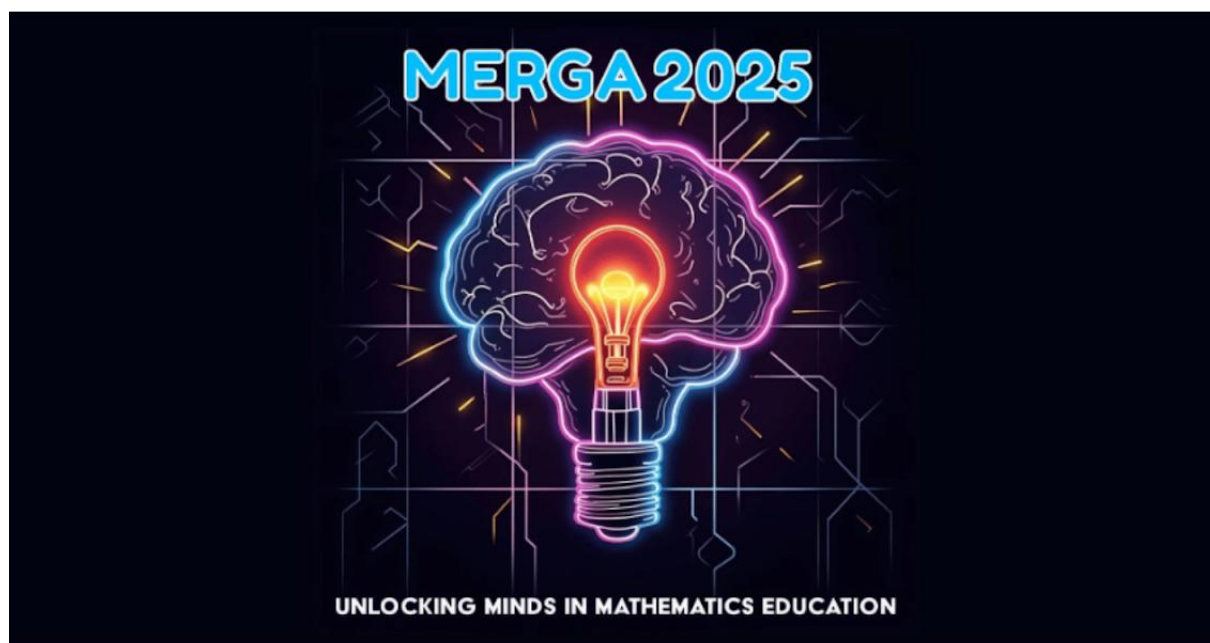




Mathematics Education Research Group of Australasia

UNLOCKING MINDS IN MATHEMATICS EDUCATION



Proceedings of the 47th Annual Conference
of the Mathematics Education Research Group of Australasia
6–10 July 2025, Canberra

Sitti Maesuri Patahuddin, Lorraine Gaunt, Danielle Harris, & Kristen Tripet (Editors)

Unlocking Minds in Mathematics Education

Proceedings of the 47th Annual Conference of the Mathematics Education Research Group of Australasia

Edited by Sitti Maesuri Patahuddin, Lorraine Gaunt, Danielle Harris, & Kristen Tripet

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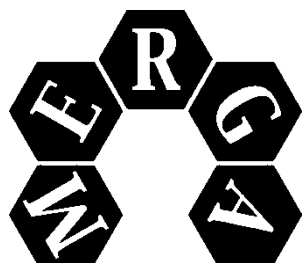
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Preface

This is a record of the Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia (MERGA). The conference was hosted by University of Canberra and organised by colleagues from University of Canberra, Charles Sturt University, and Academy of Science. The Proceedings were published online at the MERGA website.

The University of Canberra is located on the lands of the Ngunnawal people, the Traditional Custodians of the Canberra region. The Ngunnawal people have lived on and cared for this land for thousands of years, and their enduring connection to Country is acknowledged with deep respect. Established in 1967 as the Canberra College of Advanced Education and attaining university status in 1990, the University of Canberra has developed into a leading institution for professional education and research. As of November 2024, it serves a diverse community of more than 18,000 students—over 11,800 undergraduate and 6,300 postgraduates—representing all parts of Australia and more than 100 countries.

Canberra, Australia's capital, provides a unique and inspiring setting for the 47th Annual Conference of MERGA. Known for its natural beauty, thoughtful urban design, and rich cultural institutions, Canberra fosters reflection, innovation, and connection—qualities deeply aligned with the aims of mathematics education research. The conference theme *Unlocking Minds in Mathematics Education* draws on Canberra's identity as a city of ideas, policy, and education. Just as the city brings together diverse communities and disciplines to shape national direction, so too does the field of mathematics education require navigating complex and evolving pathways. This theme reflects MERGA's commitment to charting forward-thinking, inclusive, and research-informed trajectories for the future of mathematics education.

Embracing the theme *Unlocking Minds in Mathematics Education*, the local organising committee encouraged participants to approach the conference with curiosity, openness, and a willingness to explore new ideas, pedagogies, and technologies. This spirit of intellectual openness fostered a collaborative and innovative environment, where delegates felt empowered to share cutting-edge research, effective teaching practices, and transformative tools. The theme recognised the evolving landscape of mathematics education, highlighting the importance of diverse perspectives and inclusive approaches. With a broad range of topics and speakers from varied educational, cultural, and disciplinary backgrounds, the conference offered a stimulating and enriching experience. Delegates from Australia, Germany, Indonesia, Japan, Mauritius, New Zealand, Norway, Papua New Guinea, Singapore, South Africa, United Arab Emirates, and United States of America. took part, contributing to a vibrant exchange of ideas across international contexts.

The theme *Unlocking Minds in Mathematics Education* was powerfully initiated foregrounded by three keynote presentations that invited delegates to reflect deeply on pedagogical purpose, innovation, and impact. The opening keynote by Professor Susanne Prediger from TU Dortmund University, titled *Unlocking Minds by Conceptually Focused, Student-Centred Explicit Mathematics Teaching and Quantitative Evidence for Effectiveness*, focused on conceptually oriented and language-responsive instructional approaches. Her address explored how language and conceptual understanding intersect in mathematics classrooms, particularly in supporting diverse learners—a vital consideration in unlocking equitable mathematics education.

Continuing the keynote, *Comparing and Discussing Multiple Strategies to Promote Learning in Mathematics Classrooms*, was delivered by Professor Bethany Rittle-Johnson from Vanderbilt University. Drawing on more than a decade of research, her address highlighted how structured comparison of mathematical strategies can enhance learning and support students'

cognitive development. Her work underscored the importance of evidence-based instructional design rooted in cognitive science, providing practical insights for classroom application.

The Clements/Foyster Lecture was delivered by Professor Catherine Attard from Western Sydney University. In *Unlocking Minds: Exploring the Pendulum of Mathematics Pedagogy*, Professor Attard critically examined the current debates surrounding mathematics teaching practices. Drawing from her extensive experience as a teacher, researcher, and leader, she challenged narrow interpretations of evidence and advocated for a broader, more inclusive understanding of effective pedagogy. Her reflections encouraged the MERGA community to consider how its collective research can shape meaningful change in mathematics education. Together, these keynote presentations embodied the conference theme by challenging assumptions, opening space for diverse perspectives, and equipping participants to think boldly about the future of mathematics education.

At MERGA 47, the simplification of the awards nomination process led to a record number of submissions for the Beth Southwell Practical Implications Award, which recognises outstanding contributions that bridge research and classroom practice. A total of 12 candidates nominated their papers for consideration, with an additional 3 papers put forward by the review panels. From this pool, 6 papers were shortlisted and evaluated by a panel of four independent expert assessors. After a rigorous review, the panel selected a distinguished recipient for the 2025 award. The Beth Southwell Practical Implications Award recipient exemplifies the meaningful integration of research into educational practice, reinforcing the purpose and impact of the award within the MERGA community.

A highlight of MERGA 47 was the special pre-conference workshop delivered by Professor Janette Bobis and Professor Merrilyn Goos, held on the Sunday preceding the MERGA Newcomer's Welcome. Building on Janette's 2024 Clements-Foyster keynote and expanding on themes from Merrilyn's Summer Speaker Series presentation, the workshop *Balance vs Blended: Developing a Fulfilling Career in Academia* offered rich insights into navigating the complexities of academic life. Participants were invited to reflect on their career trajectories and explore strategies for sustaining long-term professional growth through strategic adaptability, the use of smart tools, and meaningful collaborations. The session fostered thoughtful dialogue and set a collegial and reflective tone for the conference.

A total of 13 self-nominations were received for the Early Career Research Award, of which 9 were deemed eligible. Following the withdrawal of one application, 8 papers were shortlisted and evaluated by a panel of expert judges, based on the quality of both the written submissions and the oral presentations delivered during the conference. Three applicants progressed to the final judging round.

As part of MERGA 47's ongoing commitment to supporting scholarly development, the Early Bird submission round provided authors with the opportunity to receive formative feedback on their work. A total of 26 papers were submitted through this process, demonstrating strong engagement with the conference's developmental objectives and a shared dedication to advancing research quality within the mathematics education community.

The formal review process commenced with the submission of blinded copies of each paper. All submissions were independently assessed by two reviewers, with additional oversight provided by a panel convenor to ensure consistency and rigour. Based on the reviewers' evaluations, papers were classified as *Excellent*, *Good*, or *Acceptable* (subject to minor revision). In some instances, papers were accepted for presentation only, in the form of a Short Communication (Short Com).

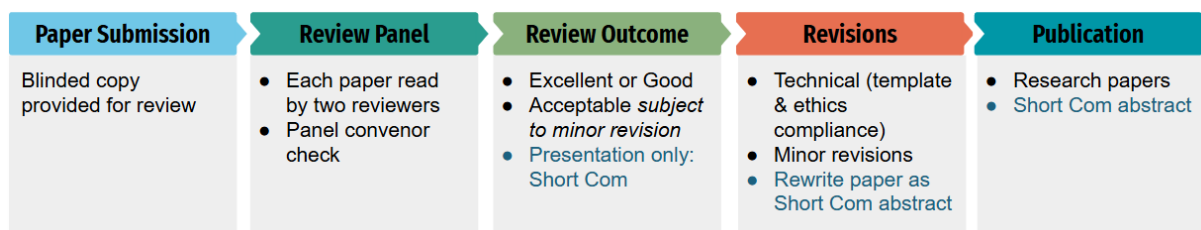
Where necessary, authors were asked to revise their submissions to address issues related to formatting, ethical compliance, or minor content revisions. In certain cases, full papers were revised and resubmitted as Short Communication abstracts. Following this multi-stage review

process, accepted contributions were published in the conference proceedings as either full research papers or Short Com abstracts.

The Proceedings of MERGA 47 comprise a rich collection of scholarly work, including plenary papers, symposium contributions, full research papers, and abstracts of short communications and round tables. All research and symposium submissions were subject to a rigorous double-blind peer review process, conducted by panels of experienced mathematics education researchers. An overview of the review process is illustrated in Figure 1.

Figure 1

Overview of the MERGA Paper Review Process



The Proceedings of MERGA 47 reflect the diversity and depth of mathematics education research conducted both within the region and internationally by members of the MERGA research community. The Editorial Team extends sincere appreciation to all contributing authors for sharing their scholarly work. We also express our gratitude to the Review Panel Convenors and Reviewers for their professionalism and dedication in providing thorough and constructive feedback. Their efforts were instrumental in maintaining the high academic standards that MERGA is known for.

We gratefully acknowledge the many individuals and groups whose contributions made MERGA 47 possible. In particular, we thank the Local Organising Committee, the MERGA Executive, the Faculty of Education at the University of Canberra, the Australian Association of Mathematics Teachers, our administrative support officers (Amanda Thomas and Vicki Steinle), our technical and formatting editor, Agustin Ernawati, and our valued sponsors: the Australian Association of Mathematics Teachers, the Australian Academy of Science, Canberra Mathematical Association, Australian Mathematics Trust, STEM Education Research Centre (SERC), University of Canberra, and Springer Publishing.

A special note of thanks is extended to Aunty Violet Sheridan for delivering the Welcome to Country, marking the beginning of the conference with respect and cultural significance.

Tracy Logan (Conference Convenor)

Sitti Maesuri Patahuddin, Lorraine Gaunt, Danielle Harris, Kristen Tripet (Editors)

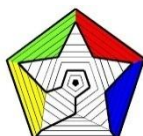


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Clements-Foyster Lecture

The Clements-Foyster Lecture acknowledges an eminent mathematics education researcher from Australia, New Zealand, or a South East Asian rim country, who is invited to present a keynote address at the annual MERGA conference. This annual keynote address is named in honour of Ken Clements and John Foyster who initiated and organised the first Mathematics Education Research Group of Australia Conference at Monash University in 1977. This led to the establishment of the organisation now known as MERGA.

Unlocking Minds: Exploring the Pendulum of Mathematics Pedagogy

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Current debates and the related impact of policymakers and educational leaders on teaching and learning in mathematics is a cause of concern and even anxiety amongst researchers and teachers alike. While these debates are playing out differently across the world, there are common challenges we face. In this lecture I draw on voices from previous Clements-Foyster lectures and use the lens of my own experiences as teacher, researcher, and leader of MERGA and AAMT to discuss and explore the current pedagogy debate in mathematics education. I argue against the use of a narrow evidence base that is fuelling the debate and I reflect on the current and potential future impact of research emerging from MERGA as a collective, and individual members.

It is a great privilege and honour to be invited to deliver the MERGA47 Clements-Foyster Lecture at what is arguably a tumultuous time in mathematics education in Australia, New Zealand and beyond. Given that the timely theme of this conference is *Unlocking Minds in Mathematics Education*, the current landscape will serve as a context for this lecture, where I will reflect on the impact of mathematics education research on education policy and classroom practice.

Previous Clements-Foyster lectures have provoked reflection on a range of topics including the quality of research in mathematics education, debates over dichotomies such as that which is currently occurring, and the positioning of MERGA as a key influence in mathematics education. For example, in the inaugural Clements-Foyster address in 2005, David Clarke discussed a range of dichotomies in education. It is twenty years on, and those dichotomies appear unchanged and perhaps more divided than ever. Ten years later, Tom Lowrie wrote: “it is also important that our research empowers people, and that our recommendations and implications improve systems, especially for the disadvantaged” (2015, p. 23). MERGA’s mission states the association is “committed to growing a research community that shapes the future of mathematics education through quality research” (n.d.). While it appears that we may not currently be exercising significant influence on policy and subsequently practice (particularly in Australia), this does not mean we should not continue to strive to influence policymakers and system leaders moving forward. It is time to pause and perhaps recalibrate and consider whose minds we want to unlock and how we should be working collectively to do this.

In his Clements-Foyster address at MERGA 37, Peter Galbraith (2014) concluded his paper by proposing that we, as individual researchers in mathematics education, write down the things about the theory and practice of mathematics that currently outrage us before sharing those outrages with each other. He wondered what might emerge, what imperatives would be suggested, and what we could do about it. In this lecture I too aim to stimulate critical reflection and discussion regarding the current landscape of mathematics education for us as a research community, and for each individual, whether researcher or teacher. I will pose questions that may not be possible to answer yet remain worthy of consideration. I draw on my experiences as a teacher, a researcher, a past MERGA president, and the current board Chair of the Australian Association of Mathematics Teachers (AAMT) to begin with a brief discussion on the current context before considering the impact of mathematics education research and the

potential and opportunities for increasing the impact of our research where it matters, in mathematics classrooms.

Evidence-Based Mathematics Pedagogy

In conceptualising the current landscape, I use the analogy of a pendulum. Within this analogy, the overhead support of the pendulum represents the mandated curriculum to be delivered and enacted in mathematics classrooms. The teacher is represented by the ‘pivot’, which connects the pendulum string to the overhead support, allowing the pendulum to swing the ‘bob’ from side to side. At opposite sides of the pendulum are dichotomies such as direct instruction (DI) versus inquiry-based learning (IBL), which represent elements of the current debates in Australia, New Zealand and beyond. At this point in time, across several school systems in Australia, the pendulum appears to have oscillated to the extreme of one side, towards a DI approach, where it appears to be somewhat stuck, and to some degree, is resulting in pedagogies that are misaligned or even clashing with curriculum expectations, potentially weakening the enactment of the curriculum in our mathematics classrooms.

Whose Evidence?

A noticeable element within the current debate is the emphasis on using evidence-based practice. In Australia, bodies such as the Australian Education Research Organisation (AERO) (Australian Education Research Organisation (AERO), 2023a, 2023b) and the Centre for Independent Studies (CIS) (Merlo, 2024) use select research as the evidence base to support their stance, with a heavy emphasis on cognitive science, applying findings in a one-size-fits-all approach across discipline areas including mathematics. This has led to a direct and significant influence on policymakers in several of our education systems, including a strong emphasis on highly structured explicit teaching in NSW (NSW Department of Education, 2024), Victorian government school systems (Department of Education and Training, 2024) and other states and systems.

There is no argument against the merits of cognitive load theory and the use of explicit teaching in mathematics education. However, this should not be highlighted at the expense of flexible and responsive teaching, student engagement, and authentic mathematical problem solving and inquiry, which appears to have been diluted in directives to teachers (e.g. Centre for Educational Studies and Evaluation, 2020). For example, teachers in the NSW Department of Education have been informed that “explicit teaching does not involve students engaging in independent learning activities and problem solving before teachers provide the necessary explanations, demonstration or modelling” (p.1), in direct contrast with findings from mathematics education research that claims these types of strategies potentially limit opportunities for higher level mathematical thinking (Ingram et al., 2020).

Moreover, what is alarming about the current situation is the lack of reference to mathematics education research and a lack of consultancy with practicing educators and academics directly involved in mathematics education, despite efforts to provide formal and constructive feedback (Mathematics Education Research Group of Australasia, 2023). This has exacerbated the issues caused by the narrow evidence base: an omission that has not gone unnoticed or unquestioned, as recorded in the Australian Government Senate Estimates (Parliament of Australia, May 2024).

Lingard (2013) states “any educational policy research of any type ought to have at the broadest level a desire to make things better in education...to improve education policy, including conceptualization and enactment” (p.116). While there is no argument the desire to improve mathematics education is present amongst educational leaders and policymakers, the narrow use of evidence currently informing policy has resulted in a range of interpretations, translations and applications across school systems that do not always promote what we in the

mathematics education research community consider to be effective, evidence-based, and responsive practice. For example, some systems have mandated the use of a specific, DI teaching program with the expectation that a one-size-fits-all approach will improve student outcomes in mathematics, with no acknowledgement of context, reducing the role of the teacher to technician rather than professional (Wiliam, 2019). Other systems are strongly encouraging prescribed lesson structures that do not allow for flexibility and responsive teaching, with no acknowledgement of teacher expertise and restricting teachers' autonomy and ability to respond to their students' needs as they arise. There is a clear gap between mathematics education research and policymaking, that have led to reforms that do not fully consider the reality of classrooms.

Best Practice?

If we consider mathematics education through the lens of the theory of practice architectures as discussed by Peter Grootenboer at MERGA44 (2022), we acknowledge that a single practice cannot address the needs of all students due to the complexities of the site where learning occurs.

...the notion of “best practice” is, at best, an unhelpful myth, and at worst, a damaging misconception that sees all learners, sites, and communities as homogenous. The site-based nature of practices, including mathematics education practices, means one can only talk of “best practices *here and now*.” (p.3)

Other evidence from mathematics education research aligns with this position (e.g. Anthony & Walshaw, 2009; Clarke, 2005) yet this evidence appears to have been ignored. Recent reports using an evidence-base sourced from cognitive science and educational psychology, espousing a “best practice” in mathematics teaching, have made false assumptions that are conveniently not supported by evidence leading to broad generalisations about pedagogy and student learning. Further, elements of the current debate have grown out of arguments that were initiated within the literacy/phonics domain (Hunter et al., 2023). For example, in the CIS report that attempts to replicate the Science of Reading within the discipline of mathematics education, *Science of Maths and How to Apply It*, Merlo claims: “Engagement happens via building competency and setting students up for success, not via relaxing requirements on correctness of answers or refraining from using timed-tests” (2024, p. 2), indicating a lack of attention to research that clearly defines engagement as more than compliance, and ignoring the multidimensional nature of engagement (e.g. Attard 2014, Fredricks etc. 2004). This is followed by the assumption that there are teachers of mathematics who do not value correct answers from students. Further, other claims in this report are equally misinformed and not supported by evidence:

Pendulum-swings in the last 40 years have gone from almost pure emphasis on procedural knowledge and rote-learning to the detriment of understanding, to conceptual understanding without building fluency and teaching procedures. (p.18)

False Dichotomies

The practice of using a select range of evidence to support a specific argument is not a new one, as noted in de Jong et al. (2022), who discuss the same pedagogical debate. Further, there is evidence that this dichotomy is a false one, and evidence exists to support the use of DI, IBL, and other pedagogies in a complementary manner. In his Clements-Foyster lecture, Clarke (2005) argued that dichotomies constructed as oppositional “offers a set of false choices, sanctifying one alternative, while demonising the other” (p.13). Clarke used the term ‘essential complementarities’ to describe how such oppositional ideas should be considered complementary. He provided the example of the dichotomy of socio-cultural and constructivist theories. He posited that while both theories can be constructed as competing, “Any conception of either theory that precludes the other is arguably inadequate” (2005, p.2). This can be applied

to the current debate and in support of this position, de Jong et al., (2023) provide an evidence-base to support the claim that IBL and DI can be complementary, stating:

Regardless of whether direct instruction or inquiry is the core of the lesson, lesson series or curriculum, additional instructional strategies and activities are usually needed to make the approach work. Different instructional strategies can coexist in a lesson. (p.8)

From the perspective of a teacher and researcher in mathematics, it is common sense that there is no one way to teach mathematics. No single practice should be employed at the expense of others, and at the expense of student attainment and engagement. As stated by Siemon (2019):

...when one approach/practice is privileged over the other on the basis of research it invariably leads to a form of tribalism that pits individual against individual and group against group in a way that is unproductive” (p. 50).

The pedagogy pendulum should oscillate in response to student needs. Context matters, and as Grootenboer (2022) states:

... *mega conditions* (e.g., national or state curricula, external assessment regimes, policies) that are overly restrictive and controlling, because they limit the capacity for mathematics education that is responsive to the unique site-based needs and requirements and conditions. For example, if mathematics teachers are to practice in a reflective and responsive educational manner, then they require scope to develop and enact their pedagogy within the guidance of curricula, and not be slavishly required to follow a detailed prescription of teaching activity. (p.5)

While there may be several reasons for the reliance on a skewed evidence base informing policy, for example, a lack of large-scale quantitative studies, this is not a new phenomenon. Berliner (2002) commented on this issue, rejecting the reliance on “scientific” research and stating that such a reliance results in a misunderstanding of educational research. Berliner posits that educational research is a ‘hard-to-do’ science due to “the power of context, the ubiquity of interactions, and the problem of “decade by findings” interactions” (p.18). Arguably it is the ubiquity of interactions alongside unique contexts, including differences in personnel, programs, teaching methods, budgets, community support and socio-economic status, that are absent from the narrow evidence base currently being presented to leaders and teachers in schools. In addition, Berliner’s concept of “decade by findings” is extremely relevant to the current landscape and the use of research findings that no longer align with the context of education in 2025. A significant amount of the evidence behind much of the current push towards prescriptive teaching approaches is drawn from research that is more than 20 years old, making it obsolete because of shifts in social, cultural, and intellectual environments.

We should never lose sight of the fact that children and teachers in classrooms are conscious, sentient and purposive human beings, so no scientific explanation of human behaviour could ever be complete. (Berliner, 2002, p.20)

Looking Forward

The current landscape has, to some extent, resulted in uncertainty for practitioners (policy makers and teachers) and researchers. Rather than lament the current situation, we must be proactive rather than reactive in our response. With this as a motivation, the AAMT recently published a position paper (2025) that draws on a broad and balanced range of research evidence to provide a clear repertoire of strategies that includes a range of practices. The paper is supported by a dynamic reading list (including evidence from both sides of the debate) to encourage informed and critical discussion. In theory this paper, alongside MERGA’s position statement on pedagogy (2025), should begin to release the pedagogy pendulum and allow teachers to control its swings according to student needs. However, there is no guarantee that broader mathematics education research will become a part of the evidence base used to inform policy and practice unless we reflect on what we are doing, and how we are doing it now, and ways we can be doing it differently to increase our impact on policy, school leadership and in classrooms, opening minds where it matters most.

The current debates are not new and should continue to be argued. In an era where educational policy often contradicts findings from mathematics education research, it is crucial for us, as a research community, to critically assess our impact. As Lowrie (2015) stated, we may have become too comfortable, and this sense of comfort has resulted in policy makers turning away from MERGA research in their bid to cause change.

Reflecting on Impact

The current landscape leads me to question why our (MERGA'S) research may not be having the intended impact. Why is our work not resulting in a pendulum that oscillates gently according to student needs and contextual circumstances? In her Clements-Foyster lecture at MERGA46 Janette Bobis (2024) discussed the importance of understanding the concept of research significance, or the 'so what' factor. Bobis presented a list of synonyms for significance, and this included the terms implications, benefits and impact. While research may be significant in generating new knowledge, it does not always translate into real-world impact beyond academia or influence educational policy post publication. Bobis also considered Hiebert et al.'s Chain of Coherence (2023) that runs through a research study and perhaps we should be considering how this model could be extended to plan for, improve and assure ongoing impact.

Further, it may be that we need to unlock our own minds and consider how we define impact and what we, as educational researchers, value as impact. While research institutions highly value citations in academic literature and journal impact scores, educational research also prioritises its influence on policy and classroom practice. However, achieving this impact is more challenging, and measuring it remains difficult. Lingard acknowledges that "the impact of research on practitioners works in multifarious, mediated, and non-linear ways" (2013, p.115). However, academics receive little institutional recognition for publishing in professional journals or producing outputs that directly reach practitioners in classrooms. As a result, there is little individual incentive to promote research outcomes beyond academia. However, there is a collective incentive to improve mathematics, and this is where MERGA's role is critical.

Over a decade ago Lingard (2013) questioned whether there is a "pressing need to reconsider the actual and desired nature of research-policy relationships in education" (p.113). Arguably, the need is more pressing now than ever before. Lingard provides a distinction between research relating explicitly to policy matters and the larger body of research that has the purpose of contributing to knowledge more generally, which perhaps describes much of the research emerging from MERGA members. The most recent RIMEA publication (Mesiti et al., 2024) includes a depth and breadth of research that not only refutes much of the evidence-base currently informing policy but has clear implications for practice. However, while we may view such findings as important, they may not explicitly address the perceived needs of those developing policies to addresses system-wide problems. Furthermore, we must consider whether readers of RIMEA are those we should be trying to influence. What more could we do with this important synthesis of research?

Increasing our Impact and Influence

Reflecting on the present landscape and adopting a proactive mindset, we need to consider how MERGA, both as a collective and as individual researchers, can progress to realign our approaches, and to unlock minds. Apart from increasing the visibility and reach of MERGA members' research outputs, we must consider how we can align our work to the work of policymakers, educational leaders, and teachers. Is it a question of simply focusing on impact, or should we reflect on whether we are asking the right questions? What has led to the use of the narrow evidence-base that is now influencing the practice of mathematics education? Whose

problems are we trying to solve, and is there a mismatch between the problems we are identifying and the problems identified by those who are driving policy?

In educational research, “What works” is usually the wrong question because almost anything works somewhere, and nothing works everywhere. A better question is, “Under what circumstances does this work”, which is why using research to improve education cannot be achieved by slavishly following a recipe dictated from...a “sovereign”. Instead, stakeholders need to become critical consumers of educational research” (Wiliam, 2019, p.137).

It is feasible that those who are driving the current agenda in mathematics education are seeking a simple, cost-effective, scalable solution to what is, essentially, a wicked problem exacerbated by a ‘perfect storm’ of events including teacher shortages, concerns over student performance, and a failure to close equity gaps. While the motivations are sincere (we all share the same goal of improving mathematics education) the solution is reductionist, ignoring the complexity and nuance of the problem that can only be solved by paying attention to individual school contexts and the seeking of solutions from a broad base of mathematics education research evidence, and including evidence sourced from practitioners.

Understanding the problems of practice from the perspective of practitioners is critical if we are to have increased impact. In an editorial focused on improving the impact of educational research, Cai et al., (2017) explore reasons for the divide between research and practice and they suggest that problems identified by school leadership or systems do not necessarily align with teachers’ problems of practice. Similarly, the problems we are trying to solve as researchers may not be aligning with the problems that need solving in systems, schools and classrooms, creating or even expanding the gap between research and practice. To this end, Cai et al. (2017) argue that we must “carefully examine the way teachers are positioned in efforts to improve the impact of research” (p. 4). They emphasise the importance of integrating practitioners into the research community, fostering collaboration to identify problems and develop contextually adaptive solutions.

In a similar vein, Lingard (2013), suggests that for educational research (research *for* rather than *on* education) to have impact and improve education policy and professional practice, educational researchers require a “pedagogical disposition”, and practitioners should have a “researchly disposition” meaning practitioners should be research-informed and research-informing. It is clear from the recent RIMEA publication that many MERGA members have a pedagogical disposition, so perhaps we should consider how we can work to increase or scale up research and initiatives such as the MERGA Teacher Reads to promote a researchly disposition, along with a more concerted effort to include practitioners as co-researchers.

Shaping the Future of Mathematics Education

In the current landscape of mathematics education, the translation of research findings into effective policies and practices presents significant challenges. In this paper I have provided a discussion on the current debates, and I have posed some questions to be considered. I now return to MERGA’s vision statement which is supported by two aims: a commitment to grow our research community, and a commitment to shaping the future of mathematics. The latter aim directly relates to the content of this paper and I conclude with some practical suggestions and thoughts.

MERGA is committed to shaping the future of mathematics education by:

- Ensuring MERGA publications are accessible to stakeholders, including policy makers, researchers and teachers.
- Seeking to influence decision makers to take account of research findings in mathematics education.
- Maintaining liaison with other organisations with interests in mathematics education or educational research and,

- Promoting social justice research in mathematics education. (MERGA, n.d.)

Finding opportunities to communicate and collaborate directly with decision makers is a significant challenge. MERGA has used the opportunity to provide feedback, as stated earlier, however without sustained communications it is difficult to exercise influence. Collaborations with other bodies such as AAMT in Australia and equivalent associations internationally (e.g. NZAMT in New Zealand) will present a united front and give everyone a stronger voice. Likewise, opportunities to conduct research with policymakers and for policy making should be a goal of MERGA.

MERGA's commitment to making publications accessible is commendable. The accessibility of conference proceedings and the format of conference papers allows all stakeholders access to MERGA research. The translation of MERGA Teacher Reads and their dissemination to teachers via the AAMT provides a valuable opportunity to introduce teachers to MERGA. These communications could be enhanced by adding actionable insights for policymakers, leaders, and teachers. Targeted, accessible summaries of research findings can be disseminated directly to policymakers. Similarly, the publication of easy-to-read syntheses of RIMEA chapters for dissemination to stakeholders would improve MERGA's standing as the lead body in mathematics education research in this region. Leveraging opportunities that are presented through the activities of other like-minded organisations such as the AAMT Strength in Numbers podcast could assist in disseminating research findings to a broader audience.

Competing political and practical priorities that result in the cherry picking of research to align with existing policy agendas make it difficult for MERGA studies to gain traction. This, in conjunction with the discontinuity that arises from the political cycle, will continue to result in challenges for MERGA, however we must continue to work towards having a stronger influence on the current landscape of mathematics education. We must work to release the pedagogy pendulum, and we must be reminded that MERGA does not simply consist of its executive members. *We* are MERGA, and together we need to work towards influencing policy and practice that is informed by a balanced evidence base that includes high quality mathematics education research.

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Keynote Presentations

Unlocking Minds by Conceptually Focused, Student-Centred Explicit Mathematics Teaching and Quantitative Evidence for Effectiveness

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Unlocking formerly underserved students' minds requires conceptually focused mathematics teaching approaches, with student-centred and explicit teaching practices for explicating meanings and mathematical structures. Convincing policymakers requires unlocking false methodological dichotomies and providing hard quantitative evidence for effectiveness of these approaches and practices. The contribution aims at convincing about these claims by giving example evidences from the authors' work.

The conference theme *Unlocking minds* offers an interesting metaphor for goals of mathematics education: all minds can grow, but some have been locked (e.g., by a low-expectation education, Boaler, 2002). The theme holds an interesting ambiguity of *whose* minds we want to unlock. First, mathematics education can develop and investigate instructional approaches that contribute to *unlock all students' minds*, with an equity emphasis on underserved students (from underprivileged family backgrounds or with mathematical difficulties). Second, this can go along with *unlocking some teachers' minds* for overcoming low ambitions for underserved students (Beswick, 2007; Wilhelm et al., 2017) and empowering them for ambitious teaching practices. Third, mathematics education research should aim at advising policymakers about promising instructional approaches and teaching practices (I write as a member of state and national policy advisory boards). As we cannot easily change policymakers' minds, this contribution suggests *unlocking our researchers' minds* by providing not only deep qualitative insights but also hard quantitative evidence, which policymakers often look for.

State of Academic and Policy Discourse

Unlocking Students' Minds Requires Overcoming False Instructional Dichotomies

One old false dichotomy was historically referred to as *conceptually versus procedurally focused* instructional approaches, prioritising either conceptual understanding or procedural fluency (Hiebert & Carpenter, 1992). In most countries, consensus emerged that both forms of knowledge need to be intertwined as they can only grow together (Kilpatrick et al., 2001). Even students with difficulties in mathematics cannot be restricted to basic skills for procedures, as conceptual understanding of basic concepts and procedures is pivotal for further learning (Gersten et al., 2009). For example, multi-digit multiplication requires place value understanding and the meaning of multiplication, and expanding fractions requires an understanding of equivalence through refining fraction bars and justifying why refining fraction bars corresponds to multiplying numerator and denominator (Prediger et al., 2022). Training symbolic procedures is thereby only sustainable after students explain actively how symbolic and graphical representations are connected.

The controversy about the second dichotomy is still ongoing: Scholars in mathematics education have often promoted *student-centred approaches* in which students are invited to discover strategies and connections themselves, ideally in independent seat or group work (de Jong et al., 2023; Scherer et al., 2016). For students with difficulties, in contrast, empirical evidence was collected through randomised controlled trials that *explicit direct instruction* is an effective

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 13–21). Canberra: MERGA.

approach, in which teachers state a goal, show the steps of what to do, and provide opportunities for practicing them (Zhang et al., 2022). Lambert and Tan (2020) problematised this dichotomy from a disability study perspective, as students with difficulties risk to be systematically denied access to high-quality instructional approaches when controlled trials restrict to direct instruction. On the other hand, completely open inquiry approaches without guidance have been shown to disadvantage students from marginalised family backgrounds (Lubienski, 2000).

Clarke (2005) already pleaded for “deconstructing the teacher-centred/student-centred dichotomy” (p. 8) and showed in various qualitative analyses that in teacher-moderated whole-class discussions, the responsibility of knowledge construction can be shared with students. Thus, the deeper structure of shared responsibility in knowledge construction, not just the surface level of activity structures, needs to be considered. Hence, conducting whole-class discussions must not be confused with direct instruction.

Regarding deeper structures of instructional approaches, empirical studies have shown that approaches are most efficient when they combine two steps: initial inquiry with student-centred work for eliciting students’ prior knowledge, followed by explicit instruction, e.g., in two-step approaches such as problem-solving before instruction (Loibl et al., 2017), other kinds of guidance such as scaffolding (NAP, 2008), and subsequent training (de Jong et al., 2023).

To sum up, there seems to be an emerging international academic consensus in mathematics education research that overcoming two false dichotomies (procedural vs. conceptual focus and explicit instruction vs. inquiry approaches) is required for unlocking *all* students’ minds, i.e., empowering students to develop full mathematical competence. However, this academic consensus has not yet reached all mathematics teachers and policymakers.

Unlocking Teachers’ Minds to Offer Rich Learning Opportunities

Academic consensus about instructional approaches is only a first step in a long way to implement them in classrooms, as not all mathematics teachers might already be prepared for it. 20 years ago, Boaler’s (2002) qualitative study revealed that teachers’ orientations toward mathematics and their underserved students’ learning influenced the richness of their teaching practices. Wilhelm et al. (2017) showed quantitatively that teachers who assign the source of students’ difficulties in mathematics to their marginalised family backgrounds offer less cognitively demanding and conceptually focused teaching than teachers who acknowledge their own teaching contribution to the difficulties. Hence, teachers’ low expectations align with creating restricted learning opportunities, which justifies the term “underserved students”. Beswick (2007) identified significant differences in how 21 teachers’ thought about students with mathematical difficulties, and provided first quantitative evidence that these teachers’ minds could be unlocked in a nine-hour professional development program, after which the teachers articulated more equitable orientations with higher ambitions. Other authors have also shown that teachers’ minds can indeed be unlocked so that they are willing to offer rich learning opportunities to *all* students, in particular those formerly underserved.

Policymakers’ Minds About Research on Mathematics Teaching and Learning

Implementing research-based instructional approaches depends not only on individual teachers, but also on policy decisions regarding standards, syllabi, textbook regulations, exams, and teacher education standards. But not all research findings about the learning needs of students and teachers receive the same attention in the policymakers’ decision-making processes. This is partially because policymakers seek advice aligned with their own positions and partially because the idea of empirical evidence is narrowed down to hard quantitative evidence from randomised controlled trials (e.g., No Child left behind act, 2001 in the U.S.; see Attard, 2025 for the Australian policy discourse). The entire spectrum of design research, qualitative classroom research, and qualitative professional development research, from which many

highly important insights into the complexities of classroom ecologies were obtained, are then at risk of being sacrificed to the so-called gold standard of quantitative research (Attard, 2025). I emphasise that quantitative research can be a promising last step *after qualitative deep dives*.

Unlocking Our Researchers' Minds and Seeking for Quantitative Evidence for Deep Phenomena and Subtle Effects Found in Qualitative Research

Clarke (2005) pleaded for the complementarity of research approaches by showing how qualitative video studies reveal deeper insights than classical black-box controlled trials and quantitative video studies if they assess mainly procedural skills (without understanding) and capture mainly surface structures (such as talk time, without deeper structures).

Twenty years later, I suggest that the *time has come* to transcend this complementarity and seek for *quantitative evidence for complex and deep phenomena and subtle effects* that were identified in qualitative research. The suggestion stems from a parallel to medicine: In German medical systems, only medical treatments are paid for which hard empirical evidence of effects is provided in randomised controlled trials. Alternative treatments from traditional healers are rarely supported, not because studies have shown they are not effective, but because no controlled trial has shown their effects, so far. However, in recent years, more randomised controlled trials are conducted for alternative treatments, and increasingly, they are accepted in the medical system (or excluded due to proven non-effectiveness).

Similarly, future educational research might aim to gain more hard evidence for complex or deep phenomena and subtle effects of teaching practices that have not yet been in focus of quantitative research on mathematics teaching and learning. For this, we need more refined quantitative instruments. Below, I will substantiate my suggestion with first (humble) examples from our Dortmund research group by which we reacted to limitations in our advisory powers.

Examples From Dortmund Research on Conceptually Focused, Student-Centred Approaches: Successive Qualitative–Quantitative Research Strategies

Example 1: Relevance of Meaning-Related Language for Explicating Structures for Students' Conceptual Understanding: Do students really have to learn that?

For students from marginalised family backgrounds, *academic language proficiency* has long been identified as background factor that strongly correlates with mathematics achievement (Ellerton & Clarkson, 1996). These large-scale findings on overall correlations mainly led to teaching approaches for simplifying language in curriculum materials (as a recent study on textbooks still reveals, de Araujo & Smith, 2022). In contrast, qualitative studies led to *nuanced insights into what language exactly is required* for enhancing all students' conceptual understanding (Adler, 2001; Erath et al., 2021). The part of school academic language that was identified as most relevant has later been termed *meaning-related language* (Prediger, 2019) and has been shown to include discourse practices and (informal yet explicit) phrases to articulate mathematical structures, such as “three groups of five” to explain the meaning of 3×5 (Anghileri, 1991) or “the part 128 of the whole 240” to explain fractions (Prediger et al., 2022). Design research studies identified those content-specific meaning-related phrases and provided cases of students who adopted the language for explicating structures and better developed the targeted conceptual understanding (overview in Erath et al., 2021).

However, when discussing syllabus changes in our educational contexts, we were confronted with sceptical questions: “Do students really have to learn these phrases?” Thus, we realised that no quantitative evidence existed that students who actively use structure-explicating meaning-related phrases have a better conceptual understanding than those who articulate ideas only implicitly or with gestures. Thus, we conducted an assessment study with 414 fifth

and sixth graders with this very particular in-depth focus in order to convince German syllabus writers. Indeed, we could show that students who actively used the structure-explicating meaning-related phrases (e.g., “three groups of fives”) to explain the connection between symbolic and graphical representations scored higher on a comprehensive assessment of multiplication and division understanding (Hankeln & Prediger, 2025). Conversely, students who tended to provide surface-level translations – identifying only the numbers but not the multiplicative unit structure (e.g., “this dot array matches 3×5 because here is three and here is five”) – received lower total assessment scores (ibid.). This quantitative evidence convinced the syllabus writers.

The underlying research strategy of conducting *focused assessment studies to test hypotheses on the relevance of specific aspects of mathematical competence* (generated in qualitative studies) rather than wide-range assessments (e.g., PISA covering mathematical competence in total) is not new. However, it could be exploited more systematically in sequential mixed-methods designs for preparing hard evidence arguments for policy decisions.

Example 2: Effects of Engaging Students in Conceptually Focused Discourse Practices in Interactions: Do teachers really need to do more than make students talk?

Early quantitative video studies already revealed substantial differences in students’ talk time across classrooms (Flanders, 1970; Stigler et al., 1999). Yet students’ talk time alone was shown not to be quantitatively predictive for learning gains (Inagaki et al., 1998; Pauli & Lipowsky, 2007). So quantitative observation protocols and video analysis tools have been criticised for covering primarily on surface structures of students’ talk (Clarke, 2005; Pauli & Lipowsky, 2007). Does this mean that students’ engagement in mathematical conversations is not as important as commonly promoted in practice-based teacher professional development?

Qualitative case studies have specified conditions of richness of talk (Lampert & Cobb, 2003; Walshaw & Anthony, 2008), suggesting to engage students in conceptually deep and discursively rich discourse practices such as explaining meanings, connecting representations, and arguing for enhancing students’ deep conceptual understanding (Moschkovich, 2013). Yet, even in professional development programs aiming at these conceptually focused discourse practices, it is not always easy for teachers to realise the necessary richness (Post & Prediger, 2024), so researchers called for long-term PD programs with a clear focus on initiating and maintaining rich discourse practices (Moschkovich, 2013; Post & Prediger, 2024).

However, policymakers’ decision-making processes regarding the focus and intensity of professional development and teacher support still seem to doubt (at least in my German context) that rich discourse practices are necessary. Instead, some state departments of education spread professional development on generic pedagogies to make students talk, while others spread curriculum materials for personalised trainings to schools, promising quick gains in procedural skills for underserved students. Again, more hard empirical evidence is required to convince policymakers (a) that conceptual understanding can be achieved by nearly all students, even those formerly underserved, (b) that engaging all student in rich discourse practices can be achieved by targeted scaffolding, and (c) that the richness of discourse practices makes a difference for students’ learning gains. Our instructional theories, developed in qualitative design research projects, generated exactly these hypotheses, but were not heard by policymakers. Hence, we felt the need to *provide harder quantitative evidence for their validity*.

We started with a *cluster-randomised controlled trial* with 589 middle school students for testing Hypotheses (a) and (b) for a language-responsive intervention on conceptual understanding for fractions and compared three conceptually focused treatments: A business-as-usual control group in which fraction understanding was taught with regular textbooks (using multiple representations without much language support; engaging students in talk, but rarely explic-

itly in rich discourse practices of explaining meanings) was compared to a *discursive intervention* in teacher-led small groups of 4-6 students guided by three design principles (DP1. Engage students in rich discourse practices. DP2. Connect multiple languages and representations. DP3. Macro-scaffolding to sequence content- and language-learning opportunities in combined trajectories). The *discursive-lexical intervention* followed the same three design principles in teacher-led small groups and additionally a fourth design principle (DP4. Build up shared meaning-related vocabulary before formal vocabulary). The analysis revealed that the discursive and discursive-lexical intervention groups had significantly higher conceptual learning gains than the control group. However, contrary to our expectations in Hypothesis (c), the slight advantages of the discursive-lexical intervention over the discursive intervention were not significant (Prediger et al., 2022). So, this black-box research strategy, which compared treatments without investigating the actual enacted interaction, could confirm the hypothesised effects of language-responsive designs to promote conceptually focused learning in rich discourse practices, but not the hypothesised effects of additional lexical scaffolding.

However, engaging students in rich discourse practices and lexical scaffolding are not determined by the curriculum material alone, but heavily depend on teachers' moderation of the interaction (Walshaw & Anthony, 2008). Thus, we decided to *open the black box of the teaching learning processes* by analysing the videos of 49 teacher-led small groups (210 students from the intervention) to *scrutinize the role of the interaction quality for students' learning gains*. By this, we were able to validate Hypotheses (b) and (c). In a huge work package, interaction quality in the intervention small groups was coded with respect to (i) students' talk time, (ii) the conceptual richness of discourse practices into which students were engaged (e.g., explaining meanings and explicitly connecting representations), and (iii) the kind of lexical scaffolding provided by the teachers' moves (not only the curriculum material). These quality degrees were then related to the measurable learning gains. A regression analysis yielded predictors for students' learning gains, showing that (i) students' talk time was not predictive for high conceptual learning gains. However, (ii) the degree to which small groups engaged in conceptually rich discourse practices (not each student's individual discursive participation) predicted conceptual learning gains for formerly underserved students. Finally, (iii) the degree to which the teachers' moves invited students to engage with the lexical scaffolds was predictive for all students' learning gains (Prediger et al., 2024).

Both study examples invite reflection on underlying research strategies: In the first study (Prediger et al., 2022), we conducted a regular controlled trial on a conceptually focused student-centred approach. Interestingly, our reviewers initially criticised that the complex fraction understanding was not *comprehensively* assessed with the simple pre-test and post-test. Fortunately, we could convince the journal editor that a study should be allowed to focus on selected conceptual aspects without suffering from not comprehensively covering *all* aspects of a complex concept. Otherwise, the fear against reducing complexity would restrict all randomised controlled trials one-sidedly to procedural skills or factual knowledge. I firmly believe that conducting more controlled trials on ambitious learning goals is strategically essential in order to provide hard evidence for at least selected conceptual aspects. A larger challenge was that our subtle design differences did not lead to the hypothesised significantly different learning gains. This challenges repeatedly occurred as the design of curriculum material alone does not determine classroom interactions, even if teachers are carefully trained to implement comparable interventions. So, it was again a research strategical decision to disentangle the interaction quality among the 49 groups who shared the curriculum material but still realised quite different interaction qualities (Prediger et al., 2024). This research strategy allowed to provide hard empirical evidence for qualitatively generated hypotheses. The coding work has been immense, yet future AI-assisted coding might lighten the coding work.

With the second study (Prediger et al., 2024), we have now first quantitative evidence for the important effects of interaction quality. This combination of qualitative deep dives and quantitative evidence helps to weaken policymakers' hope that teachers might simply be informed to make students talk with generic pedagogies. It also shows that careful teacher preparation substantially matters, as differences in the surface structure of talk time are not predictive, but conceptual richness of discourse practices is.

Example 3: Effects of Explicating Structures in Student-Centred Interventions: When teachers don't explicate structures, perhaps it is not necessary?

In the last 10 years, several research groups (not only in Dortmund) have developed various student-centred language-responsive interventions in design research studies on different mathematical contents (see Erath et al., 2021) and disseminated them in teacher professional development programs (Prediger, 2019). During this time, we have more clearly identified the key challenges that teachers face when implementing language-responsive instruction.

Many teachers in our context have overcome mindsets of low expectations described above (Boaler, 2002; Beswick, 2007; Wilhelm et al., 2017) and developed strong ambitions to create integrated concept- and language-learning opportunities for their students (especially those formerly underserved) through student-centred conceptually focused approaches. Yet, video-based qualitative analysis of their teaching practice documented a persisting challenge: a lack of explicitness in the knowledge organisation phase after students' independent inquiry (Post & Prediger, 2024). Following Selling (2016), we do not equate explicit instruction with direct instruction. Instead, we follow the two-step approach in which students first explore new ideas independently and are then supported to organise their first experiences into consolidated content and language knowledge (Loibl et al., 2017) by explicating the main aspects collectively (e.g., in whole-class discussions). For language-responsive teaching, this involves engaging students in explicitly articulating structures (e.g., part-whole-structure of fractions) while connecting multiple representations and explaining meanings. In our qualitative case studies the teachers successfully launched discussions about meanings, but they gave limited support for explicating mathematical structures in discussions (Post & Prediger, 2024).

In the implementation contexts in which we work, these case studies did not sufficiently convince the policymakers and PD program planners that the identified non-explicit teaching practices were not only single cases. Thus, we decided to gather also quantitative evidence for the non-explicitness. At the start of several parallel PD courses, we collected data from 102 teachers through a vignette-based activity, in which they were asked to explain meanings. Only 10% of the teachers explicated the multiplicative structures involved (Prediger & Wischgoll, 2023). When discussing these findings with PD facilitators, scepticism raised the following question: When teachers don't explicate structures, perhaps it is not necessary for students? And is it perhaps only important for multiplication, or for every topic?

To provide hard evidence for explicitness as necessary for students' learning, we developed an intervention on another topic: justifying the conversion procedure for measurement units (Why do we multiply by 1000 when converting kilograms to grams?). Two intervention variations both followed the design principles DP1–DP3 (from above), but differed in a fourth principle (DP4. Provide structure-explicating scaffolds). The analysis revealed that the two intervention groups yielded significantly higher conceptual learning gains than a control group. Additionally, the intervention group with structure-explication scaffolds had significantly higher learning gains than the intervention group without (Bielinski et al., submitted).

In this way, the new randomised controlled trial provided empirical evidence for the relevance of structure-explicating scaffolding. From a research strategical perspective, it is important to emphasise that we were only able to develop these two targeted interventions and the

focused comparative research design based on many qualitative studies conducted before, this is again a plea for successive mixed methods research, not for randomised controlled trials alone. We decided to focus on this subtle difference of structure-explication, as it was the aspect mostly doubted in our practical contexts and policy contexts.

In total, these sketches of our research journey provide examples of how we established qualitative insights and based upon them, hard empirical evidence for the effectiveness of conceptually focused, student-centred, explicit mathematics teaching.

Conclusion: Unlocking Qualitative Researchers' Minds to Provide Hard Evidence for Complex or Deep Phenomena and Subtle Design Elements

As a qualitative researcher, I have promoted design research approaches for many years as an ideal research approach for developing instructional approaches, curriculum materials, and for generating instructional theories about design principles and relevant learning goals. Design research approaches are powerful as they can appropriately combine constructive and reconstructive research goals and capture the complexity of learning ecologies (Gravemeijer & Cobb, 2006). However, we must acknowledge that any design principles and conditions of success we identify are only articulated in generated hypotheses.

On the other side of the assumed dichotomy of research approaches are randomised controlled trials, often criticised as too narrow in their learning goals (as standardized tests are often limited to procedural skills and factual knowledge), and in their short-term and black-box nature, which does not allow us to learn much about the complexity of student-centred instructional approaches. Nevertheless, we must not leave the field of providing hard quantitative evidence to those researchers who promote simplified learning goals and simple instructional approaches (Attard, 2025). Unlocking my mind as a qualitative researcher led me to overcome the fear of reducing complexity in quantitative designs, while still maintaining the ambition to capture complex and deep phenomena and subtle differences qualitatively identified.

Of course, the examples of successive mixed methods research briefly sketched in this paper need further elaboration (to be found in the cited papers). More importantly, they need further research as they can only give first humble examples of how deep insights from qualitative research can fuel quantitative studies of different kinds, and this is urgently needed to convince policymakers. In the future, AI-assisted data analysis will support the coding of complex phenomena, but not avoid the need to start with deep qualitative analysis, before quantification.

Finally, let me emphasise that most practicing teachers cannot be convinced by quantitative evidence. For them, we should provide opportunities for good teaching experiences that allow them to experience the power of new approaches or subtle differences themselves. These experiences can unlock minds more effectively than effect sizes and betas. Also for this field of action, there is still much more research and development needed in the future.

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Comparing and Discussing Multiple Strategies to Promote Learning in Mathematics Classrooms

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Comparison is a powerful learning process. How can we use comparison to support better learning of school mathematics within classroom settings? In 5 short-term experimental, classroom-based studies, we evaluated comparison of multiple strategies for supporting mathematics knowledge. We next developed supplemental Algebra I curriculum and professional development for teachers to integrate Comparison and Explanation of Multiple Strategies (CEMS) in their classrooms and tested the promise of the approach. Benefits and challenges emerged, leading to evidence-based guidelines for effectively supporting comparison and explanation in the classroom.

Students too often memorise ideas without understanding the ideas or being able to flexibly apply them to new contexts. For example, only 11% of 15-year-olds from around the world could work strategically using well-developed thinking and reasoning skills to solve math problems (OECD, 2016). Cognitive science research provides many insights into potential ways to improve teaching and learning in schools, but those insights infrequently make their way into classrooms (National Academies of Sciences, 2018). In the current paper, we focus on our efforts to understand how one basic learning process, comparison, can be harnessed to improve mathematics learning in schools.

We often learn through comparison. For example, we compare different brands and models of products, we compare one treatment option to another, and we compare new words, objects and ideas to ones we already know. These comparisons help us recognise what features are important and merit more attention, which can lead to deeper learning (Gentner, 1983). Indeed, research indicates that comparison promotes learning across a range of topics, including math, science and language (Alfieri et al., 2013).

In mathematics education, comparison of multiple solution strategies is a recommended instructional method in countries throughout the world (Australian Education Ministers, 2006; National Council of Teachers of Mathematics, 2014; Shimizu, 1999). Teachers are encouraged to have students share, compare and discuss multiple strategies for solving a particular problem (e.g., discuss the similarities and differences in the strategies). This recommendation is based on observations that expert teachers in countries such as the U.S. and Japan sometimes have students engage in this process, which is thought to promote deeper understanding and flexibility (Ball, 1993; Lampert, 1990; Shimizu, 1999).

However, observations in math classrooms suggest that many teachers are limited in the frequency and effectiveness with which they use comparison. In one study in the U.S., students were exposed to multiple strategies in 38% of observed algebra lessons, but teachers or students explicitly compared the strategies in only 9% of lessons (Star, Pollack, et al., 2015). Thus, research-based guidelines and curricula materials are needed to help teachers effectively use comparison.

Research Summary

Research on engaging in comparison to promote mathematics learning in the past 15 years has revealed new insights about what, when and how to compare. In this research, target learning outcomes were procedural knowledge (i.e., knowledge of what actions to take to solve problems, such as equation solving procedures), procedural flexibility (i.e., knowledge of multiple procedures and when to use each), and conceptual knowledge (i.e., knowledge of abstract and general principles, such as equivalency) (Star et al., 2016). Conceptual and procedural knowledge are typically the focus of mathematics instruction and assessment, while procedural flexibility is not, despite evidence that procedural flexibility is an important component of mathematics expertise (Star, 2005).

We have conducted extensive research to develop and evaluate the effectiveness of using a comprehensive Compare and Discuss instructional routine to deepen learning of algebra, overviewed here and reported in detail elsewhere (see also Durkin et al., 2017; Rittle-Johnson et al., 2017; Star et al., 2016). We only report results that were statistically significant.

In our initial research, we redesigned 2–3 math lessons on a particular topic and researchers implemented these lessons during students' mathematics classes (e.g., Rittle-Johnson & Star, 2007, 2011; Star et al., 2016). Across five studies, with hundreds of students, those who compared strategies gained greater procedural flexibility, often gained greater procedural knowledge, and sometimes gained greater conceptual knowledge (for study details, see Rittle-Johnson & Star, 2007, 2009; Rittle-Johnson et al., 2009, 2012; Star & Rittle-Johnson, 2009). In one study, comparing strategies was more effective for students who were familiar with one of the strategies than students who were not (Rittle-Johnson et al., 2009). To address this potential limitation of asking students with limited prior knowledge to compare strategies, we gave students more time to learn a smaller amount of material. With this added support, comparing strategies immediately supported greater procedural flexibility than delaying exposure to multiple strategies, with or without comparison of the strategies, for all students (Rittle-Johnson et al., 2012). In large part because of our research, Educator's Practice Guides from the U.S. Department of Education identified comparing multiple solution strategies as one of five recommendations for improving mathematical problem solving (Woodward et al., 2012) and teaching students to intentionally choose from alternative algebraic strategies when solving problems for improving algebra knowledge (Star, Caronongan, et al., 2015).

Given the promise of the Compare and Discuss method to promote math learning, we created the Compare and Discuss Multiple Strategies (CDMS) for Algebra materials and professional development. We then refined and evaluated this method in two large studies with teachers.

First, to help teachers use comparison more frequently and effectively in their instruction, we have developed a Compare and Discuss instructional routine. We include discussion because it helps students articulate and reflect on what they have learned and supports deeper learning from comparison (Lampert, 1990; Stein et al., 2008; Webb et al., 2014). First, students compare two examples, making sense of each and identifying their similarities and differences. The examples are often two different strategies for solving the same problem. The first phase of instruction should focus on students comparing the two examples, identifying similarities and differences. Subsequently, students discuss key points about the comparison, such as when one is better than the other or what the similarities in the examples reveal about a general idea. At the end of the activity, the teacher summarises the main points of the comparison and discussion. An overview of our Compare and Discuss instructional routine is shown in Figure 1 and is also discussed in Rittle-Johnson and colleagues (2020). This instructional routine is useful for various instructional goals such as knowing multiple strategies and why and when to use them and revising incorrect strategies and misconceptions.

In our first teacher-led study, we conducted an initial evaluation of teachers' effective use of our CDMS method with 68 teachers who were randomly assigned to implement our CDMS approach or to continue using their existing curriculum and method across the school year (see Star, Pollack, et al., 2015). However, CDMS teachers used our materials much less often than requested (i.e., an average of 20 times, for about 4% of their math instructional time, with 30% of teachers using the materials 5 times or fewer). Coding of the videos indicated that teachers implemented the compare phase as intended, but they often did not support sustained class discussion. Some teachers were not comfortable leading discussions and provided little time for students to generate explanations in response to open-ended questions or to build on each other's ideas (Star, Newton, et al., 2015). At the end of the school year, students' algebra knowledge was not higher in classrooms in which our materials were available (based on over 1,600 students). Greater use of our comparison materials was associated with greater student learning, suggesting the approach has promise when used sufficiently often. These results indicated that teachers needed more support in their implementation of our CDMS instructional routine.

We then revised our approach to better support Algebra teachers in their frequent and effective use of CDMS. Figure 1 has an overview of the revised instructional routine. We incorporated a Think-Pair-Share instructional routine to better promote discussion and critical thinking. This includes a worksheet for students to record their ideas during each phase. We also provide teachers with additional support for the lesson summary. Finally, we provided ongoing professional development to the teachers, providing feedback on lessons they had implemented, how to improve their support of the routine and planning of when to use the routine.

In this second teacher-led study, 16 Algebra I teachers received professional development and supplemental materials to support CDMS when teaching a unit on linear equation solving and 475 of their students completed assessments of their linear equation solving knowledge before and after the unit (Durkin et al., 2021). Thirteen Algebra I teachers and their 359 students were the business-as-usual control group. CDMS increased how often teachers engaged their students in comparison of multiple strategies, sustained small group work, and sustained mathematical discussions. Students in CDMS classrooms also had higher knowledge of linear equations on the posttest, particularly procedural flexibility. Thus, encouraging teachers to regularly compare and discuss multiple strategies increased students' algebra learning.

Guidelines for Supporting Learning from Comparison

We recommend two phases to instruction: a compare phase and a discuss phase. We have developed evidence-based guidelines for each phase. In Figures 2 and 3, we provide examples of materials for the compare phase. All of our materials are available for download at <https://www.compareanddiscuss.com/>.

In the Compare Phase, it is important to:

1. Select two examples that have important similarities and/or differences (Markman & Gentner, 1993). When examples are too similar or too different, students focus on obvious, unimportant features of the examples which leads to unproductive discussions. The two examples can be prepared in advance or created by students. More than two examples can be used, but it may overwhelm students to compare them without considerable support.

2. Make the examples clear and visible. In math, worked examples (a problem and step-by-step strategy for solving it) are very effective examples to help novices learn new procedures and related concepts (Atkinson et al., 2000; Sweller & Cooper, 1985). They clearly lay out solution steps and are commonly included in textbooks, so they are familiar to students. They also provide a visual record of the solution steps. Verbal descriptions of multi-step processes or complex ideas, without visual aids, can be difficult for students to process because they have to both remember and make sense of the examples (Richland et al., 2007).
3. Use a variety of comparison types, matched to your instructional goals. We primarily use three types to support math learning:
 - *Which is better?* Examples are two correct strategies for solving the same problem, with the goal of learning when and why one strategy is more efficient or easier than another strategy for a given problem type (see Figure 2 for an example). This type of comparison promotes procedural knowledge and flexibility – knowledge of multiple strategies and when to use them (Rittle-Johnson & Star, 2007).
 - *Which is correct?* One example is correct and one is incorrect with the goal of understanding and avoiding common incorrect ways of thinking (see Figure 3 for an example). The examples can be a correct and an incorrect strategy or a naïve and expert perspective. Comparing correct and incorrect strategies supports gains in procedural knowledge, retention of conceptual knowledge, and a reduction in misconceptions (Durkin & Rittle-Johnson, 2012).
 - *Why does it work?* Examples are also two correct strategies for solving the same problem, but with the goal of illuminating the conceptual rationale in one strategy that is less apparent in the other strategy. This is in contrast to the Which is better? comparisons, where the goal is to learn when and why one strategy is better for solving particular types of problems. More frequent use of Why does it work? comparisons in the classroom is related to greater conceptual and procedural knowledge (Star, Pollack, et al., 2015).
4. When engaging in comparison, present both examples simultaneously, not one at a time. Students will make better comparisons because they do not have to rely on their memory of one example while comparing (Begolli & Richland, 2015; Gentner, 1983).
5. Present examples side-by-side and use gestures, common language (e.g., terms such as equivalent, factors, etc.) and other cues (e.g., highlight key parts in same color) to guide attention to important similarities and differences in the examples. For example, students were more likely to notice that the altitude of a triangle must pass through a vertex if they studied two examples next to each other, one an example of a triangle with an added red line that passed through a vertex and the other an example of the same triangle with an added red line that did not pass through the vertex (Guo & Pang, 2011). Without supports like these, students may fail to notice important features of the examples that are similar or different, (Marton & Pang, 2006; Namy & Gentner, 2002; Richland et al., 2007).
6. Prompt students to explain, preferably to a peer. First, prompt students to explain each example individually to be sure they understand each. Then, prompt students to compare the two, using both general prompts (e.g., »What are some similarities and differences between the two examples?«) and prompts focused on specific aspects of the examples to compare (e.g., »How is their first solution step different?«). Students can do this independently or with a peer, and we recommend students talking with a peer. Generating explanations improves students' comprehension and transfer (Chi, 2000; McEldoon et al., 2013), and talking with peers improves learning and communicative competence (Johnson & Johnson, 1994; Webb, 1991).

7. Provide additional support if both examples are unfamiliar to students. It is easier to compare an unfamiliar example to a familiar example, such as comparing a new strategy to a strategy students have already learned (Rittle-Johnson et al., 2009). Students can learn from comparing two unfamiliar examples, but it requires additional support, such as providing more time for the compare phase and providing carefully-crafted explanation prompts that guide students' attention towards key ideas (Rittle-Johnson et al., 2012).

In the Discuss Phase, it is important for teachers to:

1. Prompt students to reflect on key points about the comparison (i.e., discuss connections prompts), such as when one strategy is better than the other or what the similarities in the examples reveal about a general idea. Example prompts are: »On a timed test, would you rather use Alex's way or Morgan's way? Why?« and »Even though Alex and Morgan did different first steps, why did they both get the same answer?« Prompts to discuss connections encourage students to think critically about the examples and improve learning from comparison more than generic prompts to compare (Catrambone & Holyoak, 1989; Gentner et al., 2003). In addition, when teachers use more open-ended questions that prompt students to verbalise the main ideas of the lesson, students learn more (Star, Newton, et al., 2015).
2. Incorporate a Think-Pair-Share instructional routine to support high-quality discussion, communicative competence and critical thinking. First, students think on their own for a minute about the discuss connections prompt. Next, each student pairs with another student to discuss the prompt, summarising their ideas in writing. Then, students share their ideas in a whole class discussion. Teachers should call on multiple students to answer the same question and ask students to build on each other's ideas (e.g., »What do you think about Abbey's idea?«). Such classroom discussions promote critical thinking and improve student learning and communicative competence (Lampert, 1990; Stein et al., 2008; Webb et al., 2014).
3. Summarise the main points of the compare and discuss phases. Direct instruction on the key points supplements learners' comparisons and improves learning from comparison (Gick & Holyoak, 1983; Schwartz & Bransford, 1998; VanderStoep & Seifert, 1993). We recommend students then write a summary of the main points in their own words to be sure they understood and so they can practice communicating their ideas in writing.

Discussion

Comparison is a powerful learning process. In problem-solving domains such as mathematics, comparing multiple strategies promotes conceptual knowledge, procedural knowledge and/or procedural flexibility. However, comparison requires substantial mental effort by learners, and learners can become overwhelmed by it without adequate support, especially if all of the material is unfamiliar. Supports, such as presenting examples side-by-side and using cues to guide attention to important similarities and differences in the examples, facilitate learning from comparison. To help teachers use comparison more frequently and effectively in their classrooms, curricular materials, well-specified instructional routines, and sustained professional development are likely needed. We have developed a CDMS instructional routine that is a highly structured but quite adaptable to a range of topics in mathematics.

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Figure 1

Overview of a Compare-and-Discuss Multiple Strategies Instructional Routine

Compare	Discuss Connections
 Prepare to Compare What is the problem asking? What is happening in the first method? What is happening in the second method?	 Prepare to Discuss (think, pair) How does this comparison help you understand this problem? How might you apply these methods to a similar problem?
 Make Comparisons What are the similarities and differences between the two methods? Which method is better? Which method is correct? Why do both methods work? How do the problems differ?	 Discuss Connections (share) What ideas would you like to share with the class?  Identify the Big Idea Can you summarize the Big Idea in your own words?

Figure 2

Sample Worked Example Pair (WEP) for a Which is Better? Comparison

Which is better?

Topic 2.6

Riley and Gloria were asked to graph the equation $3x - 2y = 6$.

Riley's "x- and y-intercepts" way

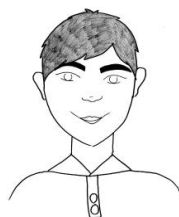
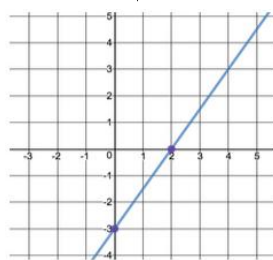
Gloria's "slope-intercept" way

First I found the x-intercept by plugging in 0 for y.

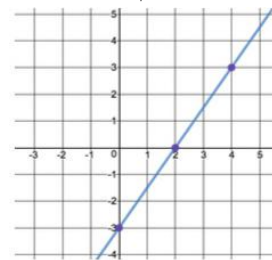
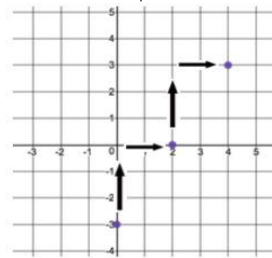
Then I found the y-intercept by plugging in 0 for x.

I plotted the intercepts and connected them.

$$\begin{aligned}
 3x - 2y &= 6 \\
 3x - 2(0) &= 6 \\
 3x &= 6 \\
 x &= 2 \\
 \text{x-intercept: } (2, 0) \\
 \downarrow \\
 3(0) - 2y &= 6 \\
 -2y &= 6 \\
 y &= -3 \\
 \text{y-intercept: } (0, -3)
 \end{aligned}$$



$$\begin{aligned}
 3x - 2y &= 6 \\
 -2y &= -3x + 6 \\
 y &= \frac{3}{2}x - 3
 \end{aligned}$$



I solved for y to put the equation in $y = mx + b$ form.

I graphed the y-intercept of -3 then used rise over run to get more points.

I connected the points to get the line.

? How did Riley graph the line? Why did Gloria solve the equation for y as a first step?

↔ Which method is better?

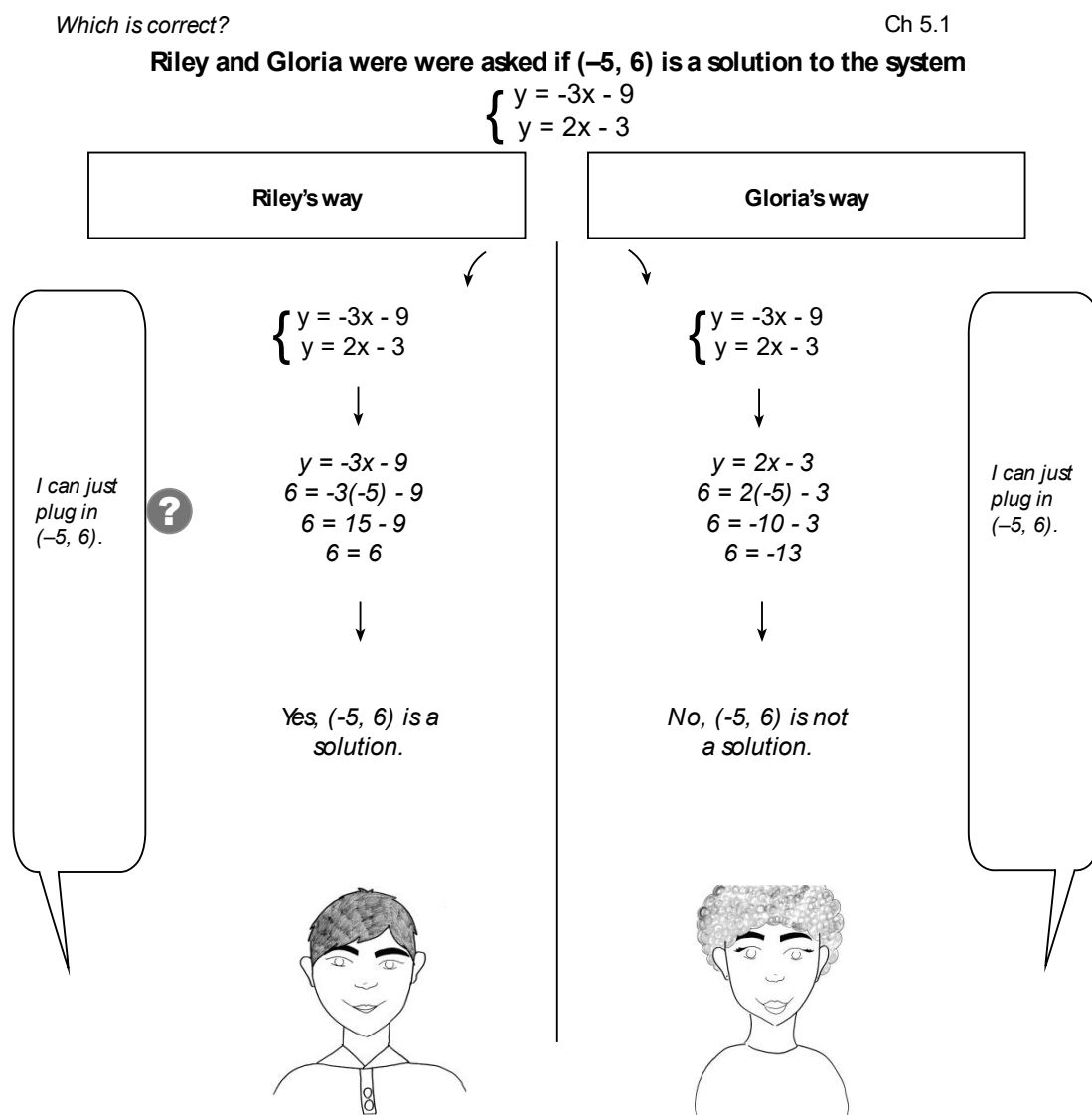
Version 2017

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Figure 3

Sample Worked Example Pair (WEP) for a Which is Correct? Comparison



- ?
 Does it matter into which equation you plug the point?
- ↔
 What is the same or similar about Riley and Gloria's methods? What is different?

Beth Southwell Practical Implications Award

The Beth Southwell Practical Implications Award was initiated and sponsored by the National Key Centre for Teaching and Research in School Science and Mathematics, Curtin University, Perth, Western Australia. Curtin sponsored the “Practical Implications Award” (PIA), as it was then known, for the first 10 years. The Australian Association of Mathematics Teachers (AAMT) now sponsors the Award. In 2008, MERGA was honoured to be able to rename the PIA as the Beth Southwell Practical Implications Award (BSPIA), in honour of MERGA’s and AAMT’s esteemed late member, Beth Southwell. The award is designed to stimulate the writing of papers on research related to mathematics teaching or learning or mathematics curricula. Application for the award is open to all members of MERGA who are registered for the conference. Applications for the BSPIA are judged against specific criteria set by a four-member panel. The panel consists of two members from MERGA, two from AAMT, and is chaired by the MERGA Vice President (Development).

Exploring the Complexity of Students' Inconsistent Relational Understanding of the Equal Sign

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Recently, the phenomenon of students holding simultaneous operational and relational understanding (SOR) of the equal sign has garnered increasing attention from researchers. Investigating this is crucial, as it helps in supporting students' transition toward a full relational understanding. This study utilised Rasch analysis and student interviews to delve deeper into the SOR stage, involving 1008 Chinese Grade 1 students. The findings validated the SOR stage and uncovered nuanced difficulties, barriers, and contributing factors underlying students' inconsistent reasoning about the equal sign.

While students' knowledge of the equal sign is a well-researched topic, new insights continue to emerge. One of these insights is the investigation of students' unstable understanding between a relational perspective (viewing the equal sign as a symbol indicating equivalence between two sides) and an operational perspective (interpreting the equal sign as a symbol denoting the calculated result). Researchers focus on the students' coexistence of relational and operational conceptions of the equal sign (e.g., Blanton et al., 2018; Lee & Pang, 2023), as well as the regression from relational understanding back to operational understanding (Molina & Ambrose, 2008; Sun et al., 2023). These findings revealed the areas that require attention during the transition as students develop their operational conception into a relational understanding. To better support students in grasping a full relational understanding of the equal sign, their conceptual difficulties, barriers, and shifts that emerged during the transition period need to be clearly examined and addressed (Lee & Pang, 2023; Sofuoğlu, 2024). However, the literature in this field, particularly regarding students' simultaneous operational and relational understanding (SOR) of the equal sign, still remains under-researched (Lee & Pang, 2023). This study utilises a sample of Chinese students in Grade 1 to further investigate the phenomenon of the coexistence of two understandings, aiming identify the specific difficulties and barriers students encounter in developing a robust relational understanding of equal sign.

Literature Review

In a seminal work, Matthews et al. (2012) proposed four developmental stages of knowledge of the equal sign: 1) *rigid operational*, viewing it as indicating results calculated from left to right (e.g., accepting only $a + b = c$); (2) *flexible operational*, recognising results calculated in either direction (e.g., accepting $c = a + b$ and $a = a$); (3) *basic relational*, understanding the equal sign as indicating equivalence (e.g., accepting $a + b = c + d$); and (4) *comparative relational*, building on the third stage, students compare relationships between numbers when solving or evaluating number sentences like $a + b = c + d$ instead of performing full calculations. Similarly, Stephens et al. (2013) recognised three stages of understanding: *operational*, *relational-computational*, and *relational-structural*. These two constructs are widely recognised as foundational frameworks for assessing students' developmental stages in understanding the equal sign. Recently, building on two constructs, more nuanced stages of students' development of the conception of the equal sign have emerged. Lee and Pang (2023) observed that while many students were able to perceive the equal sign relationally in certain scenarios (e.g. solving and evaluating $a + b = c + d$), in other cases (e.g. evaluating $a + b = c + d = e$) they demonstrated an operational understanding (e.g. considering $2 + 3 = 5 + 7 = 12$ is true). Therefore, they proposed an additional stage between *flexible operational* and *basic relational*, which is *simultaneous operational and relational* (SOR). Likewise, Blanton et al. (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 35–42). Canberra: MERGA.

(2018) also noted that some students exhibited an *emergent relational understanding*, which refers to a ‘hybrid’ of operational and relational understandings. Echoing them, Sun et al. (2023) found that about 55% of tested Chinese Grade 1 students ($n = 497$) can correctly solve the number sentences in form of $a + b = c + \_\_$, whereas only about 36% of tested students successfully solve the form $a + b = \_\_ + c$. This appeared to indicate that many students could demonstrate the relational understanding in the former case, but their conception of the equal sign seems to revert to an operational level when dealing with the latter case. Such a regression is also evidenced in other literature (Molina & Ambrose, 2008). Introducing a SOR stage potentially explains that, in two studies mentioned above, students’ conceptions of the equal sign did not ‘regress’ from the relational to the operational level; rather, they had not yet reached the pure relational level and could be considered to be at the SOR stage. It could be argued that the misconceptions students typically experienced in SOR stage could prevent these students from developing a robust relational understanding of the equal sign. Therefore, to support students in developing a full relational understanding, it is insufficient to examine their thinking solely within the dichotomy of relational and operational levels. More attention needs to be given to the SOR stage, which represents a conflation of two levels. However, as Lee and Pang (2023) showed, very few literature focussed on SOR or distinguished it from pure relational understanding. Therefore, students’ difficulties, challenges encountered at SOR stage, along with related supportive pedagogies, warrant further attention and exploration (Lee & Pang, 2023; Sofuoğlu, 2024). It is worth mention that some studies note that experienced mathematics learners or mathematicians can interpret the equal sign flexibly—operationally or relationally—depending on context (e.g., McNeil & Alibali, 2005). However, the SOR stage that will be investigated in this study does not reflect such a flexibility. Instead, it refers to specific misconceptions that can result in a problematic understanding of mathematical structures, such as interpreting $5 + 2 = 7 + 6 = 13$ as true.

Furthermore, Lee and Pang (2023) pointed out that the existing instruments commonly used to assess students’ conceptions of the equal sign, such as evaluating and solving non-conventional forms of number sentences such as $a = b + c$ and $a + b = c + d$, are difficult to identify the SOR stage. Therefore, they introduce the form $a + b = c + d = e$. Moreover, based on Sun et al. (2023), it appears that for the form $a + b = c + d$, many students demonstrated different understanding of equal sign when they were solving $a + b = \_\_ + d$ and $a + b = c + \_\_$. In this sense, adding to Lee and Pang (2023), this study considers that the number sentences solving task involving these two forms potentially have capacity to probe students’ SOR stage.

Accordingly, the purpose of this study is to explore further insights about SOR stage. First, the construct of the SOR stage as hypothesised by Lee and Pang (2023), has yet to be scientifically validated as a developmental stage in the understanding of the equal sign. Therefore, one goal of this study is to use Rasch modelling to validate the SOR stage. This leads to the first research question (RQ1): How can the SOR stage be validated through Rasch modelling analysis, and what insights does it provide into the complexities of students’ reasoning at this stage? Furthermore, the second research question (RQ2) explores what conceptions held by students identified at SOR stage and the potential sources of their confusion. By doing so, this study contributes to the knowledge of how to support students transitioning from emerging to solid levels of relational understanding of the equal sign, benefiting both research and classroom practice.

Methodology

To validate the SOR stage (addressing RQ1), this study employed the Rasch modelling (elaborated below), and used students interviews to probe their thinking (addressing RQ2). The researchers (the research grant and ethical approval see the acknowledgement) collected the data at four primary schools in Changchun City, Jilin Province, China, with the participation of

1,008 Grade 1 students, in September 2023. Sun et al. (2023) have showed that many tested Grade 1 students exhibited a mixture of operational and relational understandings. Therefore, Grade 1 can be considered as a transition period for many Chinese students in progressing towards the full relational conception of the equal sign. This underpins the choice of Grade 1 students for this study. The following sections will outline how Rasch modelling was applied and how the interviews were administered.

Rasch Modelling Analysis

Given students' responses to assessment items or tasks, Rasch modelling will match students' abilities to the assessment item difficulties, and group them simultaneously (Matthews et al., 2012). A strength of Rasch modelling is its probabilistic nature: students' abilities are determined by calculating the probabilities of their correct responses to different items, while item difficulties are similarly assessed based on the probabilities of being answered correctly by students. This can reduce the impact from unexpected distortions, for example, high ability students' incorrect response due to careless mistakes, and a low ability students' correct response due to guessing (Linacre & Wright, 2000). Therefore, Rasch modelling is less influenced by random factors and is more reliable and flexible (Matthews et al., 2012).

Research has evidenced that Rasch modelling is an effective tool for identifying and validating the developmental stages of mathematical concepts (Bao & Stephens, 2024; Matthews et al., 2012). In this study, the assessment was designed to probe students' understanding of the equal sign, so students' abilities refer to their various conceptions on the equal sign. Students' (n=1008) responses to 19 assessment items were entered into an Excel spreadsheet and then loaded into *Winsteps* software for Rasch modelling analysis.

Assessment

This study employed an adapted version of Mathematics Equivalence Assessment [MEA] (Simsek et al., 2021). Students were required to evaluate True/False for a range of number sentences, such as forms of $a + b = c$, $a = a$, $a = b + c$, $a + b = c + d$, $a + b = c + d = e$, and $a + b + c = d + e = f$, and they were required to solve number sentences such with the forms of $a + b = \_ + c$ and $a + b = c + \_$, involving both addition and subtraction. These items are commonly used in examining students' understanding of the equal sign. Notably, the forms of $a + b = c + d = e$, $a + b + c = d + e = f$, $a + b = \_ + c$ and $a + b = c + \_$ are included as the literature suggests they potentially probe students' SOR (Lee & Pang, 2023; Sun et al., 2023). In this study, the arithmetic operations involved number sentences with numbers less than 10, adapting to Chinese Grade 1 students' general mathematics ability. The specific items are shown in Figure 1 below. The reliability and validity of MEA and its adapted version are well-documented in the literature (e.g., Simsek et al., 2021; Sun et al., 2023), and it meets the assumptions of Rasch modelling (e.g., Matthews et al., 2012). In addition, as will be seen the result of the Rasch analysis of this research demonstrate this assessment instrument is suitable. Students' responses to each item were coded dichotomously (0 for incorrect or 1 for correct). With this coding scheme, a possible concern is that a student with a relational understanding might provide an incorrect answer due to calculation error, resulting in a score of 0, which may not accurately reflect their actual understanding. However, the probabilistic nature of Rasch modelling can accommodate such a circumstance, ensuring the results remain robust. The reliability of this coding convention is also evidenced in other studies (e.g., Matthews et al., 2012).

Interview

Of students who were potentially identified as being at SOR stage, 20 were interviewed by the researchers. They were asked to explain their thinking in detail when solving and evaluating

specific items, as well as to reflect on the past learning experiences that contributed to their reasoning. The interview was semi-structured, with opening questions designed to prompt responses, such as, “*Could you please explain why you filled this number here?*”, “*Could you explain why you think this one is true?*” and “*Where and how did you learn this before?*”

Results and Discussion

The *Winsteps* program evaluated the data’s fit to the Rasch model, with a key indicator being the Infit MNSQ values (IMNSQ) for test items and testers. The acceptable range for this value is 0.5 to 1.5, with values closer to 1.0 indicating a better fit to the Rasch analysis (Linacre & Wright, 2000). The IMNSQ of the assessment data of this study for students and items are 0.98 and 0.97, respectively, evidencing a good fit of this study’s data to Rasch analysis. Furthermore, the analysis showed that the SEPARATION values for students and items are 2.25 and 14.55, respectively, demonstrating the assessment performs well in distinguishing both the abilities of students and item difficulties (Linacre & Wright, 2000). Therefore, the assessment shows a good reliability and discriminative power, making it a high-quality measurement. These results are important as it provides strong statistical evidence for this study’s claim about SOR stages that will be presented later.

The Validation of SOR

The results of Rasch analysis are presented in Figure 1 below. The left column displays the

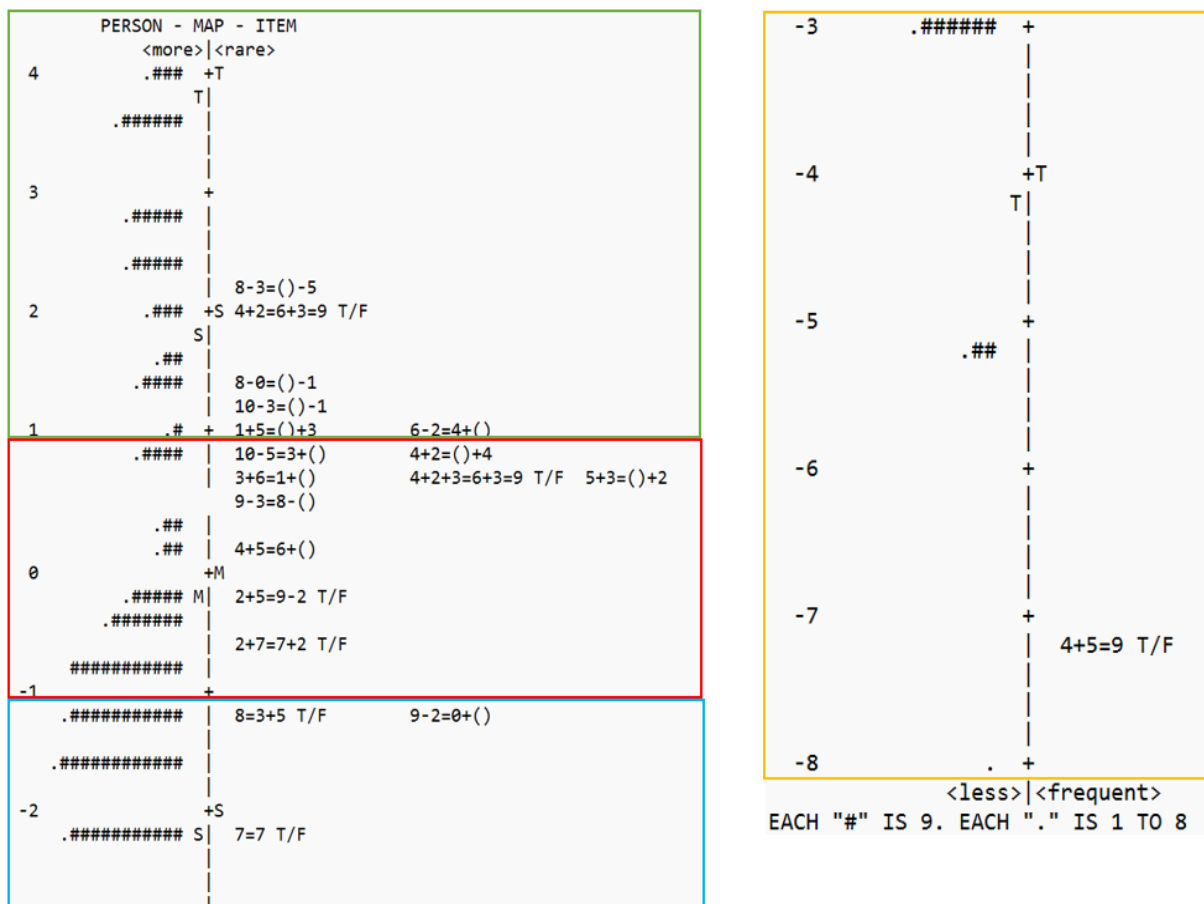


Figure 1. The Variable Map of Rasch Analysis (Analysis Results)

grouping of students based on their abilities (i.e. their conceptions on the equal sign), ranging from logit -8 to 4, while the right column displays the grouping of items according to their difficulties, ranging from logit -7 to 3. The logit values can be used to calculate the probability

of a student correctly responding to an item ($\text{Pr}(\text{correct}) = \frac{1}{1 + e^{-(\text{student logit} - \text{item logit})}}$). For instance, a student at level -3 has approximately a 11.2% chance of correctly evaluating a level -1 item, such as $2 + 7 = 7 + 2$, as true, and only an 4.74% chance of correctly finding the missing number for a level 0 item $4 + 5 = 6 + \underline{\quad}$. Both items are forms of the number sentence $a + b = c + d$, which is commonly used to assess whether a student's understanding of the equal sign has achieved relational level. This result suggested that even when a student with an operational understanding occasionally provided correct response to the items requiring relational thinking by guessing (more likely guessing correctly on evaluation items than on solving items), he/she can still be categorised in the operational understanding group (the yellow-highlighted group, will be elaborated later). This adds confidence that the identified stages accurately reflect students' genuine understanding of the equal sign. In Rasch analysis, the logit values also identify significant shifts in item difficulties, revealing the 'natural breaks' that indicate changes in cognitive skills (Griffin, 2020). In this sense, since the items in this research are designed to assess students' knowledge of the equal sign, the logit values can serve as thresholds between different stages of understanding.

The stages of students' understanding of the equal sign identified in this study aligns with developmental stages proposed by Matthews et al. (2012). The students in the yellow-highlighted group accepted the conventional form: $a + b = c$ ($4 + 5 = 9$) but failed to grasp the forms $a = a$ and $a = b + c$ ($7 = 7$ and $8 = 3 + 5$). Therefore, these students are placed in the yellow group indicating a *rigid operational* stage. Students in the blue-highlighted group understood $a = b + c$ and $a = a$ but struggled to make sense to the form $a + b = c + d$, whether in evaluating or solving tasks (e.g., True/False $2 + 7 = 7 + 2$, or solve $4 + 5 = 6 + \underline{\quad}$). These students can be categorised into *flexible operational* stage. It is noted that some students at this stage can solve $9 - 2 = 0 + \underline{\quad}$. It could be explained this number sentence is special because of the presence of '0', so students are likely to perceive this one the same as $9 - 2 = \underline{\quad}$. Later interview results supported this claim. Students in red-highlighted and green-highlighted groups can appreciate the forms $a + b = c + d$. In line with Matthew et al. (2012), they can be considered at the *basic relational* stage. For the *comparative relational* stage, this study did not include assessment items, as it was beyond the scope of the research.

However, when closely examining the *basic relational* stage, two groups can be further distinguished, with logit 1 as a threshold (i.e. green and red highlighted groups). First, it is clear that within logit 0, the number sentences are almost with the form $a + b = c + \underline{\quad}$ (red-highlighted group), and majority of items of form $a + b = \underline{\quad} + c$ are within logit 1 and beyond (green-highlighted group). This suggests the students in the red-highlighted group could confidently deal with form $a + b = c + \underline{\quad}$ but struggled with form $a + b = \underline{\quad} + c$. For instance, many students in this group filled 5 for $8 - 3 = \underline{\quad} - 5$, demonstrating operational understanding. Subsequent interviews corroborated their thinking (see below). This finding echoed Sun et al. (2023) who discovered a significant lower proportion of Chinese Grade 1 students correctly responding to items in the form $a + b = \underline{\quad} + c$ than $a + b = c + \underline{\quad}$. Sun et al. (2023) showed this result with a response distribution chart. This study takes it a step further by using Rasch analysis to confirm the existence of SOR students exhibiting such behaviours (approximately 30% of all tested students). It is noted that this red-highlighted group was likely to correctly respond to two $a + b = \underline{\quad} + c$ form number sentences $4 + 2 = \underline{\quad} + 4$ and $5 + 3 = \underline{\quad} + 2$. This study conjectured that since $4 + 2 = \underline{\quad} + 4$ can be considered as a special instance of $a + b = b + a$, and so may activate students' relational view towards this item. The interview response supported this explanation (see below). For $5 + 3 = \underline{\quad} + 2$, this study speculates that students' familiarity with addition may make a number sentence exclusively involving addition more likely to activate relational thinking for those at SOR stage. This requires a further exploration. Meanwhile, the results confirm that the True/False item $4 + 2 = 6 + 3 = 9$ is beyond the abilities

of red-highlighted group, indicating that many students who were comfortable with $a + b = c + d$, mistakenly accepted this item as true. This finding echoed Lee and Pang (2023), who proposed the existence of SOR stage by observing this misconception. Notably, a similar True/False item $4 + 2 + 3 = 6 + 3 = 9$ is located at red-highlighted group, suggesting that students in this group are very likely to provide the correct evaluation. This finding appears to show that these students hold contradictory views about the role of the equal sign in these two number sentences, demonstrating a relational understanding for $4 + 2 + 3 = 6 + 3 = 9$ but an operational view for $4 + 2 = 6 + 3 = 9$.

Taken together, the Rasch analysis shows the students in the red-highlighted group are more likely to demonstrate inconsistencies in their conceptions of the equal sign. By contrast, students in the green-highlighted group appear more robust in solving and evaluating varied forms of number sentences, exhibiting a more solid relational understanding. In this sense, this study argues the red-highlighted and green-highlighted groups should be considered as two developmental stages of the understanding of the equal sign, which align with stages of SOR and PBR (pure basic relational) proposed by Lee and Pang (2023). In addition, the Rasch analysis appears to reveal that within the SOR stage, students can be further differentiated as many students located between logit -1 and 0 are likely to exhibit relational understanding when dealing with True/False items but reverting back to operational thinking when asked to solve missing number sentence. These students' relational understanding of the equal sign is less robust than those who can solve $a + b = c + \underline{\hspace{1cm}}$ (i.e. students located between logit 0 and 1). This finding has implications for item design cautioning on over reliance on True/False items.

Students' Reasoning and Possible Source of Inconsistency

To address RQ2, this study interviewed 20 students in the red-highlighted group to explore their reasoning behind inconsistencies in solving and/or evaluating specific number sentences, as well as the factors contribute to such reasoning. The data from two students, Ming and Yang, will be presented and discussed. Ming was a typical student who could correctly responded to the form $a + b = c + \underline{\hspace{1cm}}$, but had confusions with the form $a + b = \underline{\hspace{1cm}} + c$ and $a + b = c + d = e$. Yang was a student located at bottom area of red-highlighted group (logit -1-0), who was correctly evaluate the form $a + b = c + d$ but was less confident to solve the missing number in number sentences of the same form. The interview responses from both students are representative of the patterns observed among all interviewed students.

Ming

Ming explained when he was solving $8 - 3 = \underline{\hspace{1cm}} - 5$, he filled the 5 as he saw the blank straight after ' $8 - 3 =$ '. When the researcher asked Ming whether he saw another ' $- 5$ ' follow the blank, he responded, "I don't need to care about another minus 5, I only need to calculate the answer for $8 - 3$ ". Then the interviewer asked him about the approach to $10 - 5 = 3 + \underline{\hspace{1cm}}$ and $4 + 2 = \underline{\hspace{1cm}} + 4$, Ming's response is shown below:

Interviewer: So how did you do this one, $10 - 5 = 3 + \underline{\hspace{1cm}}$, you did this correctly.

Ming: $10 - 5$ is 5 so I need to find out 3 plus another number equals 5, so it's 2.

Interviewer: So now the missing number here is not the answer $10 - 5$ not like the one in $8 - 3 = \underline{\hspace{1cm}} - 5$?

Ming: No, 3 has been already after the equal sign, $10 - 5$ obviously is not 3, so I need to think about all numbers on the right-hand side equals to the left-hand side.

Interviewer: Ok, could you please tell me what does the equal sign mean in this one [$10 - 5 = 3 + \underline{\hspace{1cm}}$]?

Ming: Two sides are the same.

Interviewer: Ok, what about this one $4 + 2 = \underline{\hspace{1cm}} + 4$, why the missing number you filled is 2, not $4 + 2 = 6$?

Ming: Because $4 + 2$ obviously is the same as $2 + 4$, I know two sides are the same.

Interviewer: Good, if you know for $4 + 2 = \underline{\hspace{1cm}} + 4$, two sides are the same. Could you please reconsider $8 - 3 = \underline{\hspace{1cm}} - 5$?

Ming: Oh, I see, the $8 - 3$ should equal the whole blank minus 5, so it should equal 10.

Ming's responses tended to reveal that in the form of $a + b = c + \underline{\quad}$, the number after the equal sign suspended his operational view and lead him to consider the equal sign relationally. However, when having $a + b = \underline{\quad} + c$, his operational view is spontaneously activated. For the special $a + b = \underline{\quad} + a$ form, the commutative law of addition made him readily switch to the relational view again. Subsequently, based on his conception on $4 + 2 = \underline{\quad} + 4$, Ming realised the same approach should be applied to $8 - 3 = \underline{\quad} - 5$ and he made a correction. Finally, Ming was asked about his evaluation of $4 + 2 = 6 + 3 = 9$. He explained that during arithmetic drill practice at home, his parents demonstrated that multi-step operations questions could be written sequentially in this manner. As a result, this approach became deeply ingrained in his thinking. This response was widely provided by many other interviewed students. For Chinese Grade 1 students, while this multi-step operations are presented in the classroom, they also practice arithmetic drill extensively at home. It is possible that on one hand, teachers in the classroom show students the correct approach, on the other hand they are influenced by parents' misleading. Consequently, students may consider both approaches (relational and operational) are legitimate.

Yang

Yang explained his thinking to evaluate two True/False items with the form $a + b = c + d$: "The calculated results of two sides are the same, so they are true". When he was asked about the strategy to solve the number sentences such as $4 + 5 = 6 + \underline{\quad}$ and $5 + 3 = \underline{\quad} + 2$, he responded, "This one ($5 + 3 = \underline{\quad} + 2$) is easy, $5 + 3 = 8$, so it's 8. But for this one ($4 + 5 = 6 + \underline{\quad}$), I am not sure, how come $4 + 5 = 6$? I am not sure" Yang's responses suggest that his relational understanding is activated when the calculated results are already provided. However, whenever he needs to perform the calculations himself, his operational thinking appears to take over, suggesting that True/False items are more likely activate students' relational understanding, as the Rasch analysis showed. When Yang was asked about the evaluation of form: $a + b = c + d = e$, his response was similar to Ming's.

Combining Ming's comments above, it appears that for students at SOR stage, number sentences where a number immediately follows the equal sign (i.e. True/False items and $a + b = c + \underline{\quad}$) are likely to encourage relational thinking. In contrast, items requiring students to fill a number immediately after the equal sign tend to lead to operational thinking. Students' responses appeared to indicate the similarity between the form $a + b = \underline{\quad} + c$ and their everyday conventional arithmetic practice ($a + b = \underline{\quad}$) contributes to such thinking. Especially for students at SOR stage, greater efforts may still be needed to prevent their relational understanding from being disrupted by the influence of conventional arithmetic drill.

Concluding Remarks: Implication for Research and Practice

This study validates the SOR stage proposed by Lee and Pang (2023), uncovering fine-grained complexities about students' conception of the equal sign during the transition period towards the full relational understanding. First, this study differentiated students' thinking by identifying their nuanced responses to various equivalence assessment items. For instance, previous literature has provided limited exploration of the discriminative power of these items. By contrast, this study confirmed that True/False items (i.e., $a + b = c + d$ and $a + b = c + d = e$ forms), together with number sentences requiring solution in the forms $a + b = c + \underline{\quad}$ and $a + b = \underline{\quad} + c$, are effective in differentiating levels of emerging relational understanding. As Lee and Pang (2023) argued, such a differentiation is important as it helps to provide targeted support for students with diverse needs. Therefore, this study could inform any future research agendas on investigating applying these nuanced items to assess and/or enhance students' conceptions on equal sign. Furthermore, this study has an implication for everyday classroom

practice as these items can help teachers pinpoint the critical misconceptions in students' understanding of the equal sign and provide tailored scaffolding.

Second, this study finds that students' emerging relational understanding of the equal sign can still be strongly influenced by their experiences with conventional arithmetic practice, which may reactivate an operational view when similar structures are encountered. This experience occurs not only in the classroom but also potentially at home, where students' parents may inadvertently use inappropriate approaches to number sentences. This highlights the need for classroom teachers to place greater emphasis on reinforcing the relational meaning of the equal sign in students' everyday arithmetic experiences. In particular, the critical misconceptions, such as the sequential approach $a + b = c + d = e$, must be addressed carefully.

Acknowledgements

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Research Symposium

Strategies That Promote Inclusive Mathematics Education

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In this symposium, we continue the discussion on attending to student diversity in mathematics education (Quane et al., 2024) by exploring strategies that promote inclusive mathematics education. In doing so, we continue to acknowledge that classrooms comprise of diverse student populations (Quane et al., 2024) with Australian classrooms experiencing higher proportions of diversity than the international average. In terms of equity and inclusion, Australia, according to a recent OECD report, is one of four OECD countries whose education systems did not have a “formal or operational” definition of inclusion (OECD, 2023, p. 25). This is problematic as there is no consensus on a common definition, and, therefore, interpretations of what inclusion means are inconsistent. In turn, a lack of a standard definition of inclusion may result in educational practices that do not align with the principles of inclusion. Of note, the Early Years Learning Framework does provide a definition of inclusion for the education of children (birth to eight years).

Cologon and Mevawalla (2018) describes genuine inclusion as comprising of “belonging, participation, opportunity and recognised and valued contribution” (p. 904). Cologon and Mevawalla (2018) continue and state that “genuine inclusive education requires embracing and addressing all aspects of human diversity through attitudinal, structural, relational, and environmental accommodations. Inclusive education is a ‘big idea’, but it is lived out in everyday moments in the interactions between people and environments” (p. 904). Slee (2019) posits that inclusive education is achieved when “excellent educational experiences and outcomes *for all children* (emphasis added) and young people” are provided (p. 8). In this symposium, we revisit the definitions of inclusion and diversity and explore strategies that promote inclusive mathematics education.

Chair: Kate Quane

Paper 1: *Universal Design for Learning for Mathematics Education*

Lorraine Gaunt, Kate Quane, Belinda Trewartha and Tom Porta

Paper 2: *The Use of Key Word Sign to Support Early Mathematical Learning*

Kate Quane, Belinda Trewartha, Lorraine Gaunt, Elaine Stigwood and Zoe Twose

Paper 3: *Let’s Throw Out the Textbooks and Streaming in Mathematics!*

Lorraine Gaunt and Matt Winslade

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Universal Design for Learning for Mathematics Education

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In this first paper of the Symposium: Strategies that promote inclusive mathematics education, we introduce the Universal Design for Learning (UDL) guidelines through a mathematics lens. Using a rapid review, empirical studies on mathematics and UDL in early years, primary, or secondary school settings were sought. Six research papers were identified from school settings with no results for early years research, thus, the Early Years Learning Framework was used to identify connections to mathematics in this context. Findings highlighted the potential of UDL to enhance mathematics education for all students creating a flexible and accessible curriculum.

Universal Design for Learning (UDL) is seen to be widely adopted as a framework for accessibility in inclusive education environments (CAST, 2024). The use of UDL to teach mathematics in inclusive settings is a progressive approach that aims to accommodate the diverse needs of all students, not just students with disabilities (Craig et al., 2023). By exploring recent research, this article aims to offer insights into the application of UDL in mathematics education, highlighting its potential benefits and challenges across early years, primary, and secondary education. Craig et al., (2023) highlighted the correlation between effective UDL implementation and student performance on standardised tests. This demonstrates the importance of examining effective examples of pedagogy that apply UDL principles to promote equity and accessibility to mathematics instruction for all students. By examining recent research, we aim to answer the question, “How can teachers use a universal design for learning approach to promote inclusive mathematics education for all students?” The significance of this study is that we hope to highlight the transformative potential of UDL in creating inclusive mathematics classrooms thus promoting just and equitable access to mathematics for all.

What is Universal Design for Learning?

Universal Design for Learning (UDL) leverages our understanding of the human brain, recognising that each brain is unique, much like fingerprints (CAST, 2024). UDL 3.0 provides guidelines to help educators cater to this diversity through three main principles:

- **Engagement (the why of learning):** Recruiting interest, sustaining effort and persistence, and self-regulation.
- **Representation (the what of learning):** Perception, language and symbols, and comprehension.
- **Action and Expression (the how of learning):** Physical action, expression and communication, and executive function.

The UDL Guidelines provide evidence-based recommendations and prompts that can be integrated into any educational setting to help support teachers to ensure that every student has the opportunity to engage in meaningful and challenging learning experiences (CAST, 2024). In July 2024, a new update to the UDL guidelines was released (CAST, 2024), addressing gaps and biases to better reflect students’ diverse perspectives and identities. The updates aim to reduce barriers by promoting more just and equitable learning environments. The revised guidelines incorporate current research and focus on fostering purposeful and reflective learner agency, collective knowledge generation, and addressing systemic biases, making them more

relevant and effective for today's diverse educational contexts. In light of the release of the new guidelines and the increasing understanding that the UDL framework can be used to support all students in all subjects, the authors reviewed recent literature to identify how UDL has been used in mathematics education across early years, primary, and secondary education.

Method

A rapid review was conducted to explore how a UDL approach can promote inclusive mathematics for all students, following PRISMA guidelines. Cirkony et al. (2022) describe a rapid review as providing timely contextual information. We used "universal design for learning" AND "mathematics" as search terms in EBSCO, focusing on full-text articles in English published from 2015. Initially, 56 articles were found, but after reviewing titles and abstracts, only six met the criteria. These included empirical research on UDL in early years to secondary school settings with data on UDL for mathematics. No pre-formal schooling papers met the criteria, so the Early Years Learning Framework 2.0 was used to connect to UDL principles. Of the six papers, three had a primary school focus (Buchheister et al., 2017; Hunt et al., 2022; Rodríguez-Ascaso et al., 2018), and three had a secondary school focus (Franz et al., 2016; Kaur & Prendergast, 2023; Root et al., 2020). In describing strategies documented in the research literature, we use the current UDL 3.0 guidelines, noting that there has been changes in the design options and language used in the reported papers.

Universal Design for Learning in the Early Years

The Early Childhood Education and Care sector (ECEC) in Australia ranges from birth to eight years of age. In this discussion, we focus on the pre-formal schooling ages of birth to five years. Whilst the UDL framework is not referred to in the Early Years Learning Framework (EYLF2.0), the principles espoused by UDL are embedded throughout the EYLF2.0 and ECEC teaching and learning strategies (AGDE, 2022). The 'universal' is reflected in national frameworks and governing bodies that view the child as a competent, capable learner that has the right to high-quality, inclusive education (AGDE, 2022). The EYLF2.0 describes curriculum as encompassing "all the interactions, experiences, routines and events, planned and unplanned, that occur in an environment designed to foster children's learning, development and wellbeing." (AGDE, 2022, p.7). In this way, the EYLF2.0 aligns with the principles of Engagement, Representation, and Action and Expression.

Engagement in ECEC is based on a pedagogy of relational and place-based philosophies, prioritising a deep understanding of the individual, families, and communities, and respect for diversity (AGDE, 2022). By working in partnership with children and their families, learner agency and opportunity to co-construct curriculum flourishes. The principles of universal access and recognising that learning is about prioritising an individual's funds of knowledge (AGDE, 2022) are at the core of this strengths-based approach. Representation is inspired by the children, the environment, and context. Provocations are informed by the children's interests and are not fixed, but open-ended. This type of curriculum acts as a catalyst to create inquiries and experiences that are multi-modal, such as exploration tables and constructive play situations, that are supported and modelled by resources that are rich in culturally appropriate and inclusive language, representative of children's one hundred languages and their multi-modal forms of expressions (AGDE, 2022; Malaguzzi, 1996). The provision of flexible workspaces and play-based learning, provides multiple entry points that cater to individual goals and facilitates co-construction for all children. Action and Expression is encouraged through careful consideration of Engagement and Representation that supports multi-modal expression and communication.

Universal Design for Learning in the Primary and Secondary Years

The identified primary papers documented two strategies. First, how games and game-based learning can cater for the individual needs of all students by giving a thorough consideration to the UDL guidelines. Second, applying the UDL guidelines to create and use videos with accessible content to teach prime numbers.

Drawing on the principles of multiple modes of presentation, expressions, and engagement, Buchheister et al. (2017) advocates games as a strategy to meet the needs of all students. Games, according to Buchheister et al. (2017) are a strategy that not only can be used to “address mathematical content, reasoning and problem solving” but do so in a way that meets the needs of individual students (p. 7). The use of games can be a strategy that encourages students to use “more sophisticated strategies” (p. 10), make connections between representations, and communicate their mathematical thinking and reasoning whilst developing positive dispositions such as perseverance. Buchheister et al. (2017) found students to be motivated by the mathematical content and the desire to develop their mathematical reasoning. Variations provided further opportunities to engage in the mathematical content of the game while building connections. In using games, Buchheister et al. (2017) recommends that teachers “anticipate our students’ diverse needs” and provide learning opportunities that have “multiple entry points so that all students can engage” (p. 12). Hunt et al. (2022) support a game-based curriculum with UDL, recommending after-game sessions for students to create, share, and revise explanations and justifications, using digital tools like Padlet and Google Jamboard. Both studies highlight the affordances of UDL in making mathematics content accessible and engaging.

Rodríguez-Ascaso et al. (2018) applied the UDL principles to producing and using mathematical videos for year 6 “non-disabled” students focusing on all three UDL principles. Rodríguez-Ascaso et al. (2018) found that videos that had “higher levels of saturation and lightness of colour” were preferred design options and that the colour red “should be avoided” (p. 13). In terms of sustaining effort, Rodríguez-Ascaso et al. (2018) reported that providing students with a written script which provided descriptions concurrently reduced the cognitive load of students.

The identified secondary papers identified two key strategies that support equitable access to all students in the mathematics classroom in secondary settings. First taking a problem-solving approach to mathematics and second, focusing on language and literacy in mathematics.

Problem-solving in mathematics education is a key foundation that supports students to work mathematically and develop the four proficiencies in mathematics as identified in the Australian Curriculum Mathematics. Franz et al. (2023) identified that key traits of problem-solving pedagogy align with UDL. For example, encouraging students to use multiple means of representation using various tools and methods, such as diagrams, tables; providing multiple means of action and expression by encouraging students to reason abstractly and quantitatively; using various methods to represent mathematical problems; and providing multiple means of engagement by encouraging students to construct and critique arguments. Franz et al. (2023) also dispel some myths of traditional instruction, such as requiring students to learn procedures before concepts, to be competent at computation before they can tackle more complex problems and assuming some students are just ‘not math people’. Particularly for students with learning disabilities, traditional mathematics instruction often neglects problem-solving skills which can hinder their progress (Root et al. 2023). By applying UDL principles, educators can create a more inclusive learning environment that supports the development of problem-solving skills for all students.

Thomas et al., (2023) explored how the UDL framework can be applied to address the language demands in mathematics education. Highlighting the relationship between

mathematics, language, and literacy, providing various ways to represent mathematical concepts, such as visual aids, manipulatives (e.g. blocks), and technology (e.g. mathematical notation software), can help students understand and engage with the material. By allowing students to demonstrate their understanding through different methods and creating an inclusive classroom environment, educators can make mathematics more accessible and engaging for all students. Kaur and Prendergast (2023) found that encouraging written reflections about their problem-solving processes allowed students to demonstrate their understanding through different means of action and expression. This led to increases in students' enjoyment and self-confidence in mathematics, enhancing students' metacognitive thinking.

Discussion

This paper highlights the potential of the UDL framework to enhance mathematics education of *all* students in inclusive settings. By providing multiple means of representation, action and expression, and engagement, UDL can create a more flexible and accessible curriculum that accommodates the diverse needs of all students. The strategies identified, whilst specific to an education sector can transcend throughout the schooling years. However, the successful implementation of UDL requires ongoing professional development for teachers and support from educational institutions. Educators must be equipped with the knowledge and skills to design and deliver instruction that meets the needs of all students. Thus, further research into the practical application of UDL principles in mathematics contexts is required. By embracing the principles of UDL, educators can create a more inclusive and effective learning environment, ultimately leading to better educational outcomes for all students.

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The Use of Key Word Sign to Support Early Mathematical Experiences

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The use of semiotic resources in the teaching and learning of mathematics has a rich history in mathematics education. This paper explores how Key Word Signs can be used as a communication strategy, providing opportunities for children and teachers to communicate their mathematical ideas. We report on the initial phase of research, which involves identifying signs that are developmentally and conceptually appropriate for preschool-aged children. In doing so, we share our findings of 20 Key Word Signs that can be used in the teaching and learning of position and direction concepts.

The use of gestures to communicate has a long history with Peet (1886) recognising gestures as a “natural language” over 140 years ago (p. 262). Peet (1886) argued that “every effort be made to communicate with the child by motions of the hands and expressions of the countenance” using recognisable signs associated with objects and concepts (p. 262). A single gesture can have multiple meanings, depending on the context and the cultural meanings attached to a gesture. For gestures to be recognisable, they need to be consistently used in terms of their production and meaning. At a similar time to Peet (1886), Peirce is acknowledged as describing this process as semiosis, involving three key elements: a sign, its object, and its interpretant (Oehler, 1987). Key Word Sign (KWS) is an augmented communication strategy that formalises and consistently uses the same gestures to represent words or phrases.

Research into the use of KWS has investigated pre-service teachers' beliefs into the benefits and suggested ways to implement KWS in early childhood settings (Cologon & Mevawalla, 2018), preschool children with Autism Spectrum Disorder (Rose et al., 2016), and children with developmental disabilities (Dark et al., 2019). More recently, KWS, has been cited in the South Australian government Preschool Curriculum Resources (DECD, 2024) as a way to support children's active participation. While KWS is a strategy to support individuals with communication difficulties, we argue that KWS is a communication strategy that would benefit all students to communicate with one another.

Key Word Sign and the Preschool Years

The Early Childhood Education and Care sector (ECEC) in Australia ranges from birth to eight years of age. In Queensland, where this research will take place, preschool is called Kindergarten and caters for children who are turning four years of age in the year before starting school. It can be delivered in stand-alone centres and long-day care, both privately and state-owned. The Early Years Learning Framework 2.0 (EYLF2.0), (AGDE, 2022) is the guiding Framework that all Australian ECEC centres must adhere to in their teaching and learning practices. It is founded on the image of the child as being a capable and confident learner and on the principles of being, becoming, and belonging.

Outcome 4 of the EYLF2.0, highlights the importance of children sharing their ideas, collaborating, questioning, and responding to others, as well as the importance of educators modelling mathematical language and engaging in mathematical discussion (AGDE, 2022). Educators are encouraged to teach skills enhancing self-expression and communication while recognising that competence transcends language, dialect, or culture. EYLF2.0 acknowledges multimodal communication as vital, stating that children use gestures, sounds, language, and visual communication, including signing. KWS provides this opportunity for access and inclusion for not only non-verbal children to communicate but all children to access another tool that empowers their communication strategies.

Key Word Sign (KWS) is a widely recognised, unaided Augmentative and Alternative Communication (AAC) strategy. In Australia, KWS integrates signs from Australian Sign Language (Auslan) alongside natural gestures to visually highlight the keywords in spoken sentences (Leon, 2024). For individuals using KWS for communication, signs and gestures can support and enhance their speech or provide an effective alternative when spoken communication is not possible. Leon (2024) found that combining manual signing with spoken language supports individuals with developmental disabilities, improves communication, and fosters speech development. Moreover, the benefits of pairing manual signs and gestures with speech extend well beyond specialised communication contexts. Studies show significant advantages for typically developing children, including enhanced literacy skills (Daniels, 2000), improved foreign language learning (Gracie-Gamez, 2023), and greater mathematical understanding and retention (Kersey 2024). This use of gesture, combined with oral and aural communication, is a semiotic bundle that adds another set of tools to allow children agency in communicating their conceptual understandings.

Key Word Sign and Developing Early Mathematical Concepts

From a very young age, spatial thinking develops, initially as a response to learning and then developing into an understanding of spatial orientation by exploring where they are in relation to the things around them and the language of position and movement. Children also engage in cue learning by using external-based systems based on the context of their environment (Sarama & Clements, 2009). According to Sarama and Clements (2009), by the age of two years, children use spatial relational words more frequently than naming objects. Position and direction were chosen as the mathematical focus in this project as they are seen as fundamental concepts that influence children's mathematical achievements in later years (Sarama & Clements, 2009). 'In', 'on', and 'under' are the first spatial prepositions young children learn at around fifteen months of age (Sarama & Clements, 2009). Vertical directionality terms such as 'up' and 'down' are next, followed by proximity words such as 'beside' and 'between' and reference terms such as 'behind' and 'in front of' (Sarama & Clements, 2009). Children often develop or are in the process of developing conceptual understandings as this language is introduced by adults, and in turn, the introduction of this language helps consolidate emerging understandings.

The Project

In this paper, we report on the initial phase of the project, identifying mathematical concepts pertinent to the development of young children's mathematical understanding. The project aims to provide communication strategies using KWS whereby children can communicate their mathematical thinking and understanding to others. Responding to the needs of children, the project will provide targeted professional learning that uses KWS to communicate core mathematical concepts related to position, direction, and movement of objects. The research aims to answer the following research questions:

- In what situations do Early Childhood Educators, Teachers and children use Key Word Sign to support the mathematical development of children?
- How do children use Key Word Sign to communicate their mathematical understanding of position, direction and movement of objects concepts?

To answer the above research questions, a preliminary analysis was needed to identify the Key Word Signs to be used and suggest contexts and situations in which they could be used.

Key Word Signs

The research team met to discuss the mathematical focus of the study. Drawing on existing literature, we found that understanding the position and direction of objects and people were fundamental concepts in the early years. The Queensland Curriculum and Assessment Authority (QCAA) identifies “using everyday language to describe shapes, directions and positions” as significant learning as part of the key focus of “exploring numeracy in personally meaningful ways in kindergarten (Queensland Curriculum and Assessment Authority, 2018, p. 30). From our review, we identified 20 signs that can be used to explore position and direction. Table 1 lists the mathematical concepts that were identified.

Table 1

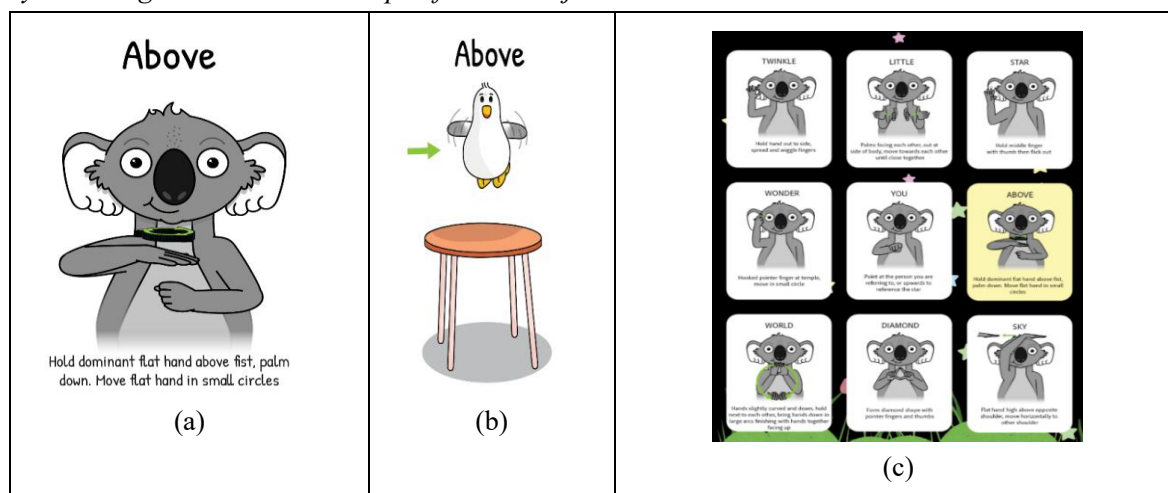
20 Key Word Signs for Position and Direction

Position and direction				
In	Under	Up	Forward	Inside
Out	Below	Down	Backward	Outside
On	Above	Open	Around	Top
Off	Over	Close	Through	Bottom

The 20 signs were verified by an Auslan interpreter who identifies as a Child of a Deaf Adult (CODA). The Auslan interpreter provided the following endorsement for the 20 signs “All illustrations used in the resource are accurate and depict each sign correctly. There are no comments on expressions as these are non-emotive signs in this context. The resource has been thoroughly researched and produced very carefully. Therefore, I would comfortably endorse the resource as well-suited for its purpose”. From the list, Sunshine Sign and Sing developed educational resources. These included posters for each KWS including a description on how to produce the sign (Figure 1a), a poster to show the mathematical concept (Figure 1b) and a range of books, videos, and posters with the sign used in context (Figure 1c).

