to duplicate numbers. By distinction, this teacher took 100 minutes, over almost two lessons to establish the contextual and mathematical background she felt necessary to ensure students had sufficient understanding to discover the importance of these aspects for themselves. Later in the lesson, this experience facilitated the students' decisions to construct a variety of representations to determine frequency for Addition Bingo—including frequency charts—which they had not previously had experience with but rather 'invented' for themselves.

The notion that students may have discomfort with not being 'told' the processes or procedures necessary to address the problem remains a consideration. However, exploring and even negotiating the boundaries of problems, discovering errors and limitations, and testing these within a context are the ways of knowing the discipline of mathematics and being 'mathematicians' (Dewey, 1938). In the excerpts above, the teacher facilitates students' involvement with a mathematical inquiry question by establishing and enhancing students' knowledge of a context in which the mathematics (or statistics) becomes relevant. Rather than privileging students with prior knowledge of the context (Kazemi & Franke, 2004), she seeks to establish common prior knowledge through engagement with the context before introduction of the problem. This includes understanding and contextual discourse.

The teacher also took time to establish prior mathematical knowledge, using the context to achieve two purposes: 1) supporting the students to discover and explore important concepts for themselves (a key shortcoming of explicit instruction (Kuhn et al., 2000), including the importance of considering the boundaries of a set and the frequency of the numbers in that set; and, 2) exploring students' mathematical knowledge and advancing potential understanding (e.g., readiness to shift from probabilistic language to fractional representations). In this way, she was able to determine the level of support and challenge that might be required to balance engagement and frustration (Sengupta-Irving & Enyedy, 2015). The teacher can also be seen to change the context slightly, from a 5x5 Bingo card with 50 calling numbers, to a 3x3 Bingo card with 25 calling numbers. By doing so, the teacher is testing for transfer—have these concepts been understood—or have the students merely become aware of their relevance as related to a single contextual instance? Again, a shortcoming of explicit instruction lies in the potential difficulty with transfer of learning (Ewing, 2011). Yet here we see transfer intentionally built into lesson design from the earliest stages.

The proposal is not that all learning must take place through GMI and we recognise that there are limitations to both IBL/GMI and explicit instruction. However, in a time when explicit instruction is supported as a sole approach, the potential for GMI to counter limitations associated with this explicit instruction must be considered. As such, we sought to identify some key practical considerations for teachers wishing to establish GMI activities with their classes: the need to support students to have common context experiences, understandings and discourse; and, the need to explore and build the boundaries of mathematical knowledge, community practices and discourse, prior to engagement with the Inquiry Question.

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