Figure 1

Key Word Sign Resources Developed for the Project



Conclusion

In this paper, we have shared the early phases of a research project that explores the use of Key Word Sign in the preschool years to explore mathematical concepts. The EYLF2.0 acknowledges that not all children communicate verbally or understand verbal communications. By formalising gestures and integrating them with spoken language, KWS promotes active participation and conceptual learning and as such an effective strategy for enabling *all* children, to express their understanding of mathematical concepts like position, direction, and movement. The project's development of resources, verified by expert endorsement, underscores the potential of KWS to transform how educators support young learners in building mathematical and communication skills, laying a foundation for lifelong learning and inclusion.

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Let's Throw Out the Textbooks and Streaming in Mathematics!

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This paper reports on an innovative project which enhanced professional learning for in-service and pre-service teachers in a regional setting. By fostering collaborative mentor-mentee relationships and co-designing professional development activities, the project improved mathematics teaching practices and student outcomes. Utilising Breakspear's Teaching Sprints model, this project emphasised continuous learning and reflection. Key findings highlight the positive impact on teacher confidence and practices and improvements in student outcomes. The project demonstrates the potential for professional development to drive pedagogical changes.

In education, the professional development of teachers is paramount to ensuring high-quality teaching and learning experiences (Australian Institute for Teaching and School Leadership, 2018). This collaborative professional learning project between a regional university and local school, aimed to address the professional development needs of both in-service teachers (ISTs) and pre-service teachers (PSTs) in regional settings (Clarke & Winslade, 2019). This article focuses on one outcome of the three-year professional development partnership. The project's innovative approach led to significant changes in teaching practices, particularly in mathematics education, and has provided valuable insights into the benefits of collaborative professional development.

Context and Background

This professional learning project operated within a regional primary school characterised by a dedicated staff with many years of experience, but a traditional approach to professional development. This setting underscores the necessity of innovative professional learning activities that engage both ISTs and PSTs. The project is built on a foundation of mutual trust and collaboration. With schoolteacher support playing a critical role in training PSTs, a partnership between school and university offers the potential for mutually beneficial outcomes (Clarke & Winslade, 2019). The regional context presents unique challenges, such as limited access to professional learning resources and opportunities, making the project's efforts even more crucial (Ferns & Lilly, 2015). By fostering a supportive and collaborative environment, this project aims to rejuvenate the professional learning landscape and enhance the quality of education in these areas.

Research has shown that staff recruitment and retention in regional and rural schools is challenging (Burke, & Buchanan., 2022) with over 65% of teaching staff in rural, regional, and remote schools being in their first two years of teaching. This has implications for the availability of appropriate professional learning for a regional context and the readiness of ISTs to provide appropriate mentoring support to PSTs.

The primary aim of the project was to develop a greater understanding of primary school stakeholder perceptions regarding shared professional learning involving ISTs and PSTs. In this paper, we report on the research question:

What impact does professional development have on ISTs and PSTs when it is co-designed by staff?

The project's dual focus on the professional growth of both ISTs and PSTs highlights the importance of collaborative learning environments. By examining the perceptions and experiences of stakeholders, we hope to provide insights into the effectiveness of co-designed professional learning and its impact on teaching practices and student outcomes.

Methodological Framework and Model

The project employs a mixed-methods approach, including the examination of school reports and documents, the collection of quantitative survey and questionnaire data, and the use of semi-structured interviews. The initial focus is on establishing local partnerships for continued professional learning in regional settings, to support the long-term retention of teachers in these areas. PSTs are prioritised for placement at the partner school, where they participate in professional learning alongside their mentor teachers. This approach ensures that professional learning is contextually relevant and tailored to the specific needs of this regional school. The professional learning model is based on the Teaching Sprints model by Breakspear and Jones (2021), which involves three key phases:

1. Prepare: Identify the focus for professional development and engage in new learning.
2. Sprint: Intentionally practice new learnings in classrooms.
3. Review: Reflect on the process, share insights, and identify next steps.

This model allows for authentic professional conversations, planned implementation, and reflection, with PSTs and their mentors learning side by side. The cyclical nature of the teaching sprints model promotes continuous improvement, enabling teachers to refine their practices and address emerging challenges effectively (Breakspear & Jones, 2021).

The Mentor-Mentee Relationship

The mentor-mentee relationship is central to the project's success. Traditional mentoring approaches often position mentors as the authority and expert, with their experience privileged over that of the mentee (Spooner-Lane, 2017). However, this project adopts an educative mentoring approach, where the mentee is an active participant in the relationship (Aspfors & Fransson, 2015; Larsen et al., 2023). This collaborative approach values the experiences of both mentor and mentee, fostering a climate of mutual learning and growth. By emphasising the reciprocal nature of the mentoring relationship, mentors are also encouraged to reflect on their own practices and learn from their mentees, creating a dynamic and supportive professional learning environment.

The Mathematics Education Journey

The professional learning model fostered collaboration among teachers, promoting continuous professional learning, building confidence in teaching abilities, encouraging innovative pedagogical and reflective practices. This led to more effective and engaging learning experiences for students. In the first year of the project, teachers identified a focus on improving mathematics outcomes and worked to identify and implement effective pedagogical practices through the use of prepare, sprint, and review (Breakspear & Jones, 2021). In the second year, the school made the decision to change their school approach to streamed mathematics groups and reliance on textbooks in the mathematics classroom and shifted to mixed-ability classes. Professional learning alongside PSTs continued. In the third year, improvements in Progressive Achievement Tests in Mathematics (PAT-M) data (Australian Council for Educational Research [ACER], 2015) validated the new approach, leading to sustained focus on mathematics professional learning.

Results

The results of this study reveal positive outcomes in three key areas. First, ISTs reported a notable increase in their confidence levels, enhancing their teaching efficacy. Second, PSTs felt more integrated into the school community, fostering a supportive and collaborative environment. Third, improvements were observed in the PAT-M assessment results, with growth data indicating increased performance across all year levels.

Initial survey and interview data reveal positive outcomes for both ISTs and PSTs. ISTs reported increased confidence in their teaching and the ability to broaden their teaching ideas through time to collaborate with colleagues and discuss classroom practices. PSTs felt more integrated into the school community and valued the opportunity to build relationships with other teachers beyond their mentor teacher. One mentor noted the value of learning together with their PST stating, "Seeing what he was pulling out from his current university studies and how it matched up with our evidence-based research, ...we learned together, and it was brilliant." Similarly, a PST noted, "We took what we learnt, and I'd implement it in class, and then he [mentor] would and I'd observe... and then we would reflect... and learn together", highlighting the importance of creating a supportive and collaborative professional learning environment that values the contributions of all participants.

School data showed improvements in PAT-M growth data when compared with state norms. Teachers felt that the shift away from traditional textbooks and streaming, combined with the Professional learning, allowed them to adopt more flexible and student-centred instructional approaches, leading to improved student engagement and achievement in mathematics. Table 1 shows a snapshot of 2022-2023 data for students in year 4 (2022).

Table 1

PAT-M Growth Data: Yrs 3 – 4 & Yrs 4 - 5

	2022		2023	
Percentiles	PAT NORMs	HPS	PAT NORMs	HPS
95th	3.1	3	6.3	11.4
75th	4.6	4.7	6.6	6.6
50th	5.7	7.3	7.4	7.4
25th	6.7	4.4	7.2	7.2
5th	8.2	5.7	2.2	1.8

Table 1 tracks the PAT-M growth of students for the partner school (HPS) from Year 4 in 2022 to Year 5 in 2023. Comparing HPS growth to the state average (PAT Norms) in 2022, HPS students generally showed equal to (75th percentile) or less growth (5th, 25th and 95th percentiles) compared to the state average. The exception was the 50th percentile (the average student), which showed a growth of 7.3% compared to the state average of 5.7%. This indicates that for most students, their growth from year 3 to 4 was at or less than was expected by the state norm. In contrast, the 2023 data indicate greater growth for HPS students compared to the state norms for all students except the 5th percentile students, where growth was slightly less. The 95th percentile showed a growth of 11.4% compared to the state average of 6.3%, and the 50th percentile showed a growth of 7.4% compared to the state average of 4.4%. This suggests that HPS students made more progress in 2023, outperforming the state average across most percentiles. The data highlights the potential effectiveness of the professional development initiatives implemented at HPS, leading to improved student outcomes.

Discussion

The project highlights the transformative potential of collaborative professional learning in enhancing teaching practices and supporting the professional growth of both ISTs and PSTs. A key finding is the importance of a collaborative mentor-mentee relationship, which fosters mutual learning and professional growth. This relationship allows both mentors and mentees to reflect on their own practices and learn from each other, creating a dynamic and supportive

environment. The positive impact of co-designed professional development on teacher confidence and teaching practices is noteworthy. By involving teachers in the design of their professional learning activities, the project ensures that the development is relevant and tailored to their needs, leading to increased confidence and a broader range of instructional strategies. The potential for professional development to drive changes in pedagogy and improve student outcomes is highlighted. Through continuous learning and reflection, teachers are able to adopt innovative and effective teaching practices that enhance student engagement and achievement. This was seen in the changes in mathematics pedagogical practices and improved student outcomes as shown by the PAT-M results.

The findings of this project had a transformative impact on teaching practices and student outcomes. The collaborative professional learning activities, grounded in Breakspear's Teaching Sprints model, fostered a culture of continuous learning and reflection among teachers. This approach enhanced teacher confidence and broadened their instructional strategies, leading to notable improvements in student engagement and achievement, particularly in mathematics.

Acknowledgements

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Research Papers

Secondary Mathematics Teachers' Understandings of Reasonable Adjustments: Is it Inclusive Teaching?

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Teachers are expected to cater for the diverse learning needs of students in their classes including by the provision of reasonable adjustments for students with disabilities that impact their learning. In this paper we problematise the notion of reasonable adjustments for students with intellectual and developmental learning difficulties. Drawing on qualitative survey data, we make the case that with few exceptions the needs of these students can be met using inclusive approaches to teaching that benefit all students. We conclude with concern that restricting teachers' freedom to provide differentiated instruction for students risks breaching legal obligations.

Access to education is recognised globally as a human right. United Nations' declarations, including the Universal Declaration of Human Rights (1948), the Convention on the Rights of the Child (1989) (Article 28), Convention on the Rights of Persons with Disabilities (2006), and Declaration on the Rights of Indigenous Peoples (2007) have led to legislation enshrining this right in signatory countries around the world including in Australia. Local legislative instruments include the Disability Discrimination Act (1992), Disability Standards for Education (2005), the Australian Curriculum (Australian Curriculum, Assessment, and Reporting Authority [ACARA], n.d.a) structured by year level, and the Human Rights Act (2019). The Disability Discrimination Act includes three key concepts:

1. The right to education on the same basis as other students.
2. Reasonable adjustments are made to ensure that students with disability have opportunities to participate in education and training.
3. Consultation between the school, parents/carers and student takes place prior to adjustments being made.

Teachers are thus legally obliged to implement reasonable adjustments when required but there exists very little guidance as to what it means to participate "on the same basis" or what constitutes a "reasonable" adjustment. Beyond legislative obligation, teachers feel a moral imperative to meet the needs of all their students and allow them to access the joy of learning. Early results of the project upon which this paper draws also suggest that students with intellectual and developmental disabilities very much enjoy experiencing the curriculum appropriate to their year level (Faragher et al., 2019). The specific research question that guided the study reported in this paper was, In what ways do Australian secondary mathematics teachers understand and implement reasonable adjustments for students with intellectual and developmental disabilities?

Reasonable Adjustments

Consistent with the three concepts that underpin the Disability Discrimination Act, an adjustment is a measure taken to assist a student with a disability to participate in education or training on the same basis as other students. Such an adjustment is reasonable if it accounts for the students' learning needs while balancing the needs of others, including the school staff and other students. Adjustments may not alter the inherent requirements of the task. For example,

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 61–68). Canberra: MERGA.

on a writing task, adjustments might include a speech to text device, so long as the task is not assessing handwriting. Consultation with parents and the student concerned is an integral part of identifying and making reasonable adjustments.

Common approaches to teaching students who are well behind their peers for any reason include providing them with work from earlier year levels, often based on a belief that it is important to start from where the student is in order to fill in gaps in knowledge, or providing completely different content characterised as functional mathematics or life skills based on beliefs about the capacities of these students to learn and assumptions about their life trajectories. These strategies are often achieved by separating the students concerned from their peers either within a mainstream classroom, through periodic extraction from the classroom, or streaming. These approaches do not meet the definitions of inclusive education or of reasonable adjustments. In addition, Faragher et al. (2016) were unable to identify any research suggesting any categories of learners that needed to be taught separately from their peers.

Differentiated instruction recognises that students learn in different ways (Gibbs & Beamish, 2021). Teachers need to tailor teaching to their students' individual needs and readiness and use flexible groupings, ongoing formative assessment to check progress, and respond to student needs as they arise. Tomlinson (2017) suggested that teachers can vary their teaching approach in four ways to cater for the diversity of students in their classes. These are varying: the content that is taught; the processes by which students learn; the products that students produce as evidence of their learning; and the learning environment with a view to creating a classroom that is safe and welcoming for all students. The latter may include practices such as the explicit teaching of social and emotional skills (Bierman & Sanders, 2021), encouraging collaboration and peer learning (Tullis & Goldstone, 2020), and practicing culturally responsive teaching (Australian Education Research Organisation, 2024; Samuels, 2018). Although most of these elements are consistent with reasonable adjustments, there is a danger in mathematics if varying the content that students are taught is too readily adopted. Learning on the same basis as other students requires beginning with content aligned with students' chronological age or year level. As such, identifying year-level appropriate content needs to be the starting point for planning teaching that will meet all students' needs (ACARA, n.d.b) including in the secondary years where the challenge appears to be greatest (Faragher, 2014).

Inclusive Mathematics Teaching

Inclusion is a broad concept that applies to all learners including gifted, culturally and linguistically diverse, of all genders and from all socio-economic backgrounds, as well as those with learning difficulties or disabilities. In inclusive environments everyone feels welcome and valued and the learning needs of all are supported (Faragher, 2015). Few would argue that such environments are not desirable but achieving inclusive education, including in Australia, has been difficult and is yet to be achieved (Boyle & Anderson, 2020). This is despite evidence of the educational, social, and economic benefits of educating all learners together (Boyle & Anderson, 2020).

Inclusion is different from integration, segregation and exclusion which are key terms with distinct meanings (United Nations Committee on the Rights of Persons with Disabilities, 2016, para 11): "*Exclusion* occurs when students are directly or indirectly prevented from or denied access to education in any form. *Segregation* occurs when the education of students with disabilities is provided in separate environments designed or used to respond to a particular or various impairments, in isolation from students without disabilities. *Integration* is a process of placing persons with disabilities in existing mainstream educational institutions, as long as the former can adjust to the standardized [*sic*] requirements of such institutions." Applied to the mathematics classroom, examples of exclusion are when students are prevented from enrolling

in mathematics options, or mathematics altogether. Exclusion of some students from higher levels of mathematics occurs for students without disabilities as well (Forgasz, 2010). Segregation occurs when students are taught different mathematics content from other students, sometimes in the same classroom, but taught separately from peers, and often by another teacher or teacher aide. Integration is in place when students are allowed to enrol in a mathematics course or class but are not supported to be successful. The old adage, “teach to the middle”, reflects an integration view – it is the learner’s job to make sense of the lessons.

Inclusive mathematics teaching rejects exclusion, segregation and integration. The learning support needs of learners are attended to, and their strengths are capitalised on for the benefit of the class. Beginning with the year level curriculum, teachers plan lessons, anticipating learning support needs from a range of sources. For example, a need for language support might come from developmental language disorder, hearing impairment, or cultural or linguistic diversity. An approach to planning, Universal Design for Learning (UDL), works on the premise that if multiple options and approaches are planned into lessons, the need is reduced for adjustments to be made later to suit some learners (Novak, 2022).

The Study

The study reported here is part of a larger study of the ways in which secondary mathematics teachers adjust year-level mathematics curriculum for students with intellectual and developmental disabilities. It included an online survey and work with mathematics teachers in six partner schools that included professional learning, co-design of adjustments, classroom observations, and interviews. This paper draws on responses to two survey items. The study received ethics approval from the University of Queensland Human Research Ethics Committee B (2022/HE001550).

Methodological Approach

A survey was distributed electronically to mathematics teachers across Australia by way of the Australian Association of Mathematics Teachers and its affiliates in each state and territory, and at conferences for mathematics teachers. It included two open-ended items (shown in Figure 1) asking teachers about their understandings of reasonable adjustments. Responses to these questions form the basis of this paper.

Figure 1

Open-ended Survey Questions.

Question 1.

We wish to know more about teachers’ thoughts on reasonable adjustments in secondary mathematics classrooms. Reasonable adjustments are required under Australian law as stated in the Disability Standards for Education 2005. One purpose of the standards is: “to ensure, as far as practicable, that persons with disabilities have the same rights to equality before the law in the area of education and training as the rest of the community”.

Please think about this, and using your experience as a secondary mathematics teacher, describe your understanding of what you are required to do in your classrooms to implement reasonable adjustments in mathematics.

Question 2.

If you have applied reasonable adjustments, please give a brief description, including which year levels/ students/ areas of mathematics were involved.

Thirty-four secondary mathematics teachers responded to the two open-ended questions on the large-scale survey. Respondents came from Queensland, Tasmania, New South Wales and Victoria, with some respondents not providing this information.

Responses to the open-ended questions were coded according to the adjustment(s) that were

mentioned. Many responses included several ways of making adjustments for students with disabilities and hence were assigned several codes. The codes were grouped into categories. We analysed the codes in each category in terms of whether they represented a reasonable adjustment or could be considered part of inclusive teaching likely to benefit all students.

Results

Five categories concerning Content delivery/pedagogy, Tasks/content, Tools, Classroom environment/management, and Assessment emerged from the responses to Questions 1 and 2. Most responses mentioned changes to the way in which content was delivered, with smaller, approximately equal numbers of responses falling into the other categories. These are discussed in turn in the sections that follow.

Content Delivery/Pedagogy

There were 33 references in responses to Q1 to modifications of methods of content delivery or presentation. Some respondents wrote that they provide additional scaffolding and others specified means by which this could be achieved. These included colour coding task steps, specifying the steps required to complete a task, using captions on videos, pausing audio-visual presentations, providing additional videos, and writing key points on a whiteboard in simple English. The provision of clear instructions was also emphasised, “Adjust the amount and complexity of spoken and written information/instruction”, along with “jointly deconstructing question prompts”, and “reminding students to use key subject-specific vocabulary”. Individual prompting and instruction were also mentioned, as was providing clear timeframes for the completion of tasks. There were two mentions of slowing the pace of content delivery and one reference to “adjusting workload expectations”.

Examples of practices offered in response to Q2 included providing, “more explanation, more visualisation, more application etc.” Other pedagogical adjustments included “not calling on students to answer publicly (usually after confirming with student support services)”, slowing the quantity of material and the pace of delivery. These were used with students across Years 7 to 12. Other examples included using pre-recorded videos of lesson content for two Grade 10 classes; providing explicit, scaffolded instructions (Year 7); individual instructions (Year 8) along with remote lessons (Years 11 and 12); and presenting information electronically or written so that copying from the board was not needed (Year 8). Chunking material and “actively ensuring I have the student's attention, attempting to use the student's interests as a hook” and making learning available online were strategies employed for a Year 12 student with attentional difficulties. Another teacher also described trying to link the content to an area with which the student was familiar as well as breaking questions into steps to follow and allowing “them to show the answer without full setting out, many students struggle to set work in a particular way but can explain”. Providing more detailed instructions, instructions one by one, and more scaffolding were mentioned by several teachers without specifying a year level.

Tasks/Content

The nine responses to Q1 in this category included the provision of different worksheets, different levels of work, the use of less complex written and oral language, different questions for “practising certain concepts”, “adjusting work requirements for students with learning difficulties”, and modifying “lesson content or expectations for each student in every lesson”. There was a concern that students be able to “engage on their level” and experience success. One mentioned the use of “low-floor, high ceiling tasks where possible”, ensuring that “ALL students can access the initial stage of the work”. Student choice was mentioned in relation to the completion of practice problems; “I allow students to choose their own pathway”, and in relation to “open-ended/research tasks” for which the provision of “structured guidance and

limited choices” were specified.

Modified worksheets were used including those providing more practice of simple familiar questions, repetition, and real-life applications. One response to Q2 described a Year 9 student, “operating at roughly a Year 6 level” studying, “a completely different (Life Skills) curriculum using specifically created resources, physical manipulatives, School Learning Support Officer support etc.” This teacher explained that, “owing to the streamed nature of classes in high school mathematics, the curriculum itself is differentiated as a result of the classes being taught and the level of mathematics pursued. Generally, I will make adjustments for lower streamed classes to enable the learning e.g., friendlier numbers...”. Two teachers mentioned starting the whole class on the same topic but then providing “alternative work at different year levels on the current topic additional scaffolding of work” or breaking into groups “to continue at their own level”. A specific Year 8 adjustment was limiting the number of variables in algebraic expressions to just two. There were several mentions of reducing expectations of the amount of work that students were to complete in class, and one said that they did not require homework.

Tools

The seven references in this category provided in response to Q1 included permitting the use of calculators, along with different representations of concepts via “diagrams, charts and graphs”, using larger fonts, and enlisting the help of a scribe. There was mention of using “modified resources”, without further explanation.

Similar examples were provided in response to Q2. In addition, the following were used: communication cards for a student in Year 10, described as non-verbal autistic; “a genre guide with explanations and word limits for each stage of the task” along with a checklist and staged completion dates; wheelchair access; copies of notes; Google classrooms revision activities, check-ins in a Year 11 class studying a less demanding mathematics subject; the use of graphical packages; stand-up desks (Year 8); digital and concrete modelling resources for students with intellectual impairments (Year 10); “Digital scaffolded summary documents” (Years 11 and 12); vocabulary lists and word walls; photographs, videos and written examples, tactile resources and addition and multiplication grids; and specific coloured paper for students with dyslexia were also mentioned.

Classroom Environment/Management

Q1 also elicited seven references relating to the classroom environment/management. These included the use of seating plans with one specific reference to allocating “seating for students according to their sensory needs in the physical environment e.g. lighting, furniture, positioning, able to move around from sitting to standing”. Relatedly, providing “access to separate learning areas as needed” was also mentioned. Visual timetables, routines, and breaks for movement and rest were also mentioned. One respondent wrote that they provide, “1-1 instruction and modelling of behaviours” and another that they, “Develop listening and speaking skills with mixed ability cooperative learning groups”.

In addition to the practices elicited by Q1, responses to Q2 included offering “lots of verbal praise”, frequent check-ins, and pre-warning of changes to routine for a non-verbal autistic student in Year 10. Having a Year 10 student with ADHD perform tasks such as wiping the board and handing out books provided opportunities for movement; organising seating to ensure space for wheelchairs; specific attention-focussing activities; opportunities for a Year 8 student with “ASD, anxiety and Sensory Processing Disorder” to participate in all class activities; and retaining a Year 10 student’s books and providing equipment were mentioned.

Assessment

Responses to Q1 included overlap in this category with strategies mentioned in relation to

content delivery/pedagogy. These included providing the assistance of scribes, extra time, and rest breaks during exams. Three of the seven references to assessment referred to different or differentiated assessment with one specifying that, “The teacher needs to be very clear on what is expected of a student in the standard of achievement. What does a student really need to demonstrate to pass a subject and be open to students not fitting a mould?”

In response to Q2, one teacher explained, “Year 11/12 students with anxiety had separate assessment routines with small group supervision, rest breaks etc. Efforts were made to reassure them; ensure they were adequately prepared for the test etc.” Another referred to a Year 7 student with dyslexia who had a reader/writer available throughout an assessment.

Discussion

Most of practices that teachers described are arguably beneficial for all students and at least do not impede their studies. Providing clear instructions, videos that can be re-watched, plain language summaries of key points, clear timeframes for tasks, ensuring that students are paying attention, allowing the use of technologies including calculators when their use does not compromise the goal of a lesson, having resources like vocabulary lists and detailed written instructions available as needed, and allowing students appropriate opportunities to rest or move are sensible for all students. All are consistent with UDL, a planning approach that assumes student variability, with multiple approaches to learning activities offered in a lesson to achieve a firm goal (Cast.org). It is important to note that UDL is not an adjustment. Indeed, prior planning with options for learners reduces the need to make adjustments for individuals.

It is noteworthy that the modifications teachers reported making fit within the universal design principle of available for all: there if you need it, and it doesn’t get in the way if you don’t. We are unable to identify in the survey responses any approaches teachers use in mathematics lessons for students with intellectual or developmental disabilities that would not fit UDL. For other disabilities, reasonable adjustments to mathematics are essential. An example is the use of tactile graphics for students who are blind or have low vision (Fanshawe & Cain, 2024). It seems that the group of learners with intellectual or developmental disabilities, often considered challenging for teachers, require good quality teaching with differentiation and learning options, consistent with the principles of UDL. These strategies and approaches are within the expertise of Australian teachers.

The extent to which adjusting the content delivery/pedagogy amounted to changing the nature of the content is unclear. It is possible for the provision of additional scaffolding, such as by providing steps to be followed to complete a task, to alter the inherent requirements of a task and hence the opportunities to learn that it provides. If this is the case, then such an intervention does not constitute a “reasonable adjustment” although consistent with Tomlinson’s (2017) suggested approaches for catering for diverse student learning needs. Certainly, slowing the pace of a lesson necessitates reducing the volume of content that is taught, as does reducing expectation of the amount of work that students with disabilities complete. These students are thus excluded from parts of the curriculum. Such teaching is not inclusive and contravenes the rights of these students (United Nations Committee on the Rights of Persons with Disabilities, 2016, para 11). Teachers’ concern for all students to experience success is a further contributor to lowering the cognitive demand of tasks offered to students with learning difficulties and is reinforced by the belief that students who are significantly behind their peers need to start from where they are, guaranteeing that they will not attain year-level curriculum. Although well-meaning, adjustments of this sort are also inconsistent with inclusion. Similarly, offering students experiencing difficulties with learning mathematics more repetitive practice further detracts from time available for engaging with new content.

A response in the survey, deeming a student to be “at year 6 level” though in Year 9, reflects

a wider held view by some teachers and a policy approach in some jurisdictions. There are many problems with this view: it presumes an even profile of learning across subjects and within subjects. This is improbable – consistent performance across all subjects of the curriculum and across domains in mathematics does not account for varying interests, aptitudes or experience. A significant risk of this view of “being” at a lower year level, is the segregation of the learner that must result, even if philosophically. They are deemed to be different from class peers, requiring different teaching approaches. This is contrary to inclusive practice.

Two teachers described approaches that involved all students starting lessons engaging with the same topic, and there was mention of tasks with low floors and high ceilings. These strategies offer the potential for all students to access year level content. Teaching students how to work effectively in heterogenous, cooperative groups is a further strategy that contributes to inclusion (Tullis & Goldstone, 2020).

There were several measures that teachers described that are established elements of effective mathematics teaching but appear to be reserved for students with learning difficulties or “intellectual impairments”. These include the use of manipulatives or tactile materials (Larbi & Mavis, 2016), emphasising applications (Stillman, 2004) and making connections with students’ interests and concepts with which they are likely already familiar (Hatisaru, 2024). All students would benefit from these strategies.

Our data remind us that efforts to achieve inclusive mathematics education are occurring in an environment in which streaming according to prior attainment means that exclusion and segregation occur at the level of class allocation. Despite the fact that streaming is intended to reduce diversity in terms of attainment, there was no evidence in our data that teachers were finding that it had made inclusive mathematics teaching easier or more common.

Most of the practices, as reported in our survey, that mathematics teachers use to support the learning of students with intellectual or developmental disabilities are examples of quality differentiated instruction. Teachers make adjustments to their lessons based on their knowledge of the learners, their knowledge of their mathematics, and their pedagogical expertise. This flexibility is essential to meeting the learning needs of their students, a right afforded to students in Australian law. In recent times in some Australian education systems, a trend is emerging to focus on whole class instruction, without variation, in some cases using scripted lessons. Such policy positions have the potential to constrain the options for teachers to meet the learning needs of students through approaches such as UDL. In such situations, it is likely that there will be a need to provide alternative learning for students with intellectual or developmental disabilities in ways that are not inclusive – either through integration or segregation. This is in contravention to Australia’s laws and our international convention obligations.

Conclusion

Australian mathematics teachers who responded to our survey indicated a range of adjustments they make to support the learning of students with intellectual or developmental disabilities. These approaches are indicative of quality teaching strategies that are beneficial for all learners or at least do not hinder their learning. Teachers, given the pedagogical freedom to plan for learning options, can support the inclusion of all students in mathematics lessons. The benefits of inclusion are well established, for students with disabilities and for those without (Hehir et al., 2016). Limiting teachers’ ability to provide learning adjustments for students risks breaching Australian law and our international convention obligations. It is critical that the expertise of teachers to adjust mathematics lessons to meet their students’ needs is affirmed, encouraged and enhanced to improve the learning experiences and outcomes for all students.

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Teacher Professional Development in Remote Papua New Guinea

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Teacher education for early childhood education in Papua New Guinea (PNG) has been minimal in the past and this paper reports on workshops in which both quality mathematics education and computers were introduced to teachers in remote areas of PNG. With a lack of funding and time, the logistics of holding the workshops are noted. There were some small but significant gains made by teachers. The paper also notes some of the issues for teachers indicating ways in which further professional development could occur.

Professional Development in Developing Countries

Much of the professional development (PD) that takes place in small developing countries occurs when curriculum changes, often at the change of government, and/or the PD is funded by an external provider under an aid project as happened with this research (Paraide, Owens, Muke, Clarkson, & Owens, 2023). One of the issues is the importation of whatever the external provider deems appropriate, generally reflecting global trends. However, in this research, the PD promoted ideas from mathematics education research on multilingual classrooms (e.g., Clarkson & Halai, 2016) and ethnomathematics (Rosa, Shirley, Gavarrete, & Alanguai, 2017; Vandendriessche & Pinxten, 2022). Language is seen as a resource and home languages and translanguaging are encouraged in the classroom (Planas, Morgan & Schütte, 2021). When the teachers work together to express the introduced (Eurocentric) mathematical terms in their own language, they gain a better understanding of the mathematical concepts (Edmonds-Wathen, Owens, Bino, & Muke, 2018; Edmonds-Wathen, Owens, & Bino, 2019). Furthermore, with ethnomathematics, there is greater social justice for all students (D'Ambrosio, Martin, Frankenstein, Moschkovich, Gutiérrez, Taylor, Kastberg, & Barnes, 2013), and improved learning, identity, and agency for thinking and working mathematically (Owens, 2014). This then reflects the importance of working together respectfully to establish Indigenous knowledges and to meet the Rights of Indigenous Peoples (UNRIP, 2007). Article 14 includes “to have access, when possible, to an education in their own culture and provided in their own language”.

The Affordances of Computers for Ongoing Professional Development

Computers have been shown to engage users in learning. They can do this by keeping the learner active, engaged, seeing the material as meaningful, and being socially interactive (Hirsh-Pasek, Zosh, Golinkoff, Gray, Robb, & Kaufman, 2015). Videos too have been shown to be engaging when they have meaning and are discussed. Furthermore, the opportunity to continue learning beyond the workshop is effective (Hirsch & Nagler, 2020) and valued (Wang, 2015).

The Setting

Papua New Guinea is an amazingly beautiful place with rich cultural heritages of 850 different language groups. It has few good roads, is swampy and mountainous. It is also relatively dangerous for a variety of reasons. In total 14 workshops were held in six provinces,

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mostly in remote areas. This paper reports on workshops run by Charly Muke in highland Jiwaka Province with teachers from a number of schools and languages and by Vagi Bino at coastal Waima close to the western border of Central Province with teachers walking or cycling up to 10 km from other schools in “the valley”. Roro is the local language. The workshop was in the new elementary school classrooms. Kay Owens assisted. Two teachers from another language area joined this workshop. The four of us were accommodated in a room of a bush material house, sleeping on the floor, with outside latrine and shower enclosures. Charly’s simple lodge accommodation made a space available for the workshops. His first workshop with Kay was for all the trainers/inspectors of the province but this paper reports on the later workshops with computers.

Approvals for the workshops had to be gained so teachers could leave their schools and come. For Waima, we brought food for participants from Port Moresby on a truck passenger vehicle supplemented by village food and prepared by the women in the extended family of Vagi’s former student. Workshop organisation relies on relationships and goodwill. In remote PNG, there are virtually no opportunities for professional development except for discussions within the school but that will be limited if teachers have virtually no teacher education which was the case for the elementary schools. These schools are now replaced by one year of early childhood education, and Grades 1 and 2 returned to fewer, more distant primary schools.

Research Aims

The aims of the research were to:

1. Use design-based research methodologies to establish a design of principles for cultural mathematical proficiencies in transition to school;
2. Develop a workshop manual expanding on the principles and incorporating an inquiry model of teacher planning, examples on different topics and ecologies, and dictionary guidelines based on past research on PNG cultural mathematics;
3. Organise and deliver workshops to evaluate the research design;
4. Develop a stand-alone interactive website incorporating the pages of the manual together with video examples for use in workshops and by elementary teacher trainers guided by a trainers’ manual;
5. Develop sustainable technology for trainers and teachers to use the manual on professional learning (PL) and to strengthen their teaching through use of the computer for classroom teaching;
6. To disseminate results within PNG especially for curriculum development and training and for other colonised communities.

This paper reports on the research to meet, in particular, Aims 3 to 5. The MERGA37 (2014) symposium (papers by Owens, Bino, Kravia, Edmonds-Wathen, & Sakopa) reported some of the results for Aims 1 and 2 along with other published papers (see reference list).

The materials in the current phase of the design research involved setting up of a webpage-like set of information that included each of the chapters of the Manual for teaching mathematics in early childhood, assessment tasks, children’s readers, material for games, mathematical terms that could be translated into local language, documents on cultural mathematics sorted by province, videos of (i) cultural activities that reflect cultural mathematics, (ii) lesson activities, and (iii) giving assessment tasks. For a teacher they were very meaningful as they enjoyed watching cultural activities and they could learn more about teaching. The manual itself covered the topics shown in Figure 1. All these were made available on a computer, powered by a lithium battery and solar panel and that combination had first to be sourced and tested.

Attendance at the workshops was high. No doubt the attaining of a computer, solar panel and battery were incentives. However, we did not lose sight of the mathematics education that was the central focus of the workshops.

Research Design

A design research methodology (Kravitz & Owens, 2014; Wang & Hannafin, 2005) was used and this paper refers to the later phases of the design development which introduced computers to teachers to review material at home and to use in their teaching both for record-keeping and as a teaching tool with self-made stimulus materials or supplied videos. Observation during the workshop and questionnaires evaluating the workshops were used to evaluate the model for professional development. The key for learning and making educational changes was that teachers need to participate in alternatives, try them out, and know where to start in planning. In order for teachers to have ownership of changed teaching practices, it is important to re-establish that teachers are able to and permitted to change. They need to be motivated to change for the good of their children, to manage inquiry from cultural mathematics to school mathematics, to assess what and how their children are learning, and have support from their managers or people that matter. There needs to be a general milieu or architecture as a context for change of practice (Kemmis, 2022).

Results of Previous Implementations

The previous implementations of the workshops (Owens, Edmonds-Wathen, & Bino, 2015) showed that:

Teachers Need to Make Greater Use of Children Using Their Home Language

Teachers need to learn how young children learn to count and work with numbers. This includes in the vernacular with the cultural counting system and later in English for base 10 arithmetic and to use the affordances of differences in the systems such as different composite groupings and different ways in which the languages might combine words and syllables to make the number word. Physical activities such as groups of children and stones with a stick to replace the group and 10-frames can enhance the understanding for students for whom English is not their first language. Teachers need mathematics readers in the vernacular and English and to learn to read to the class involving the children with questions or noticing. Teachers need to ask questions and be resourceful for small group activities so that children can develop their own understanding of mathematical concepts. Children need to do the talking of mathematics in a language they understand. This is encouraged by having culturally relevant mathematics, inquiry method for learning, and mathematics questions that might have more than one answer.

Teachers Need to Recognise Cultural Mathematics and Hence Mathematics as Reasoning

Culturally people use mathematics (patterns and relationships, numbers, spatial arrangements, measurement, reasoning) to solve problems. They have developed systems of practice that need to be recognised and linked to school mathematics. Studying the language used to describe mathematics assists teachers' mathematical understanding and helps children learn the concepts well. Teachers use good bilingual education practices to assist children to speak mathematics in their own languages before or in conjunction with English.

Teachers Need to Plan for and Practice Inquiry in Mathematics

The elementary teachers developed inquiry teaching plans. The inquiry plan for a week supports good teaching practice (Murdoch, 2022). The steps were Tune in, Decide what to find out and find out, Sort Out, Go Further, Reflect and Act. Owens had used this plan to integrate HSIE and mathematics in western NSW and trialled it in PNG incorporating cultural mathematical activities. Writing a learning plan as a group of teachers with the same or similar

languages encouraged details such as questions or things for children to notice. The teachers as a group taught the inquiry plans using their peers or a group of children if available. Teachers learned to ask open-ended questions using the readers or the cultural activity. The workshops involved open-ended questions especially in the introduction that encouraged the teachers to share and speak. They used our developed teaching design and supporting resources such as group games and readers, and they found resources or activities around the venue. The teaching design was supported by PD that covered key mathematical processes, the inquiry approach to teach, and early childhood mathematical thinking. The inquiry method encouraged teachers to go further with the concept in class, going deeper and making connections to home or other mathematics, applying to new examples, or taking action on new learning.

Implementation With Computers

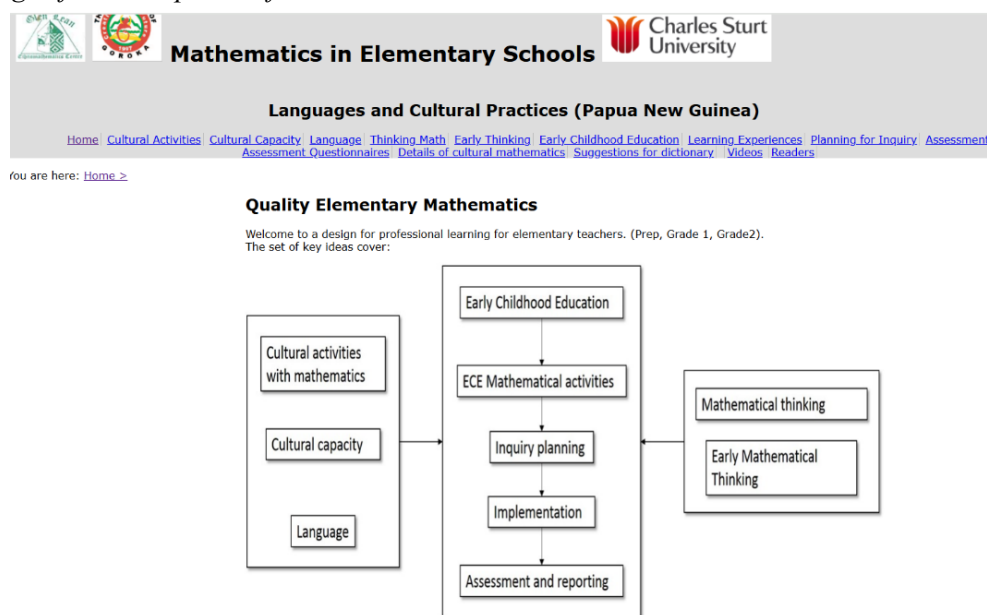
The small laptops were loaded with an earlier version of Microsoft Office that did not rely on the Cloud or yearly renewal and a freely available compatible program, Libre. The materials were presented in a website format so that each aspect could be connected by a link (Figure 1) prepared by Vagi with assistance from Kay. The batteries were all charged. The screen could be a touch screen engaging teachers through touch. However, we also taught them about files, word processing, taking photos and inserting them into a word document, and spreadsheets as teachers already kept records on paper on attendance and student outcomes.

The initial tutor's manual was prepared by Sorengke Sondo and Kay for a Madang workshop. This was checked by Vagi who had been teaching basic computing to students at the University of Goroka for some time. We began the workshop by using an introductory name tag game to show a question can have multiple answers and to note who was doing the talking, and then discussed the overview given in Figure 1. One of the features of this implementation was students using computer programs: (i) Windows Explorer: to find information such as a game, the assessment tasks, early mathematical learning, the inquiry learning plan and an example lesson plan, and they set up folders, and saved Word and Excel files, (ii) Word or its equivalent: after our introductory activity, they typed in features as an example learning plan (with headings, text, questions and dot points), and the outline of their own learning plan using the inquiry model, (iii) Excel or equivalent: recording the items on the assessment test and the results of testing their peer, and (iv) websites: as the manual presentation simulated a website, they scrolled down and used drop-down menus for more information. These were paired activities working with one computer. After the workshop, the menu and drop-down menus were placed on the left side.

Teachers first gave the purpose of the lesson (from their syllabus). They detailed the village activity related to the inquiry in the *tuning in* (introduction). Then they discussed and decided what the students were to *find out* and how and typed that in their plan. In the *sorting out* phase, they typed how students would summarise what they found out. The teachers were encouraged to add questions or mention things they wanted the students to notice. (Already this was far more than they had planned before.) In the next phase, students were to *go further* such as ask a "what if" question or apply what they learnt to another situation or problem. We emphasised this phase. Finally, students *share* their learning, plan to *do* something, *or connect* the new conceptual understanding to another part of their knowledge, usually mathematics or culture. This was to be a week of lessons. Teachers planned questions to ask students about the early readers and developed their confidence by reading as if to a class. The two or three teachers working together shared their learning plans with the rest of the class. As time was short, they partially taught the plan, describing parts that they did not have time to implement.

Figure 1

Front Page of the Computer Information With the Manual and Additional Material



Data

Charly observed lessons and he also gave six of the teachers who attended a workshop with computers a semi-structured group interview (see long quote in results) so they could hear other responses and feel comfortable sharing but he noted the individuals who commented.

The Waima workshop used an evaluation questionnaire and 18 of the teachers completed it. Key themes were drawn from their responses and summarised.

Results

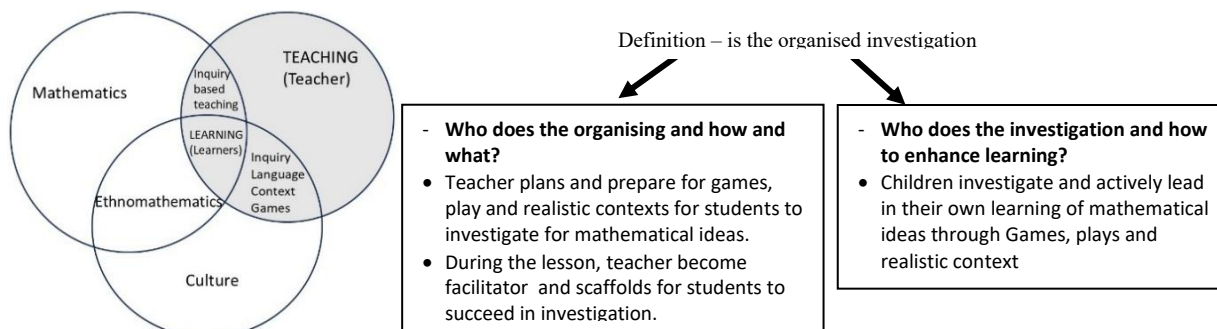
One of the results of these workshops was the ownership taken of the materials by Vagi and Charly. He reported:

The week before the workshop, I observed two lessons and videotaped them. The common teaching approach I observed was very much teacher centered. Both teachers were talking most of the time and used limited activities that involved children.

Therefore, I had to connect this experience to Inquiry base teaching and learning during the workshop. I used the following diagram (Figure 2) to emphasise that learning is the core business of a classroom context, even though layers of other factors play a part, including teaching process.

Figure 2

Models Used in the Jiwaka Workshops



I emphasised that teaching process is important but the technique used in teaching must be those that promote the learning process. Learning is the end product of a classroom and the other factors including teaching must contribute to successful learning. Teachers must facilitate and be the facilitator of learning by providing environment that engages students to learn.

The latest researched and approved teaching/learning technique that promotes learning is INQUIRY based Technique. (He told them), in this workshop, I will explore this technique in detail.

Charly used the manual to provide the key ideas in a reduced format that he knew would meet the needs of the teachers (Figure 3).

Figure 3

Summary of Key Ideas From the Manual

Teachers' Role	
• Plan – Identify what you want children to learn before setting out the resources, including: ○ Facts ○ Ideas ○ Mathematical thinking ○ Skills • Questioning ○ Ask questions – use focus questions to help children learn the appropriate: ▪ Language ▪ To make comparisons ▪ Look for multiple solutions ▪ Questions that will help children generate further questions of their own ○ Plan questions that help: ▪ Tuning in exploration to encourage children to question ▪ Specific finding out exploration and questions ▪ Going further exploration and questions ▪ Making conclusions about explorations ▪ Taking action and applying the ideas ▪ Reflecting and sharing and linking to other ideas • Make comments • Offer resources – move children to problem-solve and predict potential outcomes	• Encourage students to be: ○ Curious ○ Imaginative ○ Enthusiastic ○ Ask questions • Help children to develop the skills of: ○ Observing ○ Comparing ○ Classifying ○ Measuring ○ Communicating ○ Predicting • Provide children with TIME to: ○ Think about ideas ○ Discuss them with their classmates ○ Make observations ○ Find additional examples ○ Think of alternative explanations ○ Test ideas in situations that seem applicable ○ Think about what they themselves do, not what they see others do ○ Do things in groups where they must think critically and communicate with each other ○ Practice and master skills such as measuring, recording, and communicating
Students' Role	
• Action: ○ Play ○ Explore ○ Observe ○ Participate in dramatic play ○ Help with model	• Promotes: ○ Cooperating and interacting with other children ○ Reasoning and reflecting ○ Imagining and inquiring ○ Assessing and evaluating
Teachers spend less time talking and telling, and children spend less time memorizing facts.	

In a later workshop, Charly first asked the teachers if they had used computers before. None had used a computer before and only two of the six said they knew something about them. Because the workshops were only two days, he gave them the computers to use for professional learning and teaching as they wished. He encouraged them to find out more from other people. They came back showing they had really explored the laptops and the programs and what they asked to know more about is quite revealing of their gained knowledge. The responses were in Tok Pisin (PNG lingua franca spoken by most people including all the researchers) and translations are presented in parentheses: a) (Can we use modem to access internet), b) (We did table but making it big or small, I found it hard), c) (I found it hard to download to a flash drive), d) (We used digital camera and took pictures, and when we want to paste in the word document as a part of a report, we found hard to do that), and e) (I wanted to get pictures using the laptop camera but when I tried, it took me instead of people in front?). The comments by one teacher reflect the motivation of the teachers and how they made use of the computers.

Researcher: Morning...Okay, what we will do now, we will generally discuss about the general use of laptops. I will pose some questions...did you find it easy or difficult to get in (use it)?

Kevin (pseudonym): (When I first got the laptop, my hand was shaking and I found it hard to use it, but you went through and I was okay. It was easy for me to use the program. It looked very interesting to me)

Researcher: Yu bin creatim sampela samting? (Did you create anything?)

Kevin: Mi go inside long microsoftware na mi type. (I went into Microsoft ware and I typed)

Researcher: Yu typim wanem? (What did you type?)

Kevin: (I typed attendance record of my students and typed meeting minutes for my school. That's why, I see that it is becoming interesting in my life)

Researcher: Ol file stap inside, yu lukim tu? (Did you see the files inside)

Kevin: (Yes, I opened myself and checked the file stored inside. I saw some information inside, I took it, compared them back to my teaching and it was interesting, my children also enjoyed my lesson. Sometime I carry it to the classroom and show it to the children and they were happy)

The workshop at Waima had more teachers and more females and the computers needed to be shared (12 between 20). They were all pleased to learn to use the computers although some felt they needed more to ensure they could learn to use them well. However, they were exceptionally positive about the material on teaching with the inquiry method. They also noted that they liked the open-ended questions and getting children to find out. They appreciated that the mathematics was about the children's lives which they felt was important. When they planned their lessons, they used group work, games, and readers and had the children doing most of the talking. Typical responses were:

Purpose of education happens from the child at home, enable children's mathematical learning, e.g pair and group work helps both the teacher and the children, inquiry planning approach and role of assessment, this method makes planning more meaningful and effective.

I have learnt teacher ask questions and children do all the talking

...used games questions and drama in planning in weekly plan which was handy

The teachers wanted a longer workshop for computing and learning the new teaching way.

A previous paper has reported on the fruitful discussion using their Tok Ples (language of the place, that is the villages), Roro, to express division (Edmonds-Wathen, Owens, & Bino, 2019). In Jiwaka, teachers used their Tok Ples and Tok Pisin as well as English to teach but since the teachers came from different places, the workshop was in English and Tok Pisin and a little Waghi. The interviews were in Tok Pisin as the language of relational communication.

Conclusion

Throughout the design research, the project involved PNG researchers and co-researchers, Elementary Schools inspectors/trainers, Elementary Provincial Officers, Division of Curriculum and Assessment Development Officers, and University and Teachers College lecturers. Two trainer's manuals were prepared, trialled and finalised (one for when we had a physical copy of the manual and the other for when it was on the computer).

The design considered mathematics, early learning of mathematics, and cultural and linguistic aspects of mathematics and mathematics education. The set of principles for teaching Mathematics in early years in PNG was developed, implemented, modified with each workshop, and finalised based on theory and practice. A 100-page manual was prepared, trialled and finalised covering each principle, the inquiry method of teaching, exemplars, and questionnaires for teachers and children. The manual was relevant to PNG cultures and readable. Six books for early readers in English but with space for Tok Ples had a mathematics focus on three conceptual areas that had shown difficulties – the meaning of area and area units for measurement, place value, and equal-row representation of multiplication. All aspects were linked effectively to PNG cultural backgrounds and languages. Workshops ran on small group inquiry and active methods as models of good mathematics teaching.

We sourced, trialled, and supplied small touchscreen computers powered by solar with lithium battery. They were easy to assemble, the e-resource was relatively easy to use, and the computers functioned without the need for internet or the "cloud". Videos that had been used in the workshops with the manuals were added to the e-resource learning. The interactive, stand-

alone e-resource hyperlinked to videos and research on PNG cultural mathematics, videos of teaching and how to interview children with the questionnaires. The e-resource was provided together with Word, Excel and PowerPoint programs which they learnt to use, file and retrieve. Teachers used the e-resource, took photos and added them to files.

Unlike Kemmis' (2008) premise, government structures in the end were not supportive but teachers were assisted by Charly and Vagi to continue with inquiry learning associated with cultural practices.

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An Analysis of Place Value Visual Representations in South African, Singaporean, and Australian Workbooks

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Place value competence in early years mathematics is a precursor for success in later grades. In this paper, we analyse the place value visual representations in workbooks from South Africa, Singapore and Australia. A cross-country comparison of curricula materials provides an opportunity to understand similarities and differences in use of visual representations in teaching place value. Drawing on an adapted Visual Representation Framework, we analyse the types of place value VRs in the Grades 1-3 workbooks across the three countries. The results show that there are significant differences in the number of VR and types of VR privileged across the three countries.

Introduction

Place value is an important concept in primary school mathematics, particularly for understanding and computing with multi-digit numbers (Sari et al., 2021). Place value understanding is central to success as learners move up the schooling system. As Gebhardt et al., (2014) note, a conceptual understanding of place value supports not only whole number understanding, but also that of decimal fractions. Teachers play a central role in developing learners' understanding of place value, yet many only have a procedural understanding of the role of place value in multi-digit calculation (Ma, 1999). In such cases, teachers become more reliant on textbooks (including workbooks) to assist them in teaching mathematics. As Hoadley and Gallant (2016) note, workbooks can support the development of learners' understanding of mathematics and assist teachers in developing their mathematics and pedagogical content knowledge. Visual representations (VR) in mathematics workbooks often serve as a tool to assist learners in understanding abstract concepts (Mainali, 2021). Given this, it is important to ensure the quality and appropriateness of the VR in workbooks.

This paper forms part of a larger international research project on developing learners' and teachers' understanding of place value. To understand how the curriculum is enacted, the research team has shifted their focus to textbooks and workbooks. While shaped by differing socio-cultural and economic contexts, cross-country comparative studies offer valuable insights that potentially could lead to improvement (Huang, et al., 2022). In this paper, we focus specifically on the VRs in workbooks used by learners to support their understanding of PV. The guiding questions are: *What is the nature of visual representations of place value in the workbooks of three countries?* and *What variations exist within the type of place value visual representations in the three countries?*

Literature Review

Developing an understanding of place value is complex as it requires an understanding of multiple "big ideas" (van der Walle, 2015). Ladel et al. (2023) demonstrate this complexity in

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 77–84). Canberra: MERGA.

developing learners' place value understanding in their knowledge map which highlights the progression and connections in developing the "big ideas". These include decimal part-whole, grouping and ungrouping, bundling and unbundling, and positional notation (value of the digit, value of the number and the role of zero) (Ladel et al., 2023), as well as the additive and multiplicative structure of place value (Larkin et al., 2019). Prior to developing the big ideas, learners need to become confident with counting, part-whole structure, grouping and ungrouping objects in tens, and unitising (understanding that ten objects can become one bundle of ten). Larkin et al. (2019) suggest that the development of these big ideas can be supported by resources, for example, manipulatives, digital tools, and iconic representations. As such, the use of VRs in workbooks is likely to be key in developing learners' understanding.

Workbooks are commonplace in primary mathematics classrooms (Hoadley & Gallant, 2016), although not mandatory in all countries. Workbooks are curriculum resources that seek to link the intended curriculum with the implemented curriculum by providing a structured sequence of activities that organise teaching and learning in the classroom, and ultimately teachers' and learners' knowledge, thinking, beliefs and attitudes about the subject (Hadar, 2017). There is broad agreement in the research on mathematics workbooks that indicates that the quality thereof has an influence on student learning (Hadar, 2017). The cross-country comparison, on which this paper is based, has the potential to engage with what well-developed workbooks may look like in the context of supporting learners' PV understanding.

Workbooks incorporate various modalities, for example, text and images (Papageorgiou et al., 2019) that, if appropriate to the content taught, assist learners in developing an understanding of mathematics. VRs are external images that assist the learner in constructing internal representations (Papageorgiou et al., 2019). Teachers need to understand which VRs effectively support the teaching of a particular concept (Mainali, 2021), their characteristics, and the multiple ways learners interpret them (Papageorgiou et al., 2019). Hence, the quality of the VRs is important as this impacts their potential to support meaning-making. Mainali (2019) suggests that learners require access to a range of VRs to develop an understanding of a concept from different perspectives. However, Papageorgiou et al. (2019) take a more cautious view, suggesting that multiple representations for a single concept can cause "cognitive overload" (p.189). Thus, designers of workbooks need to carefully consider which VRs are most likely to assist learners in developing their understanding of different mathematical concepts.

Theoretical Framework

Vygotsky's (1978) three forms of mediation include tools, signs, and social interaction. This research focuses on the signs (i.e., visual representations) that mediate learners' reasoning. A VR is a symbolic tool that assists learners in making meaning of a particular concept, in this case, place value. While Kozulin (2003) insists that the facilitation and appropriation of symbolic tools is dependent on a person (more knowledgeable other), we suggest that a well-developed workbook could potentially perform the role of the more knowledgeable other. This is particularly important in situations where teachers do not have the required mathematical content knowledge (MCK) or pedagogical content knowledge (PCK).

In this research, we explore the different types of VRs in workbooks across three countries (Australia, Singapore and South Africa) to ascertain the VR used to develop learners' understanding of place value. In their comparison of textbook analyses, Huang et al. (2022) note that numerous frameworks for doing so exist. While their aim is to develop a framework for analysing workbooks across the disciplines, such a broad analysis is not the intention of this paper. We draw on the VR framework of Fotakoupoulou and Spiliotopoulou (2008), Papageorgiou et al. (2019), and Booyesen (2023) in analysing the VR. The framework used here examines the VRs based on the type, relation to content, function, relation to reality, and dimensionality. In this paper, we focus on one aspect of the VR frameworks mentioned above,

that is, the types of VR used to support PV understanding in three, Grade 1–3 mathematics workbooks.

Methodology

This study adopts an interpretivist orientation to document analysis. Activities from the workbooks were included in the analysis if they had an explicit link to the knowledge of PV.

In Phase 1, the first author analysed the South African and Singapore workbooks from Grades 1 to 3. The analysis was captured on an Excel spreadsheet. In this initial analysis the first author was able to identify the pages that contained PV VRs.

In Phase 2, each author independently analysed a workbook of the same grade from South Africa and Singapore. The analysis was guided by the Excel spreadsheet used in Phase 1. The page numbers that contained VRs were included in the first column and the categories were in the first row. Each researcher wrote a ‘1’ to indicate the presence of a VR from a specific category. During this phase, additional pages with PV VRs were identified. A Zoom meeting was held to go through the analysis and reach agreement.

In Phase 3 of the analysis process five authors re-looked at the analysis and came to the realisation that there were sub-types within each type of VR that has emerged. The authors noted that the VRs in the text went from concrete representations to being more abstract representations. The sub-types included specific variation within some of the types of PV VRs. For example, the variations in the Dienes block category were Dienes blocks (3 dimensional) and Dienes blocks (2 dimensional). It was in this phase that the Australian workbook was added to the analysis process as the data had not been sufficient for this paper. The Australian workbooks for Grade 1-3 were then analysed by the 3 authors and consensus was reached.

In Phase 4, three authors discussed and compared findings in a zoom meeting and the initial drafted paper was sent to all co-authors for commentary and further recommendations. The suggestions were taken into consideration in the final version.

Data Analysis, Findings, and Discussion

This section presents the data analysis and findings of the research. Across the three workbooks (WB), South Africa (SA) has the greatest number of PV activities (n=191), followed by Australia (Aus) (n=93), and then Singapore (Sing) (n=30). The number of VRs across the three-workbook series differed substantially with the SA WB having 296 PV VRs, compared with 248 in the Sing WB and 87 in the Aus WB. The most common VRs in the SA WB were PV cards (25%) and Dienes blocks (22%). This stands in contrast to the Sing WB and Aus WB. Most VRs in the Sing WB series are part-whole diagrams (15%) followed by bead strings (13%). The Aus WB series does not show such an extreme situation where one type of representation dominates. The most common VR in the Aus WB series are PV chart (symbolic) (11%), followed by Dienes blocks (10%) (Table 1).

Table 1

Types of PV VR

Types of VR		South Africa (%) (n=296)	Singapore (%) (n=248)	Australia (%) (n= 87)
Dienes Blocks	Dienes Blocks (3D)	22	6	10
	Dienes Blocks like (2D)	1	11	0
PV cards	Flard card	25	0	0
	Flard card like	5	0	2

Table 1*Types of PV VR (continued)*

Types of VR		South Africa (%) (n=296)	Singapore (%) (n=248)	Australia (%) (n= 87)
Numberline	Numberline	4	0	9
	Numberline like	1	0	6
	Empty numberline	1	0	6
PV chart	PV chart	2	4	2
	PV chart (symbolic)	0	0	11
	PV chart money	1	0	0
Ten frame	Ten frame	3	0	6
	Ten frame like	0	0	0
Number chart	Number chart	7	0	6
	Partial number chart	5	1	9
Money	Money (base ten focus)	1	1	0
Beads	Bead string	0	13	1
	Bead like	6	0	0
Counters	Counters (structured)	2	0	0
	Counters (unstructured)	1	0	2
Ice cream sticks	Ice cream sticks (structured)	0	0	7
	Ice cream sticks (unstructured)	0	0	0
Abacus	Abacus	1	0	3
Part-whole diagrams	PW diagram (a)	0	15	0
	PW diagram (b)	1	0	1
Arrays	Array representation (structured)	9	0	0
Total		98	51	84

Dienes Blocks: Dienes Blocks (3D) and Dienes Blocks (2D)

All of the workbooks had Dienes blocks as a type of VR for developing PV understanding. While analysing the Dienes blocks VRs, it became clear that there were two sub-types, those that looked like Dienes blocks (3D), and those that were a 2D representation of Dienes blocks (Dienes blocks-like). In SA WB and Aus WB, there were more examples of Dienes blocks (3D) (SA: 22%, Aus: 10%) and in Sing WB (6%), there were more Dienes block-like (2D) (SA:1%, Sing: 11%) VRs (Table 1).

PV Cards: Flard Bards, Flard Card Like, and Numeral Expander

Visual representations of PV cards were found in the workbooks from the different countries. The PV cards assisted learners in the decomposition of numbers in PV. The analysis found that the PV chart category had three sub-types namely flard cards (SA: 25%), flard card like (SA:5%, Aus: 2%) and numeral expanders (Aus: 3%). The flard card like VR can be explained as a VR which has the same formatting of a flard card where they are placed on top of one another to compose the number. There were more examples of numeral expanders in the Aus WB (Figure 2). These are similar to flard cards that show the number in expanded notation.

Figure 1
Flard Card Like (SA, Gr2)

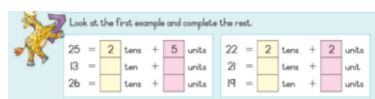


Figure 2
Numeral Expander (Aus, Gr2)

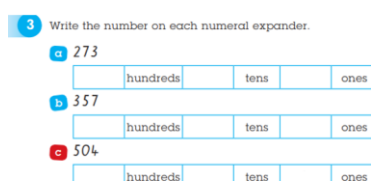
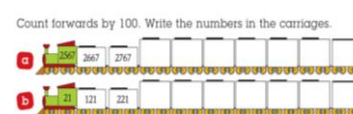


Figure 3
Numberline Like (Aus, Gr3)



Numberline: Numberline, Numberline Like, and Empty Numberline

The SA WB and Aus WB included numberlines as a type of VR for the development of PV understanding. While analysing the numberlines in the workbooks, it became clear that there were three sub-types, the numberline (SA: 4 %, Aus: 9%), those that looked numberline like (SA: 1%, Aus: 6%) and an empty numberline (SA: 1%, Aus: 6%) (Table 1). A VR that is numberline like has a similar formatting of a numberline (i.e., it is linear and has consecutive numbers with equal spaces in between). In Figure 3 the learner is required to count forwards by 100 using the carriages on the train.

PV Chart, Place Value Chart (Symbolic), and Place Value Chart (Money)

All the workbooks provided examples of PV charts as a type of VR. While analysing the PV chart VRs, it became clear that there were three sub-types, those that looked at a physical PV chart, PV chart (symbolic) and a PV chart (money) (Table 1). Most of the PV charts were found in the Sing WB (4%), while the SA and Aus WB contained a few less (2%). In the Australian workbook, there were more examples of PV chart (symbolic) (11%). This is a VR that looks like a PV chart with hundreds, tens and units but does not include a VR of Dienes blocks or counters in each column because the number serves as the symbolic representation (Fig 4). The SA WB contained PV chart (money) VRs (1%). A PV chart money is when the R100, R10 and R1 notes are used as the hundreds, tens and units in a normal PV chart. In Figure 5 the learners are required to master the concept of bundling (swapping ten R1 coins for a R10 note).

Figure 4
PV Chart (Symbolic) (SA, Gr3)

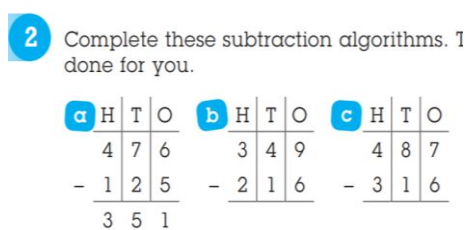


Figure 5
PV Chart Money (SA, Gr3)



Money (Base Ten Focus)

The SA and Sing WB also included money (base ten focus) as a type of VR, with an equal number of examples (SA: 1%, Sing: 1%).

Ten Frame and Ten Frame Like

The Aus and Sing WB contained ten frames as a type of VR. The Aus WB contained the most ten frames (6%), while the SA WB contained 3%. There were no examples of the second sub-type namely, ten frame like across the workbooks.

Number Chart and Partial Number Chart

All the workbooks had number chart as a type of VR for the development of PV understanding. While analysing the number chart VR sub-types were identified, the actual number chart (SA: 7%, Aus: 6%), and a partial number chart (SA: 5%, Sing: 1%, Aus: 9%) (Table 1). A partial number chart is when a section of the number chart is extracted for learners to complete.

Figure 6

Partial Number Chart (Sing, Gr1)

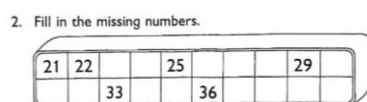


Figure 7

Bead Like (SA, Gr2)

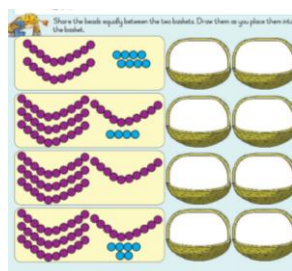
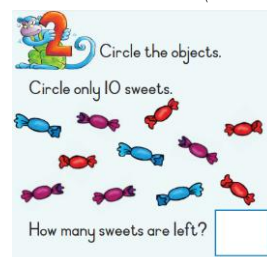


Figure 8

Unstructured Counters (SA, Gr 2)



Bead String and Bead String Like

All the workbooks had beads as a type of VR. In analysing the beads, it became clear that there were two sub-types: the beads on a string (bead string) and examples of bead-like VRs (Table 1). In the Sing WB, there were more examples of bead strings (Sing: 13%) and the SA WB that contained more bead like examples (SA: 6%). A bead string like VR can be found in Figure 7 where the beads are arranged as a necklace without the string connecting them.

Structured Counters and Unstructured Counters

The SA WB and Aus WB contained counters as a type of VR for the development of PV understanding. During the analysis process it became clear that there were two sub-types, namely structured counters and unstructured counters (Table 1). The only examples of structured counters were found in the SA WB (2%). Both the Aus WB (2%) and SA WB (1%) contained examples of unstructured counters. Figure 8 is an example of unstructured counters where the sweets are arranged randomly for learners to count and circle.

Structured and Unstructured Ice-Cream Sticks

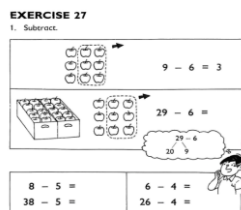
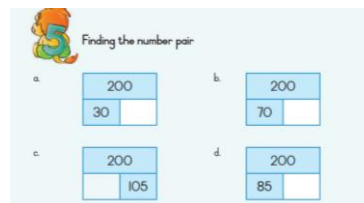
The Sing WB had examples of ice-cream sticks as a type of VR. While analysing the ice-cream sticks, two sub-types emerged, structured ice-cream sticks (Table 1), and unstructured ice-cream sticks (i.e. the sticks are arranged randomly). Singapore contained the only examples of ice-cream sticks structured (7%).

Abacus

In the analysis process it became apparent that the Sing WB (3%) and SA WB (1%) had examples of an abacus as a type of VR (Table 1).

Diagrams: Part-Whole Diagram (a) and Part-Whole Diagram (b)

All of the workbooks had part-whole (PW) diagrams as a type of VR. While analysing the PW diagram VRs, it became clear that there were two sub-types, those were PW diagram (a) and PW diagram (b) (Table 1). Figure 9 is an example of an PW diagram (a) where learners need to decompose numbers to be able to do the subtraction sum. While in Figure 10 the learners are expected to work out what the missing part of the PW diagram (b) is.

Figure 9*PW Diagram (a) (Sing, Gr1)***Figure 10***PW Diagram (b) (SA, Gr3)*

Array Representation

The SA and Sing WB contained array representations that assisted in supporting place value understanding. Most array representations were found in the SA WB (9%) (Table 1).

The above section provides insight into the sub-types of the different types of PV VR found across the workbooks of the three countries. The sub-types identified above provides a methodological contribution of adapting a framework to expose the existing variations of the different types of PV VRs found in the workbooks.

Discussion

Workbooks are replete with symbolic tools that have the potential to mediate learners' understanding of place value (Mainali, 2021). There was a substantial difference in the number of PV activities and PV VRs in the workbooks from the three countries. The SA WB had the most PV VRs and the Sing WB the least. The Sing WB had almost half of the number of PV VR when compared with the SA WB. The SA and Aus WB had examples of all the types of VRs ($n=15$), whereas the Sing WB only had 6 different types of VRs. Not evident in the Sing WB series, which were included in both the SA and Aus WB series, were PV cards, numberline, ten frames, counters, ice-cream sticks, abacus and array (Table 1).

The research highlights that there are different variations of each type of PV VR. For example, Dienes blocks were categorised as Dienes blocks (3D) and Dienes blocks (2D). Dienes blocks (3D) bore direct resemblance to the actual Dienes blocks, whereas Dienes blocks (2D) were a drawing of the front of the Dienes blocks only (no depth). All the WBs included Dienes blocks; however, these were privileged in the Sing WB series, and in particular, Dienes blocks (2D). By contrast, the dominant PV VR was the PV chart in the Aus WB series and the PV card in the SA WB series. In the Aus WB series, most of the PV chart were place value chart (symbolic) and in the SA WB series, the majority of PV card were flard cards (Table 1).

The SA WB do not provide opportunities for learners to work with PV chart (symbolic) and ten frame like, Dienes blocks (2D and 3D), ice cream sticks (structured), ice cream sticks (unstructured) and numeral expanders. Learners who make use of the Sing WB are not exposed to flard cards, numberlines, partial numberlines or empty numberlines, PV chart (symbolic), PV chart (money), ten frame, ten frame like objects (unstructured), bead like, counters (structured and unstructured), decimal blocks (3D and 2D), ice cream sticks (structured and unstructured) and numeral expander when it comes to PV VR in the WBs. The Aus WB does not expose learners to Dienes blocks like, flard cards, PV chart (money), ten frame like, money (base ten focus), bead like, counters (structured), photos, ice cream sticks (unstructured), part-whole diagram and array representation.

Given Singapore's dominance in the Grades 4 and 8 Trends in Mathematics and Science Study, and their use of fewer PV VR raises a question of whether it is beneficial for learners have access to a wide range of VR. Mainali (2019) argues that learners should be exposed to a wide variety of VR. However, this raises concern for Papageorgiou et al. (2019), who argue that multiple representations could lead to cognitive overload.

The significance of this study lies in its recognition of the variations within each type of PV VR. For example, for teachers, it is important to select the most appropriate variation based on the concept being taught, which is dependent on whether the learners are at a concrete, semi-concrete, or symbolic stage of their learning. The variations in types of VR reveal the complexity and richness embedded in each classification, offering both teachers and learners a deeper understanding of how different representations can be used in the classroom.

Conclusion

This research sought to examine the type and variations of PV VR across the WB of three countries. However, the analysis only focused on one category (i.e., types of VR) and not the complete VR framework. Further research is required to ascertain how each VR promotes PV understanding and the extent to which the use of multiple types of VR supports learners' PV understanding. This research has prompted the research team to focus on examining how the VR supports place value understanding, particularly with regard to the sequencing and progression of the “big ideas” in PV.

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Teachers' Conceptions of the Term '*Success*' in Mathematics Classrooms

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'*Success*' is a frequently cited yet ambiguously defined term in mathematics education, often carrying contradictory meanings and messages. We surveyed 139 Australian teachers of mathematics to determine their conceptions of '*success*'. This paper documents teachers' responses to an alternate word or phrase for '*success*'. Thematic analysis highlighted teachers' consistent conceptions of '*success*' as mathematical proficiency and underscored the critical omission of productive dispositions from the Australian Curriculum's mathematical proficiency. These findings open conversations about '*success*' to encourage more positive, productive engagement with mathematics.

'*Success*' is frequently cited within education research and government policies as the ultimate ambition of schooling (e.g. Department of Education, Skills and Employment, 2019; Maxwell, 2009; Peteros, 2019; York et al., 2015). However, many contradictory messages exist about what '*success*' in mathematics education is and how '*success*' is determined. For example, curriculum documents might point to the need for creative problem-solvers and critical thinkers to embrace the future's unknowns. Yet, '*success*' is often determined through assessment of knowledge and skills that can be memorised, easily quantified, and objectively measured (Klenowski & Wyatt-Smith, 2012). This example is just one illustration of how '*success*' might be conceived. Various conceptions of '*success*' in mathematics education are at the core of a research project currently underway that is interrogating the meaning of '*success*', according to teachers. The aim of this paper is to explore teachers' conceptions of the term '*success*' in mathematics classrooms by analysing their responses to an online survey item.

Literature Review

'*Success*' can be represented differently or take on multiple meanings, applications, and manifestations. Conceptions of '*success*' have been tied to students' cognitive development (Callingham et al., 2017; Maxwell, 2009; York et al., 2015) and described as synonymous and interchangeable with terms like 'academic success', 'academic achievement', and 'student outcomes' (May, 1923; York et al., 2015). These conceptions of '*success*' seem to be frequently tethered to direct or explicit instruction and students' ability to replicate procedures in pre-determined ways (Doabler & Fien, 2013; Shepard, 2000). '*Success*' also pivots around the readiness and ability to transition to the next level of schooling and/or advance to occupation opportunities (May, 1923; Peteros et al., 2019; York et al., 2015). Thus, procedures for ranking students by level of academic achievement at the end of secondary school can accompany the other conceptions of '*success*'.

According to literature sources across one hundred years (e.g., Maxwell, 2009; May, 1923), '*success*' can be evidenced through certain grading and report card practices. However, others (e.g., Clarke, 1997) contend that report card grades are inadequate for representing '*success*', as grading conveys simplified interpretations of learning as either '*success*' or lack of '*success*'. Notions of '*success*' can be seen in discussions of standardised assessment (Klenowski &

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 85–92). Canberra: MERGA.

Wyatt-Smith, 2012; Reid, 2019), such as the Programme for International Student Assessment (PISA), often seen as an indicator of the quality of a country's education system (Reid, 2019), and the National Assessment Program – Literacy and Numeracy (NAPLAN) which “helps schools identify successful programs and identify areas in need of improvement” (ACARA, 2010, p. 7).

Other literature suggests ‘*success*’ in learning mathematics is realised when students undertake a task with multiple possibilities and enact forms of critical thinking, creativity, problem-solving and self-efficacy in sharing their understandings and pathways to reach solutions (Clarke, 1997; York et al., 2015). Watt (2005) writes that “rather than someone who is able to neatly replicate a learned procedure to a routine task in a familiar context, a successful mathematics student has been reconceptualised as one who is able to devise problem-solving strategies” (p. 22), including how this problem-solving involves the identification and application of relevant strategies or procedures in different situations (Clarke, 1997). Some researchers suggest ‘*success*’ transpires when students actively navigate mathematics together and engage collaboratively, such as through real-world applications and social-cultural scenarios reflected in society (e.g., Bada & Olusegun, 2015; Watt, 2005). Conceptions of ‘*success*’ are also connected with emotions and affective domains. Grootenboer and Marshman (2016) write that studies have shown that self-belief or confidence has been a determining factor and predictor of “success in mathematics” (p. 25).

With so much meaning and yet so many contradictions embedded within one single word, there is much value in unpacking and exploring the nuances of ‘*success*’. Drawing on survey data from our larger research project, the research question addressed in this paper is: “What are teachers’ conceptions of the term ‘*success*’ in the mathematics classroom?”

Methodology

This phase of the research project aimed to gather a broad range of experiences and understandings of ‘*success*’, collected via an online survey (*Qualtrics*) consisting of short responses and Likert Scale questions. There were 139 complete or near-complete responses from teachers of mathematics currently working in Australian schools. They included 37 primary teachers, 56 mathematics-trained secondary teachers, 21 Heads of Departments or Heads of Faculty, 10 out-of-field secondary mathematics teachers, 4 Numeracy Leaders or Numeracy Coaches, and 11 participants who responded with other school-based roles. Participants’ teaching year levels ranged from Early Years (Prep-Year 2) to Senior Secondary (Years 10-12), with teaching experience varying from less than 1 year to more than 21 years.

With regard to school demographics, 84 participants were from government schools, 33 from Catholic schools, and 22 from independent schools. The geographical location of the participants’ schools was also varied: 57 participants were from major cities, 67 from regional cities and 15 from remote areas. Every State and Territory was represented: Queensland (70), Australian Capital Territory (6), New South Wales (27), Victoria (24), Western Australia (5), Tasmania (3), South Australia (3), and the Northern Territory (1).

A flyer and online survey link were shared informally through social media (e.g., LinkedIn and Facebook), emails, and via several teacher-based social media pages and various national, state or territory-based mathematics teacher associations. There was also a one-page advertisement in a teacher-based magazine, *Prime Number* (edition 02/24), distributed quarterly by the *Mathematics Association of Victoria*.

This paper analyses responses to one survey question: *What is another one-word or simple phrase to describe success in the mathematics classroom?* The analysis started by using an inductive coding approach, allowing categories to be developed from the data. However, it soon became apparent that responses tended to align with the *Australian Curriculum: Mathematics*

(v8.4) proficiency strands, prompting a shift to a deductive coding approach. While descriptions for these proficiency strands of *Understanding*, *Fluency*, *Problem-Solving* and *Reasoning* do not explicitly use the term 'success', they do mirror various conceptions of 'success' previously discussed (ACARA, n.d.a). Nevertheless, the proficiency strand descriptions in the *Australian Curriculum: Mathematics* did not provide adequate clarity to allow coding of participants' responses with confidence. For this reason, further clarification of the meaning of mathematical proficiency was sought from Kilpatrick et al. (2001), which informed the development of the Australian Curriculum's four proficiency strands.

Kilpatrick et al. (2001) identified five interwoven strands of mathematical proficiency: *Conceptual Understanding*, *Procedural Fluency*, *Strategic Competence*, *Adaptive Reasoning* and *Productive Dispositions*, the first four of which broadly correspond to the Australian Curriculum proficiency strands of *Understanding*, *Fluency*, *Problem-Solving* and *Reasoning*. Kilpatrick et al. also affirm that proficiency is not an "all or nothing" concept, instead developing over time. Providing greater detail, consistency, and nuance than the descriptions offered by the Australian Curriculum, Kilpatrick et al. gave the following definitions of each proficiency strand, which supported the analysis of teachers' responses to the survey question.

Conceptual Understanding is the "comprehension of mathematical concepts, operations and relations" (Kilpatrick et al., 2001, p. 116). Seen as the "core knowledge of mathematics" (p. 10), *Conceptual Understanding* is more than isolated facts and methods, but the "integrated and functional grasp of mathematical ideas", whereby students "understand why a mathematical idea is important and the kinds of contexts in which it is useful" (Kilpatrick et al., p.118). This proficiency strand is associated with knowledge organisation, making connections between ideas, verbalising connections, and the ability to "represent mathematical situations in different ways and knowing how direct representations can be useful for different purposes" (p. 119).

Procedural Fluency refers to "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (Kilpatrick et al., 2001, p. 121). *Procedural Fluency* is sometimes confused with applying standard algorithms correctly and quickly; however, much more is involved. *Procedural Fluency* supports carrying out procedures flexibly, accurately, efficiently, and appropriately (Kilpatrick et al.). This proficiency strand is associated with fluency, knowledge, and ways of estimating results procedures, providing tools for computing, and supporting conceptual understanding through the continuing "analysis of similarities and differences between methods of calculating" (p.121). Kilpatrick et al. write how "students can gain insight into the fact that mathematics is well-structured (highly organised, filled with patterns, predictable) and that a carefully developed procedure can be a powerful tool for completing routine tasks" (p. 121).

Strategic Competence refers to "the ability to formulate mathematical problems, represent them and solve them" (Kilpatrick et al., 2001, p. 124) and is similar to what others – including the Australian Curriculum – label as problem-solving and problem formulation (p. 124). Kilpatrick et al. explain how *Strategic Competence* also includes non-routine problems, which "require productive thinking because the learner needs to invent a way to understand and solve the problem" (p. 126) in the classroom and outside of school. This proficiency strand is associated with effective planning to solve problems and forming mental representations, including building "a mental image" of the essential components to solve a problem (p. 124).

Adaptive Reasoning refers to "the capacity to think logically about the relationships among concepts and situations" and the "careful consideration of alternatives", including the "knowledge of how to justify the conclusions" (Kilpatrick et al., 2001, p. 129) and the "determining the legitimacy of a proposed strategy" (p. 131). Interacting with problem-solving, *Adaptive Reasoning* is the capacity for logical thought, reflection, explanation, and justification.

This proficiency strand is associated with intuitiveness, deductive reasoning and inductive reasoning based on pattern, analogy, and metaphor (Kilpatrick et al.).

Productive Disposition refers to “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (Kilpatrick et al., 2001, p. 131). In addition to and developing alongside the other strands, students “must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out” (p. 131).

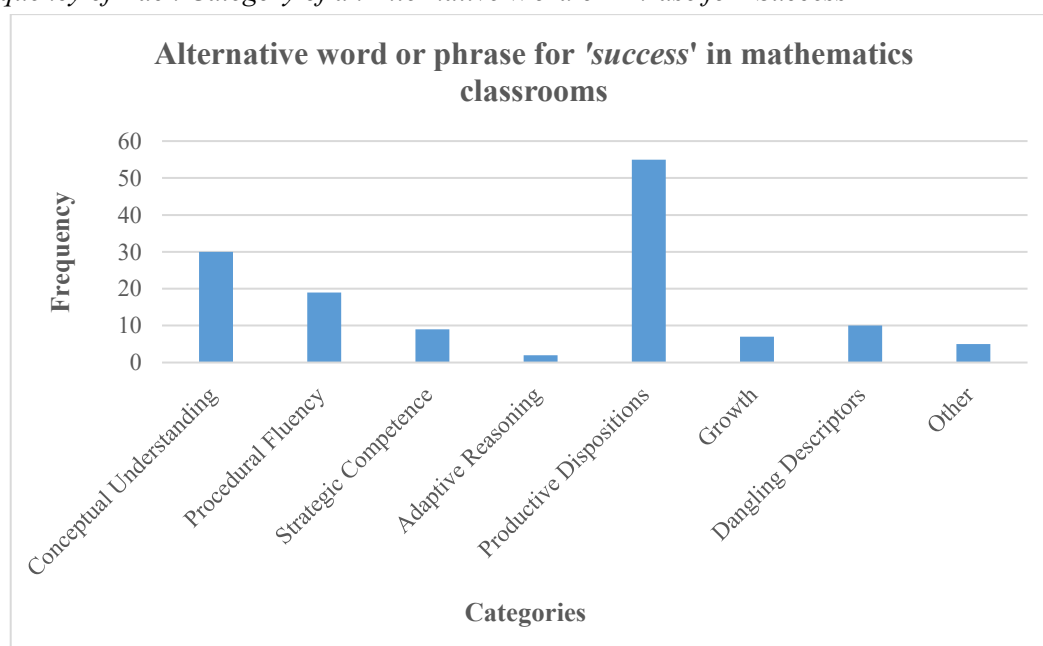
These five categories, corresponding to the five strands of mathematical proficiency, were used to code survey participants’ responses to the item asking for an alternative word or phrase to describe ‘*success*’ in the mathematics classroom. There were 137 responses. Each participant’s response was assigned one code. In rare cases where participants entered two words of varying potential themes ($n=2$), separated by either “and” or “/”, only the first word was considered to align with the survey question’s intent. Altogether, 115 responses could be coded using one of the five proficiency strand categories; inductive analysis of the remaining responses created three additional categories (*Growth*, *Dangling Descriptors*, and *Other*).

Findings

To answer our research question, response frequencies and proportions (percentage of the whole sample) for each category were recorded and are represented graphically in Figure 1.

Figure 1

The Frequency of Each Category of an Alternative Word or Phrase for ‘Success’



Productive Dispositions as an Alternative Word or Phrase for ‘Success’

Productive Disposition was the most prominent category of responses ($n=55$), accounting for 40.1% of survey participant responses. Within this category, roughly one-third of responses ($n=19$) explicitly state ‘*confident*’ or ‘*confidence*’ as an alternative word for ‘*success*’. This is also the most commonly stated word or phrase across all responses to this survey question.

Reference to a ‘*disposition*’ was explicitly stated twice. Other alternative words or phrases refer to loving or enjoying mathematics, such as ‘*love maths*’, ‘*to genuinely love it*’, ‘*enjoying it all*’, ‘*enjoyment*’ and ‘*joy*’. Some alternative words or phrases refer to ‘*resilience*’ or ‘*perseverance*’, with several explicit mentions of these terms, as well as other responses like

'never give up', 'drive to have a go' and 'grit'. Other alternative words or phrases responses associate a specific mindset to approaching mathematics (e.g., 'growth mindset' and 'Can't do it YET') or a self-belief towards mathematics (e.g., 'empowerment' and 'I can. I will try.'). A curiosity towards mathematics was another concept represented in the alternative words or phrases, such as 'curious connections' and 'following curiosity'. Many of these alternative words or phrases for 'success' can be seen within the descriptions of *Productive Dispositions* provided by Kilpatrick et al. (2001).

Conceptual Understanding as an Alternative Word or Phrase for 'Success'

Conceptual Understanding was the second most prominent category of responses (n=30), accounting for 21.9% of survey participant responses. Within this category, the word 'Understanding' (or a variation of it) was the second most frequent term (n=13) across all responses to this survey question. Other alternative word or phrase responses related to connection or being interconnected, for example, 'connections', 'making connections', and 'applying connections'. There are also alternative word or phrase responses that allude to 'solving problems' and to 'know the strategies'. Supported by data analysis within the broader research project, a distinction was made between 'solving problems' as applying known procedures and problem-solving, which emphasises navigating unfamiliar challenges. The latter will be discussed in *Strategic Competence*.

Conceptual Understanding can also be conveyed by personal affirmations of understanding. Teachers offered words or phrases of this type that are identifiable by the use of punctuation marks to portray excitement, notably an exclamation mark (!), as well as the use of first-person language (e.g., "I"). The use of "I" in these remarks is interpreted as that of students, as opposed to teachers, who were the research participants. Together, this subset of responses is referred to as students' affirmations of understanding that share similar sentiments with other words or phrases in the *Conceptual Understanding* category, such as "the aha moment", "epiphany", and "enlightenment". This category suggests that 'success' can be found in conceptual understanding.

Procedural Fluency as an Alternative Word or Phrase for 'Success'

Procedural Fluency was the third most prominent category of responses (n=19), accounting for 13.9% of survey participant responses. This includes a few direct references to 'fluency'. There was also explicit mention of 'flexibility' within this response category. Other alternative words or phrases refer to the structure of mathematics, such as 'order and patterns' and 'way of thinking'. The remaining responses within the category were quite varied expressions of *Procedural Fluency*, for example, 'most efficient', 'search for solutions', 'familiarity', 'skill acquisition', 'mastery' and 'clarity'.

Strategic Competence as an Alternative Word or Phrase for 'Success'

Although *Strategic Competence* was not the next most prominent response category, continuing to focus on proficiency strands supports a natural progression among related categories. The alternative word or phrase responses (n=9) categorised under *Strategic Competence* primarily focus on problem-solving and application, accounting for 6.6% of survey participant responses. For example, 'problem solver' was explicitly mentioned three times together with variations on application, like 'applying strategies', 'applying skills widely' and 'apply to real life'.

Adaptive Reasoning as an Alternative Word or Phrase for ‘Success’

The remaining proficiency strand, *Adaptive Reasoning*, was the least prominent category, with only two alternative words or phrase responses: ‘*reasoning*’ and ‘*analytical*’. This accounted for 1.5% of survey participant responses.

In total, 81.8% of survey participant responses were sorted into the five proficiency strand categories defined by Kilpatrick et al. (2001). A further three minor categories were established, some related to ‘*success*’ in mathematics education more than others. These additional categories are *Growth*, *Dangling Descriptors*, and *Other*.

Growth as an Alternative Word or Phrase for ‘Success’

Seven responses, accounting for 5.1% of survey participant responses, referred to ‘*growth*’ or ‘*growing*’ as alternative words or phrases for ‘*success*’. Two other responses include ‘*progress*’ and ‘*improving in maths*’.

Dangling Descriptors

The name of this category plays on the grammar term ‘Dangling Modifiers’, whereby a word or phrase is not clearly associated with a subject or noun, resulting in ambiguity or confusion for the reader. *Dangling Descriptors* (n=10) accounted for 7.3% of survey participant responses, with alternative words or phrases often describing a ‘*capability*’, ‘*proficiency*’, or ‘*achievement*’ but lacking context regarding what or the subject these descriptors are referring to. For example, the *Dangling Descriptor* does not provide information about what the student is ‘capable’ of or in. A student could be ‘*capable*’ of ‘*applying strategies*’ (a response within *Conceptual Understanding*) and is ‘*achieving*’ through the ‘*enjoyment*’ (a response within *Productive Dispositions*) of mathematics, both requiring further clarification offered in other categories. These ambiguous *Dangling Descriptors* introduce uncertainty, offering limited insight into interpretations of ‘*success*’, which was the aim of this survey question. As a result, although a category of survey responses, they do not serve as synonyms or alternatives for ‘*success*’ itself.

Other Alternative Words or Phrases for ‘Success’

The category of *Other* (n=5), accounting for 3.6% of survey participant responses, captures two distinct and meaningful subsets of responses that offer an alternative word or phrase for ‘*success*’. The first subset of responses does not align with the question posed: ‘*evidence-based*’, ‘*assessment*’ and ‘*engaged in learning*’. These alternative words or phrases may refer to what participants might think leads to or contributes to ‘*success*’, but do not answer the question. For example, it is unclear if the participant states that the concept of ‘*assessment*’ is ‘*success*’ or if they are implying ‘*assessment*’ is how they determine ‘*success*’. The former is an unclear response, while the latter does not answer the survey question that was posed.

The second subset of *Other* includes two unclear yet intriguing responses: ‘?’ and ‘*Depends.*’ For the response of ‘?’, the participant may not have understood the question, or the response of ‘?’ could denote that there is no immediate alternative word or phrase for ‘*success*’, possibly expressing its uniqueness or that ‘*success*’ encapsulates many conceptions that other terms cannot easily replace. Regardless, the response of ‘?’ should not be merely disregarded but taken seriously, despite being categorised as *Other*.

Another participant responded, ‘*Depends.*’ as an alternative word or phrase for ‘*success*’ in the mathematics classroom. This response indicates that ‘*success*’ might be contingent on varying factors or context. Looking at the other responses to this survey question, these varying factors could possibly depend on beliefs, values, perceptions of mathematics, context, interpretations, students, and pedagogical or ideological standpoints. Also worth noting is that

very few responses to this survey question included a period or punctuation in their response. If so, punctuation was included after a short phrase. '*Depends.*' was the only single-word response with a period (punctuation). This could be unintentional. Alternatively, it could be very intentional and convey a tone, emphasis, or finality to the statement. This single alternative word suggests there are broader things to consider when substituting for the word '*success*' and, consequently, interrogating the meaning of '*success*' in mathematics education.

Discussion

Analysis of teachers' responses to one survey question suggests that '*success*' in mathematics education may be represented in various ways, but is most predominantly viewed as a *Productive Disposition* within a broader superset of having mathematical proficiency. This is particularly noteworthy because, while the *Australian Curriculum: Mathematics* drew upon Kilpatrick et al. (2001), the strand of *Productive Dispositions* was entirely omitted. Yet, in the absence of a direct prompt within the official curriculum, perceptions of an alternative word or phrase for '*success*' in mathematics education held by Australian teachers of mathematics align most closely with this overlooked strand. We acknowledge that the *Australian Curriculum: Mathematics* (v9.0) (ACARA, n.d.b) has more detailed explanations of mathematical proficiency than v8.4 for *Understanding*, *Fluency*, *Reasoning*, and *Problem Solving*. The interrelatedness of the proficiency strands is also better recognised in that some strands are explicitly mentioned within the definitions of others. But Version 9 still lacks a *Productive Disposition* strand as the humanistic unifier within mathematical proficiency.

The low levels of association of higher-order thinking skills with '*success*' in the mathematics classroom emerged as a notable concern. For example, while teachers' responses that mentioned problem-solving/er and applying/application may represent the larger essence of *Strategic Competence*, the nuance provided by Kilpatrick et al.'s (2001) description for this strand is not wholly represented. The notions of planning and inventing as a part of problem-solving or using mental imagery denote higher-order thinking strategies for making sense of nonroutine problems, as opposed to simply using a known method to solve a routine problem. We cannot know if this kind of productive thinking is what participants had in mind when they responded to the survey question. But we also note that *Adaptive Reasoning*, which interacts with problem solving, was barely present in teachers' conceptions of '*success*'.

Additionally, while not a new development, the intense focus on measurability and quantification structures to determine learning has been increasingly emphasised in recent years (e.g. Reid, 2019). These ideologies are often attached to notions of objectivity and accountability (Watt, 2005). Through this lens, responses coded as *Productive Dispositions* may be viewed as too subjective or less valuable due to their ambiguity or resistance to quantification. However, in comparison, *Dangling Descriptors* – a term marked by minimal specificity and clarity – are even more indeterminate and ambiguous. Notably absent were any explicit references to scores, ranks, or performance metrics – whether in the context of NAPLAN, tertiary entry ranks, or report card grades. While the single mention of '*assessment*' (in *Other*) may be interpreted in this way, this absence of metrics suggests teachers do not widely perceive such measures as conceptions of '*success*'. This observation raises important questions about their priority and influence in the shaping of mathematics classrooms.

Implications

This paper offers insight into the nature of '*success*' and conceptions of '*success*' in the mathematics classroom, as perceived by teachers. The findings might support the navigation of multiple and often conflicting messages surrounding '*success*', enabling educators to focus on creating and fostering conditions that align with these identified aspects of '*success*' and reflecting on structures that support progress towards achieving them. The first action towards

this goal is to include *Productive Dispositions* in the curriculum with the other four proficiency strands to ensure appropriate emphasis is given to the full conceptualisation of mathematical proficiency within pedagogical practices, strategic planning, and system decision-making. These findings also suggest that, despite an array of descriptions of ‘*success*’ in the literature, mathematics teachers converge on a consistent conception of ‘*success*’ in mathematics education independently of curriculum constraints or governmental mandates. This broader understanding of ‘*success*’ unlocks greater opportunities, ensuring more students can genuinely engage with and experience meaningful ‘*success*’ in mathematics education.

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How Knowledgeable is ChatGPT 4o? Assessing the Pedagogical Content Knowledge of a Generative Artificial Intelligence Tool

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Generative Artificial Intelligence (GenAI) chatbots provide teachers with opportunities to enhance their expertise by exploring topics relevant to their practice. However, studies have raised concerns about ChatGPT's mathematical accuracy. This pilot study evaluated ChatGPT's Pedagogical Content Knowledge (PCK) and Content Knowledge (CK), revealing that while it generates varied and detailed responses indicating competence in PCK and CK, some responses are flawed or incorrect. While ChatGPT 4o can serve as a valuable professional learning resource in certain areas of mathematics education, traditional teaching methods and human insight remain essential.

The emergence of ChatGPT and other Generative Artificial Intelligence (GenAI) tools has transformed professional learning opportunities for teachers, offering innovative ways to develop their professional expertise. Where teachers may have traditionally relied on expert presenters and curated online resources, GenAI tools now enable teachers to engage in personalised dialogues with AI chatbots, exploring questions and topics directly related to their practice. However, the integration of GenAI tools presents challenges. Ethical concerns have been raised when used in education, particularly regarding the trustworthiness and reliability of AI-generated outputs. These tools are known to include “hallucinations” or inaccurate information (e.g., Williams, 2024), potentially exposing teachers to misleading or incorrect ideas. Given these considerations, it is crucial to critically examine the potential of GenAI tools to support the professional learning of teachers. This paper reports a pilot study investigating the Pedagogical Content Knowledge (PCK) and Content Knowledge (CK) capabilities of one prominent GenAI tool, ChatGPT 4o. By assessing its strengths and limitations, we aim to evaluate its potential as a supplementary resource for enhancing the professional learning of teachers, whilst also addressing the need for discernment and verification in its use.

Background

Pedagogy and PCK

Alexander (2008) defines pedagogy as the act of teaching, connecting it with educational theories, values, and evidence: “It is what one needs to know, and the skills one needs to command, to make and justify the many different kinds of decisions of which teaching is constituted” (p. 47). Shulman (1987) agrees that expert teaching involves managing both students and ideas in classroom discourse. These descriptions highlight the complexity of classroom practices where educational objectives are mitigated by specific skills and knowledge required for teaching. Alexander (2000) employs “pedagogical” as an inclusive term for classroom-related events, actions, and practices. Pedagogy is integral to education, linking good practice with educational quality (Alexander, 2015; Hardman, 2015; Shulman, 1987). Alexander’s metaphorical description of pedagogy as a “deep pool” (2015, p. 253) recognises the challenges of defining these practices.

Shulman (1987) characterises experiential knowledge as the “wisdom of practice” (p. 11), representing the accumulated wisdom of professional experiences. Educational theories of (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 93–100). Canberra: MERGA.

learning and instruction are encoded in the histories of conventionalised educational practice and are implicit in the key pedagogical terms used by each community. Schoenfeld (1999) argues that teachers' knowledge resources inform choices regarding instructional practices. "Pedagogical content knowledge" (Shulman, 1986, p. 8), a specific form of teaching knowledge conceptualised as the "transformation of subject matter knowledge in the context of facilitating student understanding" (van Driel et al., 2001, p. 979) is key in informing these choices. The development of this knowledge is facilitated by various enablers of professional learning, including access to resources, ongoing support, and collaboration. In recent years, GenAI chatbots such as OpenAI's ChatGPT, Google Gemini and Microsoft Copilot have emerged as a potential tool for teachers to use to support their professional growth. We expect this trend to continue as education systems further invest in GenAI tools to support teaching and learning.

GenAI and Teacher Professional Learning

Teachers engage in professional learning in a range of ways, including classroom-based learning, face-to-face learning and online learning (AITSL, 2020), with teachers valuing professional learning that focusses on student learning and contributes to improved teaching (Beswick et al., 2017). Recently, GenAI tools have emerged as a tool to support teacher professional learning. In their scoping survey of ChatGPT in mathematics education, Pepin and colleagues (2025) report that ChatGPT can aid teachers' understanding and ability to clearly convey mathematical concepts. For example, studies involving pre-service teachers found that using ChatGPT to analyse solutions to problems can support the critical thinking skills of pre-service mathematics teachers (Drushlyak et al., 2025). Buchholtz and Huget (2024) report similar findings regarding pre-service teachers using ChatGPT as a dialogic assistant for lesson planning and task modification. In contrast, Bagno and colleagues (2023) aimed to ascertain whether ChatGPT could function effectively as a teacher or teaching assistant for linear algebra content. While they reported promising results, ChatGPT was not endorsed for this role, citing solutions that were mathematically incorrect, contained contradictions, or included inaccurate information. More broadly, Ming and Mansor (2023) explored the potential opportunities and challenges associated with using ChatGPT in teacher professional development. The opportunities identified include increased access to resources and scholarly articles, efficiency gains, personalisation of professional learning, and support for collaboration and networking. However, challenges in using ChatGPT for teacher professional development include equity of access, ethical considerations such as academic integrity, and the reliability of outputs. Overall, while GenAI tools such as ChatGPT appear to have the potential to aid teacher professional learning, further research is required to explore the affordances and constraints of using these tools to effectively support the professional learning of mathematics teachers.

Research Design

Building on the need for further investigation into the potential of GenAI tools for teacher professional learning, this pilot study addresses the following research questions: (i) To what extent can the PCK and CK of ChatGPT 4o be assessed in the context of mathematics education? and (ii) How reliable and accurate is ChatGPT 4o's mathematics teaching advice, provided in response to PCK and CK assessment items, when compared to anticipated responses to items generated by experts in mathematics education?

The COACTIV Instrument

An existing instrument, the COACTIV Test of Mathematics Teachers' Professional Knowledge (PCK – CK) was selected for use in this study (Krauss et al., 2008). This instrument

was developed as part of the COACTIV project which aimed to assess secondary mathematics teachers' PCK and CK in Germany. The COACTIV project conceptualised PCK as “knowledge of explanations and representations, knowledge of students' thinking, and knowledge of multiple solutions to mathematical tasks” (p. 888) and CK as “deep background knowledge of school-level mathematics” (p. 888). These definitions have been adopted for this study.

The COACTIV instrument has been statistically validated and used to assess PCK and CK of mathematics teachers in a range of contexts (e.g., Krauss et al., 2008; Krauss, Brunner, et al., 2008). The 22 PCK items assess three subscales (i) knowledge of mathematical tasks, (ii) knowledge of student misconceptions and difficulties, and (iii) knowledge of mathematics-specific instructional strategies. The 13 CK items form a single subscale.

Data Collection

As this research is in its pilot phase, four items, one from each subscale, were carefully selected for inclusion (*Square*, PCK – Task; *Parallelogram*, PCK – Student; *Minus 1 times minus 1*, PCK – Instruction; *Recurring decimal* - CK). The selection was based on their availability (e.g., Krauss et al., 2008) in contrast to the full instrument which was provided by a COACTIV team member. The items and protocol are presented in Figure 1.

Figure 1

Assessment Protocol Including Tasks

Clear your memory of prior conversations

ChatGPT responds to prompt

<Item Square>

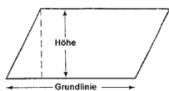
How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem (and different reasonings) as possible (and number each method).

ChatGPT responds to task

<Item Parallelogram>

The area of a parallelogram can be calculated by multiplying the length of its base by its height as shown in the attached image where Höhe: Height and Grundlinie: Base. Please sketch an example of a parallelogram to which students might not be able to apply this formula.

<Image shown below is attached>



ChatGPT responds to task

<Item Minus 1 times minus 1>

A student says: I don't understand why $(-1) \cdot (-1) = 1$. Please outline as many different ways as possible of explaining this concept to your student (and number each explanation).

ChatGPT responds to task

<Item Recurring decimal>

Is it true that $0.999999 \dots = 1$? Please explain your answer.

ChatGPT responds to task

Note: <text> indicates annotations for this paper that were not provided to ChatGPT.

As the COACTIV tests were originally designed as written tests for human subjects, they required little adaptation for use with ChatGPT due to its conversational nature. All items, except *Parallelogram*, were able to be used verbatim. The *Parallelogram* item required

modification due to the inclusion of an image in the prompt, which, for ChatGPT, needed to be uploaded separately rather than embedded within the chat.

To identify potential variations in responses, data were collected from ChatGPT 4o on five separate occasions using the protocol described in Figure 1. Each chat session began with an instruction for ChatGPT to clear its memory of prior interactions, minimising the impact of existing chats on data outputs. Items were presented to ChatGPT sequentially, allowing sufficient time for ChatGPT to respond to each item before introducing the next prompt.

Data Analysis

Each chat session underwent independent analysis using the COACTIV Codebook Test of Mathematical Teachers' Professional Knowledge (PCK – CK) as the coding framework. The analysis process began with an assessment of correctness, where responses were evaluated against codes in the COACTIV codebook. Responses aligning with a correct code in the codebook were classified as *fully correct* (see Figure 2). Responses not aligning with a correct code were either *partially correct* or *incorrect*. A *partially correct* response was defined as one that aligns with a correct code in the codebook but is either incomplete or contains an easily rectifiable error that most teachers could identify and correct. An *incorrect* response describes a response that is incorrect due to errors in the mathematics, is didactically insufficient (i.e., appeals to rules without explanation), or does not meet the requirements for a correct answer as outlined in the codebook (e.g., reasoning is not provided, variables not defined).

Figure 2

A Correct, Partially Correct and Incorrect Response to Item Square (Chat_1)

Correct response. This response aligns with the 'covariate/functional' code provided in the COACTIV codebook.	Method 5: Logical reasoning via Length Doubling 1. We know that doubling the side length ($2s$) results in quadrupling the area: $(2s)^2 = 4s^2$ 2. Now, tripling the side length is an even larger increase. 3. Instead of $2^2 = 4$, we compute $3^2 = 9$. 4. So, the area must increase 9 times.
Partially correct response. The codebook indicated that the correct use of a visual representation must include a diagram, so this response is considered partially correct.	Method 4: Visual Representation 1. Draw a square with side s, and label the area as s^2. 2. Now triple the side length to $3s$, forming a new, larger square. 3. This larger square can be divided into 9 identical original squares (each of area s^2). 4. Now we have 9 times as many original-sized squares, the area increased by a factor of 9.
Incorrect response. This response is incorrect as it is incomplete. The original length is not defined, the original area is not defined, and we must assume that the increase of side length by 3 corresponds to $3s$.	Method 6: Algebraic Expansion 1. Express the new area in expanded form: $(3s)^2 = (3s) \times (3s)$ 2. Using the distributive property: $3s \times 3s = (3 \times 3) \times (s \times s) = 9s^2$ 3. The new area is 9 times the original.

Note: Emphasis in original.

Following the correctness coding, a score was calculated for each item. In accordance with the codebook guidelines, a score of '1' was assigned for each unique correct response to an item (i.e., duplicate responses did not contribute to the score). Consequently, a higher score for an item indicates a greater number of unique correct responses provided by ChatGPT. This

scoring method allows for a nuanced evaluation of ChatGPT's performance, considering not only the correctness of responses but also their uniqueness and variety. This approach provides insights into the accuracy and breadth of AI's knowledge. By emphasising unique correct responses, the scoring captures ChatGPT's ability to generate accurate and diverse solutions to these four tasks, offering a more comprehensive assessment of its capacity in this domain.

Findings and Discussion

Figure 3 provides a comprehensive summary of ChatGPT's responses for each item across all five test sessions. The 'number of responses' column indicates a total of 46 different responses to the *Minus 1 times minus 1* item across all five chats. ChatGPT provided ten responses to this item in the first chat, compared to eight in the fourth chat. Of the ten responses to the *Minus 1 times minus 1* task in Chat 1, six of these responses were correct and of these correct responses, four were unique (i.e., responses employing different logic or reasoning). No responses were categorised as partially correct, and four were deemed incorrect. Overall, this indicates variability in how ChatGPT responds to an identical task in different chats.

Figure 3

Summary of the Correctness of Responses by Task and Chat

Item	Chat	Number of responses	Correct Responses			Incorrect Responses
			Fully correct	Unique (Score)	Partially Correct	Number
<i>Square</i>	1	8	4	3	1	3
	2	9	7	4	0	2
	3	7	6	4	0	1
	4	6	5	4	1	0
	5	6	4	2	1	1
	Total	36	26 (72%)	17 (65%)	3 (8%)	7 (19%)
<i>Parallelogram</i>	1	1	0	0	1	0
	2	1	0	0	0	1
	3	1	0	0	0	1
	4	1	0	0	0	1
	5	1	0	0	0	1
	Total	5	0 (0%)	0 (0%)	1 (20%)	4 (80%)
<i>Minus 1 times minus 1</i>	1	10	6	4	0	4
	2	9	3	3	2	4
	3	10	7	4	0	3
	4	8	5	5	0	3
	5	9	6	3	2	1
	Total	46	27 (59%)	19 (70%)	4 (9%)	15 (33%)
<i>Recurring decimal</i>	1	6	4	4	2	0
	2	5	3	3	0	2
	3	5	4	4	1	0
	4	5	3	3	2	0
	5	7	4	4	1	2
	Total	28	18 (64%)	18 (100%)	6 (21%)	4 (14%)
Overall		115	71 (61.7%)	54 (76.1%)	14 (12.2%)	30 (26.1%)

Overall, ChatGPT provided fully correct responses approximately 60% of the time, with an additional 10% being partially correct, and 25% incorrect. However, the correctness of the responses appears to be task-dependent, as evidenced by the variation in the percentage of correct responses across different items. For example, a third of responses to *Minus 1 times minus 1* were incorrect, indicating that some mathematical concepts may pose greater challenges for ChatGPT. The following section delves into the patterns identified within each correctness category, offering insights into ChatGPT's strengths and limitations.

Fully Correct Responses

A comparative analysis of the scores and the number of correct responses reveals patterns in ChatGPT's performance across different items. Notably, only two items elicited unique correct responses: the *Minus 1 times minus 1* task in Chats 2 and 4; and the *Recurring decimal* task across all chats. Notably, the *Recurring decimal* task assesses CK rather than PCK, which is the focus of the other items. These findings suggest that ChatGPT may be more adept at generating diverse, unique responses when dealing with tasks related to pure mathematical content rather than mathematics pedagogy.

The ability to explain concepts in multiple ways is a hallmark of effective teaching (e.g., State of Victoria, 2017). Therefore, ChatGPT's capacity to generate various correct responses, particularly in content-focussed tasks, presents an opportunity for teachers to develop this expertise. By reviewing the range of responses provided by ChatGPT, teachers could potentially expand their repertoire of explanations and approaches. The potential to use ChatGPT in this manner contrasts with the use of ChatGPT to create resources, such as lesson plans, where ChatGPT generated lesson plans promote a narrow view of mathematics teaching as "telling" where the teacher explains or demonstrates key procedures (Cameron & Mesiti, 2024). We suggest that prompting ChatGPT to complete tasks and explain common misconceptions and teaching strategies to remedy these may be a useful tool to develop teacher knowledge and to inform mathematics teaching and learning.

Partially Correct Responses

The *Parallelogram* item emerged as an outlier in this study, with ChatGPT failing to provide any correct responses. The item required the respondent to draw a diagram of a parallelogram where students might not be able to use the formula $\text{Area} = \text{Base} \times \text{Height}$. Across all chat sessions, ChatGPT generated images that failed to depict a parallelogram (see Figure 4), and only in one chat did it generate a description of such a parallelogram (see Figure 5). The provision of a correct written response and an incorrect image meant this was considered a partially correct response as we anticipate that a mathematics teacher would be able to understand the written response and sketch the corresponding diagram.

Figure 4

Sample ChatGPT Image for Parallelogram and Anticipated Correct Responses to Parallelogram.

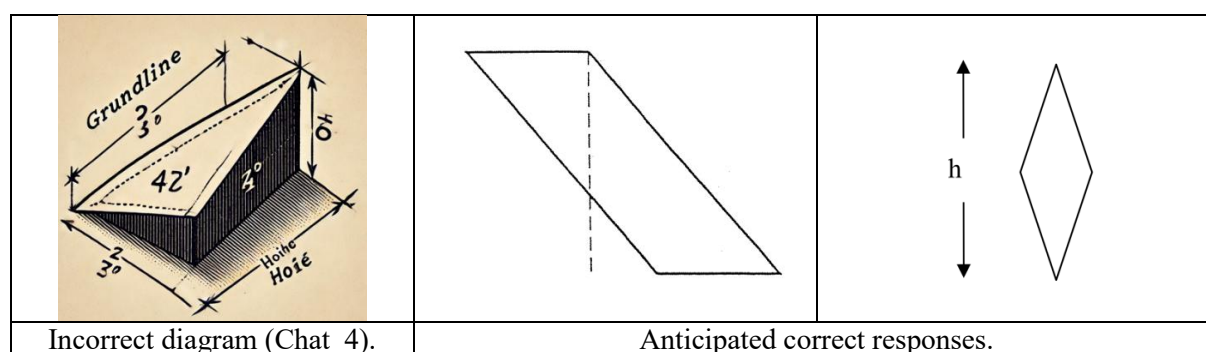


Figure 5

Chat GPT's Written Response to Parallelogram (Chat_1).

A possible parallelogram that might confuse students when applying the area formula is one where the height lies **outside** the shape rather than within it. This often happens with **obtuse parallelograms** where the perpendicular height is drawn from a vertex to an extended base.

This result, while noteworthy, is not entirely unexpected. Although ChatGPT integrates Dall-E for image generation, it is important to recognise that Dall-E creates images from text (OpenAI, 2021) in contrast to ChatGPT, a Large Language Model that creates text from text and images (OpenAI, 2022). In this case, the absence of a detailed text description, specifying characteristics of the required parallelogram, likely contributed to ChatGPT's inability to generate a suitable image. This limitation highlights a significant constraint in supporting professional learning in areas that rely heavily on visual representations, such as measurement and geometry, and emphasises the need for careful consideration when employing AI tools in mathematics. While ChatGPT can be a valuable resource in some areas of mathematics, there are others where human activities, such as drawing diagrams, cannot be replaced.

Score on PCK Tasks

Studies using the COACTIV instrument typically report PCK of cohorts using average scores for tasks within three subscales (e.g., Krauss et al., 2008). As this pilot study did not administer the entire test, we cannot directly compare ChatGPT's performance to existing studies. Krauss and colleagues (2008) report the performance of several cohorts, including 198 mathematics teachers that were "fairly representative of German 10th grade mathematics teachers" (p. 878). These teachers averaged a mean score of 18.6 points across the 22 items on the for the PCK test, indicating a mean score of 0.85 points per item. The maximum score was 37 with a mean score of 1.68 points per item. While we have not yet administered the full test to ChatGPT 4o, its mean score for these four items was 2.4 per item. However, it is crucial to contextualise these results. The human test was administered under timed conditions; participants may not have had the time or stamina to describe all possible responses to a single item. In contrast, ChatGPT is not constrained by time or fatigue, presenting an unfair advantage in this testing format. This disparity in testing conditions raises important questions about the comparability of PCK or CK between GenAI tools and humans when using written instruments administered under timed conditions. While ChatGPT's performance is impressive, it may not accurately reflect the real-world application of PCK, where teachers must make rapid decisions in dynamic classroom environments (e.g., Barendsen & Henze, 2019).

Conclusions and Future Directions

We set out to assess the PCK and CK of ChatGPT, a GenAI tool. Using the COACTIV PCK and CK test instrument and codebook, we identified correct, partially correct, and incorrect responses to both PCK and CK items. The codebook's clear delineation of anticipated responses facilitated reliable judgements. Collaboration with a mathematician supported the identification of duplicate responses despite ChatGPT labelling them as different approaches.

We were impressed by ChatGPT's ability to provide a large range of unique responses to both PCK and CK items, noting the limitation of the *Parallelogram* item for a Large Language Model. ChatGPT's diverse range of responses demonstrates its potential as a valuable resource, offering a variety of approaches to enhance their teaching strategies and support student learning. However, only approximately 60% of responses were fully correct, indicating that ChatGPT may not be sufficiently accurate to reliably develop mathematics teachers' PCK, especially for novices who may struggle to identify incorrect and partially correct responses.

This pilot study tested a limited range of tasks on one GenAI tool. Future research should expand the assessment of PCK and CK across different task and content areas. Comparing multiple GenAI tools could determine which best supports the professional learning of mathematics teachers. As these tools evolve, and new tools are released to the market, monitoring of their PCK and CK is crucial. We posit that as the reliability of a GenAI tool on a PCK and CK test increases, they may become powerful resources for tailored professional learning, potentially leading to positive outcomes for teaching and learning.

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Active Learning, Collective Participation: From Theory to Practice, Illustrations From Professional Learning About an Embodied Approach to Learning

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Effective professional learning includes the provision of opportunities for teachers to actively engage with learning in a supportive community, where ideas can be shared, trialled, questioned and revised. The professional learning designed within the Embodied Learning in Early Mathematics and Science (ELEMS) Project aimed to assist teachers in making connections between embodied learning research and teaching and learning approaches. Analysis of four online sessions during the project illustrated active learning and collective participation were key for this project's PL and were the most effective features in supporting teachers to implement a new pedagogy.

Introduction

The implied ultimate goal of effective professional learning (PL) is for improved teacher learning that in turn impacts student learning in a positive way (Beswick et al., 2017). While no set list of 'best practice' features exist to guarantee a successful PL, Desimone and colleagues' own research (Desimone, 2011; Desimone & Garet, 2015; Garet et al., 2001) and review of PL literature (Desimone, 2023) over the past 10–15 years shows that "high-quality learning experiences for teachers, linked to their curriculum and lessons, which provides active learning that includes practice with feedback, can change what teachers do" (p. 2). Five features are listed in Desimone's (2011) paper: 1) content focus—on subject matter and how students learn that content, 2) coherence—with other PL, teacher knowledge and beliefs, and school or state reforms, 3) collective participation—with groups of teachers in the same grade, subject or school to build a learning community, 4) active learning—where teachers have opportunities to be involved in observing, analysing student responses, receiving feedback, making presentations, and 5) duration—where PL activities are spread over a semester and should include 20 hours or more contact time. Desimone and Garet's (2015) paper concluded that "we need more information about specific aspects of the five features that are important in different contexts" (p. 260).

This paper is a window into the ongoing active learning process within the ELEMS project's professional learning episodes. The purpose of this paper is to provide illustrations of Desimone's five features of professional learning that supported productive teacher change. This paper explores the following two research questions:

- How do teachers' reflections and feedback illustrate Desimone's five features of effective teacher professional learning?
- Which, if any, of the features are more important in the context of the ELEMS project?

Project Background

The ELEMS research project is a synthesis of research findings from the fields of neuroscience, psychology and education indicating that a focus on haptic modes of learning (touch, body movement, gesture, tracing), and on the development of emerging mathematical and scientific drawing, can enhance children's learning by focusing their attention on essential properties, structures and relationships (for example, Alibali & Nathan, 2012; Dackerman et al., 2017). From the analysis of literature (Way & Ginns, 2024) we derived a set of key

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education principles and proposed the implications for teaching practices in Pre-school to Year 2 (4–8 years old) mathematics and science. In Phase 1 of the project, using a collective approach (Way et al., 2023), seven teachers and their students worked alongside the researchers in trialling and developing embodied learning activities related to mathematical concepts in their classrooms. At the end of Phase 1 the teaching activities were developed into a Teaching Guide, and a professional learning course was developed by the researchers for implementation in Phase 2. The learning goals of the PL in Phase 2 were for teachers to:

- Develop an understanding of embodied learning (gesture, tracing, drawing, body movement) and associated teaching design principles that underpin the ELEMS project.
- Make connections between embodied learning and teaching and learning approaches by reading research and applying knowledge of curriculum.
- Implement teaching practices focused on conceptual development of mathematics concepts in young children as part of the ELEMS research project, trialling lessons and related practices.
- Reflect on continued implementation of embodied learning principles, discussing the impact on teaching practices and students' learning, through group discussions and mini surveys throughout the project.

In Phase 2 four intervention schools received the teacher PL while four control schools did not. Students (n=294) from all eight schools were pre- and post-assessed using the Research-based Early Mathematics Assessment Short-Form (REMA-SF) (Dong et al., 2021). The diagnostic assessment data was collected to determine if there was any significant impact on students' mathematics learning in the schools where the embodied approach was implemented. Analysis received on REMA data across two timepoints (6 months apart) indicated greater learning gains for students in the intervention schools compared with the control schools.

The ELEMS research project had a design-based approach as the methodological framework. A design-based approach is focused on linking theory to practice (Reimann, 2010) and situates the research, and researchers, 'in the field'. Collaboration among researchers and practitioners is key to "make learning research more relevant for classroom practices" (Reimann, 2010, p. 37). The interactive, iterative and flexible characteristics of design-based research allowed the ELEMS project researchers to develop an embodied approach to teaching mathematics in contextually appropriate ways with participating schools.

Method

In Phase 2, 22 teachers from four intervention schools participated in the ELEMS project and attended 11 hours of PL. The teachers were teaching classes from Preschool to Year 2 (4–8 years old). The four schools varied in contexts including their levels of socio-economic advantage, percentage of students with English as an Additional Language or Dialect (EAL/D), three were metropolitan schools and one was a regional school in NSW, the regional school also had a high percentage of Aboriginal or Torres Strait Islander students.

For the analysis within this paper, a deductive approach was taken where Desimone's (2011; 2023) five features of effective PL were used as a lens to view the feedback and reflection conversations that were embedded with the ELEMS PL episodes. Table 1 presents an overview description of the ELEMS PL course and how each episode aligns with Desimone's features.

Table 1

Overview of ELEMS PL Program

Episode	Purpose and/or main activity	Very broadly, teachers will do the following	Alignment to Desimone's (2011; 2023) key features
Episode 1 Face to Face (5 hrs)	A whole day introduction to the PL program, embodied learning and the resources available to them	Understand the research base for using embodied approaches, EYLF/syllabus-linked mathematics activity examples, begin planning to integrate embodied learning into teaching programs.	<i>Content focus, Coherence, Active learning, Collective participation</i>
Implementation phase 1	Begin implementation of embodied learning approaches.	Access the support resources, try embodied approaches with their classes. Make a brief record of what they have tried and complete the assigned reading.	<i>Duration, Collective participation</i>
Episode 2 Online live (2 hrs)	Deepen understanding of embodied learning practices, focus on relationships between embodied modes and mathematical representations	Increase understanding of the application of embodied learning research, children's mathematical activity and their own teaching practice	<i>Content focus, Coherence, Active learning, Collective participation</i>
Implementation phase 2	Continue implementation ...	As at Implementation phase 1	<i>Duration, Collective participation</i>
Episode 3 Online live (1 hr)	Guided reflection, feedback and planning support	Present a brief report of their teaching experiments and contribute to discussions	<i>Active learning, Collective participation</i>
Implementation phase 3	Continue implementation ...	As at Implementation phase 1 and 2	<i>Duration, Collective participation</i>
Episode 4 Online live (2 hrs)	Deepen understanding of embodied learning practices with focus on student responses	Contribute examples of student responses, reflect on input, discussions and feedback	<i>Content focus, Coherence, Active learning, Collective participation</i>
Implementation phase 4	Continue implementation ...	As at Implementation phase 1, 2, and 3	<i>Duration, Collective participation</i>
Episode 5 Online live (1 hr)	Guided reflection, feedback and planning support	Present a brief report of their teaching experiments and contribute to discussions	<i>Active learning, Collective participation</i>

The teacher presentation sections of Episodes 2, 3, 4 and 5 are the focus for this paper. One of the objectives during these episodes was for teachers from each school to provide the researchers with feedback, reporting on and discussing embodied learning principles trialled in their classroom, sharing student work samples, providing anecdotal observations, and evaluating what further support they need for planning. Teachers were also encouraged to ask questions of the other teachers in different schools and to make connections to their own experiences during implementation. These episodes were conducted via Zoom after school and were video/audio recorded and later transcribed. The transcripts were analysed, and teacher quotations were coded to Desimone's five features. Quotations were selected that illustrated either productive teacher change, or how the teachers' experiences in the project were

influencing or impacting student learning. This paper presents a small set of examples from the larger data across the online episodes that align with Desimone's five features.

Findings and Discussion

The findings are organised using Desimone's (2011) five features of effective PL. Some descriptions are also drawn from Desimone's (2023) rethinking of teacher professional development paper. For teacher quotes, the episode number and school code [IS# Interventional School number] are used as deidentified labels, any names provided are pseudonyms.

Content Focus

Desimone (2011) states that "professional development activities should focus on subject matter content and how students learn that content" (p. 69). Within the ELEMS project, this relates to *the what* of the PL, where the content is either the subject matter content i.e., mathematics, or how students learn i.e., through embodied learning approaches.

Teachers talked about the mathematical content they were teaching in their classrooms using embodied modes such as gesture:

I definitely see as soon as you move your hands and you start to represent with your fingers, they mimic, you can see their minds are automatically engaging, they're looking at their hands and then associating the number of the words with what they see. And I think that's quite magical because that means they're really, they're wanting to understand those concepts. [Episode 3, IS2 and IS1]

I remember doing a lesson in one of the year 2 classes on arrays that it was about rows and columns and just teaching them the words vertical and horizontal to link with it and the hand gestures, they were able to get that 'vertical' and then 'horizontal' [gestures in Zoom]. [Episode 2 IS4, IS3]

The concept of counting, like your fingers, we're counting our marks but then there was a line across, and that idea that becomes a bundle of five, and it moved into a different area, and I could see that they were suddenly not disengaged but they were thinking. [Episode 3, IS2 and IS1]

Teachers likewise spoke about *how* students were learning—their confidence—when they implemented an embodied approach in lessons:

When you do the acting out of subtraction or addition number sentences, I think that's really good for conversations, and when they have to negotiate with each other what they're going to do to depict a number sentence or a word problem, I think that's really useful for them. [Episode 5, IS3, IS1]

Students started to ask to complete the activities more frequently and longer. I noticed that students, when they would come in, sometimes they go, oh, we're going to be doing, 'be a dot' today? Like they were asking for the activity. [Episode 5, IS4]

I noticed that students' confidence grew dramatically. They were willing to take more risks with their learning. They started linking mathematics into their real life and were supportive of each other. [Episode 5, IS4]

I could see the kids who are in second grade, and don't really have a sense of the backwards number sequence, ... I think oh my goodness if this had begun for that student earlier maybe that would have been a way ... they get it [Episode 5, IS2]

Several conversations in the episodes included teachers sharing how the embodied approach was supportive of students from diverse backgrounds and with different needs:

Neurodiverse kids can find it a barrier, quite difficult, to get up, ... in front of the others rather than in a group where not everyone's looking ... I think that reminds us that we need a variety of ways to engage with ideas and ways to communicate because it isn't going to suit everybody all the time ... They might find it really silly for small concepts, but as the concepts get more complex, they're going to see that it's a really useful tool. [Episode 4, all schools]

Especially with our context of the children at our school. They find it really hard to listen into people, you have to get them engaged in the learning, that's something that I've noticed [Episode 5, IS4]

When teachers reported back about the *content focus*, reflections almost always included the students, their engagement and/or eagerness to learn through embodied approaches. This

finding leads us to question whether Desimone's (2011) feature label of *content focus* fully captures this feature. Teachers reflected how students enjoyed working with their peers in more collaborative ways when haptic modes such as body movement were the focus of how the students were learning mathematics. The embodied approach was reported as being a differentiation and a communication tool, providing various ways students could show their understanding. The embodied approach was viewed as another way to scaffold learning for EAL/D students and provided an inclusive environment for learning to occur such as for neurodiverse students as mentioned in Episode 4. Teacher comments suggest that the PL supported "teachers in adapting the new ideas and practices for students who may not yet have had the opportunity to develop their foundational knowledge, who may not be native language speakers, and/or who may have special learning needs" (Desimone, 2023, p.2).

Coherence

The inclusion of coherence means "what teachers learn in any professional development activity should be consistent with other professional development, with their knowledge and beliefs, and with school, district, and state reforms and policies" (Desimone, 2011, p. 69). Coherence as a feature of PL ensures teachers can make connections from the new pedagogy or practice to their current thinking and teaching. In the ELEMS project, this relates to *the why* of the PL, the value and purpose of the approach.

Teachers reported on how the PL content was consistent with their current lesson approaches, curriculum or programs:

As I mentioned previously, the activities were easily, easy to fit into the units, and already planned lessons, and then seeing them use their mathematical language. [Episode 5, IS4]

I already had implemented touch tracing into my activities ... how to write numbers, form letters ... So that was already implemented into my curriculum, like my program, before I even started implementing the maths embodied learning. [Episode 5, IS4]

It tends to kind of fall into the structure of how our lessons go, that the embodied part is in the beginning of the lesson where they're trying to understand, like connect to the concept that we're doing. [Episode 4, all schools]

Sometimes when the activity from the DOE unit was too complex for the students or they needed just a bit more solidifying of that context, I'd always go back, and we just keep on doing the activities that were like in the embodied learning teaching guide. [Episode 5, IS4]

The embodied learning activities were easy to implement according to teachers, aligning to Desimone's (2023) reflection that the complexity of the learning matters. Teachers will use new strategies if they are "easier to learn and implement" (p. 2). The embodied learning activities aligned with teachers' current teaching programs and to the structure of their lessons.

Active Learning

Desimone (2011) suggests that "teachers should have opportunities to get involved, such as observing and receiving feedback, analysing student work, or making presentations, as opposed to passively sitting through lectures" (p. 69). The provision of active learning opportunities for teachers is important during PL so they can ask questions, share experiences, get feedback and practice new approaches (Desimone, 2023). Active learning was an overarching design element of the ELEMS project that guided the delivery, *the how* of the PL allowing space for changes made mid-project based on participant feedback.

The following interaction from Episode 3 is an example of asking questions of each other as *active learning* between teachers of the same grade:

Researcher: Is there anything that you want to ask the [IS3] teachers, for example?

IS4 Teacher: For the preschool teacher, I noticed that you were saying that a lot of it was in circle time and drawing on whiteboards. Did you take any of it outside into your environment?

IS3 Teacher: Yes, we do in the sand pit, we would draw symbols, as in the Aboriginal custodian culture.

IS4 Teacher: You'd use the eight ways symbols? Or would you use [symbols of the] Darug people?

IS3 Teacher: We're on Darug land, so we use their symbols

Another interaction from Episode 3, is an example of teachers asking questions of each other as *active learning* about how they are implementing embodied modes:

IS4 Teacher: Can I ask the IS3 another question? Cause [sic] I like it when I hear buzzwords, and I heard a number of people talk about consistent use. So can I clarify? Do you mean consistent use of the same gesture or just consistent use of like gesture or tracing or body movement at all over regularity in your classroom? If you could expand on that anyone from IS3?

IS3: It was the mode. So if I used a body movement, I'd use that for every warm-up I agree. That week, you know, four times, probably four times, I'd say, that week. That was consistent. In my eyes, that would be consistent. Yeah, so use that mode for the same thing activity.

Teachers also presented what was occurring in their classrooms, how they were embedding embodied learning into the teaching and learning cycle and linking it to research:

I think it was the article that we read that, you're not just looking for the answer, it's really hard to assess where children are at when you're just looking at pieces of paper and answers to equations, whereas when they're actually doing this embodied learning, it's far more insightful. [Episode 4, all schools]

Although teachers did not specifically comment on the delivery structure of the PL, it was clear from their interactions in each episode that they enjoyed asking teachers from another school questions about how they were implementing the tasks, to develop shared experiences and receive feedback on new practices. The researchers use of prompts within the PL episodes contributed to fostering active learning, which illustrates Desimone's (2023) note that effective PL requires "instructional leaders who are experienced and have relevant content-area expertise and know how to calibrate learning to the teachers' needs" (p.2).

Collective Participation

According to Desimone (2011), collective participation means "groups of teachers from the same grade, subject, or school should participate in professional development activities together to build an interactive learning community" (p. 69). Collective participation in the ELEMS project relates to *how* the PL is experienced by the participants.

Teachers discussed their own learning experiences with colleagues:

You know, we're all teachers in the classroom. I need the kids. They're not just learners, they need to be there to help others and teach others, and we're all in this together. It's a journey that we're all on. [Episode 4, all schools]

Teacher 1 IS1: Can I dob someone in? Tanya's gonna [sic] kill me. Tanya was talking about how she uses it for assessment.

Teacher 2 IS1: Yeah, I just said it was a useful tool for assessing where the children are at. You get a really good idea. You know, what we're noticing is, while the children are engaging in embodied activities, I'm thinking it's different. It gives us different opportunities to what we might have in more traditional style activities. [Episode 4, all schools]

What I did mostly, we planned together. [Episode 5, IS4]

The collegial discussion that came with these guys as a team, pre-doing it and then post-doing it has for me, led to a lot more beneficial discussions on where these guys can go ... there was that whole collegial discussion that these guys got that they don't get when you do it one-on-one. [Episode 3, IS4, IS3]

Collegiality and collaborative planning were specific features of *collective partition* that were valued by the participating teachers. Schools had formed learning communities, teachers shared what their colleagues were doing well, indicating that ongoing local conversations were occurring between the structured PL episodes.

Duration

The ELEMS project's PL episode structure supported the provision of sufficient time for teachers to engage with their learning, to then trial suggested activities, and integrate an embodied learning approach into their instructional routines (Desimone, 2023).

Teachers reflected on the duration of the PL and the impact an embodied learning approach was having on their students:

I feel as though, if I had another run at it, I'd probably do, I'd be recording that I did it in more lessons per week. But I'd be interested, I'm going to be very interested to see how many lessons a week has the impact. [Episode 5, IS2]

I also think that it's something that probably comes with time. The more confident the teacher is, the more it's part of their everyday practice, the more the kids are used to doing it, then it will become that natural progression ... But because I guess for the purposes of what we're doing, there's almost not that length of time needed for that transformation to become apparent. But I think it probably would naturally, because it's a great thing to do. [Episode 4, all schools]

While the teacher participants' engagement with the ELEMS project was for an extended amount of time, over 6 months. We note that, there was not a high-level of contact with the teachers. The PL included one full day of face-to-face PL for 5 hours, then two 2-hour Zoom, and two 1-hour zoom episodes, totalling 11 hours, not Desimone's (2011) recommended "20 hours or more of contact time" (p. 69). However, the ELEMS project PL included high levels of active learning, coherence and collective participation.

Conclusion and Implications

Evidence that supports Desimone's five key features was identified from the analysis of teacher feedback about implementing an embodied learning approach in classrooms, most commonly *active learning*, *collective participation* and *coherence*. For *coherence*, teachers found the activities easy to implement and aligned with their own current pedagogy and practice. Comments reflected that embodied learning approaches were beneficial for student learning, and that teachers were likely to continue with their newly developed teaching practices. Teachers seeing an embodied approach as 'easy' was an unexpected, yet exciting, observation. When asking teachers to implement a 'new' researched pedagogy, push back and challenges were expected, yet the opposite was communicated in teacher conversation. We attribute this to the *collective participation* of the PL design where during the Zoom sessions teachers were not passive but active participants, eager to hear how others were implementing the approach. The *active learning* element of the PL afforded teachers time to 'notice' practices teachers in other schools were implementing – questioning and clarifying their understanding of the embodied approach. Two of Desimone's features could be expanded on or reconsidered when designing future PL programs – *content focus* and *duration*. Teachers' responses included reference to the mathematical *content focus*, for example, arrays and counting. However, we observed that most teacher reflections did not refer to the mathematics alone but also referred to student engagement and learning of the content. We suggest that Desimone's *content focus* feature might be better labelled as '*content learning focus*'. Renaming this feature would align more to Desimone's (2023) definition that is inclusive of *how* content is learned by students, this change would also broadly include 'teacher' content learning – which is a principal objective of teacher PL. Teachers also acknowledged that time, a *duration* of time, was needed to embed the approach into 'everyday practice'. The length of time, *duration*, however, did not appear to be as important as how the PL time was structured, or used. While our PL did not meet Desimone's benchmark of 20 hours, the structure of the ELEMS PL as a continued, reflective learning community may have mitigated this feature. Future replication of this PL, or any PL, needs to question – can, or how can, high-quality shorter PL still be effective? Part of Phase 3 of the ELEMS project developed the PL into an online format. During development it

was imperative that a key feature of the online PL include sharing of ‘the learning journey’ both within- and across-schools by participating teachers implementing embodied learning in their own classrooms. Our second research question sought to identify the PL features that appeared to be more important. The interplay between an *active learning* structured PL where *collective participation* was encouraged, was most beneficial for teachers. This was evident through the high level of conversation participation, teacher-to-teacher questioning during the Zoom sessions. Future PL programs should therefore consider *active learning* and *collective participation* as essential, particularly as the prevalence of online PL increases where these features are often lost. *Active learning* merged with *collective participation* when teachers shared individually about their own classrooms but spoke using ‘we’ in reference to their school team, ‘it’s a journey that we’re all on’. The *collective participation* teachers experienced was valued more so than the amount of time, *duration*, “because I guess for the purposes of what we’re doing, there’s almost not that length of time needed for that transformation to become apparent ... because it’s a great thing to do [Episode 4, all schools]”.

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Partitive Division: The Numbers Matter to Young Children

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This paper contains an analysis of some early thinking of 94 young children aged 5 years 7 months to 6 years 5 months. These children were interviewed as part of a larger study of the multiplicative thinking of children who were midway through their first year of school in Australia. They had not been formally taught multiplication or division at school. Three task-based interview questions were posed using partitive contexts. The children attempted to solve $12 \div 3$, $7 \div 2$ and $22 \div 4$. An analysis of the responses of these children reveals many children can solve division problems even when the numbers get larger and where the divisor is not a factor of the dividend.

The Multiplication and Division Investigations project sought to understand what intuitive multiplicative knowledge young children develop before they are introduced to ideas formally at school. Our published results indicate that some children aged 4- to 6-years old can think about multiplication and division prior to school (Cheeseman et al., 2022a; Cheeseman et al., 2022b). Our recent focus has centred on children's strategic thinking about division problems.

National test results show that division is poorly understood by many Years 5 and 7 Australian students (e.g., Roche & Clarke, 2013). Years 3 and 4 students were also found to have limited understanding of the relationship between the dividend, divisor, and quotient, and limited experience of contextual problems (Downton, 2009). We believe that if we understand how young children interpret division contexts meaningfully, we may be able to help them develop multiplicative reasoning. Decades ago, Davis and Pitkethly (1990) investigated young children's interpretation of division and found that children as young as 4 and 5 years old could solve division problems using a range of intuitive strategies. We were keen to investigate current division thinking by young children. The research question we sought to answer was: How do the numbers in partitive division tasks influence young children's strategy choices?

Much of the research involving young children's understanding of division was conducted in the 1980s and 1990s and has only recently received renewed attention (e.g., Barkai et al., 2023). Earlier research concluded that children's initial ideas of division often stem from their experiences with sharing (e.g., Davis & Pitkethly, 1990; Pepper & Hunting, 1998; Squire & Bryant, 2002). These studies have shown that children as young as 3 years and 6 months can make "fair shares" when dividing quantities like 12. Partitive division is often described as a "natural" way of thinking about division, using words like "sharing" and "dealing". In partitive division, the number of recipients is known, and the task involves dividing a collection into equal parts (Fischbein et al., 1985). A key feature is that the shares must be equal. Fischbein et al. argued that partitive division arises from children's action schemas. However, others maintain that division involves more than just sharing; it also requires understanding the relationship between the dividend, divisor, and quotient. For instance, children must understand that the more parts a quantity is divided into, the smaller each part becomes (Correa et al., 1998). In a sharing scenario, this would mean that the more people there are, the fewer candies each person would receive. While previous studies have shown that children can perform division in a sharing context (Squire & Bryant), it remains unclear whether the sharing scenario specifically, or social and cultural factors, facilitates performance (Hedefalk et al., 2022).

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Young Children's Intuitive Strategies

Several earlier studies found that most children commonly used one-to-one correspondence or “dealing out” objects to allocate equal shares (e.g., Carpenter et al., 1993; Davis & Pitkethly, 1990; Desforges & Desforges, 1980; Frydman & Bryant, 1988). Dealing, defined as cyclically distributing identical objects so each group receives the same amount until no items remain (Davis & Pitkethly), was a common strategy. While children as young as three and four could share 12 items equally between 2, 3, or 4 recipients, Frydman and Bryant found that only 41% of 4-year-olds could state the number in each group and recognise the equivalence of the shares without counting each set. Desforges and Desforges observed two alternative strategies for dealing: (i) dividing the whole set into equal portions and assigning one portion to each recipient, and (ii) sharing small groups of two or three items at a time. Although children as young as three can distribute items equally in a structured way, some failed to realise that simply engaging in the act of dealing is enough to establish fair shares (Davis & Pitkethly). For instance, 93% of children in Pepper and Hunting's (1998) study used dealing, but those who struggled often lost track, resulting in unequal shares. In summary, these studies suggest that young children can make equal shares, typically by systematic dealing by ones, though they are unaware that dealing itself is sufficient for ensuring fairness. Other research, involving children aged 5-7 with formal instruction, found that children often estimated the number of items per group and tested their estimates, or used trial and error, adjusting the groups until the sharing seemed fair (Carpenter et al., 1993; Mulligan & Mitchelmore, 1997).

Unequal Grouping Situations

Three studies included unequal grouping situations (Blevins-Knabe, 1988; Carpenter et al., 1993; Desforges & Desforges, 1980). Desforges and Desforges' study described strategies when children dealt with unequal sharing situations. Strategies included: asking for one or more to make the shares equal; removing the excess to make the shares equal; breaking the “left over” into smaller parts to equalise the shares; looking puzzled about what to do with the “left over”; one to one sharing; and ignoring the “left over” (3.6 - 4.6 -year-olds). This last strategy was consistent with those used by 4-year-olds in Blevins-Knabe's study. Other studies reported that children often adjusted their distributions when they recognised the shares were unequal (Davis & Pitkethly, 1990; Pepper & Hunting, 1998). Older children, (5 – 6-year-olds) were more likely to ask for an extra item to make the shares fair or remove the excess, indicating stronger concern for fairness (Desforges & Desforges). In summary, these studies suggest that young children approach division tasks with remainders in diverse ways, using a variety of strategies and with varying degrees of success.

Selection of Numbers

Many studies reviewed for this paper used familiar numbers for both the dividend (e.g., 8, 12) and divisor (2, 3, or 4). In contrast, Desforges and Desforges (1980) used less familiar dividends (e.g., 9, 10, 11, 15, 20, 30) and varied the divisors (2, 3, 5), as did Carpenter et al. (1993) with dividend and divisor combinations such as 15, 3; 20, 4; 19, 5. Sherin and Fuson (2005) argued that young children build strong knowledge of specific numbers (like 4 and 12) through computation, which they then apply to new contexts. Mulligan and Mitchelmore (1997) suggested that children first develop strategies for familiar situations where the number facts are well known. Downton (2009) further noted that the numbers used influenced solution strategies, with smaller, familiar numbers resulting in less sophisticated strategies.

In contrast, our research included partitive division tasks with increasingly complex numbers, including those where the dividend was not divisible by the divisor. Our aim was to explore strategic thinking children use when faced with more challenging division problems.

Method

A task-based interview was considered the most suitable instrument to elicit children's existing knowledge, ways of representing their mathematical ideas, structures, and reasoning. Six interview questions were developed that mirrored the calculations from an international pencil-and-paper test (Cheeseman et al., 2022b; Tumusiime & Peter-Koop, 2019). The questions were designed with simple language, increasingly complex numbers and everyday contexts familiar to children. Three worded problems were created for each partitive and quotative division contexts to explore children's thinking about: $12 \div 3 =$; $7 \div 2 =$; $22 \div 4 =$. This paper focuses only on partitive thinking, analysing, and discussing children's responses and the strategic thinking they reveal. In Table 1 the partitive division questions as they appear in the interviewer's script and each question's intended increasing difficulty are shown.

Table 1

The Mathematical Content of the Interview Questions and the Increasing Difficulty Each Presents

Content	Question (<i>Italics instruction to interviewer</i>)	Features of the task
$12 \div 3$	<i>Place the candies (12 glass beads) on the table together with 3 jars.</i> Here are 12 candies. Share all of them out equally between the 3 jars. How many candies go in each jar?	• Small familiar numbers • Dividend is divisible by the divisor • Readily modelled
$7 \div 2$	<i>Place the "donuts" in front of the child.</i> <i>Place the 2 plates on the table.</i> Here are 7 donuts. Share all of them out equally between the 2 children. How many will each child get?	• Small familiar numbers • Divisor is not a factor of the dividend. • Not easy to model • Sharing the odd number is not straightforward
$22 \div 4$	<i>Place 4 dolls representing children onto the table place and count aloud 22 cards.</i> 4 children want to play cards. 22 playing cards are on the table. <i>Hand the 22 cards to the child.</i> Share them out equally between the children. How many cards will each child get?	• A larger, unfamiliar dividend • Divisor is not a factor of the dividend. • Not easy to model, even when dealing

Interview kits consisting of a script, record sheets and manipulatives, were provided to 10 interviewers together with video cameras. These interviewers were experienced teachers who were knowledgeable observers of young children. They were trained to conduct individual task-based interviews with children. The wording of the script could be re-read if a child required it, but not paraphrased. Interviewers were instructed to video-record the results with approximately 10 children from each Foundation class in 12 schools from whom written parental consent had been received. A total of 94 video interviews and matching interviewer written record sheets were collected for analysis. The authors created an initial coding scheme to categorise student responses. The third author independently viewed the videos and record sheets and coded children's responses on a spreadsheet for review and analysis. Inter-rater reliability measures of 85% were achieved; the codes were then refined and validated. After these processes the findings were examined.

Findings

The facility of young children with the chosen division tasks is shown in Table 2. As can be seen the tasks became considerably more difficult as the interview progressed.

Table 2

The Percentage of Correct Responses for Each of the Three Tasks

Interview questions	Number = 94 (Percent)
12 candies shared between 3 jars	84 (89%)
7 donuts shared between 2 children	68 (72%)
22 cards shared equally between 4 children	41 (43%)

The first question was designed to be accessible for 5- to 6-year-old children. It seems that the small familiar numbers ($12 \div 3$), straightforward wording, and materials that made sense to children, made the task easy to model and 89% of the children solved the problem correctly. The thinking strategies children used for this task are described elsewhere (Cheeseman, Downton & Driscoll, submitted). The main error in thinking for this task was children's ($n = 10$) lack of awareness of the need for equal groups as the dividends. Dealing by ones was the technique used by most children (37%) and a further 21% dealt by groups to arrive at a correct solution. An *estimate and adjust* technique was used by 26% of the children (see Table 3). This thinking is characterised by children dealing multiples rather than single objects. We hypothesise that the child may think that, with plenty of objects to share, it will be quicker to deal in twos (or another multiple). Some of these children kept dealing in twos for $12 \div 3$ and some switched to dealing by ones after a cycle was completed to adjust their division and to take into account the remaining objects to be shared. In Table 3 it can be seen that 24 (26%) children grouped by 3's then by 1's. For this task, approximately half (26%, 21% and 5%) of the children used a grouping strategy to find a correct solution.

Table 3

Frequency of Strategies Children Used to Solve 12 Candies Shared Equally between 3 Jars

Category	Solution strategy	Number = 94 (Percent)
Correct		84 (89%)
Dealt consistently	Shared by 1s	35 (37%)
Estimated groups and adjusted	e.g., Grouped by 3s then 1s	24 (26%)
Dealt in multiples	Shared by groups (2 & 4)	20 (21%)
Used a known number fact	e.g., said, "4 + 4 + 4 is 12"	5 (5%)
Incorrect		10 (11%)

The second interview task involved a dividend that was not a factor of the divisor ($7 \div 2$) and it provided some challenges for young children. However, 72% of children revealed knowledge of division. The correct ideas of division are detailed in Table 4.

Only 5% of interviewees solved the task perfectly correctly – with a fractional share correctly named. A further 6% halved one model donut but could not name the solution. Another 11% divided the 7 donuts correctly but when asked, "How many will each child get?" said, "Four" as their solution. We reason that these children could divide into equal quotients but did not understand fractions; therefore, they counted the number of pieces on each plate.

Table 4

The Categories of Response and Percentage in Each Category for $(7 \div 2)$

Category	Solution strategy	Number = 94 (Percent)
Correct		68 (72%)
Said $3 \frac{1}{2}$ - perfectly correct	These children broke the last playdough donut into two, gave each piece to one plate and correctly labelled the result	5 (5%)
Said split in $\frac{1}{2}$	These children could divide and understood a fraction could be a solution but could not name the shares.	6 (6%)
$3\frac{1}{2}$ said 4	This response showed good division but no knowledge of naming fractional pieces – they saw 4 pieces in each share.	11 (12%)
Left over or awareness of uneven shares	Children placed 3 on each plate then put the last donut back, saying “No-one can have this one.” Or “This one is left over.” Or “I need an extra one.”	46 (49%)
Incorrect	Most commonly children shared 4 donuts on one plate and 3 donuts on another	26 (28%)

Both of the categories of response with mis-named fractions could be interpreted as having a partial understanding of a remainder as a fraction and at least a partial understanding of partitive division. Almost half of the children interviewed (49%) understood that with 7 objects it was not possible to divide the collection into two. The comments made by the children showed their thinking: “This one is left over.” And, “I need an extra donut.” These comments echo the earlier results of Pepper and Hunting (1998) where young children understood that the number could be made divisible with adjustment. This type of response shows some understanding of division – that the resulting partitions must be equal. In coding for correctness, we said that these children could divide and get the solution of three with a remainder of one for $7 \div 2$ therefore their response was correct. Overall, only 28% of the 94 interviewed children were incorrect in their thinking about the task. The most common error was to place 3 donuts on one plate and 4 donuts on the other - to divide the objects into unequal groups.

The final partition task asked for 22 cards to be shared equally between four children. The difficulties in this task were with its more complex numbers and a more difficult division with a remainder. The wording was kept plain and we hoped that card games and dealing would be a familiar context for children. The results for this task are presented in Table 5.

Most noticeable in Table 5 is the proportion of incorrect responses to sharing 22 cards between 4 children (57%). We conjecture that the cultural context of playing cards may have overridden the mathematics of this task (Hedefalk et al., 2022). An examination of the error patterns in children’s responses may help to understand children’s thinking (Table 6). Nineteen children did not understand the task, made random shares, or only partially shared the dividend.

The most common error was in faulty dealing by one (17 children) where the children kept dealing a card to each of the four “players” until the cards were exhausted, thereby ignoring the unequal quotients forming groups of 6, 6, 5, 5 cards. The remaining two categories of errors

Table 5*The Categories of Response and Percentage in Each Category for $(22 \div 4)$*

Category	Solution strategy	Number = 94 (Percent)
Correct		41 (43%)
	Dealt by ones and put 2 away	24 (25%)
	Dealt in groups (multiples) using estimate and adjust strategies	15 (16%)
	Used number knowledge	2 (2%)
Incorrect	e.g. gave 4 cards to each and did not share the other cards	53 (57%)

involved the idea of having “left over” cards. Some children (9) estimated equal group sizes and tried several options but found none of the trials shared without remainders and said it could not be done. Eight children said that they either needed two more cards or that they would “put two cards back”. This echoes Desforges and Desforges (1980) findings. These children knew they were trying to create equal groups by sharing. Examining responses of children who correctly solved $22 \div 4$ (Table 5) the largest proportion of successful children (25%) dealt the cards by one accurately and noted that 2 cards were “left over”. However, it can be seen by comparing the proportion of the same children who could successfully deal $12 \div 3$ in Table 1 by ones (37%) that the larger number of cards was more difficult for children. Some of the complexity of the model is immediately apparent, such as when dealing cards, they tend to overlap becoming difficult to see and count.

Table 6*The Error Patterns in Response to $22 \div 4$*

Error	Examples of a child's erroneous actions	Number (n =53)
Concept division unclear	Made random groups/Made equal groups of four/ Only partially used the dividend	19 (36%)
Faulty dealing by one	Dealt by 1s until all the cards were used	17 (32%)
Faulty “estimate and adjust” thinking	Tried several groupings unsuccessfully	9 (17%)
Problems with remainders	Needed two more or two fewer cards Knew $4 \times 5 = 20$ so 22 cards cannot be shared	8 (15%)

In addition, the dealer needs to visualise the “starting place” and track one cycle of dealt cards back to the start. The dealer also needs to consider whether each share has the same number of cards when the dealing is finished and to consider that it is possible to have some cards remaining “unshared” when the dealing is finished. Taking all of these understandings into account, it is notable that one quarter of children (24 responses in Table 5) have built their knowledge of partitive division through experiences prior to school and could solve a problem which used relatively large numbers and involved remainders.

The strategic thinking used by 18% of children interviewed demonstrated their willingness to use their existing knowledge: (2%) used their number knowledge to think abstractly about the relationship between the numbers 22 and 4. One child said he knew that “four fives are 20” so he thought each person would get five cards. The strategic approaches used by a further 16% of children could be considered as a sort of trial and error or estimate and adjust thinking. In these cases, children tried dealing in equal groups of cards to see whether that would “work”. Some children started with dealing out multiples of four and adjusted to groups of five when

they had a handful of cards left over. Finding two cards still in his hand one child said, “these two we will put in the middle of the table for the children.” He was clearly thinking of the context and the numbers and the fact that some cards would remain not shared. We noted that this trial-and-error approach was taxing for some children and some (9) who attempted to deal in equal groups were not successful (Table 6).

Discussion and Conclusion

The aim of our study was to explore how numbers matter in the context of early division. While previous research examined how numbers influence young children’s mathematical thinking (Downton, 2009; Mulligan & Mitchelmore, 1997; Sherin & Fusion, 2005), we introduced unfamiliar numbers that are not easily divisible. In Australia, children are introduced to the concept of division as “sharing” in their first year of schooling, through everyday context and chiefly with numbers up to 20 (ACARA, 2023). However, our findings reveal that 43% of children correctly solved the sharing of 22 cards task, and 72% solved the donuts task. In both cases, the children demonstrated an ability to interpret remainders, showing they could consider more complex division. A subsequent finding relating to the sharing 22 cards was the children’s thinking about the size of the group and using estimation. For example, one 5-year-old estimated the group size saying, “Five might be good”, dealing out four groups of five and looked at what is in her hand, said, “only two left – I can’t use these.” Other children estimated four and then adjusted their thinking. The analysis of errors in the card task uncovered some unexpected insights into children’s strategic thinking and use of number knowledge. While they may not have fully grasped the formal division concept, we argue they showed partial understanding and sense-making of partitive division.

An additional finding was that some children’s interpretation of “sharing” was influenced by its everyday cultural meaning, which differs from the mathematical concept of division (Hedefalk et al., 2022). In everyday use, “sharing” simply means giving everyone a portion (e.g., sharing pencils), whereas mathematical division means dividing equally. This distinction needs to be made clear to children. In the card task, some children applied the everyday meaning of sharing, where it is common to deal a limited number of cards, rather than dividing the cards equally, which likely contributed to some of their errors.

Some research advice has been to consider sharing as an intuitive underpinning of division thinking (Squire & Bryant, 2002). In practice many young children are taught in school to share procedurally by dealing by ones with thinking characterised by “one for you and one for me”. However, the results of our study led us to question whether dealing by ones is straightforward as teachers often assume. While dealing by ones a person needs to consider: the total quantity to be shared, how many shares to make, giving one object to each person, tracking the cycle of sharing, watching - if not counting - the developing shares, noting the objects still to be shared, discontinuing dealing if insufficient objects remain for another cycle, and treating extras as “left-overs”. This list of things to notice is quite demanding. An unthinking use of sharing by ones leads children to the most common error we found - dealing until the dividend is exhausted. Further, these children often had no idea of the size of the quotient or whether the final shares they allocated were equal (Davis & Pitkethly, 1990).

While we acknowledge the limitations of the scope of the interview, the findings offer some insights into young children’s partitive thinking and extends earlier research. The present study found that the numbers matter when young children solve division problems. When numbers and contexts are familiar to children, the strategies they use are either: deal by ones, deal by groups, estimate and adjust or use known facts to solve the problem. Additionally, when numbers are small and contexts are familiar and the situation involves a remainder (e.g., donuts problem), most children (72%) could interpret the remainder mathematically. However, when the numbers are outside the children’s experience and the division was more complex and

involved a remainder only 43% found a correct solution. It was interesting to discover that children used similar thinking strategies with more unwieldy numbers to those they used with familiar numbers. These findings offer fresh insights into the strategies young children use to solve partitive division problems, challenging the reliance on familiar numbers and situations where the dividend is divisible by the divisor.

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Accounts of Primary Mathematics Leadership: Working With Teachers in Classrooms

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This paper reports three mathematics leaders' reflective accounts of leadership involving working with teachers in primary classrooms. A "nutshell" of each leader's interview is presented in sections: leading change, knowledge creation and relationship building. Similarities and differences in their leadership are described in terms of developing a shared purpose, building relationships, sharing knowledge and understanding change. Positive emotions engendered by leadership are acknowledged and the powerful potential of professional conversations is raised.

A key policy guiding the direction of mathematics education in Archdiocese Catholic schools in Victoria has been the *Get in2 Maths – Transforming Education in Numeracy, the 2021–2025 mathematics strategy*. This visionary strategy specified mathematics leadership that involved initiating, guiding and supporting improvements in mathematics teaching and learning. The strategy clearly placed responsibilities and expectations on leaders as the first two, of six aspirational goals, dealt with leadership. These goals stated:

Every school has an appointed mathematics leader with adequate release to enact the school's mathematics action plan.

The mathematics leader will support teachers in various ways such as analysing data in context, leading facilitated planning sessions, developing priorities for improvement and guiding the professional learning of teachers.

Every school has a mathematics leadership team to drive the learning and teaching of mathematics at a classroom level.

The mathematics leadership team will support the mathematics leader to develop, enact and embed the mathematics action plan. This team's responsibilities could include leading discussions during collaborative planning, and supporting and monitoring the learning and teaching of mathematics at a classroom level. (Melbourne Archdiocese Catholic Schools. 2020, p. 2)

The Melbourne Archdiocese Catholic Schools was the first sector in Victoria, as far as we can ascertain, to formally appoint a mathematics leader in every primary school with an adequate time allocation to fulfil the role. In the past mathematics leadership has often been under-resourced. Therefore, the initiative was widely hailed as important. The expectations that School Mathematics Leaders would "drive the learning and teaching of mathematics at a classroom level" implied the leaders would initiate teacher change through professional learning. The strategy is nearing the end of its specified timespan. We were interested to gather first-hand reflections of leaders on the nature of their work at a classroom level. The research question we sought to answer was: *What can we learn from leaders about the classroom leadership of primary school mathematics?*

Research Literature

Teachers' learning occurs in various contexts, formally and informally (Munson & Dyer, 2023). Researchers have emphasised the importance of improving teachers' knowledge of mathematics (e.g., Ball et al., 2008). Teachers have been shown to value support from mathematics leaders, especially in their classrooms (Clarke et al., 2013). Driscoll (2021) found that leaders could support teachers' development by teaching "alongside" them in their classrooms: demonstrating lessons, modelling pedagogical approaches, and providing informal feedback in an environment where teachers felt confident to take risks. As Timperley (2008) (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 117–124). Canberra: MERGA.

noted, all professional learning involves risks with a degree of challenge, and teachers need to trust that their efforts will be supported.

We consider the practice of leaders teaching alongside teachers in the classroom to be a potentially powerful opportunity to build teachers' mathematical knowledge for teaching (Kazemi et al., 2018; Munson et al., 2024). Similarly, Gibbons et al. (2021) suggested the key to supporting teacher learning is in practice-embedded contexts. Classroom support from a more knowledgeable colleague has been described variously by the listed scholars as: team-teaching; coaching; modelled instruction; observing lessons and providing feedback; and demonstrating teaching in instructional groups. Research suggests that jointly experiencing mathematics lessons can be an effective way for teachers and leaders to learn in practice (Driscoll, 2021). An implication of teaching alongside others in a classroom is that it requires leaders who: are knowledgeable teachers of mathematics; possess a high level of expertise; are flexible and adaptable; understand how adults learn; and have developed constructive relationships with teachers in their schools based on trust and respect (Driscoll, 2021).

Evidence suggests that teachers engage in learning as they enact and study practice in their own classrooms (Kazemi et al., 2018). It is through shared experiences with leaders that teachers plan, act and reflect on their work as they refine the focus of their pedagogical practice (see e.g., Ball & Bass, 2000). Interactions with leaders contribute to effective planning including goal setting and identifying teacher moves to advance student thinking (Stein et al., 2022). Typically, as teachers and leaders move fluidly and interact with one another and with students (Munson et al., 2024) they make in-the-moment decisions and respond to student thinking. As the teaching unfolds in real-time these shared experiences sharpen teacher noticing and afford teachers and leaders an opportunity to respond and act upon the impacts of their teaching on student learning (Eden, 2018).

Figure 1

A Framework for Leadership (Fullan, 2001, p. 4)



Seminal work by Michael Fullan (2001) described a framework of leadership as shown in Figure 1. This framework provides the theoretical underpinnings of our research. In our analysis and discussion of the actions of mathematics leaders we will make use of the components: moral purpose, understanding change, coherence building, knowledge creation, and relationship building. In addition, we will consider characteristics Fullan found in leaders of successful educational organisations: energy, enthusiasm and hope. and compare them to the reflections and actions of our interviewees.

Method

Following the ethics approval to conduct the research, we sought recommendations from colleagues to identify School Mathematics Leaders who were reputed to work alongside teachers in classrooms. Three School Mathematics Leaders agreed to a formal interview to help us to understand their current work of leading mathematics. Kerry collected reflections from

each leader using a semi-structured interview. The questions were designed to collect background details, and to gain a description and examples of the leaders' classroom work. Each leader was asked to describe the ways in which they worked in classrooms, the benefits and any downsides to this approach, and why they decided to work this way. The leaders were asked what they thought needed to be in place before they worked with teachers in classrooms and any advice they would give teachers and mathematics leaders.

The interviews were conducted via Zoom over a period of three weeks during November 2024. Each interview was reviewed and analysed before the next interview was conducted to ensure the independence of each set of data. Video records and transcripts were produced automatically. However, video was not reviewed unless clarification of the audio was required. The transcripts were checked and edited to remove repetitions of speech and provided the data that are described and discussed here. Each researcher read the transcripts and summarised the main points made during each interview. These summaries were used to prepare "nutshell narratives" in which quotes were used to highlight in the interviewees' voices their accounts of their leadership. Themes based on Fullan's framework features - leading change, knowledge creation and sharing and relationship building structured the findings. Similarities and differences in the leaders' approaches were then collated, summarised and discussed to provide an overview of our findings. Our aim was to collect the reflections of mathematics leaders to provide a view of school mathematics leadership. The focus of the study was "classroom work".

Findings

The main findings of each interview will be written as "nutshell narratives" where the themes that interviewees raised are matched to quotes from the transcript. Pseudonyms are used.

Harriet

Harriet was an experienced School Mathematics Leader having worked at her current school for 6 years. Her enthusiasm for mathematics education was ignited by professional learning and subsequent university study. She worked across two schools consulting and leading mathematics improvement.

Leading Change

Harriet talked about her current role "purely working with the teachers, facilitating planning, modelling in classes, and running Professional Learning Teams." When working in classrooms she would often model lessons or team teach. According to Harriet teachers preferred to have her model lessons as opposed to being observed as they taught. When modelling lessons, she used a pro forma to focus the observing teacher and to use in debriefings to help support a consistent understanding of pedagogy. Harriet considered debriefing essential, but found it difficult to arrange, as she explained "we don't have a proper structure in place where ... teachers can be released, where we can actually do a proper debrief ... for me, that's very frustrating because I think that it's not valuing the process we're trying to do."

Harriet used planning sessions to support teachers to talk about the mathematics involved in using rich, challenging tasks which would engage students. Teachers were encouraged to set learning goals based on the assessed needs of students. Harriet often modelled tasks and the pedagogical approaches she advocated to show teachers how they could work. However, when leading change Harriet believed that, "if the maths leader is coming in [teaching], then there has to be a point where modelling stops ... then maybe doing some team teaching together before doing an observation of the teacher teaching."

Knowledge Creation and Sharing

When working with teachers Harriet chose to work intensively with a team that was not cohesive. She attended this team's weekly planning meetings and supported teachers to choose engaging mathematics tasks. Harriet explained, "when you're directly planning with them and you're having discussions about the mathematics, you can see who's understanding and who's not." When teachers were unsure of how the tasks would work, she offered to teach the task in their classrooms. Harriet also talked about "feedback from teachers, particularly when they talked about their own content knowledge about how that had improved through the facilitated planning." She saw her role as supporting "teachers to understand the content and effective pedagogies to implement the content" so "children start to enjoy maths and see a value in it." Harriet believed one of the benefits of her work was seeing student's responses "first and foremost" and when teaching alongside, Harriet would often point out student thinking and say, "Did you notice this?" or "What do you think about this?" and "We will have those informal discussions throughout the lessons."

Relationship Building

Harriet's advice to leaders wanting to work alongside their teachers is about knowing your goals and knowing what you want to focus on ... and "of course, it comes back to relationships ... you can't have someone in your room if you don't have a relationship with them. I think that's the most important thing to build." When supporting teacher learning Harriet believed that communicating your aims and making them relevant was important "as a way of bringing it back to the children's learning, so it's not threatening in the sense of we're looking specifically at the teacher. But it might be something specifically focused on the learning or engagement around the students."

Jen

Jen had nearly 20 years' experience as a School Mathematics Leader. It was her tenth year at her present school where she was also leader of learning and teaching. She had a part-time role as mathematics leader and no classroom responsibilities.

Leading Change

When suggesting how teachers could improve their practice Jen's advice was to set a goal and seek "ongoing coaching." She felt rather than a "one-off lesson" it was "more beneficial because then you can move from ... modelling to team teaching to observation." Jen believed that if teachers begin this way, as they gradually build their confidence, they are more willing to be observed and seek feedback. There was no formal process for debriefing teachers after an observation; so much of the feedback that Jen provided was during the lesson, or in a "catch up for 10 minutes at the end of the day." As Jen explained, "it's more of a working conversation." Jen would give teachers "on the spot suggestions and feedback as the lesson goes on ... and where to go next with the lesson during the lesson, rather than later on."

Knowledge Creation and Sharing

A lot of Jen's work involved facilitated planning meetings, spending 90 minutes a fortnight with each team. She explained, the facilitated planning is very much professional learning for the newer teachers as this was when "the actual content knowledge developed around what they should be teaching the kids and how to teach it." At this time Jen was able to see "if there's the need for me to be in the classrooms with the teachers to be modelling or team teaching. It sort of comes from what we plan and where the needs are." At times Jen also observed lessons and provided feedback and modelled the Launch, Explore, Summarise instructional model. When describing how she modelled lessons Jen explained that the teachers, "would probably watch

me do the launch of the lesson and then ... watch what I'm doing but walking by my side." She illustrated her approach saying:

I'd often say to a teacher, Oh! Come over and have a look at what this child's doing. This would be really good to spotlight. ... Even if I'm not the one modelling to sort of walk around and say have you seen what they're doing? ... sort of just having that conversation throughout the lesson.

Relationship Building

Jen described her non-confrontational approach when working in classrooms saying things like, "Oh [I'm] just popping in to see what the kids are doing" then she would "sit down and work with them and try and show the kids that I'm there to help them as well." She explained, "that works quite nicely without putting pressure on the teachers." She found that having those "on the spot conversations" and regular classroom visits made a difference when implementing new programs. She commented, "this comes back to that relational trust and that sort of thing, which I guess I've developed, you know, having been there for quite a few years." It was apparent that Jen had developed close collegial relationships focused on the children's learning as well as the implementation of effective pedagogies.

Sara

Sara had worked as a mathematics leader and intervention teacher for the past 8 years.

Leading Change

Sara worked with teams in facilitated planning once a fortnight, which involved unpacking data, thinking about team goals and how she might support teachers "in the classroom given the opportunity." Finding time to get into classrooms, according to Sara was difficult, as she was also teaching intervention. Consequently, she made some adjustments to her time to allow this to happen. "That was one of those missing links of me, supporting teachers ... I knew it was something I always wanted to do." Reflecting on this point Sara said, "I think [working in classrooms] just helps to build the confidence of teachers. If you get it right, I think it can definitely work."

Knowledge Creation and Sharing

Sara described her classroom work as looking different depending on who she was working with and often involved modelling, parallel teaching, observing students, providing extension, giving feedback or it "could just be an extra pair of hands." Sara also chose to work with some of the newer teachers who were unfamiliar with a "structured inquiry approach." While Sara pointed out that there were a "lot of challenges around teacher confidence in mathematics" at her school. She felt that by working in the classroom she was supporting teachers and building their confidence as they observed her modelling or when she provided feedback. Interestingly Sara, was often asked to model "the summarise phase" of the lesson as teachers at her school found that the "hardest part." Sara explained that for "the majority of lesson, the teacher would be teaching" but "during the explore phase we'd be roaming around the classroom together, maybe highlighting a piece of work or noting work samples." Then for the summarise phase she "would take ownership of that part of the lesson and model how to make connections with the mathematics learning. Sara expressed the view that teachers "have particular trouble in tying it all together and knowing how to recognize the main [ideas] or concepts that they're trying to get the students to learn and draw together."

Relationship Building

Sara spoke of the importance of developing good relationships with teachers. Her advice to leaders was to "build relational trust first. I think that you have to have conversations around why, what's the purpose of being there because ... they might feel the pressure of someone

coming in to watch them.” Sara noted that while some teachers embrace this idea there can be a “bit of pushback or resistance.” She explained that “it’s providing that safe learning space where we’re working side by side and shoulder to shoulder. It’s not me saying this is how it has to be. It’s working collaboratively and collegially together that I think has the biggest impact.”

Summary and Discussion

To summarise and discuss our findings, we will examine the similarities and differences in the leaders’ interview responses. Each of the leaders gave accounts of their classroom leadership that differed. The transcripts of interview were analysed with answers to common questions coded and compared. The differences are largely explained by several relevant characteristics of each environment: the school size, mathematical strengths and interests of teachers, mathematical needs and priorities of students, and each leader and their personal approach to their leadership responsibilities. Each of the characteristics will be presented in turn with illustrative examples from each leader’s transcript.

Size of the School Population and the Number of Classrooms and Staff

In larger schools a leader may need to choose to focus on leading teachers at certain year levels or certain times. **Harriet** revealed that “it’ll be five weeks with a team of teachers like the Prep teachers ... I’ll go in for an hour and model a lesson then ... the next hour and a half ... we debrief about the lesson then we plan for the rest of the week.” **Jen** pointed out “it’s hard because you can’t be everywhere we’ve got 22 classrooms”. **Sara** stated that “there’s 21 classrooms in my school ... my load was quite heavy with intervention and facilitated planning. So, I ... cut back in those spaces which allowed more time for me to get into the classrooms”.

Different Mathematical Strengths and Interests of Teachers

The clarity of teachers’ goals, their attitudes to change, and different levels of mathematical confidence were acknowledged by the leaders. **Harriet** talked about teachers’ developing goals, “The Year 3/4 [team members] were trying to do goal setting around addition and subtraction strategies”. **Jen** advised, “start with the people who are open and then let the good news spread”. **Sara** said, “in particular there are a lot of challenges around teacher confidence in maths”.

Mathematical Needs and Interests of Students Create Different Priorities

Teachers needed different forms of support to improve students’ mathematics. **Harriet** spoke of the emotional needs of the teachers, saying, “I think some teachers find the way that I implement things is challenging because I go quite deep with the learning ... and sometimes it’s frustrating for them ... so I will go in and model”. **Jen** surveyed teachers’ needs, “to start with 95% wanted me to go in and do a modelled lesson. But that evolved over time. I asked what is it that you actually want”? **Sara** explained her work “can look different depending what the classroom teacher is requiring” and that “it’s relevant to the teachers’ needs at the time”.

The Individual and Personal Approach of Each Leader to Their Responsibilities

Each leader expressed a preference for how to work with teachers in their classrooms. **Harriet** described her actions in a classroom saying, “I don’t just sit at the back of the room either. If I’m observing a teacher, I’ll walk around and talk to the children. So, it’s kind of less threatening and doesn’t look like I’m making all these notes”. **Jen** preferred to focus “on the people who you can make a difference with” by “having a clear direction and then ... picking people who are open, who you can see the potential for it being more than just a one-off lesson, who you can work with and help to improve”. **Sara** perceived some of her teachers as, “very much used to maybe a direct instruction approach or explicit instruction where they’re used to giving all the answers. So, I have supported teachers in coming in and exploring what a

structured inquiry type approach looks like in the classroom and just sort of holding back from telling everything. ... but giving lots of steps [hints and support] to their students”.

Similarities

Despite these differences, the leaders’ interviews also revealed common aspects of their classroom leadership. These attributes are found in the literature. Each leader:

- placed importance on building relationships with teachers; (Timperley, 2008)
- expressed a vision of effective mathematics teaching; (Timperley, 2008; Fullan, 2001)
- had specific goals for improved student outcomes; (Stein et al., 2022)
- believed planning with quality tasks key to changing practice; (Stein et al., 2022)
- used planning sessions to trial tasks and model pedagogies; (Stein et al., 2022)
- analysed tasks to discuss mathematical content knowledge; (Stein et al., 2022)
- encouraged teachers to anticipate students’ responses to tasks; (Stein et al., 2022)
- taught side by side to support teacher learning (Munson & Dyer, 2023)
- modelled teaching approaches if asked to do so; (Clarke et al., 2013)
- engaged in professional conversations (Eden, 2018; Fullan, 2001); and
- consciously led teacher learning in mathematics education (Gibbons et al., 2021).

This list of features of classroom leadership can possibly be attributed to the complex forms of knowledge for teaching (Ball & Bass, 2000) developed by the interviewees during their M.Ed. degrees. However, their confidence in the productive pedagogies they were advocating was notable. These leaders possessed an assuredness that was based on their understanding of the research that underpinned the professional learning they were fostering in their teachers.

In Conclusion

In concluding we return to the research question at the centre of this study: *What can we learn from leaders about the classroom leadership of primary school mathematics?* Much of what we learned from the accounts of the classroom leadership undertaken by our interviewees had echoes in the research of earlier scholars (e.g., Fullan, 2001; Gibbons et al., 2021; Eden, 2018). Such authors wrote of developing a shared purpose, building relationships, developing coherence, creating and sharing knowledge and understanding change (Fullan, 2001) which are common characteristics listed above as attributes of classroom leadership.

In addition, the leaders we interviewed reminded us of two important aspects of their leadership work. They spoke with great enthusiasm and emotion about their teachers’ and students’ mathematical achievements. They also drew our attention to the powerful conversations they had with teachers where shared experiences led to insights.

Positive and Motivating Emotions

We found the enthusiasm, energy and hope in Fullan’s leadership framework. Harriet said, “the joy that I get from the teachers - just in passing in the corridor ... [one teacher said], “Oh, quick! Come into my room.” ... She was just so happy to talk about [what the children had done] the progress from when we first started. That’s what brings me joy.” She had a sense of fulfilment from the lasting improvements she could see in mathematics teaching and learning. The effects of working in classrooms with children were seen by Harriet who said, “a year or two later they’ll keep talking to me about their maths, which is lovely.” These comments reflect the sense of achievement these leaders felt as they reflected on their classroom leadership.

The Power of Professional Conversations

Another common feature of the classroom leadership was particularly noteworthy. Leaders described the importance of professional conversations conducted about mathematics teaching

and learning. These practical and highly relevant conversations created opportunities for sharing ideas, insights and knowledge. Leaders described these exchanges involving analysis and discussion of events. Conversations were reported as: happening prior to teaching while trialling a task, during a mathematics lesson, and in a debriefing after a lesson. Each leader reflected on the power of conversations that happened ‘in the moment’ where an action of a student or an event could be discussed in real time. For example, Jen described a team-teaching moment where it was “me walking side by side with her ... to have an on-the-go conversation about what we were noticing” and “just having that conversation throughout the lesson ... that on-the-spot discussion rather than following it up later ... sometimes the moment’s lost then you lose that impact that you can have within the lesson.” It was clear from the interviews that the leaders encouraged professional conversations about the teaching and learning of mathematics as a deliberate part of their classroom leadership. Such conversations provided powerful opportunities for professional learning to occur. These conversations may prove to be a valuable source of data for future investigations.

Acknowledgement

Monash University gave ethics approval (ID 44539) principals and teachers gave informed consent.

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Minding Emotions in Mathematics Education

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In mathematics education research, a focus on cognition in learning experiences often obscures emotion. Our ongoing research has underscored the importance of the emotional dimensions and social interactions that contribute to social solidarity. Through a sociology of emotions framing and event-oriented social inquiry, we identify salient characteristics in primary and preservice mathematics settings to show how social interactions can build social solidarity and help create an emotionally safe learning environment. We demonstrate that social solidarity includes three aspects: ritual objects; being-in/with others, and collective empathy.

Improving students' achievement in mathematics continues to be a pressing issue in Aotearoa New Zealand and internationally. A common policy lever for raising achievement is to focus on teachers' competencies for teaching mathematics. In a recent policy change, the Teaching Council of Aotearoa New Zealand (2024) now requires primary initial teacher education providers to assess the preparedness of graduates with mandated entry assessment of competence in mathematics. Year 5 students in Aotearoa New Zealand (see Caygill et al., 2024) also report that they are not confident about doing school mathematics. We take a position that attention to raising student achievement in mathematics includes building and transforming the emotional aspects of classroom learning environments.

Reframing negative narratives about emotions associated with mathematics is long overdue and approaches to responding to emotions are often counterproductive to learning. One strategy for promoting anxious learners' engagement in mathematics has been the adoption of games to cast mathematics as "fun" as part of an emotionally safe learning environment. However, efforts to "make maths fun" by simplifying the mathematics can mask and detract from students' opportunities to engage with challenging mathematical ideas (Bonne & Higgins, 2022). Mathematics anxiety has become an increasingly pervasive and unquestioned explanation for low mathematics achievement and poor relationships with the subject. The enduring and intergenerational entanglement between emotions and the teaching of mathematics (Foley et al., 2017) illuminates the challenge of transforming stubborn discourses, such as mathematics anxiety. The aim of this paper is to present insights from a body of research that focuses on how teachers and teacher educators can attend to students' emotions in mathematics classes to build emotionally safe learning environments. To do so, we unpack events that occurred in the context of students and their teachers playing whole-class mathematics games.

Considering Emotions in Mathematics Learning Environments

To date, cognition has been a primary focus in understanding education settings, however it is now important to bring emotions to the forefront to highlight the interdependent relationship between emotion and cognition, as explained in sociological theories of emotion (Collins, 2004; Turner, 2002). It is generally agreed that the place of emotions in education has been under-investigated. Zembylas (2005) suggested three reasons why this may be the case: first, that because emotions are fluid, they are therefore more difficult to describe than cognition; secondly, the dominance of cognitive psychology over education research endures; and thirdly, "there is the legacy of dualism, which has opposed reason to emotion, and accorded (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 125–132). Canberra: MERGA.

reason the high status inscribed in Western thinking” (p. 123). Indeed, a seam of research continues to explore ways to measure emotions in mathematics contexts (e.g., Lin et al., 2020; Park & Flores, 2021), even though the fluidity of emotions proposed by Zembylas (2005) could make it problematic for researchers to treat emotions as an enduring trait, rather than a temporary state. However emotions are ever-present in mathematics learning environments, and we argue they need to be integrated with considerations of mathematics achievement. In both primary classrooms and teacher preparation settings, providing an emotionally safe learning environment and also competently teaching students needs to be a focus. In mathematics learning contexts, the challenge is for educators to attend to both hearts *and* minds, that is, emotions and cognition.

Classroom studies in science education have investigated the complexities of how emotions feature in learning and teaching by using multiple framings and methods (e.g., Davis & Bellocchi, 2018; Tobin & Ritchie, 2012), yet in the field of mathematics education, few classroom studies to date have considered emotions from a sociological standpoint. Despite limited attention to emotions in mathematics education, as reported in the four-yearly *Research in Mathematics Education in Australasia* (Reid O’Connor et al., 2024), emerging research into emotions in mathematics learning includes work by Buckley and Sullivan (2023) in Australia. Notably this work investigates teacher pedagogical approaches that could help students manage anxiety and uncertainty associated with challenging mathematics tasks, with the potential to promote resilience and enjoyable learning of mathematics (Johnston-Wilder et al., 2021).

In Aotearoa New Zealand, recent mathematics education research has taken a focus on the broader construct of affect, described by Grootenboer and Marshman (2016) as including the personal constructs of beliefs, identities, attitudes, values, dispositions, and emotions. Marmur and Darragh (2024) have used the dual lenses of affect and identity to examine how significant memorable events in preservice primary teachers’ mathematics learning shape their relationships with the subject. Ingram et al. (2018) suggested effective strategies for promoting pre-service teachers’ productive relationships with mathematics included: explicitly attending to mathematical content knowledge; highlighting affective aspects of learning; and presenting practising teachers as positive role models.

A key reason why emotions are often overlooked in the learning experience is the prevalence of negative past experiences with mathematics among both primary students and pre-service teachers. To address such experiences and the emotions associated with them, we need to better understand the emotional dimensions of mathematics learning environments. Employing multiple framings and methods is critical to expanding the field of emotions and mathematics.

Theoretical Perspectives

Using a sociology of emotions framing (Collins, 2004; Turner, 2002) enables an examination of the emotional complexities associated with learning and teaching in mathematics classrooms. We draw on Turner’s (2010) stratification of a large repertoire of emotions derived from four primary emotions (anger, sadness, fear, happiness), three of which are negative. Derived from the primary emotions, Turner argues that secondary emotions “are a way to reduce the power of the three negative primary emotions into emotions that can promote social and special control” (p.181). Collins’ (2004) focus on interaction rituals is helpful for understanding social solidarity which, along with emotional energy and collective ritual objects, is one of the important outcomes of successful social interactions. We are interested in the dialectic relationship between collective | individual emotions and how these emerge and contribute to social solidarity.

Methodological Approach

Event-oriented social inquiry (Alexakos, 2015) in our program of research enables a nuanced reading of emotions in primary and preservice mathematics classrooms, illustrated in this paper by several events that we interpret as evidence of social solidarity of the groups involved. We used multiple methods, including classroom transcripts, cogenerative dialogues (cogen) (Roth & Tobin, 2002), an interactive response card system (clickers), one-to-one interviews, email conversations, and the research team's reflexive conversations. In considering how teachers and teacher educators attend to students' emotions in mathematics classes to build emotionally safe learning environments, we look back on two related empirical mathematics education studies to generate further insights about social solidarity. The first study is with a class of 34 students (10- to 13-years-old) and the two teachers who shared the class (Bonne & Higgins, 2022), and the second is with a class of 43 pre-service teachers who were taught by Joanna and Raewyn (Higgins, Bonne, & Eden, 2024). We use student pseudonyms throughout this paper.

In this paper, we trace three aspects of social solidarity that became evident when the students were participating in whole-class mathematics games. The first aspect of social solidarity is the presence of a ritual object. Collins (2004) described a ritual as "a mechanism of mutually focused emotion and attention producing a momentarily shared reality, which thereby generates solidarity and symbols of group membership" (p. 7). Following Collins' work, we see a ritual object being of an enduring nature and creating a common point of focus for interactions amongst the collective. The second of these aspects is 'being-in/with others' (Roth & Tobin, 2002) which is described as "being-in the world and being-with others" (p. 315). We have interpreted this to mean participants - regardless of their roles as teachers or students - being attuned to the same social context and physical space, providing a framing for learning and teaching. The third aspect of solidarity is collective empathy which is associated with shared emotional dimensions evident in classroom interaction rituals (Collins, 2004) and involves the collective's response to an individual's expressed emotion. The emergence of an emotionally caring safe learning environment can be seen as contingent on the generation of collective empathy.

Findings

Social solidarity is an important aspect of emotionally safe learning environments or what Zembylas (2007) described as creating caring safe ecologies, in which the whole class is concerned about everybody's emotional wellbeing. Originally, we analysed interactions during games because we had evidence that students' emotions were amplified in this context (Higgins & Bonne, 2014; Bonne & Higgins, 2022; Higgins et al., 2024). More recently, we have re-examined our work across two classroom contexts and were struck by the emergence of social solidarity associated with playing mathematics games.

Context 1: A Primary Mathematics Classroom

During our work in a primary classroom, we employed clickers to build a picture of students' perceptions of the emotional climate over the course of mathematics lessons (see Bonne & Higgins, 2022). The peaks and valleys that we noticed in the data all occurred while the class played whole-class games, in contrast with the remainder of the time, during which there was much less fluctuation in emotional climate. The students were engaged in games that are common in classrooms in New Zealand to improve mathematical knowledge related to number facts and probability: "Round the World" and "Greedy Pig", respectively (for details of the games, see Bonne and Higgins, 2022). Primarily we aim to understand how games in a primary classroom can be a site for the generation of emotions that lead to social solidarity by

exploring elements such as ritual objects, being-in/with, and collective empathy. The evidence we present here is from three cogen sessions, involving six students and Joanna and Linda.

‘The Talk’ as a Ritual Object

In one cogen session, the primary students described one of their teachers using “the talk” to help manage the behaviour of students who were “out of line” (Higgins & Bonne, 2014, p. 58). What follows is an illustration of students' experiences of ‘the talk’.

Illustration 1

“She’ll just have the talk with you”

-
24	Quentin:	Yup like she’s fun but like if you get out of line like she can give them a look sort of thing ... she’ll just have the talk with you you’ll have ‘the talk’ [makes quotation signal with one hand, students laugh]
25	Kitty:	You always know when you’re having the talk
26	Quentin:	Yeah because she’s got this [taps on the table with one finger] and these stern features in her voice and stuff.
-

Mention of ‘the talk’ was met with a ripple of laughter across the group (Turn 24), indicating a collective awareness of the teacher’s strategy for monitoring those students she deemed to be ‘out of line’. Clearly, everyone in the class knew about ‘the talk’, which we consider to be a ritual object in this class. ‘The talk’ can also be seen as a cultural practice of this classroom, however, it is important to note that its meaning is emergent and contingent across and within participants (Alexakos, 2015).

Being-in/with Others

How the students see their teacher as aligning with them is an important aspect of “being-in/with”. In an early paper (Higgins & Bonne, 2014), we observed how a teacher shifted her alignment with the students across different events in the classroom, illustrated next.

Illustration 2

“She’s a big student”

-
15	Quentin:	You kinda need a teacher that you respect like as a person because otherwise you don’t really listen to them [students all laugh] ... like say [one of their teachers] she joins in with the class whenever they’re playing games ... and she makes jokes in class and she’s a really funny teacher and stuff
16	Kitty:	She knows what kids really like
17	Scott:	She’s like one of the students
18	Kitty:	Yeah she also knows how to like kinda help and work with the kids who are being naughty in class but she: and she does it in a way that’s really good ...
19	Quentin:	Yeah she’s a big student
20	Scott:	And she knows like where the boundaries are set
21	Kitty:	Yeah so ... she has boundaries but she also knows how to have a lot of fun
-

As the illustration shows, the students recognised the teacher’s sense of fun and her understanding of what students like. We interpret the teacher’s ability to have fun with her students, making jokes and generating laughter, as contributing to the teacher being-in/with the students and in solidarity with them.

Collective Empathy

Although empathy is often associated with a response to a negative emotion, the following illustration shows the students responding collectively to an individual student in the final stages of a game and then cheering as their classmate wins.

Illustration 3

“A victorious cry”

During the final stages of a competitive mathematics game in which the last remaining student was left to challenge the teacher, Jack correctly answered a problem before the teacher, and realised he had won the game. At which point Jack raised his fist above his head and gave a victorious cry. The other students mirrored his emotion, by cheering, laughing, clapping, and surging forward towards the student and teacher, with several getting up on their knees. The teacher threw her head forward, feigning shame at losing the game.

The surge of emotional energy evident in this excerpt is what is described by Collins (2004) as the special kind of energy arising from participation in a ritual. This example demonstrates a dramatic emotional climate described variously as, for instance, collective effervescence (Collins, 2004) or high positive emotional exchanges (Davis & Bellocchi, 2018).

Context 2: A Preservice Mathematics Education Classroom

In the context of a mathematics education class with preservice teachers, we used event-oriented social inquiry (Alexakos, 2015) to analyse transcripts of classroom sessions included in an 8-week course that focussed on classroom pedagogy. We aim to understand how the interactions among students, and between students and teacher educators, highlighted the emotions involved in learning and teaching mathematics. Aligned with our work in a primary classroom context, here we specifically focus on how collective attention to emotions generated social solidarity. The evidence we present here is from transcripts of two consecutive sessions in the preservice teacher education mathematics education classroom.

A ‘Gift’ as a Ritual Object

In the pre-service classroom, a ritual object, a ‘gift’, emerged from an interaction sparked by a student’s under-the-breath playful quip “so she thinks” that potentially challenged the authority of the teacher educator, by reminding the teacher educator that they cannot know everything that is going on in the class (see Higgins et al., 2024). The term ‘gift’ was coined by the teacher educator and subsequently adopted by the class as a humorous way to highlight the emotional aspects of interactions (Eden et al., in preparation). The following illustration, focussed on number patterns, shows the first occasion on which a student uses the term ‘gift’.

Illustration 4

“I hear disagreement”

4	Charlie	But it's not...
5	Raewyn	I hear disagreement! Someone said it's not?
6	Rowan	Another gift! It's gonna go 1 2 2 cause it's still carrying on the 4 from the last time around, it's carrying on 12 12 instead....

In Turn 6, Rowan identifies “another gift”, referring to the emerging disagreement about an aspect of a pattern identified in a choral counting activity (University of Washington, 2014). The ‘gift’ as a ritual object lifts the mood of the group to enable an emotionally safe environment for disagreement about which number comes next and reduces the focus on an individual. Later in the same session the teacher educator invited the students to think about the function of the ‘gift’. The students’ responses follow in Illustration 5.

Illustration 5

"These little in-class jokes"

8	Avery	It does something towards classroom culture, these little in-class jokes that only the class knows, it makes us feel together
9	Quinn	I was going to say the same thing

The student responses show an awareness of the contribution the 'gift' makes to the social solidarity of the class. The 'gift' had spontaneously emerged and, over time, retained collective acceptance. However, the 'gift's' enduring nature as a ritual object was contingent on at least one of the students recognising its function as an "in-class joke" (Turn 8).

Being-in/with others

The function of the 'gift' was re-examined by the class when it had to be explained to a student who had been absent at the time it had emerged as an in-class joke, illustrating the notion of 'being-in/with others' as an aspect of social solidarity. This plays out in the illustration below when Joanna realised that not everyone in the class was in on the joke, and this prompted a conversation about the importance of the teacher ensuring everyone is included.

Illustration 6

"It's been a gift gift but I don't know"

11	Billie	I have no idea what the joke is! I was doing all the maths, I missed it. So this whole lesson it's been gift gift gift but I don't know!
12	Raewyn:	So that punctuates the point a little bit I guess
13	Joanna:	We've just got another gift! It's the assumption everyone was in cause everyone was here, I made that assumption ... so you're [referring to Billie] in on the gift now? [however, Billie has left the room, unnoticed by Joanna]
14	Avery	She left because she was getting frustrated. No, I'm joking! But you can see how in a class some students might stop listening
15	Joanna	Good point, ... I assumed all the people in the room were on board with the gift, it's like what building up the culture of a classroom is about.

The collective had an opportunity to reexamine the role of the 'gift' in building social solidarity, prompted by the realisation that one of the students was unaware of its meaning. We note how as teachers we rely on the continuity of classroom culture for uninterrupted learning processes and ritual ways of being in a classroom. The emergence of a shared understanding at the same time presupposes that some individuals will not be-in/with the collective.

Collective empathy

The emergence of an emotionally safe learning environment can be seen as contingent on the generation of collective empathy. The illustration below shows a separate event in the same class in which empathy is evident in the group's suggestions about modifications to the game, "Buzz", in response to one student's emotional outburst.

Illustration 7*“It’s the individual failure I hate”*

4	Quinn	[taking turns] one, two, buzz, four, five, buzz, seven, eight, buzz, ten, eleven, buzz, thirteen, fourteen
5	Helen	Fifteen. [loudly and angrily] That’s why I don’t like this game. Because it’s an individual thing, and I sucked at multiples, and I’d go out first, and everyone would make fun of me. It’s the individual failure I hate. Because it makes you feel like crap
6	Joanna	Hmm. fair point
7	Chris	Maybe you could run the game and have someone sit it out if they didn’t feel comfortable?
8	Joanna	Maybe. Let’s ask the class
9	Sam	Set boundaries earlier. No hassling. Anyone who hassles can sit out
10	Nick	We could have a referee

The students’ strategies for making the activity more inclusive (Turns 7, 9, and 10) helped to downplay the emphasis on individual performance and create a safe emotional climate. The co-construction of the environment “shapes how [students and teacher educators] are emotionally connected and engaged in learning together” (Zembylas, 2007, p. 357) and, we argue, demonstrates collective empathy.

Concluding Thoughts

The time for minding emotions in mathematics classrooms is long overdue. To help build achievement in mathematics it is important to consider how educators attend to the complexities of emotion in mathematics classrooms. Specifically, the evidence presented here shows that paying attention to individual and collective student emotion is essential to disrupting cycles of negative emotion associated with mathematics. Furthermore, the deliberate inclusion of everybody in the class was important for developing social solidarity.

We have shown that in both contexts, three aspects of social solidarity emerged: ritual objects; being-in/with others; and collective empathy. A close examination of events that all occurred during whole-class mathematics games, has illuminated a range of emotions associated with mathematics learning and teaching. This work contributes to reframing negative narratives about emotions associated with mathematics and offers new insights into building emotionally safe learning environments (Zembylas, 2007). We continue to raise questions about developing a broader explanation of how teachers and teacher educators might attend to the emotional dimensions of mathematics classrooms and foster social solidarity through ritual objects, ‘being-in/with’ others, and collective empathy.

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Argument-Based Mathematical Inquiry: Teacher Strategies for Supporting Young Students' Collective Argumentation

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Development of students' argumentative capability in mathematics supports deeper conceptual understanding and reasoning. However, young students often lack the prerequisite communicative competency. This study examines how one teacher supported students in building collective argumentation through an extended mathematical inquiry. Analysis of classroom video highlights key practices that enabled this process. When enacted coherently, these allowed students to engage authentically in disciplinary reasoning and dialogue central to mathematical communities.

Introduction

Mathematics and argumentation are innately linked, as stating and justifying claims, and exemplifying, developing, communicating and evaluating arguments are core components of mathematical reasoning (Lannin et al., 2011). Argumentation in mathematics learning supports development of students' deeper conceptual understanding and critical thinking in mathematics through the need to justify solutions and explore the validity of different approaches (Hasançebi et al., 2021). This solidifies understanding, encourages a critical thinking approach, and engages students with different perspectives (Chinn et al., 2000). The critical thinking developed has been shown to be transferrable to other disciplines (Mercer, 2000). Argumentation also develops both generalised and discipline specific competency in communication, supporting students to improve their ability to express mathematical ideas through discourse (NCTM, 2020). As students explain their reasoning, they improve their mathematical vocabulary and gain confidence in discussing complex ideas. Further, students who argue about their reasoning are more likely to retain information long-term (Hattie & Timperley, 2007).

Challenges to the teaching and learning of argumentation for both teachers and students influence its uptake. Teachers require opportunities to themselves become proficient in argumentation and this can be challenging (Francisco, 2024). Young students on the other hand, tend to find it difficult to consider the viewpoints of others (Rapanta et al., 2023). Furthermore, argumentation is both a specific genre and a discursive practice that they are likely unfamiliar with, given the emphasis in the English curriculum on persuasive genres as incorporating expressions of opinion, having the goal of 'winning', rather than building collective knowledge.

Given these challenges, an investigation of how primary teachers can integrate collective argumentation into the mathematics classroom is needed. In this paper, we focus on the practices of one primary mathematics teacher as she introduces her students to argumentation practices embedded within a guided mathematical inquiry. In response to the identified difficulties above, we ask: *How does one teacher's strategies support students' development of collective argumentation in the context of an argument-based mathematical inquiry sequence?*

Literature Review

Argumentation is a practice comprised of the two forms of argument: structural and functional. A simplified argument structure for younger students is the Claim-Evidence-

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Reasoning (CER) model (Zemba-Saul, et al., 2013), which incorporates a position statement (*claim*), data that supports the claim (*evidence*), and logic that enables the evidence to be used to establish the claim (*reasoning*). *Rebuttals* are also incorporated into the framework to provide reasoning for why alternative explanations are not appropriate. The functional aspect of argument addresses the delivery of the argument, or ‘arguing’. However, unlike debate, the purpose of the delivery is to achieve consensus or agreement or to advance knowledge.

Collective argumentation can be considered “any instance where students and teachers make a mathematical claim and provide evidence to support it” (Conner et al., 2014, p. 404). Collective argumentation recognises the discursive nature of argumentation and the constituent practices of developing, communicating and evaluating arguments as a social practice. Brown and Renshaw (2000) enumerate five key principles of collective argumentation: the necessity to communicate individual thinking about a task and may do this through use of representations (*generalisability principle*); the necessity to accept or reject ideas through logical argument and/or evidence (*objectivity principle*); the expectation that contradictory ideas or thoughts are resolved through argument (*consistency principle*); developing an agreed approach to a task as a class (*consensus principle*); and that individual and group ideas are presented to the class for discussion and evaluation (*recontextualization principle*) (Brown & Renshaw, 2000).

While young children demonstrate an inherent ability to argue and counterargue in various settings (Bubikova-Moan & Sandvik, 2022), their argumentative engagement often follows an egocentric pattern (Muller Mirza et al., 2009). Rather than aiming to enhance the plausibility of their arguments or reach mutually beneficial agreements, their primary focus is on defending personal interests and values (Rapanta et al., 2023). This cognitive egocentrism limits their ability to genuinely participate in argumentation dialogues where a shared goal is established.

A fundamental challenge in early argumentation is achieving intersubjectivity, where argumentation participants consciously pursue a common understanding of the discussion's objective (Perret-Clermont et al., 2004). Nonetheless, early informal debates may act as precursors to more structured argumentation, developing into more sophisticated argumentative reasoning over time (Iordanou & Rapanta, 2021). This suggests that fostering argumentation in young learners can be improved by scaffolding structured educational environments that encourage cooperative reasoning and challenge self-centred discourse.

Many argumentation frameworks focus strongly on cognitive and structural aspects at the expense of socio-cultural dimensions and there is more need to emphasise the social and discursive practices that occur (Francisco, 2024). Makar et al. (2015) suggest that supporting the development of argumentation practices includes: fostering a classroom in which peer collaboration, feedback and critique are encouraged; promoting the collective sharing of evidence to refine arguments; and, setting clear expectations that ideas would be examined.

Method

This study was part of a larger design experiment and was guided by the principles of Design-Based Research (Cobb et al., 2003), incorporating an iterative and reflective nature of cycles of instruction with a pragmatic focus. The first author taught the unit. The study received human research ethics clearance from The University of Queensland (clearance number 11-017) and consent from both parent and student was obtained for 22 of the 25 students. The data of the remaining three students is not included in this study.

The class was comprised of 9-10 years-old students, including five boys who were ascertained with autistic spectrum disorder and/or significant learning difficulties. The school was situated in a middle socio-economic suburb of a capital city. The class had remained intact with the same two co-teachers for two school years. These students had engaged in guided mathematical inquiry for various periods of time depending on their class placements prior to

Year 4 but with a minimum of one and a half years under the same teachers. While students had prior experience with inquiry-based learning, they had not yet been introduced to argumentation beyond the generic study of persuasive texts under the English curriculum.

The lesson sequence is centred on a geometry investigation. The class undertook a geometry unit spanning the previous week, focussing on the properties of pyramids and prisms. During this time, a student queried whether pyramids could have a scalene face. The teacher suggested the question as the basis of their next class inquiry and received enthusiastic agreement.

Data comprised video recordings of ~10 hours of learning across five days. Each video was transcribed in full, and the transcripts were examined with a view to identifying teacher supports and the associated impact on students' development of collective argumentation.

Findings and Discussion

In this section, we provide illustrations of classroom practice of each of the key principles of collective argumentation. We also incorporate a number of key practices that the researchers identified as pivotal to class involvement in collective argumentation. To facilitate the narrative for our audience, we provide an (almost) sequenced approach to addressing the principles. We argue that, in practice, the principles were consistently demonstrated across the inquiry.

Consistency Principle

This principle provides that contradictory ideas or thoughts are resolved through argument. At the outset of the lesson sequence, the teacher introduced the students to argumentation practices through a guided discussion, linking argument structure of claim-evidence-reasoning (Zemba-Saul, et al., 2013) to an inquiry framework familiar to her students (refer author, 2014), as well as to their existing knowledge of persuasive genres covered in English lessons. Through this process, the students were introduced to the notion of argumentation structure and purpose as a way of informing, shaping and supporting their conclusions, and as fundamentally different than using opinion or emotion to justify their claims. At the commencement of the lesson sequence, the opening discussion indicates the students' preparedness to bring argumentation practices into a mathematics inquiry task.

Teacher: OK. So, you need the evidence to do what?

Lucy: To convince

Connor: To persuade

Shana: With evidence you can back up, support your conclusion.

Oliver: You have to have evidence because a claim is like trying to get someone to believe what you are saying you make it stronger by giving it more evidence. You give it evidence which will make your claim better. You can make your conclusion better.

Resolving contradictory ideas through argument was also an overlaying principle throughout the lesson sequence with the teacher ushering students towards a collective argumentation practice in which all students contribute evidence and share their experiences to negotiate a common claim. Midway through the lesson sequence, each group presented their claim to the class, supported by evidence and reasoning. The resultant class critique prompted further refinement of their arguments. Through this process, students came to understand argumentation not as a final product, but as an evolving, inquiry-driven process. This empowered them to recognise their capacity to engage in disciplinary argumentation to explore and resolve questions about the world. In this instance, serving as a springboard for investigating geometric relationships, thereby deepening their conceptual understanding.

- Teacher: So, in terms of regular shapes, we could put forward a theory, just a theory, we don't know if we are right or not, that four or less sides and we can make a scalene pyramid. So, what is special about a 5-sided figure that we don't see on a four or less-sided figure? ...
- Leticia: Could it be, ...the pentagon has one line of symmetry but a square and a triangle have 2 (sic).
- Teacher: What is different about a pentagon, a heptagon, an octagon that is different from a square or triangle? And don't tell me the number of sides because that is fairly obvious.
- Delmar: On the heptagonal base and the hexagonal base and on the octagonal base aren't like all the angles the same?
- Teacher: Well, if they are regular, yes ...
- Leticia: I think if you umm ... If it is a right angle or smaller it works. If it is obtuse ...

The classroom enactment of the *consistency principle* demonstrated how students' prior experiences with persuasive texts and the inquiry framework were leveraged to build a foundation for collective argumentation. The teacher's deliberate juxtaposition of claim-evidence-reasoning (Zemba-Saul et al., 2013) with mathematical inquiry practices enabled students to reconceptualise evidence, not merely as a means of justification, but as a tool for constructing and shaping claims. Students in this study progressively learned to move beyond evidentiary assertions as static justifications toward an evolving engagement with uncertainty—a hallmark of authentic disciplinary argument (Osborne et al., 2004).

Objectivity Principle

The *objectivity principle* requires that ideas can be validated through reference to reasoning. This argumentation sequence was intended to focus students on evidence as a way of coming to know and to stress the importance of evidence quality. The purpose of the inquiry was to create evidence, and build knowledge to make a claim, and thus answer a novel question. To this end, the teacher used a balanced approach of challenge and support to deepen students' understanding of evidence. For example, she used a scenario with a series of follow-up prompts to challenge students' concepts of evidence and emphasise the role of evidence in collective mathematical argumentation. The teacher links the students' previous experience writing persuasive texts to mathematical argumentation practices, highlighting the emphasis on using elements of logical arguments and quality evidence.

- Teacher: So, when I make a conclusion, when I decide whether or not a pyramid can have a face which is scalene, what's going to be in my conclusion?
- Connor: A claim
- Teacher: So, what's a claim?
- Connor: Something that you ... [tailed off]
- Teacher: Think about the persuasive arguments that you have been writing for [co-teacher].
- Connor: Something that you say and then you give evidence.
- Teacher: Ah. So, we have a claim which is something that we believe, something that we think, something that we have concluded - from our inquiry. ... From our claim, what was the next thing that you said? [pause then prompting] For your claim you have to have?
- Connor: Evidence
- Teacher: Evidence. Why do I have to have evidence? Why can I just not say, "Can a pyramid have a face which is scalene? Yes!" ...Ahh...no someone different - Konrad?
- Konrad: How can we believe you? ... Teachers aren't always right.

The environment of collaboration and acceptance of risk-taking when sharing ideas is evident in the students' discussions concerning the acceptance or rejection of quality evidence.

In the following excerpt, Connor shows that he is open to the affordances of evidence that provide positive instances to support of the claim (“the net of a pyramid that works”), and those that provide negative instances to support the claim (“nets that didn’t work”) to formulate the logical argument. The excerpt also provides an example of teacher probing to facilitate sharing of the mathematical ideas that form the basis of students’ decisions around evidence selection.

Connor: If you were making evidence saying that it does work, you would need the net of the pyramid that works, you would need the measurements of each scalene face, and you would need the net actually built so that they know that it works. ...

Teacher: Why would you want the measurements?

Connor: If you just show the triangle, you wouldn’t know that it is scalene unless it is really wonky and all of that. But it could be like 6.9 and 7 or something like that. ... For the evidence that [the scalene pyramid] doesn't work, you would need to show a big range of different nets that didn't work, and we'd have to show each measurement that you tried that didn't work, each face your tried that didn't work and each net that didn't work.

Teacher: It sounds like it would be a lot harder to prove that you can't than it would be that you can.

Throughout the lesson sequence, teacher prompts were directed in such a way to challenge students’ conceptualisation of what counts as “quality” evidence in relation to:

- The nature of evidence, being an act of judgment as opposed to mere opinion,
- The notion of sufficiency of data in relation to the claim,
- Validity of the source of data, notably that authority does not ensure validity.

These prompts became an anchor for the teacher to which she constantly referred throughout the sequence. The teacher utilised repetitive modelling of these notions which contributed to the students becoming themselves versed in the practices of argumentation.

Consensus Principle

This principle requires that approaches to a task are agreed by the class and that all members can articulate the approach taken. The teacher ensured that, through discussion, all students understood both the problem and the need to gather evidence to address the problem. The teacher’s role was facilitative rather than directive. Students were offered choice and therefore ownership of both mode of working, and ways of progressing. The teacher encouraged collaborative work around a shared objective, and this allowed for rich interactions and cross-fertilisation of ideas. This helped students improve their openness to other people’s ways of knowing and finding out. The students moved beyond collaborative agreement of task processes to collaborative understanding of what would constitute an acceptable conclusion.

The findings highlight the importance of *consensus* as a motivating force within collective inquiry. That students co-constructed the inquiry question and chose their investigative approaches supports the view that authentic engagement arises when learners are invested in the epistemic aims of the task. This sense of ownership not only deepened their persistence but also fostered openness to multiple perspectives. Such openness is critical in developing an understanding that, while multiple interpretations may exist, they can be adjudicated through evidence (Chandler et al., 2002). Here, collective argumentation was not merely a pedagogical device but a means of cultivating what Sandoval et al. (2014) describe as epistemic cognition: an awareness of how knowledge is generated, justified, and contested.

Generalisability Principle

This principle requires that students attempt to communicate individual thinking about a task. Through this lesson sequence, the students worked predominantly in group settings in which all students were expected to contribute their ideas and understanding. The teacher introduced expectations that students would not only communicate their understandings but

also determine their own forms of representations to support this communication. For instance, the teacher asked students to envisage evidence to determine whether a pyramid might be able to have a scalene face. The students worked in groups and were encouraged to select their own mode (*consensus principle*) for articulating their evidence. All groups elected to create representations consisting of a combination of a built pyramid and/or a net. After determining the evidence, they required, and sharing their group plans in a whole class discussion (*recontextualization principle* below), students commenced creating their representations.

Teacher: Your first task is to work out ...what evidence you will need. ...I don't mind how you do it, whether you draw pictures or write words, but what evidence will we need to provide?

Salome: So, what would count as evidence?

Geneva: A model could. If you get a model with at least one scalene side, then it would be evidence because obviously it would be possible.

Lee: Maybe a diagram

Sadie: A model because it actually does show us.

Salome: [talking aloud as she writes] A model of a pyramid with one face that is scalene. And I like Lee's idea about a diagram. A diagram of a pyramid.

Teacher: OK a couple more minutes and I am going to ask each group to share what they have put down.

Salome: And a diagram of a pyramid with one face that is scalene.

Geneva: A net

Salome: But isn't the net the diagram?

Geneva: A measurement - a measurement on the diagram

Lee: A net of a scalene pyramid... A testable net

Salome: No, an already tested net.

From the findings, the *generalisability principle* was reflected in students' ability to apply discursive argumentation practices to novel problems, such as investigating whether a pyramid could possess a scalene face. This required students to envision and operationalise appropriate forms of evidence. The shift from general geometric knowledge to task-specific representation generation illustrates a refined capacity to apply learned structures in new contexts.

Recontextualisation Principle

This principle addresses the requirement for group members to present ideas to members outside the group for discussion and validation. The mathematical ideas the students were considering were quite complex. Children at this stage of the curriculum would normally identify pyramids by properties, including the shape of the base, the triangular faces and the apex. Building pyramids would normally be done from a provided net and these are universally equilateral or isosceles. Designing a functional net proved challenging. The teacher supported the students through convening whole class discussions to support group sharing and enable students to overcome hurdles that other groups had addressed successfully. For instance, the students, in groups, presented their ideas to the class; as a result, they realised their pyramid models weren't convincing evidence because the feedback suggested they were being forced. Students eventually became more rigorous about the design and production, producing quality nets to overcome original critiques. The enhanced evidence quality was driven by peer scrutiny.

The challenges students encountered in producing viable pyramid representations further activated the *recontextualisation principle*, with the teacher leveraging this difficulty as a productive site for learning. Rather than intervening to correct student efforts, she facilitated peer critique as a central mechanism for refining evidence. Through these exchanges, students

came to recognise the inadequacy of initial models and iteratively improved them in response—mirroring the communal validation mechanisms of scientific inquiry (Osborne et al., 2004).

Other Observations

Evident in the data was the role the teacher played in the development of students shared language of both the discipline and argumentation to support communication. By doing so, she facilitated a common language to support students to effectively communicate, formulate and deliver the argument. The necessity to repurpose their informal language (and learning) and develop and use formal mathematical discourse was a prerequisite for the development and support of collective mathematical argument. This was established at the outset and conclusion with whole class discussions of argumentation terms and ideas, and through consistent use of the terminology throughout. The use of argumentation and mathematical language was present throughout, with the teacher often revoicing student comments, such as when a student referred to faces meeting at the ‘top’ and the teacher repeating but reframing ‘the top’ as ‘apex’. The teacher drew the students’ attention to specific vocabulary on many occasions (e.g., when discussing properties of triangles and pyramids). By adopting discipline and argumentation specific language, the students were supported in their communicative competence by having the language needed to articulate their ideas specifically. We have tentatively named this the *Discursive Knowledge Principle*. The teacher’s approach exemplifies how epistemic and discursive practices are mutually reinforcing: as students acquired the disciplinary and argumentation language, their capacity to engage in collective reasoning also deepened.

Implications and Conclusions

Despite the many affordances of mathematical argumentation, there are still limited studies of argument-based learning practices in primary mathematics (Krummenhauer, et al., 2022). Specifically, there are few studies on teachers’ instructional approaches and ways in which teachers can support development of student argumentation practices (Campbell et al., 2020).

Our findings affirm the transformative potential of collective argumentation when intentionally scaffolded within a rich disciplinary context. Through the implementation of collective argumentation principles, this research illuminates how students can develop epistemic agency in mathematics, fostering not only conceptual understanding but also the communicative competencies central to authentic argumentation. We further proposed the discursive knowledge principle as foundational in enabling this development, particularly in bridging everyday language with disciplinary and argumentation language. Our findings support that argumentation is not only a cognitive or structural task, but also a socially situated practice mediated by language. Thus, implications for teaching underscore the need for teachers to explicitly scaffold both the epistemic processes and the discursive tools required for constructing, evaluating, and communicating arguments.

Future research is warranted to investigate how students’ discursive and epistemic practices co-develop over time, particularly in classrooms that value student agency and collaborative reasoning. Specifically, studies could examine how the development of disciplinary language and argumentation practices evolve in students across time and tasks. Longitudinal studies might further explore how students learn to internalise argumentation-specific language (e.g., claim, evidence, reasoning, rebuttal, qualifier) in relation to their capacity to engage in argument evaluation and construction. Such research could illuminate how discursive fluency supports deeper disciplinary argumentation mastery. In addition, future work could explore the multilingual and multimodal dimensions of discursive knowledge to investigate how representations, material artefacts, and translanguaging practices might contribute to students’ capacity to engage in collective mathematical argumentation.

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Evaluating Student Engagement With and Perceptions of a Flipped Classroom Design for a Large Statistics Subject

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We present the preliminary results from a project investigating a large statistics class designed for and taught using a flipped classroom model, with pre-recorded videos. The study, undertaken in 2021 during the Covid-19 pandemic, utilised student surveys in conjunction with metadata on their engagement with electronic resources. This preliminary investigation focuses on the quantitative aspects of the study. We aimed to identify which particular resources or activities were most beneficial to student learning. Overall, all engagement with the course, whether assessed or not assessed, contributed positively to students' learning.

Introduction

Active learning, where students are participants in the learning process, has been shown to be beneficial in a STEM (science, technology, engineering and mathematics) context (Freeman et al., 2014), and also specifically in statistics (Kalaian & Kasim, 2014). This occurs through focusing the responsibility of learning on the learners, and engaging students in deep learning along with thinking about what they are doing and learning. One model to increase active learning when teaching staff are present is a “flipped classroom” model (see e.g., Brame, 2013).

A typical classroom dynamic at our university involves content delivery during lectures, active learning during small-group classes (typically tutorials or computer labs) through working on questions with peers and supported by a tutor, and additional practice done individually. The “flipped classroom” model used in the course for this study reverses this dynamic, through delivering the content as short videos and quizzes completed individually, prior to attending classes, followed by active learning in an interactive lecture, tutorial and a computer lab. This allows students to work through content at their own pace, and students benefit from peer and staff interaction while completing higher level tasks. The interactive lectures are largely inspired by Eric Mazur's (see e.g., Crouch & Mazur, 2001) “peer instruction”, using structured multiple-choice questions, combined with peer discussion, in a lecture environment.

Evaluation of the flipped classroom

Several studies have evaluated flipped learning in tertiary mathematics and statistics courses. Commonly these have been quasi-experimental designs comparing the flipped classroom in one semester with a traditionally taught version in another (see e.g., Cilli-Turner, 2015), or parallel classes in the same semester each taught using either a traditional or flipped classroom design (Gundlach et al., 2015; Guerro et al., 2015). The results from these studies are mixed as to the effectiveness of a flipped classroom for educational outcomes, as measured by final grade. They are also limited by confounding factors such as time (different semesters), student agency in selecting which teaching model they enrol in, and differences in teaching staff. Even careful study design cannot eliminate some key issues identified in previous studies, such as differing staff expertise in teaching the two models (see e.g., Simmons et al., 2020).

Unlike previous research, for this study there was no natural comparison to a previous or concurrent iteration of the course taught traditionally, since the course was designed for and has only been taught using the flipped classroom model. Staff have also been able to refine the course and their teaching over time, with these data being collected the fifth time the course (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 141–148). Canberra: MERGA.

was taught. With an established course designed specifically for this teaching model, and staff who have had opportunity to develop their skills in delivering these classes, our focus was on identifying areas for future refinement and development.

Some studies have undertaken a more fine-grained evaluation of flipped learning. Survey-based studies typically have small sample sizes of less than 40 participants (see e.g., Ng et al., 2019; Johnston, 2017). Ng and colleagues (2019) attempted to evaluate the time students spent on various tasks in a course (including video and quizzes) and the students' perception of these tasks. The study was limited by a small sample size (15 students of 19 undertaking the course), only used student estimates of time spent on various tasks, and all student responses fell within a very narrow range (hence comparisons between components were not possible). Our project aimed to examine which aspects of a flipped learning model are effective for learning, using a mixture of metadata and survey responses. The research questions examined were:

- Which activities and/or resources are the most beneficial to students' success?
- Which activities and/or resources do students perceive to be beneficial to their learning in this course?

Determining the most beneficial activities and resources would provide evidence on how to improve the implementation of a flipped learning model for large-cohort teaching that uses video-based asynchronous learning. Success/effectiveness was defined here in terms of overall grades. Students' confidence in statistics and perceptions of learning were also investigated; however, these were not considered appropriate for the analysis presented here due to a lack of data for the statistical methods used for this preliminary analysis.

Research Design

Overview of methodology

Mixed-methods approaches are a powerful tool for education research (see e.g., Johnson & Ongwuegbuzie, 2003) and we sought to combine quantitative and qualitative methods suited to exploring the diversity of student experiences of learning. The full study combines phenomenography (see e.g., Marton, 1981) with a range of quantitative methods for both exploratory analysis and hypothesis testing. This is a preliminary report of the research and focuses on the quantitative results only.

Context

The course being investigated is a second-year introductory applied statistics course with a biology pre-requisite, but no mathematics pre-requisite, except that required for entry into a science degree at our institution. The course was designed to utilise a flipped learning model and has been taught in this manner since it was created in 2017. The course is primarily taken by students studying biology, statistics, environmental engineering and data science, with a small number from other fields. In 2021, 188 students completed the course.

The assessment in the course has four components: weekly quizzes (5%), fortnightly tests (15%), written assignments (25%) and an exam (55%). The weekly quizzes are short (three to five multiple-choice questions), students have unlimited attempts and are given full credit for getting at least one question correct. Fortnightly tests consist of a mixture of multiple-choice questions and scaffolded calculations/data analysis; students have a single attempt (without a time limit) and are graded automatically. All other assessment items are graded by teaching staff, and the grade is contributed proportionately. Quizzes focus on the key concepts for each week and are primarily used for informing the interactive lectures; tests assess lower-level understanding and recall; assignments focus on application of data analysis and interpretation; and the exam covers conceptual and interpretation questions.

The data were collected during 2021 and were necessarily affected by the Covid-19 pandemic. The course was offered in ‘online only’ and ‘hybrid’ modes, due to a range of limitations. ‘Online only’ students attended all classes as a synchronous online activity. ‘Hybrid’ students attended on-campus tutorials and computer labs, with all interactive lectures held online. Students normally attending their classes on-campus would occasionally attend online if they were unable to attend campus (e.g. if they had symptoms of Covid-19).

Data collection

The data collection combined survey results and a range of metadata available from the learning management system used by our university.

Survey data were collected voluntarily (implied consent) on two occasions: all enrolled students were invited to complete one survey halfway through the semester (April 2021, $N=38$, 17.9% response rate), and one immediately after the end of classes (June 2021, $N=38$, 20.2% response rate). The two surveys were identical and aimed to ascertain student perceptions of the various resources (e.g. online videos) and classes (interactive lectures, tutorials, computer labs) within the course. The design of the survey questions was based on the researchers’ previous experiences with evaluation surveys; no prior piloting or validation was undertaken. These surveys asked students about their academic background, confidence with statistics and statistical thinking, how useful they found each type of class/resource, and to respond to some specific prompts about key aspects of the classes. There were also open-ended prompts which are part of the qualitative research and not further discussed here. Details of the survey variables, each corresponding to a single survey item, can be found in Table 1. As there were no derived scales used, it is not possible to evaluate the reliability of these variables.

We also collected metadata on student interactions and engagement with all electronic resources and activities, and student results data, under an opt-out ethics approval. Details of these variables observed can be found in Table 2.

Table 1

Survey Variables Measured

Short name	Question/prompt	Response type
Recordings useful	How useful did you find the pre-recorded online lecture videos?	6-point Likert
Interactives useful	How useful did you find the interactive (synchronous) lectures?	6-point Likert
Tutorials useful	How useful did you find the tutorials?	6-point Likert
Computer labs useful	How useful did you find the computer labs?	6-point Likert
Interactives helped	The interactive lectures helped me understand concepts better	5-point Likert
Tutorials helped	The tutorials helped me answer questions and apply knowledge	5-point Likert
Teaching helped	The teaching in this subject helped me to learn	5-point Likert

Statistical Analysis

A range of statistical analyses were planned, all to be implemented using standard statistical software, *R* (version 4.3.2, 2023). They belong to three distinct categories: checking for bias in sampling, applying linear (multiple regression) models to both the survey and metadata collected, and conducting principal component and cluster analysis on the metadata for the whole cohort. Data from various sources (metadata, survey results) were linked using a unique identifier created for the purpose of this study.

Table 2*Metadata Variables Measured*

Short name	Variable	Measurement
Quizzes	Quizzes completed	Number of quizzes completed (up to 10)
Assignments	Assignment score total	% awarded, average of 3 assignments
FortnightlyTests	Fortnightly test total	% awarded, average of best 5 test scores
FinalGrade	Final grade for subject	% awarded
PageViews	Accessing materials or resources via LMS	Number of times a page is viewed
Attendance	Attendance at interactive lectures	% attendance
Participations	Participate in LMS activity	Number of assessment attempts
VideoViews	Videos watched (individual videos)	Number of videos watched
VideoViewPercent	Videos watched (total length)	% of total length watched
EngageTotal	Engagement with resources (all documents)	% of resources accessed
EngageWeighted	Weighted engagement with resources (only resources still available at end of semester)	% of resources accessed
Mode	Learning mode	Online-only or hybrid

To check for bias, final grades for various cohorts were compared. Initially these were planned to be independent samples *t*-tests; however, due to skewness in the data, Mann-Whitney *U* tests were conducted instead. One concern was differences in the mode of learning, whether online-only or hybrid, as these students may have substantial differences in the ways in which they interact with the course. Another concern is response bias, necessitating comparison between students completing at least one survey, with non-responders.

Linear models are a standard technique for assessing the contributions of a large number of potential predictors to a single outcome, and have been used for similar studies previously (see e.g., Guerrero et al., 2015). The distribution of final grades in the course exhibited a negative skew; typically, this would be remedied by transformation of the data. Diagnostic plots for the fitted models indicated this was not necessary in this case. Models were fitted to the whole dataset (using metadata only), to the subset of students who completed the initial survey (using metadata and initial survey responses), and to the subset of students who completed the final survey (using metadata and final survey responses). In each case, data were pre-processed to standardise the predictors prior to model fitting. A stepwise process based on Akaike's Information Criterion (AIC), provides an efficient way to compare a variety of potential models, incorporating the quality of the fit (measured here using R^2 , the proportion of variability explained by the model) while remaining parsimonious (favouring fewer predictors).

Cluster analysis is a useful technique to identify any groups of students in the data, based on similarities across all the potential variables in the study. To further investigate any groups identified by this method, a principal component analysis (PCA) was performed. PCA allows the data to be viewed in a much smaller number of dimensions, through analysis of the correlations between the variables in the data. As techniques only suited to large samples, cluster analysis and PCA were employed on the metadata only. Student interactions with the course were separately analysed. This consisted of formal interactions (such as individual assessment tasks) and student metadata (including viewing pages on the LMS, watching lecture recordings, etc.) with a total of 17 variables observed for the full cohort of 212 students (which includes students for whom only partial data was available). Excluding students with partial data did not alter the findings of the following analyses, so all are included for completeness.

Results

First, the ‘online-only’ and ‘hybrid’ cohorts were compared to determine if there were differences due to the mode of delivery. No significant differences were found (Mann-Whitney $U=4284.5$, $P=0.79$) based on teaching mode. The overall sample size was large ($N=188$); however, the number of students completing the surveys were much smaller (initial survey $N=38$, final survey $N=38$, with $N=17$ students completing both surveys). No significant differences were found between those completing at least one survey and the remainder of the cohort (Mann-Whitney $U=3383.5$, $P=0.27$). The small number ($N=17$) completing both surveys limited the ability to fit models utilising data from both surveys and metadata as predictors.

Linear models

Prior to fitting models, correlations between all variables were assessed. Individually, each predictor was positively correlated with the overall grade (correlations ranging from 0.211 to 0.763), as well as all predictors being positively correlated with each other (correlations from 0.199 to 0.986). With the large number of variables (21 for the metadata only, 38 each for those incorporating survey results) individual reporting of the correlations and their significance is not insightful. A stepwise process using AIC identifies the variables which provide the most information to predict the grade, while omitting those which are redundant. Linear (multiple regression) models were fitted to the metadata alone (all students who did not opt out, $N=188$) and also to the metadata combined with the survey data (only the students who voluntarily completed the surveys, $N=38$ in each case). This identified models with reasonable predictive power for all three of these scenarios. The models are summarised below in Table 3.

Table 3

Coefficients for Predictors in Best Models for the Complete Dataset and Two Subsets.

	<i>N</i>	<i>R</i> ²	Predictors in best model identified	Coefficient
Metadata only	188	62.8%	Assignments	1.37****
			FortnightlyTests	1.16****
			Quizzes	-2.68***
			VideoViewPercent	0.063*
Metadata + initial survey	38	75.3%	Assignments	3.04****
			Tutorials helped	-4.88**
			FortnightlyTests	-3.00**
			Quizzes	-4.86**
			Attendance	-14.7*
			VideoViewPercent	-14.3*
			EngageTotal	86.9*
			VideoViews	35.4*
			+ 7 others ($P>0.1$)	
Metadata + final survey	38	59.7%	Assignments	1.27***
			Teaching helped	5.42*
			Tutorials useful	3.09*
			Recordings useful	-2.86*
			+ 11 others ($P>0.1$)	

Predictors ordered by significance; level of significance indicated: **** $p<0.001$, *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

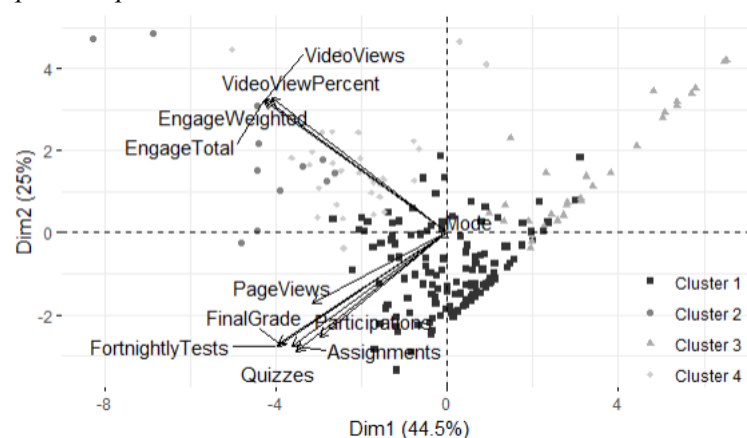
The most salient information in Table 3 is the predictors identified. The exact number of predictors is not important, as there is a tendency for overfitting on small datasets (partially mitigated by utilising a stepwise AIC process). Coefficients are provided for completeness; however, due to the high collinearity between all predictors, some coefficients are negative even though all predictors are positively related to the final score.

Cluster and principal component analysis

Principal component analysis (PCA) is an alternative analysis of the correlations between variables in a manner which complements the linear model analysis above. Cluster analysis found four distinct clusters of individuals: three larger groups (with 139, 30 and 31 individuals) and one smaller cluster (12 students). PCA was used to characterise the clusters, identifying two components being substantially more influential than all others in these data, accounting for a combined 60.1% of the variability in the data. Components based on a slightly simplified model (combining assessment items of the same type) showing the same relationships and accounting for a slightly higher 69.5% of the variability are presented here, to reduce clutter and enable easier interpretation of these graphs. The relationship between the observed variables, the components and the clusters can be seen in Figure 1.

Figure 1

Biplot Showing Principal Components and Clusters



Note. This figure shows the relationship between the clusters and the variables, using the principal components to form a two-dimensional view of the data. Points are individual students, and the arrows indicate the contribution of observed variables to the components.

The PCA in Figure 1 shows two clear groups of variables; within these groupings, variables are highly correlated with each other. The first group, towards the lower-left of Figure 1, includes all assessment items and general engagement with the course (such as number of page views on the LMS, number of assessment attempts/submissions). Students in this direction tended to do these activities more frequently than average, whereas those in the opposite direction (towards the upper-right) are those who did so less frequently than average. The second group, towards the upper-left of Figure 1, are more specific measures of engagement with content on the LMS: watching videos, and engagement with other resources. Considering both the clusters and the PCA, as can be seen in Figure 1, it appears that Cluster 1 (the largest, $N=139$) is students who were generally engaged in the course and assessment, but tended to engage less with resources and videos. Cluster 2 (the smallest cluster, $N=12$), is a group of students who heavily engaged with both assessment and all online resources and videos. Cluster 3 ($N=30$) appears to be students who tended not to engage with any aspects of the course. Finally, Cluster 4 ($N=31$) are students who tended to utilise the online resources and videos more than average, alongside below average general engagement and participation in assessment. Further work is required to fully understand these clusters.

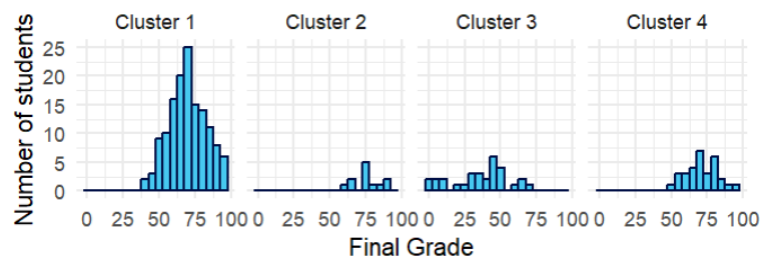
As with previous analyses, students' mode of learning (online or hybrid) was not found to have any significant impact. This can be seen in Figure 1, where Mode does not contribute to either component (in the centre of the graph near (0,0)).

Preliminary investigations of the clusters have identified that one group (Cluster 3) primarily consists of students who did not achieve high grades overall and very few obtained a

passing grade, as seen in Figure 2. These students submitted very few assessment items, viewed the LMS fewer times, and did not watch lecture recordings. This accords with previous research (see e.g., Freeman et al., 2014; Barlow & Brown, 2020), linking disengaged students with lower achievement. The differences between the other clusters show that there are a range of ways for students to successfully engage with the subject: this complicates the aim of this research to determine the most beneficial activities, as some students will benefit more than others.

Figure 2

Comparison of Final Grade for Students within the Clusters



Discussion

Positive correlations between all measures of assessment and engagement indicates that all activity within the course tended to be positively related to the students' overall grade. This does indicate that all activities and assessments were of some value to students. When considering the complete dataset, it is notable that, except for assessment, only the video percentage completed was identified as a significant predictor, although this relationship was weak. Video percentage is perhaps the most direct measure of progress through the course content available in the metadata. Notably this does not measure whether or not students are engaged with (or even watching) the videos, which is a limitation of the available data. The principal component analysis reinforces these findings, where there are two primary dimensions: one relating to engagement with assessment and the course more generally, and a second which relates to watching (or re-watching) lecture videos.

At face value, the clusters identified appear to group students based on behaviours: assessment focused students who view the LMS reasonably frequently (cluster 1); students who are highly engaged in all aspects of the course (cluster 2); students who disengage from most aspects of the course (cluster 3); and students who use a lot of resources but tend to engage less (cluster 4). The results presented here suggest this is a reasonable interpretation, but do not support this as a conclusion. Further work, such as considering engagement patterns throughout the semester rather than overall, will enable this to be investigated more fully. Understanding the differences between these groups will help with identifying which students are most likely to benefit from improving specific resources.

The primary aim of this research was to identify which aspects of the course are most beneficial. Assessed items were generally highly relevant to doing well. This is not surprising, as the final grade is at least partially composed of these results. The weekly quizzes, very low stakes assessments which are not scored proportionately but only on completion, were generally as useful as other assessment predictors, indicating that completion of regular tasks is beneficial to student learning. Notably, static resources and videos were not heavily used by most students (particularly the largest group, cluster 1). Given the substantial time and resources required to produce these relative to other learning aides, extensive work on these is unlikely to be of as much benefit. This is not a strong recommendation, as it is still unknown whether this may systematically disadvantage some students who tend to use these resources more.

The results incorporating the final survey demonstrate that students who had positive experiences of the teaching in the course (by expressing agreement with "The tutorials helped

me answer questions and apply knowledge” and “The teaching in this [course] helped me to learn”) performed better on average. This could be due to different reasons, such as students who liked the flipped classroom design being more engaged, or simply that students who responded well to the teaching both learned more and had more positive experiences. Analysis of student comments may be able to elucidate this further – we have chosen to omit any comments here in the absence of a full and rigorous analysis. The current study is only the first step in analysing these extensive data. Future plans for this research include a range of qualitative and quantitative analyses, including: further comparisons based on highly engaged/disengaged student groups; student confidence with statistics as an alternative outcome measure; analysis of student comments as to which aspects of the course they found most useful; and analysis of consistency in completing activities over the course of the semester.

Ethics Statement

Ethics approval (Project ID 13665) was granted by the University of Melbourne, and participants gave informed consent.

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Examining Gender Differences in the Impact of Instructional Clarity on Year 9 Students' Mathematics Interest in New Zealand: A Multi-Group Comparison of 2019 TIMSS Data

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Grounded in Social Cognitive Career Theory (SCCT), this study explores how teacher instruction clarity impacts Year 9 students' mathematics interest through the mediating roles of confidence and value, with a focus on gender differences. Structural equation modelling applied to the 2019 New Zealand TIMSS dataset (2,594 girls and 2,651 boys) supports the SCCT framework. While confidence and value have similar mediating effects for boys, confidence plays a more pivotal role for girls. Gender differences are primarily observed in the value-mediated pathway, underscoring persistent challenges to achieving educational equity through teachers' instruction in New Zealand.

Gender disparities in New Zealand's mathematics-intensive fields remain a persistent issue. While the proportion of female graduates in NZ tertiary education has reached an impressive 61.7%, surpassing that of their male counterparts, women's representation in STEM (Science, Technology, Engineering, and Mathematics) fields remains disproportionately low at only 39.9% (Organisation for Economic Co-operation and Development [OECD], 2023). This gender segregation trend further extends to the occupation domain, as exemplified by the University of Auckland (NZ), where female faculty members in the Faculty of Science and Engineering are significantly underrepresented compared to their male colleagues, particularly in engineering disciplines (Brower & James, 2020). Without effective intervention, the current gender imbalance may persist until 2070 (Brower & James, 2020), hindering the advancement of gender equality and posing severe challenges to the cultivation of a diverse talent pool.

Social Cognitive Career Theory (SCCT) offers a comprehensive theoretical framework for explaining gender differences in career choices. The theory posits that interest serves as a crucial source for individuals to set career goals; thus, investigating the formation mechanism of interest is paramount in unveiling gender disparities in the career decision-making process. According to the interest model of SCCT, self-efficacy and outcome expectations are the core social cognitive variables that influence interest formation. Self-efficacy pertains to an individual's belief in their capability to accomplish specific tasks, whereas outcome expectations refer to an individual's anticipation of the potential outcomes of particular behaviours and their assessment of the value associated with these outcomes (Lent et al., 1994). Bandura (1977) emphasised that self-efficacy typically wields a more potent influence on behavioural decisions. Furthermore, given that individuals tend to anticipate ideal outcomes in activities they perceive themselves to be proficient in, outcome expectations are partially determined by self-efficacy. Contextual and experiential factors, as antecedents of these social cognitive beliefs, exert direct or indirect effects on interest formation (Lent et al., 1994). Specifically, these extra-personal factors not only provide information and resources related to career development but also shape individuals' beliefs, thereby influencing the trajectory of interest development. In the process of interest formation, gender is considered a key moderating variable. SCCT posits that gender is not merely a biological attribute but also a characteristic formed through social construction. The socialisation of gender roles renders

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individuals' exposure to career-related experiences and information selective, resulting in divergent paths of influence for boys and girls in the interest formation process.

Many empirical studies support the explanations of the SCCT's interest model. Research indicates that girls' choices of STEM pathways are significantly influenced by their interest in these fields (Blotnicky et al., 2018). Moreover, the development of individuals' interest in STEM careers is directly affected by their self-efficacy and values regarding STEM occupations and indirectly influenced by environmental factors such as family, peers, and school (Turner et al., 2019). For instance, Wang et al. (2023) validated the applicability of the interest model in SCCT within the Chinese cultural context, demonstrating that self-efficacy and individuals' perceptions of career prospects, required skills, and self-development significantly enhance their interest in STEM fields. They also found that formal or informal learning experiences and social support indirectly influence the development of interest by enhancing self-efficacy and perceptions of STEM careers. Additionally, their study highlighted the role of sociocultural factors in the gender differences observed in interest development, suggesting that self-efficacy may play a more important role in the formation of girls' interest in STEM, particularly in cultural backgrounds where gender role stereotypes are more prominent.

School factors, as framed by SCCT, have increasingly become a focal point in educational research. Given that adolescents spend a substantial amount of time in school, teachers as critical agents between students and schools, significantly influence students' learning experience and motivation. Research demonstrates that teachers have the capacity to foster positive perceptions of STEM fields among girls (Smith & Evans, 2024), and that the quality of teachers' instruction is closely associated with students' learning motivation and academic performance (Ekmekci & Serrano, 2022). Moreover, effective pedagogical strategies provided by educators create conducive learning environments that enhance girls' mathematical interests (Kang & Keinonen, 2017).

Instructional clarity has been recognised as a crucial aspect of effective teaching, particularly in mathematics, where clear instruction is linked to variations in students' learning outcomes. For example, a comparative analysis of TIMSS 2019 data from Hong Kong and England was conducted by Chen and Lu (2022), revealing that instructional clarity not only improves students' enjoyment and achievement in mathematics but also reduces boredom in both jurisdictions. Likewise, Zhang et al. (2025) utilised 2019 Australia TIMSS data and discovered a significant positive correlation between instructional clarity and students' mathematics achievement, and further demonstrated that teacher instructional clarity could positively predict learning motivation. Moreover, Titsworth et al. (2015) conducted two meta-analyses, which highlighted the overall moderate positive impact of teachers' instructional clarity on students' affective learning. Nevertheless, the high heterogeneity observed in Titsworth et al.'s (2015) results suggested that contextual or individual factors may moderate the impact of instructional clarity. Notably, gender is widely acknowledged as a key moderating variable in TIMSS studies, with existing research consistently demonstrating the systematic influence of gender differences on students' learning attitudes and motivation, particularly with boys exhibiting higher confidence and intrinsic values in mathematics (Michaelides et al., 2019; Watt et al., 2012).

Building on these insights, the present study adopts the SCCT framework to examine the role of instructional clarity in fostering students' interest in mathematics. It further investigates whether there are gender differences in the pathways through which teachers' instructional clarity influences students' mathematics interests. By elucidating the critical role of instructional clarity in shaping learning motivation and promoting gender equity, this study aims to provide empirical evidence for optimising mathematics teaching practices.

Additionally, it holds the potential to uncover the underlying mechanisms that drive students' academic choices.

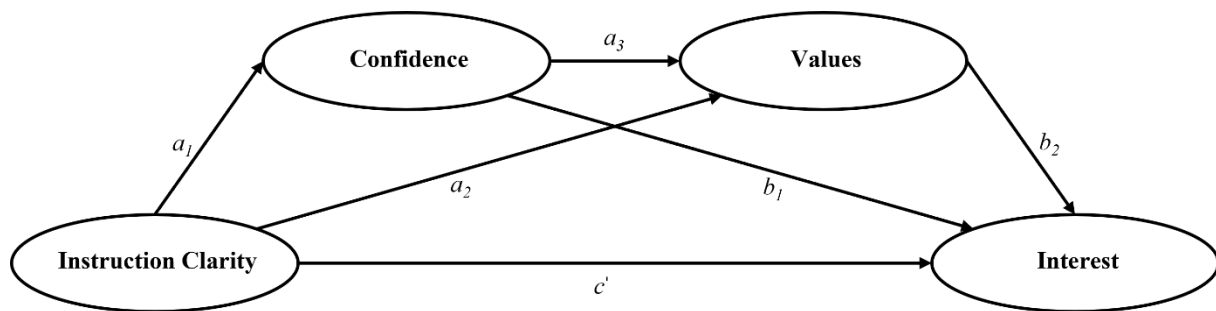
Research Questions

Guided by the SCCT, this study constructs a hypothetical model, as shown in Figure 1 and focuses on the following three core research questions:

- What are the specific mechanisms through which instruction clarity influences students' mathematics interests via confidence and values?
- Among the three mediating paths shown in Figure 1 (Instruction Clarity → Confidence → Interest; Instruction Clarity → Value → Interest; Instruction Clarity → Confidence → Value → Interest), which path has the most significant impact on boys' mathematics interest? Which path has the most significant impact on girls' mathematics interests?
- Are there significant gender differences in the three mediating paths shown in Figure 1? If such differences exist, in which mediating path are they most pronounced?

Figure 1

The Hypothetical Model Illustrating the Relation between Instruction Clarity and Mathematics Interest



Methodology

Participants

The study comprised 2,594 Year 9 girls and 2,651 Year 9 boys in the first year of New Zealand secondary education from the publicly available 2019 TIMSS dataset. TIMSS 2019 collected contextual information through the administration of questionnaires to students, their teachers, and principals.

Instruments

The 2019 TIMSS background questionnaires collected data on students' attitudes and motivation toward mathematics study, employing a theoretical framework grounded in psychological constructs, including latent variables such as confidence, interest, value, and instructional clarity which students perceive in mathematics classes. Specifically, instructional clarity was measured through five four-point Likert-type items in the 2019 TIMSS, assessing the clarity of instruction provided by teachers during mathematics lessons from students' perspectives, which were aggregated into a scale termed "Instructional Clarity" in this study. Students' confidence in mathematics was measured via six four-point Likert-type items, constituting a scale variable termed "Mathematics Confidence". Six questionnaire items captured students' mathematical values, forming a scale variable termed "Mathematics Value". Additionally, interest in mathematics was assessed through seven four-point Likert-type items evaluating students' enjoyment in learning mathematics, which were combined to create the scale variable termed "Mathematics Interest". To facilitate data processing, reverse scoring was applied to some items, ensuring that higher scores reflected higher levels of the measured items.

These four selected scales from the 2019 TIMSS can be interpreted within the SCCT framework, and their reliability is described in detail below:

The *Instruction Clarity (IC)* scale consists of five items: a. I know what my teacher expects me to do, b. My teacher is easy to understand, c. My teacher has clear answers to my questions, d. My teacher is good at explaining mathematics, e. My teacher explains a topic again when we don't understand. The internal consistency reliability of the scale was assessed using Cronbach's α coefficient, yielding a result of 0.895, indicating high reliability.

The *Mathematics Confidence (MC)* scale comprises six items: a. Mathematics is more difficult for me than for many of my classmates, b. Mathematics is not one of my strengths, c. Mathematics makes me nervous, d. My teacher tells me I am good at mathematics, e. Mathematics is harder for me than any other subject, f. Mathematics makes me confused. The Cronbach's α for this scale was 0.828, indicating good reliability.

The *Mathematics Value (MV)* scale includes six items: a. I need mathematics to learn other school subjects, b. I need to do well in mathematics to get into the university of my choice, c. I would like a job that involves using mathematics, d. It is important to learn about mathematics to get ahead in the world, e. Learning mathematics will give me more job opportunities when I am an adult, f. My parents think that it is important that I do well in mathematics. The Cronbach's α for this scale was 0.848, indicating good reliability.

The *Mathematics Interest (MI)* scale consists of seven items: a. I enjoy learning mathematics, b. I wish I did not have to study mathematics, c. I learn many interesting things in mathematics, d. I like mathematics, e. I like to solve mathematics problems, f. I look forward to mathematics class, g. Mathematics is one of my favourite subjects. The Cronbach's α for this scale was 0.930, indicating good reliability.

Method

To investigate the relations among the four latent variables, "Instruction Clarity" (IC), "Mathematics Confidence" (MC), "Mathematics Values" (MV), and "Mathematics Interest" (MI), this study employed a multiple mediator model within the framework of structural equation model using SPSS 30.0 and AMOS 30.0 software. This method offers several notable advantages (Fang et al., 2014). First, it enables the examination of the specific mediation effect of each mediator variable while controlling for other mediators, such as a_1b_1 , a_2b_2 , and $a_1a_3b_2$. Second, it provides contrast mediation effects, such as $a_1a_3b_2 - a_2b_2$, $a_1b_1 - a_2b_2$, and $a_1a_3b_2 - a_1b_1$, allowing researchers to determine which mediator variable has a stronger influence.

Results

The study assessed model fit across genders using separate samples, yielding acceptable fit indices (For boys: $\chi^2 = 2700.056$, RMSEA = 0.060, CFI = 0.940; For girls: $\chi^2 = 2796.916$, RMSEA = 0.060, CFI = 0.930) that met conventional standards (West et al., 2012). A smaller χ^2 value suggests a better model fit. The RMSEA measures the approximate error of the model fit, with values below 0.08 indicating a good fit. The CFI compares the model to a baseline (worst-fit) model, with values closer to 1 indicating a better fit.

Subsequently, a cross-gender measurement invariance analysis was conducted to examine the consistency of the questionnaire structure across different gender groups. Given this study did not involve mean or variance comparison, the following sequence of tests was performed: (1) *Configural Invariance*, ensuring that different groups have the same factor structure, including consistent factor numbers and factor-item correspondence; (2) *Metric Invariance* or *Weak Invariance*, testing whether factor loadings are invariant across groups. The configural invariance test results for boys and girls showed acceptable levels of RMSEA (0.044) and CFI (0.934), supporting configural invariance. The metric invariance test results supported metric

invariance based on traditional criteria, with $\Delta\text{RMSEA} = 0.001$ and $\Delta\text{CFI} < 0.001$, both less than 0.01, and $\Delta\chi^2 = 61.400$, $\Delta df = 20$, $p = 0.070$, which was not significant.

After confirming the good fit of the SEM, the significance of the mediation effect was tested using the bias-corrected bootstrap method. If the confidence interval of a mediation effect does not include 0, the effect is considered significant (Fang et al., 2014). It is important to note that a significant mediation effect only indicates that it is not equal to 0, and further analysis of the mediation effect size requires comparing the mediation effects of different paths.

The analysis results showed that “IC” had a significant positive impact on students’ “MI” (boys: $c' = 0.260$, $p < 0.001$; girls: $c' = 0.270$, $p < 0.001$). Additionally, IC influenced MI through three mediation paths formed by “MC” and “MV” (see Figure 1 & Table 1). For boys, the following three mediation paths were significant: IC $\rightarrow$ MC $\rightarrow$ MI: $a_1b_1 = 0.152$; $z = 8.444$; 95% CI [0.117, 0.188], IC $\rightarrow$ MV $\rightarrow$ MI: $a_2b_2 = 0.166$; $z = 11.067$; 95% CI [0.139, 0.202], IC $\rightarrow$ MC $\rightarrow$ MV $\rightarrow$ MI: $a_1a_3b_2 = 0.017$; $z = 5.667$; 95% CI [0.011, 0.025]. For girls, the three paths were also significant: IC $\rightarrow$ MC $\rightarrow$ MI: $a_1b_1 = 0.191$; $z = 11.235$; 95% CI [0.161, 0.229], IC $\rightarrow$ MV $\rightarrow$ MI: $a_2b_2 = 0.108$; $z = 8.308$; 95% CI [0.085, 0.136], IC $\rightarrow$ MC $\rightarrow$ MV $\rightarrow$ MI: $a_1a_3b_2 = 0.022$; $z = 5.500$; 95% CI [0.016, 0.031]. When comparing the mediation effects, it was found that the difference between the mediation effects of MC and MV was not significant ($z = 0.560$, 95% CI [-0.033, 0.066]) among boys. However, in the girls’ group, the mediation effect of MC was significantly greater than that of MV ($z = -3.609$, 95% CI [-0.128, -0.038]). Furthermore, in the gender difference analysis, only the mediation effect of MV showed a significant gender difference, with the mediation effect of MV being significantly greater among boys compared to girls ($z = -2.900$, 95% CI [-0.101, -0.019]).

Table 1

SEM Results of Multiple Mediation Effects of Instruction Clarity on Mathematics Interest

Gender	Mediation paths	Point estimate	SE	z	Bias-corrected percentile	
					Lower	Upper
Boys	IC $\rightarrow$ MC $\rightarrow$ MI	0.152	0.018	8.444	0.117	0.188
	IC $\rightarrow$ MV $\rightarrow$ MI	0.166	0.015	11.067	0.139	0.202
	IC $\rightarrow$ MC $\rightarrow$ MV $\rightarrow$ MI	0.017	0.003	5.667	0.011	0.025
Girls	IC $\rightarrow$ MC $\rightarrow$ MI	0.191	0.017	11.235	0.161	0.229
	IC $\rightarrow$ MV $\rightarrow$ MI	0.108	0.013	8.308	0.085	0.136
	IC $\rightarrow$ MC $\rightarrow$ MV $\rightarrow$ MI	0.022	0.004	5.500	0.016	0.031
Contrast Mediation						
Boys	MC – MCMV	0.135	0.016	8.438	0.104	0.169
	MV – MC	0.014	0.025	0.56	-0.033	0.066
	MV – MCMV	0.149	0.016	9.313	0.122	0.184
Girls	MC – MCMV	0.169	0.016	10.563	0.142	0.205
	MV – MCMV	0.086	0.014	6.143	0.06	0.115
	MV – MC	-0.083	0.023	-3.609	-0.128	-0.038
Girls – Boys	MC	0.040	0.025	1.600	-0.011	0.086
	MV	-0.058	0.020	-2.900	-0.101	-0.019
	MCMV	0.005	0.005	1.000	-0.005	0.015

Discussion

The Role of Instructional Clarity in Enhancing Students' Mathematics Interest

The present study revealed that in the New Zealand context, teacher instructional clarity not only directly enhances students' interest in mathematics but also indirectly fosters their interest by increasing their confidence and perceived value of mathematics. Notably, the improvement in the perceived value of mathematics is, to some extent, dependent on the enhancement of confidence. This finding supports the theoretical assumptions of SCCT regarding the mechanisms of interest development and is consistent with the conclusions of the majority of existing empirical studies (Wang et al., 2023). However, Cheung and Sonkqayi (2025) suggested that in Asian regions, instructional clarity exerts negative effects on students' science performance. The conflicting conclusions about the impact of teacher instructional clarity on the students' learning outcomes may be attributable to differing teaching styles shaped by distinct sociocultural contexts. In Asian regions, the teacher-centred instructional approach is prevalent, enabling educators to guide students regularly in solving academic problems. In contrast, countries such as New Zealand and Australia advocate for a student-centred approach, which typically involves less teacher intervention. Moreover, Niu et al. (2025) demonstrated that in countries with student-centred learning environments, teacher guidance significantly enhances students' academic performance compared to those in teacher-centred education systems. Therefore, within the educational setting of New Zealand, teachers' clear instruction and support may be more conducive to enhancing students' interest and achievement. It is essential that mathematics teachers in New Zealand should actively engage in relevant training programs and seek professional development support to provide students with more structured and clear guidance, which could effectively promote their interest in mathematics.

The Gender-Specific Mediating Patterns in the Development of Mathematics Interest

The present study further uncovered gender-differentiated mediation mechanisms, whereby confidence and values contribute in distinct ways to the development of interest. Specifically, confidence and values exhibit similar mediating effects in fostering interest among boys. However, for girls, the mediating effect of confidence is significantly stronger than that of values. These findings in our study are in accord with the majority of previous research, indicating that confidence plays a more important role in girls' mathematical interests, especially in sociocultural contexts with strong gender role stereotypes (Wang et al., 2023). Traditional gender role stereotypes often frame mathematics as a "male-dominated domain", which may unconsciously lead teachers to have lower expectations for girls in mathematics (Smith & Evans, 2024). Research also indicates that in coeducational learning environments, girls interact with teachers less frequently than boys (Smith & Evans, 2024). Given girls' heightened sensitivity to social interaction cues in the classroom (Reinholz et al., 2022), this disparity in interaction frequency makes girls more likely to perceive and internalise teachers' lower expectations in mathematics classes. However, confidence in mathematics may help girls develop intrinsic beliefs in their abilities, which serves as a buffer against the negative effects of gender role stereotypes or lower expectations from educators. This confidence strengthens their self-identity, enabling them to sustain their enthusiasm and commitment to mathematics even when faced with societal biases. Considering the importance of confidence in nurturing girls' interest in mathematics, teachers should be mindful of girls' psychological needs in the classroom. By creating a positive and inclusive learning environment, providing constructive feedback and encouragement, teachers can effectively cultivate girls' confidence and further stimulate their interest in mathematics.

The Gender-Differentiated Mediating Pathways in the Development of Mathematics Interest

While boys and girls demonstrate different mediating mechanisms in shaping their interest in mathematics, our study revealed that the most salient gender-specific distinction lies in their relative dependence on values rather than confidence. Specifically, the mediating role of confidence between instructional clarity and interest remains consistent across genders. However, the mediating role of values is markedly more pronounced among boys. The similar weights of confidence as a mediating factor between boys and girls suggest the relative stability of confidence in fostering mathematics interest, aligning with Bandura's (1977) assertion that robust confidence is a uniformly accurate predictor of performance under varying challenges. Conversely, the observed gender differences in the mediating effects of values on students' interest in mathematics imply that teachers' instruction may inadvertently perpetuate educational inequities in New Zealand. This finding is consistent with evidence from studies on high school students in the United States, where a disconnect between girls' values and interests in mathematics has been observed, suggesting that girls may rely less on values than boys in the development of mathematical interest (Wang et al., 2015). A plausible explanation for this gender-specific disparity in the development of interest may reside in teachers' implicit gender role beliefs, which may inadvertently foster more supportive mathematical environments for boys, thereby reinforcing the mediating role of value in shaping boys' mathematics interests. These findings in the present study highlight the potential limitations of teachers' instructional strategies in mitigating gender disparities in the formation of mathematics interests. As such, it is imperative that educators recognise and address gender differences through targeted interventions during the instructional process in New Zealand.

Conclusion

This study employed a multiple mediation model to investigate the mediating roles of confidence and value in the relationship between teacher instructional clarity and students' mathematical interest among boys and girls in their first year of secondary school in New Zealand. The findings revealed that clear instruction from teachers not only directly enhances students' interest in mathematics but also indirectly fosters their interest through the mediating effects of confidence and value. Notably, confidence emerges as a critical mediator for girls, while the primary gender difference lies in the mediating effect of value. Specifically, the value boys attach to mathematics appears to play a more prominent role in fostering their interest than it does for girls. These results highlight that, while teacher instructional clarity can effectively promote students' interest in mathematics, gender role stereotypes embedded in teachers' expectations may be transmitted to students through classroom interactions, thereby influencing students' beliefs and undermining educational equity. However, it is imperative to recognise that the influence of gender roles may vary across different sociocultural contexts. The findings of this study are specific to the New Zealand context, and further empirical research is required to confirm their generalisability.

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The Role of Compassion and Empathy When Engaging With Tasks Concerned With Social and Environmental Justice

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In this paper, we raise questions about the role of student empathy and compassion when engaging with social and environmental justice issues — significant in a world where challenges associated with such disruptions are faced daily. We draw on data from a larger, nationally funded study on enabling students' critical mathematical thinking. These data were collected in a Year 10 classroom, where a task related to the current Australian housing crisis was implemented. Data were analysed using Geiger et al.'s critical mathematical thinking framework. Findings suggest that implementation of such tasks must include elements that promote student empathy and compassion.

Introduction

There is growing realisation that students must be empowered to understand and respond to the challenges of the Anthropocene (Watson & Smith, 2022), including the impacts of global warming and social inequality. This empowerment is crucial when forming judgements and making decisions as informed, responsible, and critical citizens (Geiger et al., 2023b). Such judgements and decisions should not be a product of rationality alone but must account for consequences that impact humans, other species, and the planet more broadly (e.g., Geiger, in press)—requiring thinking for the greater good. Engaging such thinking involves the capacity to empathise (understanding others' experiences/perspectives) and feel compassion (concern for others' suffering and desire to alleviate it) (e.g., Blue et al., 2018), as responsible decision-making draws on an entanglement of rational, empathetic, and compassionate practices. Further, research in mathematics education has highlighted the entailment of emotions and emotionally charged beliefs (Beswick, 2018), that drawing upon students' empathy and compassion, has potential to enhance their mathematics learning. There is, however, little guidance for teachers regarding pedagogies that support students' engagement with the range of thinking and emotional capacities such thinking demands.

How teachers can support students' rational, empathetic, and compassionate practices is a key question in the development of informed and responsible decision-making capabilities that draw on mathematics. In this paper we begin a response to the following research question:

How can teachers support students' rational, empathetic, and compassionate practices when using mathematics to address tasks related to socio-ecological disruption?

In addressing these questions, we present a synthesis of relevant research literature, and describe the theoretical framework used to direct research activity, including data collection and analysis. We then outline the methodological approach including the task used to catalyse student discussion, mathematical activity, and decision-making. Finally, we reflect on the role of teachers in supporting students' rational, empathetic, and compassionate practices when using mathematics to address tasks related to socio-ecological disruption.

Compassion, Empathy, and Socio-Mathematical Norms

Responses to the consequences of socio-ecological disruptions require not only rational

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processes, for example, attention to statistical analysis and logistics, but must also accommodate the needs of citizens and society at large. This means that to respond in an equitable and inclusive way, decision-makers (e.g., citizens, governments) must have the capacity for compassion and empathy to enable understanding of the plight of others. Mathematics can have a role in both generating responses to problems related to disruptions and in mitigating impact on those affected. Compassion and empathy also have a role in both cases — developing human sensitive responses to challenges and in mitigating impact in ways that are fit-for-need. Integrating compassion and empathy into teaching/learning, however, has significant implications for teaching/learning in mathematics classrooms.

Teaching and Learning in Relation to Socio-Ecological Disruption

There has been an intensification of concern by mathematics educators about their role in social and ecological justice, which has led to a rapidly developing global dialogue about mathematics education, the living world, and their entanglements with the ecological health of the planet. The dialogue that has developed within mathematics education, as well as more generally, is visible, for example, in the United Nations Sustainable Goals of Development (United Nations, 2015), the International Commission on Mathematics Instruction Study 27 on mathematics education and the socio-ecological (e.g., Coles, 2025), and in special issues and other academic work (e.g., Chan et al., 2021).

This discussion has moved from arguing for the importance of social/environmental justice (e.g., D'Ambrosio, 2009) to how teaching/learning can promote thinking and action in response to ever evolving challenges (e.g., Geiger, in press). At the same time, Vasquez et al. (2022) found that there was a paucity of relevant school level initiatives (e.g., curriculum, textbook tasks) that addressed this issue. Despite this current gap between the likely impact of disruptions and responses of education systems, Ada (2020) has conceptualised six change landmarks for how action in mathematics education can take steps to address this need: (1) more holistic and interdisciplinary mathematics curriculum development; (2) values focused pedagogies; (3) empowerment as change agents through attention to critical problem solving; (4) classroom resources aimed at fostering sustainability focused learning experiences; (5) pedagogical approaches that promote understanding of participatory decision making; and (6) sustainability focused teaching materials that can be localised and made relevant to students. Traces of these landmarks are evident in recent publications (e.g., Coles, 2025; Geiger et al., 2025).

While research is being conducted into how mathematics education can address sustainability issues, and by association socio-ecological issues, we were not able to locate studies that placed affects, such as compassion and empathy, as factors in making decisions about how to respond to socio-ecological disruptions in ways that address the needs of those experiencing related hardship. We argue that affect is key to engaging students with teaching/learning experiences aimed at developing critical and sensitive decision making.

Connections Between Emotion and Rationality, Beliefs and Cognition

The connection between cognition and affect has been a focus of research for several decades with knowledge and beliefs regarded as almost entirely cognitive and distinct from emotions. Goldin (2002) and DeBellis and Goldin (2006) described emotions as constituting a representational system, akin to a belief system. That is, emotions also encode information. Although it can be analytically convenient to distinguish emotions (affective system) and beliefs (cognitive system), there is evidence these systems are connected (Beswick, 2011). The entailment of emotions with beliefs adds to the sum of their connections and hence to the strength with which beliefs are held and their resistance to change. Beswick (2011) argued that when highly connected, challenges to our central beliefs are likely to evoke negative emotions which can manifest as resistance. Conversely, when a challenge or conflict is resolved,

pleasurable emotions may result. Emotions influence attention and memory (Hannula, 2002) and can arise from and influence beliefs about the relative importance of particular facts. We argue that if teachers can engage students' affective system along with their cognition there is potential for learning to be deeper, more meaningful and longer lasting. Problems that require mathematical reasoning and that also evoke emotions such as compassion and empathy are a powerful means by which the learning of mathematics can be enhanced by being rendered more memorable, while at the same time supporting students' ability to integrate mathematical understanding with learning about broader societal challenges.

Theoretical Framework

The larger project in which this paper is situated included the development of the Critical Mathematical Thinking (CMT) framework (e.g., Geiger et al., 2023a). The framework was developed to identify and describe cognitive and affective aspects of the thinking employed when making informed and responsible decisions about real-world situations, considering potential social and environmental consequences. It was designed as a guide for teachers when engaging students with problematic real-world situations in which mathematics is used to seek solutions while cognisant of associated consequences. Derived from relevant research literature, its key features coalesce around five dimensions—mathematical capability, critical capability, evaluation capability, reasoning capability, and dispositions. In this paper, we extend the CMT framework to include the preparedness to empathise and show compassion (Table 2).

Table 2

Critical Mathematical Thinking (CMT) Framework

Dimensions	Elaborations
Mathematical capability	Power over language, skills, and practices of using and applying mathematics, for example, understanding mathematical concepts and principles, identifying patterns and relationships, manipulating mathematical symbols and expressions.
Critical capability	Examining ideas, drawing conclusions, clarifying meaning, identifying, and analysing arguments, awareness of informal and cultural knowledge (e.g., social, political, environmental) and their influence on conclusions, considering the consequences of decisions and the ethical assessment of their impact.
Evaluation capability	Assessing claims and arguments (e.g., assessing strength of evidence, quality of data, reliability of sources), generating questions, generating aligned problems (e.g., validity of a solution if the circumstances change).
Reasoning capability	Logical thinking, inferring, proposing, and checking hypotheses, generalising, interpreting different information sources, generating evidence-based arguments (including integrated mathematical and extra-mathematical practices).
Dispositions	Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, displaying perseverance, openness to alternative solutions, preparedness to empathise and show compassion.

Ernest (2002), for example, identified power over mathematical language, skills, and practices, and the critical capabilities needed to exercise participation in society as two domains necessary for mathematical competency. D'Ambrosio and D'Ambrosio (2013) further argued that critical citizenship requires wisdom when considering the suitability of specific mathematical solutions to real-world problems, emphasising the need to consider ethical and human centred responses to problems. Reasoning must be employed when developing evidence-based approaches to addressing problems and their consequences — integrating mathematical thinking and extra-mathematical contexts (e.g., Jeannotte & Kieran, 2017). This includes the capacity to construct logical arguments and draw valid conclusions from data. An

evaluative capability is needed to assess the strength of evidence (e.g., quality of data, reliability of sources) of claims and arguments—essential in the context of socio-ecological disruptions where mathematical evidence is often used to justify policy decisions, restrict or enhance personal and community freedoms, and shape public opinion (e.g., during the COVID 19 pandemic). This capability requires an understanding of how bias can be embedded in data collection and analysis. In addition, tasks based on disruptive phenomena are challenging, and require positive dispositions from students, who must persevere when progress is limited, and teachers, who must believe that students are capable of engaging with such tasks (Beswick, 2018). We see the preparedness to empathise with and feel compassion for those impacted by socio-ecological disruptions as essential when seeking to develop equitable, inclusive and just responses to socio-ecological problems.

Methodological Approach

The larger project used a design-based approach to develop, test, and refine theory about changes in teaching practice and student learning (e.g., Cobb et al., 2003). In this paper, we focus on the conditions that support the effective implementation of tasks that require students to engage with critical mathematical thinking—those concerned with socio-ecological issues.

Participants and Data Collection

In this paper, we report on one teacher working with their Year 10 students. The school was coeducational and located in an inner suburb of an Australian state capital city. We draw on video recordings of the classroom in which the teacher and students worked on a socio-ecological activity—the Airbnb task (described below). Students were provided with the task 24 hours prior to the 90-minute lesson in which they worked on the related problem. The lesson was conducted using an investigative approach aligned with Geiger et al.'s (2022) design and implementation framework for modelling tasks. Students were organised into groups of four to five of their choosing. The teacher provided initial impetus for the lesson by presenting the task, providing students with the opportunity to ask questions to clarify expectations about completing the task, and then allowing students to investigate the problem themselves. The teacher approached students during the lesson in response to questions, to monitor progress, and when he noticed an aspect of student thinking he believed required attention. At the end of the lesson students were brought back together for whole class discussion, catalysed by presentations from each group. The activity of student groups and the teacher were recorded via two stationary cameras and a mobile camera focused on the teacher. The comments of other students, in response to teacher questions or prompts, were also audible throughout the lesson. End of lesson presentations were recorded on one of the stationary cameras.

Task

The task was designed to provide teachers and students with the opportunity to exhibit behaviours that aligned with the CMT framework. The task consisted of text which described the situation and videos, drawn from the Australian Broadcasting Commission (ABC) website, that provided additional background. The text for task is presented in Figure 1.

Analysis

Video data were transcribed and analysed deductively using both critical and dispositional dimensions of the CMT framework as a lens. We sought instances of dialogue that provided illustrations of these dimensions in a process that led to the identification of sub-themes. As illustrative examples were identified, they were corroborated by a second member of the research team. Here we present examples of dialogue that reveal student orientations to compassion and empathy that influenced their approach to the task, and teacher interventions.

Figure 1

AirBnB and the Rental Crisis

Are short-stay properties like Airbnb the reason for the rental crisis?

The following video and media report discuss the apparent shortage of affordable rental accommodation because owners are making their properties available for short-term arrangements. This raises the concern that such properties are taken out of the pool of available housing for permanent residents.

Please discuss whether the Brisbane City Council (media report) made the right decision to impose a registration fee on properties that offer short-stay opportunities through online agents (e.g., Airbnb and Stayz) or are they overreacting? Justify your response using available information about the availability of affordable accommodation.

Investigate the rental prices and short stay accommodation in Gympie. Based on your findings suggest whether the Gympie Regional Council should implement a short stay registration fee as imposed in Brisbane. Who would be advantaged or disadvantaged? Suggest other possible avenues to alleviate the rental problems in Gympie. Justify all responses using available information.

Findings — Student Perspectives on Airbnb and the Housing Crisis

Soon after beginning the task, very different opinions emerged between one student, Wanda, and the rest of her group (three students). While three members of the group saw the imposition of a registration fee as part of a solution to the problem, Wanda argued that it would not help at all. The other members of Wanda's group expressed concern for the number of homeless people and those unable to get into the housing market, and while they conceded that a registration fee was not a complete solution, they argued that shifting rental properties from short term to long term would help alleviate the problem for at least some people in the short term. The importance of such a shift was highlighted by a point from a third group: "There are a smaller number of rental properties available on the market right now compared to Airbnb".

The teacher, Adam, picking up on this point, indicated that addressing the issue of the number of Airbnbs was a possible way to alleviate the housing crisis at least in part, but how this was done was important. A fourth group raised an objection to this idea, arguing: "If you own something, you should get to choose what is done with it!". Adam said he understood the point being made, but that the distinction between a *want* and a *need* was important. He asked the class if housing was a want or a need. The students responded unanimously that it was a need. Adam pursued his argument further, indicating that in the case of human need:

...governments can take steps to inhibit the free market and limit peoples' ability to freely do what they want with their own things...if it comes down to it, people need a house to live in...and if the government sees there is a problem with supply and a way to fix it, that doesn't ruin someone else's financial position, then maybe they do have some license to act in that way.

Adam then asked if anyone agreed or disagreed. One student objected along the same line as earlier—governments cannot tell you what to do with your property. In response, Adam directed the student to the task description, saying this indicated that landowners were not being forced to give up their property, rather, the government was implementing a measure to encourage a particular action. The student then agreed this was reasonable. Another student, from a different group, pointed out that governments had the authority to take back property anyway: "But how different is that from involuntary buy backs—like road reserves—this is currently in place". Adam agreed, saying that it was not uncommon for governments to resume land even though people have a deed of sale and that:

...we do not only have the free market for people to decide what they do with their land...there are already circumstances that the government sees it as necessary to take action to maintain the health and well-being of citizens.

Wanda, however, continued to argue that there was no point in applying a registration fee:

Every Airbnb owner will be unwilling to change to renting ... it is most likely you will make more money out of Airbnb. If they changed to rental, say for a beach side property, they would have to put up the price to keep the same profit...people who can't afford rental properties in Brisbane right now, will not be able to afford the Airbnbs that would then be in the rental market...

Adam accepted Wanda's argument, even though this was from a purely financial perspective, but asked her to flesh out the argument and to provide support via "some numbers". The teacher went on to say that students also needed to consider who required the most help from government: "...the family that owns three or four Airbnbs by the beach, because currently legislation serves them, or the people in the rental market that need a house?" The students all agreed that people in the rental market requiring a house needed the most help. Wanda, however, came back to their same point: "But if you are making a lot of money from an Airbnb, how can you give that up to someone who doesn't have a house?"

When Adam replied there are issues of justice on both sides, those who might make less income from Airbnbs but also for people who have less access to housing. He made the comparison between housing and water as a human need – and that it would be seen by everyone that water could not be tied up in a way that some did not have access. Most students conceded this point, but some still seemed reluctant to move away from the notion of housing as personal property to be used in any way the owner seemed fit. Adam asked students to continue thinking about the tension inherent in promoting justice on all sides when pushing forward with the problem. He also reminded them of the need to develop a mathematical argument to support their response to the task.

Animated discussion continued in Wanda's group. One student argued that if there was no housing, people should not be moving to their city, they should just go somewhere else. Another said that this was not fair, and that people who needed housing were not those who were moving from elsewhere but people who were already in their city. Another argued that the rental crisis was an outcome of COVID-19 and that such an event "doesn't happen very often", indicating that things would sort themselves out in the long run, and no drastic action was needed.

Adam approached the group, to check on their progress. The team members other than Wanda, who was working on her own, pointed out that 200,000 people had arrived in their city between 2019 and 2024, while only 40,000 more homes were built. They had also found that there were 3,600 Airbnb listings in their city and that even if all of these were reassigned as rentals, it would not solve the problem. They further argued that, in this circumstance, it would be better to keep the Airbnbs for tourism, with 85% occupied for 50% of the year, which put money back into the economy and supported other industries. Adam commended the students on their reasoning and how they had evaluated different aspects of the problem, but also posed the question – does that mean the government should stop doing anything? The three students said, no and that the government had to invest more into building more houses.

At this point Wanda re-entered the conversation. She presented a carefully constructed mathematical argument which indicated that a registration fee would make no difference to the growth of Airbnbs. Adam asked Wanda to check the reliability of her sources and pointed out that his reading of the information was that measures, such as registration fees, while not reducing the number of Airbnbs, had limited growth in the numbers. Undeterred, Wanda maintained her position. Wanda then joined in with other members of the group arguing against building more houses as it was already too crowded. She also asked the rest of the group if they were willing to compromise their own standards of living for someone else to get a house and suggested that the solution was to stop population growth (tongue in cheek).

Wanda continued to argue along the same lines, saying that the median income from an Airbnb was \$70,000 per year, which meant that if the property was returned to the rental market, the owners would have to increase the rent by \$2,000 a fortnight, which made it inaccessible to regular renters. The other members of the group agreed that returning all Airbnbs to the rental market was not the whole solution, but something needed to be done to deal with the rental crisis and especially for those who were homeless. They suggested possibilities such as placing a cap on the number of Airbnbs, so at least new properties were available to the rental market.

Wanda eventually agreed that it was important to address the issues associated with the rental crisis, and that housing should be seen as a right and not a privilege, but did not change her position that any attempt to address the impact of Airbnbs would make a difference to those experiencing hardship

Discussion and Conclusion

Across the lesson, student activity and discussion touched on all aspects of the CMT framework. Students demonstrated *mathematical capability* in making use of demographic data and financial information to generate mathematical arguments that supported their response to the challenges at the centre of the task. Through discussion and debate, they exercised *critical capabilities* in analysing each other's arguments and by considering the consequences of decisions. Students assessed the strength of evidence when *evaluating* claims prosecuted by individuals or groups. They also took initiative and displayed perseverance (*dispositions*) when engaging with the problem.

Adam, the teacher, attempted to connect students' thinking to the ethical assessment of the impacts of their decisions, as well as seeking ways to empathise and experience compassion for those who were disadvantaged by the current state of the housing market. His attempts to do so appeared to make some difference to most students. In Wanda's case, however, the issue seemed to remain a matter of financial impact on Airbnb owners, and she was doggedly determined to maintain her position that measures to move Airbnbs into the rental market would not be helpful – this is what the numbers said and that should be the end of the argument.

Wanda had addressed the specifics of the task, thoroughly and thoughtfully—we accept that they had responded to the task as it was written. We saw other students, both in other classes in the same school and in other schools, take the same approach as Wanda. In these cases, students also resisted prompts from their teachers to think more broadly and to exhibit empathy and compassion towards those who were experiencing hardship.

In light of the approach adopted by these students, however, we now ask ourselves questions about how tasks could be constructed in ways that might promote empathy and compassion. What aspects of task design need to be considered? Are there resources that could be included that might evoke concern for those experiencing hardship or disadvantage. For example, what difference might there be if the ABC video clip had featured people experiencing housing precarity? Might pedagogies involving role-play or otherwise having students put themselves in the place of particular characters affected by the issue evoke emotional responses? We also ask in what ways can the socio-mathematical norms (Yackel & Cobb, 1996) of classrooms be shaped to support the empathy and compassion that may push students to look further than simply completing the task, and to find ways in which mathematics can point to sensitive responses to the difficult life circumstances experienced by others. Data presented in this paper demonstrate how Adam attempted to foster such socio-mathematical norms. One of our challenges, as the project progresses, is to generate insight into how these norms can be developed in ways that are most effective and how long it might take to establish a mathematics classroom in which empathy and compassion are seen as essential ways of working.

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Using the Mathematics Assessment Interview and Growth Points to Identify Grade 1 Students Who Are Not Yet Thriving in Mathematics Midway Through the School Year

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Assessing and monitoring the mathematics progress of students is a key role for teachers. It is important for assessment tools to identify students who are not yet thriving and provide guidance for teachers about how to support these students. In our study, teachers assessed 396 Grade 1 students midway through the school year using the Mathematics Assessment Interview and framework of growth points. The findings describe a wide distribution of knowledge across the growth points in each of the number domains. Further analysis showed that at least 30% of students were vulnerable in each number domains, highlighting the need for mathematics interventions.

Most children thrive with learning mathematics when they transition to school, but not all. Teachers work hard to advance equity and inclusion for students who are not yet thriving and implement various teaching approaches and curricula to assist them. However, these approaches are typically *ad hoc* because Australian primary teachers are generalists who seldom have the opportunity to develop the specialist knowledge associated with diagnosing students' mathematics difficulties, nor designing associated responsive teaching (Gervasoni et al., 2021). It could be argued that initiatives in Australia for supporting students who are not yet thriving in mathematics need to include professional learning to build teachers' expertise.

The research reported in this paper is part of a larger project funded by the NSW Department of Education to advance primary mathematics intervention professional learning, specifically to support students who were mathematically vulnerable. One aspect of the study was providing professional learning about using the Mathematics Assessment Interview (MAI) (Gervasoni et al., 2011) formally known as the Early Numeracy Interview (Clarke et al., 2002; Clarke, 2013) so that classroom teachers could: (1) identify each student's current knowledge across four number domains; (2) use the data to position each student's knowledge along a framework of growth points, and then use this data to guide and differentiate teaching; and (3) identify any students who were mathematically vulnerable. A subsequent aspect of the project involved professional learning for specialist mathematics intervention teachers who would use the MAI data to prioritise students for an intervention program in semester 2, and use the growth point framework to guide their teaching while implementing the intervention. In previous research involving the MAI, students were assessed at the beginning and end of the school year, but never at the midpoint of the year. Because our study began in May, this provided the opportunity to gain new insights about a cohort of Grade 1 students' knowledge midway through the year.

Two research questions frame the findings reported in this paper: (1) What is the growth point distribution for Grade 1 students midway through the school year in four number domains? (2) What proportion of Grade 1 students are mathematically vulnerable in each number domain midway through the school year?

Background Literature

The research reported in this paper builds on previous studies that used data from task-based mathematics assessment interviews and learning trajectories to: (1) describe the range of primary students' mathematical knowledge at a point in time; (2) plan responsive teaching; and (3) identify students who were mathematically vulnerable. The following section provides an outline of this literature, with a focus on the assessment interview and growth point framework used in this study.

Interview-based Mathematics Assessment

One-to-one interview-based mathematics assessment tools that assist teachers to systematically identify each students' current mathematics knowledge along a learning trajectory (Clements et al., 2023) are helpful for supporting teachers' efforts to provide ambitious and equitable mathematics teaching (Cobb et al., 2018). Australasian research over the past 25 years has a rich tradition of developing such tools (Bobis et al., 2005; Clarke, 2013; Wright et al., 2000). Clarke (2013) claims that the experience of task-based, one-to-one assessment interviews makes a major contribution to teachers' understanding of the mathematics, the trajectories, children's minds, the obstacles, and the principles of instruction that Ginsburg (2009) argued informs the teaching of a child or group of children.

The Mathematics Assessment Interview and Growth Points

The Mathematics Assessment Interview (MAI) (Clarke et al., 2002; Gervasoni et al., 2011) is an example of a task-based one-to-one assessment interview with associated research validated growth point framework that describes key stages in students' development. The MAI was an output of the Early Numeracy Research Project (Clarke et al., 2002), and is a key tool used in our study. It requires the teacher to sit with each student, present a series of mathematics tasks that are described in a detailed script, and to observe and probe the student's thinking and strategies for solving mathematics tasks, until they have a deep understanding of the extent of the student's current knowledge. Based on a student's strategies and responses, the assessment script leads students through different tasks in nine mathematics domains, just like a choose your own adventure story. The development and details of the assessment interview, tasks, and growth points are described in full in Clarke et al. (2002).

The principles underlying the construction of the growth points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics education; reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of students who may be mathematically vulnerable (Clarke et al., 2002).

The growth point data generated from the MAI have a broad range of uses for teachers, schools, and systems in responding to students' current mathematical thinking, knowledge, and progress. For example, the MAI can also be used systematically to screen or identify students who are mathematically vulnerable. Also, the teaching advice embedded in the growth point framework for each domain (see Gervasoni, 2015) can assist teachers to focus their teaching in ways that support individual students to advance and accelerate their mathematics learning. Schools can use the assessment data and learning trajectories to identify students who may benefit from additional support, tutoring, or intervention programs.

To explain the nature of the growth points, the following are the growth points for Addition and Subtraction Strategies. These describe the strategies students use to calculate mentally.

1. Counts all to find the total of two collections.
2. Counts on from one number to find the total of two collections.

3. Given subtraction situations, chooses appropriately from strategies including count back, count-down to & count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding 9, build to next ten, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

Each growth point represents substantial growth in knowledge along paths to mathematical understanding. A student's growth point at a given point in time can also be used to identify students who are mathematically vulnerable (Gervasoni et al., 2021).

Identifying Students Who Are Mathematically Vulnerable

There is no universally agreed method for identifying students who are mathematically vulnerable or experience difficulty with mathematics. Typically, teachers or researchers use cut-off scores on tests, proficiency benchmarks, or percentile ranks such as scores below the 10th, 20th or 35th percentiles, to identify students who are not thriving (Powell et al., 2021).

As part of the Early Numeracy Research Project and subsequent research, Gervasoni (2015) used growth points generated from the MAI to identify students who were mathematically vulnerable and may benefit from intervention. Growth points in the Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies domains were used for this purpose. Through analysing February MAI data for large cohorts of students, opinions from over 200 teachers, and curriculum expectations, a set of *on the way* growth points were determined for each grade level (see Table 1). The idea is that the *on the way* growth point describes the mathematics knowledge in a domain required for students to fully engage in mathematics lessons and thrive with learning mathematics. Students who have not reached these growth points are likely to be mathematically vulnerable because they are yet to learn the mathematics necessary to fully engage with the curriculum for their class.

Table 1

February-June on the Way Growth Points for Each Grade Level (Gervasoni, 2015, p. 96.)

Number domains	F	February-June <i>on the way</i> growth points for each grade level							
		1	2	3	4	5	6	7	8
Counting	[1]	2	3	4	5	5	5	6	6
Place Value	[1]	1	2	3	3	4	4	5	5
Add & Sub	-	1	2	3	4	5	5	5	6
Mult & Div	-	1	2	3	3	4	5	5	6

As part of the Extending Mathematical Understanding (EMU) Project in 2018, Gervasoni et al. (2019) used the *on the way* growth points to identify Grade 1 students who were mathematically vulnerable and prioritise them for the EMU intervention program. The MAI was used to assess 3277 Grade 1 students from 57 schools in February. Based on the *on the way* growth points in Table 1, the results suggested that 27% of students were vulnerable in Counting, 10% in Place Value, 14% in Addition and Subtraction, and 24% in Multiplication and Division (Gervasoni et al. 2019). With the exception of the Counting results, these proportions were similar to those reported in the Early Numeracy Research Project involving 1497 Grade 1 students (Gervasoni, 2015). In the latter case, only 11% of students were vulnerable in Counting. However, it is not known what proportion of students are vulnerable midway through the school year, as the MAI was used only at the beginning or end of the school

year in the two studies described earlier. It would be useful to establish whether the proportion of students who are vulnerable reduces or increases as the school year progresses, so that teaching and interventions could be more targeted if needed.

A set of *on the way* growth points were also developed to assist teachers prioritise students for interventions in the second semester of the school year. These are described in Table 2.

Table 2

July-December on the Way Growth Points for Each Grade Level (Gervasoni, 2015, p. 96)

Number domains	F	July-December on the way growth points for each grade level							
		1	2	3	4	5	6	7	8
Counting	[2]	3	4	5	5	5	6	6	6
Place Value	[1]	2	3	3	4	4	5	5	5
Add & Sub	[1]	2	3	4	5	5	5	6	6
Mult & Div	[1]	2	2	3	4	5	5	6	7

Overall, the MAI and growth point framework provide a criterion-based method for identifying students who are mathematically vulnerable. This process draws on the distribution of MAI growth points for large cohorts of Australian students, and the expectations of teachers and curriculum documents about the level of mathematics necessary for students to fully engage with mathematics learning in their class. New insights to inform interventions are needed about the proportion of students who are mathematically vulnerable midway through the school year.

Method

The research reported in this paper drew on quantitative methods to address the research questions. The data collected for analysis was derived from the Mathematics Assessment Interview (MAI), described earlier. The Early Numeracy Research Project growth points (Clarke et al., 2002; Gervasoni et al., 2011) were used as a framework to describe the distribution of student's knowledge in each of four number domains. The Extending Mathematical Understanding *on the way* growth points (see Table 1 and Table 2) were used to identify students who were mathematically vulnerable.

MAI growth point data was collected in 2024 for 396 Grade 1 students from 15 NSW primary schools from metropolitan, regional, and rural locations. The students were assessed midway through the school year (June/July) by their classroom teachers. The teachers participated in a professional learning day via zoom to learn about the assessment interview, and how to use the tool to assess students and analyse the data to determine students' growth points in the four number domains. The professional learning included the opportunity for teachers to assess a student, and then to clarify any questions about the assessment process and method for analysing the data to determine growth points. The MAI includes a detailed assessment script for teachers to follow, a detailed assessment record sheet for recording student responses, and a scoring rubric for analysing the assessment data to determine growth points. Teachers in each school moderated the process for assigning growth points to further the trustworthiness of the findings. Any discrepancies were discussed until agreement was achieved, or were referred to the research team for clarification.

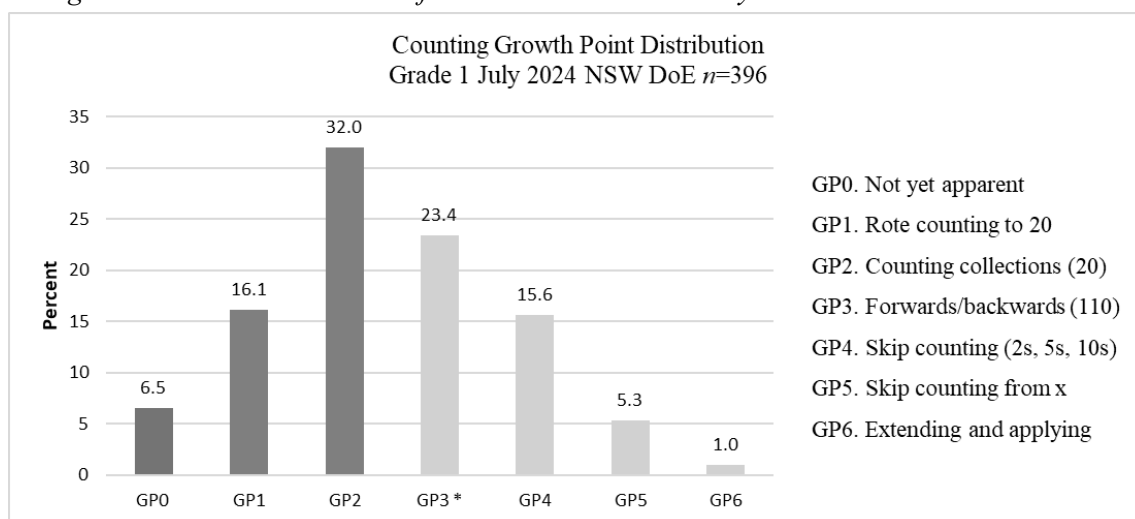
The growth points for each student were entered into an excel spreadsheet by the teacher, submitted to the education system for collation and de-identification according to approved ethical guidelines, and then submitted to the research team for analysis. Teachers used the students' growth points to plan subsequent teaching and for curriculum planning to respond to students' diverse knowledge, and to identify students who were mathematically vulnerable and who may benefit from the intervention program being implemented as part of the project.

Results

Results for this study provide insight about (1) the growth point distributions for Grade 1 students midway through the school year in four number domains, and (2) the proportion of Grade 1 students who were mathematically vulnerable in each number domain midway through the school year. Figures 1-4 show the growth point distributions for the students in Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The growth point with an asterisk in each figure represents the *on the way* growth point midway through the school year (e.g., GP3* in Figure 1). The proportion of students who have not reached the *on the way* growth point is represented in the darker grey columns in each figure. These columns represent students who were identified as mathematically vulnerable.

Figure 1

Counting Growth Point Distributions for Grade 1 Students in July 2024



The results in Figure 1 show the wide spread of knowledge and progress in Counting for this group of 396 students. The first finding to emerge is the wide spread of knowledge across all Counting growth points (GP), including that 75% of students are spread across four growth points from *rote counting to 20* (GP1) to *skip-counting by 2s, 5s, and 10s from 0* (GP4). Second is the large proportion of students who cannot yet *count forwards and backwards by ones beyond 110* (GP3, 54.6%) which is the screening growth point for identifying students who are vulnerable in Counting midway through Grade 1. Third is the proportion of students who cannot yet *count a collection of 20 objects* (GP2, 22.6%) which is also the screening growth point for identifying students who are vulnerable in counting in February. Fourth is that 6.3% of students had reached the two highest growth points (GP5 and GP6). These latter growth points involve students demonstrating early algebraic thinking and applying counting knowledge to solve problems. Overall, the findings suggest that although curriculum and teaching enable some Grade 1 students' counting to advance to the highest growth points by the midpoint of the school year, there remains a large proportion of students who are struggling to progress.

The results in Figures 2-4 follow a similar pattern to those for the Counting domain. For Place Value (Figure 2) there is also a wide spread of knowledge for the 396 students. Nonetheless, 86.9% of the students are concentrated across *reading, writing, ordering, and interpreting 1-digit numbers* (GP1) and *2-digit numbers* (GP2). More than half of the students did not yet understand 2-digit numbers, the screening growth point midway through Grade 1. Also, 7.3% of students did not yet understand 1-digit numbers, which is also the screening growth point for students at the beginning of Grade 1. A small proportion of students reached the highest growth points in Place Value. These students could interpret 4-digit numbers and greater, and apply their knowledge to solve complex problems.

Figure 2

Place Value Growth Point Distributions for Grade 1 Students in July 2024

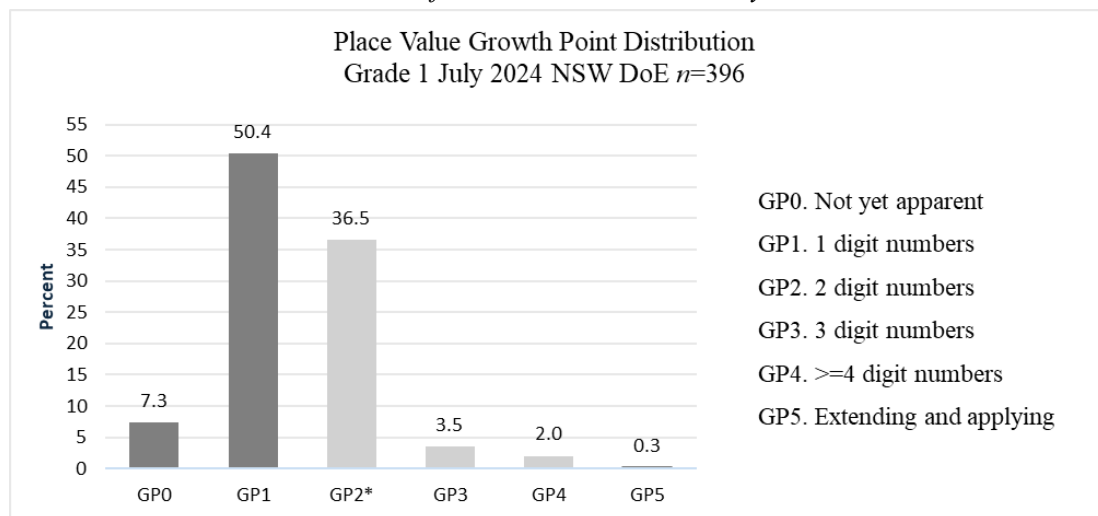


Figure 3

Addition and Subtraction Strategies Growth Point Distributions for Grade 1 Students in July 2024

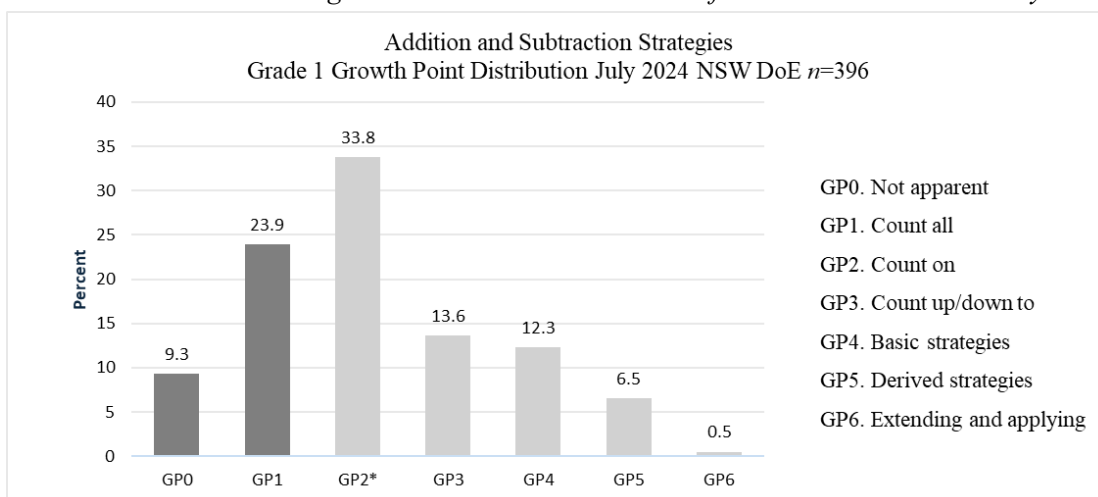
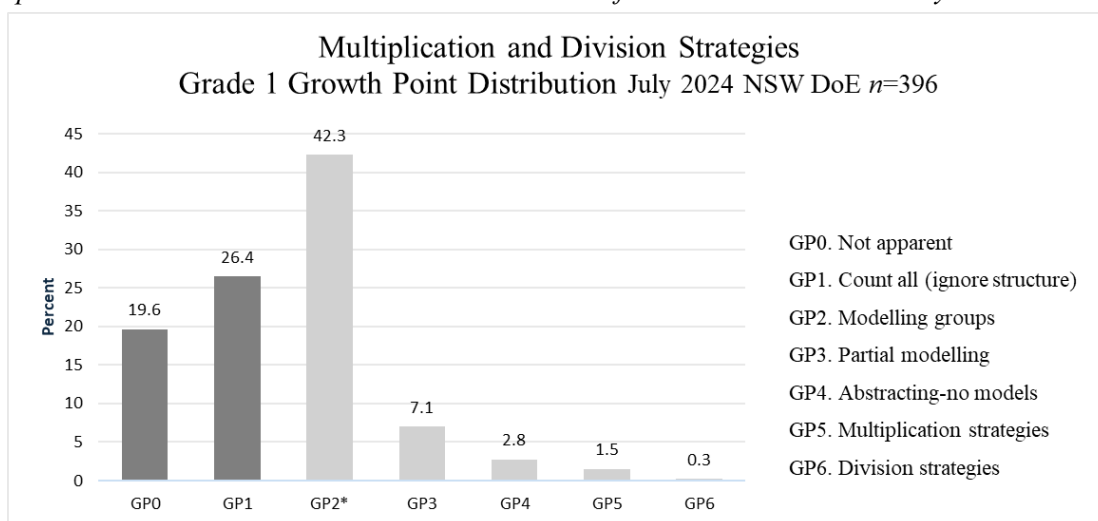


Figure 4

Multiplication and Division Growth Point Distributions for Grade 1 Students in July 2024



The results for Addition and Subtraction Strategies (Figure 3) also show a wide distribution across all growth points, with about 80% of students spread from *counting all* (GP1) to *using derived strategies* (GP5) to perform mental calculations. Most students (78.3%) used counting strategies to calculate (GP1-GP3). There were 33.2% not yet counting-on to calculate the total of 9+4 objects (with 9 objects hidden), the screening growth point midway through Grade 1. Also, 19.3% of students were on GP0 and not yet able to solve 9+4 represented with visible objects (GP1), the screening growth point at the beginning of Grade 1. Almost 20% of the students were using basic strategies (GP4) and derived strategies (GP5) to calculate mentally.

Similarly, the results in Figure 4 for Multiplication and Division Strategies show a wide distribution across all growth points, with 88.3% of students spread from GP0 to GP2 *using the multiplicative structure to calculate in situations that are modelled*. The screening growth point for students midway through Grade 1 is GP2, and 46% of students had not yet reached this growth point. It is notable that 19.6% of students, midway through the year, were not yet able to solve multiplicative problems that were modelled with objects *through counting by ones* (GP1). This is the screening growth point at the beginning of Grade 1. More than half of the students (54%) recognised and used the multiplicative structure of a situation to solve problems, and approximately 10% of students could do so in partially modelled situations (GP3+), or abstractly without models (GP4+).

Discussion

The wide distribution of Grade 1 students' number knowledge at the beginning of the year has been reported previously (Clarke et al., 2002; Gervasoni et al., 2019). The findings for this study extend this previous work to show that a wide spread of knowledge for each number domain was also apparent midway through Grade 1 for the 396 participating students. Such diverse knowledge in each of the four number domains of the MAI highlights the complexity of classroom teaching, and emphasises the need for assessment tools that can be used to identify students' current knowledge along a learning trajectory so that teachers can design targeted differentiated teaching that can advance every student's learning. It is useful for learning trajectories to include teaching advice and exemplar tasks that can guide teachers.

The findings also demonstrate that in every number domain and midway through Grade 1, at least 30% of students in the study were mathematically vulnerable with respect to the *on the way* screening growth points. It is likely that many of these students were vulnerable in multiple domains, an issue that would be useful to investigate through further analysis of these data. When comparing the proportion of Grade 1 students who were vulnerable midway through the school year with previous findings reporting the proportion who were vulnerable at the start of the year, (Gervasoni, 2015; Gervasoni et al., 2019) the proportions were much larger midway through the year. Although these comparisons represent different cohorts, the implication that the proportion of students who are vulnerable increases across Grade 1 is worthy of further research. Given that the students in this study had been at school for 18 months, it is likely that some would benefit from additional support or intervention programs earlier than midway through Grade 1. Further, without additional targeted support, these students may remain mathematically vulnerable. It is likely that early primary teachers may benefit from professional learning that assists them to understand the nature of mathematics learning associated with the first three growth points in each number domain. This additional expertise may assist teachers to design curriculum and teaching to accelerate learning for students who are not yet thriving.

Conclusion

Midway through the school year, the Grade 1 students in this study exhibited a range of knowledge in each domain that extended from the lowest to highest growth points. This finding suggests that the curriculum and teaching offered in the first semester of Grade 1 was

sufficiently challenging to enable students to extend their learning. Conversely, the fact that so many students at the midway point of the school year were struggling to progress in each of the four number domains suggests that the Grade 1 curriculum and teaching needs to be more focused to assist students who are mathematically vulnerable. It is notable that, half way through the school year, at least one third of the Grade 1 students had difficulty with counting beyond 110, interpreting 2-digit numbers, using non-counting-based strategies to mentally add and subtract, or using the multiplicative structure to calculate when a situation was modelled. It is likely that more specialised teaching of these aspects of number learning is needed to ensure that all young students flourish. Given the importance of mathematics in further education, employment and life, this situation warrants the attention of school leaders, policy makers, professional learning developers, and researchers.

Acknowledgements

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From Resistance to Reflection: The Interplay of Support, Reflection, and Practice Change

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This study examines the impact of a professional learning (PL) project on secondary mathematics teachers' professional growth in their knowledge, beliefs, and instructional practices. Initially attributing teaching challenges to external constraints, some teachers gradually recognised the need for pedagogical development through reflection and researcher-guided discussions. The study highlights the dual role of challenges as both constraints and affordances, shaping engagement through structured support and positive student relationships. The findings reveal that teachers' perceptions of outcomes significantly influence their sustained involvement in professional learning.

Introduction

The research project provides collaborative professional development opportunities for secondary mathematics teachers in Iran and explores their participation and professional learning, setting the stage for the investigation of how these teachers engage with the opportunities and the outcomes of their involvement. The main part of the study involved a collaborative PL opportunity with nine secondary mathematics teachers who expressed interest in examining changes in their instructional practices and identifying affordances and constraints in their teaching contexts. Throughout and after the intervention, several interviews—both individual and group-based—were conducted to capture teachers' experiences and reflections. While six teachers quit in various stages of the research, three of them stayed and engaged with the project until the end. This paper focuses on the participation of Arman, a teacher from the latter group. The focus is to explore how individual engagement with a PL opportunity and interpreting the outcomes of implementing lesson plans may support shifts in his perspective and practice.

Literature Review

This study, Similar to studies such as Hughes (2020) and Wilkie (2019), applied the Meta-Didactical Transposition (MDT) (Arzarello et al., 2014) model alongside IMPG as a complementary framework. The inclusion of MDT helps address two key aspects: Institutional influences on teacher change, and recognizing how school policies, assessment structures, and broader educational frameworks impact teachers' ability to implement new practices. Social interactions among teachers and between teachers and the researcher, drawing on Hughes' (2020) perspective to examine how collaboration and discourse shape professional learning experiences.

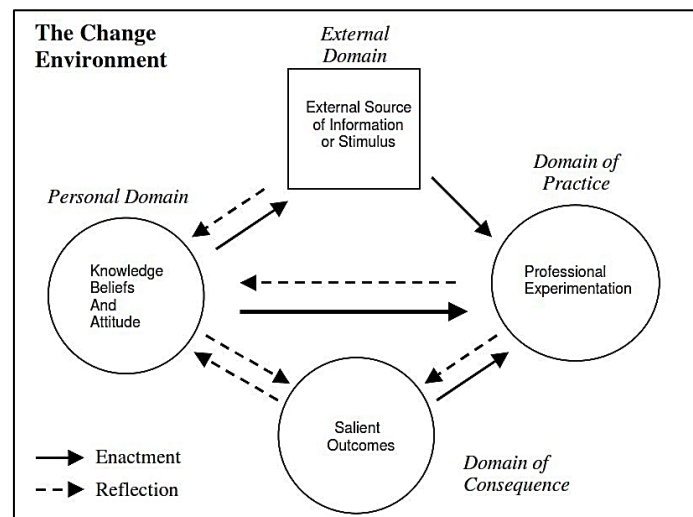
To analyse teachers' change sequences and key outcomes during their participation in professional learning (PL), this study adopts the Interconnected Model of Professional Growth (IMPG) (Clarke & Hollingsworth, 2002). The IMPG provides a cyclical and multi-entry framework that acknowledges the complex and individualized nature of teacher development. This approach highlights that teacher growth occurs through multiple pathways, shaped by external influences, personal beliefs, and professional experiences. Additionally, it underscores that teacher learning is an ongoing process, where feedback loops and reflection play a vital role in sustaining professional growth.

IMPG maps teacher learning and changes across four interrelated domains as follows. External Domain: Formal PL experiences, institutional support, and policy mandates. Domain (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 173–180). Canberra: MERGA.

of Practice: The application of new knowledge and instructional strategies in the classroom. Domain of Consequence: The impact of changes on student learning, classroom dynamics, and teacher perceptions. Personal Domain: Teachers' knowledge, beliefs, attitudes, and evolving professional identity. This study drew on Hill et al. (2008) framework of the two core elements of Mathematical Knowledge for Teaching (MKT) to conceptualise the Personal Domain, emphasizing teachers' attitudes and beliefs about teaching and learning mathematics, as highlighted by Cooke (2015).

Figure 1

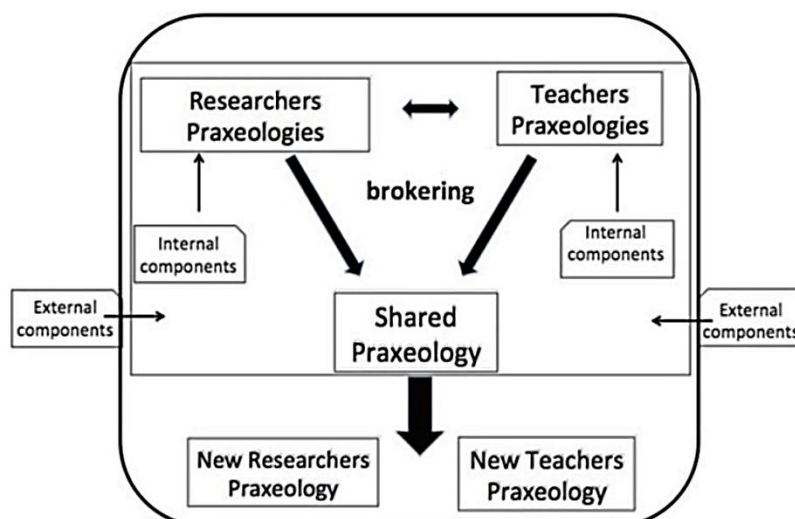
The Interconnected Model of Professional Growth (Source: Clarke & Hollingsworth, 2002)



The MDT model provides a structured lens for understanding how knowledge, teaching practices, and professional identities evolve through researcher-teacher interactions. MDT explicitly captures bidirectional exchanges between researchers and teachers, making it particularly valuable in collaborative professional learning contexts.

Figure 2

The Meta-didactical Transposition Model (Arzarello et al., 2014, p. 355)



A central concept in MDT is praxeology, which includes both 'know-how' (practical techniques) and theoretical knowledge used in instructional tasks. The model emphasises how praxeologies evolve as teachers and researchers interact, negotiate instructional strategies, and reinterpret pedagogical innovations within their unique school settings.

Table 1

MDT model features (Arzarelo et al., 2014)

Feature	Definition
Institutional aspects	The social context of the school where the intervention is conducted can influence either the plans of the researchers or the teachers' practice. These aspects also can provide opportunities or constraints to help teachers or prevent them from changing their personal domains.
Meta-didactical praxeologies	Includes tasks, approaches, and theories behind them that develop during the intervention, which can influence both teachers' and researchers' praxeologies.
Internal and external	How the external praxeologies, which the researcher brings to the project, can be a part of the teachers' praxeologies by the process of internalisation.
Brokering processes	The act of transferring the concepts from one community to another by the process of brokering. In the PL projects, since the researchers belong to both the community of teachers and researchers, they can play the role of a broker who belongs to more than one community.
Double dialectic	When teachers and researchers express contrasting interpretations (meta-didactical level) of students' personal meanings that are attached to the teaching situation (didactic level), the teachers' praxeologies tend to develop and modify to align with those of researchers.

Methodology

To explore teachers' responses to the PL intervention, their adoption of new practices, and the constraints they faced, the study employed a multiple case study methodology. A case study approach was chosen because it allows for an in-depth examination of real-world experiences within their natural setting (Stake, 2006; Yin, 2018). Given that teachers' practices are deeply embedded in their school environments, this approach enabled a context-sensitive investigation of their professional learning. Each teacher was considered a distinct case, bounded within the Iranian secondary school context, making this study an instrumental case study that sought to understand broader issues related to professional learning in this educational setting.

Data Collection

The data for this study were collected from nine lower secondary mathematics teachers working at a multi-campus private school in Tehran, Iran. In the first group meeting (the first PL session), all nine participants attended. I explained the rationale of the study and introduced example tasks designed to promote mathematical proficiencies. In the second group meeting (the second PL session), I completed the introduction and provided additional examples. I asked participants to select a problem from their prescribed problem set and modify it to better support mathematical proficiencies. Additionally, I requested that they complete a lesson plan incorporating the modified problem and share it with their colleagues and me for discussion. We had the third and fourth group meetings to reflect on teachers' participation and share our concerns. The fifth group meeting was the last one in which teachers shared their thoughts and we concluded the project. Table 2 details Arman's participation.

Data Analysis

For this qualitative study, I adopted a descriptive and interpretive approach (O'Toole & Beckett, 2013, p. 38), which aligns with my social constructivist worldview. Following Miles et al. (2014), this process involved organising, simplifying, and reducing the data to transform it into meaningful and understandable findings that could address my research questions.

In analysing the data, I employed both inductive and deductive analytic approaches. Since the study was informed by the Interconnected Model of Professional Growth (IMPG), I began with deductive analysis by utilizing the model's various elements to create a coding framework to code the data. Additional codes related to the affordances and constraints that teachers might encounter during their practice changes were derived from both the IMPG and Meta-didactical Transposition (MDT) models, as well as empirical findings from previous research (Groves et al., 2016; McCormick, 2022; Wang, 2011).

Table 2*Arman's Participation Details*

Events	Date	A brief description
Group meeting 1 (PL session 1)	24/10/2018	Describing the project as well as part of the data that I have collected from the questionnaires
Group meeting 2 (PL session 2)	31/10/2018	Elaborating on details of the project and introducing a format for designing lesson plans
Observing a typical class	12/11/2018	Observing a session of his normal class, taking some notes and reflecting on it after class
Reflecting on his normal class	12/11/2018	I attempted to link what I observed to the concepts that I had proposed in the group meetings.
Group meeting 3	19/11/2018	Reflecting on the experience of a teacher who had implemented his designed lesson plan
The first implementation of the designed lesson	24/11/2018	He designed a lesson plan individually and sent it to me the night before the implementation session.
A brief reflection	24/11/2018	Reflecting on the first implementation session
Interview	24/11/2018	After a brief reflection on the Implemented session, on the same day, I did an interview with Arman about the whole project.
The second implementation of the designed lesson	3/12/2018	Again, he had sent the lesson plan the night before the implementation session. We had the opportunity to discuss it before attending the class.
Group meeting 4	3/12/2018	Three teachers attended and we had some conversations on any issues that teachers suggested previously or in that session.
Third implementation of the designed lesson	7/1/2019	I suggested to him a few tasks related to that day's topic. He included one of those tasks in his lesson plan.
A brief reflection	7/1/2019	Reflecting on the first implementation session
Group meeting 5	23/1/2019	This was our last meeting, which five teachers including Arman attended.

Findings and Discussion

As mentioned, this paper focuses on the participation of Arman one of the teachers who engaged in the project until the end. Arman, a 27-year-old secondary mathematics teacher with a background in mechanical engineering, entered the teaching profession without formal training or exposure to structured professional development. Arman attended all group meetings and eagerly participated in discussions (Figure 1 shows his change sequence).

At the beginning of the project, Arman attributed students' learning challenges primarily to external factors rather than reflecting on his teaching practices. He believed that systemic

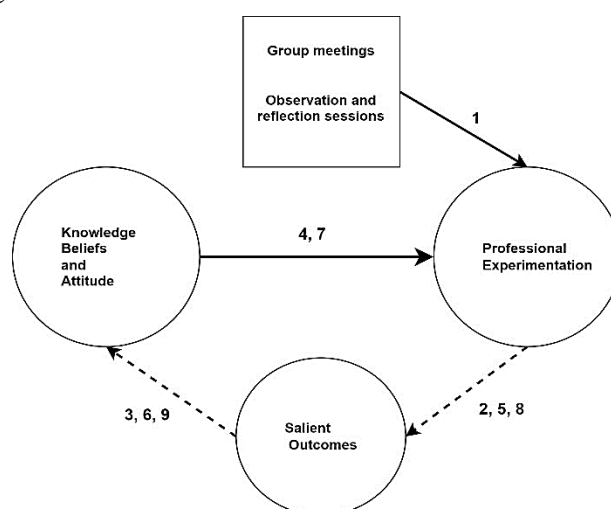
constraints, curricular limitations, and student-related factors played a greater role in shaping learning outcomes than his instructional methods. While the external factors were reported impactful (Bhutto & Rind, 2022; Chen, 2008; Wilkie, 2019), when I observed Arman's teaching, I noticed some potential areas for professional growth. During the debrief session, I talked about these aspects of his teaching and shared with him some research-informed suggestions for conducting a student-centred class such as using challenging tasks (Sullivan et al., 2015), applying group work and the five practices for orchestrating the class discussion (Stein et al., 2008). Arman mentioned finding the reflection session useful, saying,

Now I have a better understanding of the concepts that you introduced in the group meetings [the PL sessions]. I will think about the details of my lesson plans and share a few of them with you before implementing them in my classes. Please come to my classes and let me know your feedback.

Over time, Arman's engagement in the project deepened, and he began to open himself to new perspectives. As we navigated through the project, other potential gaps in his knowledge and skills surfaced which created a ground for more discussions and fostering the process of learning and internalisation of the new praxeologies.

Figure 3

Arman's Change Sequence



One of the pivotal moments appeared when he implemented a more engaging task (the swimming pool task which I had introduced in one of our PL sessions). After implementing a designed lesson plan and a brief discussion during the break time, he went to another class to teach the class with his normal approach while I was waiting in the teacher's office to have a longer discussion after his second class. After that session, he stated,

I already had started the equation topic and was willing to work on the related exercises in this session. However, after our small chat during break time and a quick reflection on my session, I realised that I only did the same teaching without any considerable change. So, I decided to include an engaging task in my teaching to see the students' reactions to it.

The noticeable improvement in student engagement in the second class led him to reflect on the impact of task selection on student motivation. He mentioned,

While the engagement of the students in the first session was not satisfying, introducing the swimming pool task significantly impacted students' engagement and motivation in the second session. Students enthusiastically worked on the task, most of whom could make sense of it. They also realised how utilising algebra could make the solutions effective and feasible.

Arman considered the difference in students' engagement in the two sessions, as a salient outcome of his classroom experimentation. This experience initiated a shift in his Knowledge

of Content and Students (KCS)², as he recognized the importance of selecting tasks that connected mathematical concepts to real-world contexts. In his second lesson plan, he attempted to design an engaging activity based on a real-life scenario. Before observing his class, he stated,

I realised that an engaging task such as the swimming pool task can be the core part of a class. Therefore, I picked up a problem from an auxiliary mathematics book that is popular among students and tried to modify it in a sense that makes students more engaged. I should use these challenging and engaging tasks more in my lesson plan in future.

It seemed that his disposition toward preparing lesson plans had been influenced by his experience of using a more engaging task. This resonates with the claim of Guskey (2002) that teachers' beliefs would be changed by experiencing students' engagement or success in the classroom. Although Arman had made an interesting opening story for this task in his second lesson plan, the limited structured guidance and formative monitoring led to moments of disengagement. This signals a gap in KCS, as teachers with strong KCS continuously assess students' understanding, ask probing questions, and provide timely interventions when students struggle or lose focus (Charalambous, 2010). By not regularly checking students' work, Arman missed opportunities to provide targeted support and adapt instruction in real-time.

Even though only some students eagerly listened to his talk and responded to his questions, Arman seemed satisfied with his actions in that class. When I provided feedback highlighting the distinction between emotional engagement and cognitive engagement, Arman was initially surprised since he seemed satisfied with the implementation of his designed lesson plan. This moment of discrepancy between his perception and mine served as a critical juncture in his professional growth, aligning with the double dialectic situation in the framework of Meta-Didactical Transposition (MDT) which could provide opportunities to develop teachers' professional competence. Through further discussions, he acknowledged that an engaging task alone was insufficient—how it was introduced, facilitated, and scaffolded played an equally crucial role in fostering deep learning. The contrast between my perception of the class and Arman's could be recognised as the salient outcome.

As Arman continued experimenting with new approaches, he showed a growing willingness to refine his lesson planning. He sought feedback on task selection, structure, and implementation strategies, demonstrating an emerging understanding of lesson orchestration. He expressed,

It seems that to get the most benefit from changing our lesson plan, in addition to selecting an appropriate task and modifying it properly, I need to think about how to present it to students and how to lead the class. I would like to attempt designing a lesson plan and implementing it another time. I'll try to show my lesson plan to you and discuss the tasks and the approach before the implementation of the session.

Arman's reflections and willingness to rethink his lesson planning and instructional approach marked a deeper shift in his Mathematical Knowledge of Teaching (MKT), particularly in Knowledge of Content and Teaching (KCT)³. One of the most significant developments in Arman's KCT was his realization that selecting an engaging task alone is not sufficient; how the task is introduced and facilitated plays a crucial role in student engagement. In his statement, he recognized that besides task selection and modification, the launch phase of a task and lesson leadership are equally important components of an effective mathematics lesson. This awareness suggests an emerging understanding of lesson orchestration, where a

²Type of knowledge that integrates knowing about students as well as mathematics (Hill et al., 2008)

³ Teachers proficient in KCT are skilled in making pedagogically informed decisions that integrate deeply with mathematical content.

teacher carefully plans the sequence of activities, scaffolds student learning, and ensures that engagement is maintained throughout the lesson (Stein et al., 2008).

Arman's professional growth was shaped by sustained engagement in reflective discussions and structured feedback. Arman required direct intervention and support to internalize new strategies and the brokering process, facilitated through one-on-one discussions and classroom observations, played a crucial role in helping him bridge the gap between theoretical insights and practical implementation. Arman was willing to design another lesson plan to reflect his recent view on it and see whether he could engage more students in his class. He stated,

This time I need to consult you in all steps including selecting the tasks, designing the lesson plan and finding a proper approach to implement them. Please send me some tasks as a resource, then I will see and select a couple of them.

This request of Arman highlighted the requirement of a 'knowledgeable other' as expressed by Sharratt (2018) to support Arman with designing a lesson plan. By seeking guidance in task selection and lesson structuring, Arman acknowledged the need for external input to refine his practice, which reflects an openness to professional growth. Arman shared his perception of constraints in designing and implementing lesson plans,

When I use my usual approach and tasks since I have taught them previously, I would be able to anticipate the difficulties that students may confront during the class. This anticipation helps me to be ready and deal with situations appropriately. However, for the new lesson plans, I am hardly able to foresee the problems and might not be ready to deal with unexpected situations and help stuck students.

As Arman mentioned in the above quote, when he encountered a new task, he found it difficult to identify which part of the task would be problematic to teach or to be understood by students. It could partially be related to the limited Specialised Content knowledge (KSC) which includes the competency of anticipating students' responses to a certain problem. Arman's statement suggests that while he feels confident using familiar tasks—where he has prior knowledge of student difficulties—he struggles to foresee potential challenges when implementing new lesson plans.

Conclusion

According to Clarke and Hollingsworth's (2002) model of professional growth, the way a teacher perceives the outcomes is shaped by their reflection on the change in practice or their personal development. However, the impact of these outcomes depends on the teacher's existing value system and how they interpret the classroom practices. The challenges Arman encountered during the project were not necessarily seen as negative; in some cases, these challenges acted as affordances for learning and growth. When the outcomes are perceived as negative or not strong enough, additional support, follow-up discussions, and encouragement appear beneficial to keep the teachers motivated and engaged with the learning process. Arman's availability of non-teaching hours, allowed for additional support and motivation to keep engaging in the project.

At the start of the project, despite being relatively inexperienced and unfamiliar with research-based pedagogical practices, Arman expressed confidence in his existing approaches. He primarily attributed his challenges to external constraints, such as standardized exams, limited time, and rigid school-set curricula, which he believed hindered their ability to adopt student-centred approaches. Over time, Arman moved from a stance of attributing student challenges solely to external factors toward a more balanced perspective—one that acknowledged the role of his instructional choices in shaping student learning outcomes. This shift was essential for his professional growth and led to self-reflection on areas where he wanted to improve. His journey underscores the importance of structured professional learning

opportunities that provide teachers with the time, space, and support needed to engage in reflective practice.

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Aestheticising Readers' Experiences of 'Reading' Research: Making Visible Emotional Trajectories in Learning

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Nathalie Sinclair's MERGA 2024 keynote challenged us to communicate video data in ways that provide the reader with a more holistic and embodied experience of the data. In this methodological position paper, we take up this challenge. Through video analysis of a pre-school child and her mother engaging about counting in threes (prompted by the layout of numbers on a television remote) we address: How might we (in articles) share learners' shifting facial expressions and body language over time to reveal their emotional trajectories? When can images be considered self-evident indicators of emotions? How can we provide a more 'sensory' experience of data?

Introduction

As teacher of mathematics, we might have noticed that a learning experience is not just about verbalizing what is known—it is also reflected in learner's emotional journeys. For example, we might have noticed frustration giving way to understanding, uncertainty evolving into confidence, or doubt followed by joy when learners achieve a breakthrough. These emotional pathways in learning, often overlooked, are not secondary to the learning experience. They are often not expressed in words yet are core to learning processes. Sinclair's (2024a) MERGA keynote 'Towards embodied validity in mathematics education research' argued the importance of 'aestheticizing' the reader's experience—adopting the term from Fuller and Weizman's (2021) notion of sensory knowing. Her work in this area (Sinclair, 2024b; Günes et al., 2024) highlights the power of Dewey's (1934) theorising about experience as characterized by its "internal integration and fulfilment" (p.42). From Dewey, Sinclair (2024b, 42) highlights that "an experience features emotional intensity together with 'cumulation, tension, conservation, anticipation, and fulfilment' (p.152)". Sinclair's work and our own experiences prompted us to relook at our analysis of a video of Lila—a five-year-old sharing her discovery of counting in threes on the television remote control with her mother (Mellony). What unfolded was an explanation of her discovery followed by a series of prompts to nudge her to continue this counting in threes beyond what was visible on the remote. Previously, drawing on Vygotsky (1978) we characterized the event as a mathematical learning experience within Lila's zone of proximal development (ZPD) and analysed the way the more knowledgeable other (MKO) shifted between Lila, the remote and the mother thus generating a multidirectional ZPD (Abtahi et al., 2017). Reflecting on the way we focused our analysis on the verbal transcript and Lila's gestures on the remote we realised we avoided, in our writing, communicating the highly visible emotional trajectory across the learning experience. We noted that we included only three drawings converted from video stills of Lila's facial expressions (see Figure 1), respectively showing her: thinking, being unsure and confidently answering 15 as next in the pattern after 12. While not explicitly stated, these images were taken as self-evident indicators of these processes and emotions.

In this paper, we re-analyse the video to focus on Lila's emotional trajectory and through this engage with the methodologically driven research questions: How might we (in articles) share learners' shifting facial expressions and body language over time to reveal their emotional trajectories? When can images be considered self-evident indicators of emotions? How can we provide a more 'sensory' experience of data? We discuss the implications of our grappling and

address what is lost when we avoid explicit analysis of emotional cues visible in the expressions and body language of children that are central to guiding effective mediation of learning events.

Figure 1

Three images of expressions shared in Abtahi et al. (2017, p.284)



Reflecting on Missed Opportunities and Omissions in Earlier Work

Figure 1 drawings were converted from video images as these were not considered of high enough resolution for journal printing. At the time of writing, we accepted that we should focus our analysis on the transcript of what was said and gestured on the remote as a mediating artefact. As our ZPD framing led us to focus on Lila's cognitive rather than emotional trajectory we considered that inclusion of images would be a 'nice to have' but non-essential part of the contribution. Our engaging with Sinclair's recent work (2024a and b) and revisiting Dewey's (1934) writing leads us to argue that this assumption is problematic for a range of reasons. First, it overlooks the extent to which our witnessing and experience of viewing the video and still images taken from it, of Lila's emotions, guided our analysis of her words and gestures. Second, it excludes analysis of the key driver of the learning event (i.e., Lila and her mother's excitement and eagerness to engage around the discovery) and excludes the way emotional energies guided the mother's choice of mediation acts. Finally, it strips our ability to strengthen the validity of our claims about learning events through sharing images that can sensibly be taken as 'self-evident' indicators of emotional drivers and generate *broader* embodied experiences for the reader in interacting with data of learner expressions. While primarily a methodological position paper, we include here re-analysis of the earlier reported learning event (Abtahi et al., 2017) with a focus on how emotional trajectories drive the learning event to illuminate the above three points and answer our research questions.

Framing and Literature Review: What is a [Learning] Experience?

Like Sinclair (2024b) we link Dewey's (1934) notion of experience to *learning* experience as constituted by emotional intensity along with cumulation, tension, anticipation, and fulfilment. A mathematical learning experience is distinctive from other experiences as it involves cognitive development of mathematical ideas (such as learning to count in threes)—this process is simultaneously shaped by cognitive and emotional dynamics, including cumulation, tension, anticipation, and fulfilment. Additionally, Dewey highlights that “an experience of thinking has its own aesthetic quality” (p. 38). This aesthetic dimension, deeply rooted in emotion, “rounds out an experience into completeness and unity” (p. 41). These emotions connect with events, objects and their movement over time. Becoming emotional involves becoming part “of an inclusive and enduring situation that involves concern for objects and issues.” (p.42). In sharing our data of Lila's emotional trajectory through movie strip style still images taken from video we hope to illuminate the way Lila's mathematical learning experience is indeed characterised by engagement and movement around mathematical objects. Lila's experience is driven by the intertwined emotional and cognitive trajectories of cumulation, tension, anticipation, and fulfilment shaped by her and her mother's ways of being

and interacting in each other's physical presence (Jordan & Henderson, 1995). Fuller and Weizman's (2021) concept of aestheticizing pushes us to explore "new conditions of knowing, seeing and doing" (p. 111). Evidence of learning is not solely defined by the facts that are presented, but also by how those facts are communicated and subsequently experienced by others. For example, Günes et al. (2024) argue for aestheticising mathematics education research pushing us to extend our lens beyond mathematics knowing to investigating the 'mathematical sensorium'. They propose using re-enactments as powerful research methods. Those at MERGA 46 will recall how Sinclair (2024) had us re-enact what we witnessed the child do in the videos. This stimulated our sensual knowing of what the child was doing/experiencing. Here, we propose (and demonstrate through sharing data from a single opportunity sample) aestheticizing the reading of mathematics education research through sharing movie type strips of learner expressions across a mathematics learning event. See Ferrara and Ferrari (2025) for their methodological contribution of ways diagrams can stimulate aesthetic and affective mathematical engagement while positioning learners at the centre of the mathematical activity.

In revisiting our data analysis, we use Sinclair's (2024) notion of embodied validity that builds on Lather's (1993) post-structural notion of voluptuous situated validity. This validity provides space for an "embodied positionality and a tentativeness which leaves space for others to enter" where authority "comes from engagement and reflexivity" (Lather, 1993, p37). Embodied validity has implications for how we interpret and share data (Sinclair, 2024a). We illuminate this in terms of how the notion of embodied validity has guided our re-interpretation of the data. By deliberately including a series of images that capture the learner's shifting expressions, we invite readers to engage with her cognitive and emotional trajectories. The visual elements further reinforce the validity of our claims regarding what constitutes a mathematical learning experience. We are thus working with the assumption that our readers 'seeing' an image that clearly captures emotional intensity will have a sensory experience linked to that 'seeing'. In the same way that journalists use emotive images to capture the human experience of various situations (joyous and desperate) based on the assumption that the viewer will have an emotive sensory reaction to the experience, we include images we consider can be reasonably assumed to be self-evident of an emotion (or range of emotions) and will likely stimulate a sensory response in the reader viewing the data. We work with this pragmatic assumption while noting the research linking exposure to visual images to emotional responses (e.g., Nanda et al., 2012) and detailing neurological mechanisms evoked when humans view emotive images (e.g., Fried et al., 1997). We further acknowledge the interdisciplinary research that has focused on interaction analysis based on multiple viewings of audiovisual recordings (e.g., Jordan & Henderson, 1995) and informs our (and Sinclair & colleagues) approach.

Method

We began our re-analysis focusing on part of Lila's mathematical learning experience from her discovery of the 3, 6, 9 pattern on the remote to her progression in counting by threes up to 18. We divided this learning experience into scenes, indicated by continuity/cumulation, tension, anticipation, and fulfilment. Following discussion and identification of these characteristics, eight scenes were identified. Every scene began with some prompting, requiring engagement toward fulfilment. Each scene ended with an indicator of fulfilment (even when the fulfilment of the experience was mathematically incorrect). One or two four-frame 'video strips' were constructed to capture the emotional trajectory of Lila across each scene of mathematical learning experience. The still images were selected and cut from the video excerpt along with the time stamp for the image. In some cases, two images are included with the same time stamp because several images can be selected from any single second of video. In cases where emotional shifts were visible within a second these were included and placed sequentially

with the same time stamp. Perhaps better software could create more refined time stamps, but we did not think this necessary for our purpose. Words spoken at the second of the image were included along with a brief comment about what was happening where appropriate. Continuity, anticipation, cumulation, tensions, fulfilment and emotive descriptors are noted across images.

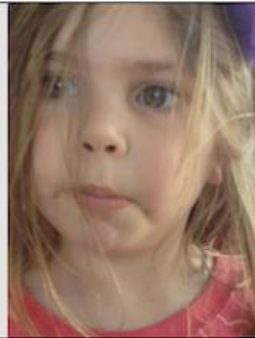
It is beyond the scope of this paper to discuss how the ZPD is stimulated and sustained equally by Lila's emotional and cognitive trajectories, or how her mother's mediation is equally prompted by Lila's emotions (seen in her expressions and tone and volume of her voice) as by what Lila says and shows on the remote (see Abtahi and Graven, in press). Here we analyse Lila's emotional trajectories across three scenes (5, 6, and 8) by sharing 'video strip' images to engage readers' sensory experience of 'reading' (and seeing images of) the learning event and thus to establish embodied validity to support our analysis. While not claiming that video strip type images of children's unfolding emotive expressions is new to mathematics education, we expect that many authors, like us, may have excluded such images for various reasons (e.g., prioritizing cognitive aspects, maintaining an exclusive focus on what is said as a marker of rigor, concerns over image quality for publication, anonymity considerations, or space limitations). Our aim is to provide sufficient sensory data to enable us and readers to consider our earlier stated methodologically driven research questions.

Analysis and Communication of Lila's Emotional Trajectory Data

(00-00:44) - Scenes 1 and 2 involved Lila responding to her mother's prompts to show her how she worked out counting in threes. Lila fulfils each scene in which Mellony requests 'show me'. In Scene 1 the experience is fulfilled as she gestures with her finger across the three rows saying 'cause everyone is three'. Scene 2 is fulfilled when she responds to 'so show me how you are counting' with gesturing across the four rows of three buttons emphasizing the 3, 6, and 9 and continues to the three non-numbered buttons—saying 12 on the last button. Scenes 3 and 4 were fulfilled when Lila responded to Mellony's questions about how she knows its 12 and why she thinks it is counting in threes, respectively. We begin our analysis from Scene 5 where Mellony asks "and what do you think comes after 12 if we are counting in threes?" The question invokes continuity and is the start of accumulation towards the fulfilment of a new task. Lila responds by spending the next 18 seconds (00:49–1:07) mouthing quietly 1, 2, 3, ...11, 12, 13. The video strips below show images across this time that indicate the cumulation, tension, anticipation and fulfilment (even while 13 is incorrect if counting in threes).

Figure 2

Video Strip Scene 5

5-a) 00:49 continuity , listening to the question <i>willingness to engage</i>	5-b) 00:49 anticipation , idea for a way towards fulfilment <i>willingness to engage</i>	5-c) 00:51 cumulation , begins mouthing 1, 2, <i>willingness to engage</i>	5-d) 00:55 continuity/ cumulation , mouthing numbers <i>willingness to engage</i>
			



Unpacking Lila's emotional trajectory we see in the two 4-frame video strips Lila's willingness to engage by listening (image 5-a) enabled continuity and anticipation of a response (image 5-b) followed by continuity/accumulation (image 5-c-f) towards fulfilment (image 5-g-h) where she happily and confidently restates 13 as the answer and looks to her mother hoping for (anticipating) a positive response. We consider that these strips provide indicators of the way in which this mathematical experience unfolded for Lila and the way in which her emotions are integral to this mathematical learning experience. We expect the reader to experience some kind of sensory reaction to viewing these images while acknowledging that individual responses may vary. Embodied validity lies in the degree to which readers and our interpretations of the images' characteristics and emotions resonate. Lila's anticipation of a response (5-h) shows willingness to continue to engage around this. She has not shown that she wants to leave. This emotional cue leads Mellony to note the logic of her answer '13' (indeed Lila counted from 1-13 effectively) and nudge her to finding the next number when counting *in threes*, saying at the start of Scene 6: "13 is the number after 12. That is right, but that is one number after 12, what if you were counting in threes?"

Mellony's prompt invokes continuity and the start of accumulation towards another fulfilment of the same task. The video strips below show new sets of images across the time that indicate the Lila's emotional trajectory: cumulation, tension, anticipation and fulfilment.

Figure 3

Video strips Scene 6



We see in this scene Lila's *unease* (image 6-a) along with continued *willingness* to engage. She maintains eye contact and does not walk off. Lila experiences tension in being unsure

(appearing frustrated/disappointed) when Mellony says 13 is ‘one number after 12’ (image 6-b). The tension is followed by continuity/accumulation (image 6c-d) towards some kind of fulfilment. Lila is unsure but engaged (image 6-d). Fulfilment comes when at 1:15 Lila says quietly and slowly ‘I’m not sure’ (see second line drawing in Figure 1). Again, in this scene, we see that emotions are an integral part of Lila’s mathematical learning experience. In Scene 7 Mellony suggests using the remote. Lila responds that there are no more buttons beyond the three under 7, 8, 9, labelled ‘menu’ ‘0’ and ‘help’. At the start of Scene 8 Mellony asks: Can we pretend that there are three maybe?” Mellony points to the remote as she prompts Lila to reintegrate the TV remote into the mathematical learning experience, suggesting ‘pretending’ there are three. This fosters continuity and further carries anticipation for accumulation toward a newer (and more mathematically sound) fulfilment. Mellony’s inquiry is guided by her perception of Lila’s willingness to engage captured in her expressions and body language.

Figure 4

Video strips Scene 8

8-a) 1:24 cumulation uses remote, moves to place middle fingers over the 1,2,3 buttons <i>willingness, tension</i>	8-b) 1:31 cumulation , soft um while three fingers are on 1,2,3 <i>willingness, unsure</i>	8-c) 1:37 cumulation , says ‘twelve’ as three fingers are on buttons 7,8,9 <i>willingness</i>	8-d) 1:42 cumulation, anticipation , mouths 13,14,15 faintly audible, fingers aside remote, <i>pleased</i>
			
8-e) 1:42 fulfilment , repeats ‘fif’ jumping up off screen (hence blurred) <i>excited</i>	8-f) 1:43 fulfilment , says ‘teen’ landing from jump <i>excited, confident</i>	8-g) 1:43 fulfilment , says ‘fifteen’ <i>confident, happy and proud</i>	8-h) 1:44 fulfilment , looking at mother while she says: ‘My word my angel, that is brilliant’. <i>confident, happy, proud</i>
			

In Scene 8, we observe Lila’s willingness and engagement intertwined with moments of uncertainty (images 8-a and 8-b). Cumulation and continuity unfold as she focuses on the remote and moves her fingers on them. Her wanting to complete the task culminates in a breakthrough, marked by a radiant smile (image 8-d). Fulfilment becomes strikingly apparent as Lila, brimming with pride, nearly leaps off the screen (image 8-e), triumphantly declaring 15 as the next number (images 8-f to 8-h). Again, after the scene’s fulfilment, Lila stays, maintaining eye contact, showing willingness to continue engaging. This leads Mellony to ask what comes after 15 with new scenes similarly reaching fulfilment with Lila’s answer of 18.

Discussion

We have shown the use of sequential 4-frame stills cut from video to create 'video strips' of evolving expressions and body language across a series of linked learning experiences. Images are time stamped and categorised as an indication of continuity/cumulation, anticipation, tension, and or fulfilment along with a listing of the emotion/s visible—interspersed with transcripts of what is said and an explanation of the episode. We propose this as a productive way to analyse and interpret data more fully (than focusing only on words and actions as evidence of cognitive drivers) and that these provide the reader a broader experience of the data. Indeed, across these scenes few words are spoken and sharing the images is to engage the reader in an experience that highlights how emotional trajectories and cognitive drivers are inseparably intertwined. We have taken the images shared as self-evident indicators of certain emotions. We concede that viewers presented single images would likely give different, though synergistic, descriptions of the emotions visible (e.g., for 8-a 'pleased', 'assured', 'excited'), raising the question of how closely our descriptions of Lila's emotions must align with the readers'. This is a complex issue requiring further investigation. However, we argue that the embodied validity emerges not in the reader's confirmation or agreement with emotional labels for each image but with the emotional categorisation—as a whole.

We argue that separating the analysis of emotional trajectories from the cognitive drivers of learning (as we did earlier) is problematic on theoretical, methodological, and pragmatic levels. Theoretically, the separation disregards the central role of emotion in learning events. We have shown here how emotions are not merely peripheral to cognitive processes; rather, they actively shape and drive learning interactions. Our earlier cognitive focus overlooked how Mellony's choices were mediated not only by Lila's verbal expressions but also by witnessing and experiencing the full spectrum of her emotional trajectory. The interplay between emotion and cognition is fundamental, as emotional responses can reinforce, hinder, or redirect cognitive engagement. Methodologically, separating these aspects fails to account for the ways our observation and experience of viewing the video and its still images—capturing Lila's emotions—shaped our interpretation of her voice and gestures. Treating cognition and emotion as distinct risks misrepresenting the holistic nature of a learning process. Further it constrains our ability to strengthen the validity of our claims about learning events. Without the inclusion of images that serve as 'self-evident' indicators of emotional influences, we miss an opportunity to provide readers with a more embodied and immersive experience, allowing them to engage more fully with the affective dimensions of the learning process. Pragmatically, separating these aspects is problematic because if we render invisible this aspect of learning in our writing and our work with prospective and in-service teachers, we inadvertently send messages that children's emotions do not need consideration when mediating mathematical learning.

This begs the question of why we have not focused on learner emotive expressions and body language in our earlier writing. As noted, we have struggled to find appropriate theoretical tools to analyse emotional trajectories. Sinclair's recent work, however, introduced us to a range of valuable theoretical perspectives that support our efforts in addressing this gap. However logistical issues also threaten to hamper our future intentions to bring emotional trajectories centre stage in our analysis of mathematical learning events. Beyond the issue of the quality of images that journals may require is the issue of abandoning anonymity for the child in the study. If we consider children's expressions important to our analysis and to establishing embodied validity, then blocking out faces or eyes would defeat our aim as it would weaken the sensory experience of the reader. In the first author's institution many research proposals indicate they will not capture images of learners faces and will only video their hands, what they write, draw etc. This is a way to address the anonymity and vulnerability of the learners and can increase the likelihood of ethical clearance being granted. We argue that much is however lost when we

avoid capturing and communicating images of learners' emotional trajectories guiding learning. We lose sharing a richer, more human, understanding of the nature of learning interactions among participants and the central role of emotions in guiding mediation with our field. However, drawing on the adage across multiple languages that "a picture is worth 1,000 words"—meaning multiple ideas can be succinctly communicated in an image saving lengthy wordy descriptions—we can enrich readers' experiences. We propose this adage be increasingly *exploited* in support of embodied validity and readers' aesthetic experiences.

Concluding Remarks

In this paper we revisited our earlier analysis (Abtahi et al. 2017) of a learning event where we looked at an entirety of a learning experience, through what was said. Here, we highlighted that emotions are not secondary to learning; they actively shape interactions and decision-making. Ignoring emotions in video analysis not only reduces the depth and validity of our interpretations but risks sending implicit messages that children's emotional experiences are at best secondary and at worst irrelevant in mathematical learning. We understand that ethical restrictions related to anonymity limit the ability to analyse and share emotional expressions effectively. Despite these challenges, we argue that incorporating visual data of human expressions into research writing enriches our understanding by providing a more humanized and embodied perspective on learning. The integration of emotional and cognitive analysis offers a more nuanced and holistic view of learning interactions, enabling researchers and educators to recognize the deeply interconnected nature of emotion and cognition. Embracing this perspective, our field can move toward a more comprehensive understanding of learning.

Ethics Statement

This paper involves re-analysis of already published data. This said, Lila, now an adult, read the paper and gave consent for the inclusion of her images and data in the paper.

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One Teacher's Unique Experience Teaching Online During the Covid-19 Pandemic

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This paper explores the reflections of a primary teacher, Lisa, who began teaching her class just as Victoria entered a state-wide lockdown, prompting a shift to Emergency Remote Teaching. In professional interviews, Lisa evaluated the strategies she used during online instruction. She concluded that live sessions, interactive activities, small group workshops, and hands-on problem-solving tasks enhanced student participation and fostered more positive attitudes toward mathematics. Her experiences underscored the importance of building strong interpersonal relationships and adopting student-centred approaches to mathematics teaching.

In 2020, Victoria experienced its first lockdown due to the Covid-19 pandemic, leading to the closure of schools across Victoria. This lockdown prompted a sudden shift from face-to-face teaching to online learning, commonly referred to as Emergency Remote Teaching (ERT). ERT is defined as a temporary transition from standard teaching methods to online instruction in response to an urgent and crisis-driven situation (Hodges et al., 2020). ERT was a unique situation for everyone involved. While most teachers continued working with students they had previously taught in face-to-face classrooms, this article shares the story of one teacher who had a different experience teaching her students during ERT.

Lisa (pseudonym) had a unique experience teaching online, as she had not previously taught her students. Her school employed Lisa to teach her Year 5/6 class from the start of Term 2 in 2020, which coincided with the first day of Victoria's first lockdown. Although she had not taught the students before, she had spent an hour in their classroom at the end of Term 1, meeting them alongside their previous teacher. During this brief visit, Lisa noticed negative attitudes towards mathematics among the students. Lisa explained her school used Google Classroom, and Google Meets as their platforms during ERT. Online learning for her included live teaching, pre-made and teacher-created videos, and small group work.

Given her unique situation, Lisa prioritised student engagement. While other teachers could draw on their prior knowledge of their students to create an engaging environment online (Green, 2024), Lisa had limited information about her class. During an online professional interview, she described how she prioritised building relationships and getting to know her students to enhance their engagement and support them in developing a more positive mindset towards mathematics.

Lisa shared stories and anecdotes highlighting strategies she employed to build relationships and provide positive mathematical experiences. This article aims to support all mathematics teachers of young children (not just those teaching in online environments) by demonstrating how Lisa's focus on building strong relationships with her students first enabled her to implement strategies that encouraged more positive attitudes towards mathematics. Her story highlights that, regardless of how deeply ingrained negative attitudes may be, they can be transformed.

Background Literature

Student engagement has long been a focus of research. Over the years there have been many different definitions of engagement. For example, Bond et al. (2020, p. 3) described engagement as “the energy and effort that students employ within their learning community, observable via

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 189–196). Canberra: MERGA.

any number of behavioural, cognitive, or affective indicators across a continuum.” Pianta et al. (2012) reported on links between interactions and engagement in the classroom, noting that implementing strategies to increase the quality of relationships between teachers and students can increase student engagement and, in turn, their learning and development.

The loss of student-teacher and student-student interactions has been identified as one of the biggest challenges in teaching online (Ferri et al., 2020; Kalogeropoulos et al., 2021; Russo et al., 2021). With fewer opportunities for connection, teachers faced the task of actively creating meaningful interactions to foster engagement. Ferri et al. (2020) highlighted a clear link between interaction and student engagement, noting that the absence of social connection often led to disengagement. The goal was to maintain students’ active participation in mathematics by encouraging collaboration and interaction to support successful online learning.

Engagement was a focus for teachers as they made the transition into online learning. Given this was a new experience for all, it was unknown how students would adapt to this new way of learning. Since the Covid-19 pandemic, several studies have been published that have brought attention to how students engaged in online learning during ERT (e.g. Aguilar et al., 2022; Bissessar, 2021; Khlaif et al., 2021). Khlaif et al. (2021) highlighted the impact student engagement has on students’ learning during ERT. Factors such as teacher presence, quality of content, student attitudes, and home learning environments affected students’ engagement online. These challenges were further amplified for a teacher with no prior knowledge of their students, making engaging students online even more complex.

Aguilar et al. (2022) and Kalogeropoulos et al. (2021) recommended the use of live, synchronous instruction to enhance student engagement and foster a sense of connectedness between teachers and peers in online learning environments. Synchronous sessions provided more immediate opportunities for interaction, allowing teachers to respond to students in real time and enabling peer-to-peer communication, both of which were limited during ERT. Aguilar et al. (2022) found a strong correlation between live instruction and increased student engagement in online settings. However, concerns about student engagement in mathematics extend beyond the context of ERT. A substantial body of research has explored the persistent challenges of engaging students in mathematics (Attard, 2012; Jansen, 2020; Skilling et al., 2021). These studies examine various dimensions of engagement, behavioural, emotional, and cognitive, and often link them to students’ achievement levels. Despite these differences, a consistent message across the literature is the critical need to support and promote meaningful mathematics engagement for all students.

Negative attitudes toward mathematics can make it more challenging to engage students in the subject. Rojas et al. (2018) defined student attitudes toward mathematics as an individual’s overall interest, curiosity, and contentment with the subject, focusing on emotional rather than cognitive dimensions. These emotional responses were particularly strained during ERT, when students’ perceptions of learning were generally poor. Schmitt-Cerna et al. (2024) found that students reported low satisfaction with their online learning experiences during ERT, alongside similarly low attitudes toward mathematics. Such negative attitudes are not new. Not all students enjoy learning mathematics, and the experience of ERT likely exacerbated this for many. Reduced daily access to teacher support and limited opportunities for peer collaboration, both vital for engagement and confidence, may have deepened feelings of disconnection and frustration, especially for students who already struggled with the subject.

Research consistently shows that negative emotions toward mathematics tend to increase over the school years. By the end of primary school, more than half of students may present at-risk profiles in their relationship with mathematics (Hanin & Gay, 2023; Russo et al., 2023). While the timing and causes of this shift remain unclear (Hanin & Gay, 2023), Boaler et al. (2023) found that negative feelings toward mathematics are common. For some students, these

feelings can escalate into maths anxiety, which may lead to avoidance, reduced confidence, and diminished performance in the subject (Suárez-Pellicioni et al., 2016). To address these issues, teachers are encouraged to adopt a range of strategies that foster positive mathematical experiences and promote sustained engagement. These include connecting learning to students' interests, encouraging autonomy, providing effective feedback, linking mathematics to real-world contexts, and building strong, trusting relationships (Boaler et al., 2023; Skilling, 2014).

The Study

This paper is drawn from my doctoral study (Green, 2024) which investigated teachers' reflections on noticing as a key action of informal assessment in the primary mathematics classroom. The aim of the study was to identify what teachers notice that is meaningful for mathematical learning in different contexts and how the information gained through these moments can be used to enhance student learning. The study adopted a qualitative approach grounded in an interpretivist epistemological framework (Walter, 2019), emphasising the understanding of individuals' subjective experiences and the meanings they construct within their social contexts. The interpretivist lens provided depth and context, and highlighted the uniqueness of individual perspectives, aiming to uncover the complexities of human behaviour and social interactions.

Early in 2020, Victoria experienced its first lockdown due to the Covid-19 pandemic, which led to the introduction of online learning, commonly referred to as Emergency Remote Teaching (ERT). The eight teachers in the study participated in two semi-structured interviews via Zoom; the first in September, while teachers and students were still involved in ERT during Victoria's second lockdown of 2020, and the second in November, a few weeks after teachers and students had returned to the face-to-face classroom. These two interviews allowed for exploration and comparison of the two different learning environments.

Each interview was guided by four main questions, prompting teachers to recall recent mathematics lessons, highlight particular moments, describe any changes they had made, and share observations of specific students. The interviews were recorded, transcribed, and then coded using NVivo software. The aim was to identify instances where participants shared a response to a noticed moment. Thematic analysis was conducted to identify recurring themes and patterns in the data. Data was analysed using two noticing frameworks: the *Learning to Notice* framework (Van Es and Sherin, 2002) and the *Professional Noticing of Children's Mathematical Thinking* (Jacobs et al., 2010), as well as utilising a framework for engagement (Attard, 2012).

Three key themes emerged from the data: Adaptations to mathematical instruction—Teacher responsiveness, Student participation and dispositions, and Communication with families. This article focuses specifically on the theme of Student participation and dispositions, examining it across two contexts: the online environment during Emergency Remote Teaching (ERT) and the face-to-face classroom after ERT. These contexts are presented in this sequence, reflecting the order in which data were collected. Since no data were gathered prior to ERT, it is unclear whether the face-to-face data collected after ERT accurately represents pre-ERT classroom experiences. While all participants were teaching online during the first interview, Lisa was the only participant who had not met previously taught their students.

Lisa

This paper focuses on one participant from my study, Lisa, an experienced primary teacher with over fifteen years in the profession. Lisa began teaching her Year 5/6 class at the start of Term 2, which coincided with the onset of Victoria's first lockdown. As a result, Lisa entered ERT without having previously met her students, a situation that posed unique challenges. At the time, she was working at a government primary school in South-East Melbourne. The data

presented here are drawn from two interviews with Lisa. In the first, she reflected on her experiences during ERT, describing the strategies she used to get to know her students and engage them in mathematics, supported by illustrative examples. In the second interview, conducted after the return to face-to-face learning, Lisa discussed her continued focus on student engagement and how insights gained during ERT informed her classroom practice.

Finding and Discussion

Interview 1

Lisa's first interview took place late September in 2020, by which time Victoria had been in lockdown for over twelve weeks. The interview was conducted online via Zoom and focused on four key questions related to recent mathematics lessons and Lisa's observations of her students during those lessons. At the beginning of the first interview, Lisa discussed the challenges of teaching her students in an online setting, particularly as she had had not previously taught them in a face-to-face classroom setting:

It's hard, because if you don't - you don't know their situations, do you? Some are saying, we're doing it, but it's not submitted and what do you do, you know... I don't know these kids. I don't know how to push them.... I've picked up on it, obviously, who's high and low and all that - But, to begin with it's harder to give feedback because I don't know if that's your best... I don't know if you should be doing the extension maths stuff or, yeah, I don't know if you are a better speller than that or whatever-you know? So, that kind of stuff was hard. The feedback was hard. But I do know where they sit now.

Lisa's uncertainty about whether the work submitted represents a student's best effort or aligns with their actual skill level highlights the difficulty of assessing students authentically without the context provided by face-to-face interactions. She was cautious about pushing too hard, as it could risk further disengagement from her students. This quote ultimately highlights the critical role of relationships and context in effective teaching and assessment, reinforcing the importance of connection and understanding in fostering student growth and success, particularly in challenging environments like ERT. Despite these challenges, Lisa commented in her first interview that she felt like she had really gotten to know her students. She stated that she had gotten to know them as individuals, learning about their families, their interests and who their friends are.

Lisa had worked hard at the beginning, wanting to get to know her students and build strong relationships with each one. She reported implementing specific strategies, including running fun sessions every day such as social meets, games and "minute to win it" challenges, activities designed to bring joy, reduce stress, and foster a sense of community. Ferri et al., (2020) reported that implementing such strategies can enhance student engagement by promoting emotional connection, increasing motivation, and helping students feel seen and valued. Lisa noted that the online environment required minimal behaviour management beyond reminders like, "Don't write that in the chat box", allowing her to focus on interacting meaningfully with her students and fostering connections. Lisa's primary focus during this time was less on teaching specific content and more on establishing positive relationships.

Lisa had met the students for just one hour before officially starting with them. During this brief interaction, she learned that they exhibited particularly negative feelings toward mathematics. After asking Lisa what her thoughts were on the subject, they stated, "Oh, we hate maths". Knowing these students had a negative mindset towards mathematics, Lisa wanted to change that. She described recognising the importance of building strong, trusting relationships with her students first (Skilling, 2014). One example that Lisa shared was a student she believed was more capable than his report had stated. Lisa met with the student and let him know that she could see that the mathematics tasks were relatively easy for him. She invited him to a

mathematics extension group she was running, afterwards he came along to every session. Lisa commented:

Every time we do the maths extension group; I leave them with an additional task. So, we do a couple of activities and then I say, right, well, if you want an extension of what we just did, here it is. If you want to go ahead and try it and try and find the rest of the answer at home. And every time, without fail, he's done it. And so, I've dropped off a certificate at his house every week.

Lisa explained that she wanted to recognise and reward his work.

Once Lisa felt those close relationships were established, she focused on building the students' curiosity and interest, showing the students how enjoyable mathematics could be. Lisa shared examples of activities she implemented, including games, problem-solving tasks and hands-on lessons such as when teaching angles. One example was an inquiry project she implemented over two weeks where the students designed and constructed a box to hold two pyramids. Lisa had created a story to frame the task, explaining that the box was to send a package to their previous teacher, who was on holiday. She commented that it was well received, and the students really enjoyed it. Russo et al. (2023) noted that interesting tasks can be engaging for students and can lead to them seeing mathematics as useful and important.

When asked to share a highlight from an online mathematics lesson, Lisa spoke about their Google meets. Every day at 11:30, she offered a "maths meet" or "drop-in session" where students could work on their tasks together and receive help if needed. This highly successful daily meeting of students echoed the findings of Aguilar et al. (2022) who found a strong correlation between live instruction and student engagement. Although initially, attendance was sparse, over time, more and more students began to join. Her goal was to create a space where students could feel comfortable participating, could enjoy the experience, and not perceive it as extra work. This links to the work of Boaler et al. (2023), who stated that small group work can lead to deeper engagement and more positive experiences with mathematics.

When asked if her strategies were effective, Lisa expressed confidence that they were and reflected on an email she had received from a parent regarding the small group sessions:

One of the mums sent me a message just going, you know, he loves – he can't wait for his 11:30 maths meet... he's actually teaching his brother how to do fractions, who's in Year 7, and he's in Grade 5... he enjoys the maths now.

Another story Lisa shared was regarding Lucy, who she mentioned was probably the most negative of all the students when it came to mathematics:

We've got a little girl, Lucy, hates maths. That was the first thing she told me. She was really reluctant to come along to the group sessions... really didn't understand that asking for help was a good thing. So, she came and... it was interesting to watch her become more confident of herself, just through coming, and being open to, well, I don't know, I'm going to come to the group...and then she was one of the ones who would actually share her ideas the quickest.

Lucy's initial reluctance and dislike of mathematics illustrate how negative attitudes toward a subject can create barriers to participation and learning. By building a connection through small group sessions and providing a supportive environment, Lisa was able to engage Lucy in mathematics, cultivate a sense of belonging, and nurture her growing confidence.

Lisa also spoke about a student named Felix. The previous teacher had informed her that he was quite strong in mathematics, so she was surprised when his parents mentioned he was struggling with the work at home. Felix began attending the 11:30 "maths meets", and it did not take long for him to get back on track and regain his confidence. Even after overcoming his initial struggles, Felix continued to join these sessions:

He would just jump on at 11:30 every day and go... 'I've done it today', or 'It was so easy today', you know, or 'I've only got this little bit to go'. And he would share what he's done, every day. And then I'd go, 'Oh, that's awesome. Well, do you want to hang around and hear it again or are you,

you alright to go?’ And he’d go, ‘No, I’m going but I just wanted to tell you that I’ve done it’. You know, so, he was checking in every day, just proud of himself that he’d done it.

Although Felix no longer required the support, he still wanted to connect in with Lisa and was excited to tell her about his progress. By implementing these strategies, Lisa shared that she was able to get to know her students individually and build strong relationships with them while teaching online during ERT. Lisa went on to say:

It will be interesting.... at least I know now who to pull down for a small group... straight away I know I’ve got two boys in there who are Year 7 plus. They’re in Grade 5. I do know who’s pretty confident in it [mathematics] and I know who isn’t. I will definitely be able to take what I’ve gained from remote [learning], back in there [face-to-face classroom]. It’s not like starting over- I’ll know straight away who to look out for.

She reflected on what she had learned about her students and how this information could be applied in the face-to-face classroom.

Interview 2

Lisa’s second interview took place in November 2020, several weeks after schools in Victoria had returned to face-to-face teaching. Like the first, this interview was conducted online via Zoom and followed a similar structure, with four main guiding questions. Lisa reflected that her efforts to connect with students during Emergency Remote Teaching had been successful, noting that the transition back to the classroom was eased by the fact that she already felt she knew her students well:

I felt like I knew them coming back and yeah had that connection with them, so that all felt really good. I guess it was weird the first couple of weeks seeing who their friends were because they told me, but it was hard for me to connect the friends, and so then seeing who are chatty with and who they get along with and yeah seeing them in a social way was a bit different.

She continued to prioritise engagement, considering the students’ previous mindset towards mathematics and the extended period of time learning from home. Lisa was determined to show her students how enjoyable mathematics could be:

I have been trying to make maths fun because they’re all so negative about maths. One girl who is the most negative, Lucy, I said- ‘you are my gauge, and every lesson I am going to ask you’ and she is still really reluctant to give me much even though I know that it’s better that she is liking it more. She will give me a bit, but she doesn’t want to get too excited, but others are saying ‘Oh maths is fun. I didn’t realise maths could be like this.’ So that’s been really good.

Lisa shared a few specific examples from enjoyable mathematics lessons she had implemented in the face-to-face classroom:

...the Lego shop they loved that... that was a massive hit early in the term and then like you know volume and capacity – took them out in the sand pit – always a fun thing to do. Just trying to make it – show them that it can be interesting.

Lisa continued looking for ways to include more hands-on lessons, problem-solving tasks and games to engage the students (Russo et al., 2023). Lisa used the game “Buzz” as an example where students stand in a circle and go around the circle saying numbers in a counting pattern. After introducing this game to the students, they would ask to play it every day. When asked if she felt she was winning the students over, Lisa responded: “Getting there, they are at least commenting ‘That wasn’t so hard’ or ‘That was fun’... yeah there’s a bit of a way to go. Obviously, they have had this [negative] mindset for a long time.”

Although it took some time to get up and running, Lisa continued to run the extension group she initiated during ERT in the face-to-face classroom. The students thoroughly enjoyed these sessions:

They sit out on this table together and they work through their own, the rest of the lesson and then myself or the integration aid will touch base with them at the end, and they love it. They are just like ‘Oh we love maths so much now’. I said to Dean, ‘Have you learnt something new?’ I said, ‘Was

there something today that you didn't know already?' and he is like 'Yep' 'Oh are you sure? Because I don't know if there is much that you don't know' and 'He's like yep I know' – he's like 'Yep there was something' and they have just been thriving off it.

By offering tailored opportunities that both challenge and excite, Lisa successfully engaged the students in meaningful mathematics learning (Boaler et al., 2023). This approach fostered a genuine love for the subject, with students thriving as they felt challenged, supported, and valued in their progress. Establishing a positive and supportive learning environment was key to transforming their attitudes toward mathematics.

Feedback can assist in developing connections between teachers and students and lead to enhanced motivation. Lisa found it important to debrief with the group after the lesson. She shared a story from a conversation after one particular lesson:

Adrian was like we spent 10 minutes on one question talking about it, and I said, 'You guys were on task talking for 10 minutes about one question?' and he was like 'Yes'. I said 'That's amazing. I am so happy to hear that'. I said, 'Really? Were you guys on task talking about that for 10 minutes?' – 'Yes, because we were all different on it and we had to get like our ideas out' and yeah, they just were really thriving off it and I contacted a couple of parents and said 'Look I am doing this for them' and they were like, 'Yeah we know we've heard – they come home and they are so excited about it'.

These students enjoyed their mathematics experiences so much that they went home and shared them with their families. Lisa recounted a story about one student who joined the group after overhearing a classroom conversation. Curious about the activities, he asked a peer what they were doing and eventually approached Lisa himself to ask if he could join. Lisa recalled being surprised, as this was a student who had previously done only "the bare minimum" during ERT. She was impressed by his willingness to challenge himself and his confidence in asking to join. At the end of his first lesson, she offered him feedback: "That was really great that you wanted to do that. You challenged yourself, stayed focused, and stuck with it without getting distracted." This moment highlighted his growth, showing his willingness to step outside his comfort zone and take ownership of his learning. Lisa had created a safe and supportive environment where he felt comfortable enough to seek out a new challenge.

Conclusion

Lisa's experience highlights the critical importance of building strong relationships as a foundation for student engagement, particularly in challenging and unfamiliar contexts like ERT. Despite not having met her students prior to lockdown, Lisa prioritised connection and trust, using live interaction, engaging activities, and personalised support to shift her students' negative attitudes towards mathematics. Her success demonstrates that meaningful engagement is possible even in online settings when teachers are responsive, reflective, and committed to understanding each student as both a learner and an individual.

Positive teacher–student relationships are central to fostering engagement and learning, and this is especially critical in mathematics, where student attitudes and self-beliefs can be fragile (Boaler, 2016; Attard, 2011). Lisa's story offers a powerful reminder that fostering positive dispositions towards mathematics begins with seeing and valuing students, something that holds true in both online and face-to-face settings. While her experience is just one example and cannot represent all contexts or student experiences, it does illustrate the potential impact of relational teaching. Every interaction, whether virtual or in person, is an opportunity to notice, connect, and nurture a love for learning mathematics.

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Exploring How Mathematics Teacher in Indonesia Perceive Culturally Responsive Pedagogy (CRP) in Mathematics

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Contextualisation in mathematics offers a solution for bridging mathematical concepts with the real world. However, the limited attention given to cultural elements raises the question of whether these contexts are truly relevant to students' lives. Furthermore, the restricted implementation of cultural integration in mathematics teaching only through ethnomathematics highlights the need to explore teachers' understanding of the connection between mathematics and culture. The findings indicate that teachers perceive culture mainly as tradition and generally view mathematics as a culturally neutral discipline, which justifies the potential of Culturally Responsive Pedagogy.

Introduction

Contextualising mathematics to make it more relevant to real life has long been suggested to enhance student engagement and understanding (Boaler, 1993). However, its implementation faces several challenges, including the frequent use of superficial, “camouflaged” contexts that fail to foster critical engagement (Boaler, 1993; Wijaya et al., 2015), insufficient attention to cultural relevance (Spooner, 2023), the incorporation of cultural elements in a purely decorative manner (Laurens et al., 2018; Madrazo & Dio, 2020), and an overreliance on teachers' assumptions about students' familiarity with the context (Boaler, 1993; Widjaja, 2013). While the Realistic Mathematics Education (RME) approach offers potential solutions (Freudenthal, 1991), the question of how “realistic” a problem truly is, without considering the deeper cultural meanings behind these contexts, means many approaches remain culturally neutral despite their aim for authenticity.

The lack of attention to cultural elements in mathematics teaching often stems from teachers' perceptions of mathematics as culturally neutral and overlooking the influence of societal values on its development and instruction (Bishop, 1990). Additionally, cultural integration in mathematics is frequently limited to ethnomathematics, defined as the cultural anthropology of mathematics that explores the math embedded in a cultural group's practices and problem-solving strategies (Ambrosio, 1984). Unfortunately, many practices of ethnomathematics are restricted to the framing of local cultural practices within existing mathematical knowledge, and it only including traditions from certain communities (see the literature review on ethnomathematics from Hidayati & Prahmana (2022)). This narrow focus creates a situation where a student who is not part of that specific community has their own cultural dimension overlooked and not integrated into their mathematics teaching.

Research consistently demonstrates that students learn more effectively when instruction is connected to their environment and lived experiences (Dewey, 2013; Boaler, 1993; Wijaya et al., 2015). While disparities in mathematics achievement in Indonesia are evident, with national exam results showing students in Java outperforming those from eastern regions like Papua and Maluku (Suputra et al., 2021), these gaps cannot be attributed solely to socioeconomic (SES) factors (Aditomo & Faridz, 2019). This paper argues that a more fundamental issue lies in how the educational system engages with the nation's rich cultural diversity. Research has shown that students, particularly those from low-SES or marginalised backgrounds, often struggle when their personal backgrounds and lived experiences are not explored or valued in academic settings (Ladson-Billings, 1995). When mathematics is presented as a culturally neutral discipline (Bishop, 1990), it overlooks the vast potential of Indonesia's diverse linguistic and (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 197–204). Canberra: MERGA.

ethnic landscape to serve as a foundation for meaningful learning. Therefore, the need for Culturally Responsive Pedagogy (CRP) emerges not just as a tool to close a generic achievement gap, but as a direct pedagogical response to this cultural disconnect. CRP is an educational approach that fundamentally recognises and values the cultural backgrounds, experiences, and identities of students as essential to the learning process (Gay, 2018). It aims to bridge the divide between formal, in-school mathematics and the mathematics present in students' daily lives by integrating their cultural knowledge and lived experiences directly into the curriculum (Gay, 2018). By doing so, CRP seeks to empower students, nurture their cultural competence, and foster their academic success (Ladson-Billings, 1995).

Indonesian students' rich linguistic and cultural diversity profile which involves 718 different local languages, and 1340 ethnicities (CIPS, 2025) offers great potential for creating inclusive and effective teaching practices, yet how teachers understand and utilise this diversity in mathematics education remains unclear. While ethnomathematics provides a framework for cultural integration, its application in Indonesia often lacks depth and fails to fully address the complexities of students' cultural identities (Hidayati & Prahmana, 2022). This study seeks to bridge this gap by exploring how teachers conceptualise and implement cultural integration. Re-examining teachers' understanding of what it truly means to integrate culture into mathematics teaching, particularly in the Indonesian context, has become increasingly important. This study aims to address this gap by exploring how teachers conceptualise cultural integration and examining its implications for more culturally responsive mathematics education.

Background Literature

Research of the relationship between culture and mathematics education highlights that mathematics is a cultural product which is shaped by human experiences, social interactions, and cultural values. Looking from anthropological perspectives. Wilder (1978) argued that mathematicians themselves are cultural bearers, as they create and transmit mathematical knowledge within their own cultural frameworks. Furthermore, White (1959) connected culture and mathematics by defining culture as a system which relates humans to both their environment and to one another. Bishop (1990) expanded on this by proposing principles for understanding mathematics as a cultural phenomenon, emphasising values, symbolic technologies, and the enculturation process in mathematical knowledge. However, mathematics education often fails to integrate and acknowledge cultural dimensions, leading to the misconception that mathematics is culturally neutral and ignoring the influence of societal values on its development and instruction (Bishop, 1990).

D'Ambrosio (1984) then introduced the concept of ethnomathematics to address this gap by exploring and acknowledging the mathematics as embedded in cultural groups' practices and problem-solving strategies, further defining it as the cultural anthropology of mathematics education, called ethnomathematics. Despite these perspectives, cultural considerations in mathematics education are often limited to surface-level cultural references without fully exploring how mathematical knowledge and values emerge from social contexts (Acharya et al., 2021). This is also supported by Boaler (1993), who argues that the connection between culture and mathematics education should go beyond simply learning about specific cultural groups. Instead, it should involve exploring students' own ethnomathematical practices: their coping mechanisms, how they navigate life, and how they address everyday problems. Boaler (1993) also recognised that many mathematics teachers have approached culture in ways that may not fully capture its richness or relevance to students' lives. By broadening these approaches, educators can better harness the potential of cultural connections to make school mathematics more meaningful and connected to students' everyday experiences

It is therefore essential to revisit how mathematics teachers perceive the relationship between mathematics and culture. Equally important is the adoption of pedagogical approaches that value culture beyond traditional practices and artifacts of specific communities. As emphasised by the National Council of Teachers of Mathematics in its position statement on the intersection of mathematics and culture, schools are encouraged to embrace culturally responsive mathematics instruction that reflects and respects the diverse experiences of all students (NCTM, 2024).

Culturally Responsive Pedagogy in Mathematics

Culturally Responsive Pedagogy (CRP) is an educational approach that recognises and values the cultural backgrounds, experiences, and identities of students as essential components of the learning process (Gay, 2018). It seeks to create inclusive and equitable learning environments by integrating students' cultural knowledge, traditions, and lived experiences into the curriculum and teaching practices. CRP aims to empower students, particularly those from historically marginalised or minoritised groups, by fostering academic success, cultural competence, and critical consciousness (Ladson-Billings, 1995; Gay, 2018). At its core, CRP challenges traditional, Eurocentric educational practices that often exclude or devalue the cultural assets of diverse student populations, instead promoting a pedagogy that is both affirming and transformative. Originating in the U.S. (Keazer & Nolan, 2023), CRP benefits all students by making learning more meaningful, for instance, by connecting in-school mathematics with students' out-of-school experiences (Nasir et al., 2008; Ladson-Billings, 2006).

From a sociocultural perspective (Vygotsky & Cole, 1978), culture is defined as the organised daily practices (Wertsch, 1998) and social interactions mediated by tools and symbols (Nasir & Hand, 2006). This extends beyond traditional values and language to include communication and learning styles (Gay, 2018). CRP emerged from the need for teaching strategies that honour the cultures of students of colour and bridge their home and school knowledge (Ladson-Billings, 1995). The core aims of CRP are to develop students' academic skills, nurture their cultural competence, and foster a critical sociopolitical consciousness. This approach allows students to maintain their cultural integrity while achieving academic success and empowers them to recognise and critique social inequities.

In the subject of mathematics, CRP becomes the guideline for building the mathematics instruction in the classroom which consists of three components: 1) building on students' informal mathematical knowledge through informal and cultural and experiential knowledge; 2) developing tools of critical mathematical thinking and critical thinking in general; and finally, 3) orientation to students' culture and experience (Gutstein et al., 1997). The study highlights that teachers should use what they understand and know about their students' cultural background to improve students' learning in mathematics and helping students to develop the critical approaches toward the knowledge.

CRP research, particularly foundational work in the United States, has focused on solving the achievement gap and issues of educational access between students of colour and white students (Gutstein et al., 1997; Ladson-Billings, 1995). In Indonesia, a similar achievement gap exists, typically seen in how Javanese students often outperform non-Javanese students, particularly those from eastern regions of Indonesia (Kismiantini & Setiawan, 2020). This indicates a clear need for CRP in Indonesia to address these disparities in mathematics education. However, before CRP can be implemented effectively, it is essential to first understand how Indonesian teachers currently connect culture to teaching and learning mathematics. Without this foundational insight, efforts to promote culturally responsive practices may lack relevance or fail to address the unique challenges faced by teachers and students in the Indonesian context. Therefore, this study seeks to explore the perceptions and

experiences of teachers in Indonesia regarding the integration of students' cultural backgrounds into mathematics instruction.

The research questions guiding this study are:

- What do teachers perceive about learning mathematics using students' cultural experiences?
- How do teachers experience connecting cultural experiences to learning mathematics in the classroom?

By examining these aspects, the study intends to provide insights into the potential for more culturally responsive mathematics teaching practices.

Research Method

This study aims to answer two research questions using a qualitative research design to capture both perceptions and practices through in-depth interviews with two experienced Mathematics teachers. These interviews will provide insights into current teaching practices in Mathematics and the teachers' perceptions of culturally responsive pedagogy (CRP). The data from the interviews will be analysed using thematic analysis (Braun & Clarke, 2006) to reveal how teachers perceive and implement CRP in their Mathematics instruction.

Teacher Backgrounds

Both teachers have extensive experience teaching high school Mathematics, with over 5 years each. The first teacher (T1), an ethnic Butonese, was born in Maluku, an eastern province in Indonesia, known as "Spice Islands", for its historical role in the global spice trade. Maluku is comprised of numerous islands with rich cultural diversity, and a mix of ethnic groups, languages, and traditions. In addition to that, ethnic Butonese culture is known for seafaring traditions and maritime skills due to the proximity to the sea. T1 has been teaching for 9 years in a vocational high school specialising in computer engineering. The second teacher (T2), from East Java, has been teaching in the region of Surabaya for 13 years since graduating from university. East Java, one of Indonesia's most populous and economically dynamic provinces, is home to Surabaya, the nation's second largest city after Jakarta and a hub of education and industry. She teaches in a private high school in Surabaya, where students often come from more urban and economically advantaged backgrounds.

The two teachers were purposefully selected to compare perspectives from regions that mirror the national achievement gap. T1 from Maluku and T2 from the major educational hub of Surabaya, East Java, which represent contrasting socioeconomic and cultural contexts. This selection provides a rich basis for exploring how these differences may influence their perceptions and practices of CRP. Finally, given their extensive teaching experience, both teachers have encountered and managed students from diverse backgrounds and navigated various dynamics in teaching Mathematics. They have each lived in their current communities, where they also teach, for over five years, which implies familiarity with the local context. This diverse background contributes to a unique perspective that influences their teaching approach. The varied backgrounds of both teachers suggest that their different experiences and cultural perspectives may impact their perspectives on integrating culture into mathematics education.

Results and Discussion

RQ 1: Perception of Culturally Responsive Pedagogy

Both teachers were asked about CRP, translated to Bahasa Indonesia as "*Pendidikan tanggap budaya*." They admitted that they had not heard the term before but provided their interpretations of it. T1 and T2 explained that CRP involves integrating local culture into education. They perceived "culture" as relating to the specific traditions of ethnic groups, such

as the Javanese or Butonese, which include traditional clothing, food, dance and customs. Interestingly, while T2 expressed that mathematics and culture have a connection, T1 expressed the belief that culture has little relevance to Mathematics education, viewing it as more pertinent to subjects like history and social science. Additionally, T1 mentioned that no competency standard of mathematics in curriculum has stated that mathematics has a connection with culture. This viewpoint is illustrated in the following verbatim statement:

Translation:

History and social studies teachers often integrate culture into their lessons, but in mathematics, it's different... well, because of the demands of the Basic Competencies (KD) or Learning Outcomes (CP).

This reflects T1's understanding that Mathematics is culturally neutral, aligning with Bishop's (1990) findings, which asserted that many Mathematics teachers still perceive the subject as unrelated to culture. It is ironic that a finding from 35 years ago remains relevant today. This perception leads teachers to overlook cultural differences among students, assuming these differences do not impact educational achievement. In fact, many underachieving scores come from students with minority backgrounds, who have their own cultural differences (Abdulrahim & Orosco, 2020).

T1 also noted that cultural engagement varies by region. Some areas are still rich in culture because people there adhere to traditions and practice them daily, while other areas have lost touch with their cultural heritage. This observation underscores the teacher's justification for the lack of awareness about integrating culture into Mathematics education.

Translation:

Well, you can't really say I'm Ambonese, nor can you say I'm Butonese, because even though I was born and raised in Ambon, Maluku, the cultural aspect here is a bit different. In Ambon, it's not like, for example, Yogyakarta, which, even though it's a city, still has a very strong cultural identity. Here, it's different—people don't really hold onto traditional culture as much anymore.

This excerpt demonstrates that the teacher viewed culture mainly as local traditions, which may pose a challenge to CRP. Culture is a broad concept encompassing values, beliefs, communication styles, and language (Gay, 2018). For instance, in mathematics, teachers must bridge the gap between students' diverse everyday language and formal mathematical language to be effective (Haryani & Harso, 2023). When a teacher feels disconnected from their own culture due to lapsed traditions, it reveals a lack of cultural self-awareness. This self-awareness is a foundational prerequisite for culturally responsive practice; without it, a teacher cannot effectively adjust their teaching for diverse students (Howard-Hamilton, 2000). Therefore, this perspective does not characterise a culturally responsive teacher but rather highlights the urgent need for one.

Regarding the T2's perception of CRP, initially, she believed that culture referred only to traditional local customs specific to certain communities. However, when the researcher inquired about the backgrounds of her students, she revised her understanding. She acknowledged that culture encompasses the backgrounds and situations familiar to the students, which should be integrated into the learning process. She elaborated by noting that her students come from various islands across Indonesia, such as Sumatra and Kalimantan, bringing diverse traditions from their ethnicities. This realisation highlighted that culture includes the immediate contexts related to the students, making learning more meaningful by connecting with their surroundings. This perspective is evident in her following verbatim statement:

Translation:

Initially, I thought culturally responsive education was mainly about integrating local traditions into teaching. But after reflecting on my students' diverse backgrounds, I realised it's more about connecting learning to their everyday environments and experiences. Since my students come from

various regions in Indonesia, involving their surroundings in projects makes the learning more meaningful and relevant to them.

The T2's perception aligns with the rationale behind introducing CRP, as it addresses the gap between school mathematics and the mathematics used in everyday activities outside of school (Civil, 2016; Nasir et al., 2008; Gay, 2018). By making mathematics more relatable to students' backgrounds and experiences, CRP aims to engage and motivate students to learn more effectively. The next discussion will focus on the practice of teaching mathematics to identify the alignment with CRP.

RQ2: The Current Practice of Teaching Mathematics

The two participating mathematics teachers demonstrated distinct pedagogical approaches, particularly in how they contextualised lessons and engaged with students' backgrounds. T1 described her teaching method as contextual, aiming to make mathematics more relatable to daily life. For example, she used a shopping scenario in the market to teach systems of equations and the distance between the school and a local tourist destination for lessons about scaling. T1 believed this approach helps students understand concepts more easily and visualised the problem-solving steps. However, her methodology revealed a critical limitation when viewed through the lens of CRP. T1 explained that she consistently introduces the general mathematical concept first, and only then asks students to apply it to a surrounding environmental context. She described this process for a lesson on trigonometry:

Translation:

For example, if I want to teach trigonometric ratios, I usually start by teaching the general concept first. After that, I encourage them to imagine scenarios like, 'What if we were to measure a building?' Then, we go outside because our school area is quite small. We head to the neighbourhood, go downstairs, and I ask them, 'Can you measure the building using ratios? How would you apply the ratios?' We already have the tools prepared, so then we start using them.

While this approach aims to make mathematics meaningful, its structure is teacher-centric. By starting with a pre-determined concept and then finding a single context for all students to explore, the method imposes a uniform learning experience rather than drawing from the students' diverse backgrounds. This "one-explanation-fits-all" model contrasts with the principles of CRP, where instruction should be based on "mining students' thinking" and helping them see their own cultures valued in the classroom (Ladson-Billings, 1995).

This disconnect from students' personal contexts was evident during the interview. When asked about her students' family backgrounds, such as their parents' occupations, T1 could only offer a vague possibility, stating that some "*might work as daily labourers, but I'm not sure.*" This response reveals a lack of specific information about students' family contexts and suggests that communication with students about their lives is not currently being used to inform and adjust classroom instruction. Instead, the teacher's questioning is focused on determining if prerequisite material has been achieved. This approach, which prioritises content over personal inquiry which acts as a barrier to culturally responsive teaching as teachers do not understand their students' diverse backgrounds (Gay, 2018; Rychly & Graves, 2012).

A significant challenge T1 identified was connecting mathematics to the vocational context of her students. She stated her primary difficulty was "*connecting mathematics with [my] vocational major... computer and network engineering.*" This difficulty in finding applications suggests a potential gap between her knowledge of mathematical procedures and the deeper conceptual understanding required to adapt concepts to specialised fields. Finally, her struggles point to a need for a more reflective process, beginning with understanding her own culture and learning about others, which are emphasised as key characteristics of a culturally responsive teacher (Dewey, 2013; Howard-Hamilton, 2000).

In contrast to T1's method, T2 described a problem-based approach where she presents a problem before formal instruction. Using linear programming as an example, she explained that even without prior knowledge of the topic, students use their prerequisite skills to devise solutions. This method allows for trial-and-error and fosters lively student debate as they work to solve the problem. As T2 described:

Translation:

Initially, I don't start by explaining. I usually give them a problem first, like, 'What do you think, is this a profit or something else?' And a few times, I've tried using English. On average, when we take English problems, there are some questions that don't clearly indicate whether it's about profit or loss. So, I encourage them to think about it, like, 'As a business owner, how would you determine if it's a loss or a profit?' Eventually, they try to figure it out, and it turns into a fun debate among the kids, arguing whether it's a profit or a loss. It's quite engaging for them.

T2's pedagogical decisions appear to be rooted in constructivist, inquiry-based approach to foster conceptual understanding. Rather than begin with explaining a concept, T2 presented the authentic problem situated in a daily life situation, which requires student to have "the productive struggle", while also providing an engaging activity. By reflecting on her students' context and adapting her teaching, T2 takes the initiative to create more meaningful learning opportunities, a key practice for a culturally responsive teacher (Ladson-Billings, 1995; Dewey, 2013).

Finally, T2's primary professional challenge was "finding on wider knowledge on how to teach mathematics even better [*sic*]". While this reflects a commitment to growth, this study's findings suggest that such knowledge cannot be limited to mathematical content alone. Rather, it must also include a deeper understanding of the characteristics of a culturally responsive teacher to fully address the needs of diverse students.

Conclusion

This study sheds light on how teachers in Indonesia perceive CRP in mathematics. Both teachers viewed culture as limited to traditions, overlooking students' diverse backgrounds. Additionally, they found integrating culture into mathematics teaching and learning to be challenging, not only due to a lack of guidance but also because the government curriculum does not emphasise it in the Core Competencies (CP) and Basic Competencies (KD). Furthermore, *Permendikbud* (Government Law) No. 79 Year 2014, which addresses the integration of local contexts into teaching and learning, is primarily limited to art, dance, and social sciences, leaving mathematics perceived as culturally neutral. This perception impacts how teachers approach classroom instruction, often assuming one context fits all students, which can lead to a lack of context for student learning and subsequent student disengagement. This study highlights the need for greater awareness of the importance of culturally responsive mathematics classrooms. Future studies could analyse government textbooks, as the primary teaching materials, to assess whether they are culturally responsive or not.

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Examining the Pedagogical Practices of a Teacher in a High Mathematical Wellbeing Classroom: An Exemplary Case Study

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This study advances mathematical wellbeing (MWB) research by mapping specific pedagogical practices associated with high class-level MWB. Through examining a Year 5 classroom demonstrating high MWB in a sample of 350 classes (4137 students; 160 teachers), the analysis revealed five key pedagogical practices: substantive conversations, challenging tasks, high expectations, social norms, and mathematical practices. The teacher emphasised student-centred and collaborative approaches while minimising teacher-directed instruction. These findings highlight ways to create enabling classroom environments where students can thrive mathematically.

Mathematics is an important school subject and a predictor of future educational and employment opportunities; however, students often report mixed feelings towards it - either loving or hating it. Associating negative emotions (e.g., anxiety) with mathematics inevitably contributes to lower achievement and limits students' pathways beyond school (Watt et al., 2019). While students typically begin primary school with a positive disposition towards mathematics, this often declines as they progress through their education (Grootenboer & Marshman 2015; Watt et al., 2019). This suggests that it's not the discipline of mathematics per se but rather how students experience mathematics (i.e., pedagogies and teacher practices) that impacts their disposition towards mathematics. This study offers an innovative contribution to the field by examining the relationship between pedagogical practices and student mathematical wellbeing (MWB).

MWB is defined as the fulfilment of values whilst learning mathematics accompanied by positive feelings (e.g., enjoyment) and functioning (e.g., engagement, accomplishment) in the discipline. Recent international studies purport that students have high MWB when given opportunities to fulfil seven core – or 'ultimate' – values (i.e., accomplishments, cognitions, engagement, meaning, perseverance, positive emotions, and positive relationships) through the mathematics learning process (Hill et al., 2021; 2022). I developed a MWB framework in response to calls for wellbeing to be integrated into existing subject disciplines (e.g., Waters, 2021) and addressing the often deficit and binary discourse (e.g., "I am/am not a maths person") prevalent in mathematics education. Over several decades, research has documented pervasive negative trends and experiences in mathematics education, including mathematics anxiety, disengagement, declining or stagnating achievement, and negative attitudes toward mathematics (Grootenboer & Marshman, 2015; Hill & Hunter, 2024; Watt et al., 2019). While addressing these challenges remains important, an emerging strengths-based MWB perspective argues for greater attention to understanding and promoting positive mathematics experiences and outcomes (Hill et al., 2021).

While growing research explores student MWB through self-reported surveys, there remains a limited understanding of how pedagogical practices influence class-level MWB. The present study addresses this gap by focusing on a classroom demonstrating high class-level MWB. Through detailed analysis of classroom observations using a productive pedagogies framework, teacher and student surveys, and an email, we examine the practices of Karla (pseudonym), whose Year 5 class demonstrated one of the highest MWB scores across a large sample (350 classes, 4137 students). By investigating Karla's classroom as an exemplary case, I aim to identify specific teacher characteristics and pedagogical practices that may support

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MWB. In the current study, I respond to the following research question: *What teacher characteristics and pedagogical practices are evident in a classroom demonstrating exemplary levels of student mathematical wellbeing?*

Background and Theoretical Framing

Mathematical Wellbeing

Mathematical wellbeing (MWB) stems from value fulfilment theory, which suggests that wellbeing occurs when individuals successfully pursue and fulfil their core values (Tiberius, 2018). Rather than solely addressing negative experiences like mathematics anxiety, a value fulfilment approach emphasises proactively supporting students to thrive by aligning classroom experiences with what they value most in mathematics learning. Through a systematic review of mathematics education values literature and various international empirical studies, Hill and colleagues (Hill et al., 2021, 2022, 2024) identified seven ultimate values that consistently support students' positive mathematical experiences. These include accomplishments (e.g., mastery, achievement), cognitions (e.g., mathematical understanding, skills), engagement (e.g., interest, focus), meaning (e.g., relevance, utility), perseverance (e.g., sustained effort, productive struggle), positive emotions (e.g., enjoyment, pride), and positive relationships (e.g., teacher support, peer collaboration).

Concerningly, students' MWB significantly declines as students progress through school (especially positive emotions and engagement), with the sharpest declines during the primary to secondary school transition (Hill & Hunter, 2024). This trend suggests a need to better understand how teachers can proactively cultivate enabling classroom environments and pedagogies that support students in fulfilling these seven values to foster MWB. These seven ultimate values can be fulfilled in various ways. For example, a teacher might support students' relationship values through collaborative tasks and an inclusive classroom climate while fulfilling students' valuing of meaning through contextual problems connected to students' lives. The way teachers position themselves and their students (e.g., power dynamics), their personal characteristics (e.g., patience), specific pedagogical choices (e.g., group work), or mathematics content selection (e.g., meaningful tasks) all potentially influence students' opportunities to fulfil these ultimate values.

Importantly, a MWB approach offers a preventative rather than remedial strategy. Instead of waiting to address the anxieties, negative emotions and attitudes after they develop, focusing proactively on supporting students to thrive in mathematics may help prevent the disaffection and anxieties students experience. This approach aligns with broader shifts in psychology toward strength-based approaches that emphasise building and affirming positive capacities rather than solely addressing deficits (Seligman, 2011). The MWB framework provides a useful tool to understand what aspects are working well for students (i.e., values that are fulfilled) and what areas need more attention (i.e., values that remain unfilled or are conflicted). This framework also provides a foundation for examining how specific teacher practices and characteristics might support student enjoyment, engagement, persistence, and performance in mathematics.

Productive Mathematical Pedagogies

The productive pedagogies framework offers a structured approach to examining quality teaching practices across three key dimensions: Intellectual quality, cultural connectedness, and supportive environment (Lingard et al., 2003). This framework has been adapted for mathematics education to document specific pedagogical practices that promote deep learning while supporting student diversity and wellbeing (Hunter et al., 2024). The intellectual quality dimension focuses on embedding high-level mathematics content through practices like

challenging tasks and substantive discourse. Cultural connectedness encompasses practices that recognise and build upon students' cultural knowledge and experiences. The supportive environment dimension attends to creating inclusive classroom contexts where students feel valued and supported to participate (Hunter et al., 2024).

The choice of this observational tool was deliberate, as it was developed to connect the theoretical frameworks of culturally sustaining, ambitious, and wellbeing-oriented pedagogies. The three dimensions align closely with the underpinnings of MWB. For example, the intellectual quality practices support students' cognitive engagement and sense of accomplishment, while supportive environment practices foster positive relationships and emotions. However, no research has explicitly examined which specific pedagogical practices most effectively promote class-wide MWB. The present study addresses this gap by investigating the pedagogical practices (as measured through an adapted productive pedagogy framework) of one teacher of upper primary students whose class survey results indicated an overall high class-level of MWB.

Methods

Study Design

This study employed a mixed-methods exemplary case study design to examine the pedagogical practices and teacher characteristics associated with high class-level mathematical wellbeing (MWB). The case study approach was selected to provide an in-depth understanding of how one teacher, Karla (pseudonym), creates and maintains a high MWB classroom environment.

Participants and Case Selection

Karla was purposefully selected as an exemplary case from a larger longitudinal study (2020-2023) that investigated changes in teachers' pedagogical practices and their impact on student outcomes across 350 classes in New Zealand. The broader study included 4,137 students and 160 teachers from 31 schools from which Karla's 2023 Year 5 class ($n = 20$ students) was identified as demonstrating exceptionally high MWB levels. Karla's Year 5 class was selected as an exemplary case based on three criteria: (1) her class-level MWB score was in the top 5% of the sample ($M = 8.46$ versus sample $M = 6.6$), (2) she had consistently high class MWB across three consecutive years despite different student cohorts, and (3) she was teaching at the critical Year 5 transition point where MWB typically declines (Hill & Hunter, 2024). Karla is a diploma-qualified primary school teacher with over 30 years of experience, currently teaching in a public primary school (Year 1-6) serving a low socioeconomic New Zealand neighbourhood. She has taught across Years 4-6 at this school for the past 9 years. Her school serves approximately 140 students who identify as Māori (36%), Pacific Island (59%), Asian (21%), and Pākehā/New Zealand European (27%).

Data Collection

Data collection involved three complementary sources to triangulate findings about Karla's pedagogical practices and their relationship to high class-level MWB:

Quantitative Data

Student MWB was measured using a 21-item survey previously validated with this population (Hill & Hunter, 2024), assessing levels of MWB across seven ultimate values: Accomplishments, cognitions, engagement, meaning, perseverance, positive emotions, and positive relationships. Students rated each item (e.g., *I feel like my maths learning has a purpose and is meaningful to me*) on an 11-point scale (0 = not like me at all, 10 = completely like me)

Classroom observations were conducted using the productive pedagogy profiling tool, assessing 20 pedagogical practices across three dimensions: Intellectual quality, cultural connectedness, and supportive environment. For each lesson ($n = 2133$ lessons over three years), two observers independently rated each practice on a five-point scale (1 = a total absence of the pedagogy in the lesson, 5 = a strong feature of the pedagogy during the entire lesson), considering both quality and duration throughout the lesson. Each teacher was observed on at least three occasions over a 12-month period. Observers then engaged in moderation discussions to agree on final scores (Hunter et al., 2024).

The teachers also completed a 50-item self-report survey rating the frequency of various pedagogical approaches (e.g., *I use questioning to elicit students' reasons and explanations*) on a four-point scale (1 = never, 4 = every or almost every lesson). They also responded to several open-ended questions (e.g., *What are your strengths and weaknesses as a math teacher?*)

Qualitative Data

To gain deeper insights into Karla's pedagogical practices, she responded to three open-ended questions via email: *How do you support students to feel good and function well in your maths class?* *Can you describe a specific moment in your class when you enabled a student to feel good and function well?* and *Is there anything specific that you do to support students who don't enjoy doing maths?* These questions were designed to elicit Karla's understanding of her practices and their impact on students' MWB.

Analysis

Analysis began with examination of the overall study data to identify patterns in class-level MWB before focusing on Karla's exemplary case using a convergent mixed methods approach. Individual student MWB scores were calculated by averaging responses across the 22 survey items, then aggregated to create class-level mean MWB scores for all 350 classes across the three-year period (2021-2023). Descriptive statistics and visualisations were generated using Excel and SPSS to examine the distribution of class-level MWB across year levels and identify patterns in the broader dataset. Karla's Year 5 class was identified as an exemplary case by comparing her class-level MWB score against the distribution of all classes in the sample. Classes demonstrating MWB scores in the upper quartile were examined, with Karla's class selected based on consistently high performance across multiple years (2021-2023). For the pedagogical practices, Karla's observational scores were compared against group means to identify distinctive practices, with differences of 1 point or more on the 5-point scale considered substantial. The five practices with the greatest positive differences were identified as Karla's most distinctive pedagogical characteristics (Table 2). Self-report survey responses were similarly compared against group means to identify Karla's distinctive self-reported practices.

Karla's email responses and open-ended survey questions were analysed using a deductive thematic approach, aligning her responses with the seven MWB values (accomplishments, cognitions, engagement, meaning, perseverance, positive emotions, and positive relationships) (Hill et al., 2021) and the productive pedagogies framework (Hunter et al., 2024).

The quantitative and qualitative findings are integrated below to develop a nuanced profile of Karla's pedagogical practices and how these may support student MWB.

Results in Discussion

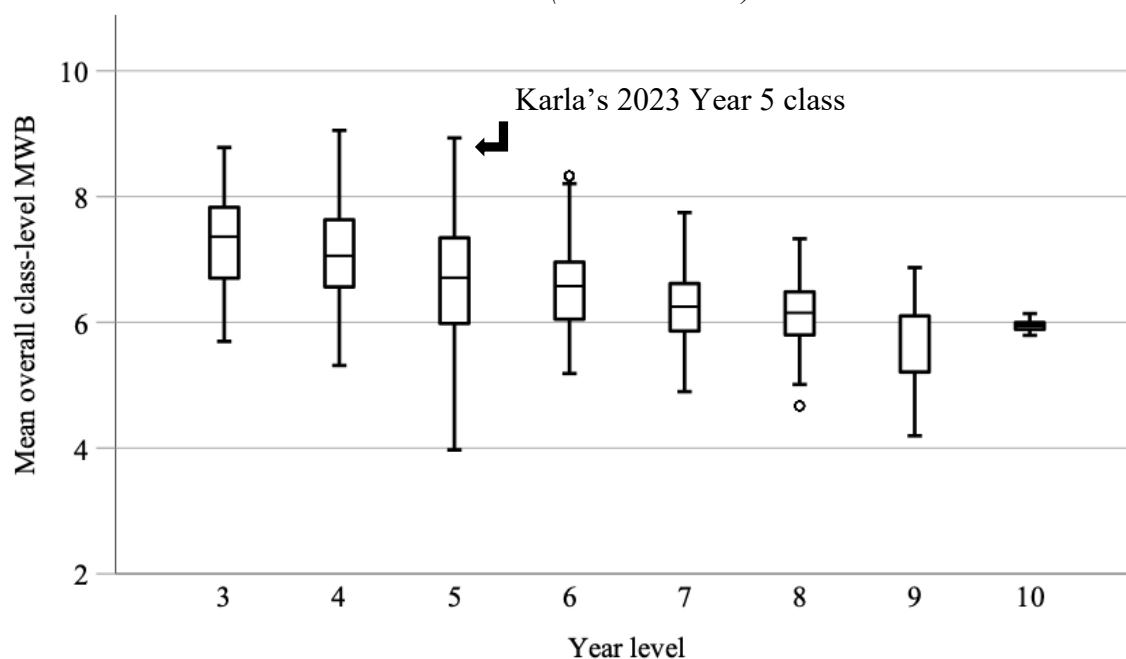
Karla's Class Level MWB in Context

As shown in Figure 1, Karla's Year 5 class from 2023 ($n = 20$ students) demonstrated substantially high MWB ($M = 8.46$), placing it in the uppermost range of all Year 5 classes and well above the overall sample mean ($M = 6.6$). This finding is particularly notable given that

the Year 5 classes showed the largest variability in MWB scores (ranging from $M = 3.97$ to 8.46), suggesting that Karla's practices may offer important insights for supporting MWB during this upper primary transition period. Also notable was that Karla's class level MWB was consistently high over the three years she was observed (M ranging from $8 - 8.46$) despite teaching different students across each year.

Figure 1

Class Level MWB Scores between 2021 – 2023 ($n = 350$ classes)



Karla's Distinctive Pedagogical Practices

Analysis of the classroom observations revealed five pedagogical practices in which Karla substantially exceeded sample means in her Year 5 class, as shown in Table 2. Sample mean scores of $2.6 - 2.8$ represent moderate implementation across the broader study, making Karla's scores of $4.0+$ particularly distinctive and indicating consistent, high-quality enactment throughout entire lessons. These practices formed a pattern reflecting her student-centred, relationship-focused teaching approach that fostered productive struggle.

Table 1

Pedagogical Practices in Karla's Year 5 Classroom

Pedagogical practice	Karla's mean	Sample mean	Difference
Substantive mathematical conversations	4.22	2.66	1.56
Challenging tasks	4.11	2.65	1.46
High expectations	4	2.62	1.38
Social norms	4.11	2.87	1.24
Mathematical practices	4	2.8	1.20

Note. Karla's Year 5 class was observed three times in 2023. Sample mean includes 350 classes.

Substantive Mathematical Conversations

Substantive mathematical conversations involve productive verbal/non-verbal communication that promotes sustained intellectual interactions between teacher to students and students to students, focused on mathematical ideas and thinking (Hunter et al., 2024).

Karla was observed consistently facilitating these conversations that focused on students' thinking and reasoning. Her survey responses revealed she asks students to *develop a mathematical explanation including why and how their solution works* in every or almost every lesson, compared to never asking them to *listen to her explanations*. In her email, she noted: "Listening to my students more rather than 'telling' them. Developing a strong maths culture in the classroom where all my students [sic] ideas are valued." She also revealed the intentional nature of this practice:

I often get group members to share with each other what skills they bring to the task. This can range from being a good listener, being able to ask thoughtful questions, know their number knowledge, show their thinking using diagrams etc. I stress that everyone brings skills to their group.

Karla's approach creates opportunities for all students to contribute meaningfully to mathematical discourse, directly supporting the MWB values of positive *relationships* through collaborative sharing, *cognitions* through developing understandings, and *positive emotions* from enjoying working collaboratively. The emphasis on student discourse and mathematical explanations creates opportunities for students to engage deeply with mathematics while building understanding through peer interaction (Tsuei, 2012).

Challenging Tasks

Challenging tasks are group-worthy teaching tasks that create opportunities for higher-order thinking and/or applying mathematics—tasks which are open-ended with multiple pathways and/or have no immediately obvious solution (Hunter et al., 2024). Karla was observed maintaining consistently high expectations while providing challenging mathematical tasks. Her survey responses showed she asks students to *work on challenging problems and develop their own solution strategies* in every or almost every lesson, while never asking them to *memorise procedures*. In her email, she described her practice to "move to developing student capability through challenging group tasks" and noted that "when I provide more challenging tasks, students are more active participants and I draw on students for my problems." Using challenging mathematical tasks supports the MWB values of *accomplishment* through meaningful challenge, *cognitions* through deep mathematical thinking, and *perseverance* through productive struggle (Hill et al., 2021). Challenging tasks serve as entry points for developing deep mathematical understanding while supporting student engagement and positive mathematical disposition (Hunter et al., 2024). In turn, teachers who incorporate challenging tasks often experience greater enjoyment from observing their students overcome challenges and those "light bulb" moments of understanding (Russo et al., 2023).

High Expectations

High expectations involve teachers expressing encouragement and affirmation of students' positive mathematical disposition towards intellectual challenge while using mistakes as learning tools (Hunter et al., 2024). Despite working in a low-demographic school with predominantly Māori and Pacific students, which is often associated with low teacher expectations (Rubie-Davies, 2017), Karla was observed consistently maintaining high expectations and supporting agency for all her students. Her survey responses showed she never uses *ability grouping*, instead implementing *mixed-ability grouping*. In her email she noted:

I think that students feel they are successful when they know how things go. e.g. How to work in their group, expectations during independent learning. It's kind of 'how we do things' It's not a secret from the students and they do have a say.

These responses demonstrate her belief in all students' capabilities, building relationships and ensuring a clear understanding rather than lowering expectations to support struggling students. This pedagogical approach supports MWB by fostering students' *accomplishments* (e.g., mathematical self-concept) and *relationships* (e.g., care and respect) (Hill et al., 2021).

High teacher expectations towards students are also linked to mathematical performance and engagement (Rubie-Davies, 2017), which can further support MWB.

Social Norms and Respectful Relationships

Social norms refer to establishing and discussing behavioural expectations where pro-social behaviour is evident in student interactions, creating inclusive classroom environments (Hunter et al., 2024). Karla's strong emphasis on social norms was evident both observationally and in her responses. For example, her survey showed she facilitates *students to work in groups* and *share solutions with the class* in every or almost every lesson. In the survey, she described her commitment to "Ensuring my students' cultures are reflected in all mathematics learning" and "Making it relate to real-life scenarios and integrating word problems that relate to their cultural background." She described how "the group norms play an important role as we discuss them before every maths session" and emphasised that:

Firstly, it is the relationships that I develop with the students in my class. I take time to learn about them as people rather than students and I focus on their culture and what they bring to our class.

Her approach to supporting students who don't enjoy mathematics centres on relationships over remediation:

Again, it's relationships. Finding out why, discussing as a class where, when and how we use Maths. Group dynamics and recognition of all skills needed to solve problems - group norms - showing an ethic of care by not letting them opt out and contributing in any way possible.

This relational focus directly supports the MWB value of positive *relationships* while creating conditions for students to experience *meaning* through discussion of mathematics relevance and cultural connections, *engagement* through inclusive participation, and *positive emotions* through supportive interactions (Hill et al., 2021). Such social norms encourage respectful and diverse ways of knowing and participating in mathematics, aligning with the relationship value of MWB and culturally sustaining pedagogy that builds on students' cultural assets while maintaining high academic expectations (Hunter & Hunter, 2023).

Mathematical Practices

Mathematical practices involve students generating ideas and sense-making through disciplinary practices such as developing explanations, justifying, conjecturing, representing, and generalising (Hunter et al., 2024). Karla was consistently observed supporting her students' engagement in these mathematical practices. Her survey responses showed she asks students to *justify their reasoning* and use *questioning to elicit reasons and explanations* in every or almost every lesson. Her email responses revealed how she structures sharing opportunities: "At the end of the working time, I asked their group to share, ensuring that they all had a role and contributed to the sharing." Students regularly explained their thinking, justified their reasoning, and made mathematical connections—practices that support the *cognitive* and *accomplishment* MWB value by fostering skills, understanding, and confidence (Hill et al., 2021). These disciplinary practices are key aspects of ambitious pedagogy and support deep conceptual understanding while enabling students to construct mathematical arguments and solve authentic problems (Lampert & Ghouseini, 2012).

Conclusion

This study makes a significant contribution by mapping both observed and self-reported pedagogical practices to high class-level MWB, moving beyond individual student perspectives to examine classroom-level factors. While these findings cannot establish causal relationships due to the nature of the case study analysis, they highlight a pedagogical profile associated with a high MWB class. The findings suggest that integrating intellectually challenging content, holding high expectations, emphasising mathematical discourses, and providing strong social-

emotional support can foster student MWB. By identifying concrete teaching practices associated with high class-wide MWB, this research provides valuable insights for teachers and teacher educators seeking to implement preventative rather than remedial approaches that support students to thrive mathematically. Future research will use statistical models to interrogate the relationships (including potential causality and directionality) between the seven MWB values, specific pedagogical practices, and academic achievement. Teachers and students in high MWB classes will also be interviewed to provide a more nuanced understanding of their classroom practice and discourses.

While mathematics education research often focuses on documenting negative experiences, this study offers a solution-focused and strength-focused approach and contributes to the field by exemplifying how teachers can develop classroom environments where all students can flourish mathematically.

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Dynamic Learning Artefacts: A Methodological Approach for Analysing and Interpreting Student Thinking in Mathematics

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Opportunities to closely monitor and evaluate students' mathematical thinking enables accurate interpretations of their conceptual understanding. This methodological paper reports on a novel and versatile analysis approach to extract student thinking during mathematical games through the generation of Dynamic Learning Artefacts (DLAs). Illustrative examples drawing upon lesson observation notes, interview transcripts and photographs describe the process of generating DLAs to better understand student thinking. The implications for adopting this method of analysis to support teachers' knowledge for instruction are presented and discussed.

There is wide recognition that gaining accurate insights into the thinking processes students adopt when engaged in mathematics is a key component to improving student learning outcomes (Cohors-Fresenborg et al., 2010). The ways in which students are able to access, combine and build upon their prior knowledge reflects their conceptual understanding and comprises more interconnectedness and depth than the mastery of facts and procedures (Lester & Cai, 2016). The multifaceted pathways students take in the space between delivered instruction and their output production can be difficult for teachers to navigate yet becomes critical in being able to plan instruction that will best meet learning requirements (Hiebert & Stigler, 2023). Many studies have reported on the ways in which examining extracts of student learning has supported teachers to become better attuned in noticing the mathematical thinking that occurs throughout problem-solving lessons (e.g., Franke et al., 2001; Jacobs et al., 2010). However, accurately capturing the fleeting, yet vivid mathematical thinking students exhibit during other rich mathematical contexts, such as when playing games, into translatable extracts for analysis beyond the classroom, is not straightforward (Goldsmith & Seago, 2010). The purpose of this methodological paper is to present illustrative examples of a developing analysis approach known as Dynamic Learning Artefacts (DLA) designed to document clear and coherent examples of the thinking processes students demonstrate when playing mathematics games.

Background Literature

Mathematical thinking is a broad term generally used to describe various cognitive processes students undertake when constructing or applying mathematical knowledge. For teachers, accurately monitoring student thinking is critical in being able to authentically respond to and appropriately guide the next steps of learning (Thanheiser & Melhuish, 2023). When teachers take genuine interest in what students say and do as part of the learning process, they are instigating cycles of constructive feedback and signalling to students that the public sharing of thinking will lead to more tailored support and guidance (Ryan et al., 2023). Moreover, these interactions offer instant evaluative feedback to teachers in terms of how effectively their instruction is being received (Matos et al., 2018). However, as Franke et al. (2001) recognised, teachers often require professional learning support to initiate and establish these cycles of noticing effectively as part of their instructional practice. Their longitudinal study showed that when teachers are supported to notice and interpret student thinking for conceptual understanding, their beliefs shifted from a perception that students are incapable of thinking without direct instruction towards an orientation in which students' thinking became the starting point for instruction (Franke et al., 2001). Framing the benefits of interpreting and evaluating (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 213–220). Canberra: MERGA.

student thinking in this way is distinct from traditional practices that focus on output derived from written assessments, narrowly reflecting learning experiences of students (Wiliam, 2007).

There is a myriad of methods reported within the literature that document and distil student thinking for analysis that extend beyond simple output production to include: work samples (Jacobs et al., 2010); video recording (van Es et al., 2014); transcripts of lesson dialogue (Sfard, 2001); and student interviews (Bobis et al., 2005). However, as Goldsmith and Seago (2010) caution, the artefacts themselves do not inherently magnify student thinking processes and care should be taken to consider *what* is being analysed and by *whom* (Hill, 2019). For example, Jacobs and Empson (2016) identified that when evaluating the transcripts of mathematical interviews conducted with early years students, experienced teachers were able to notice and expand on students' thinking more effectively than novice teachers who tended to accept incomplete or procedural explanations. Other studies concur that experienced teachers often have developed their noticing skills to engage with student thinking as a sense making process rather than an evaluation of procedural application (Thanheiser & Melhuish, 2023). As Mason et al. (2010) noted, it is prudent to consider how different levels of pedagogical content knowledge may lead to varying interpretations of the same artefact.

Video extracts and work samples are two prominent methods used to evaluate student thinking beyond the classroom, as they often simultaneously detail the mathematical content of the task as well as offer insights as to how students may have solved the tasks (Santagata, 2010). However, as Goldsmith and Seago (2010) noted "video has multiple channels of information but also multiple channels of distraction" (p. 186). This was evident in one study utilising video footage to discern effective ways to leverage student thinking throughout a lesson, reported by Thanheiser and Melhuish (2023). In this particular instance, viewers inadvertently attended to the procedure written on the board by the teacher rather than the audio detailing the student's explanation. The authors concluded that this misdirection was a result of the visual being more readily perceived than the audio (Thanheiser & Melhuish, 2023). Similar challenges are apparent in the evaluation of written work samples where what students say may differ from what they actually do (Sfard, 2001). When this occurs, it is difficult to accurately discern the thinking students actually applied during the task itself (Jacobs & Empson, 2016), and it is common for teachers to then revert back towards expected performance standards rather than try to discern nuanced solutions or unpack the evident misconceptions (Hill, 2019). Therefore, it is necessary to ensure that the source and selection of artefact are closely aligned to the intended focus of analysis from the outset (Santagata, 2010).

Much of the literature reporting on student thinking processes is contextualised through problem-solving approaches to mathematics (e.g., Franke et al., 2001; Mason et al., 2010; Lester and Cai, 2016; Schoenfeld, 2016). One justification for this co-occurrence is that problem-solving tasks offer ample scope for students to adapt their existing knowledge to new situations and to think creatively rather than procedurally (Schoenfeld, 2016). However, an alternative context in which students are also encouraged to think flexibly and demonstrate mental agility is through engagement with mathematical games (Russo & Russo, 2023). Often requiring no written output at all, mathematical games are considered rich and engaging contexts for students to readily apply critical and creative thinking skills (Applebaum, 2025). In an analysis of questionnaire data from 248 Australian teachers, Russo et al. (2021) reported that the majority of participants readily identify the value of mathematical games to support the development of mathematical proficiency, recognising the value of games to develop reasoning, problem solving and conceptual understanding in addition to fluency. Yet, there is limited literature reporting on the thinking processes students undertake when playing mathematical games, with no evident studies analysing student thinking through artefacts extracted from the learning itself. Therefore, exploring the potential of suitable analysis processes to support the

noticing of student thinking when engaged in mathematical games is warranted, and forms the basis of this methodological paper.

Dynamic Learning Artefacts (DLAs)

DLAs, originally developed for capturing student behaviours when working on challenging tasks (Hubbard, 2023; 2025), offer a novel approach for extracting and analysing student thinking in mathematics as they combine multiple data sources into a single and coherent graphical representation of learning. DLAs are generated by taking a work sample and overlaying it with time-related annotations to provide greater detail of what occurred, and when, over a selected lesson excerpt. In addition to the observational data, where relevant, the student's reasoning—derived from interview transcripts—is also weaved into the annotations to portray the experience from a holistic perspective. Essentially, the core aim of documenting accounts of student thinking in this way is to simultaneously communicate what is seen and heard from both the student and observer's perspective into a single unit for analysis.

The notion of triangulating multiple sources of learning to generate the DLAs was informed by Schoenfeld's (2016) study that used video data to document when and how effective problem-solvers used various types of thinking when completing tasks. In his study, over 100 college students were video recorded, with changes in behaviour tracked according to time points throughout the lesson. The findings reported that students who readily and efficiently solved the tasks demonstrated more frequent shifts in strategy selection over the timeframe than those students who persisted, unsuccessfully, in the application of a single rehearsed strategy (Schoenfeld, 2016).

The impetus for the current iteration of DLA design reported next was in response to the limitations associated with relying on either video data or written work samples reported throughout the literature (e.g., Goldsmith & Seago, 2010; Jacobs & Empson, 2016; Thanheiser & Melhuish, 2023). Hence, the intention was to draw upon the strengths of video data by underpinning the DLA with a visual image, whilst also ensuring that student explanations were contextualised through time-bound annotations. As the thinking in mathematical games is often not visible, determining how this could clearly be communicated through a visual graphic to underpin the DLA posed further challenges that had not been addressed elsewhere. Critical to the design was being able to maximise focus on student thinking and minimise unnecessary information that could divert attention away from the strategies students used in each turn. This malleable construction process offers a versatility to the DLA method that is not afforded through video extracts or work samples.

An Illustrative Example

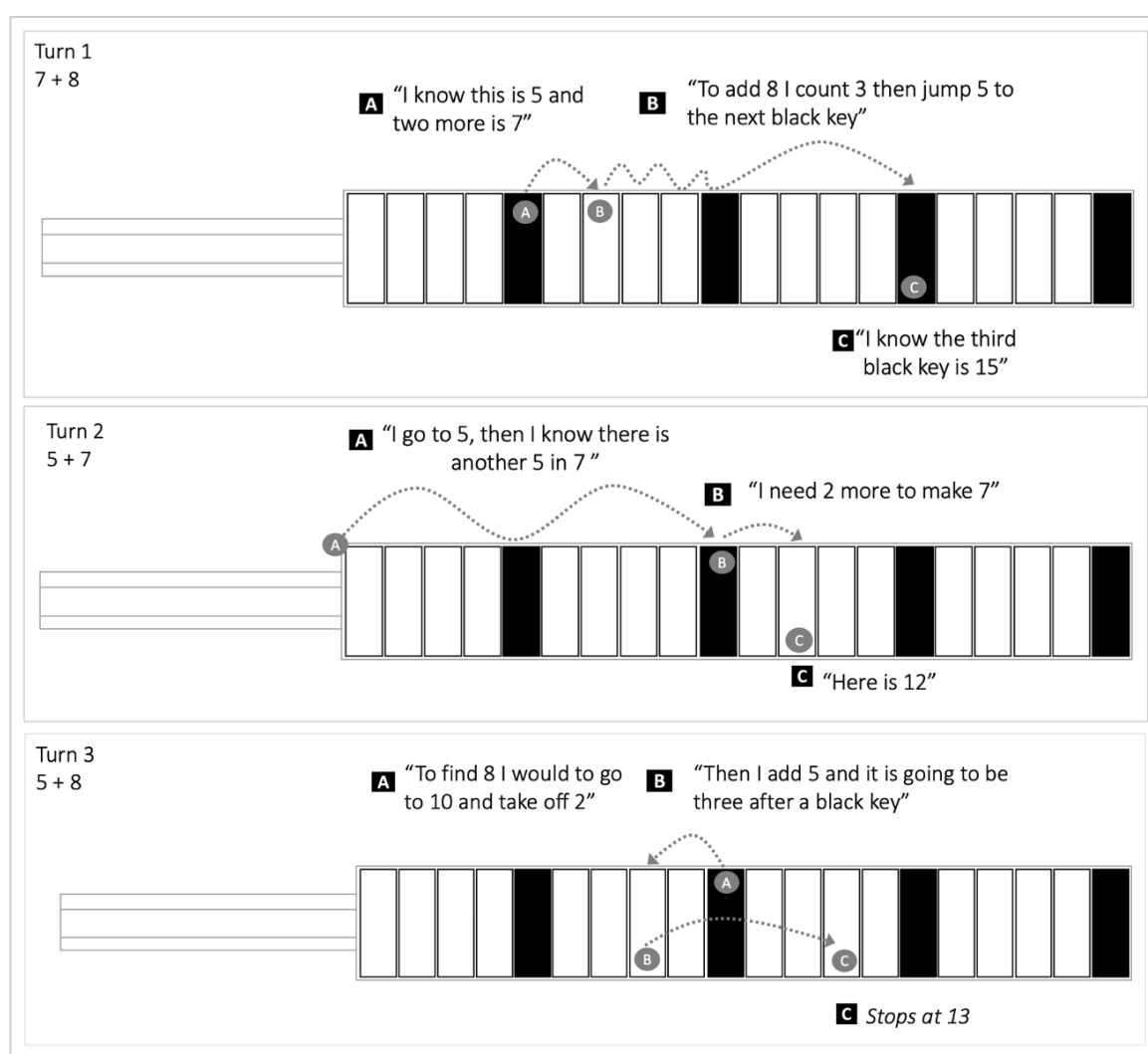
The illustrative examples of DLAs reported in this paper were generated as part of a larger research project focused on how Year 2 students utilised and applied their number sense knowledge when playing mathematical games supported by a manipulative known as The Keyboard (see Hopkins et al., 2025). This project consisted of teaching a series of lessons over a two-week period, where students were introduced to The Keyboard representation and used it in various ways to play a range of mathematical games to focus on their mental computation strategies for addition and subtraction. Given this context, The Keyboard formed the basis of the visual prompt in which the DLAs were generated. Further information from the lesson such as time bound observation notes, photographs and student explanations were layered onto this image to convey the subsequent game moves students performed throughout a series of game turns. To make explicit the strategies students demonstrated with each turn, a template of The Keyboard underpinned each observation. From there, grey dotted arrows were used to depict the sequential moves observed for each game turn, and these were labelled A, B, and C to correspond to the respective verbal explanations offered by students as they played the game,

or retrospectively, once prompted in the post-lesson interview. Two exemplar DLAs, representing the game plays of two different students (i.e., Student 1 and Student 2) are described next, demonstrating potential affordances of adopting this methodology for analysing student thinking when playing mathematical games.

The two DLAs presented next make visible the thinking strategies that would otherwise be indiscernible as students took turns in playing a mathematics game involving single-digit addition. Figure 1 shows a DLA from focus Student 1, comprising three consecutive game turns observed over a 5-minute period of the lesson. Between each turn, the other student pair was completing their turn of the game, which is not documented in the DLA shown here. The grey arrows and letters represent the active moves made by the student, which were documented through the lesson observation notes. The quotations reflect relevant excerpts of the audio recording transcripts that aligned to the respective turn being observed.

Figure 1

Student 1: Consecutive Turns Using Multiple and Flexible Strategies



The DLA presented in Figure 1 shows the various thinking strategies Student 1 demonstrated over multiple turns when solving single-digit addition sums in relatively quick succession. To accurately interpret Student 1's thinking strategies for each turn, it is necessary to integrate both the visual tracking on The Keyboard (grey arrows) with the associated student explanations. Attending solely to the arrows makes it difficult to directly link the moves on The

Keyboard with the associated sums being solved. For instance, Turn 2 could be misinterpreted as $10 + 2$ instead of $5 + 7$, as the arrows show Student 1 skip counts in 5s to 10 and then adds on 2 more. However, following the arrows in conjunction with the explanations enables clear and informative interpretations of student strategies to be identified. For example, in Turn 1, by focusing solely on the arrows, it is unclear why Student 1 starts on 5, moves 2, counts 3, and moves 5 when the sum they needed to solve was $7 + 8$. When reading this turn with the accompanying explanations, it becomes clearer that this student had decomposed 7 and 8 (into 5, 2 and 3, 5 respectively) and used this decomposition strategy to flexibly work with The Keyboard benchmarks (i.e., the black keys). Deconstructing each of Student 1's turns to this extent shows that over the three turns, a range of processes are used such as skip counting, benchmarking, and decomposition. Furthermore, it provides clear evidence to support the assertion that Student 1 has established a level of conceptual understanding whereby they can flexibly and efficiently manipulate single-digit numbers with proficiency. The ability to not only notice, but also understand and follow Student 1's mental computation knowledge to this extent is a strength of the DLA design.

The second DLA, presented in Figure 2, is focused on Student 2 and was constructed to extract and visually replicate just a single turn lasting less than 2 minutes. In this instance, repeated Keyboard images have been used (i.e., part A and B) to highlight the two separate thinking processes this student demonstrated to solve $7 + 6$ in one turn.

Figure 2

Student 2: Single Turn Requiring Two Different and Disconnected Strategies

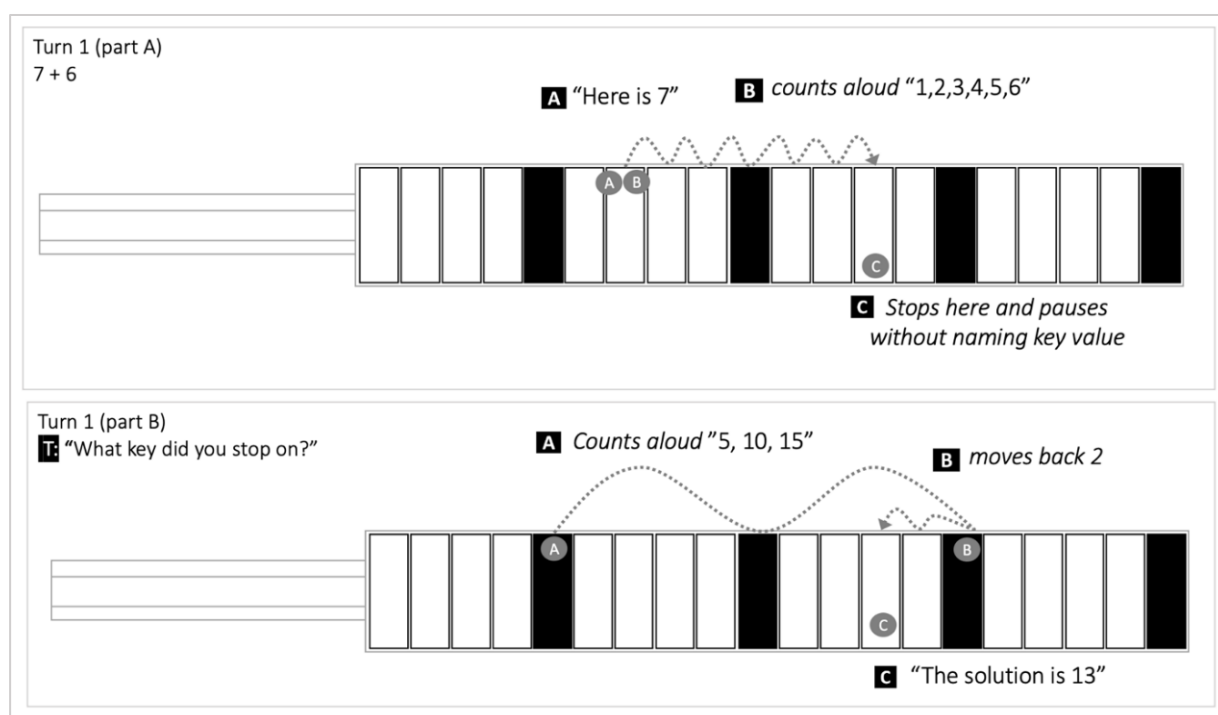


Figure 2 presents the deconstruction of a single game turn which offers critical insights into how Student 2 had to use two different and disconnected strategies to first solve the single digit addition sum and second to show the key to representing the sum solution. Had this same game turn been observed using a conventional method, it is unlikely that these subtle yet important aspects of Student 2's learning would have been identified. For example, even though this student does demonstrate a correct process for adding $7 + 6$ in Turn 1, Part A (i.e., starting on 7 and counting up 6 more), the annotated comments provide evidence that this has been done

procedurally rather than conceptually, as there are no connections made between how the parts of 7 and 6 comprise the whole of 13. Further evidence of procedural application can be identified in Turn 1, Part B, when Student 2 demonstrates a process disconnected from $7 + 6$ by applying skip counting by 5s to work out the value of the 13 key. Again, this reinforces the interpretation that Student 2 is not identifying important connections between the problem strategy and the answer. Being able to discern this level of detail about students' understanding and proficiency offers potential to more accurately plan the necessary targeted instruction for future learning.

These two exemplar DLAs show how multiple sources of qualitative data can be compiled to create simple yet cohesive and comprehensive accounts of individual student thinking. In addition, there are affordances in using these data collectively to evaluate the diversity across cohorts and consider how this will influence teachers' future instruction. For example, being able to compare and evaluate the different strategies utilised by Student 1 and Student 2 highlights that, even though both students were able to correctly answer the addition sum, there was a considerable distinction in how each outcome had been reached. Student 1 demonstrated productive and agile computation strategies, whereas Student 2 had to rely on more superficial and procedural application of knowledge. As the class teacher, having the means to recognise such discrepancies beyond the correct solution is critical in planning for instruction that caters to the diverse needs of the class. Moreover, developing an acute awareness of how some students may appear to be thinking, but are actually replicating procedures without understanding, will better prepare teachers to anticipate and respond to students authentically in the actual lesson. Such levels of evaluation have previously been associated with experienced teachers who have been supported to develop their pedagogical knowledge for in-lesson noticing. Therefore, an affordance of using the DLA methodology in the ways described here is that it creates greater accessibility for teachers of all levels of ability to engage with student thinking in meaningful and productive ways.

Concluding Thoughts

Presented here was a novel and alternative method for evaluating student thinking in mathematical games through the generation of Dynamic Learning Artefacts (DLAs). Capitalising on the strengths of existing data collection processes—such as visual perception enabled through video footage (Thanheiser & Melhuish, 2023) and reasoning from student interviews (Jacobs et al., 2010)—this approach allowed for the salient aspects of the learning experience to be leveraged in a 'fit for purpose' unit of analysis (Santagata, 2010). Constructed from a pictorial base and overlaid with observational annotations, DLAs reflected students' active thinking trajectories across a static image, offering accurate insights into a moment of learning through combining what was said with what was done (Sfard, 2001). The careful selection and alignment of lesson observations, photographs and student descriptions offered vivid accounts of the mental computation strategies students accessed and applied when solving single addition sums, enabling distinctions between procedural and conceptual understandings to be identified (Schoenfeld, 2016). Previously, the ability to make such specific evaluations about student thinking processes has been challenging, as classroom observations are generally less definitive, requiring inferential conclusions to be made beyond output production.

This work extends on the research reported by Hubbard (2023; 2025) in narrowing the focal point of the DLA to a granular level (i.e., 3 to 5 minutes), as opposed analysing student learning over the duration of a lesson. Modifying the unit of analysis in this way enabled the focal point to be conveyed more clearly than would be possible in attempting to replicate all of the game turns throughout the entire lesson. Pre-determining the focal point and constructing the DLA around the specific moves made on The Keyboard provided a consistent interpretation of student thinking, not only across each student's turns, but also between different students.

Attending to these moves, rather than the sum being solved, addresses concerns raised by Mason et al. (2010) that too often when evaluations of student thinking are made, priority is given to the mathematical content instead of the strategies students use that demonstrates their conceptual understanding. The simplicity and precision with which such thinking can be communicated makes DLAs an ideal mode to introduce less experienced teachers to the importance of developing their practices of noticing to underpin their mathematical instruction (Hill 2019; Jacobs & Empson, 2016).

Several limitations exist regarding the generation of DLAs and their broader applicability that are important to consider. For instance, the initial development process may be perceived as resource intensive, requiring time commitment to conduct the observations, synthesise the data then generate the DLAs. In addition, it is critical to recognise that without a specific pre-determined learning focus, making sense of the copious information sources readily available within classrooms may be overwhelming for an untrained observer. This reiterates Santagata's (2010) concerns—that without accounting for a specific focus, the analysis of the DLA may be compromised.

In conclusion, the use of DLAs has also been shown to be highly effective in conveying the various thinking strategies students demonstrated during mathematical games, and how these strategies extend beyond fluency recall to emulate the proficiencies of reasoning, understanding, and problem solving (Russo et al., 2021). Given the frequency with which teachers readily use games as part of their mathematics programs, there is merit in considering how DLAs can further support the identification and analysis of students' mental computational strategies within such contexts. Just as developing teachers' noticing skills has improved their pedagogical content knowledge for problem solving (Franke et al., 2001), closely attending to the thinking of students within a mathematical game context may enable teachers to readily recognise the potential for meaningful interactions that are supportive in building conceptual understanding. As the DLAs reported here were based on a game that used a specific manipulative (i.e., The Keyboard), there is scope for further exploration and research into the ways in which this approach can be adapted within broader game contexts that draw upon different visual representations and models when supporting student thinking processes in mathematics.

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Although this was a non-empirical paper, it is important to note that data collected for the purpose of illustrating the DLAs took place within a broader study, for which ethics approval was granted by Monash University [Ref: 36277], and participants gave informed consent.

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Giving Underachieving Students a Voice: Exploring Student Perspectives to Improve the GRIN Program

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The Getting Ready in Numeracy (GRIN) program supports underachieving students by preparing them for upcoming mathematics lessons through small-group tutoring. While prior research suggests positive outcomes, student perspectives remain underexplored. This study thematically analyses responses from 311 GRIN participants (Year levels 1-9) on improving the program. Three-quarters of participants suggested changes, with key themes including more or longer sessions, more games, scheduling, degree of challenge and group composition. Findings highlight the need for flexibility in GRIN implementation and the importance of student voice in refining intervention programs.

Mathematics education is pivotal in cultivating students' critical thinking and problem-solving skills. Despite its importance, many students encounter difficulties with the subject, resulting in disengagement and diminished achievement. This often perpetuates a cycle of lost confidence and declining motivation (Skilling et al., 2021; Ma & Xu, 2004). Targeted interventions like the Getting Ready in Numeracy (GRIN) program have been developed to enhance both students' mathematical skills and their attitudes toward the subject. GRIN prepares students for forthcoming mathematics lessons through small-group tutoring that activates relevant prior knowledge, introduces and clarifies important language, and seeds new concepts. This preparation enables students to engage more effectively in upcoming lessons. By promoting peer interaction, the program also helps students view mathematics as approachable and within their capacity to succeed, ultimately boosting their confidence (Kalogeropoulos et al., 2020a).

Previous research has revealed that the GRIN program has been effective for reengaging students in mathematics and improving their attitudes towards the subject (Kalogeropoulos et al., 2020a; Russo et al., 2025; Sullivan & Gunningham, 2011). However, most studies have focused on educator perspectives and program implementation, with limited attention given to student voice in shaping the program's development. In response, recent iterations of the program included a post-program questionnaire inviting students to suggest specific changes to enhance their GRIN experience. This paper thematically analyses these qualitative responses to better understand their suggestions for enhancing GRIN at both the program and individual school levels, concluding with considerations of the practical implications of these findings.

Overview of the GRIN Program

The GRIN program is an educational initiative aimed at re-engaging students with mathematics through targeted support ahead of their scheduled classes. Conducted several times weekly, these small-group tutoring sessions cater to students identified by their teachers as underperforming, though not those with severe learning difficulties. The program's structure is built on foundational principles that address cognitive load by priming students with simplified versions of lesson material, cognitive-affective strategies to reduce fear of failure and enhance engagement, and social-cognitive elements that utilise peer interactions to improve understanding and application of mathematical concepts (Kalogeropoulos et al., 2020a).

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Each GRIN session is carefully structured to activate prior knowledge, enhance language skills, and familiarise students with the key mathematical concepts they will encounter in upcoming lessons. These sessions progress through phases of reflection, targeted fluency practice through mathematical games, language immersion, and finally engagement with relevant mathematical problem-solving tasks. The approach is dynamically adjusted to meet student needs, incorporating enabling prompts and access to concrete materials to make the learning material more accessible and less intimidating (Sullivan & Gunningham, 2011; Sullivan et al., 2006). This not only prepares students for immediate upcoming lessons but also aims to foster long-term academic growth and confidence.

Schools commit to the program for at least a year, requiring significant preparation and professional development for teachers and designated GRIN tutors, often teaching assistants with specific training. Feedback mechanisms, such as regular tutor-teacher planning and review meetings, shared document and resource access, and informal debriefs, are all designed to help coordinate the tutoring with classroom activities (Kalogeropoulos et al., 2020b). Preliminary studies and ongoing assessments suggest that GRIN supports student engagement and confidence in mathematics (Kalogeropoulos et al., 2020a), with some evidence indicating benefits for older students in keeping pace with their peers (Sullivan & Gunningham, 2011). While these findings are promising, further research is needed to explore GRIN's long-term impact on mathematical learning outcomes.

The Role of Student Voice in Refining GRIN

Underachievement in academics often reflects a disparity between a student's potential and their actual performance. While traditionally associated with gifted students, this phenomenon extends across all cognitive levels, affecting both individual well-being and societal productivity (Preckel et al., 2006). Underachievers often exhibit negative attitudes towards their educational environments and a diminished motivation to succeed, making them a critical focus for educational interventions (Lamanna et al., 2019). The GRIN program specifically targets underachieving students in mathematics who show potential for improvement but are currently performing below their peers without severe learning disabilities or significant behavioural issues. These students are typically disengaged due to gaps in content knowledge and are identified by their teachers as capable of catching up with their peers with appropriate and timely support (Kalogeropoulos et al., 2020a, b).

Self-Determination Theory (SDT) provides a useful framework for understanding student engagement and motivation, particularly for underachieving and disengaged learners. According to SDT, individuals have three basic psychological needs: autonomy, competence, and relatedness, which, when satisfied, foster intrinsic motivation and engagement, and when thwarted, lead to disengagement and amotivation (Ryan & Deci, 2016). Autonomy refers to the need for a sense of control over one's actions and learning, competence reflects the need to feel effective and capable, and relatedness involves feeling connected and valued within a social context (Deci & Ryan, 2015). In educational settings, students who lack agency in their learning, struggle to experience success, or feel disconnected from their teachers and peers are at greater risk of becoming disengaged (Ryan & Deci, 2016). Many underachieving students, such as those targeted by the GRIN program, are likely to have experienced repeated frustration of these needs, contributing to negative attitudes towards mathematics and a reluctance to participate in learning experiences. Consequently, interventions designed to (re)engage these students should seek to support autonomy by providing meaningful choices, (re)build competence through structured opportunities for success, and foster relatedness through positive interactions with teachers and peers. SDT thus offers a valuable lens for examining how GRIN can be refined to better meet the needs of its participants and improve their engagement with mathematics.

Incorporating student voice into educational programs provides a mechanism for fostering the three basic psychological needs outlined in SDT: autonomy, competence, and relatedness. When students are given opportunities to express their perspectives and shape their learning experiences, they are more likely to feel a sense of ownership over their education, which supports autonomy (Reeve, 2012). Additionally, involving students in program refinement can enhance their sense of competence by validating their insights and positioning them as active participants in improving their own learning conditions (Wilding, 2015). Moreover, providing students with opportunities to voice their perspectives can mitigate the detrimental effects of self-silencing, thereby fostering a deeper sense of relatedness and reducing the power imbalances that hinder genuine teacher-student connections (Patrick et al., 2019). Research has demonstrated that when students perceive their schools as genuinely responsive to their voices, they exhibit higher academic achievement, better attendance, and reduced disengagement (Kahne et al., 2022). Studies have highlighted that tokenistic approaches where students are asked for feedback but see no tangible impact can be counterproductive, reinforcing a sense of powerlessness rather than empowerment (Gillett-Swan & Baroutsis, 2024). Given that many GRIN participants have historically experienced disengagement and frustration with mathematics, embedding authentic student voice mechanisms into the program could help fulfil their psychological needs while also refining GRIN's effectiveness in addressing their learning barriers.

While prior research has demonstrated the benefits of student voice in shaping educational interventions, there is limited evidence on how underachieving students perceive and experience such programs. Given that many GRIN students have a history of disengagement, listening to their perspectives may provide valuable insights into how the program can better meet their needs. In the current study, we sought to capture student voice by inviting participants to reflect on their experiences in GRIN and identify one specific change they would suggest to improve the program. By thematically analysing these responses, we aim to explore how student feedback can inform refinements at both the program and school level. Through centring student perspectives, this research moves beyond traditional measures of effectiveness to provide actionable insights on how GRIN can be refined to better align with the needs and experiences of underachieving students.

Method

Study participants included primary and secondary school students across all school sectors (government, Catholic, independent) who undertook the GRIN program in 2023 ($n = 97$) and 2024 ($n = 214$). Table 1 displays a breakdown of participants by year level. It is apparent that approximately three-quarters of participants in both years were in middle-upper primary school (Years 3-6), although there were comparatively more participants from lower secondary school in 2024 (25%) than 2023 (19%). Across both years, the majority of participants were girls (2023 = 63%; 2024 = 64%).

Table 1

Number of Participants by Year Level

Year level	Not included	1	2	3	4	5	6	7	8	9	Total
2023		1	6	27	9	27	9	14	4	-	97
2024	2		12	41	40	42	24	22	23	8	214
Total	2	1	18	68	49	69	33	36	27	8	311

Following their participation in the GRIN program in 2023 and 2024, students were asked a range of questions to get at both their experience of GRIN and their attitudes towards mathematics more broadly. The current paper is focussed on an analysis of student responses to one particular open-ended question: “If you could change one thing about the GRIN program, what would it be?”

The responses were analysed using qualitative line-by-line coding, following the approach outlined by Braun and Clarke (2012). The analysis involved several stages: familiarisation with the data; generation of initial themes; merging responses that could be accommodated within a single theme or expanding them into multiple sub-themes for greater clarity; defining and naming themes; and producing a table with illustrative quotes for each theme. Most responses (73%) were coded to a single theme, while some were assigned to multiple themes. For example, one Year 2 student commented, "More games and more sessions." This response was coded under both *Activities – More games* and *Duration – Longer sessions or more sessions*.

Findings

Table 2

Frequency of Themes that Capture How Students Would Change the GRIN Program (n = 311)

Themes	Number of students (%)
Change nothing/I don't know	80 (25.7%)
Duration	71 (22.8%)
Longer or shorter not ascertained	1 (0.3%)
Longer sessions or more sessions	61 (19.6%)
Less time or sessions	9 (2.9%)
Activities	63 (20.2%)
More games	36 (11.5%)
Other activities	9 (2.9%)
Use of manipulatives/technology	7 (2.2%)
Content	16 (5.1%)
Scheduling	47 (15.1%)
Amount of challenge	25 (8.0%)
Make it harder	16 (5.1%)
Make it easier	9 (2.9%)
Working arrangement- size and makeup of the group	18 (5.8%)
More in the group	5 (1.6%)
Less in the group	7 (2.2%)
Makeup of the group	6 (1.9%)
About being taught	12 (3.8%)
More fun	8 (2.6%)
Working environment	4 (1.3%)
Not doing maths	2 (0.6%)
Utility	1 (0.3%)

*Note that 5 responses (1.6%) were unclear/irrelevant and not assigned to any theme.

Table 2 presents the findings from the thematic analysis of what participants would change about the GRIN program if given the opportunity. We elaborate on the most prevalent and salient themes below.

It is noteworthy that approximately one-quarter of participants would not change any aspect of the program. For some participants, this may have been indicative of the difficulty of bringing to mind potentially desirable modifications when completing the questionnaire; they may have been more forthcoming had the data been collected through a more interactive format (e.g., semi-structured interviews) or if they had taken more time to consider the question. However, for other participants, this response was clearly intended as an endorsement of their experience of GRIN, as illustrated by comments such as “Nothing—I enjoyed it just as it was” (Year 2), “I don’t want to change anything about it” (Year 6), and “I wouldn’t change anything because it’s helping my learning” (Year 6).

A further tacit endorsement of GRIN is evidenced by the fact that the most frequently identified change to the GRIN program related to its duration, with around one-fifth of participants specifically requesting either more GRIN sessions or longer GRIN sessions. This sentiment is well captured by a Year 6 student who stated, “Do GRIN every day and make more people do GRIN because it really helped me and it was great for my maths confidence”. Given that students are typically invited to participate in the program because their teachers view them as disengaged from mathematics learning, the prevalence of these themes—to change “nothing” or to increase the “duration” of GRIN—is consistent with previous findings that GRIN is effective for (re)engaging students in mathematics (Kalogeropoulos et al., 2020a) and improving attitudes towards the subject (Russo et al., 2025).

In terms of putting forward changes to the substance of the GRIN program, around one-fifth of students requested changes to the mix of activities. Most frequently, this comprised requests to play more mathematical games during GRIN sessions: “Even more games” (Year 3); “Spend more time playing fun games” (Year 8); and “To play more math games to include times tables” (Year 4). This aligns with broader research findings that games are widely regarded as an effective means of supporting students’ cognitive, behavioural, and affective engagement in mathematics (Russo et al., 2024). The reference to multiplication tables is particularly noteworthy, as it was the most frequently mentioned topic in students’ requests for changes to GRIN’s mathematical content. This trend was evident even among younger GRIN participants, as reflected in comments such as “Strategies on times tables” (Year 3) and “I would like to have more times tables practice” (Year 2).

Scheduling GRIN sessions has always been a challenge, given the need to balance participation in the program with ensuring students do not miss out on other valued learning experiences. Since GRIN sessions are scheduled immediately before maths lessons, it is unsurprising that scheduling emerged as a theme in students’ suggested changes to the program, with around 15% of students identifying it as an issue. Many of these concerns were highly context-specific, reflecting the unique circumstances of individual students and schools, making it difficult to draw a definitive takeaway. Indeed, some responses were in direct tension with one another, as illustrated by the contrast between a Year 6 student’s comment, “The only thing I would want to change is to make the time of when we do GRIN not in the morning during the roll call”, and a Year 3 student’s preference, “I like to do GRIN in the morning.” More generally, concerns were raised about missing out on other subjects, as captured by a Year 6 student who stated, “If I could change one thing about the GRIN program it would be avoiding letting students miss out on important learning while at GRIN.” These findings align with previous research, which has identified timetabling and coordination as persistent challenges in GRIN implementation (Kalogeropoulos et al., 2020a).

While scheduling challenges and requests for more games were among the most commonly cited areas for change, a smaller but still notable proportion of students (8%) expressed a desire for adjustments to the level of difficulty in GRIN sessions. Interestingly, these perspectives were somewhat contradictory, with some students requesting more challenging work—“I want harder questions, because sometimes it feels a bit boring and basic” (Year 6) and “I wish it could be a little bit harder for me” (Year 5)—while others expressed a preference for easier work—“For the work to be a bit easier” (Year 5) and “Not as hard questions” (Year 6). Notably, approximately twice as many students asked for GRIN work to be harder than those who wished for it to be easier. This finding aligns with broader research suggesting that, when given the opportunity to comment on challenge, students are more likely to request greater difficulty rather than reduced difficulty, even if they sometimes struggle with the material (Russo & Minas, 2020). At the same time, the fact that only a small minority of students raised concerns about difficulty suggests that, for most participants, the level of challenge in GRIN was generally appropriate.

Similar to the differing perspectives on challenge, students also expressed a range of preferences regarding the size and composition of their GRIN groups. While some students wanted larger groups, such as “to have more students in the group” (Year 5) and “a few more students in the sessions” (Year 4), others preferred smaller, more individualised settings. One student commented, “Less people in a group” (Year 4), while another articulated a more detailed rationale for preferring fewer peers: “To have less people in my group so I can be more involved. When I have just one teacher and one other kid, I feel like it makes me learn better, I have to participate more, and I learn more” (Year 4). Beyond group size, some students expressed preferences regarding group stability and composition. For instance, one student wanted continuity in working with the same teacher across multiple years, stating, “Doing it with the same teacher in grade four and five and six” (Year 3). Another highlighted the importance of grouping students by similar learning needs: “Have people in a group that have the same understanding and things we don’t understand” (Year 6). These varied perspectives reinforce the idea that while students’ input should be considered in shaping their learning environments, there are no universal rules for structuring GRIN sessions that will satisfy all students. Instead, responsiveness to individual needs remains critical.

To summarise, the findings highlight both broad student endorsement of the GRIN program and a diversity of perspectives on potential improvements. Approximately one-quarter of students indicated they would not change any aspect of GRIN, reflecting overall satisfaction. Among those who suggested modifications, the most common request was for more frequent or longer sessions, reinforcing the program’s perceived value. Many students also expressed a desire for more mathematical games, particularly for practicing multiplication. Scheduling emerged as a context-specific challenge, with students raising concerns about missing other subjects but differing in their preferred timing of sessions. Similarly, preferences for task difficulty varied, though twice as many students requested harder work than easier work. Students also had differing opinions on group size and composition, with some preferring larger groups and others favouring smaller, more individualised settings. Collectively, these findings reinforce the idiosyncratic nature of student preferences, suggesting that while student input is valuable, no universal structure will suit all learners. Instead, an emphasis on flexibility and responsiveness is key to optimising GRIN’s impact.

Conclusions and Implications

The findings of this study reinforce that the GRIN program is broadly well-received by students, with many expressing a desire for more or longer sessions, a strong preference for games, and general endorsement of its current structure. However, the diversity of student

responses highlights that no single approach will suit all learners, reinforcing the need for flexibility in implementation at both the program and school levels.

At the program level, these findings align with previous research showing that successful GRIN implementation requires careful scheduling and teacher-tutor coordination (Kalogeropoulos et al., 2020a). While previous recommendations have encouraged schools to minimise content-area conflicts, student responses suggest that scheduling needs greater adaptability rather than a one-size-fits-all approach. Additionally, the strong student preference for more games and interactive activities suggests that games should be embedded as a central instructional strategy rather than treated as an occasional feature of GRIN sessions.

At the school level, findings from both this study and previous research highlight the importance of tailoring GRIN to student needs (Kalogeropoulos et al., 2020b). Student preferences around group size, session structure, and level of challenge varied widely, reinforcing the need for regular reflection on program delivery. While some students wanted larger groups for social engagement, others preferred smaller, individualised instruction. Similarly, the preference for increased challenge over easier work suggests that GRIN should maintain a strong focus on differentiation and appropriately high expectations for student learning.

Although past recommendations have favoured a group size of three students plus a tutor (Kalogeropoulos et al., 2020a), recent research suggests that there is no clear relationship between group size and intervention outcomes. A meta-analysis of mathematics interventions found that student achievement gains were not significantly influenced by group size, indicating that factors such as instructional quality and intervention fidelity may play a more critical role in determining effectiveness (Rojo et al., 2024). These findings are consistent with the varied preferences expressed by GRIN participants in this study, where no single group size emerged as universally optimal. Accordingly, schools may benefit from adopting a flexible approach when structuring GRIN groups, allowing for adjustments based on student needs, logistical constraints, and available resources rather than adhering to a prescriptive model.

Beyond group size, tutor consistency and collaboration with classroom teachers also emerged as key considerations. Prior research has highlighted the importance of strong relationships between GRIN tutors and students (Russo et al., 2020b). While a small number of students in this study expressed a preference for working with the same tutor across multiple years, the broader implication is that GRIN tutors play a central role in fostering engagement and confidence. Although they receive structured training as part of program-wide professional learning, their deployment is determined at the school level based on local needs. This includes decisions about consistency, collaboration with classroom mathematics teachers, and the groups they support. Ensuring that tutors are well-integrated with the classroom mathematics program and provide targeted support that responds to students' learning needs remains a priority for effective program implementation.

Ultimately, these findings reinforce that there is no universal 'best way' to structure GRIN sessions. Instead, the most effective approach may be to build in flexibility at the school level, ensuring that GRIN is tailored to individual student needs, school contexts, and available resources. By keeping the program adaptable, schools can better meet the needs of the diverse learners GRIN is designed to support while retaining its core strengths as a tool for re-engaging students in mathematics.

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Examining the Impact of Tutorial Activity Engagement on Undergraduate Students' Collaborative Preferences

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This study examines the impact of tutorial engagement on Collaborative Preferences for Learning Mathematics (CPLM) in a tertiary context. A two-way mixed ANOVA analysed these preferences over a semester in a sample of undergraduate students. As expected, collaborative engagement had a significant main effect, with students who collaborated more reporting stronger preferences for working with their peers (higher CPLM). The absence of an interaction effect between the nature of tutorial engagement and time suggests CPLM differences remain stable. This may indicate that familiar modes of tutorial engagement may reinforce existing collaboration preferences.

Collaboration is a central component of mathematics learning, particularly in tertiary education, where collaborative and active learning approaches are increasingly emphasised, and students differ in their preferences for working with others. These preferences, termed Collaborative Preferences for Learning Mathematics (CPLM), reflect the underlying cognitive, social, and motivational needs and dispositions that shape how students engage in mathematical tasks (Kim & Evans, in press). While extensive research has explored strategies that promote collaboration (e.g., Leitner & Gabel, 2024; Lo et al., 2017; Oates et al., 2024; Oates et al., 2016), less is known about how students' engagement with mathematics influences their preferences for working with peers over time. Although existing studies highlight both the benefits and challenges of collaborative learning, the role of tutorial participation in shaping these preferences remains underexplored.

This study examines the role of tutorial engagement—whether students engage primarily individually, in a mixed format, or collaboratively—in shaping CPLM. Using a two-way mixed ANOVA, we investigate changes in CPLM across a semester, exploring whether tutorial participation fosters shift in students' collaborative learning preferences. Specifically, we address two research questions: (1) What are the CPLMs among undergraduate students based on tutorial engagement, and (2) how does tutorial engagement (individual learners, mixed learners, collaborative learners) influence students' CPLM over time? By understanding these dynamics, this research contributes to ongoing discussions in mathematics education regarding the interplay between instructional contexts and students' learning preferences.

Literature Review

Collaborative Preferences

Preferences students have within the domain of mathematics are deeply connected to the values and interests they uphold. These inclinations to work in particular ways may reflect underlying cognitive and affective needs that stem from their past experiences and motivate behaviour and effort expended by the students (Goldin et al., 2016; Saadati & Reyes, 2019). As a driver of behaviour of short- and long-term habitual practices, student preferences are likely to have an influence on the ways students engage in mathematical thinking and communication and, hence, have an indirect impact on their learning outcomes. One key dimension of these preferences is their disposition toward collaborative versus individual learning environments. While some students thrive in collaborative settings, valuing the opportunity to share diverse perspectives, others prefer solitary study to maintain focus and control over their learning process.

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Mathematics education research provides further insight into the complexities of students' collaborative behaviours and learning preferences. Studies examining undergraduate mathematics students have shown that, while many hold positive attitudes toward group work, they often still prefer independent learning (MacBean et al., 2004). Informal group interactions outside of the classroom offer flexibility in how students can collaborate, and the unsupervised nature of it can foster a sense of camaraderie as they share experiences in problem-solving (MacBean et al., 2004). Formal group activities taking place during class hours can be valuable as instructors can have greater oversight over the students, allowing them to monitor student progress and offer guidance. Nevertheless, they also present challenges, including unequal participation and difficulties working with unfamiliar peers (D'Souza & Wood, 2003; Karau & Williams, 1993; MacBean et al., 2004; North et al., 2000). Further illustrating these complexities, a study conducted in an advanced-level mathematics course found that students perceived the greatest benefit of group work to be the exposure to diverse perspectives (Sheryn & Ell, 2014). Not all students viewed group work positively. Many who were initially apprehensive about it later described the experience as "interesting," even if they did not necessarily find it useful (Sheryn & Ell, 2014, p. 875). D'Souza and Wood (2003) reported on the cognitive advantages of learning with peers, such as fostering higher-level thinking skills and improving the performance of weaker students when paired with higher-achieving peers. These benefits, however, were accompanied by challenges, such as disengaged participants, which resulted in more engaged peers expressing frustration. Research in broader educational contexts echoes these concerns, noting that while collaboration can enhance learning, unproductive group dynamics and inefficiencies are recurring challenges (Salomon & Globerson, 1989; Sofroniou & Poutos, 2016; Tucker & Abbasi, 2016).

Considering these insights, the decision to incorporate group work requires careful thought about both student preferences and class dynamics. This complexity highlights that students' preferred learning environments do not always align with those that best support their learning outcomes. For example, Deslauriers et al. (2019) found that students often misjudge the efficacy of different learning approaches, favouring methods that create an illusion of learning, which may not always lead to greater understanding. Additionally, while early research suggests that aligning instructional practices with student preferences may enhance engagement and learning in certain contexts, this approach requires careful consideration (Okebukola, 1986).

Beyond immediate educational contexts, research in psychology and education has identified patterns between students' collaborative preferences and individual traits, including gender and personality, suggesting that these preferences may be shaped to some extent by innate factors. For instance, Owens (1985) found that girls generally exhibit a stronger preference for cooperative learning than boys, with this tendency becoming more pronounced as they age. Regarding personality traits, Chamorro-Premuzic et al. (2007) observed that extroverted students are more likely to favour collaborative learning, while those with higher neuroticism scores tend to prefer independent study. While these traits may influence students' predispositions, they do not define them, as students' preferences for collaboration can also be shaped by their broader social, cognitive, and educational experiences.

Theoretical Grounding

The familiarity principle, also known as the mere-exposure effect, posits that individuals tend to develop a preference for stimuli they encounter frequently (Zajonc, 1968; Zajonc, 2001). In educational contexts, this suggests that repeated exposure to specific instructional practices can influence students' preferences and perceptions of their effectiveness. For instance, when students consistently experience collaborative learning environments, they may develop a stronger preference for such settings due to increased familiarity and comfort (Macaluso et al., 2022). This repeated exposure can lead to increased comfort and perceived efficacy in group

settings, thereby reinforcing positive attitudes toward collaborative learning. Conversely, students who predominantly experience individualised learning environments may become more comfortable with solitary study methods, thereby reinforcing preferences for independent work. This alignment between exposure and preference reflects the broader influence of familiarity on perceived learning effectiveness. A study by Vroom et al. (2022) provides additional evidence of the mere-exposure effect within the context of mathematics education. Their findings show that students often perceive instructional practices they encounter frequently as more beneficial to their learning. Specifically, students in this study reported that interactive classroom characteristics, such as group work and peer support, were helpful when experienced on a regular basis. These findings emphasise the role of familiarity in shaping students' attitudes towards particular teaching methods and suggest that the repeated use of collaborative or individual approaches can influence how students evaluate their effectiveness.

In the context of this study, the familiarity principle is particularly relevant for examining how students' preferences for collaborative or individual learning in mathematics might develop based on their cumulative instructional experiences. By exploring the relationship between repeated exposure to specific learning modes and students' collaborative learning preferences, this research aims to contribute to a deeper understanding of how instructional practices can shape affective factors, such as engagement, comfort, and motivation, in mathematics education. This theoretical framing provides a foundation for the research questions and aims, which seek to investigate how exposure to collaborative or individual learning influences students' preferences and whether these preferences relate to academic performance and broader affective outcomes.

Data

The data used in this study were collected during the second semester of 2024 from a second-year undergraduate mathematics course at the University of Auckland. The course covered three primary topics: Calculus II, Linear Algebra II, and Differential Equations. Over the 12-week semester, students were expected to engage in weekly one-hour tutorials focused on problem-solving with a tutor—either the lecturer or an experienced graduate student—who was available to help students with their progress. Students were encouraged, though not required, to collaborate with peers. This study employed a quantitative, survey-based methodology to examine students' collaborative learning preferences and their self-reported tutorial engagement. Data collection involved two primary measures: (1) a Collaborative Preference for Learning Mathematics (CPLM) scale, which assessed students' preferences for individual versus collaborative learning, and (2) a self-reported measure of tutorial engagement, which captured the extent to which students worked individually or collaboratively throughout the semester. The self-reported CPLM data and tutorial engagement data were collected at three points in the semester (the first week, after the midsemester break, and the final week). Of the 294 students enrolled in the course, 201 completed the surveys at all three time points, and these responses were used in the analysis.

A validated 5-item scale was used to capture students' CPLM (Kim & Evans, in press). This scale captures students' preferences for collaborative versus individual learning across a range of scenarios. Each item was presented as a slider-based, close-ended question, with responses recorded on a scale from 0 (indicating a preference for individual learning) to 100 (indicating a preference for collaborative learning). The five items were designed to evaluate preferences across different contexts, including learning new concepts, studying for assessments, and engaging in problem-solving activities. The prompt for the scale was: "Consider yourself learning mathematics. By moving the slider, state the extent to which you prefer to do it from 0 = Individually to 100 = Collaboratively for each of the items." The five items were as follows: (1) "What is the most effective way for you to learn mathematics?" (CPLM_1), (2) "What is

the best way for you to make sense of mathematics?" (CPLM_2), (3) "What is the most effective way for you to study for high-stakes maths assessments (e.g., exams)?" (CPLM_3), (4) "In what social setting do you prefer to be exposed to novel concepts?" (CPLM_4), and (5) "In what social setting do you prefer to engage in problem-solving in low-stakes assessment (e.g., homework, practice exercises)?" (CPLM_5). To quantify CPLM, a composite score was calculated for each student at each time point by averaging their responses across all five items. This approach ensured that students' collaborative learning preferences were captured holistically rather than being based on a single item. Averaging across multiple items reduced variability and improved reliability, providing a more stable measure of individual differences in learning preferences.

Collaborative tutorial engagement was assessed through a self-reported survey item designed to capture students' actual behaviours during tutorials over the semester. Students responded to the following prompt: "Please indicate below to what extent you worked individually or with others in your tutorials during the entire semester." Responses were recorded on a five-point Likert scale: 1 = Always individually ($n = 18$), 2 = Mostly individually ($n = 38$), 3 = Sometimes individually and sometimes with others ($n = 61$), 4 = Mostly with others ($n = 41$), and 5 = Always with others ($n = 43$). To ensure more comparable group sizes for analysis, responses were recoded into three broader categories: individual learners, comprising students who responded "Always individually" or "Mostly individually" ($n = 56$); mixed learners, comprising students who responded "Sometimes individually and sometimes with others" ($n = 61$); and collaborative learners, comprising students who responded "Mostly with others" or "Always with others" ($n = 84$). This recoding facilitated meaningful statistical comparisons while preserving the integrity of students' self-reported tutorial engagement.

Analysis

Descriptive statistics were reported for students' CPLM scores, and a two-way mixed ANOVA was conducted to examine how CPLM scores changed over time and whether this change varied based on students' tutorial engagement. The between-subjects factor was collaborative tutorial engagement, with three levels (individual, mixed, and collaborative learners), while the within-subjects factor was time, measured at three points during the semester. An interaction term was included to assess whether the change in CPLM scores over time differed depending on tutorial engagement style.

A two-way mixed ANOVA was appropriate for this analysis as CPLM scores were measured on a continuous scale, allowing for the comparison of mean differences both within and between groups while accounting for repeated measures. As the test assumes sphericity for repeated measures, Mauchly's test of sphericity was conducted to verify this assumption. The test revealed a violation to the sphericity assumption for the two-way interaction, $\chi^2(2) = 9.654$, $p = 0.008$. To address this, Greenhouse-Geisser corrections ($\epsilon = .954$) were applied to adjust the degrees of freedom for the F-tests.

Results

Descriptive Statistics

Table 1 presents the descriptive statistics for CPLM scores based on the recoded tutorial engagement groups—individual learners, mixed learners, and collaborative learners. Individual learners, who primarily worked alone in tutorials, consistently reported the lowest mean CPLM scores across all three time points, with a decline from 43.68 at Time 1 to 38.36 at Time 3. Conversely, collaborative learners, who predominantly worked with others, demonstrated the highest mean scores at each time point, increasing from 53.10 at Time 1 to 56.03 at Time 3, reflecting a strengthening preference for collaborative learning over time. Mixed learners, who

engaged in both individual and collaborative approaches, exhibited more dynamic changes, with their mean CPLM scores rising from 46.08 at Time 1 to 51.56 at Time 3. These trends suggest that students who engaged more collaboratively in tutorials increasingly favoured collaborative learning, while those who worked individually showed a decline in collaborative preference over the semester.

Two-Way Mixed ANOVA

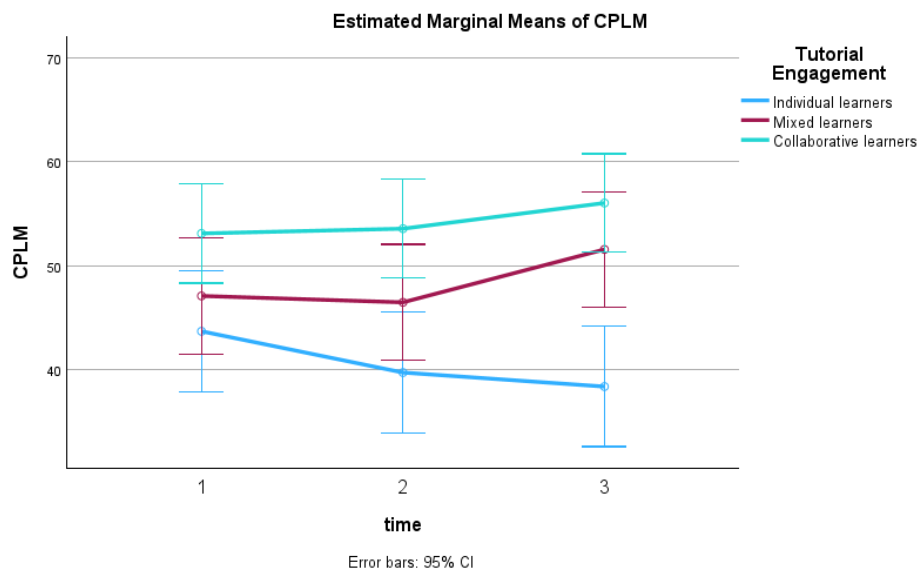
The interaction between collaborative tutorial engagement and time approached statistical significance but did not meet the conventional threshold, $F(3.817, 377.926) = 2.127$, $p = 0.08$, partial $\eta^2 = 0.02$. This suggests a potential, but not definitive, difference in how CPLM scores changed over time depending on tutorial engagement style.

The main effect of time was not statistically significant, $F(1.909, 377.926) = 1.066$, $p = 0.343$, partial $\eta^2 = 0.005$, indicating that overall CPLM scores did not exhibit a uniform change across the semester. However, the main effect of collaborative tutorial engagement was statistically significant, $F(2, 198) = 8.901$, $p < 0.001$, partial $\eta^2 = 0.082$, indicating a medium effect size (Cohen, 2003). This finding suggests that students' preferences for collaborative learning in mathematics differed significantly depending on their tutorial engagement style, with collaborative learners consistently reporting the strongest preference for collaboration.

Table 1

CPLM Descriptive Statistics at each Time Point Based on Collaborative Tutorial Engagement (three levels)

Time	Collaborative tutorial engagement	Mean	Std. deviation	N
1	Individual Learners	43.68	22.48	56
	Mixed Learners	46.08	20.69	61
	Collaborative Learners	53.10	22.95	84
	Total	48.65	22.41	201
2	Individual Learners	39.71	24.90	56
	Mixed Learners	46.47	19.90	61
	Collaborative Learners	53.56	21.62	84
	Total	47.55	22.72	201
3	Individual Learners	38.36	25.19	56
	Mixed Learners	51.56	19.60	61
	Collaborative Learners	56.03	21.31	84
	Total	49.75	23.07	201

Figure 1*Change in CPLM Scores Over Time for Individual, Mixed, and Collaborative Learners*

Discussion

This study investigated how collaborative tutorial engagement influences students' collaborative preferences for learning mathematics (CPLM) over time. The findings reveal that students who engaged collaboratively reported higher CPLM scores and exhibited a positive upward trajectory, whereas individual learners experienced a steady decline in their collaborative preferences. These results suggest that educational contexts and structured peer interactions may play a pivotal role in shaping and reinforcing students' learning preferences over time. Regarding the second research question, while no significant interaction between collaborative tutorial engagement and time emerged, the main effect of collaborative tutorial engagement was significant. Specifically, students who actively worked with peers throughout the semester demonstrated higher preferences for collaboration by the end of the semester compared to those who predominantly engaged in individual learning.

One interpretation of this result is that the nature of students' interactions and engagement during tutorials may reinforce beliefs and preferences that align with their experiences. This interpretation aligns with the familiarity principle (Zajonc, 1968; Zajonc, 2001), which suggests that individuals develop preferences for contexts and experiences they encounter frequently. Regular exposure to peer interactions and collaborative problem-solving may have fostered a sense of comfort, confidence, and perceived efficacy in collaborative settings for students who participated in group activities. Conversely, individual learners, with less exposure to collaborative opportunities, may have increasingly relied on solitary learning strategies, thereby reinforcing their preferences for independent study. These dynamics highlight the importance of considering how repeated exposure to specific educational practices can shape students' learning preferences and behaviours over time.

This finding is consistent with prior research, such as Vroom et al. (2022), who found that students often perceive the instructional practices they encounter most frequently as the most beneficial for their learning. Specifically, Vroom et al. (2022) noted that students viewed classroom characteristics as helpful when they facilitated deeper conceptual understanding, built on prior knowledge, supported its application in new contexts, and provided opportunities to explain concepts to peers. In the present study, students engaged in collaborative tutorials may have experienced similar benefits through interactions with peers, which could have

reinforced their preference for collaborative learning. On the other hand, students who predominantly engaged in individual learning may have found value in refining independent strategies that aligned with their preferred learning style, strengthening their inclination towards solitary study.

Broader implications of collaborative learning preferences should also be considered. Students who prefer collaborative learning may benefit from shared problem-solving experiences and opportunities to enhance social and communication skills. Conversely, students who prefer individual study may develop strengths in self-regulation, independent thinking, and autonomous learning. This study does not advocate for one preference over another, nor does it propose definitive recommendations for practice. Instead, it emphasises the importance of understanding how learning preferences are shaped by repeated behaviours and exposure to particular instructional contexts. Future research should explore the long-term implications of these preferences, particularly their impact on academic outcomes, educational trajectories, and students' capacity for lifelong learning. Understanding how preferences evolve in different learning environments and how they influence engagement, achievement, and broader skill development may further enhance educational practices aimed at supporting diverse learners.

Acknowledgements

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Understanding Changes in Teacher Dispositions Towards Mathematics Teaching Using an Epistemic Network Analysis

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This study seeks to understand the experiences of a teacher who has participated in professional learning that focuses on the values, beliefs, and enjoyment that primary school teachers experience about mathematics teaching. Using a quantitative ethnography methodology, an epistemic network analysis was applied to understand these changes experienced by a teacher over the professional learning period.

Many Australian primary school teachers do not enjoy mathematics teaching (Russo et al., 2023). This situation is likely to have negative consequences for learning and teaching mathematics in primary school. Professional learning for primary school mathematics teachers commonly focuses on developing teachers' curriculum and pedagogical content knowledge (Beswick et al., 2016), but little attention is given to developing teachers' positive attitudes and enjoyment for teaching mathematics (Lamb, 2010).

The focus of this study is to investigate and describe any impacts associated with participating in the Mathematics Identity (MICubed) professional learning activity which investigates the values, beliefs, and emotional dispositions that primary school teachers experience with mathematics teaching. The research reported in this paper is part of a broader PhD study involving teachers from two primary school communities in Victoria, Australia. For the purposes of this paper, results and findings are reported for one participating teacher. This case describes how one teacher's self-perceptions change over time and the influence of this change on their mathematics teaching.

Background Literature

The MICubed professional learning was designed using evidence-based practices for teaching primary school mathematics, grounded in pedagogical content knowledge (PCK) (Shulman, 1987) and mathematical knowledge for teaching (MKT) (Ball et al., 2008). Before examining the role of teacher dispositions towards mathematics in the implementation of effective teaching practices, it is useful to first examine this evidence base.

Bobis et al. (2012) argue that effective mathematics teaching practices are a combination of curriculum content knowledge, beliefs about mathematics, and the social/cultural contexts from which these are being drawn. This builds upon the work of Sullivan et al. (2009), highlighting the importance of teacher pedagogical content knowledge (PCK) for being effective practitioners. In response to the foundational work of Shulman (1987), which took a cross curricular perspective to PCK, Ball et al. (2008) sought to further refine the concept of pedagogical content knowledge for mathematics teachers, arguing that the field of mathematics education requires a more specialised strand of PCK. They refer to this as mathematical knowledge for teaching (MKT). This focus on the specifics of mathematical knowledge is an important foundation for this study. With Ball et al. (2008) recognising the distinct understandings and practices required for the effective teaching of mathematics, it is reasonable to wonder if a distinct set of emotional dispositions are also necessary for achieving this outcome in primary mathematics classrooms.

Considering these discussions with an eye to school improvement through the lens of professional learning actions, key questions arise: What is more important, teacher knowledge, or confidence in their knowledge? Will a teacher with less specific content knowledge but lots

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of confidence and positivity towards the subject, be more effective in creating a learning pathway for student achievement? Lee (2005), through a survey questionnaire completed by 81 randomly selected teachers in Indiana, USA, produced correlations between teacher attitudes to the teaching of mathematics and student learning outcomes. Teachers who showed high enjoyment and positivity towards mathematics teaching tended to favour a more investigative, problem-solving approach that encouraged student independence, whilst teachers with negative attitudes towards mathematics teaching were more likely to teach lessons focused on single answer outcomes and spend less time teaching mathematics as part of the overall curriculum.

With Lee's (2005) argument that teacher positivity and enjoyment towards mathematics can lead to them taking an investigative and problem-solving approach within their teaching practices, we must consider the literature to understand the value of this approach for improved student outcomes. Problem-solving is a particular aspect of MKT and has potential to influence both mathematics learning and engagement. This is highlighted by De Bortoli and Underwood (2025) in their discussion of the most recent Programme for International Student Assessment (PISA) results. For this reason, problem solving was an important aspect of the professional learning that was conducted as part of this study. Not only is problem-solving demonstrated to be a key pedagogical practice in many countries around the world, including high performing PISA (De Bortoli et al., 2023) jurisdictions, it is important to investigate for this study as it can be a trigger for mathematics negativity in primary teachers (Lamb, 2010).

Problem-solving links mathematical content knowledge and procedural understanding with social and emotional functions such as collaboration, engagement, challenge and creativity. Gervasoni et al., (2012) assert that the collaborative process of finding answers and solutions to problems results in the development and display of more sophisticated thinking in individuals. In addition to Hunter (2009), these authors also draw these conclusions from the work of Askew (2012) and Williams (2008), noting Williams' linkage of student mindsets towards problem solving (specifically their levels of optimism) as a key consideration for teachers when they are scaffolding group interactions.

Looking at mathematical teaching practices through an emotion lens, Russo and Russo (2019) claim that teachers who have greater autonomy over curriculum and engaging with their own interests and areas of enjoyment as well as the interests and enjoyment of their students, are more likely to achieve a positive outcome for student learning. These authors highlight that there is an iterative relationship between teacher and student enjoyment. When teachers enjoy what they are teaching, students enjoy their learning more. Teachers enjoy teaching when students are engaged and responsive (Russo & Russo, 2019; Frenzel et al., 2018).

Extending the argument for the positive correlations between teacher enjoyment and student outcomes by Russo and Russo (2019), Frenzel et al. (2018) in their longitudinal study, asked teachers and students to assess their levels of enjoyment in lessons over the period of a term, with assessment taken at the beginning, middle and end of the time period. Students were also asked about their perceptions of the teacher's enthusiasm. This study indicates that it is the teacher perception of the levels of engagement and enjoyment of their students that is the greatest driver of sustained teacher enjoyment and enthusiasm. Essentially, teachers derive enjoyment from the enjoyment of their students. Indeed, this is a reciprocal process rather than a teacher driven model of enjoyment for both parties. Essentially, as Russo and Russo (2019) found, positive emotional responses are reciprocal in classrooms – teachers and students feed off each other. On the surface, this conclusion seems rather obvious. In responses to the fact that many primary school mathematics teachers do not enjoy teaching mathematics, the study presented in this paper investigated a professional learning approach to assist schools to proactively build enjoyment of mathematics and teaching mathematics for teachers, and ultimately feed into the reciprocal loop of enjoyment of mathematics between teachers and

students. Indeed, it is likely that approaches for improving students' enjoyment of mathematics often need to ensure that teachers are enjoying themselves too.

There is a clear thread through the literature of agreed practices for effective mathematics teaching, the importance of problem solving as a pedagogical approach within this, and that the emotional dispositions of teachers towards mathematics can influence the implementation of these. This study adds to these findings through an exploration of professional learning that focuses on developing teachers' positive emotional dispositions towards mathematics teaching and linking this with the established knowledge base.

Context for the Study

MICubed is a professional learning activity that was designed for this study in which teachers participated in group-based mathematics problem-solving activities. This professional learning had a dual purpose: developing the mathematical and pedagogical knowledge of teachers; and fostering enjoyment, confidence and engagement with the doing and teaching of mathematics. MICubed sought to develop and enhance positive dispositions of teachers towards both the doing and teaching of mathematics, using evidence-based practices. MICubed involved four, 60-minute sessions which were conducted in schools over the course of two terms. In each session, participating teachers engaged in discussions about their mathematical identity and beliefs, what they valued in a mathematics classroom, and how they can foster positive mathematical interactions through inquiry-based problem solving with their students. MICubed was conducted in two primary schools in Victoria, Australia.

The findings discussed in this paper are drawn from the experiences of a single teacher, Alice from Hawke Primary School (pseudonyms). From Hawke Primary School a total of 12 teachers participated in the MICubed professional learning (including the principal). These teachers had diverse experiences with and attitudes towards mathematics.

Method

Mixed methods, including a longitudinal ethnographic case study, was selected as most appropriate for this study. Both qualitative and quantitative elements of data collection and analysis were employed through a quantitative ethnography approach (Shaffer, 2017). Quantitative ethnography enables elements that form the basis of a learning culture (such as teaching), to be analysed in a way which gives visual and statistical representations of both qualitative and quantitative data. Kaliisa et al. (2021) describe these elements as ways of listening, talking, reading, writing, acting, interacting, valuing, feeling and believing that are specific to a learning culture. These elements are then coded and identified within data sets and mapped using an Epistemic Network Analysis (ENA) to analyse the connections between elements within the data.

ENA is a methodology designed to identify and quantify the connections among elements in coded qualitative data (Shaffer, 2014). These connections are visualised in dynamic network models. The generated models are useful for analysing how connections between codes change and evolve, which is relevant when studying professional learning processes and outcomes. An advantage of using ENA to produce networks of connections, is that results can be compared visually and statistically. ENA software (<https://www.epistemicnetwork.org/>) is used to display coded data elements as nodes within a network and shows connections between these nodes to represent their co-occurrences or associations. Thicker connecting lines between nodes represent more frequent co-occurrences or associations.

The use of ENA is relatively new in education, and particularly as a tool for analysis of teacher emotional dispositions towards primary school teaching and curriculum areas. The following explanation is included as a guide for the interpretation of findings made in this study.

Epistemic Frame Theory is the basis from which the learning and understandings from an ENA are derived. Epistemic frames model how a community of practice might think, act and be (Shaffer & Ruis, 2017). The community of practice refers to a group with a culturally distinct repertoire of methods, beliefs and actions. Kaliisa et al. (2021) made the following link between epistemic network analysis and its basis within epistemic frame theory, "ENA stems from the operationalization of epistemic frame theory, a learning theory that models learning as ways of thinking, acting, and being in the world of some community of practice" (p. 5).

Data Collection Instruments

Data was collected via three different instruments across the cycle of the study. These were: (1) Teacher beliefs surveys (pre and post participation) using the Mathematical Beliefs Instrument (MBI) developed by Peterson et al. (1989); (2) Mathematical identity graphs that investigated emotional dispositions towards mathematics across a lifetime. These involved participants identifying particular experiences, periods or events in which they recalled a specific feeling towards the doing of mathematics; and (3) Individual interviews (pre and post participation). The aim was for the data to provide insight about how teachers arrived at their current dispositions towards mathematics teaching, how these dispositions changed over the course of their participation in the professional learning, and any impacts this process had on their mathematics teaching values, beliefs, and practices.

Data Analysis

The data generated and collected in this research were coded by the researcher to identify practices, beliefs, values, and dispositions related to teaching mathematics. Using a thematic analysis approach (Braun & Clarke, 2006), a specific sequence of data analysis was undertaken. This involved an initial familiarisation with all of the collected data, and then the generation and identification of codes. From these codes, themes were identified, defined and named. Once his process had been reviewed, links between themes were generated for the individual teachers using the ENA software. This process mapped connections between coded teacher practices, dispositions, values and beliefs. The thickness of lines in the network models indicate the strength of these connections.

Results and Findings

Alice, the teacher whose experiences have been investigated in this research, is a generalist primary school teacher, and in 2024 taught a composite Year 3/4 class. Alice had been a teacher for 8 years; however, teaching is not her first career. Despite formally becoming a teacher in her forties, Alice indicates in Interview 1 that she has always had an affinity for the profession. "I think I've always been a teacher, just not in this capacity".

Figure 1 is an ENA generated from data collected from Alice's pre- and post-interviews, her mathematical identity graph and her MBI surveys. These data have been analysed using the codes listed above in Table 1. The connections between data codes from Interview 1 are represented by the red lines in Figure 1, and the connections between data from Interview 2 are represented by the green lines.

In Interview 1, Alice's strongest connections (indicated by the thickest lines) are between her Mathematical Confidence, Mathematics Positivity and Mathematical Enjoyment. These were clearly strong themes that Alice spoke about repeatedly in her first interview. Alice referenced her own personal enjoyment of engaging with mathematical procedures as a student herself, stating, "*I love maths, it just kind of made sense. It was like everything is just add, subtract, multiply, divide.*" It is interesting to note that despite this love of mathematics, there is a much weaker link between these sentiments and the nodes of teaching confidence and positivity.

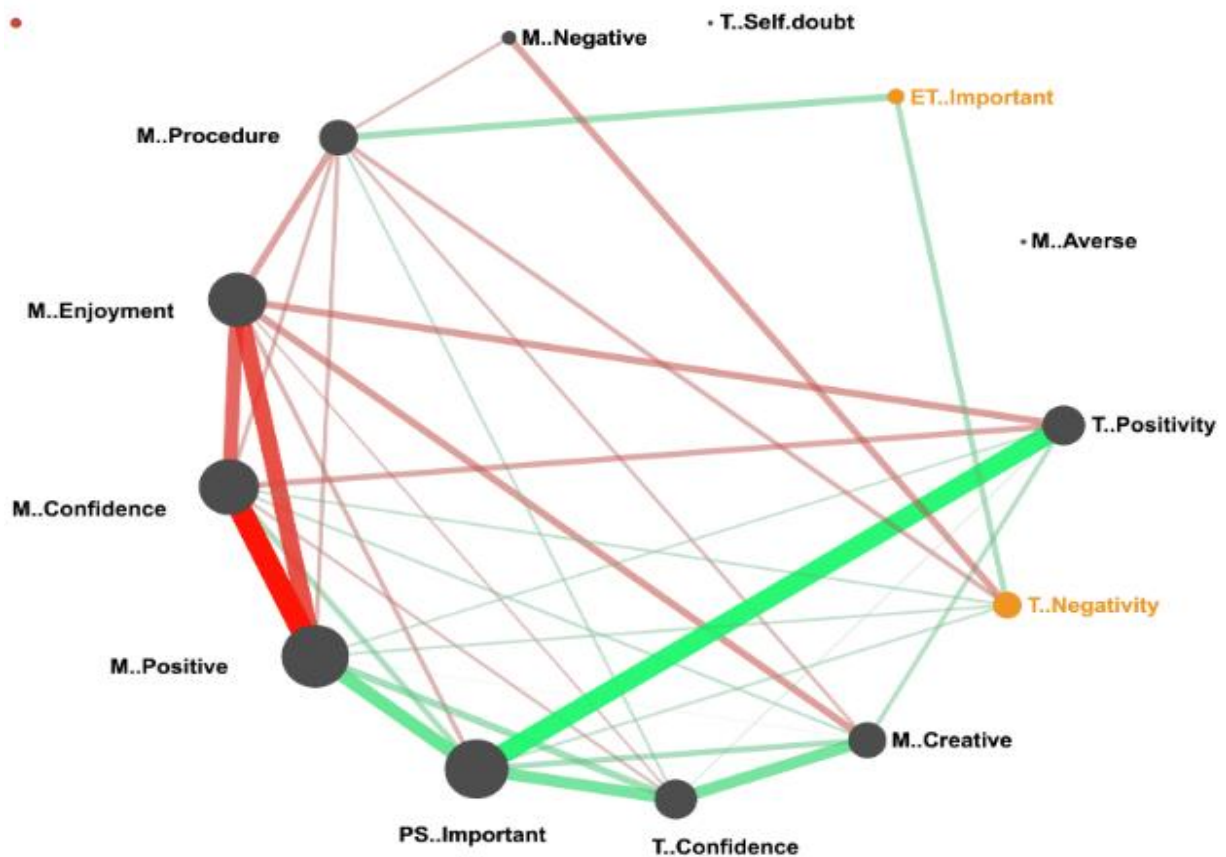
Table 1

Participant Interview Data Codes

Participant interview data codes
1. Problem solving is important (PS. Important)
2. Explicit teaching is important (ET. Important)
3. Mathematical enjoyment (M. Enjoyment)
4. Mathematics averse (M. Averse)
5. Mathematical confidence (M. Confidence)
6. Mathematics creativity (M. Creative)
7. Mathematical procedures (M. Procedure)
8. Mathematical positivity (M. Positive)
9. Mathematical negativity (M. Negative)
10. Teaching confidence (T. Confidence)
11. Teacher self-doubt (T. Self-doubt)
12. Teaching positivity (T. Positivity)
13. Teaching negativity (T. Negativity)

Figure 1

ENA Model: Alice



Mathematical Creativity and Mathematical Enjoyment are also clearly connected for Alice. These red lines tell the story of a teacher who loves mathematics, who understands algorithms and formulas (Mathematical Procedures). It tells the story of a teacher who loves mathematics but is still building her confidence as a mathematics teacher, *“finding extension tasks that don't just feel like more work, yeah. It's really tough”*. It is notable for interview 1, that there is only a minimal connection made between Problem Solving is Important and Mathematical Enjoyment. There were no connections in Interview 1 data between Problem Solving is Important and Teaching Confidence, Teaching Positivity or Mathematical Creativity.

For Interview 2, represented in *Figure 1* by the green lines, there is a visually different map of connections between the data codes. We can now see very strong connections between Problem Solving is Important and Teaching Positivity, as illustrated by the following excerpt for Interview 2, *“So that has changed and I'm able to incorporate (problem solving) activities that help all the kids shine”*. Strong connections are also shown between Problem Solving and Mathematical Positivity / Teaching Confidence. An utterance from Interview 2 which supports this is, *“I love kids showing the way they get answers, because they all get it differently”*. This indicates a possible change in dispositions and beliefs for Alice towards mathematics teaching over the four-month period of professional learning.

This correlates with some changes in Alice's responses to the pre and post Mathematical Beliefs Instrument survey. After initially agreeing with the statement in Item 1 of the MBI pre-survey; *Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum*, in the MBI post-survey, Alice changed her position. This was confirmed in Interview 2. When asked to describe her enjoyment levels with mathematics teaching compared with prior to the professional learning, she responded, *“I'd say it is definitely different. I definitely enjoy it more”*. When asked about how her teaching practices have changed, Alice talked about how problem-solving activities had been effective for encouraging students to work independently and in a differentiated way.

The pre-post differences in connections describing Alice's beliefs and dispositions highlighted in the ENA were also reflected in another response in the MBI post-survey compared to the MBI pre-survey. For Item 20; *Good mathematics teachers show you the exact way to answer the maths question you will be tested on*, Alice initially strongly agreed with this statement, however, in the post survey, Alice disagreed with this statement. This is a direct correlation to the increased emphasis on problem solving represented by the green lines in the ENA in *Figure 1*. With this in mind, we can return to the underlying research question of this study; How does participating in MICubed impact primary teachers' enjoyment, beliefs, and pedagogical values when teaching mathematics? Based on this analysis of the data generated for Alice, there is some evidence to suggest that her participation in MICubed may have contributed towards increasing her enjoyment and confidence when teaching mathematics as well as her value for problem solving as an effective pedagogical approach.

Discussion and Conclusion

This research highlights the potential of the MICubed approach in improving teachers' emotional dispositions towards mathematics, which could foster a culture of high mathematical achievement in Australian schools. Empowering teachers to create engaging environments can promote more positive attitudes and self-belief in both teachers and students. While existing professional learning in Australia often focuses on student engagement and teacher PCK, the findings from this study suggest that improving a teacher's dispositions through professional learning, centred on mathematical problem-solving, can be effective.

Using ENA to understand different phenomena within teaching pedagogies and practices (Elmoazen et al., 2022) has emerged over the past decade as an effective data analysis tool.

ENA models offer a unique way to interpret and understand different data sets concurrently. The use of ENA to describe and analyse data from the research presented in this paper is unique in its application towards understanding the emotional dispositions of primary school teachers towards mathematics teaching, and for describing changes in these dispositions following professional learning.

This paper is an early contributor to the knowledge base of quantitative ethnography using epistemic network analysis in this particular area of study. The identification, understanding and description of the emotional dispositions of teachers is an inherently qualitative process, commonly approached through interviews and observations. By employing ENA within a quantitative ethnographic approach, statistical rigour is applied to the analysis of these qualitative data sets, and nuanced analysis can be made. The findings presented in this paper, whilst limited by its analysis of only a single teacher, indicated that ENA has the potential to describe changes in teachers' beliefs and values towards, and enjoyment of, mathematics teaching. Further research is needed to investigate whether ENA is useful for describing changes in these areas for other individual teachers, and also for describing changes across larger groups of teachers who participate in professional learning.

Through this analysis of Alice's participation in the MICubed professional learning, we can see that she has experienced some positive changes in her dispositions towards the teaching of mathematics. Alice indicated potential changes in her beliefs about what constituted good mathematics teaching, as well as what she valued in the mathematics classroom. After participating in the MICubed professional learning, Alice applied problem-solving as a key pedagogical practice in her mathematics lessons, and the ENA strongly linked this to positive feelings towards the teaching of mathematics.

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Exploring the Integration of Data Visualisation Through Self-Quantification: Insights From School Education

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In an era increasingly defined by the proliferation of big data and its transformative impact on decision-making across disciplines, the ability to interpret and engage with complex datasets has become an essential skill for future generations. This literature review reconceptualises data visualisation through the lens of school education and proposes a framework. By framing data visualisation as both a product and a process, the paper explores how data visualisation be utilised in activities framed within the context of self-quantification.

Background

Big data, which has been re-shaping modern lives through the way we think and act, significantly expands the amount of information available for use in various aspects of life, enabling innovative ways through which people engage in data across different fields (Cukier & Mayer-Schoenberger, 2013). One such innovation is the development of wearable devices and mobile apps that track individuals' physical and emotional activities. By collecting and analysing real-time data, these technologies offer valuable insights into health, behaviour, and well-being, illustrating the potential of big data to transform everyday experiences into quantifiable metrics - a process known as *self-quantification* (Maltseva & Lutz, 2018). Self-quantification relies on data visualisation, which enables users to interpret their data, track changes, and make informed decisions through real-time visual displays. From the perspective of mathematics education, this process of self-quantification provides an engaging and authentic context for students to interact with real-time and personal and dynamic data via dynamic data visualisations.

Given the potential of self-quantification and data visualisation to enhance students' engagement with real-world data, integrating data science projects grounded in the context of self-quantification emerges as a valuable endeavour (Lee, 2019). However, integrating innovative data science projects into an already overcrowded curriculum can be challenging for many school educators. One possible reason is that data science draws on content knowledge from multiple disciplines, including mathematics, computer science, and statistics, as well as domain-specific knowledge of different subjects (Donoho, 2017). The interdisciplinary nature of data science makes it complex to be integrated into one single subject area without collaboration between educators from different disciplines (Herro et al., 2022). Additionally, curricula often lack sufficient detail on how data can be effectively visualised as a tool for students to make sense of in data-rich investigations (Australian Curriculum and Reporting Authority [ACARA], 2022; Ministry of Education Singapore [MOE], 2021). This gap in curricular guidance contributes to challenges in equipping teachers with the necessary skills and confidence to integrate data visualisation into their teaching practices. A recent study on over 3,000 primary and secondary teachers' perceptions of teaching data science indicate that many teachers do not feel confident in using data visualisation as a tool to support students' learning due to their lack of content knowledge and experience in data science (Filderman et al., 2022). To begin to address the problems discussed above, this paper aims to investigate following research questions:

RQ1 – How has the literature defined data visualisation relevant to school education?

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RQ2 - How can data visualisation be utilised in activities framed within the context of self-quantification?

The paper is structured in two main parts. The first part presents a literature review that explores data visualisation within the context of school mathematics curriculum, with a particular focus on how it connects individuals to the broader world via self-quantification. The second part provides a simple example of how data visualisation can be used a tool to facilitate students' exploration of data within the context of self-quantification.

Literature Review

Given the exploratory nature of the research question, a narrative literature review was conducted to synthesise existing research and theoretical perspectives on data visualisation, self-quantification, and their applications in mathematics education. The first section of the literature review examines the concept of data visualisation and presents a conceptual framework of data visualisation, and the second section uses the framework to explore the application of data visualisation in various forms of self-quantification.

Data Visualisation within the Context of School Mathematics Curriculum

In mathematics education, visualisation acts as both a *product* and a *process* that facilitates understanding (Arcavi, 2003). As a subset of mathematical visualisation, data visualisation embodies these dual roles, serving as both a *product* (e.g., a visual representation of data) and as a *process* (e.g., a process of reasoning with data). Beyond its role in mathematics, data visualisation is an essential component in data science, where it enables the exploration, analysis, and communication of data (Donoho, 2017). As mentioned in the Background, the concept of data visualisation is not explicitly illustrated in curricula. For example, the Australian Curriculum Version 9.0 mentions the visualisation of data through presenting it in summarised forms; however, this description remains insufficiently comprehensive to fully capture the scope and depth of data visualisation as a tool for analysis, exploration, and interpretation of data. To address this gap, we explore the concept of data visualisation in its role as both a product and a process.

Data Visualisation Acting as a Product

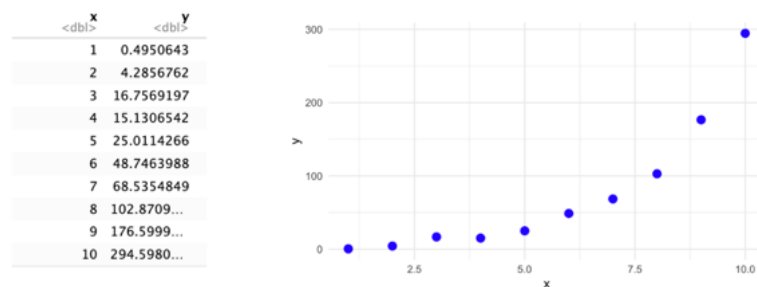
As a product, data visualisation transcends the role of a graph into a data graphic. A mathematical graph is an abstraction of observations or thoughts about the world (Wilkinson et al., 1999). *Data graphics* are broader, including visual representations of information, data, or knowledge designed to communicate complex relationships or patterns clearly, efficiently, and aesthetically (Tufte, 2001). That is, a visual representation is carefully designed and accomplished by *aesthetic attributes* such as frameworks (e.g., axes), visual dimensions (e.g., the lines on a line graph), labels (e.g., title and keys), and backgrounds (e.g., colouring and grids) (Friel et al., 2001). These attributes enable data graphics to *reveal* and *communicate* complex information, transforming abstract data into visually interpretable forms that enhance understanding and insights (Tufte, 2001). The use of these aesthetic attributes may vary depending on the purpose of the data graphic within a data-rich investigation. For instance, when a data graphic is generated for presentation purpose (i.e., presentation data graphics), significant focus is placed on its aesthetic attributes, as these are critical for effectively displaying and explaining the information conveyed by the data (Unwin, 2020). If aesthetic attributes are not used appropriately, the data graphic may fail to aid in understanding the data or even mislead by obscuring or distorting the intended message. Misleading visual representations are commonly found not only among novices creating data graphics but also in media, where intentional or unintentional distortions can (mis)shape perceptions of the data. To date, much of the work in data graphics has focused primarily on presenting information, with

less attention given to what can be inferred or deduced from it (Chen et al., 2008; Hudson et al., 2024). Within primary and secondary mathematics education, the appropriate application of aesthetic attributes remains an underexplored and insufficiently addressed aspect.

In addition to creating presentation-focused data graphics, data scientists frequently examine existing datasets from various perspectives, rapidly generating multiple exploratory data graphics to uncover new insights (Unwin, 2020). Compared to presentation data graphics, which are typically created for audiences not directly involved in the data investigation, exploratory graphics are primarily intended for use by data scientists themselves. As a result, less emphasis is placed on refining aesthetic attributes, with more time and effort dedicated to generating graphics that reveal hidden patterns in the data from various perspectives. For example, the scatter plot in Figure 1 reveals the exponential pattern of the dataset, a feature that is not immediately apparent in its tabular form. This use of exploratory data graphics as a tool also highlights and links to another key feature of data visualisation - its role as a *process* for exploration and analysis of data.

Figure 1

The Use of Exploratory Graphics to Reveal the Pattern of Dataset



Data Visualisation Acting as a Process

Data visualisation is a powerful *process* that allows us to interpret and represent what data appear to convey (Tukey, 1977). In data-rich investigations, the approach used to explore and analyse data extends beyond using data solely as evidence to confirm a hypothesis – a method known as *confirmatory data analysis*. Instead, these investigations often begin with *exploratory data analysis* (EDA), which encourages delving deeper into data and data graphics to uncover new insights. The notion of delving deeper goes beyond identifying patterns and outliers within a dataset. Data visualisation as a process invites us to ask a critical question: what exactly are we visualising? Although data may appear to be the most obvious answer, a cursory examination of data limits our ability to uncover deeper and more nuanced insights. Within the context of classroom teaching and learning, students are seldom given opportunities to critically engage with the process of generating and wrangling with data (Rubin, 2021). Instead, they often work with pre-prepared, neatly organised datasets that lack the complexity and messiness of real-world data (Gould et al., 2017). To address this gap, learning activities are needed that enable students to actively collect, prepare, and analyse data for exploratory data analysis. By engaging in the entire data lifecycle - from data generation and collection to cleaning, organising, and visualising - students can develop a deeper understanding of the complexities inherent in real-world data (van Borkulo et al., 2023).

Advancements in data science and technology have made innovative *data graphics* increasingly being used in the process of data visualisation. Examples include, but are not limited to, heat maps, dynamic bubble charts, and decision trees. Innovative data graphics expand the possibilities of what can be visualised in exploratory data analysis. For example, a decision tree is more than just a visual display of a dataset; it serves as an algorithm of the

reasoning process involved in modelling with the dataset (Erickson & Engel, 2023). In other words, data visualisation not only shows trends and patterns of data but also reveals *information* which might be deduced from data (Chen et al., 2008). The view of data visualisation as a process reflects how data scientists deal with big data, using visualisation to explore the complexity of datasets and then create algorithms for machine learning models (Godsey, 2017). For instance, data scientists often rely on visual tools to identify patterns, test hypotheses, and refine predictive models, making data visualisation an integral part of the iterative process of data analysis and model building. However, this dynamic and process-oriented view of data visualisation is not sufficiently emphasised in the school curriculum, where data visualisation is often reduced to static representations of numerical or categorical data (Lim et al., 2023). Students are rarely exposed to the idea that data visualisation can serve as a bridge between raw data and actionable insights, nor are they encouraged to explore how visualisation tools can be used to develop algorithms or predictive models.

Data and Data Visualisation in School Education

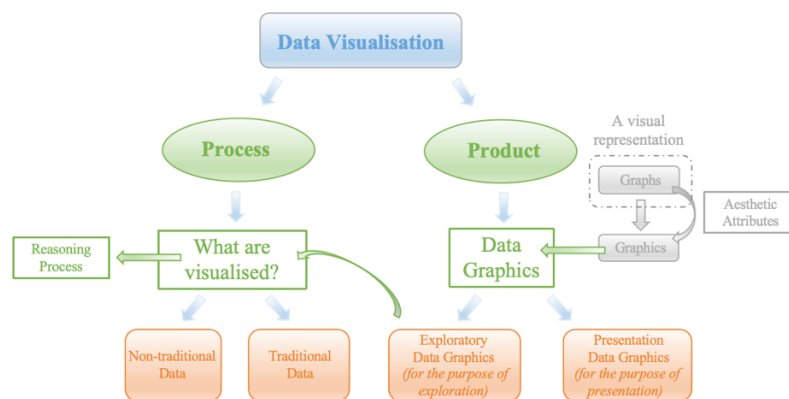
Big data differ from traditional data in three keyways: *volume* (the large size of datasets), *velocity* (the rapid speed at which data are generated and processed), or *variety* (the wide range of data types) (Laney, 2001; McAfee & Brynjolfsson, 2012). Within the context of school education, a dataset may be classified as non-traditional if it demonstrates any of these three defining characteristics. Researchers have underscored the significance of integrating non-traditional data into school curricula to improve students' ability to reason with these emerging forms of data (Noll et al., 2023). Recent research on the use of technology-based data visualisation tools, such as CODAP, has demonstrated the possibility of engaging primary and secondary students in the exploration of large datasets (referred to as *volume*) (Hudson et al., 2024; Zapata-Cardona, 2023). Compared with traditional data such as numerical and categorical data, non-traditional data take diverse and unstructured forms (referred to as *variety*) such as sound, images, text, and other multimedia representations (Arnold et al., 2021). Newer forms of data are generally analysed using advanced technology such as Natural Language Processing for text and Computer Vision images and other multimedia representations. While teaching students these technologies to translate text or image data may not be feasible, it is highly possible to introduce diverse data forms and related creative visualisations (Fergusson & Pfannkuch, 2024). Furthermore, the final characteristic of non-traditional data—velocity—can also be integrated into school education. For instance, a recent study illustrated how students utilised data visualisation tools to examine and analyse real-time temperature data collected by sensors installed within the vicinity of their school community (Fleischer et al., 2022).

The rise of big data has enabled the transformation of many previously unquantifiable aspects of the world into data - a process known as *datafication* (Cukier & Mayer-Schoenberger, 2013). An example of datafication is the use of facial expression data to analyse emotions. Advanced algorithms can capture, quantify, and interpret various facial features, such as the curve of a smile, the furrowing of eyebrows, or the widening of eyes (Martinez & Du, 2012). These features are then analysed to determine emotional states like happiness, surprise, or frustration. By translating these subtle physical cues into data points - such as the distance between the eyebrows and the mouth baseline - this process turns complex human emotions into measurable information that computers can interpret. To achieve this, experts must first identify the *features* (c.f. feature engineering) which refer to any values that can be computed from data for the purpose of modelling the problem (Duboue, 2020). Capturing such features requires individuals to *be aware of*, and able to visualise, the existence of data in real-world, often messy, scenarios. A recent study on data awareness reveals that even students without formal statistical backgrounds can identify potential variables within real-world phenomena,

despite not always being explicitly aware of the existence of data (Sherwood & Makar, 2024). To facilitate this process, *engaging and accessible contexts* are needed for students, ensuring that these contexts promote datafication as well as encouraging the exploration of non-traditional data through the incorporation of data visualisation. For instance, self-quantification serves as an exemplary context, as it naturally integrates these elements while fostering student interest. In this section, we have explored the concept of data visualisation and data within the context of K-12 mathematics education, drawing on a review of relevant literature. To consolidate our findings, a framework for data visualisation is presented in Figure 2.

Figure 2

A Framework of Data Visualisation



Data Visualisation in Self-Quantification

A key challenge for many mathematics educators lies in identifying effective strategies to integrate this comprehensive and innovative view of data visualisation into classroom instruction. To address this challenge, we draw upon the concepts on data and data visualisation included in the framework (Figure 2) and explore the role that data visualisation plays in different types of self-quantification – plugged or unplugged.

Self-quantification is a process that involves personally collecting and interpreting data about various aspects of one's own life (Maltseva & Lutz, 2018). The primary advantage of using digital devices for self-quantification is their ability to transform personal data into visual displays in real-time, providing users with immediate insights (Lupton, 2013). These visual displays exemplify the concept of *data graphics for both presentation and exploration*, as outlined in the framework presented in Figure 2. For instance, an individual might establish daily physical exercise goals using a smartphone application. At 3:00 PM, they could review real-time data graphics displaying metrics such as energy consumption and steps taken, interpret the visualised information, and make informed decisions about their activities for the remainder of the day. In this example, real-time data graphics, which are initially designed for presenting information about individuals' physical activity, serve as a medium for communicating insights derived from the data. This process highlights how self-quantification tools not only generate *data graphics for presentation* but also facilitate immediate, actionable insights through the *process* of data visualisation.

Due to advancements in self-tracking technology, such as smartphones and smartwatches, self-quantification has emerged as a new field of research. Self-tracking can also be conducted without the use of technology, as individuals may engage in manual methods of recording and reflecting on their own experiences. For instance, data creatives Lupi and Posavec undertook a one-year self-quantification project, during which they exchanged hand-drawn data visualisations on a specific topic each week (Lupi et al., 2016). In unplugged approaches to

self-tracking, individuals must thoughtfully identify and define the *features* that can be quantified to effectively capture data relating to the chosen aspects of one's life. With the abundance of non-traditional data available today, teaching students how to *recognise* and *identify* features of non-traditional data becomes crucial in school mathematics education.

Whether plugged or unplugged, self-quantification offers valuable contexts for designing meaningful, data-rich projects. Wearable technologies empower learners to gather real-time data and interpret dynamic visualisations, gaining insights into their physical, mental, and emotional selves (Wilson, 2012). Over the past decade, researchers have explored the use of self-tracking devices to engage students in collecting and analysing data about their own lives (Lee et al., 2021). Findings from these studies indicate that incorporating real-life and real-time data enhances students' statistical reasoning and deepens their understanding of key concepts in statistics (Lee, 2019). Unplugged self-quantification complements plugged approaches by offering learners more opportunities to transform real-life topics into measurable features and manually create different types of data graphics to effectively explore and present these features. A study focusing on data visualisation found that when given a research topic, students were able to quantify it by identifying and measuring features from their own life (Krekhov et al., 2019). However, these students used data visualisation as a tool for passively representing data (data graphics for presentation), rather than actively using it as a process for exploration and analysis (data graphics for exploration). One possible explanation is that schools often teach data visualisation primarily as *data graphics* to display data, and current mathematics curricula place less emphasis on its role as *data graphics for exploration*. This highlights that incorporating real-life contexts alone does not always result in rich learning experiences with data (MacGillivray, 2023). Instead, designing meaningful learning activities that engage students with authentic contexts and complex data requires careful planning and significant time investment. Consequently, it is essential to provide K-12 mathematics educators with a greater number of guidelines and illustrative examples to facilitate their implementation, experimentation, and provision of constructive feedback to learners.

A Simple Example of Incorporating Data Visualisation

In this section, we demonstrate a simple example to explore the question: *How can data visualisation be utilised in activities framed within the context of self-quantification*. The design of a simple example was informed by insights and theoretical perspectives derived from the literature review.

In this learning activity, students identify, collect, explore, analyse, and present data from their own life to answer the following question: "How do I connect to people surrounding me?". First, students quantify the research question by brainstorming possible measurable features as variables (Van Dijck, 2014). Next, they plan for data collection via *self-tracking* by creating a data journal to document the process, addressing who, how, when, where, and why they connect to family members, friends etc. (Rubin, 2021). During data collection, they gather data on the identified variables over 1 to 2 weeks, using appropriate *data graphics* to organise the data. When organising the data, students use *data visualisation as a process* to note issues like missing data or outliers and adjusting their approach for the remaining collection period. Afterward, they analyse the data at an individual level, identifying patterns and variations while considering the context. They then expand their analysis to the class level, comparing their findings with peers and selecting appropriate statistics for interpretation. Finally, they present and reflect on their findings by creating an infographic (*data visualisation as a product*), evaluating how self-tracking has impacted their lives and ensuring the data graphic effectively communicates their understanding of non-traditional data with *variety* and *velocity*.

The purpose of this sequence of learning tasks is to offer K-12 mathematics educators a practical approach to integrating key elements of data science into their regular teaching

practices. The question “How do I connect to people surrounding me?” is intentionally open-ended and uncertain, offering students the opportunity to draw on their everyday experiences to identify measurable features of a phenomenon, which can then be conceptualised and analysed as data within the context of self-quantification (Lee, 2021). The focus on data preparation aligns with the concept of “data-ing,” which refers to the process of enhancing data quality, organising it, selecting relevant features, and visualising it effectively to prepare for analysis (Gafny et al., 2024). This learning activity is designed to emphasise the dual role data visualisation plays (as both *a process* and *a product*) at various stages of a data-rich investigation. While the sequence of learning tasks offers educators an idea for designing projects that incorporate non-traditional data, successful implementation of the learning activity hinges on effective teaching practices.

Conclusion

Big data continues to shape various aspects of our daily lives, yet research in the field of data science education is still in its early stages (Gafny et al., 2024). Given the novelty of this research area, it is crucial to explore the practical applications of key data science elements. This paper reinforces potential of data visualisation in primary and secondary mathematics by framing it as both a product and a process. Moreover, we proposed a framework of data visualisation through the lens of school education. However, challenges such as working with messy, real-world data and the interdisciplinary nature of data science underscore the need for thoughtful curriculum design and targeted professional development for educators. To address these challenges, we employed the data visualisation framework as a guiding structure to review literature on self-quantification, which serves as an authentic context for enabling students to explore non-traditional data through data visualisation. Future research should focus on the implementation of this learning activity to investigate the affordances and constraints relating to how students engage in self-quantification through using data visualisation as a tool.

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Private Tutoring and Mathematics Education: A Review of the Current Research Landscape and Future Directions

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The tutoring market has expanded dramatically over the past decade particularly in mathematics. In this paper, we problematise the growth of mathematics tutoring given its societal impacts and the current lack of evidence for its effectiveness. In addition to gaps in research on its effectiveness, there exist no comprehensive frameworks for conceptualising effectiveness in tutoring. To address this gap, existing research on the effectiveness of tutoring will be considered to inform the development of a holistic evaluation framework. This theoretical paper will facilitate identification of directions for future research to foster a more nuanced understanding of tutoring's impact.

Introduction

In recent years, the tutoring industry has expanded rapidly, becoming an integral part of education systems worldwide, particularly in countries such as Australia, China, South Korea, and Japan (Bray, 2009; Guo et al., 2020; Zhang et al., 2021). Commonly referred to as 'shadow education', tutoring consists of fee-based supplementary lessons outside regular school hours, aimed at enhancing students' academic performance (Bray, 2009; Zhang et al., 2021). Its growing prevalence reflects the increasing pressure on students and parents to secure academic advantages in highly competitive educational environments (Zhang et al., 2021).

Among the various subjects, mathematics tutoring constitutes a significant share of the market, underscoring its perceived importance in academic success and its role as a foundation for STEM-related career pathways (Zhang et al., 2024). Despite the potential benefits of tutoring, its rapid growth has sparked considerable debate. Proponents highlight its role in addressing learning gaps and improving student outcomes, particularly in mathematics (Zheng et al., 2020). However, as access to high-quality tutoring or tutors that have high qualifications often correlates with socio-economic status (Ömeroğulları et al., 2020), critics point to concerns about an increasing reliance on tutoring potentially exacerbating inequalities. Additionally, there are concerns about tutoring's possible disruption of teaching in normal schooling (Yung, 2020). While research has examined these dynamics in various international contexts, there remains a notable gap in studies focusing on the Australian education system. In this paper, we aim to critically examine the global research landscape on tutoring, unpack how its effectiveness is understood in the tutoring context, and conceptualise what effectiveness means in tutoring.

Tutoring: Current Issues and Socio-Economic Impacts

Despite existing research on the academic benefits of tutoring (Guo et al., 2020; Zheng et al., 2020), increasing attention is being given to its broader socio-economic and policy implications. The societal impact of tutoring is particularly evident in China, where families spend an average of USD 5,000 annually, with students dedicating approximately 10 hours per week to private lessons (Zhang et al., 2021). Concerns over its consequences on educational inequalities and burdens on families' educational expense led to the introduction of the 'Double Reduction' policy, which prohibited tutoring agencies from providing instruction aligned with school curricula (Zhang et al., 2024). Similarly, South Korea implemented strict regulations on (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 253–260). Canberra: MERGA.

private tutoring between the 1980s and 2000, aiming to reduce education inequalities linked to socio-economic disparities. However, after the policy was lifted in 2000, participation in tutoring surged, with approximately 75% of students engaging in private lessons, contributing 2% to the national GDP (Kim & Park, 2012). Beyond Asia, similar trends have emerged. In Germany, 8.2% of primary students receive mathematics tutoring before entering high school (Benckwitz et al., 2022). In the UAE, 27% of students are tutored, with 83% of these students being tutored in mathematics (Rocha & Hamed, 2018). In the Australian context, there is a lack of comprehensive national data, which makes it difficult to quantify student participation in tutoring. However, estimates suggest that tutoring in Australia is a billion-dollar industry (Australian Tutoring Association, 2022), with over 45 thousand tutors formally identified by Jobs and Skills Australia (Australian Government, 2024). However, the true size of the Australian tutoring industry is unknown, as the work of many tutors goes unreported. Data gathered by major tutoring providers suggests that approximately 40% of Australian students engage in some form of private tutoring (Apex Tuition Australia, n.d.).

Research claims that the expansion of tutoring is largely driven by societal pressures, including parental expectations and peer competition (Guo et al., 2020). Mathematics, in particular, is often regarded as a high-stakes subject, prompting parents to invest in tutoring as a strategic tool for securing academic advantages (Zhang et al., 2021). However, this shift has raised concerns that tutoring has evolved beyond remedial support into a competitive mechanism designed to outperform peers, particularly in high-stakes examinations (Guo et al., 2020). Critics argue that this transformation undermines the original purpose of tutoring, which was to support struggling students, particularly in challenging subjects such as mathematics (Yung, 2020). Given tutoring is widely used for academic advantage, tutoring exacerbates social inequalities, as wealthier families have greater access to more experienced tutors who have higher qualifications, which ultimately widens the educational divide (Bray & Kwo, 2014; Yung, 2020). This disparity is particularly evident in mathematics education, where access to skilled and qualified tutors significantly influences students' conceptual understanding and problem-solving abilities (He et al., 2021). Additionally, the widespread reliance on tutoring raises critical questions about the adequacy of formal education. Some scholars argue that mainstream classrooms often fail to provide sufficient individual attention or accommodate diverse needs, prompting families to seek external support (Benckwitz et al., 2022).

Considering these global trends, tutoring presents significant challenges in relation to parental anxiety, equity, academic competition, and formal education. While the format of tutoring varies across countries, a fundamental question remains central to scholarly debate: Is tutoring truly effective? Correspondingly, what does 'effective' mathematics tutoring mean?

Researching the Effectiveness of Tutoring

Given the concerns raised and common perceptions of tutoring's ability to enhance students' academic performance, it is important to be able to measure the efficacy of tutoring in meeting the perceived needs. Research on the effectiveness of tutoring in improving student learning outcomes remains inconsistent across countries and educational contexts (Guo et al., 2020; Ömeroğulları et al., 2020). Scholars have identified multiple factors contributing to this variability (Guo et al., 2020; Zhang et al., 2021; Zheng et al., 2020). One major challenge lies in the context-dependent nature of effectiveness, as its conceptualisation varies significantly across different educational settings and stakeholders who view tutoring as a commodity for satisfying their own needs. Bray and Kwo (2014) highlights that research design and findings are highly sensitive to local contexts, meaning that stakeholders from different education systems and policy environments may have divergent criteria for determining what constitutes effective tutoring. Factors such as government policies, school structures, and learning environments shape these perceptions, making it difficult to establish a universal definition of

tutoring effectiveness. Moreover, this contextual diversity has led to a lack of a standardised conceptual framework for reliably evaluating student learning outcomes and teaching quality (Zhang et al., 2021). While many studies focus primarily on academic achievement as a measure of effectiveness, others adopt a broader perspective, assessing the development of cognitive skills, problem-solving abilities, and student engagement in mathematics (Guo et al., 2020; Ömeroğulları et al., 2020; Zhang et al., 2024; Zhang et al., 2021). Given these challenges, this section examines how scholars have assessed tutoring effectiveness, identifies limitations in existing research, and highlights key areas for further investigation in the Australian context.

Mixed Results in Global Research on Effective Tutoring

Despite the widespread belief in tutoring's ability to enhance student performance, research has yielded mixed findings regarding its effectiveness (Zhang & Liu, 2022; Zhang et al., 2021). While some studies report significant academic benefits, others suggest more modest or even negligible impacts. One common approach to assessing tutoring effectiveness involves comparing students' standardised test scores before and after receiving tutoring (Zhang et al., 2021). This method, though methodologically valid, has produced mixed results depending on factors such as tutor qualifications, tutoring intensity, quality of instructions, and programme structure. For instance, a meta-analysis by Nickow et al. (2020) found that tutoring programmes generally have substantial positive effects on student achievement, particularly when delivered by trained teachers or paraprofessionals. Similarly, Zhang et al. (2021) highlighted that tutoring is most effective (as measured by school examination scores) when conducted during school hours and structured into frequent, short sessions. However other studies (Zhang & Liu, 2022; Guill et al., 2022), suggest that private tutoring's impact on student achievement is relatively weak, implying that the benefits may be overstated or contingent on contextual factors. Additionally, White et al. (2023) argue that the effectiveness of tutoring depends on instructional quality, tutor-student relationships and the alignment of tutoring with students' academic needs. These conflicting findings underscore the need for a nuanced understanding of tutoring's impact. While tutoring can enhance academic performance, its effectiveness is influenced by multiple factors, including student motivation, programme design, and socio-economic context (Bray & Kwo, 2014). Moreover, the definition of effectiveness varies across studies (Zhang et al., 2021; Zheng et al., 2020), with some emphasizing short-term gains in test scores and others considering broader educational outcomes. This complexity leads to a crucial question: how should effectiveness in tutoring—particularly in mathematics—be defined?

Issues With Defining Effectiveness in Mathematics Tutoring

As discussed earlier, any study evaluating tutoring effectiveness must first establish a context-informed understanding of what constitutes effective tutoring. Defining effectiveness requires a critical review of existing literature to examine how scholars have conceptualised and measured tutoring outcomes. Many studies begin by identifying the motivations driving students and parents to seek tutoring. For instance, Guo et al. (2020) and Zheng et al. (2020) highlight that students engage in tutoring for a variety of reasons depending on their personal or local contexts (e.g., prior achievement, motivation in learning mathematics, pressure from examinations, local educational system and resource distribution), and these motivations shape how effectiveness is perceived. Some families prioritise tutoring as a means to improve performance on high-stakes examinations. For these families, higher test scores and improved class rankings serve as primary indicators of effectiveness. In contrast, other families adopt a more holistic perspective, valuing tutoring for its role in enhancing conceptual understanding, problem-solving skills, and fostering a genuine interest in mathematics.

While defining effectiveness through student and parental motivations provides valuable insights, this approach overlooks perspectives from other key stakeholders, such as teachers

and curriculum developers, whose work is also influenced by tutoring. Although academic performance aligns with parental expectations, in the Australian education context, deeper mathematical thinking, reasoning skills, and conceptual understanding are often more emphasised in school instruction by teachers and curriculum developers (He et al., 2021). If tutoring effectiveness is measured solely by short-term academic gains, it risks neglecting broader educational objectives. That said, He et al. (2021) identified that tutors often ignore the holistic aims of (mathematics) education, that they prioritise student's fluency in replicating procedures and overlook the enhancement of skills like reasoning, communication, and logical thinking, which are critical skills for successful mathematics learning. Additionally, affective factors, such as a students' confidence in solving mathematical problems and their mindset toward learning, are difficult to quantify but essential in evaluating tutoring's role in mathematics education, as those factors may directly or indirectly contribute towards students' long- and short-term success in learning mathematics. Effectiveness in tutoring is further complicated by variability in student backgrounds, including prior knowledge, levels of achievement, and learning needs. As a result, tutoring outcomes can be both short-term (e.g., immediate test performance) and long-term (e.g., sustained mathematical competency and independent problem-solving skills). These complexities highlight that effectiveness in tutoring cannot be reduced to a single metric but should instead be understood and defined through a multidimensional framework that considers academic, cognitive, and affective development.

Key Factors Influencing Tutoring Effectiveness

Ömeroğulları et al. (2020) identified three key factors that influence the effectiveness of mathematics tutoring: socioeconomic status (SES), parental involvement, and motivation and prior performance. SES is a key determinant for access to high-quality mathematics tutoring, with those from higher SES backgrounds having access to experienced tutors with advanced qualifications (Ömeroğulları et al., 2020) and others from lower SES families having limited access to qualified tutors (Guo et al., 2020). These disparities indicate that there remains inequitable access to mathematics tutoring. Active involvement by parents and students is another determinant of tutoring effectiveness (Zheng et al., 2020). Students who receive active encouragement from their families tend to perform better in mathematics and obtain benefits from tutoring (Zheng et al., 2020). Motivation and prior academic performance also play pivotal roles in gaining benefits in mathematics tutoring (Benckwitz et al., 2022; Zhang et al., 2021). High-achieving students, who are more motivated, can achieve better results in standardised assessments from tutoring sessions (Zhang et al., 2021). Conversely, lower-achieving students may require longer periods of support to see meaningful progress. This highlights the influence of individual learner characteristics in impacting the effectiveness of tutoring.

While the above factors can actively impact the effectiveness of tutoring, it is also essential that scholars realise that those factors can also be the driving reason of why students and parents choose tutoring in mathematics. For instance, students and parents may choose tutoring to boost their confidence in tackling difficult questions, to get better results in high-stake assessments which influence their career pathways, or to demonstrate more autonomy involving making educational decisions. As such, those factors can not only influence how tutoring's effectiveness is evaluated but can also shape the impact tutoring has on many different aspects in students' lives which may eventually be used to evaluate the effectiveness of tutoring. As a result, how should the scholars comprehensively evaluate the effectiveness of tutoring in the context of Australia becomes a key consideration for any research to take place.

Evaluating the Effectiveness of Tutoring in Australia: Developing a Conceptual Framework

In the previous sections, this paper has outlined the existing challenges and complexities associated with tutoring, emphasising the necessity for further research in the Australian context. While international studies have explored the multifaceted impacts of tutoring, there remains a substantial gap in the development and application of conceptual frameworks specifically designed to assess its effectiveness. This section aims to address this gap by proposing key considerations that scholars should incorporate when formulating conceptual frameworks for tutoring research in Australia. As outlined previously, Bray and Kwo (2014) highlight the importance of adopting a context-informed conceptual framework, recognising that tutoring operates within distinct educational, socio-economic, and policy environments. As such, it is necessary that scholars assess the contexts in which tutoring is shaped in Australia, such that a conceptual framework can accurately evaluate the effectiveness of tutoring.

Defining Effectiveness in Tutoring Research

A foundational step in evaluating tutoring effectiveness is to establish a clear and multidimensional definition of what constitutes effectiveness. As explored previously, existing studies often conceptualise effectiveness by solely focusing on academic performance (Zhang et al., 2021). However, this approach is limited as it fails to account for interest of other stakeholders and longer-term educational benefits such as cognitive development, problem-solving abilities, and student autonomy in learning (Guo et al., 2020). A robust conceptual framework should consider three key dimensions of effectiveness:

1. Academic Outcomes – measurable improvements in student performance, including standardised test scores, class rankings, and subject-specific proficiency, particularly in mathematics and STEM-related fields.
2. Cognitive and Affective Development – gains in student confidence, self-efficacy, and motivation toward learning, which are critical for sustained academic engagement.
3. Equity and Accessibility Considerations – the extent to which tutoring mitigates or exacerbates educational disparities, particularly in relation to socio-economic background and access to quality instruction.

A framework that incorporates these three dimensions could allow for a more holistic assessment of tutoring effectiveness, moving beyond simplistic measures of academic success to consider broader educational and social implications.

Theoretical Perspectives in Conceptualising Tutoring Effectiveness

To build a strong foundation for evaluating tutoring, established learning theories and educational models can be considered concurrently to holistically conceptualise the effectiveness of tutoring. One commonly cited framework is Carroll's (1963) Model of School Learning, which emphasises the relationship between instructional time, student aptitude, and learning outcomes. While this model provides a useful starting point, it has a narrow focus on instructional input, and neglects psychological, motivational, and socio-economic factors that influence tutoring effectiveness. An alternative perspective is Vygotsky's (1978) Sociocultural Theory, which highlights the role of scaffolding and social interaction in learning. In the tutoring context, this suggests that one-on-one or small-group tutoring may be most effective when it aligns with students' zone of proximal development (ZPD)—the range of tasks that learners can complete with guided assistance but not independently. This perspective underscores the importance of adaptive instructional methods, wherein tutors tailor their approaches to meet the diverse needs of individual learners. Another relevant theory is Self-Determination Theory (Deci & Ryan, 2012), which posits that students are more likely to

succeed academically when their basic psychological needs for autonomy, competence, and relatedness are met. Tutoring that emphasises student agency and active learning may therefore be more effective in fostering deep learning compared to rote memorisation-focused models.

Key Factors Influencing Tutoring Effectiveness in Australia

Based on prior research evidence, several contextual factors must be accounted for when designing a conceptual framework for tutoring research in Australia. These include socio-economic status (SES), student motivation and prior academic achievement, parental and institutional involvement, and the format of tutoring (e.g., online versus face-to-face).

Research conducted internationally consistently indicates that SES plays a crucial role in determining access to high-quality tutoring (Ömeroğulları et al., 2020). Families from higher-income backgrounds can afford experienced tutors with advanced qualifications, while lower-income students often rely on less-qualified tutors or may be unable to access tutoring at all (Guo et al., 2020). To address this global trend and examine whether if a similar trend exists in Australian education setting, a conceptual framework must account for these disparities, and consider how tutoring effectiveness varies across different SES groups.

Student motivation and prior achievement are also critical determinants of tutoring success. From a self-determination theory perspective (Deci & Ryan, 2012), students with high prior achievement are more likely to engage with learning and tutoring with intrinsic motivation. High-achieving students, who often seek tutoring for enrichment rather than remediation, tend to show greater academic gains in standardised test results (Zhang et al., 2021). Conversely, students who struggle with learning might participate tutoring under peer or parental pressure, this may yield in more extrinsic motivated learning in tutoring. Consequently, those students may require more sustained and targeted interventions to see meaningful engagement with learning and improvements in standardised test results. Therefore, research frameworks should differentiate between tutoring's impact on high-achieving vs. low-achieving students by taking into account the diverse motivation and prior achievement exist among students being tutored.

Active parental support has been shown to enhance tutoring effectiveness, as students who receive consistent encouragement tend to perform better (Zheng et al., 2020). Additionally, the relationship between formal schooling and tutoring is critical. If tutoring complements school instruction, students may experience significant benefits. However, if tutoring operates in direct competition with schools, as seen in some Asian contexts, it may undermine classroom learning. Understanding this dynamic is essential for an effective conceptual framework.

The Australian tutoring landscape has undergone significant shifts with the rise of online tutoring platforms, particularly following the COVID-19 pandemic. Studies have shown mixed results regarding the effectiveness of online tutoring compared to traditional face-to-face instruction (Benckwitz et al., 2022). A conceptual framework must differentiate between these modalities and assess their respective impacts on student learning outcomes.

Future Research Implications

This paper has highlighted contemporary issues related to tutoring and identified a significant gap in research within the Australian context. To advance this field, further exploration of tutoring's effectiveness and broader societal implications is important. Research on tutoring in Australia remains limited, both in scope and in the development of a conceptual framework suited to the local context. A well-defined framework is crucial for evaluating tutoring's impact and understanding its role in Australian education. Thus, scholars should build on the key issues identified in this paper to research and study tutoring comprehensively.

Several critical research gaps persist. First, no comprehensive Australian study has systematically compared students' academic performance before and after receiving tutoring.

While international research has yielded mixed findings depending on the country and context (e.g., Bray & Kwo, 2014), the absence of similar studies in Australia underscores the need for robust empirical investigation. Second, as previously noted, tutoring may have long-term effects on students' learning trajectories and career progression (Zhang et al., 2021). Longitudinal studies are necessary to determine whether tutoring influences students' study habits and academic success over time. Although this area of research is still developing globally, it has received more attention internationally than in Australia. Third, there is a lack of research on how external tutoring and formal schooling can coexist in a mutually beneficial manner. A deeper understanding of this relationship could help shift stakeholder perceptions and foster collaboration between tutoring providers and schools, ultimately enhancing student learning experiences. Lastly, given the absence of regulations in Australia's tutoring sector, further research is needed to examine its implications for education policy and government intervention. This is particularly important given the Australian tutoring market appears to be resistant to attempts to bring in place national accountability measures (Dhall, 2024).

Addressing these gaps would contribute to a more nuanced understanding of tutoring in Australia. It would also support the professionalisation of tutoring and strengthen its relationship with formal education—ultimately prioritizing student learning and achievement.

Conclusion

The expansion of tutoring has led to concerns and debates regarding its effectiveness, accessibility, and impact on formal education globally. While tutoring may be perceived as an efficient tool for academic improvement by many stakeholders, this paper has demonstrated that current research findings on tutoring's effectiveness remains inconsistent across various contexts and regions. Furthermore, a clear and comprehensive framework for evaluating tutoring effectiveness is currently lacking, particularly within Australian education settings. This gap highlights the need for a more nuanced understanding of tutoring's role in students' learning beyond simple measures of academic performance.

At the same time, we have identified key challenges in understanding, defining, and measuring the effectiveness of tutoring. Though a variety of indicators have been used, there is still yet an agreement on what constitutes effective tutoring. Additionally, factors such as socioeconomic status, student motivation, prior achievement, tutor qualifications, and the relationship between tutoring and formal schooling further complicate the evaluation of tutoring effectiveness. These complexities further indicate the necessity for an integrated, context-informed framework that can accommodate multiple dimensions of effectiveness of tutoring.

In the Australian context, research on tutoring remains lacking, and the lack of national data further obscures our understanding of its role in students' achievement and its implications on society. To bridge this gap, future research should focus on systematically assessing both short- and long-term impacts of tutoring on students, schools, and society, especially how tutoring interacts with formal schooling, and implication of tutoring on the regulatory landscape. A well-defined conceptual framework tailored to the Australian education setting would greatly assist in advancing our understanding of this discourse and informing evidence and research-based policy decisions. This can therefore foster a more rigorous, equitable, and informed approach to tutoring research, which can contribute to the professionalisation of the industry and ensure that tutoring serves as a meaningful complement to schools, prioritising student development.

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Perceived Benefits of Collaborative Learning When Solving Cognitively Demanding Tasks

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Research suggests that collaborative learning may enhance the benefits to learners when solving cognitively demanding tasks. However, there are concerns about how students with different achievement and engagement levels perceive these benefits. This study examines how students with varying achievement and engagement levels respond to collaboratively solving cognitively demanding tasks. Findings reveal that collaboration enhanced peer learning and enjoyment, particularly for low and moderate achievers, and increased engagement among less engaged students. The findings highlight the potential of collaborative learning to improve problem-solving experiences for diverse learners.

There is a consensus that high-quality mathematics instruction should involve tasks that promote reasoning and problem solving (National Council of Teachers of Mathematics (NCTM), 2014). However, primary classrooms often rely on teacher-directed instruction, despite evidence linking problem solving and cognitively demanding tasks to higher student achievement (Le Donne et al., 2016). One possible explanation for this discrepancy may be that some teachers believe complex and cognitively demanding tasks are less suitable for students with lower achievement and/or engagement (Sullivan & Mornane, 2014). Research into students' views on challenging tasks is growing but studies rarely explore differences in student perspectives based on achievement or engagement levels. Generally, students have reported that they can both enjoy and learn from these experiences (Russo & Hopkins, 2019; Sullivan & Mornane, 2014) but some authors have suggested integrating other instructional practices, such as collaborative learning, to further enhance student engagement and participation (Russo & Hopkins, 2019).

Collaborative learning, defined as students' collective efforts to attain shared objectives or complete group tasks (Gillies, 2016), is widely recognised for enhancing student achievement and engagement (Gillies, 2003). Given its influence on student achievement and engagement, it is reasonable to assume that integrating opportunities for problem solving and collaborative learning would be productive for learning. There is some existing literature which examines the link between collaborative learning and problem solving (e.g., Watson et al., 2001), however, there is a scarcity of studies on student perspectives regarding the merge of these strategies.

An aim of the current study was to examine whether upper primary students with varying achievement and engagement levels perceive the benefits of collaborative problem-solving in cognitively demanding tasks differently. Examining these differences may offer insights into how learner characteristics shape their experiences in this context. Additionally, exploring the intersection of collaborative learning and cognitively demanding tasks can inform effective classroom implementation. Consequently, this study was guided by the following research question: How do students with varying levels of mathematics achievement and engagement perceive the collaborative solving of cognitively demanding tasks?

Literature Review

The emphasis on challenge and the use of cognitively demanding tasks is grounded in the inherent nature of mathematics being a dynamic, problem-solving discipline. One of the primary goals of teaching mathematics is to enable students to use mathematical thinking to solve problems (Stacey, 2006), and so the solving of cognitively demanding tasks is central to

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mathematics and essential to its learning. The role of challenge can be examined through Vygotsky's (1978) social constructivism theory. Vygotsky posits learning as a continuous progression from the current level or zone to a more advanced one. He describes the Zone of Proximal Development (ZPD) as the distance between a learner's current level and their potential development. Learners should be challenged within their ZPD to enhance their understanding. The ZPD also highlights the role of social interaction and collaboration, particularly with more knowledgeable peers or adults. The intersection of collaboration and cognitively demanding tasks within ZPD perspectives is significant. Cognitively demanding tasks present challenges just beyond an individual's capability, while collaborative settings enable collective problem solving, enhancing understanding and facilitating learning.

Cognitively Demanding Tasks and Collaboration

Cognitively demanding tasks refer to tasks that require significant mental effort and complex thinking processes. Leaning on existing literature into task design and challenging tasks (e.g., Smith & Stein, 1998; Sullivan & Mornane, 2014), these tasks involve students:

- using complex, non-algorithmic thinking with solutions not immediately apparent,
- processing multiple pieces of information,
- devising their own approaches and methods involving multiple steps,
- finding multiple solutions or solution strategies,
- exploring and connecting multiple mathematical concepts,
- solving problems with considerable cognitive effort and persistence
- explaining and justifying their thinking and reasoning.

Many researchers advocate that substantial learning gains occur when students work on tasks which require high levels of thinking and reasoning (Russo et al., 2020). In addition to learning gains, research has identified links to improved student motivation, participation and enjoyment (Russo & Hopkins, 2017; Sullivan & Mornane, 2014).

Also important to the current study is collaborative learning. Gillies (2016) defines collaboration as students working together towards a common goal. Collaborative learning is well-documented for its potential to enhance student achievement (Gillies, 2016; Slavin, 2014). Through collaboration, students learn to give and receive help, exchange ideas and consider diverse perspectives (Gillies, 2016; Webb et al., 2023). It can also improve language and social skills and boost motivation through active participation and behavioural engagement (Gillies, 2016; Johnson et al., 1985; Tunnard & Sharp, 2009).

The link between collaboration and problem solving is fundamentally tied to the nature of the task. Open-ended, discovery-based tasks inherently promote collaboration by encouraging discussion, evaluation and collective problem-solving efforts (Cohen, 1994). Consequently, cognitively demanding tasks can be effective in facilitating collaborative problem solving.

Concerns about low achieving and disengaged students are prevalent in both collaborative learning literature and studies on cognitively demanding tasks. Some teachers hesitate to use challenging tasks with these students (Sullivan & Mornane, 2014), and there are views that novice learners may struggle with self-constructed solutions (Kirschner et al., 2006). Similarly, debates persist about low achievers in collaborative settings, with some studies suggesting these students may not engage or may be overshadowed by high achievers (Mulryan, 1992). However, emerging evidence offers a contrasting perspective. For instance, Russo et al. (2020) reported that teachers were surprised by low performing students' success with challenging tasks. Additionally, Webb et al. (2023) found that low achievers could benefit equally from collaborative environments. This is promising, especially given the heterogeneity of Australian schools. Given the importance of allowing students to solve complex mathematical problems, it is crucial to explore whether collaborative learning supports all students in this context.

Methodology

An exploratory case study involving qualitative and quantitative data collection was conducted to investigate student perspectives on collaborative cognitively demanding tasks.

Intervention Lessons

Five intervention lessons were created, each featuring a cognitively demanding task that was solved collaboratively. Each lesson ran for 60-minutes and was conducted by the researcher who is a qualified primary teacher but not their regular classroom teacher. Figure 1 provides an example of a cognitively demanding task that was used during the second lesson and adapted from Sullivan (2018). This task was deemed to be appropriately cognitively demanding because it involved processing multiple pieces of information, required students to devise their own approach, allowed for multiple solutions, connected multiple mathematical concepts and solutions were not immediately apparent to students. For all tasks, students self-selected their partners to collaborate on solving them, resulting in approximately 9-12 heterogeneous groupings. This approach was chosen to ensure students felt comfortable with their partners and to encourage diverse achievement pairings. Support and extension scaffolding, based on Sullivan's (2018) enabling and extending prompts, were employed to cater for varying achievement levels within the class.

Figure 1

Example Task from Lesson 2



You have a box that needs 270 cm of string to tie it up like this.

You need 30 cm to make a bow. What might be the dimensions of the box? Give three different possibilities.

Support Scaffolding	Extension Scaffolding
If the box was a cube and had side lengths of 10 cm? How much string would you need to tie string around the box, with an extra 30 cm to make the bow? What if the box had side lengths of 20cm?	How many different solutions can you find?

Participants, Data Collection, and Analysis

A heterogeneous Year 6 class from an independent co-educational school participated in the intervention. Baseline data on students' achievement and engagement characteristics were collected prior to the intervention for the purpose of identifying the learner characteristics of students in the class. This included quantitative data from Progressive Achievement Testing (PAT), a researcher-developed assessment on addition and subtraction and Martin's (2007) Motivation and Engagement Scale (MES). After students were categorised by low, high or moderate achievement and engagement levels, 10 students with varying characteristics were selected to participate in the semi-structured interviews. Consent forms were obtained from all students according to the ethics protocol HREC 2022/746. The semi-structured interviews sought students' feedback on their experiences, behaviours, reactions and engagement during the lessons. The interviews were conducted immediately after each lesson, video recorded and transcribed. All intervention lessons were also video recorded. Researcher field notes were taken and student work samples were collected after each lesson. Braun and Clarke's (2006) hybrid thematic approach was employed to identify patterns within the qualitative data. Deductive followed by inductive methods were used to identify tags. An inductive process was then used to organise the tags into themes.

Results

Across the semi-structured interviews, 37 quotations were tagged where students referenced the collaborative solving of cognitively demanding tasks. These quotations were analysed and organised into six themes: opportunities for peer learning, increased efficiency, the support of peers, promoted behavioural engagement, provided enjoyable learning experiences and increased cognitive engagement. Evidence in support of these themes are presented in this section and draw from the perspectives of students with various mathematical achievement and engagement levels. Pseudonyms are used to protect student confidentiality. Students' achievement and engagement levels are included. For example, Maddie, a student identified by moderate achievement and low engagement, is classified as (Mod A, Low E).

Opportunities for Peer Learning

Observations of peer learning were most evident among low and moderate achieving students. For example, Maddie (Mod A, Low E) observed, "Since I was with a partner, they were able to explain it to me and help me with it" (Lesson 4). Hannah (Low A, High E) noticed learning opportunities through the exploration of diverse perspectives: "It's an opportunity to increase knowledge because, especially when you are working with other people, they know different stuff to what you know" (Lesson 2). This was also observed by Edward (Low A, High E), who learned efficient strategies from his partner: "He shared answers with me that I wasn't thinking of, and I could realise how that would work better" (Lesson 5). In addition to learning directly from peers, some students also found it productive when able to explain their thinking aloud to others. For example, Sally (Low A, Low E) stated, "It was good to explain why. It gives me a better view" (Lesson 1). High achieving students reported a different perspective on peer learning. Victor (High A, Mod E) valued the assistance of peers when finding additional approaches: "They helped me think of the second strategy" (Lesson 5), and Laura (High A, High E) noted learning when comparing different solutions: "We had lots of time to think and compare our thinking with the other person" (Lesson 4).

Increased Efficiency

Nearly half of the case study participants, all with low or moderate achievement and engagement, viewed the collaborative solving of cognitively demanding tasks as an efficient process compared to individual problem-solving processes. Sally (Low A, Low E) expressed her enjoyment in sharing the workload with her group members, stating, "I enjoyed it because I didn't have to do all the work" (Lesson 1). Maddie (Mod A, Low E) echoed this sentiment, describing the equitable process of solving tasks with a partner: "Since I was with a partner, we would both take turns putting them down and finding the answer" (Lesson 4). Similarly, Connor (Mod A, Mod E) highlighted the collaborative approach after the final lesson, stating, "My partner helped with the problem. They helped me complete it" (Lesson 5).

Support of Peers

Students with all achievement and engagement levels reported receiving peer support, mainly social affirmation and positive encouragement. Two high achieving students appreciated having someone there to validate and check answers. For example, George (High A, High E) stated, "If you don't know a question, you can turn to someone and say and ask for an answer" (Lesson 1). Similarly, Laura (High A, High E) said, "I could share my ideas with someone else and I could get more clarification ... If you are in a pair with someone who works with you, you can figure out stuff together and correct each other's mistakes" (Lesson 2). This was also observed by moderately achieving students. For example, Leah (Mod A, Mod E) shared, "I could tell my answers to the people around me and see if we both got the same number" (Lesson 3). Connor (Mod A, Mod E) spoke about relying on collaborative partners

when stuck: “If I couldn’t think of something, the person next to me could think” (Lesson 4). Leah (Mod A, Mod E) also felt supported by peers to reduce the likelihood of errors: “She explained how to do it and if I got it wrong, she was able to correct my mistakes and explain to me why that’s wrong and I would get the answer right” (Lesson 5). Two low achieving student identified peer support when their partners guided them through the task. For example, Oscar (Low A, Low E) claimed “That’s how I found out how to do most of the work” (Lesson 4).

Promoted Behavioural Engagement

All responses relating to increased engagement came from high or moderately engaged students. Reasons for increased engagement included interacting and discussing mathematical ideas aloud. For example, Leah (Mod A, Mod E) remarked, “I was sharing my ideas, not keeping them to myself. While in other pieces of work, sometimes I would keep them to myself and not really share them. When I say it to my partner, I’m getting involved” (Lesson 2). Edward (Low A, High E) highlighted the benefit of talking through ideas: “I liked it when we sorted the things as a group, but it wasn’t much fun when we did it by ourselves and we sorted out the problems. When we did the group one, we got to talk more and share our ideas. We put our heads together” (Lesson 1). He further elaborated, “I was most engaged in the group one because I was suggesting things and deciding. I thought I was part of the group” (Lesson 1).

Another social aspect noted by participants was the necessity to reach agreement on solutions or strategies. George (High A, High E) shared, “The partner work, we had to agree on something. If I think something is harder than the other and he thinks a different option, we have to use different maths sentences to sort them out” (Lesson 1). Edward (Low A, High E) similarly mentioned: “We both had different ideas ... we saw which one was right and we were listening to each other” (Lesson 2). Hannah (Low A, High E) spoke about the role of her partner to actively engage her through questioning and dialogue, stating, “I was most engaged when I was asked questions and challenged by [partner] during the challenging task” (Lesson 2).

Enjoyable Learning Moments

Enjoyment was a common response experienced by the students. Comments relating to enjoyment of these experiences came from low or moderately engaged students. Oscar (Low A, Low E) expressed high levels of enjoyment during three different lessons when given the opportunity to solve problems with peers. He remarked, “It was more fun when you do it with a partner” (Lesson 1), “It was fun because we were with our friends. It’s more enjoyable because you are with your friends and you work it out with them” (Lesson 2). In a later interview, he added, “because it is more fun with a partner. We worked together to get the answer, and it was a bit better” (Lesson 3). Similarly, Victor (High A, Mod E) shared his enjoyment of working with others to gain different perspectives, stating, “I found it fun that you had to see the different perspectives of your partner” (Lesson 1).

Enhanced Cognitive Engagement

Maddie (Mod A, Low E) consistently reported increased motivation when solving cognitively demanding tasks with others. After the third and fifth lessons, she highlighted the significant role a peer played in maintaining her motivation and persistence “I find it easier to work through difficult things with a partner because when I’m usually by myself, it makes me give up more easily. But if I’m with a partner, we both put our ideas, and we boost each other up” (Lesson 2). Additionally, she commented on the positive impact of peer support on her sense of accomplishment, “I find myself more engaged than if alone. If I did the first one, I would have given up, but since my partner was there, I felt if I got the answer, she would be proud of me and also, we could work together during it” (Lesson 4).

Discussion

Findings from this study affirm that students can perceive benefits when collaboratively solving cognitively demanding tasks. Six key findings were evident from student responses:

- Cognitively demanding tasks that were solved collaboratively promoted peer learning, especially among low and moderate achievers who benefited from partners explaining concepts and sharing perspectives. High achievers valued peer assistance for finding additional approaches and comparing solutions.
- Low and moderate achievers found the process of solving cognitively demanding tasks more efficient in a collaborative environment compared to an individual setting as they enjoying the shared workload and taking turns.
- Students of all achievement and engagement levels reported receiving social affirmation and positive encouragement in the collaborative setting. High achievers appreciated validation and clarification, while moderate achievers relied on support and error correction, and low achievers found guidance crucial.
- High and moderately engaged students reported increased engagement through interaction and discussion, valuing the need to agree on solutions and on-task engagement through questioning and dialogue.
- Collaborative cognitively demanding tasks were found to be more enjoyable, particularly for low achievers who appreciated working with peers.
- Low engagement students reported increased motivation and persistence when working with peers.

Affirming both Russo and Hopkins' (2017) and Sullivan and Mornane's (2014) findings, varied responses to cognitively demanding tasks were present. However, the collaborative setting in the current study seemed to play an instrumental role in positively influencing students' responses. In the following discussion, the research question has been split into two sub-questions to address trends in responses relating to students' achievement and engagement levels. Due to space constraints, only key findings are discussed.

How do Students with Varying Mathematical Achievement Perceive the Collaborative Solving of Cognitively Demanding Tasks?

The learning benefits perceived by low and moderate achieving students are noteworthy, given that concerns about cognitively demanding tasks and collaborative learning often focus on these learners. Peer learning, a well-established benefit of collaborative learning (e.g., Gillies, 2016; Slavin, 2014; Tunnard & Sharp, 2009), was frequently mentioned by these students, challenging the belief that complex tasks are unsuitable for them. Many low achieving students noted that their peers helped explain tasks, offered different perspectives, or modelled better approaches, providing learning opportunities that they might have missed independently. For instance, Sally (Low A, Low E) experienced increased learning through discussing and explaining approaches with her partner, aligning with Webb et al.'s (2023) finding that low achieving students can benefit from sharing and engaging with others' ideas. Similarly, Maddie (Mod A, Low E) found it helpful when her partner explained the task. Sullivan and Mornane (2014) emphasised the importance of task comprehension with challenging tasks, with Maddie engaging more deeply in the collaborative setting as her partner clarified the requirements.

Additionally, Kirschner et al.'s (2006) argument that novice learners struggle with self-constructed solutions is countered by findings that low and moderate achievers found the process of solving cognitively demanding tasks more manageable and efficient when working with peers. These students reported dividing the workload, taking turns, or seeking help when stuck, indicating that solution construction was a shared process rather than an independent one.

Importantly, high achieving students also reported learning benefits. They found it advantageous and enjoyable to explore multiple solutions and compare different problem-solving approaches. Additionally high achieving students valued peer support for validating and confirming responses. Supporting the findings of Johnson et al. (1985), high achieving students can also experience benefits when collaboratively solving cognitively demanding task.

How do Students with Varying Mathematics Engagement Perceive the Experiences of Collaboratively Solving Cognitively Demanding Tasks?

Moderately and highly engaged students reported increased behavioural engagement when collaboratively solving cognitively demanding tasks. They attributed this engagement to active dialogue, which positively contributed to their participation and on-task focus. Contrary to Mulryan's (1992) findings, there was evidence that low achievers actively engaged in the collaborative setting. Quotations from Edward (Low A, High E) and Hannah (Low A, High E) illustrated their high engagement and peer interaction during shared problem solving, showing that their achievement levels did not hinder their engagement during the task. Further research is needed to support low engaged students when collaboratively solving cognitively demanding tasks. However, this study provides evidence that contradicts the belief that low achieving students will disengage from complex tasks. It may be the case that students' engagement characteristics were more influential than their achievement levels in supporting behavioural engagement when students collaboratively solve cognitively demanding tasks.

Finally, the case of Eloise (Mod A, Low E) was promising as she reported increased cognitive engagement, particularly regarding her persistence and motivation, when working on such tasks with a partner. Persistence and motivation are both important behaviours for students when solving challenging tasks (Russo & Hopkins, 2017; Sullivan & Mornane, 2014). Eloise's remarks about giving up without her partner illustrates the potential the collaborative setting can have on increasing the persistence of low engaged students.

Implications, Limitations, and Conclusion

Student responses highlighted the potential benefits of collaboratively solving cognitively demanding tasks for learners. The findings are particularly encouraging for teachers with heterogeneous classes, as low achieving students reported numerous benefits, including peer learning, support, increased enjoyment, and motivation. Importantly, the collaborative setting appeared to be advantageous for all learners.

The study raises important questions about the role of student engagement, revealing that engagement levels had a greater influence on students' perceptions than achievement levels. Although low engaged students showed increased enjoyment and cognitive engagement, they reported fewer moments of engagement when collaboratively solving cognitively demanding tasks compared to moderately and highly engaged students. Engagement remains a barrier to optimising student learning and warrants further exploration in the context of cognitively demanding tasks.

Findings from this study highlight several practical implications, despite limitations such as a small sample size, short intervention duration and reliance on qualitative data. Two key implications for practitioners include:

4. Students' perceptions of cognitively demanding tasks vary with their achievement and engagement levels, necessitating pre-planned modifications and scaffolding to meet diverse needs. Adjustments for students with moderate and lower engagement are particularly important to encourage participation and persistence.
5. Despite concerns about the suitability of such tasks for low achieving students, the study found that these students can benefit from engaging in these tasks collaboratively.

Teachers should integrate cognitively demanding tasks into lessons to foster deep understanding and problem-solving skills for all learners. Collaboratively solving cognitively demanding tasks can enhance enjoyment, peer support, active engagement and motivation, positively impacting students' experiences in mathematics. Therefore, leveraging the intersection of collaborative learning and cognitively demanding tasks can create an inclusive, supportive environment that promotes mathematics as a problem-solving discipline.

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The Development of a STEM Literacy and Numeracy Program for the Foundation to Year 2 Years of Schooling

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This paper describes design and content decisions undertaken in the development of the Early Years STEM Australia (ELSA) Foundation-Year Two (F-2) school program. The ELSA F-2 program aims to foster critical STEM literacy and numeracy skills in young learners by providing an integrated approach to science, technology, engineering, and mathematics (STEM). The paper reviews the existing literature on the STEM Practices framework and the Experience-Represent-Apply (ERA) pedagogical approach that underpinned the design of learning activities and digital tools to ensure concentrated topic coverage of STEM related outcomes from the Australian Curriculum V9.

The Early Years STEM Australia (ELSA) program has received ongoing support from the Australian Government since 2016. The aim of the program is to enhance Science, Technology, Engineering, and Mathematics (STEM) engagement and numeracy learning from preschool (see Lowrie & Logan, 2019) through the early years of school (i.e., Foundation-Year 2). The school program will be the focus of this paper. Although STEM engagement has had a presence in schools for more than a decade, the term and its practice(s) have not been clearly defined (English, 2016). To some degree, this is because classroom teachers and educational leaders cannot reference a curriculum document to support the development of authentic learning activities. Instead, should they choose to follow a STEM approach, they must plan integrated STEM activities drawing from three different curriculum documents – Mathematics, Science and Digital Technologies, with the additional limitation that there are no curriculum documents supporting the integration of Engineering into STEM activities.

The conceptual framework underpinning ELSA is built on STEM practices, a departure from traditional, content-based approaches that dominate early years education. This framework provides a foundation for STEM literacy and numeracy that encourages active engagement with real-world problems, allowing young students to explore, apply, and represent their understandings in innovative ways. Instead of focusing solely on discipline-specific content, the ELSA framework nurtures skills and processes related to STEM thinking, which is crucial for building a more holistic understanding of the world.

This paper explores the conceptual framework of ELSA, drawing on existing research in STEM education and early childhood pedagogy, including the work of Larkin and Lowrie (2017), Lowrie et al. (2018), and Lowrie and Larkin (2020). In the following sections, we will outline the STEM practices framework, the Experience-Represent-Apply (ERA) pedagogical model, and report on how these theoretical and pedagogical constructs influenced the design of the ELSA F-2 program.

STEM Practices as a Design Brief

A practice is defined as an application or use of an idea, method or value (Lowrie et al., 2017). Such practices are enabled in situations when content is both contextualised and contested within the classroom—as students not only learn about discipline content but also understand how to use and apply knowledge and skills in various contexts. The focus on

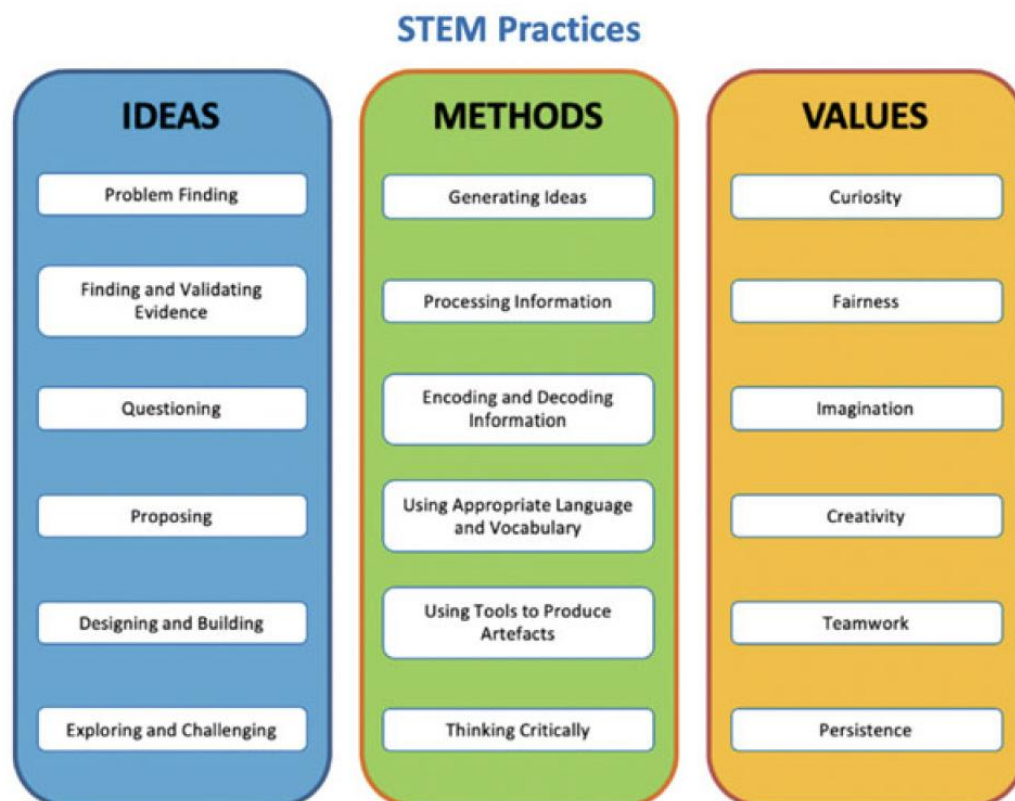
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practices ensures that understandings are related to the real-world contexts that are enacted through participation and engagement (Kemmis, 2008). Larkin et. al., (2022) maintained that STEM education should go beyond simply teaching the content of science, technology, engineering, and mathematics. Instead, STEM should focus on the practices that are characteristic of these disciplines. This approach aligns with the idea that STEM is a way of thinking and doing, rather than a body of knowledge that can be compartmentalised into isolated subjects.

Lowrie et al. (2018) discuss how these practices involve concept development and formation, problem-solving, critical thinking, and collaboration. STEM practices also emphasise real-world applications, where students can engage with complex problems and come up with solutions that have tangible outcomes. This approach allows students to not only acquire content knowledge but to also understand how to apply it in various contexts. In working with educators in the ELSA program, the following 18 ideas, methods and values were established as important for early years STEM learning (see Figure 1).

Figure 1

STEM Practices Underpinning the ELSA Program (Note: Permission Granted by SPLAT-Maths to Use This Image).



By focusing on STEM practices, the ELSA program encourages young learners to engage with the core principles of STEM through hands-on activities that promote exploration and creativity. This aligns with research that suggests that early exposure to STEM practices supports the development of critical thinking skills and helps students become more confident and competent in their problem-solving abilities (Brenneman, et al., 2019).

The Experience-Represent-Apply (ERA) Pedagogical Framework

The Experience-Represent-Apply (ERA) framework, theorised by Lowrie and Larkin (2020), is the second key component of the ELSA program. The ERA framework emphasises three distinct phases that are integral to STEM learning: experience, represent, and apply. This framework builds on the understanding that learning is an active process that involves not only encountering new information but also making sense of it and applying it in meaningful ways.

- **Experience:** In the first phase, students engage with real-world problems or scenarios that stimulate curiosity and inquiry. They are encouraged to explore concepts through hands-on, experiential learning activities that connect abstract ideas to tangible experiences.
- **Represent:** In the second phase, students use various tools and strategies to represent their understanding of the concepts they have explored. This may include using visual representations, mathematical models, diagrams, or digital technologies to express their ideas.
- **Apply:** The final phase involves students applying their knowledge to solve problems or create solutions in real-world contexts. This phase encourages students to think critically and make connections between their learning and the world around them.

The ERA framework supports the idea that learning is an iterative process that involves not just absorbing content but also actively engaging with and applying that knowledge. By incorporating this framework into the ELSA program, students are guided through a structured process of engagement that encourages them to think critically, make connections, and develop practical solutions to problems.

Scope of the School Program

In conceptualising the scope of the program, we were especially mindful that in many rural and remote areas across Australia children are learning in classrooms with peers from different year levels. Consequently, we envisaged developing ‘content bundles’, which could be accessed by children across various learning arrangements in the early years of schooling (e.g., single year level classes, combined year level classes, home schooling, distance learning). Such an approach fits with our conception of STEM practices and provides teachers with flexibility in the delivery of contextually relevant content. The content bundles address learning outcomes from the discipline areas (mathematics, science, digital technologies) in ways that promoted the STEM practices. Within each of the units (a four-week block of activities explained below), relevant STEM practices were outlined for the teachers. For example, in the Foundation Unit on Location and Transformation, student learning was underpinned by the following STEM Practices: *Ideas* - Exploring and Challenging; *Methods* – Processing Information and Appropriate Language and Vocabulary; *Values* – Teamwork.

Our main vehicle for ensuring contextual, community based, and culturally appropriate learning experiences was to ensure that theme-based content had some form of authenticity and connectivity to children of this age, irrespective of their geographical location. The Program Advisory Group, established by the Department of Education, was especially helpful in providing advice on this aspect of the program scope. The STEM-practice themes included:

- **Foundation:** Local store and local garden
- **Year 1:** Fire and ambulance emergency services
- **Year 2:** Community radio station

To make learning more accessible to children in rural and remote areas, themes were not aligned to traditional or commonly perceived STEM professions such as engineers or mathematicians or scientists. Rather the themes supported an understanding that STEM is

present in much of the daily lived experiences of students, such as a local garden or the emergency services in their town or community radio. These broader themes also provided teachers with opportunities to tailor the learning to suit the learning context they were in.

The Design Process

Using the ERA Framework as our guide, we developed four integrated STEM units for each year level – Foundation, Year One and Year Two. Within each of these 12 units there were four topics, giving a total of 48 topics across F-2. Some examples of the topics covered in the program include *sorting objects based on spatial and numerical attributes; aligning numbers to spatial patterns; and give and follow directions using landmarks* (Foundation); *measure and estimate distance with informal units; decoding representations using maps; and sorting and ordering visual data* (Year 1); and *graphing using and X- and Y- axis; Fraction Families – Wholes, halves, quarters and eights; and transformations and rotations* (Year 2).

Using the ERA framework as a scaffold, each topic included experience, represent and apply activities that would comprise approximately 1 to 1.5 hours of learning (this included time spent using the digital devices – approximately 12-15 minutes per topic). It was recommended to the teachers that they deliver one topic per week and that at the end of the four weeks they take an additional week to consolidate learning within the unit. The units were mapped to STEM Outcomes from the Australian Mathematics, Science and Digital Technologies Curriculums V9 and included learning outcomes and success criteria, digital and non-digital resources, and skills progressions via the digital games. The teachers also had access to a digital dashboard that provided learning analytics (updated daily) on each student's learning journey as they interacted with the digital games.

Digital Learning Experiences

The ELSA programs uses both digital and non-digital learning materials as part of the ERA learning experience outlined above. Here we focus on the design of the digital learning materials created in the program, as they played an important role in our vision for flexible learning experiences for children and teachers participating in the Program. The place of digital tools in children's learning has been well established in the research literature (Hirsh-Pasek et al., 2015; Moyer-Packenham et al., 2019).

In the development of the ELSA Program, the ERA Framework was also used to guide the design of the digital components of the ELSA Program. It is necessary to indicate here that, although digital design and coding experts (professional app developers) were employed to code the 48 learning games in the program, they worked under the close and very explicit pedagogical direction of the ELSA pedagogical team. This ensured that the digital games were explicitly linked to the weekly ERA learning activities that we created for students in F-2. This design process regularly resulted in very robust discussions between educators and app developers to ensure that the apps were philosophically aligned to a play-based approach to learning (Arnott et al., 2018) and pedagogically aligned to the EYLF and intentional teaching (Lowrie & Larkin, 2020).

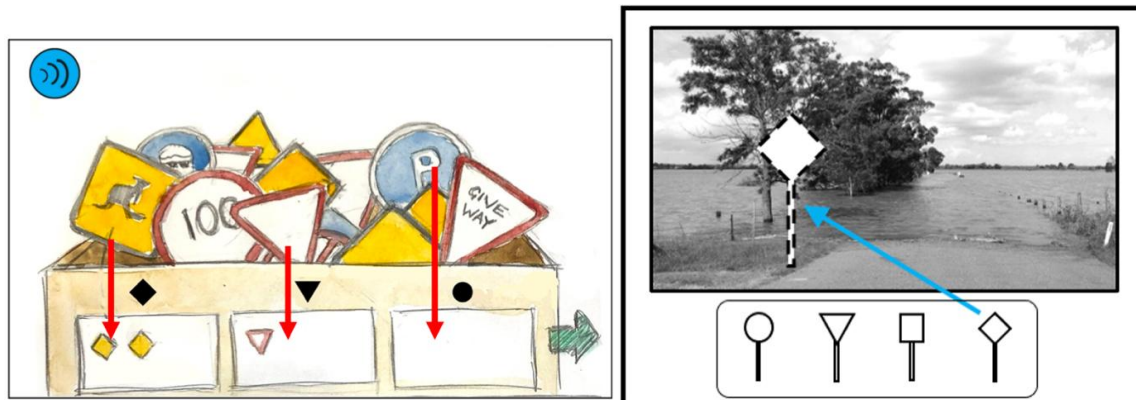
Thus, the digital games were designed to reinforce the learning from each ERA topic cycle and were also designed to be played by children without teacher support. Each game increases in complexity as the children experience success in the game with scaffolding provided in game. Below we provide an example of one digital task from each year level.

Foundation

The first set of tasks is for students *to order objects (groceries) based on a specified attribute (e.g., height, width, number)*. Children are in a local store and are looking at a set of shelves. There are groceries that need to be packed onto the shelves. Students place the grocery

Figure 3

Decoding Data (A) and Encoding Data (B)

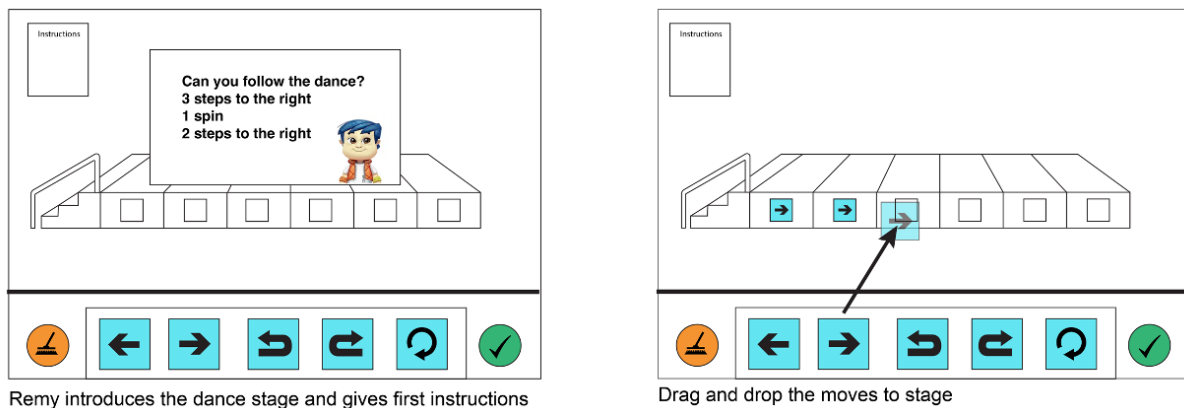


Year 2

At this grade level, students complete tasks related to the functionality of a community radio station. Students will transition from computational thinking activities in the radio station (e.g., debugging errors in how a sound mixer is set up by moving sliders and turning dials and designing simple musical devices using coding blueprints they have chosen) to a dance studio where they will teach an ELSA character to dance. The series of dances incorporate, with increasing complexity, translations, internal rotations, external rotations, and combinations of translations and rotations. The increasing complexity will be achieved by modifying the number of moves to be choreographed, by changes to the representations of the moves (from pictorial through to symbolic), and by including numerical instructions whereby the character can perform loops of instructions. The children move the character into the position they would be at the end of the instructions and then hit submit (see Figure 4). The levels of complexity include:

- Movements left and right and right to left using various numbers of steps (e.g., three right, two left and two right).
- Movements forwards and backwards and backwards and forwards using various numbers of steps (e.g., three forwards, two backwards and one forward).
- Combinations of a and b above.
- Combinations of a, b and c above, with numerical symbols indicating loops of actions to be taken.
- Combinations of all of the above, but with abstract representation (e.g., dance shoeprint style).

Built into the game design is an important element of spatial reasoning as the children will have the option, later in the game, to change the perspective from which they see the dance performed. In this change of perspective scenario the students will code the dance, as per the instructions above, but will then have the option to choose to watch the ELSA character dance the instructions either from the perspective of the crowd (the perspective they will see as they learn to code the dance throughout the early levels), or from the perspective of the ELSA character performing the dance. In the later scenario, this means that instructions to move left or turn clockwise from the view of the crowd are, from the ELSA character's perspective an instruction to move right or turn anticlockwise. Research has indicated that perspective taking predicts mathematics achievement, and is especially important for girls (Harris et al., 2025) and those from rural/regional communities (Harris et al., 2022).

Figure 4*Encoding and Decoding Translations to Develop a Dance Sequence*

Conclusion

The Early Years STEM Australia (ELSA F-2) program represents an innovative approach to STEM education in the early years of school. By focusing on STEM practices as a theoretical stance, and the Experience-Represent-Apply (ERA) as a pedagogical model, the program supports teachers in providing an integrated STEM experience in authentic contexts that can be modified to suit the needs of their students, whilst at the same time fostering essential STEM literacy and numeracy skills in young learners that are mapped to the Australian Curriculum. The program has also been designed based on a robust evidence base, highlighted by a national-wide randomised control trial (RCT), which assessed the efficacy of the preschool program under experimental conditions (Resnick & Lowrie, 2024). The RCT revealed that engagement with a play-based spatial program led to better overall spatial reasoning and transferred to better numeracy understanding for students compared with a business-as-usual control. The results underscore the importance of embedding spatial learning within strong pedagogy and authentic learning contexts, supported by the use of engaging digital games that can be played independently by the students with their learning automatically reported back to teachers, via a digital dashboard, at individual and cohort levels.

The continued development and implementation of the ELSA program will contribute to the broader goal of enhancing STEM education in Australia and beyond. In the next phase of research, we intend to report on a randomised control trial that was conducted during the 2024 school year. We hope to report of these findings at a forthcoming MERGA conference.

Acknowledgements

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Experiences of Pre-Multiplicative Thinking Junior Secondary School Students

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The Australian Curriculum suggests that students have transitioned from additive to multiplicative thinking upon entry into secondary school. However, research shows that many students are thinking additively, placing them at a disadvantage to their multiplicative thinking peers. In this paper, early findings from a broader research project describe the mathematical experiences of pre-multiplicative thinking students in a secondary school context. Critical realism was applied to initial interview data, and a proposed model emerged to represent the pre-multiplicative thinking research space.

Multiplicative thinking sits within the study of mathematics. It is an essential cognition that is defined as the use of multiplication and division strategies to solve a range of numerical-based problems (Young-Loveridge et al., 2012). First appearing in the *Year 2 Australian Curriculum* (ACARA, 2024), multiplicative thinking is progressively developed across the primary school mathematics curriculum. Students are expected to think multiplicatively upon entry into secondary school (ACARA, 2024) because multiplicative thinking lays the foundation for accessing mathematical concepts prescribed by the secondary school curriculum (Beckmann et al., 2008). Unfortunately, research indicates that between 30% and 55% of Australian Year 8 students are not thinking multiplicatively (Siemon et al., 2019). This statistic points to a problem in mathematics education, implying that a significant number of middle school students may be struggling to negotiate the age-appropriate curriculum. The need arises for secondary mathematics teachers to contemplate how they work with pre-multiplicative thinking students to ensure these students can access learning at the age-appropriate level.

In this paper, preliminary findings from a secondary school project are shared to explore the research question: What do pre-multiplicative thinking students in the junior secondary school experience when learning mathematics? Participants chosen for the project were positioned at the cognitive threshold between additive and multiplicative thinking. These participants were representative of middle school students having some understanding of multiplication and division but not yet able to apply robust multiplicative thinking to answer numerical questions. Critical realism (CR) advocates for researchers to listen to those at the centre of the research (Sayer, 2010). The findings arising from CR data analysis on participants' initial interview responses are presented. As many middle school students are operating pre-multiplicatively, it will be meaningful to consider participants' thoughts and responses in relation to learning mathematics that are shared in this paper.

Critical Realism and Its Application in Research

CR originated through the writings of Bhaskar (2008) who challenged social scientists to consider whether people-centred research should be conducted using the same methods that scientists apply to the natural world. Sayer (2010) elaborated that when studying human beings and their actions, it is impossible to isolate variables as scientists do when studying natural phenomena. CR offers a useful way to study entities within a social context (Sayer, 2010). While CR has not yet been used in mathematics education research, its application has been suggested (Nunez, 2015). In this paper, students, teachers and learning, within a junior secondary school mathematics setting are viewed through the theoretical lens of CR.

According to CR, an entity is of a human, physical, or conceptual nature (Venkatesh et al., 2013). Every entity owns a set of properties and, possesses a causal power or liability which may, or may not, be exercised (Sayer, 2010). For example, a teacher (entity), with a toolkit (property) holding many and varied pedagogical techniques, would be considered to possess a power. If the teacher effectively accesses techniques from their toolkit to assist students with learning, then the teacher is exercising their power. Because the teacher is a complex human entity, there is the possibility that the teacher does not exercise their power to effectively use the available pedagogical techniques in their toolkit. Alternatively, a teacher with a limited toolkit may possess a liability because the teacher may not be equipped to ably assist students with learning. Initial interview data identified examples of teachers who have exercised their power or liability in the classroom. Some of these examples are presented in this paper.

CR specifies the analysis of entities at different levels (Easton, 2010) so that researchers can identify the connectedness of an entity's data. Emergence is the name given to this analytical process because data at a higher level of an entity may emerge because of data at a lower level (Stylianou, 2019). During data analysis of initial interviews, the following levels were identified: Level 1 - student (L1); Level 2 - learning (L2); and Level 3 - policy (L3). This type of layered analysis elicits the following example: It emerges that a student may be reluctant to answer questions in front of the class (L2), because the student is feeling confused about the learning (L1). In CR, emphasis is also placed on relationships that exist between entities. Relationships help to explain events that occur in the CR research space (Easton, 2010).

Three levels of reality are defined in CR. The *empirical* level defines events that are visible through experience or observation (Easton, 2010), such as how a student feels when learning mathematics. The *actual* level represents events that are harder to measure as they may be concealed or observed but not clearly understood (Easton, 2010). An example of an actual event is a student's observation of their teacher's disposition to teaching mathematics. The observation is made from a second-hand perspective so is not fully understood by the student. The third level of reality, named the *real*, represents hidden factors, known as causal mechanisms (Easton, 2010). These mechanisms, like an entity's power or liability, may, or may not be exercised. The aim of CR is to discover mechanisms that, when exercised, may cause the research event of interest. While not easy to identify and examine, critical realists work to study actual and real events that occur as undercurrents in their research. Thus, a future research goal will be to identify mechanisms that support middle school students' transition into multiplicative thinking.

The Multiplicative Thinking Continuum

An eight-zone multiplicative thinking continuum, describing students' shift from additive to multiplicative thinking, was developed during the *Scaffolding Numeracy in the Middle Years* (SNMY) project with students from Years 4 to 8 (Siemon et al., 2006). The continuum, named the *Learning and Assessment Framework for Multiplicative Thinking* (LAF), included key ideas and teaching advice for each LAF zone, from Zone 1-*primitive modelling* through to Zone 8-*reflective knowing*. During the SNMY project, assessment tasks were created to determine a student's zone position on the LAF, and targeted teaching resources were developed to assist with students' growth along the multiplicative thinking continuum. SNMY researchers identified that many middle school students find it difficult to transition from additive thinking (Zones 1-4) to robust multiplicative thinking and beyond (Zones 5-8).

Recent research (Siemon et al., 2019) reveals that there has been no significant change in the multiplicative thinking landscape compared with the earlier study (Siemon et al., 2006). It remains that many middle schoolers are operating pre-multiplicatively (Siemon et al., 2019). The critical juncture between additive and multiplicative thinking lies at LAF Zones 3 and 4. Participants described in this paper are Year 7 students positioned at this critical multiplicative

thinking juncture. Zone 3 participants: use helpful numbers, such as 2 and 5; use doubling and halving strategies; and are beginning to work with large numbers by applying additive thinking. Zone 4 participants: use common two-digit multiplication; use additive thinking for larger numbers and decimals; can begin proportion problems; and are only able to provide minimal mathematical explanations (Siemon et al., 2006).

Research Design

The research design describes the CR data analysis that was conducted on the interviews of ten pre-multiplicative thinking participants from a broader research project. At the start of the project, participants from a Queensland Year 7 mixed-ability mathematics class were assessed against the LAF. From this original participant group, six participants at LAF Zone 3 and four participants at LAF Zone 4 were chosen to represent students at the critical juncture between additive and multiplicative thinking. These individuals formed the project's focus group, and their mathematical experiences expressed in the initial interviews, are presented in this paper.

Each participant undertook a 30-minute initial interview which involved answering 16 questions. Verbal and written responses were video recorded and transcribed. The initial interview protocol was informed by a literature review as recommended in CR research (Hoddy, 2019). Additionally, a grounded theory interview protocol (Kennedy, 2011) was referenced during question development. Kennedy's (2011) protocol was designed to capture interactions between disengaged middle school students and their teachers. Initial interview questions aimed to capture participant voice because CR places importance on social scientists listening to the understandings of their participants (Sayer, 2010). Therefore, questions encouraged participants to express their feelings and perspectives in relation to learning mathematics. The first interview question asked participants to draw a picture or use words to describe how it feels when learning mathematics. Two further questions illustrate how participant voice was captured: What challenges do you face in learning maths? If you could give your maths teacher one piece of advice to assist students with their learning, what would it be?

Due to a lack of methodological guidance in the literature on how to apply CR, the suggestion to use grounded theory methods was taken (Hoddy, 2019). Audio data from initial interview video recordings were transcribed. Open coding from grounded theory was applied which broke apart the qualitative data and allowed for the creation of eleven initial codes to represent its discrete parts (Ho et al., 2025). After 10 open coding iterations, 34 codes were identified, each holding a bank of descriptive data from the 10 participants. Next, axial coding from grounded theory was iteratively applied to rebuild the data in a meaningful way by identifying how the codes were connected through general themes (Ho et al., 2025). Finally, axial coding, applied through a CR lens, assisted with synthesising the data into general themes to produce a CR model representative of the research space. In a future research stage, selective coding will be applied, through the lens of CR, to identify causal mechanisms which will point to reasons for students' multiplicative thinking delay.

Participants' Accounts of Learning Mathematics

Open coding and primary axial coding exposed four general themes in the data: participants' affective responses; participants' experiences; participants' insights about teachers; and participants' advice to teachers.

Participants' Affective Responses

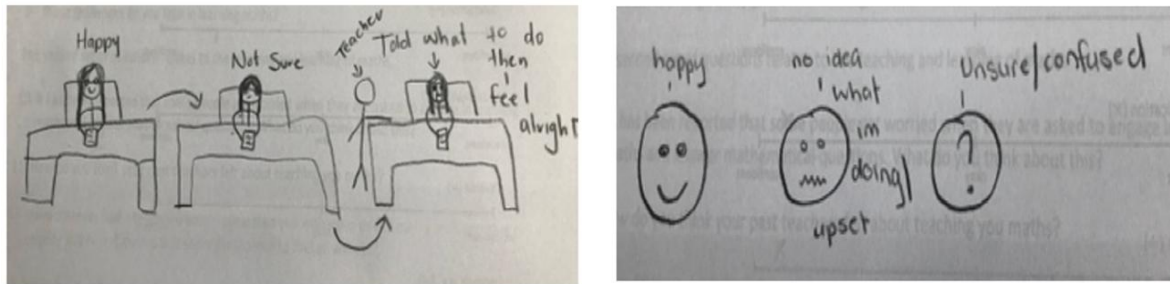
Participants reported two types of affective responses in relation to learning mathematics: *emotional responses* and *responses to the cognitive challenge*.

Nine participants reported *emotional responses* when learning mathematics. Most reported feeling discomfort (8), including responses of "stressed" and "anxious" when learning

mathematics. Other discomfort responses included feeling overwhelmed and feeling worried about passing mathematics. Three of the eight participants who had reported feeling discomfort, also reported experiencing a series of emotions. In response to the first question of the initial interview, Karen and Indy (pseudonyms) drew pictures to show the series of emotions they have experienced during a mathematics learning episode (see Figure 1). In addition, many of the participants believed that some people get worried when learning mathematics. Three other *emotional responses* were recorded. Participants said they felt “happy” when understanding the learning (2), and “annoyed” (1) and “bored” (1) when learning mathematics.

Figure 1

The Series of Feelings That Karen and Indy Experienced when Learning Mathematics



(a) Karen

(b) Indy

Responses to the cognitive challenge of learning were also articulated. Seven participants used the word “confused” to express how they have felt when learning mathematics. Five of these participants described times when they had not understood the learning being delivered.

Participants’ Experiences

Participant voice revealed the following codes: *learning mathematics is difficult*; *time affects uptake of learning*; *reluctance to answer questions in front of the class*; *reluctance to ask for help when not understanding*; *factors that support learning*; and *working with others helps with learning*.

Learning mathematics is difficult was further coded into *difficulty with using large numbers and generalising*; *significant cognitive load*; *mathematics is harder than other subjects*; and *trying to understand the question and getting stuck*. Seven participants articulated how *learning mathematics is difficult*, with five reporting *difficulty with using large numbers and generalising* in the SNMY assessment. Uma retells her difficulty with moving from a count-all approach to one that requires generalising a number pattern.

I was getting used to drawing the tables and the people sitting at the tables. When it came to the larger numbers, I tried to add up the tables and times the tables, but I got them incorrect cos I didn’t do something right.

Five participants reported a *significant cognitive load*. Casper said, “when I’m thinking too much, I put too much effort in, and I get a headache”. Narelle reported her challenge.

Sometimes you [the teacher] go through all three [problems] at once and it gets too much. It’s like my brain is going a hundred k’s an hour. You just can’t organise all three things. It comes in one big clump.

Abby reported feeling overwhelmed when trying to copy the work and listen to her teacher’s explanation at the same time. Four participants believed that *mathematics is harder than other subjects*, while three participants reported *trying to understand the problem and getting stuck*.

Seven participants articulated that *time affects uptake of learning* with reports of not having enough time to grasp the learning, and responses of “too fast”, “too much”, “can’t get the work done in time” and “can’t catch up”. Bonny described not being ready for the harder questions

as she had not yet grasped the concept. Uma explained she needs time to read, reread and interpret worded problems. Indy described how it can take several visits, over time, to grasp a concept. When Indy ultimately grasped the concept, she reported it was “whoa, mind-blowing”. Abby shared a positive learning experience: “I like Maths Pathway because you can spend more time thinking on the questions and have more time to answer them.”

Three participants reported a *reluctance to answer questions* in front of the whole class. Bonny described how her nervousness to answer questions overshadows her learning: “Sometimes I’m scared to say the answer in case I get it wrong, then everyone laughs.”

Three participants articulated a *reluctance to ask for help* when not understanding the learning. Narelle reported worry that she will anger her teacher if she asks for help.

“Oh, we just learned that. You didn’t listen clearly.” (quoting a teacher) When the test comes, I don’t know how to do it. I didn’t understand that question and I didn’t ask for help. I would pretend it’s all okay. When it comes to the exam, I don’t know anything.

Participants identified *factors that support learning* (6). Karen discussed how doing and visualising assisted with conceptualisation of the Cartesian plane.

We were doing Cartesian plane, and we did battleship. We had to play the game and then you had to try and figure out the coordinates to sink the other person’s ship. We had to do that on a piece of paper so we could actually see the Cartesian plane. Playing battleship probably made it a lot more fun to learn, and it probably made it easier, cos you actually visualise what it was.

Participants reported that *working with others helps with learning*, specifically that talking with others helps (5). Working in small groups was reported as valuable (4). Karen detailed working with her peers: “I like one-on-one. See how they do theirs. Ties into what I am doing.”

Participants’ Insights About Teachers

Participant voice revealed that students notice *teachers’ disposition to teaching mathematics*, and students are appreciative of *teachers who articulate their desire to help, followed by enacted help*.

Four participants recalled *teachers’ dispositions to teaching mathematics*, with three describing teachers who liked teaching mathematics. Nikki shared her observations: “My Year 6 teacher was, I don’t know whether you’d call it a mathematician, but she studied maths. So, we did lots of maths.” Two participants reported variation in teachers’ dispositions. Bonny articulated the contrast in disposition between teachers: “We’d pretty much only do maths [Year 6 teacher]. But then other teachers, they tried to avoid maths as much as possible. So that’s probably where they’re stronger at, where they like to teach.”

Two participants, Indy and Casper, reported they were appreciative of *teachers who articulate their desire to help, followed by enacted help*. Indy spoke about two teachers who had revealed this quality, a primary teacher and her current Year 7 mathematics teacher.

She [primary teacher] taught me a lot about math and helped me understand. I think she felt like it was a good thing that she could help me learn better. He’s [Year 7 maths teacher] always trying to help us understand. He always shows us diagrams and stuff to help us understand it [the learning].

Participants’ Advice to Teachers

Participants provided advice to teachers around the following themes: *provide clarity around learning; use models to represent a concept; differentiate learning; check in with students; use different strategies to teach a concept; make time for learning; get students to reflect on mistakes; and do not use worksheets to teach*.

Five participants recommended that teachers *provide clarity* around learning. All five used the term “explain” during the interviews. Uri said, “explain it more, in more detail”. Keith suggested, “do it with me or do it on the board; walk us through it”.

Five participants requested that teachers *use models*. Two participants noted the use of place value blocks. Two participants suggested how the array representation was useful. Bonny described how the use of concrete props to represent variables and constants when developing an expression was helpful. She also described how visual cues were helpful.

He [teacher] drew pictures on the board like the crocodile and a boat and it would show you where they were swimming. It was just a number line but with more pictures, so you'd understand it. That helped. I think for some of the class, like me, [who are] much more visual, like watching that [picture] so then they can focus on it better.

Participants requested that teachers *differentiate learning* (3). Karen gave detailed advice.

Work with them [students] in groups. It helps. You [teacher] get a whole group of students who are trying to figure out the same thing and then do some more different ways of trying to do it. And it helps in ways cos there's less people. They [teacher] have more focus on you [the student] particularly, and they kind of see what you need help with, so then they can actually help you with whatever you're learning.

Three participants recommended teachers *check in with students* rather than assuming students will let teachers know when they need help. Indy focussed on non-verbal cues: "Look back at them [students] to see if they're doing it. Watch their facial expressions, and if they haven't written anything down [then they don't understand]."

Three participants suggested that the teacher *use different strategies* to teach a concept. Karen described a teacher whom she believed could not help students who were struggling because he was unable to explain the concept in a way that the students could understand.

Two participants recommended that teachers *make time* for learning. Abby noted, "give students more time to copy the work; if you don't get all the information down, you'll have to copy off someone else's book or you just don't get the information at all".

Participants suggested that teachers *get students to reflect on mistakes* they have made (2). Narelle remembered how reflecting on her mistakes had been useful.

I got a question wrong, and it was like four times eight and I remember I got 16. Then he [her teacher] knew I was trying to do double double, but I had only done double. He told me to reflect on that. I think it made me learn times four better because I know it's double double now, not just double.

Two participants advised: *do not use worksheets to teach*. Bonny reported that worksheets put students in a "bad mindset". Keith described his experience of being given a worksheet and told by the teacher to "learn this". Keith advised, "that doesn't help; go through it on the board".

Analysis and Discussion

After open coding had initially been applied to the interview data, identifying 34 codes, it was noted that instances of repeating participant descriptions occurred across some codes. For example, *time assigned to learning* appeared under the codes of *participant challenges* and *advice for teachers*. Examples of these repeating participant descriptions are shared in the qualitative data presented above and emphasise the importance of the associated messaging in participant descriptions.

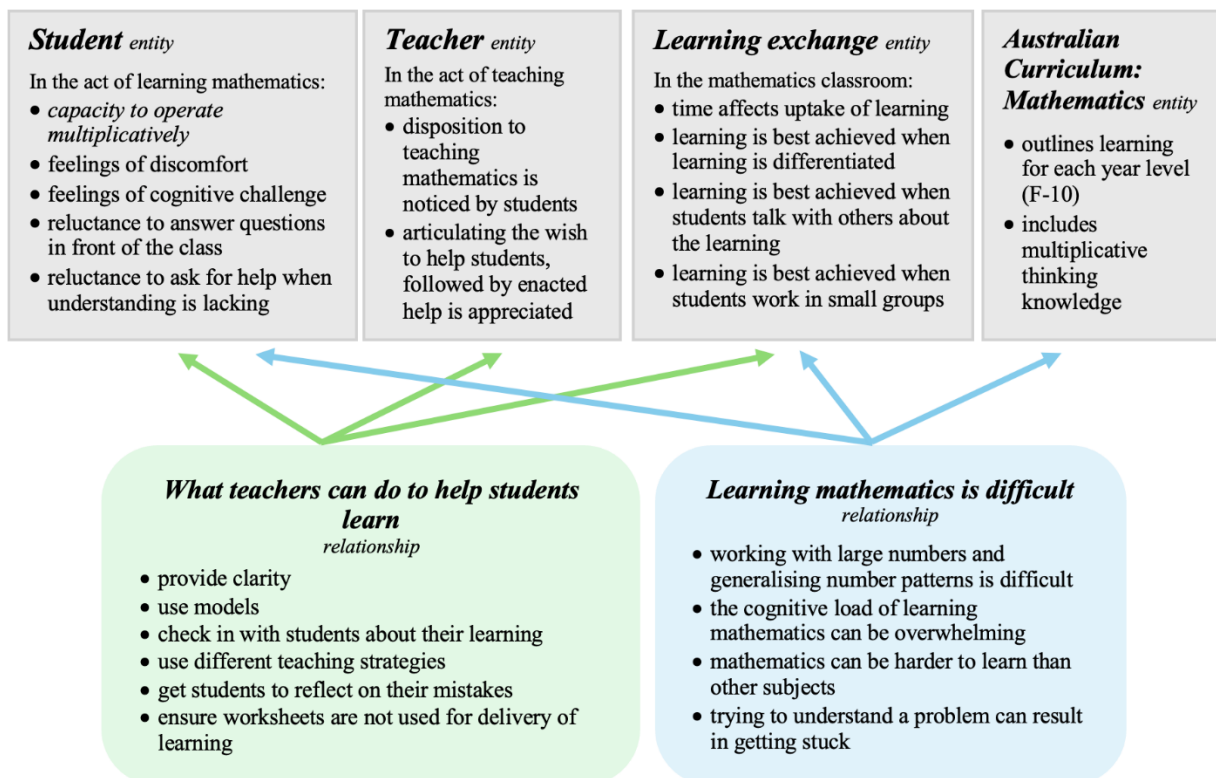
After primary axial coding had been conducted to develop general themes (Hoddy, 2019), secondary axial coding was undertaken to place the qualitative interview data into a proposed CR model (Easton, 2010), representing the experiences of pre-multiplicative thinking students in a junior secondary school mathematics setting. Four emerging entities and their associated properties, as prescribed by CR, were identified to represent the research space: *Student*, *Teacher*, *Learning exchange*, and *Australian Curriculum: Mathematics*. Easton (2010) used an exchange entity to represent the ongoing communication between buyer and seller in his CR study. Hence, the *Learning exchange* entity was used to represent ongoing classroom interactions between the student and teacher. The *Australian Curriculum: Mathematics*, while

not explicitly discussed in the initial interviews, was deemed a necessary entity as it conceptually underpins actions taking place between other entities in the research space. Secondary axial coding assisted with the appropriate placement of repeating participant descriptions and emerging connections in the data. Two themes that surfaced could be viewed as further entities in the proposed model but were instead classified as key relationships between existing entities: *What teachers can do to help students learn*, and *Learning mathematics is difficult*. In CR, a relationship that has been found to have come into existence and interacts in the research space may operate as an entity with its own set of properties (Easton, 2010).

Figure 2 depicts the proposed CR model that represents the pre-multiplicative thinking secondary school research space from a student's perspective. The four entities of *Student*, *Teacher*, *Learning Exchange* and *Australian Curriculum: Mathematics* are shaded in grey. The key relationships are coloured green and blue. Matching arrows show each relationship's connectivity with other entities in the research space. Participants' experiences that were axially coded into general themes using grounded theory were appropriately positioned across the proposed CR model. The secondary axial coding process using properties of entities and relationships was presented in bulleted lists. The first property of the Student entity has been italicised because it is the focus of the broader research project, and depicts the Student entity's power or liability, according to CR. At the time of the initial interviews, all participants possessed the property, *capacity to operate multiplicatively*, however the capacity had not yet been realised, therefore presenting as a liability. If, over the course of the project, a participant shifts into multiplicative thinking, then their liability will transform into their power.

Figure 2

Proposed CR Model of Pre-Multiplicative Thinking Junior Secondary Students' Learning Experiences



Conclusion

The findings presented in this paper describe the experiences of pre-multiplicative thinking junior secondary school students when learning mathematics, viewed for the first time from a

CR perspective. Detailed qualitative interview data provided the following key findings of students' mathematical learning experiences: feelings of discomfort; difficulties with understanding; challenges with cognitive load; and struggles with time constraints. In addition, students identified that talking with others and working in small groups supports their learning, and they identified ways in which their teachers can help them learn.

A proposed CR model depicting pre-multiplicative thinking junior secondary school students' learning experiences was developed. Four entities (*Student*, *Teacher*, *Learning Exchange*, and *Australian Curriculum: Mathematics*) and two relationships (*What teachers can do to help students learn*, and *Learning mathematics is difficult*) emerged. In addition to capturing key findings from the initial interview data, the proposed CR model illustrates elements and connections in the secondary mathematics classroom, from the perspective of the pre-multiplicative thinking student. Secondary mathematics educators are encouraged to consider the findings presented in this paper because they provide a unique insight into the experiences of pre-multiplicative thinking students when learning mathematics.

Acknowledgements

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Mind the Gap: Investigating Misaligned Mathematics Study Intentions and Perceived Capabilities

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Strengthening lower secondary students' attitudes toward mathematics is key to addressing declining enrolments in higher-level courses. This mixed-methods study examined the alignment between students' perceived mathematics capability and study intentions, as well as demographic and attitudinal differences. Results suggest that students with aligned intentions had more adaptive attitudes than those with an intentions-capability gap. Many who intended to study below their perceived capability cited low utility value, suggesting that enhancing mathematics' perceived utility could increase higher-level enrolments.

Over the past two decades, the proportion of students participating in advanced senior mathematics courses in Australia has declined, falling from 34.9% in 2008 to 26.7% in 2022 (AMSI, 2024). In most Australian states, calculus-based courses are available at intermediate and higher levels with a 50% gender balance in intermediate courses, but only 37.5% representation for females in higher level courses (Marchant & Kennedy, 2024). There is also evidence of declining enrolments in mathematics-based university level courses (AMSI, 2024) despite predictions of growth in industry demands for STEM-skilled employees (Office of the Chief Scientist, 2014). To address declining enrolments in senior mathematics courses it is important to examine student attitudes and enrolment plans in the junior years of high school, well before students make subject selections and potentially opt out of mathematics study.

The intrinsic value, utility value, ability beliefs, and cost constructs from expectancy-value theory (EVT) are widely used in motivational research to understand student attitudes and intentions towards STEM subjects in secondary school and beyond (Wigfield & Eccles, 2000).

Intrinsic value measures students' enjoyment and interest in the subject, with enjoyment a predictor of students' intentions to continue studying a subject once it is no longer compulsory (Mackenzie et al., 2024). *Utility value* refers to students' perceptions of the usefulness, importance, or applicability of a subject in other contexts such as a future career (Berger et al., 2020). Utility value is an important predictor of gendered enrolment patterns for university courses, for instance, student's expectations of a mathematically oriented career in adolescence explains approximately 15% of the gender gap in mathematically intensive majors (Law, 2018). *Ability beliefs* refer to students' views of competence in a subject and may be operationalised several ways, including confidence, self-efficacy, and self-concept (Berger et al., 2020). In this paper, we use perceived talent as an indicator of ability beliefs. Higher ability beliefs about STEM subjects have been associated with intentions to pursue mathematically intensive senior subjects, university degrees, and careers (Berger et al., 2020; Law, 2018).

Costs are students' perceptions of the sacrifices needed to succeed or engage in a task and can include effort, opportunity, and psychological costs (Lee et al., 2022). In this paper, we use mathematics anxiety as a well-known measure of psychological costs. Mathematics anxiety is a "feeling of tension and anxiety that interferes with [...] the solving of mathematical problems in [...] ordinary life and academic situations" (Richardson & Suinn, 1972, as cited in Dowker et al., 2016). Mathematics anxiety is associated with poorer performance and opting out of

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 285–292). Canberra: MERGA.

mathematics at higher levels, with girls also tending to express more mathematics anxiety than boys (Dowker et al., 2016).

Students' perceptions of their ability to cope with the academic challenge of mathematics may also influence their intentions to study higher level courses. *Academic buoyancy* refers to students' everyday resilience in dealing with academic challenges that occur in the ordinary course of schooling (Martin & Marsh, 2008). Academic buoyancy is moderately related to achievement in mathematics (Colmar et al., 2019) but importantly is also related to greater persistence and lower anxiety (Martin et al., 2010). Thus, academic buoyancy may serve as an important protective attitude towards pursuing mathematics at higher levels, especially given that mathematics is often perceived as being challenging and difficult (Li & Schoenfeld, 2019).

During the junior years of high school, adolescents build a view of what level of mathematics they are capable of studying in the senior years. This view may be informed by internal (attitudes, aspirations, prior achievement) and external (advice from others, school structures) factors (Hine et al., 2024). While previous research has examined how attitudes are related to students' intentions to study mathematics (e.g. Sheldrake et al., 2014), we were unable to locate any studies that have investigated the extent to which the level of mathematics that students perceive themselves as *capable* of studying is aligned with the level of mathematics they *intend* to study. Theoretically there may be two groups of students: those who intend to study the same level of mathematics that they feel capable of (aligned aspirers) and those who intend to study a level different to that they feel capable of. This second group demonstrates a mathematics study intention and perceived capability gap which indicate under-aspiration (studying below the level of perceived capability) or over-aspiration (studying above the level of perceived capability). Under-aspiring students may be a group with whom there is an opportunity to intervene to support increased enrolments in senior mathematics. On the other hand, over-aspiring students may require additional supports to increase their chances of success in courses that are more difficult than they believe they are capable of studying.

As such, in this study we aimed to investigate differences in the demographic profiles and attitudes of secondary students who do and do not display a mathematics study intention and perceived capability gap. Further, given that under-aspiring students may be a group in which interventions to increase mathematics enrolments can be targeted, we examined their reasons for selecting a lower level of mathematics. As such, the research questions for this study are:

- (i) How do demographic profiles and attitudes differ between groups that intend to study a mathematics level (i) lower than their perceived capability, (ii) equal to their perceived capability, and (iii) higher than their perceived capability?
- (ii) Why do adolescents choose a lower level of mathematics study lower than their perceived capability?

Method

Participants

The participants in this study were 1390 students (544 boys, 796 girls, 50 other or preferred not to disclose) drawn from 21 public secondary schools across NSW, Australia. There were 452 students in Year 7, 329 in Year 8, 296 in Year 9, and 313 in Year 10, and 30% reported that they received mathematics tutoring outside school. The schools that participated in this study ranged in the Index of Community Socio-educational Advantage (ICSEA) from 863 to 1147 ($M = 1007.29$, $SD = 76.89$), suggesting that the sample was diverse in terms of relative educational and socioeconomic status. 856 (61.6%) students attended comprehensive schools, while 185 (13.3%) attended selective and 347 (25.0%) attended partially selective schools. Most of the sample ($n = 1114$, 80.1%) attended schools in metropolitan areas and 273 (19.6%) attended schools in regional and remote areas.

Measures

Mathematics Study Intentions and Perceived Capability

Participants were asked which level of senior (Year 11-12) mathematics they intended to study using four options: no mathematics/numeracy, standard mathematics (lowest), advanced mathematics (moderate), or extension mathematics (highest). They were also asked to identify the highest level of mathematics they felt capable of studying using these options.

Perceived Talent and Intrinsic Value in Mathematics

Perceived talent and intrinsic value in mathematics were measured using four and three items respectively from Watt et al. (2019) (sample perceived talent item: *Compared with other students in your class, how talented do you consider yourself to be at maths?*; sample intrinsic value item: *How interesting do you find maths?*). Participants were asked to respond on a 7-point scale from 1 (not at all) to 7 (extremely). Both scales demonstrated excellent reliability: perceived talent $\alpha = .91$, $\omega = .91$; intrinsic value $\alpha = .94$; $\omega = .94$.

Utility Value

Participants' level of utility value in mathematics was measured using five items adapted from PISA 2006 (OECD, 2007) (sample item: *I study maths because I know it is useful for me*). Participants were asked to respond on a 7-point scale from 1 (strongly disagree) to 7 (strongly agree). This scale demonstrated excellent reliability ($\alpha = .94$; $\omega = .94$).

Mathematics Anxiety

Mathematics anxiety was measured using eight items from Bai (2010) (sample item: *Maths makes me feel nervous*). Participants were asked to respond on a 6-point scale from 1 (strongly disagree) to 6 (strongly agree). This scale demonstrated excellent reliability ($\alpha = .94$; $\omega = .94$).

Academic Buoyancy in Mathematics

Academic buoyancy in mathematics was measured using four items adapted from Martin and Marsh (2008) (sample item: *I think I'm good at dealing with maths schoolwork pressures*). Participants were asked to respond on a 7-point scale from 1 (strongly disagree) to 7 (strongly agree). This scale demonstrated good reliability ($\alpha = .84$; $\omega = .84$).

Reasons for Intending to Study Mathematics Level Below Perceived Capability

Participants who reported that they intended to study a level of mathematics below the level they felt they were capable of were asked to provide reasons for this decision (*Why aren't you studying the highest level that you feel capable of?*) in an open comment field.

Procedure

Purposive sampling was used to recruit participants from diverse school contexts under the direction of the funding body. First, the principal of each identified school was contacted to seek consent for their school's participation. After this consent had been received, parents/caregivers of all students in Years 7-10 at each school were provided with a participant information sheet and asked to provide informed consent for their child to take part in an online survey. Students with parental consent to participate completed the survey via the Qualtrics platform during a timetabled mathematics lesson.

Analysis Strategy

Statistical analyses were conducted in SPSS 29. First, participants were placed into one of three groups using their responses to the mathematics study intentions and perceived capability items: *under-aspirers* indicated their intention to study a level of mathematics lower than their

perceived capability, *aligned aspirers* indicated that they intended to study a level of mathematics equal to their perceived capability, and *over-aspirers* indicated their intention to study a level of mathematics higher than their perceived capability. Following group assignment, five chi-square tests were conducted to compare the demographic profiles of each group with a Bonferroni correction of $p < 0.01$ applied to control for Type 1 error. Standardised adjusted residuals were used as post hoc tests to identify significant differences between the groups with a Bonferroni correction applied. All attitudinal variables demonstrated acceptable levels of skewness and kurtosis, indicating approximately normal distributions. This enabled attitudinal differences between the groups to be examined using five ANOVA tests with a Bonferroni correction of $p < 0.01$. Eta-square was used to determine main effect sizes with those under .01 deemed to be small, between .01 and .06 to be moderate, and over .06 to be large (Cohen, 1988). Welch's ANOVA was used in place of a standard ANOVA when Levene's Test for homogeneity of variance was violated. Post hoc tests were then conducted to identify where groups differed (Tukey if variances were equal, Games-Howell if variances were unequal).

Qualitative responses provided by under-aspirers were coded independently by two members of the research team, initially examining the responses for evidence related to the constructs measured in the survey (perceived talent, intrinsic value, utility value, mathematics

Table 1*Chi-Square Tests Comparing Group Demographics*

Demographics	X ²			
	Frequencies (percentage of group)			
	Under (<i>n</i> = 189)	Aligned (<i>n</i> = 943)	Over (<i>n</i> = 258)	Total (<i>n</i> = 1390)
Gender	X ² (4, 1390) = 18.09, $p = .001$			
Male	84 (44.4%)	382 (40.5%)	78 (30.2%) [↓]	544 (39.1%)
Female	99 (52.4%)	521 (55.2%)	176 (68.2%) [↑]	796 (57.3%)
Other	6 (3.2%)	40 (4.2%)	4 (1.6%)	50 (3.6%)
School type	X ² (4, 1388) = 44.00, $p < .001$			
Comprehensive	147 (77.8%) [↑]	572 (60.8%)	137 (53.1%)	856 (61.7%)
Partially selective	36 (19.0%)	247 (26.2%)	64 (24.8%)	347 (25.0%)
Selective	6 (3.2%) [↓]	122 (13.0%)	57 (22.1%) [↑]	185 (13.3%)
School location	X ² (2, 1388) = 13.41, $p = .001$			
Metropolitan	134 (70.9%) [↓]	764 (81.2%)	217 (84.1%)	1115 (80.3%)
Regional	55 (29.1%) [↑]	177 (18.8%)	41 (15.9%)	273 (19.7%)
Year level	X ² (6, 1390) = 37.48, $p < .001$			
Grade 7	47 (24.9%)	293 (31.1%)	112 (43.4%) [↑]	452 (32.5%)
Grade 8	49 (25.9%)	216 (22.9%)	64 (24.8%)	329 (23.7%)
Grade 9	56 (29.6%) [↑]	191 (20.3%)	49 (19.0%)	296 (21.3%)
Grade 10	37 (19.6%)	243 (25.8%) [↑]	33 (12.8%) [↓]	313 (22.5%)
Tutored outside school	X ² (2, 1390) = 29.84, $p < .001$			
Tutored	25 (13.2%) [↓]	303 (32.1%)	89 (34.5%)	417 (30.0%)
Not tutored	164 (86.8%) [↑]	640 (67.9%)	169 (65.5%)	973 (70.0%)

Note. X² = chi-square; [↓] = significantly fewer than expected; [↑] = significantly more than expected.

anxiety, academic buoyancy) while allowing additional themes to emerge. Some responses were coded to more than one theme if applicable. The initial independent coding across eight categories generated a pooled kappa of 0.87. Differences in the application of the codes were discussed, and codes were added or deleted until agreement was reached on all codes.

Results

Most students (67.8%) were aligned aspirers, indicating that they intended to study the level of mathematics that they felt capable of. The remaining students reported a mathematics intention-capability gap: 18.6% were over-aspirers and 13.6% were under-aspirers.

Demographic Profile Differences

Results of five Pearson chi-square tests investigating demographic profile differences between the groups are shown in Table 1. All five tests were significant at $p < 0.01$, indicating that there were differences in the demographic profiles of the three groups. In the under-aspiring group, students from comprehensive schools, regional schools, in Year 9, and those not being tutored outside school were overrepresented. Conversely, students from selective schools, metropolitan schools, and those being tutored outside school were underrepresented. In the aligned group students from Year 10 were overrepresented. A significant gender difference was observed in the over-aspiring group, with girls overrepresented and boys underrepresented. Students from selective schools and in Year 7 were overrepresented in the over-aspiring group, while students in Year 10 were underrepresented.

Attitudinal Differences

Results of five ANOVAs examining attitudinal differences between the groups are shown in Table 2. All main effects were significant with effect sizes (η^2) ranging from small (mathematics anxiety) to moderate (perceived talent, intrinsic value, academic buoyancy) to large (utility value). Post hoc tests revealed that over-aspiring students had significantly lower levels of perceived talent and higher levels of mathematics anxiety than aligned students. Under-aspiring students exhibited significantly lower levels of intrinsic and utility value than aligned and over-aspiring students. Finally, aligned students reported significantly higher levels of academic buoyancy than under-aspiring and over-aspiring students.

Table 2

ANOVA Results and Group Means of Attitudinal Variables

Attitude	ANOVA	<i>M (SD)</i>		
		Under	Aligned	Over
Perceived talent	$F(2,1337) = 9.02, p < .001, \eta^2 = .01$	4.36 (1.44)	4.60 (1.44) ^a	4.18 (1.53) ^a
Intrinsic value	$F(2,1337) = 24.70, p < .001, \eta^2 = .04$	2.99 (1.52) ^{ab}	3.97 (1.76) ^a	3.86 (1.71) ^b
Utility value	$F(2,395.38) = 41.79, p < .001, \eta^2 = .06^{\wedge}$	4.44 (1.40) ^{ab}	5.36 (1.31) ^a	5.55 (1.16) ^b
Mathematics anxiety	$F(2,1337) = 5.60, p = .004, \eta^2 = .01$	3.37 (1.26)	3.15 (1.23) ^a	3.41 (1.20) ^a
Academic buoyancy	$F(2,1337) = 13.81, p < .001, \eta^2 = .01$	4.43 (1.52) ^a	4.74 (1.38) ^{ab}	4.45 (1.31) ^b

Note. η^2 = eta-squared; $\wedge$ = Welch's ANOVA due to unequal variances; ^a = significant difference between groups; ^b = significant difference between groups.

Reasons for Selecting Below Perceived Capability

Of the 189 students in the under-aspiring group, 41 (21.7%) planned to study no mathematics or numeracy, 81 (42.9%) planned to study standard mathematics, and 67 (35.4%) planned to study advanced mathematics. These students were invited to provide reasons why they were intending to study a level of mathematics lower than their perceived capability. After coding, eight themes emerged with 252 codes allocated across 189 open-ended student

responses. The frequency of responses coded within each category is detailed in Table 3 along with example responses for each category.

Table 3

Frequency of Reasons for Choosing Level of Mathematics Below Their Capability and Example Responses

Reason	Frequency (female <i>n</i>)	Example responses
Utility value	56 (29)	Because I don't really need advanced maths in the job I want to do (female, Y10)
Perceived talent	41 (28)	I know that I am not as talented in mathematics compared to other subjects (male, Y7)
Intrinsic value	39 (19)	Because I hate maths (male, Y9)
Cost (time)	28 (16)	I feel like I could do it, but I also think it would take a lot of energy and time that I'm not willing to give up (male, Y10)
Difficulty of subject	28 (20)	I know they will complicate it so what's the point of going through that difficulty? (female, Y9)
Mathematics anxiety	21 (12)	I don't want to put too much effort into something I'm not all that passionate about, so I don't burn myself out or get too stressed over nothing (female, Y9)
Academic buoyancy	11 (7)	Because it will be too hard and I won't be able to handle it unless I get extra help (female, Y8)
Other	28 (14)	e.g. I don't know; I'm not sure why; I don't want to.

Students most frequently (29.6% of responses) cited reasons related to the perceived lack of utility of higher mathematics levels for their lives and/or future careers. Further, students cited their perceptions of their ability to do mathematics at higher levels as influential on their aspired level of mathematics (21.7%) followed by the extent to which they liked the subject (20.6%). Many students felt that the substantial time that it would take to succeed at higher levels was a deterrent (14.8%), as was the perceived difficulty of mathematics as a subject (14.8%). Just over 10% of students expressed being influenced by anxiety, stress and pressure related to mathematics and almost 6% were concerned about coping with higher levels.

Discussion

Education systems across Australia have agreed on the aim that “young Australians of all backgrounds are supported to achieve their full educational potential” (Education Council, 2019, p.5). For this aim to be achieved students must be provided with opportunities, support and encouragement to fulfil their potential across the curriculum. This is particularly important in mathematics, which acts as a gatekeeper discipline for many post-school educational and career pathways (Li & Schoenfeld, 2019). When students opt out of more advanced levels of mathematics, especially when they feel capable of participating, they may inadvertently curtail future options of interest. It is important, therefore, to understand which students are opting to take lower levels than they are capable of, and their reasons for doing so.

Our study found that the aligned group displayed the most adaptive attitudes towards mathematics. For example, these students had higher levels of academic buoyancy than either of the other groups, indicating they felt capable of handling minor setbacks experienced during their study of mathematics. While it is positive that most students were in this group, it is of concern that, the average intrinsic value score indicated that most students do not enjoy or find mathematics interesting ($M = 3.97$ on a 7-point scale). Given the critical nature of intrinsic value for continued study of mathematics (Watt et al., 2012), interventions to increase students' intrinsic value may be useful for all students regardless of their intention-capability alignment.

Our demographic profile analysis of the groups in this study found that students in comprehensive schools, in regional areas and who were not receiving tutoring, were over-represented in the under-aspiring group. Conversely, students in academically selective schools were over-represented in the over-aspiring group. On one level, these findings may be explained by the levels of support available to these groups of students, either financially in the form of extra tutoring or in terms of encouragement and high expectations that may be more prevalent in selective and/or metropolitan school communities.

In terms of attitudes towards mathematics, students in the under-aspiring group indicated that they liked mathematics less and found it less relevant to their lives and futures than other students. These quantitative results were also reflected in the open-ended comments made by these students, where utility value emerged as the most frequent reason given for choosing lower levels. It could be that students are making accurate assessments of the expected mathematical demands in their future careers and lives. It may also be the case that they are misinformed in this regard, potentially impacting negatively on their range of career opportunities. In both cases, the findings imply a role for schools to ensure that students are accurately informed about the levels of mathematics required for various careers, and an imperative for teachers to foster more positive attitudes to the subject for all students.

For those students in under-aspiring group, further insights were gained from their open-ended responses asking them to explain their reasoning. Mathematics was perceived by many in this group as an overly time-intensive subject at the higher levels, potentially taking time away from other subjects that they were more interested in. Relatedly, it was also viewed as a subject that was difficult in comparison to other subjects and students feared the stress and pressure than might result from attempting higher levels.

While the under-aspiring group raises important issues for schools to address, other issues arose for the over-aspiring group. These students were more anxious and perceived themselves to be less talented in mathematics than those in the aligned group. This may be explained in part by the fact that girls were overrepresented in this group: in previous research girls have been found to experience higher levels of mathematics anxiety (Dowker et al., 2016) and lower levels of perceived talent than boys, even when their attitudes towards mathematics are very positive (Berger et al., 2020). Given that mathematics anxiety can negatively influence mathematics performance (Dowker et al., 2016), these findings suggest that interventions designed to address mathematics anxiety and strengthen ability beliefs may be particularly useful for over-aspiring students (especially girls).

Stereotypically, mathematics is viewed as a challenging and difficult subject (Li & Schoenfeld, 2019) and this view continues to be represented in our study, especially for those students in the under-aspiring group. In addition to viewing the subject negatively they also see little use for mathematical knowledge in their futures, possibly reducing their future options prematurely. The over-aspiring group suffered from higher levels of mathematics anxiety than the aligned group. This suggests that both groups with an intentions-capability gap are potentially at risk for different reasons, pointing to a role for schools and teachers in ensuring that students are feeling adequately prepared for higher levels of mathematics and sufficiently informed about the mathematics knowledge that they will need for their future lives.

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Teachers as Change Agents: The Role of Social Capital

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Teachers play a crucial role as change agents in schools; however, research rarely positions teachers as the implementers of reform. Teachers in classrooms understand the complexity of their context and are well positioned to adapt professional learning to implement change in their schools. Multiple case study, one in Australia and one in Norway was used to investigate the most significant change and the role of social capital in implementing this change. Findings showed when teacher leaders had social capital, they could support colleagues to implement changes in mathematics teaching. This highlights the importance of nurturing networks of teachers' relationships.

Teachers play a crucial role in implementing change within their schools; however, research describes this change as generally imposed upon them by school leaders, curriculum designers or policy makers rather than being instigated by teachers themselves (Brown et al., 2023). Teachers in classrooms understand the complexity of their context and are well positioned to adapt professional learning to implement change in their school. In this project the teachers, one in Norway and one in Australia sought the professional learning themselves to implement change to mathematics teaching in their own school.

Reports from Australia (Merlo (Centre for Independent Studies), 2024; Australian Education Research Organisation, 2023; Hunter et al. (Grattan Institute) 2025) are urging policy makers to adopt more traditional teaching methods. This aligns with a broader international trend mandating explicit instruction (e.g. State of New South Wales, 2024) and more traditional teaching practices. Also, Australia's severe shortage of mathematics teachers means that less than 25% of year seven to ten students have a qualified teacher each year and the situation is worse in rural and remote locations (Australian Mathematical Sciences Institute, 2018). These out-of-field teachers need support with "relevant, high-quality training and educational materials to assist their ability to inspire, empower and educate students" (Australian Academy of Technological Sciences and Engineering (ATSE), 2022, p.14) mathematically in contextually and culturally appropriate ways particularly in regional, rural, and remote locations. (ATSE).

Queensland's Department of Education's (DoE) *M in STEM* project includes mathematics educators working with clusters of schools across Queensland to improve mathematics teaching, particularly supporting out-of-field teachers to engage students. Explicit instruction conflicts with the aims of *M in STEM*. The first author was a critical friend in *M in STEM*.

The Norwegian Directorate for Education and Training introduced a Teacher Specialist program where mathematics teachers studied at the university to lead professional development in their schools. In Norway's egalitarian society there is a flat organisational structure allowing teachers autonomy to choose how to best teach the mandated curriculum, hence these Teacher Specialists were not well received. Hoem et al. (2017) argued that Teacher Specialists need to be supported during their studies to become change agents in their schools. Although teachers have autonomy in how to teach, the mandated Norwegian curriculum requires development in competences like inquiry, reasoning, and argumentation, which conflicts with explicit instruction. The second author was a teacher educator with this Teacher Specialist program.

The research investigated the teachers' role as change agents. Collecting significant change stories from participants in these projects has allowed the identification of not only what the change to mathematics teaching was but also what contributed to this change, including the

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participants' social capital. The research questions addressed are: *What was the most significant change arising from the teacher's choice to engage in the Professional Learning? How did social capital support the changes in these schools?*

Theoretical Background

Teachers play a crucial role as change agents in schools, influencing educational reform beyond their classrooms. However, current literature predominantly positions teachers as implementers of top-down change, neglecting bottom-up reform possibilities (Brown et al., 2023). Effective teacher leaders can create new cultures of support, lead collaborative inquiries to improve teaching effectiveness, and maintain ambitious instructional practices (Brondyk & Stanulis, 2014). These leaders may be looking to either influence, "indirectly altering another's practice by informing their thinking in ways that shape what they do" or change by intentionally steering others to implement a particular practice (Cooper et al., 2016, p.88). When teacher leaders are teaching in classrooms, they understand the complexity of teaching and so are well placed to model and adapt content specific pedagogies to their context (Wenner & Campbell, 2017) but need a 'means to influence' as well as 'engage in actions' that will support colleagues to change their practice (Cooper et al., 2016).

Research suggests that social capital plays a crucial role in supporting teachers to implement change and improve their pedagogical practices. Social capital can be defined as "the mutual sense of reciprocity and trust which enables groups of people to live and work together successfully" (Australian Institute of Family Studies, 1998). Within schools, social capital comes from the networks of relationships between the teachers. In Li & Choi's (2014) study social capital, including collegial trust, access to expertise, and willingness to take risks, had a strong direct effect on teachers' self-perceived changes in their pedagogical use of technology. The trust built between teachers means that they can rely on one another for support and social influence may emerge from the interactions between teachers (Uekawa et al., 2006). Comprehensive school reform can increase social capital among teachers, which in turn mediates the influence of reform on teachers' pedagogy (Uekawa et al., 2006). Cooper et al. (2016) suggested that teacher leaders guide change by focusing on teaching and learning, productive and trusting relations with both formal and informal support.

Teachers who are most capable of acting as change agents have social capital (Brown et al., 2023). There are two types of social capital 'instrumental' social capital which includes the ability to give advice, problem-solve, and can support colleagues to achieve specific goals, and 'expressive' social capital - trust, encouragement, and support (Tomanovic, 2012). These teachers can depict their desired change as attractive to and achievable by others (Brown et al., 2023). Participation in professional learning networks allows teachers to cultivate new pedagogies which they share with colleagues. (Wenner & Campbell, 2017).

Research has shown that professional learning is more effective when it takes place within school settings, allowing teachers to test and refine ideas in their own mathematics classrooms while collaborating with supportive colleagues to discuss challenges and solutions (e.g., Loucks-Horsley et al., 2003). This was particularly so when the professional learning focussed on content knowledge and ways to support students learn this content, provided opportunities for participants to plan how to implement ideas in the classroom using the curriculum, and discussed outcomes with colleagues (Garet et al., 2001).

Research Design

Multiple cases can shed light on a phenomenon (Stake, 2006). In this study, we present two cases of teachers who have enacted bottom-up change to mathematics teaching in their schools: one conducted in Australia (Case 1) and the other in Norway (Case 2). The study compares the

findings from these two projects to explore similarities and differences in the ways teachers drive change within their respective educational contexts.

Context

Case 1 is investigating *M in STEM* run by DoE in Queensland. Teachers in schools identify a problem of practice in mathematics teaching within their school, or cluster of schools and register to participate and enact change in their school. Professional learning is run by DoE and each cluster of schools is assigned a university-based researcher as a critical friend. *M in STEM* aims to support the improvement of mathematics teaching, particularly for out of field teachers.

Case 2 is a professional development course in the Teacher Specialists Programme in Mathematics in Norway. The participants work in schools whilst taking this master level course, *Selected professional-relevant methods and perspectives on learning*. Participants must have at least five years teaching experience and a minimum of one year study in mathematics. The course aims to provide insight into various ways of leading professional development among colleagues, in addition to relevant learning theories and research methods in schools.

Participants

Sarah who led the project in her school was from a rural school in Queensland participated in a 20-minute interview. She is a primary trained teacher who had moved to teaching in secondary schools and was new to acting head of mathematics. Two out of field teachers, Anne and Chloe, and an experienced mathematics trained teacher, Ben, participated in a 25-minute focus group. Pseudonyms have been used.

Mary (pseudonym) is one of the students in the Teacher Specialist programme in Norway. She is an experienced teacher working at a Year 5-10 school in Norway. After the Teacher Specialist programme, she continued with a master's thesis in mathematics education.

Data Collection and Analysis

A semi-structured interview with Sarah and a focus group with three teachers who had worked with Sarah were conducted face-to-face using the Most Significant Change (MSC) methodology (Davies & Dart, 2005). This methodology asks interviewees to tell a story that illustrated the most significant change they observed whilst participating in the project; why the story was significant to them; and how participation in the project contributed to a change in their or the school's practice. Focus group participants were then asked to build a single story that best described what aspect(s) of their involvement with *M in STEM* led to the most significant changes in their own mathematics teaching practice or mathematics teaching within their school. The audio recordings were transcribed by otter.ai and checked for accuracy.

Mary's written reflections during the Teacher Specialist course and during her master's thesis are used in this paper. In addition, the second author interviewed her about Significant Changes from her perspective.

The data were analysed thematically using Braun & Clarke's (2022) six steps. In this paper themes related to problem of practice, most significant change, and social capital are presented.

Findings

Results are presented under problem of practice, most significant change, and social capital.

Case 1

Problem of Practice

Sarah saw the *M in STEM* project advertised and thought it was an "opportunity to improve the results in maths" by getting support for the school's problem of practice; students were not

engaging in mathematics; hence the school's results had been falling for several years. Sarah aimed to involve the feeder primary schools to smooth the students transition to the high school.

Most Significant Change

Sarah described the most significant change as the enthusiasm of staff which boosted student engagement. The teachers introduced Problem Solving and Modelling Tasks where students completed real world problem solving or mathematical modelling tasks. An excursion was included in each of Year 7, Year 8, and Year 9. The Year Nine students were investigating the statistics in football and travelled to Brisbane (100 km by bus) to see an evening game in the National Rugby League competition involving the Brisbane Broncos.

I think the enthusiasm of the staff. So, because we focused on engagement, ... and introducing, like the football trip to boost engagement, the talk amongst the staff in the department has become more positive and wanting to be involved, ... hoping to translate through to the students. (Sarah)

Anne thought the most significant change was Sarah's willingness to include new ideas so that her teaching now included group work, contexts, and less use of textbooks.

Sarah being so involved in it, her filtration down to us, and her willingness to incorporate new things, like Building Thinking Classrooms ... My pedagogies have changed, more group based, more context-based type stuff, just like less textbook. ... Students responded to them. (Anne)

There was also discussion about the connection with the primary schools, even though they did not participate for long, that helped the secondary teachers to get a better understanding of where the students had come from mathematically that made the difference in their practice. Anne said she was "a little bit more patient" now since hearing about behaviours and "how they structure their maths" in primary school. Chloe said she was "thinking more about, like, what the gaps were ... instead of just assuming that they [students] knew it".

Reflecting on her engagement with the project Sarah said the best part was the teachers and the students enjoying and engaging with mathematics.

I enjoy maths but seeing it in my teachers and seeing it in the kids. Even getting back to the football trip, you know, very simple idea to engage students to see real life context of the maths (Sarah)

Ben reflecting on the project said it was teachers "being willing to accept that there are other ways to do things" which has led to more engagement of teachers and students. Anne's reflection included teachers "more willing to learn, more willing to try new things, ... maths having this changed mindset that's focused on engaging with kids."

Social Capital

It was Sarah's relationship (social capital) with the Year 9 teachers that encouraged them to respond to the email expressing interest. The principal's trust (social capital) in Sarah allowed her to introduce the mathematics excursions including taking all the Year 9 students to an evening game of football.

Project teachers at the school received support from the DoE facilitators of *M in STEM* over the two years through professional development and follow-up support for Sarah. In this way they were extending their network (social capital).

I've spoken to [the facilitators in DoE] a fair bit emails just coming across certain questions I've had, and then they've been in meetings where it's like, Oh, hey. Quite supportive, like the Peter Liljedahl stuff [*Building Thinking Classrooms* (Liljedahl, 2020)], they were there. I meet up with them, so in terms of their support, like the encouragements there, like, how's everything going?

There was also support from a university-based critical friend as part of the project with conversations, resources, readings, and professional development (PD) targeted to the needs of teachers in the cluster schools, once again extending their support network (social capital). Sarah acknowledged the value of her critical friend and how she should have utilised her more while the teachers acknowledged the usefulness of the critical friend's PD.

I love [my critical friend] ... You're actually coming out and seeing us in our environment, and meeting more than just me, ... you're giving us the additional information and the prodding for thinking, and the support that you know, the offer of support that it's there, (Sarah)

You did have your PD that you have with us one staff meeting that was, that was useful. (Chloe)

Sarah and the teachers, Anne, Ben, and Chloe describe the expressive aspects of social capital (Tomanovic, 2012) - trust, encouragement, and support that there is within the teachers in the mathematics department as well as the network with the feeder primary schools. *M in STEM* provided opportunities for Sarah to build her own instrumental social capital by being able access the *M in STEM* project team and a critical friend for support.

Case 2

Problem of Practice

Mary is a teacher working at a small primary school in Norway. Mary chose to participate in the Mathematics Specialist Program as mathematics is the subject she enjoys teaching the most, and she has previously had some mathematics coordinating roles amongst her colleagues.

I hope that the study will make me a better supervisor for colleagues and contribute to my ability to develop mathematics as a subject at the school I work at. I also hope that the study will make me better equipped to tackle the new and exciting curriculum. (Mary)

Mary's description of her work environment was positive and collaborative. Students in this course described their schools' problems of practice, and Mary described an issue that some colleagues were not as committed as others and how many changes were introduced at once.

Too many focus areas or new projects at once leave little time in a hectic everyday life to really get to grips with the new. This often means that you only do the minimum of what you must, and do not fully get the "new" under your skin. (Mary)

Mary's description of her workplace was a good collaborative environment, and a principal who wants her to take a leading role among colleagues. This seems to provide a solid foundation for her to utilise learning from the Teacher Specialist course where she gained more insight into how she can lead her coworkers in developing their mathematics teaching.

The Most Significant Change

Linda noticed significant change during her collaboration with Mary, both as a teacher educator during the Teacher Specialist course and as her supervisor of her master's thesis. During the Teacher Specialist course, the students were expected to lead Lesson Study (Lewis et al., 2019) development in mathematics at their workplace. This way, the students learned about a method for professional development at the same time as they conducted it in their workplace. Lesson Study was chosen, because it is a bottom-up change where the teachers themselves are challenged to identify what part of their teaching they want to further develop. Since leading Lesson Study development in their workplace was a compulsory task in the course, they were given the opportunity to deviate from the method if they could reflect on what they gained and what they lost by these choices.

In Lesson Study teachers work together to improve their teaching and share responsibility for teaching and outcomes. In Mary's case, she was one of the students that chose to deviate from several of the key components of Lesson Study by: doing the teaching herself instead of drawing among all teachers who would teach in the end and providing all the theory and resources for the group of teachers. According to Mary, she made these choices so as not to burden her colleagues unnecessarily with her assignment. However, as she reflected on it, she realised that her colleagues lost ownership of the Lesson Study.

After the Teacher Specialist course Mary decided to write a master's thesis. She made it clear from the beginning that the research would have to be something that could benefit not

just herself, but also her school colleagues. She chose to conduct action research in collaboration with five of her colleagues, where the aim was to further develop their competence in orchestrating productive whole class discussions. When Mary was evaluating how this research went, she described her colleagues' reactions as positive. They experienced it as useful and that it increased their professional and pedagogical knowledge.

From the Lesson Study assignment in the Teacher Specialist course, to how she led professional development at her workplace in the master thesis, is considered a significant change. Although both interventions had the possibilities for teachers' autonomy in the project, Mary was more worried about adding to her colleagues' burden, than to seeing the value that such a collaboration could be for them. This had changed when she started on her master's thesis. To get five colleagues at a small school to put in the work and effort as they did in the action research, is impressive. There seemed to have been a change where the teachers wanted to work on this type of professional development together.

Mary described this change as related to time. When they were doing the Lesson Study, they did not have allocated time for the project from their principal. However, when she conducted her master project, she included colleagues and leadership in the process and decision making from the beginning. They had time set aside, and the action research was conducted over a three-month period, which gave the participants time to delve into issues on how to lead productive whole class discussions in a mathematics classroom.

Social Capital

Mary describes a school where the expressive aspects of social capital is part of the work environment and she therefore has trust, encouragement, and support among colleagues. However, she seems to have less security in her own instrumental social capital. The change from trying to lessen the workload on her colleagues during Lesson Study to involving them deeply in her master's project, means she is gaining confidence that she can provide advice and support colleagues to achieve specific goals. This is the second type of social capital described by Tomanovic (2012). At the same time, when Mary describes how her colleagues talked about the master's project and why they felt it was successful, they are mostly referring to issues related to expressive aspects of social capital. Mary's colleagues point to how this project felt safe and had a low threshold, because it was led by a colleague. According to them, external resources do not always understand what it is like at their school. Mary is also experiencing that her participation in the Teacher Specialist course has provided her more trust among colleagues, because she can bring something to the table.

Discussion and Conclusions

This multiple case study investigated how teachers can lead bottom-up change in their schools. We found that these teachers utilised their social capital to lead change in their schools.

In case 1 Sarah had identified the problem practice as the lack of student engagement and hence the school's falling mathematics results. In Case 2 Mary identified the problem of practice as a principal who signs the school up for too many initiatives, leading to some teachers choosing only to do the bare minimum. In both cases there was a lack of engagement by some of the teachers of mathematics. In the Norwegian case the principal was directing too many projects for the staff to engage with whilst in the Australian case, the principal was not directing any. In both cases the teachers chose to participate in a teacher-initiated project that made a difference in their teaching practice similar to Cooper et al. (2016).

In Case 1, by implementing real world problem solving and excursions for the younger students (Case 1), teachers' enthusiasm for mathematics increased and was followed by student engagement. Sarah had built the trust and relationships (social capital) (Li & Choi, 2014;

Uekawa et al., 2006)) with the Year 9 teachers so that when they were asked to participate, they readily agreed. These three teachers had a tight network (social capital) which meant they joined the project together to support each other. Together the teachers decided that it was Sarah's social capital - leadership, willingness to take a risk and encourage teachers to try different pedagogies, including real world problem solving and modelling, group work, and less use of textbooks that was the most significant change from the project. Accessing professional learning for her teachers through *M in STEM* which included the *Building Thinking Classrooms* (Liljedahl, 2020) PD and their critical friend, Sarah ensured they had the support (social capital) they needed to make changes to their mathematics teaching. The changes to teaching practice led to more engagement of students and teachers. Sarah had the confidence and expressive social capital within her school initially and developed her instrumental social capital through *M in STEM* by utilising the project team and her critical friend for staff professional development and advice to support her teachers change their practice.

In case 2 the significant change was in Mary's confidence and social capital to lead an action research project on orchestrating whole class mathematical discussions. Originally, she had tried to support (social capital) her colleagues by relieving the burden of Lesson Study (Lewis et al., 2019) but realised that it reduced their opportunity for learning. With the action research project Mary supported her colleagues (social capital) to participate fully. Mary's instrumental and expressive social capital are intertwined. While she described elements from expressive social capital as being part of her workplace prior to Lesson Study, these are further emphasised as Mary developed her instrumental social capital in the Teacher Specialist course.

Social capital supported the teachers in these studies to successfully lead change in mathematics teaching within their schools. These schools had networks of productive and trusting relationships between the teachers which Cooper et al. (2016) suggested leaders need to enact pedagogical change. These schools also had environments where teachers were allowed to take risks and try different pedagogies and there was access to expertise through the teachers leading the change similar to Li and Choi's (2014) study which highlighted the importance of social capital (collegial trust, access to expertise, and willingness to take risks) on teachers' self-perceived changes in their pedagogies. These schools were not in a situation where a specific pedagogy was being imposed (for example, Australian Education Research Organisation, 2023; State of New South Wales, 2024)

These findings are significant as they highlight the importance of both expressive (trust, encouragement, and support) and instrumental (ability to give advice, problem-solve, and goal achievement support) social capital (Tomanovic, 2012) for teachers leading bottom-up change as identified by Cooper et al. (2017) and Brown et al. (2023). The teacher leading the change needs an environment of trusting and supportive relations between themselves and the teachers that allows for risk taking as well as the ability to seek and give advice, problem-solve, and support colleagues achieve the chosen goal. Without this social capital leading change within a mathematics department would be very difficult. Social capital also has implications for those who support teachers to enact change in the schools. We acknowledge this is a small study and we are continuing to further research.

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Wondering, Wandering, and Desiring in Mathematics

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Traditional mathematics teaching approaches result in inequitable outcomes for students outside the dominant community. One reason for this is a school system that prioritises overly narrow ways of knowing, being and doing. One example of this is a teaching approach that prioritises very specific learning goals and highly structured lessons that focus on performance and product to be judged as a success or failure by a teacher. This paper explores an alternative teaching approach that opens possibilities for intellectual movement and exploration, recognising additional ways of being, knowing and doing to be valued equally to more traditional ways of knowing, being and doing.

Introduction

Much research has been done into mathematics education as it pertains to equity. Under this theme, we might consider the inequity of learning outcomes for students, for example, de Freitas (2008) points to school mathematics as a form of cultural capital that is distributed along lines of class, race, and gender among other categories. Thus, mathematics education is “implicated in the social stratification of communities” (p. 43). In thinking about why this is the case, one important aspect to consider is the type of knowledge that is valued in the classroom. Aslan Tutak, Bondy, and Adams (2011) explain that mathematics within schooling and society is commonly viewed as apolitical, standardised, and universal with little space for interpretation. This traditional view of mathematics puts the focus on the students learning a fixed set of facts and ignores the background and identity of the students (Louie, Adiredja, & Jessup, 2021). However, Yolcu (2019) suggests that equitable mathematics learning outcomes must consider both the knowledge that is privileged and the learning approaches that are made available to students. Looking at the Australian context, Spillman and Wilson (2024) suggest that the contemporary schooling system generally “works to prioritise a far too-narrow set of ways of knowing, being and doing, and requires serious rebalancing” (p. 78). Pedagogical practices shape possibilities of what kinds of learning and knowing are able to happen within a classroom. Lesson planning gives indications of what teacher’s value as mathematics and mathematics learning. It is fairly standard practice for teachers to ensure each lesson includes a clear learning intention and success criteria that can be used to measure if students have performed successfully (William, 2017). In this paper, I reflect on whether the expectation that every lesson has a clear purpose and goal may be a narrowing influence on student possibilities for knowing, being and doing. In particular, it narrows possibilities of knowing down to just those available in the standardised curriculum, and it narrows possibilities of being and doing down to a performance and the creation of a product to be assessed based on the standardised curriculum.

This reflective paper came from the early stages of analysis of a PhD study. The PhD study is a self-study that explores the interactions between primary pre-service teachers and a mathematics teacher educator. During the early stages of analysis of this study, I found myself reflecting on the tendency for both school and university structures to focus on performance and product rather than seeing learning as an emergent process that students go through, and rather than seeing that process as having value separate from any ensuing performance or product. This paper was shaped by a combination of ideas from academic sources and data from the study. It tentatively proposes an alternative teaching approach for mathematics based on a literacy teaching approach called literacy desiring. This approach attempts to offer

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opportunities for students to be able to be, do and know in ways that are process driven rather than outcomes orientated. The PhD study is investigating the research question:

- How might primary pre-service teachers and a mathematics teacher educator deconstruct their positionalities and consider possibilities for equitable mathematics teaching practice in a neoliberal education system?

This paper then uses data and reflections from this research focus to consider the question:

- How might a move away from a focus on achievement outcomes open up possibilities for additional ways of knowing, being and doing in mathematics for primary school students?

The educational system in Australia is built around an emphasis on standardised achievement outcomes, similar to the situation in North America, as described by Francis (2019). Learning intentions are ubiquitous, if not standard, for the majority of teacher planning, and are a tangible illustration of this emphasis. Hattie (2009, 2023) discusses the role of learning intentions and structured lessons as a contributor to learning. He is a key advocate for learning intentions in the Australian landscape, and his perspective is influential, particularly with government agencies. His work continues to influence education decisions. Hattie explains that the purpose of learning intentions and success criteria is to “make students understand what the teacher is using as the criteria for judging their work, and, of course, to ensure that the teacher is clear about the criteria that will determine if the learning intentions have been successfully achieved” (Hattie, 2009, p. 169). The picture built in relation to learning intentions is one of rigidity. Each lesson must be purposeful and structured around a specific goal, and there is no space and time “to pursue the marvelous (*sic*) leads introduced by the students themselves” (Regenspan, 2002, p. 583). Francis (2019) suggests that the over-emphasis on standardised achievement outcomes reduces the potential for joy and play in mathematics and is detrimental to the majority of students, including causing trauma for some. Do lessons that are overly structured and focused on specific achievement outcomes fail to leave space for students to develop creativity and bring their ideas in mathematics, thus hindering multiple ways of being, knowing and doing?

Methodology

The methodology for this study is an inquiry informed by self-study of teacher educator practice (self-study). This methodology uses a range of qualitative methods and involves a practitioner-researcher inquiring into themselves and their practice in order to bring about positive transformation (Feldman, 2016). Those who undertake self-study stress that its focus is on the personal and professional self and is usually undertaken by teacher educators (Bullough & Pinnegar, 2001; Feldman, 2016). Self-study provides a dual purpose within the study, looking reflectively at the teacher educator’s practice while at the same time looking at the context in which the teacher educator practices – teacher education more generally. In this study, I focused on my own practice as a mathematics teacher educator – looking at both my work with the participants, and also my work more generally as a primary mathematics teacher educator, as relevant. Thus, the more general focus on teacher education looks at the participants’ learning through the study, and also at the context which impacts their learning and my teaching through the study – this includes the relevant teacher education degree, and then broader aspects such as Initial Teacher Education (ITE) accreditation and curriculum, the Australian Curriculum and the political and social context that shapes these.

Context and Participants

The study involves two pre-service teacher (PST) participants, Olivia and Annie, who are both PSTs attending an Australian university in an ITE program of four years duration. They are undertaking a Bachelor of Education (Primary R-6), and both were in the second year of

their study when they commenced their participation with the study. Both PSTs involved in the study are female. I am the third participant in the study. I work as a mathematics teacher educator at the same university. The participants were identified using purposeful homogeneous sampling. Homogeneous sampling involves the researcher purposefully sampling individuals or sites based on possession of certain characteristics (Creswell, 2012, p 208). The recruitment was conducted within a primary mathematics education course, ensuring the participants had some common experiences.

The positionality of all three of the participants is important to address. As white women, our educational experience as learners was one that privileged and mirrored our beliefs, values and cultural norms. The ways of knowing, being and doing valued in schools have aligned with our cultural norms in many ways, as we formed part of the dominant group of society. Thus, it is important to acknowledge the privilege and whiteness that informs all three participants' perspectives as we engaged together and as I reflected individually. Pough and Willey (2022) highlight that whiteness refers "to the attitudes and behaviors that serve to maintain systemic racism and white dominance" (p. 135). They note that it is standard for white teacher educators and PSTs to bring unexamined biases to their engagement in the educational context. Additionally, both Annie and I had experienced achievement and success with mathematics learning, while that was not the case for Olivia. Regardless, all three of us had our identities and perception of our values shaped by the education system's focus on meeting achievement outcomes, and its narrow view of success and performance. Olivia experienced feeling "dumb", and like maths was not something she could do during her time at school. Annie had a more positive experience when learning maths, but she still acknowledges that her experience nonetheless created pressure to be successful: "I got it. Therefore, that was my value, you know... And if I didn't maintain that, then I wasn't intelligent." And my own experience aligned with what Annie described.

Methods

- Eight 1-hour meetings across 10-month period
- Different short articles, chosen by researcher, read together within each meeting
- After reading, participants engaged in reflective dialogue together
- Researcher undertook reflective journalling as relevant

In this study, I have been exploring the process of deconstruction and of imagining possibilities for social justice in mathematics teaching practice undertaken by a group containing two primary pre-service teachers and me as a mathematics teacher educator. I was interested in the experiences of all participants as we sought to interrogate and deconstruct our personal positionality and consider possibilities regarding mathematics teaching practice for social justice. The three participants met across eight one-hour occasions. Encounters would begin with reading of a different short article each time, which then gave impetus to shared reflective dialogue. The meetings were minimally structured and deliberately intended to allow flexibility to follow the reflections of the two PST participants. My role was to find relevant articles and to be a participant in the discussion, to suggest themes, and bring in alternative perspectives. These meetings were separate from any formal learning undertaken by the PST participants. I also worked to interrogate my own practice as a teacher educator as I undertook my role in this research. My individual reflections took the form of journal entries. The foci for the journal entries included reflection on my encounters with the participants in the study, and additionally, any classroom interactions and practice choices that caused thinking about my teaching practice, especially related to mathematics as cultural. This means that the focus for my journal entries encompassed a wider context than the meetings with the PST participants. The data used for this paper take the form of transcripts of the group meetings, and excerpts from my own journal entries.

The data analysis frameworks used for this study are thematic analysis and critical discourse analysis (CDA). Thematic analysis is used to explore themes that I notice within the data. Thematic analysis (TA) helps with identifying and then analysing patterns (Clarke & Braun, 2017). Braun and Clarke (2006) write that thematic analysis can have commonalities with discourse analysis, considering “the sociocultural contexts, and structural conditions, that enable the individual accounts that are provided” (p. 85). Thus, it can be suitable to use in conjunction with CDA. Within CDA, data must be considered in the context that shaped its meaning. MacDonald-Vemic and Portelli (2020) state that, “[c]ritical discourse analysis in educational research involves highlighting the coexistence and intertextuality of the multiple discourses that circulate in the educational sphere” (p. 298). As this is a reflective paper, the data included has informed my reflective writing. However, data analysis is incomplete. Thematic analysis has commenced and has informed this paper. CDA is incomplete and has not informed this paper.

Discussion and Findings

In reading through the transcripts from the meetings with the participants, I found myself noticing and reflecting on a binary perception of learning – someone has succeeded or failed, someone has achieved or not achieved. Connections can be made from this back to Hattie’s discussion of learning intentions and success criteria. He sees learning intentions as important to allow the teacher to judge the students’ work, and to know when they are successful. This suggests that his view of learning is as destination or as a performance. In talking about success criteria, he even says, “Too often students may know the learning intention, but not how the teacher is going to judge their performance” (Hattie, 2023, p. 313). This theme also came up within my reflective journal entries (RJE). The first time I wrote about it in a journal entry, I called it learning as process vs destination, and I connected it to the current focus in learning on assessing students against set learning intentions or outcomes. In the journal entry, I note that having fixed learning outcomes “make[s] everything learning and goal oriented” (RJE, 12/7/22). In a later journal entry, I state, “learning is a verb - to learn. But I wonder how much we treat it as a noun, a product, rather than recognising it as a process, and more importantly a relational process” (RJE, 23/7/22).

I see connections between this idea of learning as focused on process and becoming, that moves away from fully planned and controlled learning activities, and a discussion that took place during one of the meetings with the participants. During a discussion about teaching mathematics in a way that connects to other curriculum areas, one of the participants, Olivia, was concerned that the necessary mathematics may not come through strongly when teaching in a cross curricular way. She says, “I suppose to me it looks like it could potentially become a little bit wishy washy about what the [mathematics] concept is.” In thinking about breaking down boundaries between learning areas, she wondered, “how far do you go down each area, you know, before you get lost?”

This comment struck me as being strongly shaped by an “assumption that exploration as part of schooling/education might result in getting lost, because it starts with the framing of needing to meet certain goals” (RJE, 2/12/24); that is goals as defined by the curriculum. A teacher may be less concerned about getting “lost” if there was not the same pressure to perform a certain way as a teacher, and to “get your students to learn”, as Olivia expressed. She explained, “you’ve got certain goals to meet and, you know, assess the standards and it’s trying to in your head making sure that you’re ticking... ticking those boxes and making sure it’s achieved.” Does an expectation that teachers create learning intentions and structured lessons result in teachers who feel they must rigidly adhere to lesson plans? In a neoliberal context, the drive to ensure efficiency and meet certain outcomes are key factors (Connell, 2013). Learning intentions that focus solely on performance along with a lesson structure intended to optimise

time on task are both shaped by a neoliberal discourse about education. Neoliberal discourse in education comes from a Western way of knowing, being, and doing. It is a significant shaping force in our current education system. How might we rise to the challenge from Spillman and Wilson (2024) and bring in some additional ways of learning, knowing, being and doing to work alongside what is currently prioritised.

Thinking about lostness made me recall a quote from *The Lord of the Rings*: “Not all those who wander are lost” (Tolkien, 2001, p. 167). This led to the question, why is wandering as part of education seen as getting lost? Why is it a bad thing to go off the trail? Wandering is a way of moving – it equates to there not being a clear purpose or direction in the movement. However, it is also used to describe thoughts and conversation that leave the original trail. Dominguez et al. (2020) talk about movement, both physical and intellectual, as being connected to creativity. They state, “Learning bodies do not move aimlessly; they mobilize material and human resources purposefully across time and space as part of mobilizing mathematical concepts” (p. 80). What is wandering if not a form of movement. Surely following wonderings can be intellectual movement. Dominguez et al. (2020) also suggest, “repetitive and regimented movement recreates a space to which children come back each day to do the same. A body that moves the same way again and again is a constricted body often producing what is expected not what [it] is capable of creating” (p. 81). The same is surely true for the mind. Rigid goals and structure within learning may need to be augmented with opportunities for both mental and physical movement and in ways that are less regimented and repetitive to bring in opportunities for creative potential.

I wonder if opportunities to wonder/wander, and even perhaps get lost, might open space for students to move past producing what is expected, towards surprising and creative possibilities. Through the study I am undertaking, I have come to see mental wandering as an important part of my own learning. I’ve found myself going down rabbit holes and undertaking side quests as my curiosity is piqued. And those side quests so often have shaped and enriched my thinking. Conversely, I experience negative pressure from existing in a system that feels like it is constantly requiring performance and a finished product. I state: “School feels like it's always about making a product; there's never the space to just be. It's about performing all the time... rather than the space to learn and learning to be a process... and also the space just be yourself in that” (RJE, 1/7/23). The wondering/wandering process for me acted as a compost to fertilise (Rozitis, Debono, Brady-Clark, Jarrett, & Walker, 2025) my thinking about both research possibilities and pedagogical possibilities. Fertilise can be used with dual meanings here. The process of wondering/wandering was a fertilisation where fertilise means both to implant new ideas and also to enrich and nourish existing ideas. All this leads me to speculate about the value of a process of wondering/wandering that acts as a fertiliser and allows students to focus on the process and on becoming (mathematicians) within a mathematics class. I suggest that this may allow a move in mathematics from “an outcome-focused curriculum to one based on flourishing, which I feel would be more kind and loving” (Francis, 2019, p. 80).

Kuby and Rucker (2020) discuss an approach to teaching and learning literacy that they call literacy desiring. They describe it as a “both/and” approach, which both provides “spaces for traditional learning objectives” (p. 32) and also opens space for student exploration and interaction with materials and artefacts in ways that challenge traditional ideas of what constitutes writing and literacy, and can constitute valid knowing, being and doing. They are particularly concerned with those students who are Othered “based on abilities, race, language proficiencies and so forth” (p. 17) and the way certain students are seen as requiring interventions because they are judged as being at a deficit when compared to school ways of knowing and being, which tend to be based on a White, middle-class, abled bodied standard. They argue that “if we continue to conceptualize literacies in the ways we always have, we will

continue to produce and tame children as not fully human and lively – perhaps inhuman – in their relationships with the world coming to be” (pp. 30-31).

In this approach, students are told to go be writers. But that does not have to mean alphabetic writing. Literacy is seen as moving beyond the alphabetic to include interactions between people and also interactions between humans and materials or artefacts. Kuby, Rucker, and Kirchhofer (2015) seek to open up possibilities for learning experiences that are much more interactional, and focused on “processes and becoming, not necessarily on a future product” (p. 398). They see the materials as playing an active role – it is through interactions with materials and artefacts that learning emerges, and the materials themselves are agents that shape learning possibilities. From their observations, students “who were hesitant to write (traditionally speaking) at the beginning of the year flourished when desiring with materials and with conventional writing as the year progressed” (p. 415). This approach celebrates other ways of knowing, as knowing is seen as an embodied, relational and interactive process – considering interactions between humans within the classroom, and also between humans and artefacts and materials. It opens possibilities for student and teacher being and doing that is not focused on achievement, performance, and success or failure.

Might this be a beneficial approach to explore within the mathematics space? What might it look like? Kuby et al. (2015) provide an example when discussing literacy desiring that shows clear potential connections to mathematics learning. Within the literacy class, students were told to go and be writers, and that was flexible enough to allow students to explore, design and build physical artefacts, working with a range of physical materials. This process of working with materials would sometimes turn into finished products, but that was never an expectation placed on students. One group of students when engaging in the process of literacy desiring built 3D bird houses. Another time, reading a book about animal bones inspired the creation of a life-size giraffe out of paper. This happened because the students learnt from the book that humans and giraffes have the same number of bones in their neck. There are some clear connections from these tasks both to measurement during the creation of the artefact, and also the use of mathematics including, but not limited to, measurement once the artefact has been completed. This approach, valuing the mathematics learning afforded by artefacts, has similarities to Percival (2001) description of the use of ancient artefacts to drive mathematics inquiry. However, I argue that the removal of the pressure for a product or a specific curriculum sanctioned outcome, may be one alternative to allow students kinder and more joyful interactions with mathematics, as advocated by Francis (2019). To facilitate connections between the engagement with materials and mathematics learning, teachers will need to develop their own confidence to see mathematics in the everyday. Annie describes this as “learning to use the muscle to find maths in other things”. This must be part of our work as teacher educators – to support PSTs to develop their muscle of noticing mathematics.

It is important to recognise that there would be tensions and concerns for many teachers at the thought of bringing a pedagogy such as this into their classrooms. In discussing a creative interpretation of curriculum within a New Zealand school, Tweed (2021) acknowledges that any teacher choosing to open up possibilities in this way opens themselves up to criticism from school leadership, other teachers, and parents. It may be that others “cannot recognise that mathematics is happening” (p. 61). These tensions would also arise due to the requirement to assess students against a formal curriculum. Olivia expressed this tension clearly when discussing a pedagogical approach that is more student-centred: “I don't know whether the time is given to teachers and students in especially those curriculum things, because it's like, do this, do this, do this, do this, and now you've marked off a piece of work.” In another meeting she says, “You've gotta be able to show at the end of the year that you've done this, this, this and this. So so I... quickly quick, I haven't done... It's not very collaborative. Look, it's like teacher at the front...” In one of the meetings, I also recognise these tensions related to bringing more

openness into a teaching-learning situation. I say, “I think as a teacher, if you’re like and now we’re doing maths, and maths, looks like this and I can control it, it feels safer”. Here, I am drawing on my own experiences engaging with a less controlled and structured way of teaching-learning. Engaging in a less structured approach may cause doubts when compared with a traditional conception of the role of teacher as instructor and explainer (Lo, 2021). Teachers are likely to feel tension between the pressure for efficiency and a pedagogical approach that allows students to wander and wonder. And moreover, that tension is likely to be magnified when contrasting a wondering/wandering approach with something more structured and controlled.

Conclusion

I can’t help but wonder what mathematical ideas might emerge if students were given the chance to wonder/wander with materials, including creating their own artefacts or working with mathematical artefacts. What other ways of knowing and doing mathematics might students come to recognise as valid? What other ways of being might be able to be validated and celebrated? Can we as teachers engage in a both/and approach. This would mean that at times there is a focus on specific learning goals, meeting curriculum standards, and end products that will enable students to meet formal educational goals (Kokka, 2015), while other times students are given the chance to bring their whole selves, take time with their learning, and encounter other ways of being, knowing and doing. Could this provide opportunities for students to be fully human, not tamed and produced in constricted ways within mathematics learning? Might this allow their wondering/wandering to act as compost and fertiliser for creativity and intellectual movement that can both enrich the more formal educational goals, and also bring in unexpected creations and ideas? Future research into a mathematics desiring teaching approach is warranted. This could initially involve a study conducted in partnership between a researcher and a primary classroom teacher to explore the practical impact of this approach when working with a diverse range of students. This would provide evidence of whether it opens opportunities for additional ways of being, knowing and doing within mathematics.

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“I Felt Sad, Stupid, and Annoyed”: Characterising Primary Pre-Service Teachers’ School Mathematics Experiences Through Achievement Emotions

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Pre-service teachers’ (PSTs) past mathematics experiences shape their future teaching. We analysed 266 Australian PSTs’ written recollections of positive and negative school mathematics experiences using Pekrun’s Achievement Emotions Framework. Negative memories evoked shame (41%) and anxiety (32%), while positive ones reflected enjoyment (50%). We interpreted these emotions through the lenses of self-efficacy and mathematical wellbeing to understand how PSTs’ emotional experiences might influence their capacity to flourish as mathematics educators. Findings reveal the lasting impact of negative experiences and the importance of supporting PSTs’ self-efficacy.

Introduction

Mathematics anxiety is a common issue among primary preservice teachers (PSTs), often linked to negative past experiences and the belief that being good at mathematics equates to being inherently smart (Jablon Stoeck, 2017; Lo, 2021). These perceptions can lower PSTs’ confidence in both learning and teaching mathematics. The findings of this study aim to deepen understanding of how PSTs’ mathematics anxiety and self-efficacy beliefs develop, in turn helping teacher educators consider how to design supportive programs that build PSTs’ self-efficacy, confidence and effectiveness for teaching mathematics as well as their mathematical wellbeing. By examining the interplay between achievement emotions, self-efficacy, and mathematical wellbeing, this study provides a holistic understanding of how PSTs’ past experiences shape their future capacity to flourish as mathematics educators.

Past learning experiences in mathematics create a filter through which PSTs view what they learn during their initial teacher education programs (Bloomfield, 2010). Negative experiences when learning mathematics, including poor grades, uninspiring instruction or unsupportive teachers, can contribute to anxiety, negative attitudes and lower self-efficacy (Mazana et al., 2019). These experiences also undermine mathematical wellbeing in PSTs, which encompasses feeling positive and functioning optimally when learning or teaching mathematics (Hill et al., 2021). Conversely, positive school experiences of learning mathematics can positively impact PSTs’ engagement with mathematics during teacher education programs (Chalkiadaki, 2018). When PSTs encounter positive mathematics learning experiences, they tend to be more motivated and exert greater effort to persist with challenges, which increases the likelihood of increasing their self-efficacy for teaching mathematics (Schunk & DiBenedetto, 2020). In

recognition of the importance of considering prior experiences in influencing mathematical wellbeing and self-efficacy, our study aimed to address the following research questions:

How do PSTs describe positive and negative experiences with learning school mathematics?

What positive and negative emotions are evident in PSTs' descriptions of their learning experiences?

Literature Review and Theoretical Framing

This study drew upon literature and theories related to achievement emotions, self-efficacy, and mathematical wellbeing to interpret PSTs' emotional experiences in learning mathematics at school. These three theoretical frameworks are interconnected and important for understanding how PSTs' past mathematical experiences may shape their future teaching practices. Specifically, achievement emotions provide the lens for categorising and analysing emotional responses, self-efficacy explains how these emotions translate into beliefs about teaching capability, and mathematical wellbeing encompasses the holistic impact on PSTs' long-term flourishing when teaching and learning mathematics.

Achievement Emotions

Achievement emotions are feelings people experience in relation to academic activities, such as learning mathematics, or in response to achievement outcomes, such as success or failure (Pekrun, 2019). These emotions arise when a person's performance is evaluated according to standards of competence. Achievement emotions can be intensely experienced and impact people's learning and wellbeing (Pekrun, 2006). Given that performance evaluations are a central feature of education settings, classrooms are common settings where these achievement emotions are frequently experienced (Pekrun, 2019).

Pekrun (2006; 2019) developed a *Control-Value Theory of Achievement Emotions* (CVT) framework for conceptualising achievement emotions, which we draw upon in this study to guide our analysis. Rather than focusing on all three dimensions of CVT, we specifically utilise Pekrun's eight emotion categories (enjoyment, hope, pride, anger, anxiety, shame, hopelessness, and boredom) as our analytical framework. These categories enable systematic classification of PSTs' emotional recollections, providing insights into the valence and nature of their mathematical experiences. The three dimensions of CVT are: Valence classifies emotions as either positive (enjoyment, hope, pride) or negative (anger, anxiety, shame, hopelessness, boredom) based on subjective pleasantness. Activation differentiates emotions by arousal levels: activating emotions (enjoyment, pride, anger, anxiety) increase arousal and prepare learners, while deactivating emotions (boredom, hopelessness) decrease arousal and may cause withdrawal. Temporal classification categorises emotions by occurrence: prospective (before tasks), activity-related (during engagement), and retrospective (after completion).

Using CVT facilitates the analysis and interpretation of achievement emotions and supports understanding of how these emotions are experienced by learners. Analysis and interpretation can focus on how emotional experiences modulate cognition, information processing capabilities, motivation, and self-regulatory actions in learning situations (Pekrun, 2006; 2019).

Self-Efficacy

Bandura (1994) defines self-efficacy as a person's belief in their ability to perform a task to achieve a required or desired outcome. In the context of this study, self-efficacy serves as the bridge between PSTs' emotional experiences and their future teaching confidence. When PSTs experience negative achievement emotions such as shame or anxiety about mathematics, these emotions can diminish their self-efficacy beliefs when teaching mathematics effectively (Jablon Stoeck, 2017). Several factors contribute to a person's self-efficacy development, including mastery experiences (successful teaching opportunities), vicarious experiences (observing effective teaching), verbal persuasion (encouragement from mentors and peers), and

physiological factors (emotional responses to teaching challenges) (Bandura, 1994). Similarly, past experiences of learning mathematics can significantly shape PSTs' self-efficacy, influencing their confidence and approach to teaching the subject.

Mathematical Wellbeing

Mathematical wellbeing encompasses fulfilling one's core values whilst teaching and learning mathematics, contributing to positive feelings (e.g., enjoyment) and functioning (e.g., meaning, belonging) (Hill et al., 2021). This construct provides an overarching framework for understanding how emotional experiences influence PSTs' capacity to flourish as mathematics teachers. While achievement emotions capture immediate responses and self-efficacy addresses confidence and beliefs, mathematical wellbeing represents flourishing in mathematics, finding purpose, belonging, and sustained engagement. Research identifies seven core values supporting mathematical wellbeing: accomplishments, cognitions, engagement, meaning, perseverance, positive emotions, and relationships (Hill et al., 2021; Hunter & Hill, 2024). Pedagogies grounded in these values create enabling environments where students engage, perform well, persist through challenges, and feel a sense of belonging. Understanding PSTs' emotional recollections reveals how their mathematical wellbeing was shaped, potentially influencing their future teaching. In this study, enjoyment, hope, and pride indicated positive wellbeing, while shame, anxiety, anger, and hopelessness represent barriers. Hill and Hunter (2024) found positive emotions decline sharply from primary to secondary school, highlighting the critical importance of emotional experiences in mathematics classrooms.

Pre-Service Teachers' Past Experiences Learning Mathematics

Through recognising how prior learning experiences shape beliefs, attitudes, self-efficacy, and wellbeing in mathematics, we identify factors that could influence PSTs' pedagogical choices for teaching mathematics. PSTs' beliefs about teaching mathematics are likely shaped by their own mathematical learning experiences. They may employ familiar teaching practices they have observed or ignore them if they see little value in them (Brady, 2012). Similarly, when PSTs recall supportive, patient and enthusiastic teachers, they are more likely to associate mathematics with positive emotions such as enjoyment. This extends to teacher educators as well, with research showing that positive and enthusiastic role models and teachers at the tertiary level play a key role in enhancing PSTs' self-efficacy for teaching mathematics (Küçükalioglu & Tuluk, 2021). Teacher educators can facilitate PSTs' development of strong self-efficacy in mathematics teaching by encouraging reflection on past learning experiences and modelling effective pedagogies.

Methodology

Following full ethical approval, a total of 266 PSTs from two Australian Universities across six campuses and four states or territories gave informed consent to participate in the study. Data were collected from 14 different cohorts of PSTs, who were enrolled in their first mathematics education pedagogy unit within an initial teacher education (ITE) degree for primary school teaching. All PSTs attended classes on campus and were asked to provide written responses to the following two prompts during their first class:

Describe an *unpleasant* experience you can remember associated with mathematics during your school years. Try to describe why you felt as you did.

Describe a *pleasant* experience you can remember associated with mathematics during your school years. Try to describe why you felt as you did.

The prompts were deliberately open-ended with 'school years' meaning they could recall experiences from primary or secondary school, or both. Minimal direction was provided by the respective tutors, other than explaining the purpose of the study and administering the

information sheet. PSTs were given 10-15 minutes to write their answers, but only responses provided by consenting participants were submitted to their respective tutors. Responses were anonymous and handwritten. Demographic details requested included first name (optional), gender, age range, and state or country of schooling. Initially, the data were inductively coded (Braun & Clarke, 2022), using NVivo to name categories and identify different themes. A codebook was developed, and a selection of 20 responses was coded by three authors. Discrepancies were discussed, some additional codes were added, and others were refined. Once reliability was established, the full dataset (n=266) was coded by the seventh author, with periodic reviews conducted by the first author to ensure accuracy and reliability. For this paper, all data were then deductively coded (Braun & Clarke, 2022), with responses coded to the eight categories of emotions as identified by Pekrun (2019).

Results

The results are reported in two parts: the frequency of codes and categories used to describe PSTs' written responses, and examples of positive and negative experiences.

Frequency of Categories of Experiences

When coding PSTs' pleasant and unpleasant experiences associated with learning mathematics at school, we were interested in analysing their written descriptions. Specifically, we focused on identifying the emotions associated with each description, categorising the recalled events as either positive or negative. Using Pekrun's (2006) achievement emotions framework, we conducted a frequency count of instances of each reference to the eight categories of emotions. Table 1 provides a brief description, a coding sample, and the frequency of responses that were coded positive (n=201) and negative (n=315). Some participants' responses were coded to more than one category (e.g., anger and hopelessness), and some responses were not coded to any of the eight categories.

Table 1

Emotion Categories and Frequency

Category	Description	Sample of coding	Frequency	Percentage*
Enjoyment	Happiness and delight	Hands on activities	132	50%
Shame	Guilt or embarrassment	Not know my times tables	110	41%
Anxiety	Unease or worry	Being put on the spot in front of the class	84	32%
Pride	A sense of accomplishment	Getting better with practice	62	23%
Anger	Displeasure or hostility	Frustrated and annoyed	56	21%
Hopelessness	Belief improvement is unlikely	Struggled to understand	38	14%
Boredom	Lack of interest or engagement	Doing boring worksheets	27	10%
Hope	Expectation of a positive outcome	Teacher cared about my learning	7	3%

* The percentage was calculated based on a total of 516 coded instances

Table 1 shows three positive emotions (enjoyment, hope, pride) and five negative emotions (anger, anxiety, boredom, hopelessness, shame) as reported by PSTs. Overall, enjoyment was the most frequent category for positive experiences (50%), and shame was the most frequent

category for negative experiences (41%). Negative emotions appeared more frequently than positive emotions, partly due to having two more categories. Additionally, the responses tended to be longer when recounting negative experiences.

Positive Experiences

Enjoyment was the most popular category (50%) for positive emotions. Examples included teachers who were encouraging and expressed pride in students' work, created caring environments, offered activities that enhanced understanding, avoided textbooks, and accommodated learning needs by providing multiple explanations from the teacher.

Teachers' dispositions were frequently noted:

Having a passionate mathematics teacher who encouraged discussions about mathematics with friends made the subject more engaging.

Knowing that the teacher genuinely loved mathematics inspired a deeper appreciation for the subject, helped make the subject enjoyable.

Enjoyment was sometimes associated with rewards such as:

If everyone in our class completed our times tables up to nine, we could have an ice cream party. This motivated me to do my best.

My teacher used monopoly money ... when you completed tasks/homework You could pay for things like lollies, moving seats, free time etc.

Interactive mathematics activities were also associated with enjoyment. Four PSTs remembered enjoying games, including when using technology, multiplication and group games, while another PST remembered enjoying learning times tables through songs.

Overall, enjoyment was associated with supportive teacher attitudes, creative and interactive learning experiences, and motivational strategies that made mathematics feel accessible, engaging, and rewarding.

While *hope* was not often explicitly mentioned (3%), some responses referred to enjoyment and implied feelings of hope. One student described themselves as scared of high school, but their Year 8 mathematics teacher "... *made me feel smart and included and that if I put my mind to it, I could do it.*"

Hope was associated with the expectation of positive outcomes, such as "... *the teacher believed in me.*"

PSTs, as students, also liked to feel they belonged:

.... I had a few teachers over the years who supported my curiosity about the subject and made me feel confident that even though I wasn't the smartest maths student, I still had value in their classroom.

Pride concerns a sense of accomplishment, and 62 (23%) recollected experiencing pride.

PSTs felt pride when receiving a high score on a mathematics test or knowing solutions that others did not.

I studied really hard and shocked my teacher with a 90%.

I met with my friends to discuss homework and felt proud when I knew something they didn't.

As with enjoyment, pride and hope can also be closely associated, as reported in the above responses.

Negative Experiences

Shame. Of the 266 unpleasant responses, 41% were classified as shame. They often included a direct reference to embarrassment, humiliation or feelings of guilt. The following illustrative comments reflect the types of experiences described:

When others understood the maths concepts, I felt ashamed and scared to ask questions.

When the teacher divided high-ability and low-ability learners, and I wasn't chosen for the high-ability group. I felt ashamed as I was usually chosen for the high-ability group.

In Year 9, I was moved to a lower maths class. I remember feeling embarrassed.

Anxiety. Perhaps not surprisingly, 32% recalled unpleasant experiences related to feelings of anxiety or worry. Many of these experiences were characterised by being singled out, having to perform or demonstrate in front of the class or when completing timed tests, especially in primary school. For example, students reported:

When we had to do timed times-table tests in Grade 3 ... the other students would finish ... yell out 'done' and it would increase my stress ... I would sit there crying rather than completing it.

Mathematics was also associated with anxiety in secondary school, especially with not understanding the content or feeling unsupported by the teacher. For example:

[Specialist Year 11] I struggled with understanding the content but was too afraid to ask the teacher. I was falling behind. I felt scared and worried ... I wanted to catch up, I didn't know where to start.

Anger. Responses classified as 'anger' stemmed from a sense of injustice and/or frustration at not receiving support or nurturing from the teacher in learning mathematics topics. For example, one student recounted an unpleasant experience in primary school when:

My teacher would only go over certain topics once, like fractions, and then not repeat the learning. I felt frustrated and annoyed and I felt disadvantaged due to her teaching.

Limited teaching approaches added to their frustration:

One of my teachers only taught one way ... it was very frustrating if you didn't get something.

Others experienced anger at themselves for not performing successfully on tasks:

Year 12 ... failing my maths class. I did my work homework, got tutored and studied before exams ... but would still fail each exam. I felt sad, stupid and annoyed and I hated maths more than ever.

Hopelessness. This was manifested in responses that indicated a belief that effort was futile and improvement unlikely. Responses such as "struggled with maths in primary school and never really understood it," with feelings of "being left behind" or told to "get a tutor because I'm not going to help you", removed the sense of hope of achieving success in mathematics. For many of these PSTs, it seemed that if they had received additional support rather than being 'disregarded', their learning experiences might have been more pleasant. Responses coded to hopelessness often included references or implications of anger. For example:

Struggling to grasp concepts of algebra, calculus in later levels. I felt it lost relevance for me as I couldn't see the point of it in real life context. I felt frustrated and dumb and I fell behind.

Boredom. Recounted experiences classified as 'boring' often included references to worksheets and textbooks. For example:

Throughout year 10 I was in a mathematics class with a teacher who did not explain the content well and always gave us boring worksheets to complete. I always went to class bored and left feeling I had learnt nothing.

During secondary school, the teaching was boring, disengaging and mostly rote learning with worksheets.

Most of my high school experience was using the textbook in class and just doing textbook questions rather than having more interesting and engaging tasks.

Discussion and Conclusions

This study was designed to explore PSTs' emotional responses to their mathematical learning experiences during their schooling. In total, 266 PSTs provided responses that were categorised as corresponding to eight emotions as identified in Pekrun's Achievement Emotions Framework (2019). Our analysis reveals that achievement emotions can serve as the foundation for understanding PSTs' mathematical experiences, which may, in turn, subsequently influence their self-efficacy beliefs and overall mathematical wellbeing. This imbalance suggests that

negative emotional experiences in primary and secondary mathematics classrooms may leave more enduring impressions than positive ones, reinforcing the need for targeted attention within teacher education for our PSTs.

Impacts on Self-Efficacy, Mathematical Wellbeing, and Teaching Practices

The prevalence of shame (41%) and anxiety (32%) aligns with previous studies on mathematics anxiety and low self-efficacy beliefs among PSTs (Jablon Stoehr, 2017; Hill & Hunter, 2024). The negative experiences described by participants, particularly being singled out, humiliated, or unable to grasp concepts despite effort, represent significant barriers to developing positive self-efficacy (Bandura, 1994). These negative achievement emotions directly undermine the development of self-efficacy, as PSTs internalise their emotional responses into beliefs about their mathematical capabilities. These findings support Lo's (2021) assertion that past experiences shape not only PSTs' engagement with their mathematical learning but also impact their self-efficacy for teaching mathematics. Our findings suggest that pedagogical approaches that PSTs encountered during their schooling, including the dominance of worksheets (identified in the boredom category), the lack of differentiated instruction (noted in the anger category), and the public nature of mathematical performance (highlighted in anxiety and shame categories) all contributed to negative emotional associations with mathematics. These instructional approaches not only created negative emotions but also failed to create enabling environments for mathematical wellbeing to emerge. The accumulation of negative achievement emotions and diminished self-efficacy ultimately compromises PSTs' mathematical wellbeing, limiting their capacity to find meaning, purpose, and enjoyment in mathematics teaching—essential elements for flourishing as educators. Consistent with Brady's (2012) findings, these experiences likely shaped the PSTs' views of mathematics education and may potentially influence their future teaching practices.

The Role of Positive Teacher-Student Relationships

A counterpoint to the dominance of negative emotions was found in the enjoyment category, comprising 50% of positive responses. These reflections frequently emphasised caring, supportive, and passionate mathematics teachers. These responses support Chalkiadaki's (2018) views that positive learning experiences are influenced by how teachers construct lessons and the teaching strategies they employ. Conversely, many negative experiences centred on feeling unsupported or even disregarded by teachers. The prominence of relationship-focused responses suggests that the quality of teacher-student interactions may be a powerful contributor to positive mathematical learning experiences and mathematical wellbeing (Hunter & Hill, 2024).

Implications for Teacher Education, Limitations, and Future Research

The experiences recounted by PSTs provide a valuable lens which likely shape their future teaching practices. By explicitly acknowledging and discussing PSTs' prior experiences with mathematics, teacher educators can help PSTs critically reflect on how their experiences are likely to influence their relationships with their future students. The findings also highlighted the importance of developing PSTs' awareness of the emotional dimensions of mathematics learning. Understanding how practices such as public performance, ability grouping, and limited teaching approaches can evoke shame, anxiety, and anger in students may help PSTs create more emotionally supportive classroom environments that foster positive mathematical wellbeing (Hill et al., 2021; Hunter & Hill, 2024).

While this study offers valuable insights into PSTs' mathematical experiences, it was limited in that only two open-ended questions provided a snapshot of PSTs' self-reported experiences. Future research might include follow-up focus groups or one-on-one interviews to gain deeper

insights into PSTs' experiences and their perceived impact on teaching intentions. Additionally, the study primarily focused on emotional recall rather than analysing how these experiences might translate into teaching practices. Future research could explore how beginning teachers' practices evolve, investigating whether they replicate or diverge from the positive and negative experiences they encountered as students.

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Promoting Spatial–Mathematical Connectivity (SMC) Through a 20-Week Spatial Reasoning Design Study in Grade 4

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A Spatial Reasoning Mathematics Program (SRMP) was implemented over 20 weeks with 28 mathematically-able 4th-graders as the second phase of a classroom design study. The SRMP embedded transformation skills in 28 interrelated mathematical tasks comprising 2-D/3-D relationships, geometric growing patterns and sequences, directionality, area and perimeter, and perspective-taking. Improved spatial reasoning was found across the Pattern and Structure Assessment-Early Mathematics (PASA-2), and the Pattern and Structure Assessment–Spatial (PASA-Sp) measures at post-SRMP. We observed ‘Spatial-Mathematical Connectivity’ (SMC) as bi-directional: spatial reasoning and conceptual understanding proved complementary.

The relationship between spatial reasoning skills and mathematics achievement has been well established across a range of studies from early childhood to secondary school (Hawes et al., 2022). Spatial skills have been found predictive of mathematics achievement, are malleable and can be developed from early childhood (Gilligan et al., 2020; Lowrie et al., 2017; Mix et al., 2021; Uttal et al., 2013). Studies on the association between early patterning and spatial reasoning skills, and between patterning and mathematics achievement, have also aimed to establish causal relationships and to inform pedagogical approaches (Mulligan et al., 2020a; Rittle-Johnson et al., 2019). However, studies in the elementary school years do not yet have sufficient scope and depth, or application at scale, to permit generalisation about the effects of different spatial-training programs on mathematics learning more broadly, or *how* and *why* transfer may occur (Harris et al., 2019; Lowrie et al., 2020; Young et al., 2018).

Theoretical Approach and Literature

The development of mathematical concepts has been shown to develop hand-in-hand with an Awareness of Mathematical Pattern and Structure (AMPS) which can emerge from or underlie mathematical concepts and relationships (Mulligan & Mitchelmore, 2009). In a sequence of interrelated intervention and design studies focused on ‘pattern and structure’ we described mathematical conceptual development as a complex, dynamic system of interconnected components. This system is fundamentally dependent on underlying patterns, spatial structures and interrelationships rather than on isolated numerical and computational skills (Mulligan et al., 2020a). For example, conceptual connections can be observed in the interrelationship between a pattern of squared numbers, its spatial representation and multiplicative structure. Adopting this theoretical view, we proposed that spatial structuring was integral to the development of AMPS and indeed, the development of number concepts may be primarily spatial in origin. Thus, our more recent investigations have developed pedagogical approaches to support the role of spatial structuring in conceptual connectivity. For this purpose, we coined the term ‘Spatial-Mathematical Connectivity (SMC)’.

A range of studies, including controlled intervention and classroom design studies have found that spatial reasoning skills can be trained under widely varying learning conditions and contexts (Davis et al., 2015; Harris et al., 2019; Hawes et al., 2017; Lowrie et al., 2021;

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Mulligan et al., 2020b). Indeed, a recent meta-analysis of 28 controlled studies found that spatial training improved mathematical understanding and performance across populations and ages and increased with age from 3 to 20 years. Transfer was observed under several conditions including duration and type of intervention, when assessment tasks were tailored to the training, and when particular resources and representational strategies were used. However, a more coherent understanding of how spatial training effects mathematics learning and achievement needs to be established and more theoretically-guided studies are recommended (Hawes et al., 2022, p. 1). Furthermore, systematic analyses support the prioritising of spatial skills in curricula, and call for sustainable, spatial reasoning programs in practice (Woolcott et al. 2020).

Studies of spatial reasoning in the elementary school years have often focused on specific spatial skills, connected to, but not necessarily applied to a range of mathematical concepts (Gillman et al., 2020; Lowrie & Logan, 2018). Intervention and correlational studies on spatial reasoning have been limited by assessment of mathematics performance using standardised tests or these have focused on isolated concepts. However, study of mathematical-spatial relationships is multi-faceted where differences in findings may be attributed to particular spatial concepts and different task types (Mix, 2019). For example, a study of 455 students from grades 4 through 9 found distinctions between item types with different spatial skills (Harris et al., 2019).

Background to the Design Study

The *Connecting Mathematics Learning through Spatial Reasoning Project* comprised a 2-year intervention study employing 61 students in Grades 3 through 4 (Mulligan et al., 2020b). A Spatial Reasoning Mathematics Program (SRMP) was developed and implemented to investigate student's spatial reasoning skills, spatial/mathematical interrelationships and the influence of the SRMP on mathematical performance (see Table 2). The intervention study found significant gains by the experimental group on spatially-oriented interview-based assessments post-SRMP. However, the study was limited in establishing *how* and *why* students improved in spatial reasoning, and why there was minimal gain on standardised measures of mathematics achievement. Subsequently, we focused our attention on in-depth study of student's grasp of interrelationships between particular spatial skills and mathematical concepts across a range of tasks through a classroom-based design study.

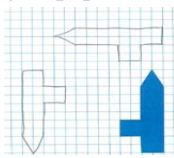
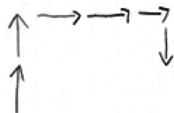
Our study aims to advance our understanding of the influence of particular spatial-mathematical interrelationships on problem solving in order to inform effective pedagogical approaches. In this paper we address one key research question: *How did a Spatial Reasoning Mathematics Program (SRMP) support students' Spatial Mathematical Connectivity (SMC)?*

Design and Method

Employing design-study principles (Bakker, 2018), a two-phase classroom design study of the SRMP was developed and implemented in Grade 4 at an independent school in Sydney, New South Wales (NSW), Australia. An entire cohort of 46 Grade 4 students participated in Phase 1 of the study, and a sub-group of 28 mathematically-able students (12 female and 16 male) were retained in Phase 2. Participants were aged between 8y10mo and 9y10m at Grade 4 entry. The study of these students afforded the analysis of SMC with mathematically-able students who had developed spatial and mathematical interrelationships.

Three assessments were utilised: standardised Progressive Achievement Tests of Mathematics, PATMaths4 (Stephanou & Lindsey, 2013); interview-based: Pattern and Structure Assessment [PASA-2] (Mulligan et al., 2020b); Pattern and Structure Assessment-Spatial [PASA-Sp] (rotation, reflection, equipartitioning, directionality, perspective-taking). Table 1 describes the PASA-Sp interview that assessed key spatial concepts: students explained and represented their solutions and described how they used 'mathematical ideas'.

Table 1*Pattern and Structure Assessment– Spatial (PASA-Sp) Items*

Item	Stimulus and response example	Item description
<i>Rotation & reflection directionality; translation</i>	An arrow-like shape placed on grid paper at middle RHS. 	Imagine the shape turned 90° left. Where would you place it. Why? Draw the shape. Imagine you repeated the turn; draw it. Explain the relationship between the original shape and the final one.
<i>Equipartitioning & congruence; equilateral, hexagonal structure</i>	A drawing of a regular hexagon and one equilateral triangle with the same side length. 	Imagine the small triangle placed inside the hexagon. How many will fit exactly into the hexagon without overlapping? Draw these in the hexagon. Explain how you visualised and drew the parts. Did you use other mathematical ideas? If so, how?
<i>Directionality & rotation; navigation and mapping</i>	Blank A4 or A3 paper (or graph paper or digital device (iPad)). 	Stand in position. Visualise walking two steps forward, turn right; walk one step forward. Take that walk. Draw a diagram. Imagine you repeated this walk in a different direction. Draw a diagram to show your steps.
<i>Perspective-Taking & orientation; 3-D/2-D/Representation</i>	A model 'building' of four cubes. 	Draw what you see [front on]. Draw what you think I can see when I look from above. What direction are the blocks facing? If I turned the blocks what would you see? Is it the same or different?

Spatial Reasoning Mathematical Program (SRMP) Design and Implementation

The SRMP integrated patterns and spatial structures with a range of mathematical concepts categorised as six components (see Table 2). Transformation processes (rotation, reflection, spatial orientation and spatial visualisation) were embedded and interrelated in the components informed by recent intervention studies (Hawes et al. 2017; Lowrie et al. 2017; Lowrie et al., 2021) and pedagogies focused on spatialisation of mathematics learning (e.g., Davis et al., 2015; Lehrer & Schauble, 2024). The development of the SRMP also reflected patterning and spatial tasks embedded in our earlier interventions (Mulligan et al., 2020a; Mulligan et al., 2020b). Moreover, SRMP tasks were designed to elicit SMC within and between components.

A 'Modelling-Representing-Visualising-Generalising-Sustaining' pedagogical model was employed based on the Pattern and Structure Mathematics Awareness Program (PASMAT) (Mulligan et al., 2020a). This model shares similarities with the Experience-Language-Pictorial-Symbolic-Application (ELPSA) framework (Lowrie et al. 2021). Both approaches emphasise visualisation and representational (pictorial) processes and symbolic application of concepts. In the SRMP, students were guided to focus on patterns and relationships between models and representations, use visualisation and visual memory, and to make simple generalisations. The use of modelling with manipulatives, including the use of robotic toys was central to the pedagogical approach. Establishing interrelationships between geometric models and number patterns was featured in several components.

Table 2*Components of the Spatial Reasoning Mathematics Program (SRMP)*

Component	Sub-component	Sample tasks
2-D and 3-D relations	Nets of boxes, pyramids/ solids	Create nets for 3-D shapes using grid paper. Use polydron to check nets. Identify patterns in the formation of nets. Visualise the net of a cube given three different views.
Growing patterns	Square patterns	Count and describe square number pattern of a single face of an expanding cube pattern.
	Cubic patterns	Construct and describe number pattern of an expanding cube. Predict and identify patterns using different coloured blocks.
	Graphing functions	Graph functions to represent repeated linear and growing patterns.
Fibonacci sequences	Fibonacci areas and spirals	Use a Fibonacci sequence to construct adjacent squares on a grid, form a spiral, and mirror image.
Directionality	Bee-Bot paths	Develop and record a coding system to describe movements
Shape and perimeter	Rectangles with a fixed perimeter	Use the Bee-Bot to determine all possible rectangles with a perimeter of 8, 10 and 12 steps.
	Polygons with fixed perimeter	Create as many polygons as possible with a perimeter of 12 units using right-angles. Check using geoboards.
Perspective-taking	Visualising different views	Complete the missing views if front and left views are given.
	Drawing top views	Draw the top view of complex models.
	Drawing different views	Draw a 2-D representation of an L-shaped block from four different perspectives.

Procedures and Analyses

The ‘teacher as researcher’ (author 2) collaborated with the classroom teacher to review, implement, evaluate and refine the SRMP components and tasks, and contributed to the collection of assessment data, classroom observations and interpretation of student responses. Twenty lessons were implemented over two sequential 10-week school terms in a cyclical process of planning, teaching, review and refinement. Lesson duration was between 50 and 65 minutes. The SRMP lessons replaced about 25% of the regular Grade 4 mathematics program, by substituting and extending Mathematics syllabus content on Patterns and Algebra, Geometry and Measurement. The research team provided guidance on lesson scope and progression, although the teaching dyad enjoyed considerable autonomy in determining how the SRMP lessons were scaffolded and aligned with individuals’ learning. Each concept was explored in depth over multiple lessons, with opportunities provided for students to discuss, reflect, self-correct and provide feedback to peers and teachers. In concert, the teachers ensured that the lessons met syllabus requirements and school programming and reporting.

The assessment data: PATMaths4 (28 tests), PASA-2 (28 x 16 items) and PASA-Sp (28 x 4 items) were scored and coded by three members of the research team and interrater reliability (0.91) obtained. PASA-2 and PASA-Sp responses were coded as one of five increasing levels of spatial structuring (see Mulligan et al., 2020a). SRMP classroom data for each lesson were compiled systematically, including student representations, videos of task engagement, observation notes, teacher and student reflections and self-assessments. Transcripts were consulted to validate the coding and to interpret spatial reasoning. Student representations and digital recordings of problem-solving processes were examined for indicators of SMC.

Results and Discussion

We present a summary of findings from the assessment data and illustrations of how students used spatial reasoning and SMC to solve the SRMP tasks.

Students' PATMaths4 mean scale score of 138.2 pre-SRMP approximated the 90th percentile. While there was an increase in performance for students in the 80-90th percentile, PATMaths4 scores remained relatively consistent pre-post-SRMP (post-mean scaled score of 145.6) which may be attributed to the high performance at pre-SRMP. PATMaths4 did not challenge the students sufficiently, or include suitable items, to show gains in performance.

PASA-2 data showed a moderate increase in total scores at pre-post-SRMP (mean scores 56 (pre) to 71 (post) from a total score of 80). However, there was marked growth in spatial reasoning post-PASA-2: responses coded at an advanced level, requiring a range of complex spatial skills and were found for more than 12 of 16 items. For example, tasks involved interpreting and visualising arrays and grids and extending rotational growing patterns.

The PASA-Sp assessment focused on four groups of spatial concepts (see Table 1). Responses were assigned a score of 4 out of 5 (level 4 spatial structuring) and 5 out of 5 (level 5 advanced spatial structuring) based on prior analyses (Mulligan et al., 2020a). Mean scores increased from 16 to 19 out of a possible 20, showing a gain post -SRMP, but more importantly growth from well-developed spatial concepts to more advanced spatial structuring and reasoning. Analyses of spatial skills and SMC from the PASA-Sp data are not reported here.

Illustrations of Spatial Mathematical Connectivity (SMC)

The comparative analysis of data sources (video, student explanations and representations, and teacher observations), showed that students solved the SRMP tasks by utilising spatial and conceptual resources, but student's solutions and explanations did not necessarily reveal SMC. However, we can infer that SMC may have been internalised through spatial-visualisation processes and was not made overt in the solution process. Analysis of responses by task, and of individual profiles of response, showed that while there were some common patterns for particular tasks and for students of varying ability, individuals made connections between concepts by using spatial resources in different ways: the type, depth and strength of SMC varied and may have been bi-directional, i.e., spatial-reasoning skills guided mathematical conceptual understanding, or vice versa. A key finding was that visualisation skills were inextricably linked to SMC and problem solutions. Mental rotation skills were found common to solutions across tasks and were often combined with pattern recognition but applied in different ways (e.g., nets of solids, growing patterns and perspective-taking).

We provide examples of three students' responses to Growing Cubic Patterns, Fibonacci Sequences and Directionality tasks respectively. Figure 1 shows the student first constructed a cuboid of 2 x 2 x 2 and extended it using a pattern of three different-coloured blocks. One aim of the task was to observe how the student visualised and interpreted the edges and faces of the cuboid. Another aspect was observing the cuboid in different orientations and from different perspectives. Observing the number and pattern of one face of the cuboid was the initial step in SMC. The student completed a table of data showing the numerical relationships between the different-coloured blocks from the front face (Figure 1). Some students were able to visualise, generalise and draw the pattern for each face of the cube. The following excerpt exemplifies the teacher's (T) and student's (S) mathematical and spatial reasoning.

T: Why doesn't the number of green cubes (on the corners) change?

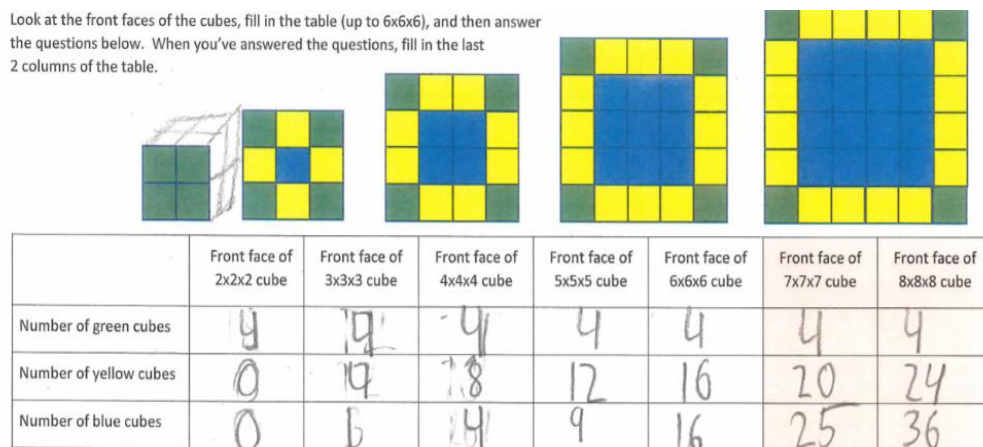
S: Because the green squares always touch a corner; there's always four corners, front-back, top-bottom...you can't count the corners twice when you work out the number of cubes.

T: How many yellow squares do you see on *one edge* of each of the 2x2x2 to 6x6x6 cuboids? Does it make a pattern? How? Describe it.

S: It's growing like double but then its 3, then 4 so it's adding one each time, but you have to times it by 4 sides to get all of the yellow ones on the front face each time...I think this is same on the other faces.

Figure 1

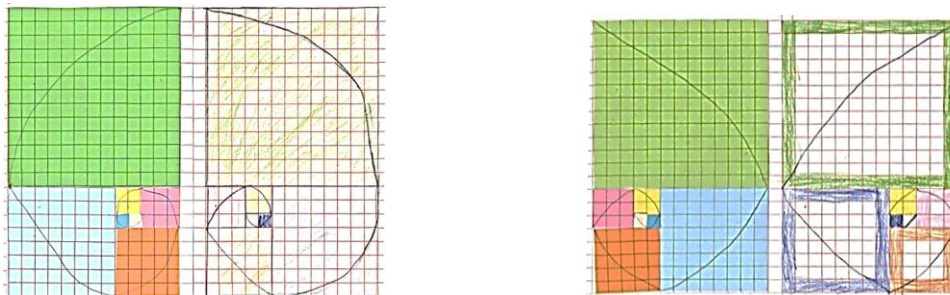
Number Pattern Created by Identifying Colours of the Growing Cuboid Front Face



The Fibonacci task initially depended on students' prior observations of visual examples and identification of the recursive number pattern and; the ability to extend the pattern to visualising a 2-D area model linked to the number pattern (squared).

Figures 2a and 2b

Mirror Image Representations of a Fibonacci Area Model and Spiral

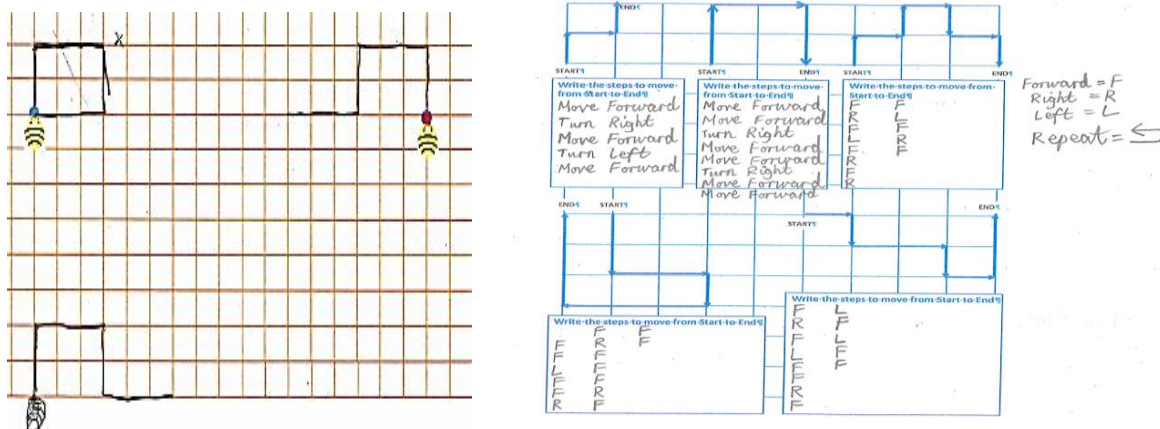


The tasks were scaffolded through the alignment of coloured squares, visualisation and drawing of the spiral freehand on grid paper and then creating a mirror image; all requiring spatial reasoning, and in this case, symmetry (Figure 2a). Some students used SMC to represent perimeter/area relations (Figure 2b). In all responses, students demonstrated and explained SMC of the sequence and the area model but needed support to visualise and connect the spiral.

The tasks on Directionality involved the use of Bee-Bots as iterable units that afforded students the opportunity to predict, visualise and form area/perimeter relationships (determine all possible rectangles with a perimeter of 8, 10 and 12 steps). This task focused on SMC particularly in the connection between dynamic movement and rotation, and visualising the shape and area created by the perimeter. In another task students' informal descriptions of Bee-Bot directions were developed, symbolised and represented as a coding system (Figures 3a, 3b). Students' then explored mirror patterns, maze designs and navigational strategies, and estimated and measured distance and area in units, numerically, using Bee-Bot steps. SMC was essential in the solution process. Some students solved the tasks without grid paper and relied on their visualisation skills and the tangible modelling with the Bee-Bot.

Figures 3a and 3b

Representations of Bee-Bot Pathways, and with Coding System



Implications and Conclusions

Our findings support current research that indicates spatial intervention programs can support spatial-reasoning skills and mathematical understanding (e.g., Hawes et al., 2017; Lowrie et al., 2021). We found that SMC may be bi-directional, meaning that strong spatial-reasoning skills support mathematical conceptual understanding and vice versa, and advantages problem-solving solution processes. Secondary analysis of data for measurable differences in SMC, and patterns in responses for the cohort and individuals, and by task type, is necessary to provide a reliable measure of, and developmental indicators of SMC. The SRMP pedagogical approach supported the transition from modelling to visualisation essential for mental rotation; establishing an optimal pedagogical approach is challenging in classroom design studies but can provide new insights into the mechanisms of SMC.

Despite the valuable insights gained into the complexities of SMC we were not able to draw any conclusions about task type or the sequencing of SRMP components that would promote growth and transfer. Employing a broader range of spatial-mathematical concepts may yield more conclusive findings. Nevertheless, we gained more evidence into how to promote SMC with elementary students with the view that SMC must also influence transfer and the robustness of that transfer (Harris et al., 2019; Lowrie et al., 2023; Resnick & Adams, 2024).

Other issues remain about how research findings are articulated, accepted, and applied in practice, and the role of teacher mathematical content knowledge. The implications for pedagogy, curricula and assessment are to emphasise and prioritise spatial reasoning and embed spatially-oriented programs within practice. More conceptually-advanced and explicit descriptors of spatial/mathematical interrelationships would be needed to support pedagogy. Based on the extant literature and our prior and current findings, we purport that both a reliable measure, and micro-genetic analysis of SMC in longitudinal studies, and at scale, may establish a more coherent, and complementary, interdisciplinary view of SMC.

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Introducing Dialogic Pedagogy in Early Years Mathematics: Contrasting Authoritative and Persuasive Discourses

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The study presented in this research report aims to compare and contrast the pedagogical approaches in early years mathematics. We present transcript data from a baseline and end lesson from one teacher as part of a broader study to introduce dialogic pedagogy and collaborative group work. The pedagogical approaches are analysed in relation to Bakhtin's notions of authoritative and internally persuasive discourses and on the teachers and students' use of language. We highlight how language can balance authoritative and persuasive discourses and influence the way students collaborate and assimilate the mathematics.

Dialogic pedagogy is loosely defined as teaching that prioritises talk and interaction to engage students, stimulate thinking and advance understanding (Alexander, 2004). There is now a substantial body of literature in this field generally and in mathematics specifically (e.g., Mercer & Sams, 2006). There is also evidence that there is a relationship between the nature of classroom discourse and student outcomes (Howe, et. al, 2019) with high levels of elaboration and querying, typical of dialogic pedagogy, being positively associated with test scores. Despite this maturing knowledge, adoption of dialogic approaches in mathematics classrooms remains limited (Kibler et al., 2020) and more transmissive approaches related to the fixed authority of the teacher are often more prevalent. Teachers' concerns focus on their students' access to content knowledge and meeting curriculum objectives. As such a more authoritative approach may be adopted to transmit knowledge. A question arises how an authoritative teaching approach compares to a dialogic teaching approach that prioritises talk and interaction in supporting students' access to the intended learning. Such a question is of particular concern in early years mathematics classrooms where young students are less likely to engage in talk and collaboration.

In this paper, we focus on Bakhtinian notions of dialogicity, that refer to authoritative and persuasive discourses, and on Pimm's (1987) work that positions mathematics as a language. We examine the pedagogy of one teacher, Cathy and her Grade 1 and 2 (6- and 7-years) students, from two lessons that were part of the data collected for a larger study to support talk in early years mathematics classrooms. Our research questions are:

- How do the two lessons compare in relation to authoritative and persuasive discourses?
- How is the teacher language reflected in the two discourses?
- In what way do the discourses in the two lessons influence student collaboration in group work?

Literature Review

Despite the maturing knowledge regarding dialogic pedagogy and the evidence that collaborative dialogue with students in small groups suggests academic achievement, longer term retention and application of critical thinking (Mercer & Sams, 2006), the adoption of such practices in mathematics classrooms remains limited (Kibler et al., 2020; Moser et al., 2022).

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In the literature review, we consider how dialogic pedagogy has been defined and then review studies that have investigated the introduction of dialogic pedagogy in mathematics classrooms.

Dialogic pedagogy has been defined by several theorists, including Alexander (2004) who characterised it as teaching that focuses on talk to engage student interest, stimulate thinking, and advance understanding. As such, Alexander positioned dialogic pedagogy as shared enquiry and contrasted this to the teacher-centred dominance of Initial-Response-Evaluation (IRE). Other theorists, such as Wegerif (2006), have gone beyond the notion of shared enquiry to define dialogic pedagogy as a multivocal discourse where meaning emerges within an open dialogue. Common to these two definitions is that dialogic pedagogy is multivocal, emphasising inquiry and problem-posing. As such, knowledge is treated ambiguously, creating space for interaction and rearticulation of thoughts with others.

In these regards, dialogic pedagogy is typically countered in opposition to transmissive, univocal pedagogies where knowledge is presented as fixed and presented in an authoritative manner either by the teacher and/or textbook. Mathematics teaching is often seen as objective, relating to established facts and processes (Artigue & Blomhøj, 2013), and a teacher's focus on objectives may defer to authority so that students gain the correct learning for the lesson (Murphy, et al., 2021). As such, teachers may not be prepared to tolerate students' engagement with ambiguity that is more evident in problem-solving or inquiry-based approaches.

Hence, tensions arise between teacher-centred dominance and the student-centred inquiry approaches in mathematics classrooms. Studies that have investigated the use of professional development in supporting dialogic pedagogy have uncovered such tensions in relation to the teacher's role in scaffolding learning (Klemp, 2020; Murphy et al., 2021) and the decentralisation of authority (Campbell & Teo, 2022). Other tensions relate to the positioning of mathematics (Ng, et al., 2021) and the need to attend to precision (Otten et al., 2015). These tensions arise despite the evidence from Howe et al. (2019) that dialogic pedagogy supports student performance. As such, the debate relating to authority structures and the need to achieve a balance, incorporating both dialogue and univocal discourses continues.

Theoretical Perspectives

In relation to the definitions of dialogic pedagogy above, we refer to Wegerif's theory of multivocal discourse and open dialogue where knowledge is treated ambiguously through inquiry and problem-posing, creating space for interaction and rearticulation of thoughts with others. Wegerif (2011) further proposed that dialogue has both an outside and an inside nature and these relate to Bakhtinian (1981) notions of authoritative and internally persuasive discourses. Bakhtin (1981) referred to an authoritative discourse as an inflexible kind of assimilation. The authoritative voice is seen as outside and one that forces acceptance or rejection. It presents a single, unyielding perspective, without acknowledging alternative viewpoints or engaging in meaningful dialogue. In the case of education, the authoritative voice would relate to learning by rote, recitation or following a set inflexible method unquestionably.

Wegerif contrasts the authoritative voice with Bakhtin's notion of an internally persuasive discourse as one that "enters into the realm of my own words and changes them from within" (Wegerif, 2011, p. 181). The discourse is "half ours and half-someone else's" (Bakhtin, 1981, p. 345). A key tenet of dialogic theory relevant in teaching is how students learn to think (Wegerif, 2011). In this regard, education is about generating new meaning from others through social language (written or spoken), and this requires dialogue to be both inside and outside. In mathematics education, Williams and Ryan (2020) referred to the internally persuasive nature of dialogised discourse. Whilst there is an endpoint in the learning, there are opportunities for dialogue in the discourse of both teachers and students' multiple subjectivities.

In sum, Wegerif's (2011) perspectives distinguish between the authoritative voice and the persuasive voice. The authoritative voice directs you to do something and comes from the outside whereas a persuasive voice enters the inside. Wegerif viewed dialogue as the opening up of new spaces of meaning that include participants within it, that is as a "dynamic continuous emergence of meaning" (p.4) rather than a static space. Such a space allows for the persuasive voice. In this regard, Wegerif posited that, if education is about generating new meaning from others, then education is about being both inside and outside. In this paper, we refer to this position in relation to the language and discourse used by Cathy in her two lessons.

The Study

Research Design

This paper reports on data collected from the first year of a large-scale study which investigated talk in early years mathematics teaching. The first year of the study involved a school-based participatory design (Hennessy, 2014) with four teachers engaged in four cycles of teacher-researcher workshops. Video data of mathematics lessons were collected as baseline and endpoint data as well as interim video data during the four cycles. The first teacher-researcher workshop focused on strategies to promote collaboration and talk in small groups, based on Mercer and Sams' (2006) strategies to promote talk. In consultation with the research team, the teachers were encouraged to try out resources in relation to problem solving tasks. Subsequent workshops then reviewed the teachers' practice including use of resources and tasks based on the videoed lessons and identified the next steps forward. We affirm that the research participants and the children's legal guardians provided informed consent for publication of the non-identifiable data.

In this paper, we focus on Cathy's lessons from the baseline data and the endpoint data collection. Cathy's lessons were selected as they provided a consistent data set with evidence of her implementation strategies to promote student talk and collaboration. In this short paper, we include transcript extracts of Cathy's whole class introduction from two lessons, one before the design intervention and one at the end of the four cycles. We then include the transcripts from one pair of students in each lesson as they collaborate on an independent task. These data were deliberately chosen to illustrate differences in the discourse between the baseline and the end lesson and to consider the impact of the teacher discourse on the students' paired work.

The transcripts were analysed qualitatively to explore Cathy's pedagogical approach and how the students' interaction reflected the approach. Mercer's (2005) sociocultural discourse analysis (SCDA) is used to understand how the teacher used spoken language as a tool for introducing the mathematics content and how the students used language to share their understanding in their paired group work. Transcripts are presented and, as part of SCDA, analysis is provided as commentaries to determine language uses in the discourse. We then interpret the use of language in relation to the teacher's authoritative and persuasive discourses and to the shared thinking in the groups. To distinguish between the discourses, we refer to language-based perspectives. Of particular concern is Pimm's (1987) focus on use of the pronoun *we* as a linguistic practice and how it relates to either an authoritative or persuasive discourse. For example, a teacher stating "in mathematics we..." is referring to the authority of mathematics, whereas a teacher stating "we are going to explore..." suggests a shared endeavour. Another concern is the teacher and student use of modality, that is the expression of certainty, possibility, judgement or request (e.g., we might..., you could..., we should). This use is contrasted with the active verbs (e.g., we are going to..., we know that...).

The Baseline Lesson

In the baseline lesson, Cathy intended to build on students' existing knowledge of part-whole thinking and number bonds to ten and introduced bridging ten as a mental strategy. In the beginning of the lesson, Cathy revised number bonds to ten. She then introduced the bridging ten strategy, first using the numerical example $9 + 5$, and then modelling the strategy using ten frames. These examples were displayed on the interactive screen at the front of the class with students on the carpet.

Cathy : We are learning to use an addition strategy called bridging ten. We can use this strategy whenever we are adding two numbers together when the answer is greater than 10. We start by adding on to make up ten and then we add the left-over parts. Let's have a look. So here is a sum. (Displays $9 + 5$.) What is the sum?

S1 : Nine add five.

Cathy : Good boy, nine add five. Is that answer going to be bigger than ten?

Several students: Yes.

Cathy : How do we know that?

S2 : Because it's fourteen.

Cathy : I don't want the answer. That's not what I am looking for. How do we know it is going to be bigger than ten?

S3 : Nine like one more is ten

Cathy : So, there's only one more and we've got ten, haven't we?

In this introduction, Cathy used the pronoun 'we' several times, mostly to denote that, as a class, we are doing or knowing a piece of mathematics. As a class, the students will follow the mathematics and will be directed to a known strategy that is going to be taught. Active verbs were used.

Next, Cathy referred to the Little Thinking Man (a thinking emoji displayed on the screen) saying that the Little Thinking Man knows that nine and one is ten because he knows his ten facts and that the man then knows that four are left over. She then asks where the four left over came from. Students gave various responses: "From out of pocket; from fourteen; nowhere." To further support the students, Cathy wrote $9 + 5$ on the board and then $9 + 1$ underneath. She pointed to the 5 and 1. One student then answered that the four is from the five.

Cathy : From the 5, because we only added one and we want to add five. So, the difference between five and one is four. Then we know we've got ten because nine and one is ten. So, we have to do ten and four more is... Do we know what it is?

Several students: Fourteen.

The Little Thinking Man could refer to an outside voice, representing an authority in how the mathematics is done. Cathy does then open the discourse in asking how the man knows that four are left over. Student responses seemed random, and it is not until Cathy presented the sums and directly pointed to the numbers that one student provided the required answer. Cathy then used 'we' to indicate 'what we do' or even 'what we have to do' suggesting a requirement. Next, Cathy displayed two ten frame images for the same $9 + 5$ problem with nine yellow counters on one frame and five blue counters on the other frame.

Cathy : How many have we got in the yellow one?

S5 : Nine

Cathy : How many have we got in the blue one?

S6 : Five

Cathy : So, if we want to add nine and five together our bridging ten strategy shows us we are going to make ten first. What would we do to make ten? Would we move the yellow ones into the blue one or the blue ones into the yellow one? Which would be the most efficient?

A student suggested moving the yellow ones and Cathy used the response to consider if this would be more or less efficient. Students agreed that moving the blue would be more efficient. When modelling with the ten frames, Cathy used ‘we’ to indicate what is evident in the mathematics. Whilst she used modality “What would we do...? Which would be more efficient?” she was eliciting a required response. She then continued to use ‘we’ in modelling use of the counters.

Cathy : We’re going to take one of those blue counters into the ten frames. So what is this part of the sum?
(Points to the yellow frame with the one blue counter).

S7 : Ten

Cathy : But what’s the sum. Nine and...?

S8 : Four

S9 : It’s fourteen

Cathy : I’m not up to that step yet.

S10 : Nine and five

S11 : Nine and one

Cathy : Yes, nine yellow and one blue. We now know we’ve got ten and our more. Ten and four equals fourteen.

In this last step, Cathy again required specific responses from the students relating to the strategy. Her use of ‘we’ referred to the taught strategy, but she also used the singular personal pronoun in referring to her process. Active verbs were used throughout.

The students then worked in pairs each using their own worksheet and supply of yellow and red counters. The worksheet was printed with two ten frames and addition problems within twenty. The transcript from one pair of students, P1 and P2 is presented. They were looking at the problem $9 + 4$.

P1 : So, if we move one from here (moves one from four) that would be ten and three.

P2 : No, it would be nine and one. (Counts out in the nine red and one yellow counter in ones).

P1 : We don’t have nine add one. (Looks at the problems on the worksheet and writes the answer 10 next to $9 + 4$ on her worksheet).

P1 and P2 attempted to follow the strategy by moving counters to fill the ten frames for the sum $9 + 4$. Their use of “If we...” suggested a modality in trying out the strategy. They then referred to a quandary (“we don’t have...”) suggesting they were confused how to record the intermediate step, $9 + 1 = 10$, on the worksheet provided by the teacher.

P2 : Ok, so what does it equal? (Counts out all counters in ones to get thirteen.) No, it equals thirteen. Ten and three.

P1 : What does it equal again?

P2 : I’m not going to tell you.

P1 : Tell me

P2 : Let me do it first. (Takes pencil from P1 and writes 13 as the answer to $9 + 4$ on his worksheet.)

P1 : Oh, thirteen. (She rubs out the 10 and writes 13 on her worksheet).

Despite having the ten frame P2 used count-all strategies to determine the answer was thirteen. The students’ dialogue also suggested disputation. The use of pronouns shifted to ‘it’ in referring to the problem and to the singular personal pronoun “I”. So, whilst students were directing each other to the problem their thinking was individual.

The End Point Lesson

The focus of the endpoint lesson was partitive division. The lesson was based on the book *The Doorbell Rang* (Hutchins, 1989). Images from the book were displayed on the interactive screen. Cathy started the lesson by reading the first pages where the mother presents a plate of

cookies to the two children. Cathy then read the next page, “That’s six each said Sam and Victoria... and then the doorbell rang”. Cathy had prepared dynamic images of plates and cookies to model sharing the cookies for the students and showed an image of an array of twelve cookies (four rows by three columns) on a plate.

Cathy : How will we share six each?

S12 : Halves

Cathy : We’re going to halve them that is exactly right because there are two halves.

S13 : Two equal parts

Cathy : Yes, two equal shares because we want to be fair.

Cathy : How might you go about doing that?

S14 : You put a line in the middle.

S15 : You could put a line straight through the middle to split them in half. We’ve been doing that.

Cathy : Think about what you would do if you were sharing lollies with your friends.

S16 : Give them halves.

Cathy : Yes, you are going to halve them but think about how you are going to halve them? How would you make sure they are a fair share?

Cathy’s used the pronoun ‘we’ authoritatively (“we are going to...”) and in the phrase “we want to be fair” it is possible she meant “we need to be fair”. She then shifts to using the pronoun ‘you’ and modality, “how might you ...?” When the students responded they also use the pronoun you and modality.

Cathy : Let’s have a look at how I would do it. (Cathy moves one cookie at a time to each plate on the display board? and randomly positions them on two plates.)

Cathy : Am I sharing fairly?

Collectively students respond yes.

Cathy : Was that an efficient way to make sure that Sam and Victoria had the same amount?

Students murmur yes and no.

Cathy : If you have a better way keep it in your head because, guess what, you’re going to do exactly the same activity.

Cathy had used the pronoun ‘I’ indicating her method for equally sharing the cookies. She then asked students to consider if they can think of a better way. Her reference is to their method rather than a prescribed method.

The students then worked in pairs. They were given two paper plates and 12 cookie picture cards to use between the pair. Their task was to find a way to share the cookies equally onto the two-plate remembering that ‘we want to be fair.’ In the following example, P3 and P4 were working together. P3 placed 12 cookies in the array modelled by the teacher on the carpet in front of him.

P3 : So, I would have that one, that one, that one, that one, that one and that one. (He points to six cookies - two in each of the first three rows). So, these can come out the way...and put them on my plate. (He removes the third column of four cookies) and places the four cookies on the plate.

P4 : You know what I would do...

P3 : Plus, these two. (He takes the two from the bottom row to add to his plate.) And you can have six. He picks up the 6 remaining cookies and gives to P4 who places the six cookies on his plate).

P3 used his own strategy and counted in ones but then he took one column of four and the additional two. P3 used the modal phrase “I would...” providing a suggestion. P4 similarly used modality to provide his own suggestion. The dialogue then followed as the students checked.

P4 : Put them like this to make it fair. (He arranges his cookies on a plate to show two rows of two and then two additional either side.)

- P3 : (Continues to rearrange the cookies on his plate.) I want to make sure there are no cookie shortages. (He rearranges his cookies into a two by three array.)
- P4 : (Pointing at his plate.) You can do it like this.
- P4 : Let's count how many each have. One, two, three, four, five six. (P4 counts his own cookies, then they then both count P3's cookies together.)
- P4 : Now that's fair.

In this exchange, P3 and P4 continued to rearrange the cookies to help to determine if they had six each. P4 used an active verb "Put them like this..." but he also used modality in suggesting "You can do it like this...". They arrive a shared view that the arrangement of cookies was fair.

Discussion and Concluding Remarks

In the baseline lesson, Cathy broke the bridging ten strategy into small steps with explicit modelling. She referred to the students' knowledge of number facts and connected the numerical model to the use of counters with the ten frames. Her use of language suggested an authoritative discourse. The pronoun 'we' referred to mathematics in an authoritative manner and Cathy predominantly used active verbs. Furthermore, she referred to the Little Thinking Man as an authority as the way to think through the problem. Apart from when a student suggested moving the yellow counters, she did not attempt to acknowledge other thinking. Even though several students already knew the answer, her intention was to model the strategy so that students would independently copy the mathematics as directed. There was some use of modality in the paired student work but there was also evidence of disputation. However, there was limited evidence of any shared understanding, and it appeared that the students were struggling to assimilate the authoritative discourse.

In the endpoint lesson, Cathy set a context that was accessible to the students. She used modality in asking students how they might share the cookies. This shift related to how they could do the mathematics. Whilst there was a requirement to be fair and share equally, students were encouraged to explore their methods. When they worked in a pair, the students rearranged the cookies into arrays, coming to a shared agreement that it was fair. The students emulated modality in their language, suggesting that students' assimilation was both outside and inside. Whilst there was authority in the mathematics, the use of language within the context suggested a persuasive discourse.

In the baseline lesson, Cathy maintained an authoritative voice, often characterised by use of the pronoun 'we' in appealing to the authority of the mathematics in modelling the bridging ten strategy. The language of the students suggested they were focused on their own thinking in an attempt to assimilate the outside discourse. In the endpoint lesson, Cathy's language was persuasive in inviting students to investigate partitive division. The language of the students as they worked together suggested they were engaged in an intentional act, understanding what the other intended (Gallagher, 2012). Hence, there was a shared understanding or "insideness" as they worked on the task by coming together to check equal sharing with the potential to encounter and incorporate change (Wegerif, 2011). Dialogue was both inside and outside and the thinking became half the students' own and half the others.

The data in this paper is limited to only two lessons with one teacher so claims are not generalisable. However, the data illustrate the potential for contrasting teacher and student discourses within mathematics classrooms. Further analysis of teacher and student talk that focuses on language use in authoritative and persuasive discourses could help reveal some of the nuances associated with classroom discourse and student performance. Nevertheless, a key point to make is that, if we are to balance teaching and then teachers could be encouraged to incorporate both dialogic and univocal discourses in relation to persuasive and the authoritative

voices. Instruction can be seen as both outside and inside. In this way, pedagogy can be both persuasive and authoritative in maintaining student-centred approaches that focus on skills and knowledge. Such a balance could move away from the dichotomous perspective of authoritative explicit teaching or dialogic problem-solving (Otten et al., 2015).

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Analysis of Common Errors in Simplifying Algebraic Problems among Grade 10 Learners

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This paper analyses learners' common errors in simplifying algebraic problems. 102 Grade 10 learners from three rural schools in South Africa participated in the study. Following a quantitative approach, content analysis of learners' responses to algebraic tests revealed that while learners commit several errors in algebraic problems, encoding and transformation errors were the most prominent. These errors are mainly due to the inability to utilise correct methods, misapplication and overlooking algebraic rules, and failure to provide final answers to problems. The study highlights the need for remedial intervention, which has implications for improving mathematics teaching and learning.

Algebra is a discipline characterised by its abstract nature and governed by a set of rules, principles, and theories. However, errors and misconceptions arise when these rules, principles, and theories are disregarded or improperly implemented. In the South Africa's Further Education and Training phase (Grade 10 - 12), solving algebraic expressions and equations typically requires the understanding and integration of various mathematical concepts, including equations, factorisation, division, and exponents (Baidoo, 2019). As such, it is important to recognise that understanding algebra is beneficial for simplifying and solving mathematical problems in various subjects, including functions, calculus, trigonometry, financial mathematics, and others (Luneta & Makonye, 2010). This is because proficiency in algebra is essential for achieving success in higher-level mathematics.

Notwithstanding the South Africa National Diagnostic Report (2022) stating that algebra and equations make up 30% of the curricula for Grade 10, 25% for Grade 11, and 17% for Grade 12, learners exhibit deficiency in solving algebra. In fact, Grade 10-12 learners sometimes struggle to demonstrate fundamental and elementary algebraic skills that should have been gained in the early years of education, resulting in difficulties responding to algebraic problems (National Diagnostic Report, 2022). Similarly, researchers across several countries have reported that learners have challenges in comprehending the simplification of algebraic problems (Adeniji & Baker, 2023; Luneta & Makonye, 2010). This could be due to the requirement of performing multiple mathematical algorithms concurrently and utilising additional concepts such as exponents, factorisation, and division. The inability to remember algebraic rules or the inappropriate application of the rules forms the basis for encountering difficulties in solving algebraic problems, leading to several errors or misconceptions (Luneta & Makonye, 2010; Mamba, 2012; Pournara et al., 2016). The present study examines the causes of learners' errors in algebra and the possible solutions to avoid them. Specifically, this paper answers the following research question: *Which errors are the most prevalent when solving algebraic expressions among Grade 10 learners?*

Literature Review

The role mathematics plays in our everyday lives and in understanding other subjects of the curriculum cannot be overemphasised (Ndemo & Ndemo, 2018). Algebra is the bedrock of advanced mathematics, and it is related to all branches of mathematics (Department of Education, 2011; Moru & Mathunya, 2011). Agustyaningrum et al. (2018) stated that algebra (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 333–340). Canberra: MERGA.

consists of axiomatic theories that provide opportunities to consider various mathematical systems, whether special cases or not. In addition, Adeniji and Baker (2023) stated that solving algebraic equations plays a crucial role in developing learners' critical thinking and supporting learners' journey to higher education mathematics. Despite its significance, the performance of learners and the errors they make have raised concerns across the world (Ndemo & Ndemo, 2018). Several studies, such as Akhtar et al. (2020), Mncube (2016), and Ndemo and Ndemo (2018), revealed that learners' limited understanding of algebra often results in a negative attitude and poor performance. The National Senior Certificate (2022) diagnostic report also highlighted that South African learners still lack algebraic skills because many learners lack fundamental skills that should have been acquired in lower grades. Based on Newman's Error Analysis (NEA), this study is limited to the prominent types of errors identified by Makonye and Hantibi (2014), specifically, systematic errors, transformation errors, and encoding errors. First, systematic errors are those common errors that are being made by learners continuously for a long period, usually as a result of failing to understand specific concepts, rules, or operations (Mncube, 2016). Learners sometimes construct incorrect theories, assumptions, and misconceptions that they become attached to, and those theories and misconceptions interfere with learning (Mbewa 2013). For instance, Mulungye et al. (2016) argued that when most learners were given a task such as $(a + b)^2$ majority of the learner's answer was $a^2 + b^2$ instead of $a^2 + 2ab + b^2$. In this instance, learners fail to use the distributive property and the rules of exponents correctly.

Second, transformation errors occur due to the inability of learners to correctly choose the right mathematical solution methods during the process of solving, simplifying, or rearranging variables in algebraic problems (Abdullah et al., 2015). An example of transformation error is illustrated in a study by Moru and Mathunya (2022), where learners misapplied the rules of exponents. For instance, the learners were given $y^4 + y^4$, and the answer that was given by learners was y^8 . The learners argued that they added the exponents because "when the bases are the same, we add the powers" (Moru & Mathunya, 2022, p.136), which means learners misunderstood the rule of adding algebra to that of multiplying exponents. Lastly, Abdullah et al. (2015) defined encoding errors as mismatch between the actual relationships in the problems, misusing mathematical symbols, confusing dependent and independent variables, mixing units and scales, and inability to express the final answer. Furthermore, Fitriani et al. (2018) added that encoding errors usually occur when learners are not used to writing conclusions in math problems. For instance, in a study conducted by Abdullah et al. (2015), learners were required to solve this problem: *studies show that $\frac{5}{6}$ of the students play sepak takraw. $\frac{1}{2}$ of the students who play sepak takraw also play badminton. If there are 132 pupils, what is the number of students who play sepak takraw and badminton?* The findings of the study showed that many learners got the answer 55 right, but they failed to provide a concluding statement that relates the answer to the problem context as 55 students played both sepak takraw and badminton.

Research Methodology

This study employed the positivist paradigm and quantitative research approach to analyse learners' errors in solving algebraic problems. According to Kaboub (2008, p. 343), positivist paradigm asserts that real events can be empirically observed and explained with logical analysis. For this study, data were collected from learners in three schools from the Amathole West district in the Eastern Cape province of South Africa. The schools were purposefully selected from the rural district, and Grade 10 mathematics learners served as the participants. Given that learners at the Further Education and Training (FET) phase in South African schools are at liberty to choose between mathematics and mathematical literacy, the researcher

purposefully selected schools with higher mathematics enrolment. As such, the sample of mathematics learners in schools X, Y, and Z was 23, 23, and 56, respectively, making a total of 102 participants.

Data were gathered through a content-based test and an examination of the document released by the Department of Education. The content-based test enabled participants to make errors related to algebraic expressions. The structured questionnaire was employed to reveal the fundamental causes of the errors made in the written test. Based on the researchers' understanding and experience, a protocol detailing descriptions of the three types of errors committed in algebraic problems was created. All the instruments for data collection were validated by two Professors of mathematics education.

Data Analysis

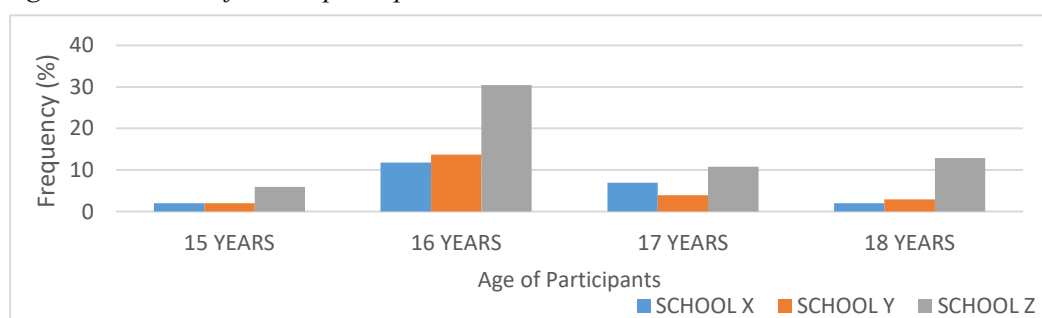
A quantitative approach to content analysis was employed to answer the research question. The use of content analysis in this study provides opportunities to systematically analyse and interpret learners' responses to algebraic problems (Rourke & Anderson, 2004). It can be utilised to discern similarities and contrasts, patterns, and relationships. Content analysis was employed for the present study because it allows the researchers to handle large samples and draw more robust conclusions, enhancing the external validity of analysing learners' errors in solving algebraic problems. Additionally, descriptive statistics provide information about the distribution, relationship between variables, and frequency variability (Cohen, Manion & Morrison, 2017). The statistical analysis involves calculating the frequencies and percentages of correctly answered questions and identifying common errors among the samples.

Results and Discussion of Findings

First, participants' characteristics in terms of gender and age from the three schools are presented. In this context, the majority of the learners are female, with 63 (62%) participants being girls and 39 (38%) being boys. Specifically, the breakdown of female participants in each of the three schools was as follows: School X 56.5%; School Y 60.9%; and School Z 64.3%, which aligns with the general population breakdown in our societies. Additionally, Figure 1 illustrates the age distribution of the participants from each participating school. According to the South African Schools Act, 84 of 1996, the statistical age norm per grade is the grade number plus 6. As such, the grade 10 learners' average age is expected to be $10+6 = 16$ years. Accordingly, in this study, a total of 9.9% of the participants are aged 15 years in all three schools, while the majority of the participants fall within the appropriate age of 16 years old (55.9%). About 34.4% of the participants were over the required age, with 21.6% being aged 17 years and 12.8% being 18 years, which is a stipulated schooling age (South African Schools Act 84 of 1996, Amended 1998).

Figures 1

Age Distribution of Participants per School



The analysis of the content-based test was presented below based on the types of errors committed.

Systematic Errors in Algebra

The content-based test consisted of three questions, each testing a type of error. For example, Question 1, which has five sub-questions (1.1-1.5), tests systematic errors. Participants were allowed to provide solutions to the algebraic problems on paper provided for them. The participants illustrated various approaches below based on a variety of questions.

Question 1: Simplify $2(x - 2y)(x^2 + xy + y^2)$

For this question, learners are expected to solve algebraic expressions by first understanding the basic components of algebra, such as variables, constants, and coefficients. Then, simplify the expression by eliminating the parentheses using the distributive property, applying exponent rules, combining like terms, applying the order of operations, and factorisation. All these align with the South Africa Department of Education's (2011. p.59) expectations of Grade 10 learners.

Table 1 revealed that learners in schools X, Y, and Z performed well (i.e., 100%) on question 1.1. However, on question 1.2. Only learners in schools X and Z have responses that are error-free at 100%, while school Y performed at 60.86%, depicting that 39.14% of the responses have errors. Table 1 further revealed that 41.17% errors were committed across all three schools with question 1.5 posing a greater challenge to the majority of the learners. From the research findings, the majority of the learners employed an inappropriate use of the distributive property. Similarly, some of the learners misapply properties, neglect negative signs, apply signs in one direction, and fail to combine like terms properly, which often leads to incorrect solutions. Hence, the primary reasons learners struggle with algebra are inadequate conceptual understanding and sometimes inattention. This finding is similar to those of Mulungye, Miheso, and Ndethiu (2016), who iterate that some systematic errors that are made by learners in algebra include the inappropriate use of the distributive property, which states that $a(b + c) = ab + ac$.

Table 1

Comparison of Learners' Responses and Systematic Errors Committed in Answering Question 1

Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
1.1	23	100.00	23	100.00	56	100.00	00	0.00
1.2	23	100.00	14	60.86	56	100.00	09	8.82
1.3	21	91.30	19	82.60	51	91.07	11	10.78
1.4	19	82.60	10	43.47	38	67.85	35	34.31
1.5	22	95.65	11	47.82	27	48.21	42	41.17
Ave. %		93.91		66.95		81.42		

Figure 2 illustrates a sample of learners' responses showing the error committed in question 1. This response shows a systematic error in line 5 where the learner forgets to appropriately distribute $-4y$ across the terms. Even though the steps were clearly shown, many learners commit this error, neglecting some terms when applying the distributive property, leading to systematic error.

Analysis revealed that when some learners were given a positive smaller number to be subtracted from a negative bigger number or vice versa, they consequently ignored or failed to apply the rule that states that the answer takes the sign of the bigger number. Furthermore, errors committed were based on a faulty line of thinking, which is likely to recur since they were built on false ideas. This resonates with the argument of Mncube (2016), who argued that systematic errors are the result of learners' failing to understand concepts or rules. As such, Mbewa (2013) stated that learners construct the rules and misconceptions that they become

attached to, and those rules and misconceptions interfere with learning. Systematic errors in algebraic expression are common among many mathematics learners because, in many schools, algebra is often taught as a series of procedural steps rather than a coherent system built on logical principles. As such, learners frequently memorise algorithms without grasping the underlying concepts, leading to systematic errors when they encounter problems that require flexible thinking. For many learners, algebraic symbols are abstract and lack meaning, making it challenging to view equations as representations of real-world situations (Mbewa, 2013; Mulungye et al., 2016; Biney et al., 2023). Similarly, learners often face difficulties with the transition from arithmetic to algebra, a shift that necessitates a more abstract way of thinking. The ability to manipulate variables and understand their relationships is complex and can be overwhelming for learners who have not fully internalised basic mathematical principles. This lack of understanding often manifests as errors, particularly when learners are asked to simplify expressions or solve equations that deviate from familiar patterns.

Figure 2

Sample of Response to Question 1

Handwritten work for Question 1: Simplify $2(x-2y)(x^2+xy+y^2)$. The student's work shows the following steps:

$$2(x-2y)(x^2+xy+y^2)$$

$$(2x-4y)(x^2+xy+y^2)$$

$$2x \times x^2 + 2x \times xy + 2x \times y^2 - 4y \times x^2 - 4y \times xy - 4y \times y^2$$

$$2x^3 + 2x^2y + 2xy^2 - 4xy^2 - 4xy^2 - 4y^3$$

$$2x^3 + 2x^2y + 2xy^2 - 4xy^2 - 4xy^2 - 4y^3$$

$$2x^3 + 2x^2y - 2xy^2 - 4xy^2 - 4y^3$$

A bracket on the right side of the work is labeled "Systematic Error (Inappropriate use of distributive Properties)".

Transformative Error in Algebra

Question 2 aims to identify and analyse transformative errors in algebraic products involving various operations. When faced with such questions, the algebraic product may involve terms that contain variables and constants and may require the application of the distributive property, factoring, and simplifying. Thus, question 2, which comprises of algebraic product, has five questions (2.1 – 2.5). Analysis indicated that participants illustrated various approaches to solve this kind of question; however, below is a sample of common errors, identified as transformative errors.

Question 2: Simplify $(-2y^2 - 4y + 11)(5y - 12)$

Figure 3

Sample of Response to Question 2

Handwritten work for Question 2: Simplify $(-2y^2 - 4y + 11)(5y - 12)$. The student's work shows the following steps:

$$(-2y^2 - 4y + 11)(5y - 12)$$

$$-2y^2 \times 5y - 4y \times 5y + 11 \times 5y - 12 \times (-2y^2) - 12 \times (-4y) + 11 \times (-12)$$

$$-10y^3 - 20y^2 + 55y + 24y^2 + 48y - 132$$

$$-10y^3 - 20y^2 + 55y + 24y^2 + 48y - 132$$

$$-10y^3 - 20y^2 + 55y + 24y^2 + 48y - 132$$

A bracket on the right side of the work is labeled "Transformation Error (Inability to choose the right mathematical method)".

Table 2 revealed that only school Y performed well (i.e., 100%) in question 2.1. However, learners from all three schools (X, Y, and Z) struggled with questions 2.3 – 2.5 with varying causes of transformation error. Based on the average percentages presented in Table 2 below, all three schools managed to perform a little above 50%. For learners to be able to solve algebraic products, they are expected to develop a clear understanding of algebraic concepts, the arithmetic and algebraic rules, and how to apply these to simplify or manipulate the expression.

However, many of the learners across the three schools struggle with questions 2.1-2.5 in general as a result of their inability to choose the right solution methods. Table 2 further revealed a 70.58% committed errors across all three schools with question 2.4 posing a greater

challenge to the majority of the learners. From the research findings, many learners seem not to fully grasp the properties of arithmetic and algebra, while some learners over-generalise specific strategies or rules, neglecting the nuances of different algebraic expressions.

Table 2

Comparison of Learners' Responses and Transformative Errors Committed in Answering Question 2

Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
2.1	16	69.56	23	100.00	44	78.57	19	18.62
2.2	16	69.56	13	56.52	39	69.64	34	33.33
2.3	13	56.52	09	39.13	28	50.00	52	50.98
2.4	12	52.17	06	26.08	12	21.42	72	70.58
2.5	09	39.13	08	34.78	32	57.14	53	51.96
Ave. %		57.38		51.30		55.35		

Furthermore, findings revealed that even when learners already understand the needs of the question, they still fail to choose appropriate mathematical operations. For instance, when a learner mishandles arithmetic rules, a multiplication sign is taken as an addition sign and vice versa or confused about whether to apply addition or multiplication when dealing with exponents which leads to transformative errors. In addition, most of the learners fail to observe the rules for their ordering, which normally goes by the acronym BODMAS, meaning "Bracket, Off, Division, Multiplication, Addition, and Subtraction". This finding corresponds with those of Abidin and Ali (2015), who argued that learners make transformation errors when they already understand the needs of the question but still fail to choose the correct mathematical operations that are involved. For example, in a study conducted by Moru and Mathunya (2022), learners were given $y^4 + y^4$ and the answer that was given by learners was y^8 . According to the authors, learners argued they added the exponents because "when the bases are the same, we add the powers," which means learners mistook the rule of adding algebra to that of multiplying exponents. As such, Abdullah, Abidin, and Ali (2015) argued that when learners lack the ability to choose the right mathematical solution method when solving algebraic expressions, they often commit transformative errors.

Encoding Error in Algebra

Question 3 focuses on algebraic products involving several variables and coefficients. It consists of five questions (3.1-3.5). The participants illustrated various approaches, and below is a sample of common errors made based on a variety of questions given.

Question 3: Simplify $(3a - 5b)(3a + 5b)(a^2 + ab - b^2)$

Figure 4

Sample of response to Question 3

$$\begin{aligned}
 & c. (3a - 5b)(3a + 5b)(a^2 + ab - b^2) \\
 & 9a^2 + 15ab - 15ab - 25b^2 \\
 & = (9a^2 - 25b^2)(a^2 + ab - b^2) \\
 & 9a^4 + 9a^3b - 9a^2b^2 - 25a^2b^2 - 25ab^3 + 25b^4 \\
 & = 9a^4 + 9a^3b - 16a^2b^2 - 25ab^3 + 25b^4
 \end{aligned}$$

Table 3 revealed that none of the schools had 100% error-free responses in all five questions asked (i.e., from questions 3.1-3.5). Based on the average percentages presented in Table 3, all three schools managed to perform a little above 50% (in all the five questions), which indicates that learners from all the sampled schools struggled with algebraic expressions involving various variables and coefficients. Table 3 further revealed a 66.66% committed errors across all three schools with question 3.3 posing a greater challenge to the majority of the learners. From the research findings, the errors majority of the learners encounter stem from a

misunderstanding of symbols where learners confuse similar-looking symbols or use incorrect symbols, ambiguity in notations, and complexity of expressions in producing final answers.

In South African schools, learners are typically taught how to simplify algebraic expressions with multiple variables and coefficients through a structured and methodical approach. They were also taught how to factorise third-degree polynomials (Department of Education, 2011. P. 49).

Table 3

Comparison of Learners' Responses and Encoding Errors Committed in Answering Question 3

Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
3.1	12	52.17	22	95.65	38	67.85	30	29.41
3.2	16	69.56	13	56.52	51	91.07	22	21.56
3.3	13	56.52	07	30.43	14	25.00	68	66.66
3.4	09	39.13	12	52.17	27	48.21	54	52.94
3.5	11	47.82	09	39.13	19	33.92	63	61.76
Average %		53.04		54.78		53.21		

However, most of the learners struggled with these types of questions because of their inability to express the final answer. Even when learners can solve problems following the appropriate mathematical steps, they still fail to produce the correct answer. This finding is in line with Abdullah et al. (2015), who argued that encoding errors occur when learners can solve problems following the right mathematical steps but still fail to produce the right answer. Furthermore, the study findings also revealed that some learners see the final answer as incomplete because it has a plus or minus or more than one term. This finding corroborates those of Bayos (2020) and Fitriani et al. (2018), who assert that encoding errors usually occur because learners are not used to writing conclusions in mathematics problems. Hence, researchers (e.g., Baidoo, 2019; Khalo et al., 2022; Delastri & Lolang, 2023; Fumador & Agyei, 2018; and Iddrisu et al., 2017) urged that mathematics teachers should identify all the errors made by learners when solving algebraic questions and use this to enhance their teaching focus in algebra lessons.

Overall, this study reveals that learners often commit more transformation and encoding errors than systematic errors. Specifically, findings indicated that 19.01% of the learners made systematic mistakes, 45.09% made transformation errors, and 46.47% made encoding errors. This suggests that learners frequently make transformation and encoding mistakes due to the complexity of the cognitive processes involved in simplifying algebra. These results call for remedial intervention targeting the common errors identified.

Conclusion

The findings in this paper highlight that, despite algebra being a foundation for several other topics in mathematics, many Grade 10 learners commit various errors when solving algebraic problems. Two of the prominent errors identified are encoding and transformation errors, largely caused by the selection of inappropriate methods, confusion in applying algebraic rules and manipulating symbols, inaccurate representation of symbols, and unclear answers. The study highlights specific difficulties learners encounter when solving complex algebraic problems involving multiple steps. The findings of this study extend existing results from the literature and contribute to the need to improve learners' understanding of algebra to meet the expectations of the South African Department of Education. It is therefore recommended that mathematics teachers should pay close attention to the errors commonly committed in algebra and encourage constant practice, double checking operations when applying distributive law, factoring and isolating variables, appropriately breaking down complex problems by taking a step at a time, and reviewing and cross-checking steps leading to final answer if improved learning outcomes in algebra are desired.

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Why Ontologies Are Important in Mathematics Education

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Charles Sturt University had the motto “For the common good” but listening to the wisdom of the voices of the Wiradjuri Elders, *Yindyamarra Winhanganha* – the wisdom of knowing how to live well in a world worth living in – now guides us. We teach and research in mathematics with a deep-seated belief about mathematics and teaching it, and its purpose and value. While we are concerned about the selection of content and the way we teach and research, the underlying reason for our work directs us. This paper draws together key ideas for grappling with our ontologies, our purpose, and beliefs.

Ontology is an important part of preparing for a research project but it is often not discussed in terms of our personal development. In this paper, I reflect on the lived development of my ontology. The reflection is interrogated by the works of Pinxten and Skovsmose. The interrogation strengthens the argument that ontologies are important for mathematics education. This is a philosophical paper and not a report of an empirical study.

Starting Points

February 2025 was the 60th Anniversary of the Freedom Ride, NSW, organised by SAFA (Student Action for Aborigines). I belonged to this organisation and the Evangelical Union at the University of Sydney. I learnt as much in my university societies as I did in formal lectures. My ontological journey and purpose in the world began. SAFA was a group, led by Aboriginal activist Charly Perkins, of communists, other socialists like Fabians, humanists, anarchists, Christians, top medical students, top law students, top history and sociology students, scientists, an engineer, and only a couple of country students (including Aboriginal Gary Williams). They prepared well for a year to get publicity for the blatant discrimination and poor living conditions of Aboriginal people in country NSW, things of which city dwellers were totally unaware. I was a keen, socioeconomically and politically ignorant first year student, scared of failing and knowing the unspoken of how much it meant to my parents that I was there on scholarship. I was enthralled by learning new ideas. I learnt about a variety of socialist perspectives and different Christian perspectives. After a stint teaching in NSW, I worked in an Australian colony, PNG, for 15 years with amazing people from across the world and the country. We kept our Aboriginal and PNG connects and friendships over 50 years.

I can say my life has been so enriched and directed as it builds on my often non-described, almost unplanned, lived perspective. It took me to marry a like-minded person and to live in Papua New Guinea (PNG). My perspective permitted me to study and teach mathematics but to learn about mathematics differently, similarly health education about which I was also passionate and taught in Australia and PNG and which influenced my mathematics education in terms of cultural values and group learning. Over my 30 years of teacher education, I have actually taught all subject areas (except Physical Education) for early childhood and primary, and mathematics education and mathematics from early childhood to tertiary. The highlights were teaching Literacy and Numeracy together, Mathematics and Creative Arts together, and Mathematics and Human Society & Its Environment (with SAFA as an example of People Power) to the same students so I could integrate their learning. My ontology encouraged me to listen and work with Indigenous people in Australia and PNG and to make many global transcultural research colleagues.

At our Discussion Group at ICME15 on overcoming neocolonialism, our Maori colleagues pointed out that if cultural practices are to be used in mathematics, then the cultural practices must be learned first and culturally because the culture is lost if mathematics comes first in the students' learning. For example, the wall messages of the meeting house might have mathematical shapes and relationships but the cultural meaning should be learned first. Indigenous cultures are rich, holistic and integrated rather than deconstructed into mathematics and art. This paper establishes and analyses my ontology discussing various philosophical perspectives to show the significance of ontology for mathematics educators.

Moving Away from the Limitations of Educational Psychology

Most of us would adhere at least to social constructivism but that is not where I started. I did psychology at university and behaviourism was strong. We did 'experiments' and I learnt how to write scientific reports, design instruments for assessment, about validity and reliability, and over the years testing with t-tests and then factor analysis and various other multivariate analyses (thanks to computational tools). When I was at school, I could do maths if I understood the words (poor English or maybe the emphasis on rote learning hampered me⁴) and I loved the exercises about triangles and calculus – they were like a game of chess working out which method to use. But I wondered about the purpose of mathematics and the reason for spending so much time on mathematics. I came from a blue-collar working-class family who had no idea. After three years of university mathematics, I still did not know the answer to my question. When I started to teach mathematics at the PNG University of Technology, I finally found out how different areas of mathematics were important for a whole variety of technologies from electrical engineering and communications, to surveying and valuation, to accounting and business studies, to architecture and building management, to forestry and fisheries, to mining and food technology. Colleagues planned mathematics programs to suit each technology. Furthermore, I learnt how calculators and then computers could assist students to understand probability and statistics, linear programming, and queuing theory, and to write computer programs for carrying out calculations and for presenting moving graphs. How grateful was I for my mathematician colleagues and an advanced university whose computer usage for teaching was better than the university I returned to in Australia.

However, I also learnt that my PNG students:

- Had multiple counting systems and often not base 10.
- Were excellent at taking risks to solve problems.
- Could visualise, thinking spatially, and find their way travelling capably across land, rivers, and sea without a compass.
- Came from a diversity of rich and interesting cultures with beliefs and abilities to make extraordinary houses, boats, tools, artefacts, headdresses, bags, songs, and dances.

This then is the background for my developing ontology, something I cannot easily express but it guides my living, teaching and research. It's a lived ontology rather than a written one and maybe I need not make apologies for that.

Ontology

Modernism and mathematics might suggest that what is true or acceptable can be developed by reasoning from an axiomatic base. It might suggest that irrational thoughts such as a God that does not seem to follow rational thought should be dismissed. However, as Skovsmose

⁴ I often smile that we used counters for a few days to develop multiplication tables, which I was good at, but it took me until first year university mathematics on set theory to actually know the meaning of that long word, multiplication.

(2024) and Ernest (2018) point out, mathematics without ethics can lead a person to use mathematics for immoral purposes as occurred in World War II. Skovsmose's (2024) critical philosophy of mathematics avoids absolutism to put the logic of mathematics above all else. As mathematics is constantly being constructed, it does not reach a definitive form but rather it is merely a construction. For example, at school we learnt geometric proofs of a particular Euclidean kind but once dynamic geometry software developed, proof took on other meanings for geometry. Mathematics can develop new concepts, proofs, topics, and applications. Mathematical rigour is contested and new topics develop. However, mathematics can also create power and culture (Ernest, 2012, 2018).

Mathematics can have powerful roles, untruthful roles, money-making roles, and profiteering roles. However, it can bring truth, justice, and improvement of society. My Swedish friend and colleague Annica Andersson completed a doctorate on teaching school students, who were majoring in social science, upper-level mathematics. She showed that their projects on social issues, in which they needed to use mathematics and share it with the school in posters, created agency for students. They had agency not only in *doing* their mathematics and creating something mathematical, as opposed to following someone else's mathematics and reproducing it, but also they were *doing* mathematics for truth, justice, and improvement of society. That teaching reflects Annica's ontology and epistemology (Andersson, 2010).

Our ontology is our perspective and understanding of the world around us and it influences our epistemology, that is our focus on how we know and even the limits of our knowing. This paper presents some of the challenges from mathematician philosophers. For me there is the grappling with Christian views and humanistic views. Ethnomathematics has brought for me a large number of different perspectives on mathematics and ontologies and many research friends. Hence it is interesting for me to read their reasoning too.

NEED Humanism and Ethnomathematics

One person who has influenced my thinking over the past 35 years has been Rik Pinxten who has also spent a long time listening to the voices of his own Belgian family and community, First Nations in America, migrants and displaced people. He recently published a book, *Humanism Revisited*, which portrays its author Rik Pinxten the person grappling, as he does, with an issue that is intellectually troubling him, not just for himself but for the world and others. Rik Pinxten the story-teller reveals sufficient of his personal life and experiences to give a humanness to his discussion. As one reads this book, it is as if he is sitting discussing his thoughts with you, what is troubling him. And yet this is no ramble, it is a clearly presented argument which has been carefully crafted to present his points. He bases his arguments on lived realities and other scholarly works. These works he also presents clearly for the reader. The last time I read a book that made me feel part of a conversation was one by Bill Barton (2008) on *Languages of Mathematics*. Again, at least for mathematics educators in Australia, Aotearoa, and the Pacific, this is essential reading with an equally well-developed argument for perceiving mathematics as being about quantitative, relational, and spatial aspects of life but not necessarily as European/school mathematics has developed it. Rather he has seen mathematics incorporating the perspective of Maori.

Rik Pinxten intentionally avoids losing the reader at the start by not discussing the nuances of past humanism and enlightenment philosophies. He clarifies the purpose of his book, and the context of the world today that disturbs him, and should disturb all humankind. It is time to grapple with our ontologies, to recognise the plurality and value of these rather than being left behind in some former humanistic ontology that does not meet the world's current needs. He clarifies that the dualistic ontology based on Jewish, Christian, Islamic thinking about which humanism reacted had, in fact, insidiously led to the idea of one ontology of humanism, that of a dualistic humanism/enlightenment - of *us* who know and have, and *them* who need these

things. This ontology led to colonisation by missions, educators and governments, not forgetting of course the power, greed and wealth that was taken or the destruction that it caused. Indeed, this Eurocentric humanistic ontology needs changing.

Pinxten encourages us to recognise that people and individuals are a product of their society. In discussing freedom, he mentions the impact of social media, and the rich, deal-making men who continue to bring societies to the brink of destruction. However, he could have expanded on the choices that people make to find information. People have a choice to listen to Sky News or ABC but they do not see freedom as meaning a responsibility to be well or unbiasedly informed (Steve Thornton, personal communication). However, Pinxten notes that not all people have choice. Nevertheless, those from a capitalist background, Islamic, or Chinese-obedience society background, need to recognise how the dualistic thinking of the group who think they are right, supported by members of institutional, powerful groups, for example, churches or political parties are encouraging far-right fears without concern for others and heading people and the earth into catastrophe. The ontological stance must move from the neocolonial in which race, gender and cultural identity forgets the class struggle, poverty and interdependence. It is time for an anthropologically inspired ontological turn.

Pinxten encourages us to look at ontologies that are not based on European humanism. He carefully unpacks his **NEED** humanism that recognises **Non-Eurocentric** humanism, **Ecological** humanism, and **Durable** humanism. Indeed, each is utterly important for this world of ours especially with the current autocracies and plundering of the earth's resources and the high proportion of people without basic needs, rights and justice. He does not forget mathematics and mathematics education in this commentary for they have and continue to pursue a mathematics centred on European studies alone without regard for the diversity of mathematics that may be found among First Nations, disposed and other voiceless or landless people of the world.

I read Rik Pinxten's current book after reading a book by Garry Worette Deverell (2023) on Indigenous theology. Deverell draws together two knowledges to discuss Christ as country (analogously), to see Christ as cosmos, seed, staple food and drink, an ancestral voice, and teaching custodian. This is an example of what the Australian Universities suggest as an Indigenous standpoint in merging ontologies and perspectives and knowledges as ways of doing:

In the language of the academy, the concepts of First Nations ways of Knowing, Being, Doing might be translated respectively as ontology, epistemology and methodology (for the research community) or pedagogy (for teachers and learners). (Charles Sturt University, 2025, p. 18)

Ontologies of Indigenous cultures whether they be those of Australia (the great south land Gondwana), *Ngurra*, or the First Nations of the Americas, or others, the point is these ontologies enlighten people who have western/European/Christian (or Jewish, Islamic, or Chinese obedience society etc) backgrounds. Pinxten tells the ontology of the Diné (Navajo Native Americans) whom he personally knows and appreciates from living, yarnning (talking with), grappling with ideas, and working with them on mathematical ideas and curricula (e.g., Pinxten, van Dooren & Harvey, 1983; Pinxten, van Dooren, & Soberon, 1987, Pinxten & Francois, 2011), and Quechua supported by Descola's (e.g., 2016) work. He notes the more recent discussions by scientists and alternative thinkers about animals and plants, even the world (Gaia), in terms of community, care and empathy. Another mathematician philosopher, Natalie Sinclair (e.g., 2024), has also discussed similar ideas in which mathematical ideas and tools can be used to generate new spatial imaginaries about the world, that is, new ways of imaging the relations that pertain to existence and experience.

Pinxten provides some of the historical and recent developments that bring the world into crisis where greed is favoured over people, and nation alliances bring solidarity but also less

freedom while other nations' leaders flaunt but extinguish democracy. These are new crises: ecological destruction (some mass population deaths), new inequalities (riches out of reach of government, and encouragement of greed), exhaustion of resources, and population mismanagement. With his sketched historical background and crisis summaries, Pinxten presents his arguments for NEED humanism and provides ethnomathematics as an example.

The strength of Pinxten's argument, while he notes political manipulations, is to indicate that the extreme right voice for 'freedom' whilst it sounds humanistic (often backed by 'Christian' influences) is indeed a Eurocentric view of humanism and one that puts individuals before other issues such as the environment and what it means to live in poverty. The 'others', as opposed to us, mentality is used for political gain but for Pinxten, the situation is more deep-seated where the world's other ontologies must be recognised by humanists. Humanists need to avoid individualism and grapple with these issues as a group of humanists. It is a timely reminder that identity is individual, but of a group for action, and of a community for establishing values as interacting groups. Is not MERGA such a community? Perhaps its recent position statement on mathematics education indicates such a move. Pinxten's example is the story of Hannah Arendt and her move from individualistic humanism to recognising the impact of the group in Germany. However, he also shines a spotlight on the degree of change in thinking with the more recent interaction of Europeans with various other cultural practices, particularly awareness of North American Native cultures but also those of the East, Confucian practices. These interactions of groups and thought changes are present in the humanistic realm of Europe. His story this time is of Turpo who continues to follow his Quechuan group identity despite danger compared to the individual, greed-influenced president of Peru and a new wave of those who consider group humanism as 'primitive' and not to be followed. Neocolonial countries like Papua New Guinea face similar dilemmas. Hari (2023) who refers to many thousands of examples of non-Eurocentric humanism while Pinxten continues to focus on his own 'neighbourhood' that once incorporated poor Belgium-born families but now consists of poor from multiple cultural backgrounds with some similar and some different needs and gifts.

Importantly, he notes that the power of rich and powerful individuals, needs humanism to go *beyond individualism* that arose in opposition to the collective strength of the churches with political power *to* recognising the importance of the global and collective. This might be seen as a new stand for humanism to emphasise the collective, "a type of updated, respectful, postcolonial humanism" (p. 79), to be more political. I was surprised by the thought that humanists might not have already taken on this cause considering the strength of the humanist society at my own university in Australia in the 1960s to recognise Aboriginal rights (along with all 'left' political groups and Christian churches). However, he also notes that philosophers and theologians play their role in disconnecting ideologies from reality since for them "empirical reality is not relevant, and logical consistency or coherence is the primary value" (p. 84). Mathematics education is not immune from this and hence the reason for us to take a strong stand on our ontology.

Rik Pinxten recognises the need for practical implementation. He shares the key tenets of others such as Nussbaum (2011) on life, physical health, physical immunity, chances to observe, imagine and think, feelings, practical reasons, other biological species, play and creativity, built and political environments. All of these involve mathematics. In moving forward, Pinxten says:

On all ten rubrics a general engagement for justice and equal opportunities for all has priority over any juridical or ideological value or norm. When looking at the real world, such an emphasis on human capabilities goes deeper than ... basic human needs; ... universally agreed-upon moral values (must come) first, prior to juridical regulating (p. 149).

Ethnomathematics shows how the values and ontologies of non-European peoples can influence this new humanistic worldview. For ethnomathematicians, Pinxten presents a sound ontological basis for their studies and expounds on this by saying:

Ethnomathematicians started studying the out-of-school formal concepts and procedures in the particular linguistic and cultural environment of the child. The technologies developed and used successfully in many cultures – the architectural sophistication, the cosmologies, the formal aspects in rituals and in music, and so on – all these showed a huge variety of formal or abstract reasoning. Ethnomathematical educationalists would then go on and start organizing further learning processes on the basis of these local or culture-specific skills. ... no longer will the teacher erase the supposedly wrong or ‘undeveloped’ notions and practices of the pupils in order to plant the one true doctrine in their heads. Rather, the educational process will develop as a dynamic, interactive relationship where the local categories are taken to be valuable concepts and procedures on which more and other notions and views can possibly be grafted. Or, to put it differently, education resembles more a process of intercultural negotiation than one of assimilation. Thus, a holistic approach at the intuitive level is respected and can gain the same status as the dualistic view of the westerner, if not a better entry in the field of formal reasoning (Baker, 2023, p. 153)

Pinxten’s own work with the Diné is his lived ontological approach and Vandendriessche and Pinxten’s (2023) book is a companion to this book *Humanism Revisited*. Thus Pinxten presents both a theoretical and logically reasoned revised humanism. It is non-Eurocentric, ecological and durable in terms of not only saving humanity and the planet but also a way of living, thinking, and philosophising. This truly is a humanistic ontology needed in all aspects of the world but essential for education including mathematics education.

We need to relearn living together and with the whole of nature. There is no time dimension to this, it is essential. It recognises differences in humanity and takes these as foundational capabilities for pluriversal humanism (Tim Ingold, in Pinxten, 2023). Mathematics then becomes an understanding of all humanity with “quantitative, relational, or spatial aspects of human experience” (Barton, 2008, p. 10). This perspective, often referred to as the QRS system, has been built upon for epistemology by others such as Alangui in the Philippines and Trinick in Aotearoa (NZ) and their colleagues. For example, Trinick et al. (2023) refer to the Cultural Symmetry Model in which the cultural practices and knowledges are forefront in education.

The first step is to describe the cultural knowledge and identify the cultural values connected to the practices and artefacts under investigation, which is best done in collaboration with elders, who are knowledgeable about these practices and artefacts. As traditional practices are tightly connected to Indigenous language, there is a need to ensure that those discussing the practices know the language to engage in discussions of practices and artifacts. The second step is to examine the cultural practices and discuss them from a range of perspectives, of which mathematics would be one. The third step involves considering how mathematics can add value to cultural artefacts and practices, without detracting from the cultural understandings. (p. 304)

Critical in this argument is that cultural understanding should come before the mathematics because, if the mathematics comes first, education does not share the culture properly.

Logical Mathematics

Many views of mathematics surround the issues of logic. This may lead to absolutist views and link to dualistic views of the world – what is right and what is wrong. Such logical views were the basis in the late 1800s of mathematicians such as Klein and Hilbert (Skovsmose, 2024). However, at this point Skovsmose (2024) discusses the idea of metamathematics using four criteria:

independence of mathematical axioms, the *consistency* of a mathematical theory, the *completeness* of a mathematical theory, and the *decision problem* concerning the possible existence of an algorithmic procedure for determining whether a given mathematical formula is a theorem or not in a given mathematical theory. (pp. 78-79)

These might be interesting for assessing systems from ethnomathematics or critical mathematics education.

The formalism of mathematics is tempered by noting:

Formalism advocates an if-then absolutism with respect to mathematical statements. The situation is quite different with respect to the truth of a mathematics theory. A mathematical theory can be applied in order to describe some empirical phenomena. It can provide more or less precise and reliable descriptions. The degree to which it does so is an empirical question. Formalism advocates an empirical graduation of truth with respect to mathematical theories. (pp.83-84)

This led to mathematics as being almost a game without regard for its purpose, an ethical issue. Furthermore, the structural view of mathematics by the Boubaki Modern Mathematics Movement developed many strands of mathematics from sets or the group. However, it ignored, as it did in its implementation into school mathematics (1960s and 1970s), other aspects of mathematical understanding. Yet set theory led me to understand multiplication.

Critical Mathematics Education

One little-recognised comment from van Hiele (1986), was his description of intuition that it might well be prior experience. The phrase ‘teaching from the known to the unknown’ has often been used in education but are students actually allowed to do this, outside of early childhood play. It is worth noting developments in mathematics education:

Both Freudenthal and Brouwer argued that formal structures did not capture the essence of mathematical thinking. While Brouwer highlighted that mathematics is a mental activity, Freudenthal repeated many times that mathematics is a human activity. They first of all saw mathematics as an activity and not as a structure. While the Modern Mathematics Movement tried to teach students mathematical scores, Freudenthal tried to engage them in playing the music of mathematics. (Skovsmose, 2024, p. 101)

Critical mathematics education has taken the stage.

Although the modern philosophies of mathematics—logicism, formalism, and intuitionism—have had a profound impact on mathematical practices in research, application, and education, they have remained oblivious to any such impacts unconnected to other forms of knowledge as well as to any socio-political issues. (Skovsmose, 2024, p. 102)

Mathematics has resulted in discrimination against students who have not the same background as Europeans of the modern world, especially through colonisation or power elites that have often been racially or linguistically identified. Nevertheless, mathematics may be used to address critically sociopolitical issues through reflective inquiries into critical issues.

Conclusion

I began my journey into critical mathematics education from teaching Sociology of Education and HSIE with an emphasis on recognitive social justice (that is people having a say in their future, for example, how government money is used to support them rather than the government bodies deciding). I read Gruenwald’s view of critical theory and place-based education (2008; Gruenwald & Smith, 2007) and applied it to mathematics education (Owens, 2014). An ecocultural pedagogy both aims to decolonise and reinhabit mathematics education, particularly for Indigenous students, but it also takes account of different geographies. In particular, education must take account of the kind of places we inhabit and leave behind for future generations. In these places, the learner takes initiative, poses questions, experiments, solves problems, makes decisions, and is accountable for the results. Places and actions in places are visualised and people reason holistically about places that they visualise (Pinxten et al., 1983). This visuospatial reasoning is central to mathematics learning and thinking and this perspective emphasises cultural links to land as embedded in relationships and language.

Research with Indigenous communities needs the community to decide on what should be researched, how it should be researched and who should do the research, and they continue with ownership of the research. As UNDRIP, Article 14 (United Nations, 2007) says, the right to “have access ... to their education in their own culture and provided in their own language”. Their ontology involves the land, as Shaun Williams and NSW Aboriginal and Education Consultative Group (AECG) say “Healthy country, healthy culture”. First Nations mathematics lies in their long-time culture. My role, as non-Indigenous, is to discuss ideas that can become of keen interest to the community with their ideas and knowledge that go well beyond my thinking. The final product needs to be their voice, shared as they see appropriate. Seeing mathematics differently to their colonised perspective needs discussion and more than one voice. Digging deeply into culture and cultural relationships means recognising culture and relationships as priorities; language, especially non-verbal and by others of the group, as complexities, within which the ontology, the research, the mathematics, and the education survive.

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Context-Based Line Graph Comprehension: Do Teacher Education and Experience Make a Difference?

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This cross-sectional study examines participants' comprehension of line graphs at different career stages, comparing first-year and third-year pre-service teachers (PSTs) with in-service teachers (ISTs). A 23-item line graph task aligned with Curcio's framework—*Reading the data*, *Reading between the data*, and *Reading beyond the data*—reveals persistent challenges in *Reading beyond the data*. ISTs outperformed PSTs, suggesting practical experience enhances interpretation. However, no significant differences emerged between first- and third-year PSTs, challenging assumptions that teacher education fosters progressive development in graph interpretation skills.

Introduction

The ability to accurately read, interpret, and infer from graphical representations is a fundamental aspect of critical thinking and problem-solving in a data-rich world (Glazer, 2011; Romero Ariza et al., 2024). Graphs, charts, and plots, omnipresent across disciplines from science to social studies, are pivotal for visualising complex information, making informed decisions, and discerning trends. These skills are crucial for students to effectively navigate and interpret the information-rich world around them (Rodriguez & Jones, 2024). The importance of these skills extends beyond student learning to the educators responsible for their development.

Teachers, as primary facilitators of learning, must possess the competence to interpret and teach graph-related content. This ability is a key component of their pedagogical content knowledge, influencing their effectiveness in guiding students through the intricacies of graph comprehension (Freedman & Shah, 2002; Friel et al., 2001). Teachers not only need to understand graphs themselves but also require the skills to effectively teach this content, helping students to construct key concepts, interpret graphs in relation to contextual situations as well as overcome difficulties and misconceptions (Glazer, 2011; Kerslake, 1981). As a matter of fact, studies indicate that both in-service and pre-service teachers often struggle with interpreting and teaching of graph-related content (Glazer, 2011; Patahuddin & Lowrie, 2019). Research in this area, therefore, is invaluable to understand the particularities that make graph comprehension accessible or constraining to both preservice and in-service teachers. This study also contributes to provide a developmental perspective of graph comprehension from pre-service to in-service education and enhances our current understanding of graph comprehension.

The current investigation advances our understanding of context-based line graph interpretation skills in a sample of 256 pre- and in-service teachers. It aims at exploring the knowledge growth of teachers by analysing the developmental trajectory of graph interpretation skills across 3 cohorts—Year 1 and Year 3 pre-service teachers, and in-service teachers. The following research question guided the study: *How do graph interpretation skills differ across first-year and third-year pre-service teachers and in-service teachers, and what does this reveal about their developmental progression?*

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The Research Gap: Understanding the Developmental Progression of Line Graph Interpretation from Pre-Service to In-Service

While extensive research has been conducted on enhancing in-service teachers' graph interpretation skills through various interventions, there remains a notable gap in understanding how these abilities are developed during pre-service teacher education. This stage is critical as the competencies formed during pre-service training profoundly influence future educational practices. Incorporating pre-service teachers into graph interpretation research is essential to evaluate their initial skills and misconceptions, providing a basis for targeted educational strategies that could profoundly impact their future teaching effectiveness.

The literature consistently highlights the importance of graph interpretation skills, suggesting that these competencies are crucial for teachers from the onset of their careers due to the increasing complexity of graphical data in educational settings (Glazer, 2011; Leinhardt et al., 1990; Zeuch et al., 2017). However, studies specifically focusing on the development of these skills during pre-service teacher training are sparse. The work of Ates & Stevens (2003) indicates that different teaching modalities can influence skill development, pointing to the need for a diverse range of instructional approaches. Thus, investigating how pre-service teachers acquire and refine these skills can close a significant research gap and provide essential data for enhancing teacher education curricula to better meet the demands of contemporary education in data literacy.

Additionally, previous research has identified persistent challenges in graph interpretation among pre-service and in-service teachers (e.g., Alacaci et al., 2011; Zeuch et al., 2017), suggesting that misconceptions do not necessarily diminish with experience. Studies reveal that in-service teachers, like their students, often misinterpret line graphs by treating them as literal representations of physical events rather than abstract representations of data (Glazer, 2011; Leinhardt et al., 1990). This 'iconic graph difficulty' leads to fundamental errors in understanding the variables and relationships depicted, indicating a deep-rooted misconception that could be addressed beginning in pre-service training.

Moreover, the persistence of these interpretation challenges among experienced teachers (Patahuddin & Lowrie, 2019) underscores a lack of effective professional development in this area. It suggests a broader systemic issue in ongoing education that allows these misconceptions to perpetuate, affecting both teaching and learning outcomes. Research focusing on the inception and correction of these misconceptions during pre-service education could therefore play a pivotal role in breaking this cycle of misunderstanding and enhancing overall educational quality.

The study by Patahuddin & Lowrie (2019) focusing on in-service teachers' graph interpretation skills found three key insights: firstly, teachers struggled with tasks that required 'reading beyond the data,' indicating difficulties in interpreting graphs beyond their explicit content. Secondly, these challenges were more pronounced in teachers teaching only one grade level. Thirdly, the study discovered that these difficulties occurred irrespective of the teacher's gender. The study adds to the growing body of evidence that misconceptions in line graph interpretation are not limited to students but extend to teachers as well, affecting their ability to impart these crucial skills to their students.

Overall, there is a need for research that probes the development of graph interpretation skills at the pre-service level to in-service teachers. Such studies would not only fill a critical gap in the academic literature but also potentially lead to significant improvements in educational practice by equipping future teachers with the necessary skills to handle the complex data-centric challenges of the real world.

Theoretical Framework for Interpreting Graphs

Various theories and taxonomies have been proposed to understand the processes involved in graph interpretation. Shah and Hoeffner (2002) suggested that successful graph interpretation involves three steps: (1) identifying and encoding graph features, (2) interpreting general relationships, and (3) relating these relationships to the disciplinary context. Bertin's (1983) semiotic theory divides graph reading into three stages: external identification (recognising graph features like labels and units), internal identification (understanding components and visual arrangements), and perception of pertinent correspondences (comprehending data through the interaction of external and internal features). Curcio's three-tiered framework (1987) considers factors such as prior knowledge, mathematical content, and graph form, correlating to three levels of interpretation ability: (1) reading the data (extracting information directly from the graph), (2) reading between the data (identifying relationships within the graph data), and (3) reading beyond the data (making inferences and predictions from the graph data).

Arteaga et al. (2015) used Curcio's theory to assess prospective primary school teachers' abilities to read and construct graphs. They found that while more teachers could construct complex graphs, fewer could interpret data at the highest level, 'reading beyond the data'. Similarly, Jacobbe and Horton (2010) found that competent graph interpreters among U.S. primary school teachers struggled with 'reading beyond the data'. These studies highlight the need for improvements in teacher training and professional development programs. The commensurability of the current research with the aforementioned studies as related to teachers, motivated the selection of Curcio's framework.

Methods

Research Context and Participants

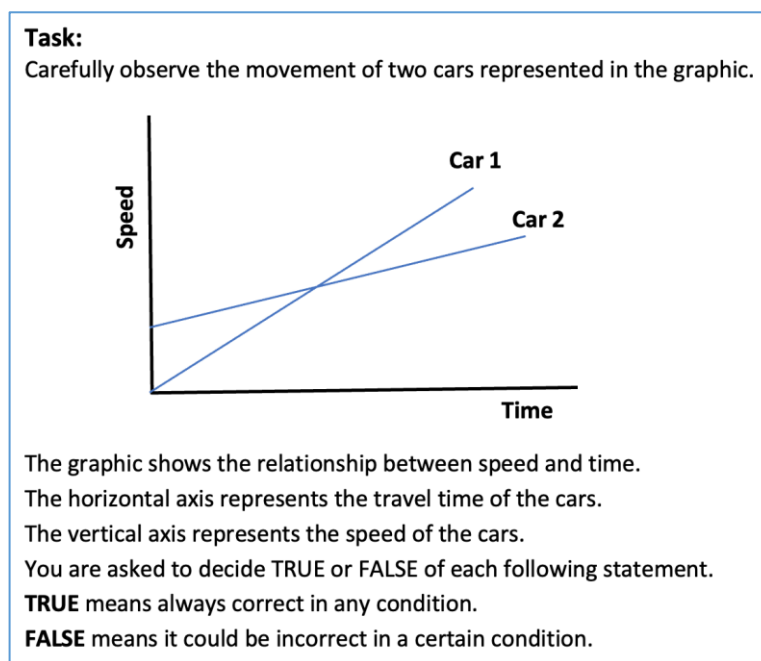
This cross-sectional study examines line graph interpretation skills among 256 participants in Eastern Indonesia, divided into three cohorts: **Cohort 1** (85 first-year pre-service secondary teachers), **Cohort 2** (105 third-year pre-service secondary teachers), and **Cohort 3** (66 in-service secondary teachers) with an average of 9.4 years of teaching experience (Years 7–9). These cohorts represent different stages of teacher education, with pre-service teachers at varying levels of training and in-service teachers having completed at least a four-year undergraduate degree in education. The first-year preservice teachers had studied two semesters of mathematics content courses such as Calculus I and II, Trigonometry, Statistics, Elementary Algebra, Geometry and Number Theory along with other education-related modules. On the other hand, the third-year preservice teachers had studied Discrete mathematics, Real Analysis, Mathematical Modelling among others. It should also be highlighted that the university established their curriculum based on National Standard of Higher Education (NSHE) and the Indonesian Qualification Framework (Abadi & Chairani, 2020). All participants provided informed consent voluntarily. They were made aware of the study's aims, procedures, and potential implications prior to their participation, and their anonymity has been preserved throughout.

To evaluate graph interpretation skills, a 23-item True-False test (Refer to Table 1) was used, sourced from a graph test, empirically designed and validated to assess teachers' conceptualisation of graph comprehension in relation to line graphs depicting variable speeds of two cars (Patahuddin & Lowrie, 2019). The complexity of this task emanates from the fact that the speed of two cars have to be compared through two intersecting line graphs. Each test item was classified into one of Curcio's levels of graph interpretation—*Reading the data*, *Reading between the data*, and *Reading beyond the data*—by a team of eight researchers, yielding a Fleiss Kappa inter-rater reliability of 0.7, suggesting substantial agreement.

The tests, conducted in Bahasa Indonesia, took approximately 30 minutes and were administered in a paper-and-pencil format to maintain consistency across sessions. Figure 1 shows the task context. The students were presented with a set of 23 statements and they had to rate whether it is true or false.

Figure 1

Task Associated with Context-Based Line Graph Interpretation Skills (Patahuddin & Lowrie, 2019, p. 790)



Results

The items in the questionnaire have been categorised according to the three dimensions of Curcio's framework. The last category 'Reading beyond the data' was further subdivided into Type I and Type 2. Type 1 requires an understanding of the graph's content that impacts its interpretation (such as interpreting the slope as indicative of changes in speed; understanding distance in terms of speed and time) while Type 2 entails pre-existing ideas regarding the contextual setting or personal experiences related to the content of the graph, which influence interpretation (for instance, perceiving a connection that is not directly perceptible but has to be inferred).

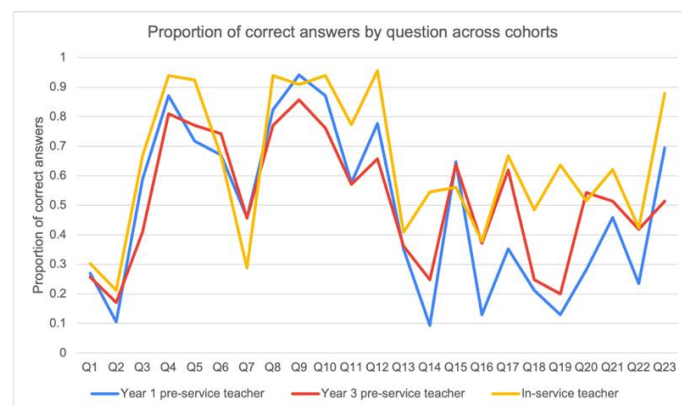
Table 1 shows the proportion of correct responses from three groups of respondents. Notably, the data illustrates a progressive improvement in the correctness of responses from Year 1 pre-service teachers to in-service teachers. It highlights the enhancement of interpretation skills through experience and education. Moreover, Figure 2 shows that certain questions, notably Q1, Q2, Q7, Q13, Q16, Q18, and Q22, were challenging for all cohorts, with less than 50% accuracy. These items either require relatively higher order interpretation or are intuitively disorienting. Additionally, the varying difficulty level across different categories is evident, with particularly lower success rates in the "Reading beyond the data Type 2" questions (e.g., Q1, Q2, Q14, Q19), suggesting these require higher interpretation skills. Specific questions, such as those concerning the initial car speed and speed changes between the cars (i.e., Q4 & Q5), show markedly higher correctness rates, indicating some concepts might be more intuitively grasped by the respondents.

Table 1*Progression of Line Graph Interpretation Skills Among Pre-Service and In-Service Teachers*

Item	True-false statement	Key answer	Proportion of correct answer		
			Year 1 PST	Year 3 PST	IST
Reading the data					
Q4	Car 1 and Car 2 have different initial speeds	T	0.87	0.81	0.94
Q5	Car 1 had an initial speed of 0 whilst Car 2's initial speed was greater than 0	T	0.72	0.77	0.92
Reading between the data					
Q6	The speed of Car 1 is always greater than the speed of Car 2 at any time	F	0.67	0.74	0.67
Q8	Car 1's speed will be greater than Car 2's speed after a certain time	T	0.82	0.77	0.94
Q10	Car 1's speed continuously increases	T	0.87	0.76	0.94
Q11	Car 2's speed is constant	F	0.58	0.57	0.77
Reading beyond the data Type 1					
Q3	Car 1 and Car 2 will meet if they travel at the same speed for the same duration.	F	0.59	0.41	0.67
Q9	The change in speed for Car 1 and Car 2 differs	T	0.94	0.86	0.91
Q12	The change in speed of Car 1 is greater than Car 2	T	0.78	0.66	0.96
Q13	The change in speed of both cars are constant	T	0.35	0.36	0.41
Q16	The distances covered by Car 1 and Car 2 are different	F	0.13	0.37	0.38
Q17	At all time, Car 1 covered a greater distance than Car 2	F	0.35	0.62	0.67
Q20	The initial position of Car 1 and Car 2 cannot be identified from the graph	T	0.28	0.54	0.52
Q21	The route travelled by Car 1 and Car 2 cannot be identified from the graph	T	0.46	0.51	0.62
Q22	The graph does not show whether or not Car 1 and Car 2 meet at a certain point	T	0.24	0.42	0.42
Q23	The final destination of Car 1 and Car 2 cannot be identified from the graph	T	0.69	0.51	0.88
Reading beyond the data Type 2					
Q1	Car 1 and Car 2 meet in one place	F	0.27	0.26	0.30
Q2	Car 1 and Car 2 meet at the intersection	F	0.11	0.17	0.21
Q7	Car 1 will catch up to Car 2 then overtake it	F	0.46	0.46	0.29
Q14	The initial location of Car 1 and Car 2 always differ	F	0.09	0.25	0.55
Q15	Car 1 left a few minutes after Car 2	F	0.65	0.64	0.56
Q18	Car 1 and Car 2 drove to the right	F	0.21	0.25	0.49
Q19	Car 1 had a steeper route than Car 2	F	0.13	0.20	0.64

Figure 2

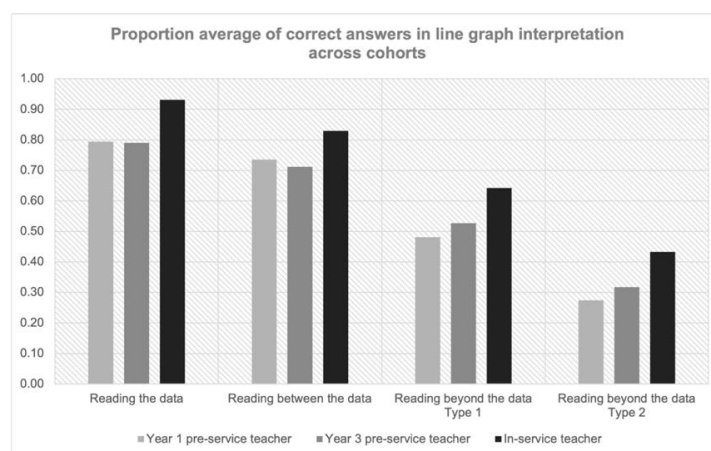
Line Graph Comparing the Proportion of Correct Answers for Each Item Among Three Cohorts



Moreover, the bar graph (Figure 3) compares the average proportion of correct answers in the line graph interpretation across the three cohorts and along the three-tiered framework of Curcio. In-service teachers consistently showed higher proportions of correct answers across all categories, indicating superior graph interpretation skills. Year 3 pre-service teachers performed better than Year 1 pre-service teachers, particularly in the more complex categories, suggesting a developmental progression in their graph interpretation abilities.

Figure 3

Comparative Analysis of Graph Interpretation Skills Among Pre-Service and In-Service Teachers



A Kruskal-Wallis test indicated a significant difference in performance across the three categories of teachers $\chi^2(2) = 50.0$, $N = 256$, $p < 0.01$. The median scores were 11, 12 and 14 for Cohorts 1, 2 and 3 respectively. Post-hoc comparisons indicated significant differences between Cohorts 1 and 3 ($p < 0.01$) and Cohorts 2 and 3 ($p < 0.01$). However, there were no significant differences between Cohort 1 and Cohort 2 ($p = 0.05$). These findings suggest that in-service teachers significantly outperformed pre-service teachers, with the largest gap between Year 1 pre-service and in-service teachers. Importantly, no significant differences were obtained between Year 1 and Year 3 pre-service teachers.

Discussion and Conclusion

This study aimed to explore the differences of line graph interpretation skills across three distinct groups: first-year pre-service teachers (PSTs), third-year PSTs, and in-service teachers (ISTs). The results showed a progression in line graph comprehension from first-year PSTs to

ISTs, but no significant improvement between the first- and third-year PSTs. The findings underscore the importance of practical teaching experience in enhancing graph interpretation skills, although the robustness of this knowledge is questionable as could be inferred by the relatively low proportion of correct answers from the ISTs for some of the items. The lack of significant differences between first- and third-year PSTs raises questions about the effectiveness of the current teacher education curriculum in fostering graph comprehension skills. Despite their exposure to increasingly advanced mathematical content, third-year PSTs did not show marked improvements over their first-year counterparts. This suggests that the existing curriculum may not be adequately addressing the specific pedagogical strategies or content knowledge necessary for developing strong graph interpretation skills. One possibility is that while the curriculum provides mathematical and statistical content, it may not focus sufficiently on the graphical literacy aspects that are crucial for teaching and interpreting data in the classroom. In the furtherance of teacher education programs, it may be beneficial to integrate more targeted instruction on graph interpretation and analysis, emphasising real-world applications of graphing skills (Friel et al., 2001; Shah & Hoeffner, 2002).

Importantly, this study brings into focus some insightful features of linear speed-time graphs, demonstrating why graph comprehension embedded in context can be demanding. Although we did not conduct interviews, yet the questionnaire's design—focusing on various interpretations—allow us to offer potential explanations for deductions made by participants. In general, there is an intuitive tendency to make direct inferences from the graphic display without moving to the next level of thinking as could be inferred when the respondents had to read beyond the data. For instance, Question 3 posits that Car 1 and Car 2 will meet if they travel at the same speed for the same duration. However, the fact that their paths intersect on the graph in Figure 1 does not imply a physical meeting. Thus, an additional level of deduction is required to realise that the point of intersection shows the time at which the two cars have the same speed, and this does not imply that they meet physically. A numerical example serves to instantiate this relationship. For instance, after 10 mins, both car A car B may have a speed of 60 km/h but that does not mean they would have covered the same distance. Thus, a common misinterpretation relates to the point of intersection, mistakenly perceived as physical locations, rather than understanding them within the context of relationships like speed and time, where location is irrelevant. This observation also rejoins what Kosslyn (1985) referred to as visual perception (the visual image of a graph) and graphic cognition (converting a visual image into meaningful information). The teacher education program of the current sample of teachers may not have adequately addressed this important component of teacher knowledge, a point also supported by Jacobbe & Horton (2010) who argued that lack of exposure to content influences graph comprehension.

Similarly, the low proportion of correct answers in Question 19 (Car 1 had a steeper route than Car 2) can be explained by the direct intuitive association that the respondents may have made from the visual appearance of the line graph of car 1 being higher than that of car 2. Furthermore, the research corroborates earlier studies which found that not only students but also teachers frequently interpret graphs as literal representations of events rather than abstract quantitative data (Bell & Janvier, 1981; Leinhardt et al., 1990; Patahuddin & Lowrie, 2019). This misinterpretation underscores the need for educational strategies that enhance the understanding and interpretation of context-based graphical data among teachers, improving their ability to convey abstract concepts effectively.

The findings from this study not only inform improvements in teacher training but also lay the groundwork for future research aimed at enhancing the analytical competencies essential in our data-driven world. However, given the study's focus on a specific graphical task and a regional participant pool, further research involving a broader range of tasks and demographics is necessary. Expanding the scope of research could provide a more comprehensive

understanding of how different educational systems and cultural contexts influence graph interpretation skills, contributing to the development of effective pedagogical strategies. While this study offers insights into graph interpretation across teaching career stages, its cross-sectional design limits the ability to examine individual development over time or identify factors behind group differences. Future studies using qualitative methods, such as interviews, could help explain the lack of progression observed in teacher education.

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Supporting 5 Year Olds to Represent Their Mathematical Reasoning

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This paper illustrates how young students aged 5 years old were supported to develop their mathematical representations. In seven lessons, students progressed from using concrete manipulatives to representing their mathematical ideas in more sophisticated ways. The teacher utilised specific instructional actions including holding high expectations for all students to engage with collaborative discussion, paying close attention to the students' reasoning and representations and drawing on these to extend them further. The findings offer potential for how teachers of young students can support them to develop reasonable mathematical representations.

The New Zealand Mathematics Curriculum (Ministry of Education, 2024) outlines expectations that students are supported to engage in mathematical practices such as explanation, justification, generalisation and representation. New Zealand children begin school when they are 5 years old and bring with them a wealth of prior life experiences and skills. These include making informal representations to show “how many” or “which are longer or shorter”, however, the process of connecting these to formal mathematical concepts requires support (Bobis & Way, 2018).

Mathematical representations can be made in a range of different ways, for example, using concrete materials, manipulatives, or drawing pictures or diagrams, or by using number sentences or formal equations. The purpose of representing mathematics in these ways is to show or explain mathematical reasoning (Bicknell et al., 2016; Bobis & Way, 2018; Pape & Tchoshanov, 2001). Accurately representing mathematical thinking is particularly challenging for young students as representing mathematical reasoning is a complex process requiring students to transfer internal thought to an external medium (Pape & Tchoshanov, 2001). For young learners, there is often an emphasis placed on using some type of materials to support mathematical learning. However, depending on how and why certain materials are used, some students may find it challenging to reasonably represent their mathematical ideas.

Teachers often use material representations when modelling or showing students how to do certain things such as counting using counters or blocks or using tens frames to count and group to ten (Bicknell et al., 2016; Obersteiner et al., 2014). In their study, Donovan and Alibali (2021) investigated how students aged 7-8 years-old used non manipulatives (toys) as “tools for doing maths” (p. 296). While the study showed success in the students manipulating these materials, this occurred through the researcher (in the teacher role) specifically instructing the students to use the toys as tools for mathematics. Donovan and Alibali concluded that it is necessary to make explicit to students *how* materials are to be used to represent mathematical reasoning. A New Zealand study by Bicknell et al. (2016) examined how the teacher made deliberate decisions about which manipulatives 5-year-old students would use in a mathematics lesson involving multiplication and division. For example, when working out the number of egg cartons required for a selected number of eggs, egg cartons and counters were provided to support the students in their representation of division. In a follow-up interview, the teacher in Bicknell et al.'s study shared that at times the students would revert to using representations

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that they were comfortable with rather than engaging with the formal representations modelled by the teacher. To support the students the teacher would either co-construct number equations with the students or remind the students to represent using equations. Pape and Tchoshanov (2001) emphasise the important role teachers play in facilitating students to reasonably represent their reasoning. They highlight that students require regular exposure to multiple representations and the opportunities to practice applying these mathematical representations in mathematical activity. Additionally, Gravemeijer (1999) found that careful teacher scaffolding is required for students to develop their representations from early models to more formal models.

For young learners, the use of drawing as a mathematical representation is a powerful way to articulate thinking (MacDonald, 2018). A study by Roche et al. (2020) with 5-year-old students highlighted how students drew ducks to represent their emerging reasoning, however what was not evident in the drawings was the exact mathematical understanding the students held. Several researchers (e.g. Bicknell et al., 2016; Bobis & Way, 2018; Cartwright, 2023; MacDonald & Murphy, 2019) have shown that while drawings and mathematical representations can provide teachers the opportunity to understand students' reasoning, at times deeper insight is required from the students to completely understand their representation. Additionally, within a sociocultural framework, Pape and Tchoshanov (2001) highlight how representations can be used as tools for explaining mathematical reasoning and that teachers should aim for a balance between student representations and their verbal explanations. In an Australian study with students aged 6-years-old, Herbert and Williams (2023) focused on one teacher's actions to elicit mathematical reasoning. The teacher used in-the-moment-questioning to focus the students to make connections between the language and representations required.

Building on this research literature, we aim to add to the field by investigating how young students just beginning school can learn to develop mathematical representations. In this paper, we outline how one teacher facilitated young students across three mathematics lessons to develop several mathematical representations. The aim of this paper is to highlight how, over a short period of time, young learners can be supported to make and extend reasonable mathematical representations. Our research question is: How can young students be supported to develop mathematical representations?

We situate this research study in a sociocultural framework. Sociocultural theories of learning emphasise the dynamic nature of learning, with key components highlighting how students learn through interaction with others and their setting. Within mathematics classrooms, a sociocultural framing is characterised by students working collectively on mathematical activity and engaging in mathematical discourse to reason about important concepts (Boaler & Sengupta-Irving, 2016).

Research Methods

This qualitative study was conducted in one New Zealand primary school classroom with 10 students aged 5-years-old. The teacher was an experienced junior primary school teacher. The data used for this paper were drawn from these students' first three formal mathematics lessons at school and formed part of a six-week-long investigation examining teacher actions to support young learners to engage in mathematical practices. From their first lesson, the students were both expected and required to communicate their thinking with their peers. All ten students were grouped together heterogeneously. Each mathematics lesson followed a similar structure. Firstly, a mathematics task was presented to the group. The teacher read the problem to the students and then used talk moves, such as repeating, revoicing, adding on, wait time, and reasoning (Chapin & O' Connor, 2007) to engage the students in discussion. This part of the lesson (usually 5 minutes) continued until the teacher was sure all students understood the context and knew what they were expected to find out. The students then worked in pairs

to solve the task together. While the groups worked, the teacher monitored their activity, intervening when she thought necessary to scaffold, prompt, or support them. After 5 minutes working together, the students formed a bigger group where ideas were shared, while the teacher recorded explanations on the board. To conclude the lesson, the teacher connected students' thinking to highlight important mathematical concepts.

Data collection involved classroom observations which were video recorded and later transcribed. These provided opportunities for the researchers to review the data multiple times. In addition, field notes and photographs of student work were collected. Thematic analysis was used to analyse the data. Thematic analysis involves seeking meaning from the data to answer the research questions (Clarke & Braun, 2017). Additionally, comparison between the field notes and video observation occurred to ensure all sets of data were interpreted accurately. For this paper, the authors undertook an initial individual examination of the video recorded observations to identify emerging themes. This included identifying the teacher actions both before and after the representations presented here. The authors then cross checked for validity of emerging themes. The subsequent analysis involved the comparison of the developing themes to ensure they addressed the research question.

Findings and Discussion

In this section, we present the teacher actions that created opportunities for students to develop mathematical representations. They also demonstrate the shifts in teacher actions over the lesson sequence that lead to the students successfully representing their thinking. These teaching moments occurred over seven lessons during an algebra patterning unit.

Initial Teacher Modelling

In the first lesson, the teacher presented a task which required the students to copy and extend an ABCABC pattern using Unifix cubes. The teacher instructed the students to finish their pattern at the 12th cube. While working in pairs, the teacher noticed one group sharing ideas, talking together saying, "yellow, pink, purple, yellow, pink, purple" and using the cubes to support their ideas. The teacher selected the pair to explain how they had constructed their pattern. The two students counted out loud together saying, "one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve" as they connected each cube to the next. While the students explained and showed their thinking to the big group, the teacher wrote the count on the board. By notating the count on the board, the teacher provided an opportunity for all students to see the count and connect what they could hear to the written form.

The teacher then drew the other students into a discussion, stating "I could hear lots of groups counting as they were making their patterns, well done." This statement explicitly emphasised to the students that what they were doing was useful for their mathematical reasoning. As the other groups had successfully copied and extended the pattern to 12 with the cubes, the teacher sought to extend their initial ideas. While drawing squares in a modelling book and colouring these in the colours of the cubes, the teacher explained what she was doing saying, "look this is what we can do. I am drawing a square to represent the cube, then I am colouring it in to show what colour each cube is". Her action of explicitly modelling for the students is similar to what Herbert and Williams (2023) found in their investigation. Additionally, the teacher modelled how to translate a pattern from concrete materials to a diagram. This supports what Gravemeijer (1999) advocates that careful teacher scaffolding is required for students to develop their representations from early models to more formal models. The teacher also notated numbers under the cubes to support the mathematical ideas that each colour represents a position in the pattern, reinforcing their counting prior.

Immediately following her modelling, the teacher then asked the pairs to think about how they could represent their own patterns as a drawing. Figure 1 shows one group's representation.

Figure 4

Student Representation - Drawing



The visual representation highlights how the group were able to extend their representation from materials to a drawing. While all the groups had successfully made their patterns with the materials, this group was the only one to successfully extend to a drawing.

Notably while all the groups were attempting to extend their representations to a drawing, the teacher consistently validated their reasoning saying, “you are all so clever, you have represented your pattern in two different ways; you have represented it with materials and now you have represented it by drawing”. Validating the students’ ideas emphasised the usefulness of representations expressing mathematical reasoning. Additionally, by always naming the mathematical practice of representation, the teacher’s actions support what Selling (2016) found, that naming the mathematical practice facilitates students to learn and use them.

Extending to a Mathematical Equation

Continuing in lesson three, the teacher asked the students for the total number of flowers in one unit of repeat. Chloe responded by saying, “five”. The teacher validated her response by saying “wow, you are such a good mathematician”. Using Chloe’s response, the teacher modelled how their drawings of the unit of repeat could be extended to a number sentence. By revoicing Chloe’s reasoning the teacher emphasised the unit of repeat saying “so you thought if we had four [draws a line under the four red flowers on the whiteboard] and one more [draws a line under the one yellow flower] in our unit of repeat we have five, is that right?” (Chloe nods). These actions highlight the teacher publicly validating Chloe’s reasoning and providing a model for other students. The teacher then wrote numbers under the lines and flowers stating “so four plus one equals five”. Figure 4 shows the teacher’s model on the whiteboard.

Figure 5

Model of Mathematical Equation



The teacher had modelled how the students’ drawing of the unit of repeat could be represented as a mathematical equation. By paying close attention to how the students represented their reasoning, the teacher was able to use this as an opportunity to extend mathematical thinking. She made the students’ implicit understanding of $4 + 1 = 5$ explicit through the modelling of the equation. These actions are similar to what Bicknell et al.’s (2016)

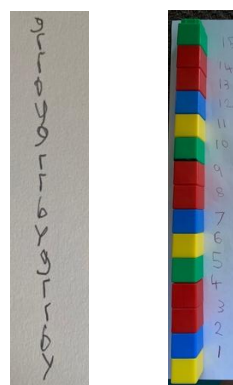
found where the teacher purposefully modelled representations as equations. As the lesson continued, the teacher asked the students to represent the total number of flowers in three units of repeat. Whilst two groups reverted to 1:1 counting, the other two groups attempted the use of an equation. This highlights the potential of exposing students to formal representations early in their mathematics education.

These first three lessons served as a foundation the teacher built on in the subsequent four lessons. Throughout lessons four to seven the teacher continued to be purposeful in her public praise as students represented their mathematical reasoning in various ways. For example, as students were working in their groups solving their task, the teacher could be heard saying “Wow, I love how this group have represented their thinking in two ways”. In addition, the teacher continued to specifically use the term “represent”, for example, she asked a group “share your ideas with us and I will represent it on the board” or “what did we do last week to represent our thinking? We showed our pattern in colours and letters”. The ongoing naming of the mathematical practice of representation supported the students to know and use this practice effectively.

The progression of learning to know and use mathematical representation was highlighted in lesson seven. In this lesson, the students were working on a pattern task that involved working out further points within a pattern (ABCCD). The students were provided one unit of repeat (yellow, blue, red, red, green) and then asked to find the sixth colour and 15th colour of the pattern. During the launch of the task the teacher asked the students “can you work out the colour of the sixth and 15th block and record it – represent your thinking”. This was an open-ended prompt, as the teacher did not specify how the students should solve and represent their reasoning. The students began working in groups, all making the same pattern (yellow, blue, red, red, green) with Unifix cubes. Immediately following their use of Unifix cubes, all students began drawing their patterns in their books. As they did this, the teacher praised one group saying, “I love how Lukes’ group is representing, they are drawing squares to show each block”. This statement prompted a response from Luke, “we don’t know what colour they are.” The teacher replied, “well, we only have pencils today so what could we do?” In their groups, all students immediately began discussing how they could show which block represented which colour. Rather than telling or showing the students how to resolve their dilemma, the teacher provided an open-ended prompt that initiated active engagement from all students and provided an opportunity for them to find a reasonable way to represent their thinking. Figure 5 show two group’s representations.

Figure 6

Group One and Group Two's Representations



Group One Group Two

Group One’s representation shows a tower of letters, each letter representing the coloured blocks y = yellow, b=blue, r=red, g=green. As this group shared their representation with the group, they counted each letter aloud, stating that fifteenth block would be green. Group Two’s

representation shows the tower carefully aligned to numbers that they had written. Both representations reflect the progression from the students' initial representations with concrete materials (Lesson One) to more sophisticated mathematical representations.

Conclusion and Implications

As prior research has found (Pape & Tchoshanov, 2001), supporting students to accurately represent their mathematical thinking is particularly difficult for younger students, as it requires the transference of internal thought to an external medium. However, the progression of sophistication of mathematical representation over a short period of time evident here, highlights how teachers can purposefully support young students to make sense of mathematics. To begin with, the teacher provided concrete materials to support the students to sense-make by representing a pattern. The students were then shown how to transfer from one representation model to another. These findings add to prior research (e.g., Bicknell et al., 2016; Donovan & Alibali, 2021, Obersteiner et al., 2014) focusing on scaffolding students to use concrete materials as tools for learning mathematics. To achieve progression in learning, the teacher then provided explicit models for students to replicate that built on their prior understanding and lead them to further explore ways to show their reasoning. Furthermore, adding to what Pape and Tchoshanov (2001) emphasised, the teacher in this study provided consistent exposure to multiple mathematical representations and opportunities for the students to explore these.

This paper has outlined one model of how young students entering school can progress from representing their mathematical ideas using materials, to using drawings and diagrams to represent reasoning to equations. To achieve this, the teacher used a range of pedagogical actions. From the start, the teacher held high expectations for students to engage in collaborative discussion while working on mathematics activity. Noticeably, she paid close attention to the students' reasoning, drawing on their reasoning and building on the students' ideas to extend them. The teacher carefully scaffolded new learning by explicitly modelling how to transfer concrete representations to visual representations. This meant that by the third lesson, the teacher had afforded the students an opportunity to see how concrete and visual representations of mathematics could be represented numerically.

The data presented in this study are based on a small-scale study. Therefore, more research is needed to explore how teachers can support young students to represent their mathematical thinking in their first few weeks of school across multiple settings. Nevertheless, the study provides a strong exemplar of how teachers can build upon student generated representations to develop mathematical understanding in the moment.

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Benefits of Cognitive Offloading Tools in Mathematics Education

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Mathematical problem-solving places heavy demands on children's developing working memory capacity. This review examines how offloading numerical information using embodied (e.g. finger counting) or external tools (e.g. manipulatives) can reduce cognitive load and improve mathematical task performance. Strategic offloading emerges in childhood; however, it is less often observed in children with lower working memory or mathematics ability. Scaffolding and encouraging children's use of cognitive offloading strategies may promote mathematical competency.

Mathematical problem-solving is heavily reliant on the ability to simultaneously store, manipulate, and update numerical information in working memory (WM) (Peng et al., 2016). Indeed, the relationship between WM capacity and mathematical skill is well-established (Friso-van den Bos et al., 2013; Lee & Bull, 2016), with highly replicated evidence demonstrating that WM capacity predicts mathematical competence across the school years (Lee & Bull, 2016; Raghubar et al., 2010; J. Zhang et al., 2024). WM is a temporary storage space subject to capacity limits that increase with age (Ahmed et al., 2022; Reynolds et al., 2022). Managing the demands of mathematical problem-solving can therefore be challenging for children (Spiegel et al., 2021), particularly when learning unfamiliar and increasingly complex applications of mathematical concepts (Raghubar et al., 2010). This challenge is reflected in research demonstrating that WM is heavily taxed when children possess a novice understanding of a mathematical concept across all years of their schooling (Peng et al., 2016; Spiegel et al., 2021). This review focuses on the role that cognitive offloading, the outsourcing of mental demands to external tools (Risko & Gilbert, 2016), plays in reducing children and adolescents' WM load during mathematics.

Benefits of Strategic Cognitive Offloading

Cognitive offloading is a highly effective method of reducing WM load during learning and memory tasks across age groups (Armitage & Gilbert, 2024; Gilbert et al., 2023). For instance, when tasked with recalling letter strings of increasing length, the accuracy of letters recalled by 11- to 12-year-old children increased when offloading was permitted by writing down letters during their presentation (Iley & Medimorec, 2024). A similar pattern of results was observed in adults, wherein the accuracy of letters recalled from WM increased when given the opportunity to offload (Risko & Dunn, 2015). There is also evidence that for children aged 8 to 11 with low WM capacity, reducing extraneous load (i.e., load on WM imposed by the presentation of information in the environment) improves performance in a WM task (Berry et al., 2019). Ball et al. (2022) demonstrated that young adults with lower WM capacity perform equally well as their counterparts with higher WM during prospective memory tasks when permitted to offload. This result was replicated in later research which concluded that offloading has the potential to circumvent the impact of low WM capacity in tasks requiring participants to remember information in prospective and retrospective memory (Richmond et al., 2025).

Although the benefits of offloading in alleviating WM demands are well-established, developmental research indicates that the ability to strategically offload to mitigate the impact of insufficient cognitive resources does not emerge until the ages of 9–10 (Redshaw et al., 2018). In fact, research shows that younger children typically fail to spontaneously and strategically use compensatory strategies to overcome cognitive limitations (Armitage & Gilbert, 2024), despite the ability to accurately evaluate the difficulty of a task relative to (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 365–372). Canberra: MERGA.

personal ability emerging during the preschool years (Gardier & Geurten, 2024). Although the use of offloading strategies becomes more sophisticated with increasing age (Armitage et al., 2020; Armitage & Redshaw, 2022), there is also evidence that children aged 11–12 occasionally overlook the benefits of offloading (Iley & Medimorec, 2024). Furthermore, children with low WM capacity do not always not recognise the benefits of offloading without explicit instruction (Berry et al., 2019). Overall, this research suggests that until adolescence, children are likely to require scaffolding to support their use of offloading strategies when performing tasks that place a heavy burden on their WM.

In the context of mathematical problem-solving, offloading information could alleviate demands on WM by redistributing numerical computations to external referents (Kirsh, 2010). For example, instead of holding sequential solutions in WM during active problem-solving, children could represent them on their fingers (i.e., offloading via embodied cognition) or even write them down (i.e., offloading to the environment via external artefacts) (Pouw et al., 2014). These strategies are a form of ‘extended cognition’ that free WM resources for immediate use (Clark & Chalmers, 1998). Given the evidence that some mathematical operations may impose larger demands on certain domains of WM (i.e., visual or verbal) at different ages (Y. Zhang et al., 2023), the utility of offloading tools may differ depending on age and problem-solving strategy. Understanding this is important as WM resources can become depleted during complex and successive computations (Chen et al., 2018), especially in early childhood when capacity is most limited (Cowan, 2016). The purpose of this review is therefore to consider how children use embodied cognition and external artefacts in mathematics and determine their utility as cognitive offloading tools to alleviate WM demands during active problem-solving.

The Utility of Embodied Cognition as an Offloading Tool

Extensive research has documented the utility of embodied cognition in learning, problem-solving, and instructional practice (Castro-Alonso et al., 2024; Shapiro & Stolz, 2019). For instance, head tilting has been shown to increase task efficiency by alleviating the cognitive demand of mental rotation tasks (Armitage & Redshaw, 2022; Risko et al., 2014) and gesturing (e.g., pointing, mimicking, signalling) has proved highly effective in enhancing the retention and application of learning material across educational contexts (Castro-Alonso et al., 2024). Representing numerical quantities on the fingers (Crollen & Noël, 2015) is the predominant way embodiment can be used as a cognitive offloading strategy in mathematics.

As finger use is ingrained in early mathematics instruction cross-culturally (Bender & Beller, 2012; Fischer et al., 2024), educational psychologists have long sought to determine the usefulness of finger counting for early mathematical learning. Cumulative evidence indicates that although not essential (Lafay et al., 2013), finger counting is highly-beneficial for the mastery of arithmetic skill and the efficient adoption of internal calculation strategies between the ages of 5 and 8 years old (Neveu et al., 2023; Thevenot et al., 2025). Embodied cognition researchers have attributed this benefit to a sensorimotor view of numerical cognition, which proposes that the abstract concepts of number sense, ordinality, and cardinality become grounded within sensorimotor neural pathways when counting on the fingers (Sixtus et al., 2023; Soylu et al., 2018). Repeated finger use efficiently internalises these concepts (Soylu et al., 2018), before finger counting is abandoned in favour of mental strategies (Poletti et al., 2022). Although well-established, this interpretation does not consider whether children use finger counting as a method of reducing WM demands during mathematics tasks.

Compared to the emergence of strategic offloading in psychological tasks outside the educational context around 9 to 10 years old (Armitage & Gilbert, 2024), there is evidence that finger counting is strategically used to manage cognitive load in early childhood. For example, at the earliest stage of learning mathematics operations, children’s WM capacity is heavily burdened by the need to hold a problem in mind while concurrently recalling number sequence

and steps to be completed (Noël et al., 2004). At age 5, children reduce this burden during single digit addition by instantiating the quantities on separate hands (e.g., $5 + 3$ is represented with three fingers raised on one hand and five on the other) and counting all raised fingers to find the total (Beller & Bender, 2011). As ordinality and cardinality become internalised, children begin to recognise the inefficiency of this method and shift to using their fingers to count onwards from the larger number (e.g., $5 + 3$ is solved by raising three fingers one by one and counting 6, 7, 8) (Dupont-Boime & Thevenot, 2018). As children with higher WM capacities adopt the latter strategy before children with lower capacities (Dupont-Boime & Thevenot, 2018), this indicates that finger counting methods reflect internal capability, with more efficient strategies being adopted as WM capacity increases in early childhood (Thevenot et al., 2025).

In addition to increasingly sophisticated finger counting methods, there is evidence that *when* children use finger counting also becomes adaptive alongside increasing age and ability. For instance, a recent study showed that although finger counting is non-systematic when solving arithmetic problems at age 5, children begin to selectively count on their fingers when they recognise that internal strategies (e.g., arithmetic fact retrieval) are likely to be insufficient for problem complexity between the ages of 6 and 7 (Ollivier & Lemaire, 2024). Furthermore, longitudinal research has shown that finger counting is generally abandoned around age 8 or earlier for children with high WM capacities as mental calculation becomes automatised (Poletti et al., 2022). This indicates that children this age can already engage in a cost-benefit analysis when deciding to use their fingers whereby the cost of laborious finger counting is weighed against the benefit of solving a complex problem faster (Ollivier & Lemaire, 2024). On a similar note, neuroimaging research has shown that the neural networks corresponding to finger use increase in activation when subtraction problems increase in complexity (Berteletti & Booth, 2015). There is also research demonstrating that children aged 5- to 6-years-old are less able to keep track of continuous computations when finger counting is inhibited, particularly if their WM capacity is low (Crollen & Noël, 2015). Collectively, this research indicates that strategic offloading via finger counting emerges much earlier in development than other offloading strategies measured outside the mathematics context, possibly because finger counting is used in culturally specific ways from very early childhood, and because the neural circuits underpinning finger use may support numerical abilities (Sixtus et al., 2023).

The Utility of External Artefacts as Offloading Tools

The utility of external artefacts (henceforth referred to as manipulatives) in learning and instructional practice has been a focus of mathematical educational research for decades (Byrne et al., 2023), with particular interest in the role of concrete objects in learning abstract mathematics concepts (McNeil & Uttal, 2009). Historically, some researchers have advocated for a reduction of concrete supports due to concerns that they limit the transfer of concrete-to-abstract mathematical knowledge (Brown et al., 2009; Kaminski et al., 2008). However, research on whether manipulable objects can improve mathematical competence is inconsistent, with positive findings for primary (Carbonneau et al., 2013; Domino, 2010; Fennema, 1972; Holmes, 2013; Lafay et al., 2019; Sowell, 1989), and secondary school students (Carbonneau et al., 2013; Holmes, 2013), but high heterogeneity and low-quality methodologies across studies (Byrne et al., 2023).

The benefits of using manipulatives in mathematics were traditionally attributed to a need for concrete support prior to the maturation of abstract thinking (Bruner, 1966; Piaget, 1952). However, as benefits have been observed in older age groups (Carbonneau et al., 2013; Holmes, 2013), external objects may support mathematical cognition in other ways (Mix et al., 2010). Whilst several functions of manipulatives have been described in the literature (Pouw et al., 2014), the focus here is on their utility as cognitive offloading tools. From this perspective, manipulatives could be used to represent information that would typically occupy children's

limited WM and serve as a physical referent of otherwise abstract information (Kirsh, 2010; Manches & O'Malley, 2012). Although there were concerns that using manipulatives could prevent children from recognising more efficient mental problem-solving strategies (Manches & O'Malley, 2012), education research has since highlighted the vital role of concrete support as a “learning scaffold rather than a crutch” (Stull & Hegarty, 2016, p. 1). This has reframed the goals of early learning, with more research focusing on the benefits of using manipulatives to develop mature conceptual thinking, rather than on the acquisition of purely abstract knowledge (Mix et al., 2010). Cognitive psychology research aligns with this updated view, with assertions that using the external environment to support internal computation is an integral component of human intelligence (Kirsh, 1995).

From an offloading point of view, several studies have demonstrated that interactivity, that is, the coupling of internal cognitive computations with the physical manipulation of external objects (Vallée-Tourangeau, 2013), can improve arithmetic efficiency by alleviating WM demands resulting from problem complexity or competing task demands (Guthrie & Vallée-Tourangeau, 2015; Ross et al., 2020; Vallée-Tourangeau, 2013; Vallée-Tourangeau et al., 2016). A key aspect of these studies is that rather than assessing the reduction of extraneous cognitive load, i.e., the load imposed by how a problem is presented as in some earlier research (e.g., Martin & Schwartz, 2005), the focus was on the deliberate act of cognitive offloading. That is, the conscious decision to outsource information to reduce internal computation demands (Risko & Gilbert, 2016). It can therefore be said that manipulatives have a role in alleviating WM demands during mathematics if offloading is considered a conscious choice, rather than a result of environmental manipulation.

Rather than focusing on *how* or *whether* children use manipulatives to alleviate their cognitive load, most studies have assessed the utility of interventions involving manipulatives across age groups and mathematics tasks (Byrne et al., 2023; Lafay et al., 2019). Moyer and Jones (2004) did note that children aged 11–14 years old spontaneously use external mathematics tools during mathematics lessons. However, whether children were using manipulatives adaptively to reduce WM load was not analysed. One study has shown that adolescents aged 13–15 years with low levels of mathematics achievement do not spontaneously use manipulatives because they are unfamiliar and they under-value the use of these materials (Hodgen et al., 2024). Indeed, other research has found that children aged 7 to 9 with higher WM capacity and higher intelligence show the most sophisticated use of manipulatives to enhance their mental arithmetic when working under a high cognitive load (Ross et al., 2020). This indicates there may be individual differences in the need for explicit instruction and scaffolding when using manipulatives in the classroom, with the most scaffolding likely needed to support children and adolescents with low mathematical abilities and WM capacities. This is consistent with broader developmental offloading research showing that older children and those with low WM capacities may not consistently recognise the benefits of offloading without explicit instruction (Berry et al., 2019; Iley & Medimorec, 2024).

The utility of different types of manipulatives at different ages has been seldom researched. However, there is evidence that particular written offloading strategies may be more effective than others in helping reduce procedural problem-solving errors for students with different mathematical abilities (Jazby & Pearn, 2015). This idea is yet to be investigated empirically, though it does indicate that there may be individual differences underpinning the utility of written offloading strategies. Additionally, despite embodied cognition researchers asserting that the fingers offer a unique sensorimotor advantage in early mathematics learning (Sixtus et al., 2023; Soyulu et al., 2018), a recent study found that manipulatives are equally as beneficial as finger counting for 4- to 5-year-old children's arithmetic skills (Lê et al., 2024). There is also evidence from adult research indicating that using tokens is more efficient than using prepared drawings or gesturing when solving addition calculations (Guthrie & Vallée-Tourangeau,

2015). Nonetheless, these results do suggest that there may be age, ability, and context specific factors underpinning the utility of different manipulative tools in mathematics. Further research is needed to help educators understand how to foster the benefits and optimal use of different manipulatives across all stages of mathematics education.

Optimising Cognitive Offloading in Mathematics during Development

This review has presented evidence that embodied cognition strategies and external artefacts can support mathematical problem-solving from a cognitive offloading perspective. Despite current accounts of children's offloading behaviours outside of the educational context indicating that strategic offloading emerges around ages 9 to 10 (Armitage & Gilbert, 2024), children appear to strategically use embodied cognition strategies, namely finger counting, to support their calculation skills as early as 5 years of age (Ollivier & Lemaire, 2024). While the abandonment of finger counting around age 8 suggests that finger strategies would be best promoted for offloading purposes in the early primary school years, it is important for educators to consider individual differences in children's developing WM. Indeed, children with low WM capacity and mathematics difficulties show a more persistent reliance on finger counting throughout primary school (Hodgen et al., 2024; Neveu et al., 2023). Discouraging finger use too early could worsen their arithmetic performance by forcing a reliance on internal calculation strategies that deplete WM resources (Crollen & Noël, 2015). It is also important to consider the impact of different mathematical operations on cognitive load as there is evidence that subtraction problems involve the use of procedural counting strategies for longer than other mathematics operations (Barrouillet et al., 2008). This indicates that individual and task factors are likely to influence the optimal use of finger counting as an offloading tool in the classroom.

Less is known about the strategic use of offloading via manipulable objects. One study found that children aged 7 to 9 with higher WM capacity showed the most sophisticated use of manipulatives during mathematical tasks (Ross et al., 2020). This suggests that like finger counting, children appear to be engaging in strategic offloading earlier than generally observed in other psychological tasks, though this is likely dependent on their cognitive abilities. Qualitative research indicates that adolescents with lower mathematics achievement are likely to require explicit instruction to use manipulatives and representations during mathematics problem-solving because they under-value or are unfamiliar with the benefits of these tools (Hodgen et al., 2024). Given that there is a relationship between low mathematics ability and low WM (Peng et al., 2016), this finding is somewhat reflective of research showing that children with low WM capacities do not engage in strategic offloading in non-educational tasks, even when they have benefitted from offloading strategies in the past (Berry et al., 2019). Further research is needed to replicate these findings to investigate the strategic use of manipulatives in mathematics from an offloading perspective, and how to foster its use in children with lower WM and mathematics abilities. There is also a need to determine which offloading artefacts (physical manipulatives, pens and paper, etc) are likely to be most impactful for students of different ages and cognitive abilities in the context of mathematics education.

Overall, using embodied cognition and external artefacts as cognitive offloading tools are useful methods of managing WM demands during mathematical tasks. Whilst more research is needed to determine how to promote and support the use of these tools across mathematical education, the literature suggests that children, particularly those with high WM capacities, show a propensity for strategic offloading when utilising their fingers and manipulatives during active mathematics problem-solving. In contrast, children and adolescents with lower WM and mathematical capacities tend to show a less sophisticated use of these offloading strategies and under-value their utility in reducing cognitive demands. Further research is needed to better understand how to cater for these individual differences and foster the adaptive use of offloading strategies across the developmental period to promote mathematical competency.

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Becoming a Mathematics Teacher: Tracing Teacher Identity Development Throughout an Initial Teacher Education Degree

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In this qualitative case study, we trace the journey of one preservice secondary mathematics teacher through each of their placement experiences during their initial teacher education degree. The purpose of tracing this journey was to examine the way in which teacher identity develops throughout an entire preservice education degree. The findings of this case study reveal how formative and transformative school-based placement experiences are and begin to shed light on the key placement experiences that shape a positive mathematics teacher identity. Practical implications in relation to quality supervision of mathematics preservice teachers are also revealed in the data.

Teacher identity significantly influences teachers' motivation, commitment to the profession, and their resilience in navigating the challenges of the profession (Day, 2018). It is a construct shaped by personal beliefs, professional experiences, and the contexts in which teachers work (Cross Francis et al., 2018). Consequently, understanding how teacher professional identity is formed and sustained is fundamental to addressing key educational issues such as the high attrition of early career teachers (Wyatt & O'Neill, 2021) and the current shortage of Australian mathematics teachers (Australian Teacher Workforce Data [ATWD], 2021). Therefore, it is important to identify initial teacher education experiences that lead to the development of positive teacher professional identity for secondary mathematics educators to ensure graduates are well situated to remain in the profession. This study takes a unique longitudinal view of PST identity development using the theoretical and analytical frames of Self-Determination Theory and 3-Dimensional Space Narrative Structure respectively.

Preparing Preservice Mathematics Teachers for Career Success

Teacher Identity

It is essential that initial teacher education (ITE) ensures preservice teachers are well situated to flourish and remain in the profession. We deliberately use the verb flourish to evoke the literal meaning of the word – “to grow or develop in a healthy or vigorous way, especially as the result of a particularly congenial environment” (Oxford English Dictionary, 2023). With this definition in mind, we are concerned about understanding the impact of ITE experiences on beginning teachers' capacity to flourish in the profession, and thus consider that a flourishing early career teacher is one who has developed a positive mathematics teacher identity.

Teachers' professional identity is dynamic, multidimensional, and contextual (Beijaard et.al., 2004), reflecting “the degree to which a person categorises her/himself personally and occupationally as someone who enacts the roles required of a teacher, engages with the social ties of the profession, and is committed to the career in the future” (Richardson & Watt, 2018, p. 39). The development of teacher professional identity is constantly shaped by interactions between teachers and their environments (Yang et al., 2021). The interplay between multiple teacher identity elements has seen some research distinguish between personal and social identity (Tice & Baumesiter, 2001). Personal identities refer to individual expressions of attributes, goals, values, competencies, and self-concepts (Richardson & Watt, 2018). Social

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identities refer to interpersonal relations and group memberships, and are associated with the group norms. Personal and social identities are related in that group identification is dependent on how well personal identity aligns with group membership requirements. Lasky (2005) describes teacher identity as “how teachers define themselves to themselves and to others” (p. 901), capturing the relationship between personal and social identity. Identity is formed by the network of interpersonal relationships in an individual’s environment; therefore, teachers’ professional identity is something personally held but socially constructed.

A flourishing, positive mathematics teacher identity is shaped by personal and social motivators. Tice and Baumeister (2001) posit that identity begins with motivation and suggests social motivators (the need to belong) are central. The importance of belonging has been emphasised in key motivation theories such as self-determination theory (SDT) which describes how teacher professional identity is influenced by their sense of autonomy, belonging (relatedness), and competence (self-efficacy) (Ryan & Deci, 2017).

In relation to identity development, it is necessary to differentiate between teachers and preservice teachers (PSTs). This paper focus on the identities of PSTs who bring with them values, beliefs, and experiences as students, which may differ with what they experience in the classroom as a teacher. PSTs have a dual role and identity as a student and a teacher, and part of developing a positive teacher identity is the rectification of experiences as a student with becoming a teacher. This makes placement experiences a critical time for identity development, and thus we focus on these experiences in this study.

Self-determination Theory (SDT) and Teacher Identity

Whilst not initially designed to explore teacher identity development, SDT has since been applied in educational research to understand identity formation (e.g., Wong & Liu, 2024). SDT is applicable to understanding teacher identity given it pertains to social and personal influences on individual’s motivations. An individual’s pursuit to personally fulfil the three basic psychological needs of autonomy, belonging, and competence (Ryan & Deci, 2017) can be supported or hindered by their social environments (contexts) in which they work. Autonomy describes control over one’s actions and having ownership over their work and decisions (Ryan & Deci, 2017). Belonging, which some consider the primacy of the interpersonal self (Tice & Baumesiter, 2001), encompasses being connected to those around us and is related to feeling a sense of group membership and group identification. The alignment between personal interests and values and those of the group can also support a sense of belonging. The need for competence describes the need to feel capable, effective, and knowledgeable (Ryan & Deci, 2017). As well as internal influences, Ryan and Deci (2020) propose that environments that encourage embracing challenge, give constructive feedback, and allow for growth opportunities support a sense of competence. The three psychological needs proposed in SDT will be used as the theoretical frame to analyse identity development in this study. Analysis will focus on understanding how specific experiences influence PSTs sense of autonomy, belonging, and competence and therefore their overall identity development.

School-based Placements, Teacher Professional Identity, and SDT

In this study, placements are situated as fundamental ITE experiences as they are opportunities for PSTs to apply their knowledge and develop their professional identify in a way that cannot be replicated at university. The interactions with mentors and students, and the critical self-reflection from authentic teaching experiences that occur during placement help PSTs align their initial beliefs with the realities of the profession (Torres-Cladera et al., 2021). The perception of being recognised as a “teacher” in a school also plays a symbolic role in solidifying teacher identity. Drawing on the theoretical framing of the study, placements should allow for the development of autonomy as PSTs engage in making instructional decisions,

competence by refining their teaching skills, and belonging as they form professional relationships. Diverse placement experiences are also important as they enable PSTs to confront and adapt their pedagogical beliefs and values, thereby fostering a stronger professional identity (Turner & Jumatova, 2020), which explains why ITE programs typically offer multiple placement experiences. During a placement, experiencing and overcoming challenges can be formative for identity. Emotional challenges such as insecurity and perceived failure are common but can act as catalysts for reflection and professional growth (Prabjandee, 2019). The ability of the PST to navigate these challenges with psychological and logistical support can facilitate the internalisation of a positive teacher identity. Thus, placement experiences involve important social interactions and opportunities for personal development that can critically shape a future teacher's identity. The research question that will be answered in this study is: *What experiences during school-based placements influenced a preservice mathematics teachers' sense of autonomy, competence, and relatedness?* Answering this question will add to our understanding of critical experiences that foster a positive mathematics teacher identity.

Research Design

The stories we tell about who we are shape how we interpret the past, understand the present, and who we will become in the future. Our stories (narratives) are how we come to understand and give meaning to our lives. Narratives are not just reflections of experiences but are active constructions that shape how individuals perceive and respond to their world (Clandinin & Connelly, 2000). Given this, the dynamic development of teachers' professional identity is explored in this case study through a narrative research design. Narrative inquiry as a qualitative research design typically involves following the story of an individual (Cresswell, 2012) allowing for a deep understanding of how individual PSTs negotiate their professional identities by making sense of their placement experiences through the stories they tell.

To examine individual experiences told through stories in a narrative inquiry, Clandinin and Connelly (2000) describe a Three-Dimensional Space Narrative Structure (3D-SNS). In this structure, the three dimensions include interaction, continuity, and situation. This framework enables researchers to examine the complex interplay between personal experiences, social contexts, and the temporal aspects of identity construction (Clandinin & Connelly, 2000). Interaction describes the personal (looking inwards) and social (looking outwards) interactions situated in stories, aligning with the view that identity is both an individual and a social phenomenon. Continuity refers to time and how the past, present, and future can be referenced in a story. For PSTs, their narratives can reveal how they rectify their beliefs as a past student, as a present student-teacher, and as future teachers. Situation refers to place and context in which the stories are taking place. The situational dimension underscores the significance of the placement school environment and the available relational and material resources in shaping PSTs' professional identities.

As an analytic frame, 3D-SNS connects to the study's theoretical framing (SDT) by recognising how teacher identity dynamically develops through over time (continuity), through interactions, within specific contexts (situations) to shape an individual's sense of autonomy, belonging, and competence. For example, the 3D-SNS dimensions allowed us to identify how 'interactions' that supported or hindered the fulfilment of a PSTs SDT needs—such as when positive supervisory feedback enhanced a sense of competence. Similarly, 'continuity' helped trace how shifts in autonomy unfolded over time, revealing how confidence and identity are co-constructed. By focusing on the story's PSTs tell about their placement experiences, this study aims to provide a nuanced understanding of how particular 'interactions' and specific 'situations' that occur over time ('continuity') during placements can enhance autonomy, competence, and relatedness, thus fostering a positive professional identity. In analysing the data, deductive codes based on interactions between SDT and 3D-SNS included the following:

autonomy-continuity (past/present/future), autonomy-situation (context), belonging-interaction (school community, students, supervising teacher), competence-continuity (past/present/future), competence-interaction (other teachers, personal, students), competence-situation (context). Coding was carried out in NVivo and findings reported using a narrative structure. One participant was chosen for the case as the goal was to deeply understand the complexity of PSTs experiences during placements throughout an ITE program (given each placement has unique characteristics as expectations of teaching expertise increase). The participant, Mei (pseudonym), was an international student undertaking a 2-year postgraduate degree in secondary mathematics education in Australia. Mei was interviewed each week of her first placement (4 week placement, 4 interviews total), at the midway point and end point of her second placement (4 week placement, 2 interviews total), and at the conclusion of her final placement (4 week placement, 1 interview total).

Findings

Mei's first placement was a time of nervousness and excitement. During the first week of the placement, Mei experienced small threats to her competence when experiencing an Australian high school for the first time. For Mei, the unfamiliar Australian school context and teaching in her non-native language were key challenges. Mei felt these difficulties were exacerbated by being placed in an all-boys school, where she felt that female teachers had their authority challenged ("it's tough for female teachers"). Mei tackled these challenges through positive interactions with other teachers, who reassured her that managing behaviour in classrooms was a constant challenge even for expert teachers. During this week, Mei's positive interactions with her supervising teacher and students reinforced her sense of belonging and she felt cared for by her supervisor ("my supervisor will ask me every day are you feeling comfortable"). Mei also reported feeling competent when she had positive student interactions despite feeling challenged by their behaviour – "when I help them to finish one question, they then smile at me and say, 'thank you Miss'. I feel a sense of achievement".

Similar themes continued into week two as Mei continued to wrestle with her autonomy in finding her own teaching style whilst developing her competence in managing classroom behaviour. When struggling to gain student attention ("they just don't obey my instructions"), Mei was self-aware of her emotional response when her competence was challenged and noted "it is hard, it is a real skill to try not to take it personally...it's a real funny thing of learning how to act angry but not feeling angry or frustrated or disappointed". To overcome this, Mei was supported with positive encouragement from her supervisor. At this midway point through placement, Mei liked the high levels of structure her supervisor was providing through the provision of a specific daily timetable as they helped ease anxiety. This indicates that Mei did not yet seek autonomy but rather appreciated specific direction. This week included a turning point where Mei started to consider making her own pedagogical choices, such as including group work and ICT, rather than following the explicit instruction model shown by her supervisor. However, she lacked confidence in these decisions and felt it would threaten her ability to manage the class: "They might not do the tasks, they might keep always chatting". Her supervisor reinforced these fears and Mei expressed frustration at this: "my supervisor just suggests for us to write on the whiteboard...but I think when we receive the teaching from uni, we are encouraged to use technology". This highlights a tension between her training and growing desire to make pedagogical decisions, and the direction from her supervising teacher.

In week three, Mei grappled with her perceptions of how a classroom should be run and her current teaching context. For example, Mei expressed frustration that colleagues did not check homework, showing she was developing her autonomy and competence in her ability to confidently express her own teacher beliefs. Mei had strong opinions about this based on observations: "they didn't have the solid foundation...most of them are struggling with how to

calculate a negative number". This shows Mei beginning to consider her own autonomy and realising that mathematics classrooms have diverse learners. Overall, Mei's identity was still developing as she was changing her teaching style depending on who was observing her.

By the conclusion of her first placement, Mei transitioned from feeling overwhelmed to a growing feeling of confidence and fulfillment. Her competence improved as she reflected "I have gradually found out how to deal with students, how to communicate with students". Mei reflected on how proud she felt when the students said they would miss her. She also felt a great sense of competence when seeing a struggling student perform well on a test – "I was so excited when I saw his quiz. It's just like, wow". The stories Mei chose to tell upon concluding her placement demonstrated how social interactions with students supported both her sense of self-efficacy and belonging. This was a shift from week one where Mei sought substantial reassurance from her supervisor. Given classroom management remained her primary concern, this was a stated goal for placement two. Mei's other goal was being "confident when I introduce the topic" due to language challenges. Whilst Mei was shifting her concentration from internal to external matters, her primary focus remained internal. Regardless of challenges, she affirmed the experience was enjoyable and that she had transformed from "nervous, also even panic...but after the placement, I think the worries, the nerves are just gone".

For her second placement experience (~6 months after the first placement), Mei attended a coeducational public school. Mei reported midway through that she was enjoying the experience and felt well supported by her supervising teacher. Her sense of competence was growing in relation to managing diverse student groups ("I know I have to use different strategies for different students") with the support of her supervisor ("She's very good. Taught me a lot about how to manage students in different classes...it's very useful"). Mei's growing self-efficacy was expressed through her reflections that "I think it's a bit better than my first prac...I know how to deal with students and how to organise my lesson plans". During this placement, Mei was developing her own teaching style and began experimenting with teaching strategies but recalled challenges integrating tools like ICT: "Some of the students are actually playing, but...I feel like it's not so helpful for those students who are not", showing critical reflection. At the conclusion of the placement, Mei reflected "I did enjoy the placement, I learned a lot...I think I progressed a lot.", and highlighted a memorable experience: "They were very sweet...I said this is going to be our last lesson, and some girls they just said 'no, Miss, I'm going to miss you'. I said, 'I'm going to miss you too'." This shows Mei becoming an appreciated part of the school community. Her goals going into her final placement remained centred on managing classroom behaviour, as she was still negotiating the tension between authority in the classroom and creating positive student relationships ("I don't want them to hate me, but sometimes I have to do it").

For her final placement (~4 months after placement two), chose to remain at the same school as she enjoyed her second placement and felt familiar with the systems. During this placement Mei was given greater independence to teach classes without her supervisor in the room. Her supervisor expressed to her "my standards will be stricter" as it was Mei's final placement, and Mei stated "I feel a bit stressed" due to this. Mei noted this stress before some lessons as said, "every lesson I will take a deep breath before I went into the classroom". Mei told a story about one challenging lesson: "one boy just said 'you are not even my teacher, why should I listen to you'...heartbreaking". Incidents like these highlight the difficulties PSTs face in establishing credibility, particularly in mathematics where content authority can be contested. During these experiences, Mei felt supported by her supervising teacher and noted that her needs as a teacher with English as an additional language were supported by her supervisor providing her with specific wordings to say in response to particular situations "so if I am in the same situation, I can just remember the sentence". This shows a shift in Mei from prioritising student approval, to understanding professional responses to situations and the value of consistency in responses

to students. By the conclusion of all placements, though some classes specifically challenged Mei, she did feel that she continued to progress in her ability to independently manage classes, with the advice of her supervisor. However, her perceived competence still made her feel that “for my future career, I might start with just casual or part-time, as full time is quite hard for me”. As well concerns over managing classes independently, Mei’s worries were also driven by workload observations – “I noticed my colleagues from [the] maths staff room, they were always busy with writing reports, marking their papers, preparing the lessons, and some of them might have full periods on the same day...I don’t think I can handle it”. Despite these challenges, Mei’s highlight of placement was described as follows: “every lesson I feel a sense of achievement...after every lesson they would say thank you Miss, see you next time...I feel so good every time when they said it to me”, showing how the interpersonal interactions continued to be key for Mei.

Discussion

Mei’s placement experiences throughout her ITE program show growth from a nervous beginning teacher to an increasingly confident mathematics teacher, demonstrating how dynamically formative and transformative placement experiences are for teacher identity development (Torres-Cladera et al., 2021). In this section, the key themes related to autonomy, belonging, and competence will be discussed. In unpacking Mei’s experiences, 3D-SNS elements of interaction, situation, and continuity can also be seen.

As Mei’s confidence increased and she gained greater control over her teaching choices, her journey reflected an identity shift from hesitation to autonomy. Autonomy was closely tied to her sense of competence, emphasising the relationships between SDT elements (Ryan & Deci, 2017). Across placements, Mei’s confidence in making pedagogical decisions increased as she gained experience in seeing a range of schools and classes. However, Mei’s experiences also showed tensions for PSTs in negotiating classroom authority. Rather than being solely negative experiences, these challenges proved to be formative catalysts (Prabjandee, 2019).

Through her placements, Mei’s autonomy was supported by gaining greater independence from her supervisor. By the final placement, Mei felt she could begin to make her own decisions about pedagogy and classroom management, and was more focused on her own teaching decisions rather than seeking student approval. However, as expectations of independence increased, it brought raised levels of stress as supervisor oversight lessened. This finding is consistent with literature on the psychological demands of early-career teaching, which emphasises the need for structured yet gradual release of responsibility to avoid burnout (Wyatt & O’Neill, 2021; Yang et al., 2021). Mei’s growing autonomy came in conjunction with the realisation of critical professional challenges, such as recognition of workload demands, which caused a rethinking of future career plans. This emphasises what is already understood in research about teacher identity influencing commitment to the profession (Day, 2018).

In relation to Mei’s sense of belonging, her placement experiences illustrated a journey from seeking approval to professionalism. Belonging was critical to Mei’s teacher identity development, showing the importance of the social aspect of identity (Tice & Baumesiter, 2001) and how identity is shaped by the context in which teachers work (Cross Francis et al., 2018). Mei’s experiences demonstrate the importance of a collegial environment for PSTs on placement. Mei’s story is one of growing professional confidence that was substantially supported by strong, positive supervisor relationships and strong collegial support. Social interactions with teachers and students were key sustaining experiences for Mei, highlighting the importance of embedding PSTs meaningfully in the school community. Navigating classroom authority with students was not a simple task for Mei, and it was an aspect of teaching she constantly reflected. What Mei came to learn, through supervisor guidance, was the importance of balancing consistency and discipline with developing student relationships. For

PSTs, the lack of natural authority they have during placements is a tangible challenge (Vetter et al., 2013) particularly if a supervisor is no longer present in the classroom, and supervisors need to be aware of this issue when mentoring PSTs. As a result, they may need to give practical strategies that differ from how an established teacher might gain authority in a classroom.

Mei's sense of competence similarly grew through each placement and was shaped by internal factors as well as external interactions with her supervisor, other teachers, and students (Ryan & Deci, 2020). Whilst Mei's self-efficacy grew substantially, questions about teaching mathematics remained at the conclusion of placements, highlighting that early career mathematics teachers are not yet the "complete package" when entering the profession (i.e., still have developing competence and confidence to teach mathematics). There were additional challenges to Mei's competence as she had to quickly understand the Australian educational system and each unique school context. This draws attention to the potential need for universities, schools, and supervisors to provide additional support to international students. Mei's competence centred around two differing themes: managing student behaviour and teaching mathematics. Classroom management was Mei's top concern and focus during the early weeks of her first placement, highlighting that it is difficult for some PSTs to concentrate on the intricacies of teaching mathematical content during the first placement. By the conclusion of Mei's placements, she did feel substantially more confident in managing behaviour, but understood that her skills were still developing. Despite this, Mei slowly shifted from frustration to strategic thinking about behaviour management. In relation to teaching mathematics, Mei developed from reluctant compliance with her supervisor's pedagogical directives to experimenting with pedagogy. This shift was fostered by her growth in autonomy, which fed a sense of competence in making her own teaching decisions.

Whilst Mei's journey demonstrated a PST who was gaining confidence in their ability to teach mathematics, this in turn caused new concerns over the sustainability of a full-time role. This was driven by a change in Mei's focus on the immediate present situation to reflection on the future, showing how past experiences shape future aspirations and actions. Understanding the demands of the job combined with judgements on her perceived competence influenced Mei's future professional decisions before graduating from ITE. Previous research has identified the influence of self-efficacy on teachers' career commitment (Richardson & Watt, 2018). This emphasises how critical placements are for establishing a robust workforce in secondary mathematics education, and how carefully we must treat placement experiences. In Mei's case, three positive placement experiences in collegial supportive environments still resulted in an outcome of questioning how feasible her teaching career was. If preservice secondary mathematics teachers are already rethinking their capacity to remain in the profession *before* entering the profession, this is a key matter for ITE programs and further research to consider and address proactively. In Mei's case, being highly self-aware of not being the "complete package" prior to graduation was shaping her future career plans. A limitation of findings presented in this paper is the exploration of one teacher's unique experiences, thus further research could attend to diversity of PSTs narratives using 3D-SNS.

Conclusion

If we are to address critical global concerns regarding recruitment and retention of secondary mathematics teachers, it is important that we pay attention to how we are situating PSTs to flourish as they enter the profession. School-based placements are a crucial element of ITE, offering PSTs opportunities to integrate content knowledge with practical experiences in real classroom settings (Turner & Jumatova, 2020). Positive placement experiences, characterised by supportive mentorship, play a significant role in shaping a robust professional identity (Cross Francis et al., 2018). As teacher identity dynamically shaped by contextual and

experiential variables, the extent to which ITE fosters identity growth will impact early-career teachers' resilience and long-term commitment to teaching (Beijaard et al., 2004).

This narrative study of one preservice teacher's placement experiences demonstrated a growth journey in terms of autonomy, competence and belonging which was supported by effective supervision and collegial school environments. Mei exhibited a transformational trajectory from apprehension about her ability to teach mathematics and gain classroom authority, to increased confidence in making pedagogical decisions and managing classes. However, it should be again stressed that PSTs like Mei are still developing a robust teacher identity upon exiting ITE. As previous research has highlighted, early career teachers require continued support to solidify their professional identity and sustain motivation in teaching (Wong & Liu, 2024). Without structured induction and professional learning opportunities, the positive gains made during ITE may be at risk of erosion due to the pressures of the profession (Wyatt & O'Neill, 2021). Overall, this study reinforces the argument that well-supported placements are crucial for sustaining a resilient mathematics teaching workforce (Day, 2018).

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Scripted Solutions for Mathematics? Rethinking Outsourced Curriculum Planning and Teacher Workload Challenges

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Curriculum reform is an opportunity to refresh programs to meet the needs of current students, but this is not without significant investment from teachers. In this exploratory multiple case study, semi-structured interviews captured teachers' processes of curriculum interpretation for mathematics planning and teaching in a school using scripted curriculum materials. Findings show that when using the scripted materials, the teachers in this study still felt compelled to engage in additional planning to prepare adequately for teaching their students. This finding has implications for proposed teacher workload solutions that suggest teacher curriculum planning can be outsourced.

Curriculum change is a constant feature of the Australian educational landscape. As teachers adapt to each new curriculum, they engage in a process of interpretation to determine the most effective ways to engage their students and teach the outlined concepts and skills (Ross, 2024; Superfine, 2008). The literature often highlights two key dimensions of this process: the intended curriculum and the enacted curriculum (Ben-Peretz, 1990; Ellis, 2004; Reid, 2008). The intended curriculum encompasses the materials or policy documents provided to teachers, usually by curriculum agencies or government bodies. In contrast, the enacted curriculum refers to the collective learning and teaching interactions within the classroom, shaped by the teacher's intentions and students' responses. Australian curriculum discussions have widely supported this dichotomy (Reid, 2008). However, they do not capture the process of interpretation that a teacher must undertake to enact the intended curriculum.

Curriculum reform offers opportunities for teacher professional learning (Moore et al., 2021) but also adds to teacher workloads. Many reports are emerging with descriptions of the current state of work intensification in teaching (Creagh et al., 2023; Wilson et al., 2020), with Creagh et al.'s (2023) synthesis indicating that interventions must address the root causes of workload to tackle teachers' time poverty effectively. While governments propose solutions to ease teacher burdens, curriculum work is being framed as an administrative task (Stacey et al., 2024), leading to calls for outsourcing curriculum planning. However, research shows that outsourced curriculum materials can reduce teacher satisfaction (Poulton & Mockler, 2024), and many teachers value lesson planning as a key part of their role (Stacey et al., 2024). This paper examines five teachers' experiences teaching mathematics with scripted curriculum materials to explore whether they truly saved time.

Literature Review

In seeking to conceptualise the process of curriculum enactment, Remillard and Heck (2014) developed a framework of the curriculum policy, design and enactment system to provide a heuristic for studies that investigate the "relationships among elements of curriculum and influences on them" (Remillard & Heck, 2014, p.714). Their framework (see Figure 1), specific to decisions made with mathematics curricula, acknowledged the various curricular elements contributing to mathematics curriculum development, design and enactment (Jameson & Bobis, 2023). The research presented artefacts of mathematics curriculum interpretation decisions, such as the teacher-intended curriculum and student outcomes, highlighting the distinction between the intended (official) and the enacted (operational) curriculum. Their definitions describe the official curriculum as specified by governing authorities and setting out

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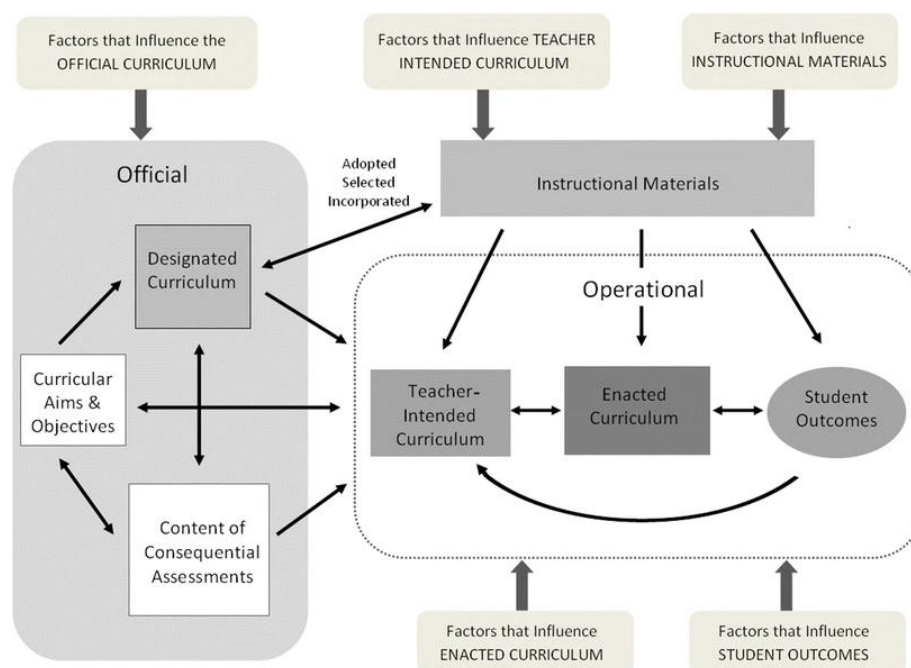
expectations for students' learning. The operational curriculum represents the transformation of the official curriculum into teachers' personal plans, and how these plans play out in the interactions between teachers and students in the mathematics classroom.

Remillard and Heck (2014) highlighted factors that influence elements of the official curriculum and the operational curriculum: these factors “may be social, political, cultural, structural, or cognitive” (p. 714). Influencing factors may include societal needs, values, expectations, and beliefs; views of individuals and groups wielding power; research on learning, teaching, and assessment; teacher knowledge, beliefs and practices; teachers' access to resources and support; contextual opportunities and constraints; and a range of student characteristics and cultural resources. These factors interact with each other in complex ways, and their influence on the curriculum system may be either direct or subtle (Ross, 2017; 2024).

Remillard and Heck's (2014) framework articulates other curricular elements that shape the official and operational curriculum for teachers. The official curriculum includes the Designated Curriculum, defined by Remillard and Heck to refer to the set of supporting materials for mathematics teachers informed by the official curricular aims and objectives and specified by a governing body. Their elaboration of this curricular element encompasses approved textbooks or packages of teaching resources to mandated system-wide assessment. These materials are distinct from Instructional Materials in the framework, which are positioned outside of the official or operational curriculum boxes to denote that these materials are typically not authored by those developing the official or operational curricula.

Figure 1

Visual Model of the Curriculum Policy, Design, and Enactment System (Remillard & Heck, 2014, p. 709)



Teacher-Intended Curriculum

Teacher planning is an important phase of teaching, giving teachers time and space to make decisions about the content and instruction that they will use to shape student learning in mathematics (Superfine, 2008; Ross, 2024). The concept of teacher curriculum planning, referring to the process by which teachers interpret the curriculum into a plan for enactment, has been inconsistently addressed in the literature (Ross, 2017). It has been described as aligning with the intended curriculum, as the teachers' intention of what they will cover (Ellis,

2004; Sherin & Drake, 2009), while other researchers have positioned it as an aspect of curriculum enactment (Solomon, 2009). Curriculum planning supports teachers in enlivening and making the curriculum content accessible to the students in the classroom; the blueprint, which the teacher devises for covering the intended curriculum, but not the act of teaching the curriculum in the classroom. It is an interpretation developed by the teacher to unpack the curriculum and tailor it to meet the needs of the students in their classroom (Kurz et al., 2010; Solomon, 2009).

Remillard and Heck (2014) described the teacher-intended curriculum as an interim step between the official and the enacted curricula, representing the process of interpretation and planning that teachers must engage in to envision and plan mathematics classroom instruction (Ross, 2017). While effective classroom instruction begins with thoughtful planning (Bieda et al., 2020), Remillard and Heck's (2014) studies of mathematics teachers suggested that the teacher-intended curriculum is often the hardest to observe, as it is in its most detailed form within the teacher's mind. However, growing accountability measures require teachers to produce planning artefacts in various forms and degrees of specificity from as far back as their initial teacher education (Ross, 2024). These processes are now under scrutiny as governments try to solve teacher workload issues with a frequent trend towards providing teachers with scripted curriculum materials to ease the burden of curriculum interpretation (Mockler & Stacey, 2024).

Scripted Curriculum Resources

There is a current trend in the Australian educational landscape to position curriculum planning as an administrative burden of teachers, with growing suggestions that a potential solution to workload issues in teaching can be found in outsourcing curriculum planning (Stacey et al., 2024; Ross, 2024). Shulman (1983) used the phrase “remote control of teaching” to refer to situations where teachers feel they have limited autonomy to make decisions in their classrooms, essentially making them feel like they are following instructions from a remote control. These “classroom-ready” resources provide the day-by-day approach to teaching that can leave teachers feeling as though they no longer have agency in planning for their specific group of students (Poulton & Mockler, 2024). Further, removing the teacher engagement with the curriculum may have the effect of de-professionalising teachers by ceasing their need and capacity to work with curriculum policy to develop student pathways that best support student development, as they frequently take the form of a “one-size-fits-all approach to instruction” (Remillard & Reinke, 2012, p.3). These highly structured curriculum guides have been referred to as “scripted curriculum”, positioning teachers more as performers than professionals.

What happens, then, when the Designated Curriculum replaces the Teacher-Intended Curriculum? In Queensland, during the implementation of the Australian Curriculum: Mathematics (published by the Australian Curriculum, Assessment and Reporting Authority (ACARA)) (n.d.), government schools were given the Curriculum into the Classroom (C2C) materials by the Department of Education (DoE) (2015). The C2C materials, a comprehensive suite of whole school, year-level, unit-level, and lesson plans, and assessments with pre-written marking criteria, were provided to schools to ease curriculum reform fatigue by fast-tracking the interpretation process (Ross, 2017). The C2C materials were delivered with mixed messages about their use, which were refined to an “adopt or adapt” mantra (Ross, 2024). Teachers in schools that “adopted” the C2C materials were told to use them as written. In this circumstance, the Designated Curriculum is being provided to the teacher in place of their process of curriculum interpretation with the specific aim of replacing the Teacher-Intended Curriculum as their intended plan for enactment. Do teachers feel compelled to enact the scripted curriculum unaltered? The following question frames this paper:

- What process of curriculum interpretation is undertaken by primary teachers using scripted curriculum materials to teach mathematics?

Research Design

Context and Participants

This paper describes part of the research from a larger Doctoral study analysing teachers' processes of curriculum interpretation to implement the Australian Curriculum: Mathematics (ACARA, n.d.). Exploring Remillard and Heck's (2014) framework based on studies of mathematics curriculum enactment, this paper focuses on the curriculum interpretation process undertaken by primary teachers of mathematics using scripted curriculum.

The five teachers in the study were part of the teaching staff at a government primary school in Southeast Queensland. The teachers expressed interest in participating in the study, were all female, and each had over 10 years of teaching experience, having taught mathematics across several primary school year levels. As the school had decided to *adopt* the DoE's C2C (2015) materials, the teachers were enacting curriculum plans developed to support them in implementing the Australian Curriculum: Mathematics (ACARA, n.d.).

Data Collection

Using an exploratory multiple case study design, this project sought to capture the process of teacher curriculum decision-making while enacting a primary mathematics unit of work. Pre- and post-unit semi-structured interviews aimed to capture the processes of preparation for and reflection on the enactment of the unit. Semi-structured interviews enable open questioning to capture detailed narratives of the participants. Yin (2012) highlighted their value in eliciting richer data and identified interviews as a key data source in case studies.

Reflective journals kept by teachers during the mathematics unit sought to provide a continuous capture of the curriculum decisions that teachers made during the unit and the influences on those decisions. Journals allow the participant to reflect upon events as they occur rather than utilising memory after some time has passed. It also provides a mechanism to triangulate data collected via other means within the study (Janesick, 1999). The teacher journals were used during the post-unit interviews as a stimulus for reflection.

Data Analysis

The process of thematic analysis employed was based on Braun and Clarke's (2014) six phases of thematic analysis. The data gathered for each participant was considered as a whole and analysed individually; that is, all three items for one participant (pre- and post-unit interview transcripts and unit journal) were analysed before analysis commenced on the items from another participant. This paper reports findings from the theme, *curriculum interpretation*, which was derived from teachers' references to their process for engaging with the curriculum.

Data coded to the theme *curriculum interpretation* was used to develop diagrammatic representations of each teacher's pathway from the Australian Curriculum: Mathematics (ACARA, n.d.) to its enactment in their classroom, thereby representing their curriculum interpretation and enactment process (all representations can be seen in Ross, 2017). The representations were developed for comparison purposes.

Results and Discussion

The diagrammatic representations of each teacher's curriculum interpretation and enactment process provided a picture of their sequential path from the official curriculum to enactment in their mathematics classrooms. Comparisons of the representations provided an opportunity to synthesise the general process of curriculum interpretation and enactment used

by the teachers to implement the Australian Curriculum: Mathematics (ACARA, n.d.). The results of this paper present the general process of curriculum interpretation and enactment and discuss the implications of this process in relation to Remillard and Heck's (2014) framework.

Scripted Curriculum Materials as the Official Curriculum

When the teachers commenced their implementation of the Australian Curriculum: Mathematics (ACARA, n.d.), their school had made the decision to adopt the C2C (DoE, 2015) materials as written. While the messaging from the DoE was to "adopt or adapt", meaning changes could be made to the scripted materials, many schools felt compelled to implement the materials as they were written (Ross, 2017). As schools recognised that the C2C materials had been developed from the Australian Curriculum, they saw the C2C materials as a feasible starting point for their curriculum implementation. Hillary described that it provided her with all she needed to engage in curriculum planning, "[I] mainly stay with the C2C. You don't have a lot of timeframes. All the other subject areas that you're working on...too much diversion...it [C2C] seems to cover it" (Pre-unit interview: Hillary). Hillary did not see a need to consult the Australian Curriculum on ACARA's website. Similarly, Miriam did not feel the need to consult the Australian Curriculum: Mathematics, as she felt the C2C materials and other accompanying supports were sufficient in cascading the information to teachers "I don't go to ACARA [website] much at all because ... we're fortunate that we've got Amy (Curriculum Coordinator) and Lesley (Deputy Principal), and they both do a fabulous job of passing that information through to us in our planning sessions." (Pre-unit interview: Miriam).

Miriam's comments are indicative of how the teachers in this study engaged with the materials. Recognising the C2C (DoE, 2015) materials were an interpretation of the Australian Curriculum: Mathematics (ACARA, n.d.), C2C materials were consulted as if they were the curriculum, illustrated by Lyn's comment referring to the C2C materials as the Australian Curriculum: Mathematics "I mean, we are doing time again this term, but if you are not doing it all the time, they forget so quickly. That is another - a little problem I find with the maths curriculum." (Post-unit interview: Lyn). The Australian Curriculum presents Curricular Aims and Objectives by year level rather than specifying topics in terms, as Lyn has referred to. In the same way, Winnie stated: "I know I keep talking of Australian Curriculum, but that's just what we follow, the C2C." (Post-unit interview: Winnie), denoting that the teachers see the C2C materials as interchangeable with the Australian Curriculum: Mathematics. The consideration and treatment of the C2C materials by teachers in the study positions them firmly as an aspect of the official curriculum in the teachers' minds.

Scripted Curriculum Materials as the Operational Curriculum

While the teachers may have viewed the C2C (DoE, 2015) materials as though they were the official curriculum, the intention of the materials, when adopted, was to provide schools and teachers with ready-made planning documents. The C2C materials were developed to fast-track the process of interpretation and provide teachers with the plans ready for enactment. As Hillary describes, the C2C materials were quite prescriptive.

They [the C2C materials] specifically guide you through with the expectation of the objectives. You just follow those through. If you get the time to do every single lesson, it's all extremely prescriptive for it to guide through to the final guides to making judgements (Post-unit interview: Hillary).

While, as Hillary expresses, the mathematics materials were available for teachers to enact as written, all five teachers in the study felt it necessary to develop a classroom plan based on the C2C (DoE, 2015) materials. Most of the teachers in the study cited differentiation or preparing students for assessment as the key reasons for their adaptation to the C2C materials. Lyn explained her concerns for a lack of opportunity to differentiate the content: "There is a lot of directed teaching in it [the C2C materials], which makes it hard to then differentiate because

those children are not getting your modelling ... If you are modelling to one group what are the others doing?" (Post-unit interview: Lyn). Miriam shared these concerns, stating, "I don't think that there has been enough time for us ... to model that [process] for them [the students]" (Post-unit interview: Miriam). The teachers felt compelled to alter the C2C materials to provide opportunities to differentiate for their students.

Winnie also suggested that time constraints meant she struggled to work through all the mathematics content, instead opting for a more direct route to ensure her students had the necessary learning for assessment success:

I just made sure that I covered everything that needed to be covered for their assessment. I know that's sort of teaching to a test in a way, but I'm not going to put something in front of them that we haven't worked at together. (Post-unit interview: Winnie).

Abigail shared Winnie's concerns about time constraints in teaching mathematics content, but described a path that was led by her students' prior knowledge of the content or topic

Time restraints was the biggest one...I guess it's not a huge deviation, but it's more a condensing of everything...choosing the most important parts to teach and having to...let go of a few other bits that might not have been as worthy or that were too time-consuming. (Post-unit interview: Abigail).

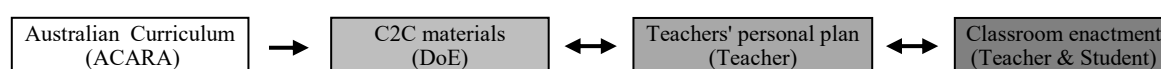
Each teacher felt it necessary to develop a classroom plan based on the C2C materials. C2C was provided to teachers with the message that it would fast-track curriculum planning. This did not eventuate. While the intention of the C2C materials may have been to minimise the curriculum planning time that teachers were required to invest, in practice, they were still devising personal curriculum plans. The C2C materials did not "fast-track" the process of interpretation by removing the requirement to unpack the curriculum. Instead, the teachers simply began the same process from a different starting point. The teachers began planning with an interpretation of the intended curriculum rather than from the intended curriculum itself.

Impact on Curriculum Interpretation

While the teachers felt the C2C (DoE, 2015) materials needed adaptation before enacting them in the classroom, they did not feel compelled to engage with the original official curriculum, that is, the Australian Curriculum: Mathematics (ACARA, n.d.), to begin their process of curriculum interpretation. Acknowledging that C2C had been developed from the Australian Curriculum, they instead opted to use C2C as their starting point for planning. Figure 2 represents a model of curriculum interpretation and enactment of the teachers in the study.

Figure 2

General Model of Teacher Interpretation and Enactment



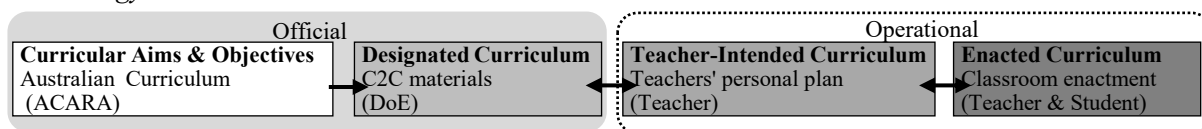
The model (Figure 2) denotes the Australian Curriculum: Mathematics (ACARA, n.d.) has been used to develop the C2C (DoE, 2015) materials by the DoE, but the arrow is unidirectional, as once this interpretation has taken place, the Australian Curriculum was not included in the process of curriculum interpretation undertaken by the teachers in this study. The C2C materials were instead used as the basis for teachers' interpretations of their classroom intentions, rather than the Australian Curriculum. These aspects align with curricular elements from Remillard and Heck's (2014) framework but are not necessarily in the same configuration. The general model developed from the teacher experiences in this study (Figure 2) has been overlaid with Remillard and Heck's terminology in Figure 3.

The Australian Curriculum: Mathematics (ACARA, n.d.), developed by ACARA, provides the Curricular Aims and Objectives for teachers to teach. A different government agency (the Queensland DoE) developed the C2C (2015) materials as a set of instructional materials to

support teachers' interpretation of the Curricular Aims and Objectives. Thus, the intent of the C2C materials aligns with Remillard and Heck's (2014) definition of Designated Curriculum.

Figure 3

General Model of Teacher Interpretation and Enactment with Remillard and Heck's (2014) Terminology



In Remillard and Heck's (2014) framework, the Curricular Aims and Objectives and the Designated Curriculum comprise elements of the Official Curriculum together (see Figure 1). The arrows connecting these elements in their framework allow for engagement with both elements, in addition to the Content of Consequential Assessments, describing flexibilities inherent in the different systems of curriculum policy, design and enactment the framework is designed to represent. Whereas, in the way teachers in this study used the C2C materials and not the Australian Curriculum, the Designated Curriculum (C2C materials) became an intermediary element between the Curricular Aims and Objectives and the Teacher-Intended Curriculum, as teachers felt the need to adapt the C2C materials to support the needs of their students. In doing so, teachers created their Teacher-Intended Curriculum from the C2C materials. To an extent, Remillard and Heck (2014) pre-empt this eventuality, describing systems where "nested governing agencies" (p.710) specify different components of the official curriculum. As is the case in Australia, the state-based agencies are responsible for implementing a federally developed curriculum. Thus, Remillard and Heck's (2014) framework accommodates this relationship.

Conclusion and Implications

There is a key resource implication that emanates from the relationship between the Curricular Aims and Objectives and the Teacher-Intended Curriculum highlighted in this study, are these curricular elements supporting teachers with their curriculum making, or duplicating processes undertaken by teachers that assist with developing their knowledge of curriculum and students? Either way, current teacher workforce issues are leading to calls for more scripted curriculum materials to ease the burden of curriculum interpretation. However, the notion that materials like C2C will remove the necessity for teacher curriculum planning was not seen in the teachers' actions in this study. The teachers in this study still needed to invest time in developing their plans to best meet the needs of their students (Superfine, 2008). Teachers value this time to prepare for their students (Stacey et al., 2024); thus, teacher curriculum planning cannot be "fast-tracked" as easily as has been described in the current curriculum landscape.

The evidence presented in this study is based on a small sample of teachers from one school involved in the implementation of the Australian Curriculum: Mathematics (ACARA, n.d.). Consequently, further research is needed to explore the impact of scripted curriculum materials in other schools and circumstances. Additionally, further research could be undertaken to compare the processes and products of curriculum support across teacher backgrounds and jurisdictions implementing the Australian Curriculum: Mathematics. Regardless, this study provides strong support for the importance of teacher curriculum planning to ensure they feel adequately prepared to support the needs of their students.

Acknowledgements

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"Anyone Can Teach Maths": Workplace Perspectives on Out-of-Field Teaching of Mathematics

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The pervasive shortage of mathematics teachers in Australia has resulted in a critical reliance on out-of-field (OOF) teaching, significantly impacting student performance. This study shares insights from a scholarly inquiry in progress aiming to map the complex educational ecosystem that perpetuates OOF teaching. This paper examines the positioning of OOF mathematics teachers within (or absent from) the current policy and organisational strategy documents that are supposed to support them. It explores analyses and interpretations using methodologies relating to the professional capital of the researchers considering them, with implications for future research.

The escalating teacher shortage across Australia and globally has precipitated a crisis, compelling school leaders to implement emergency measures to staff classrooms. An increasingly widespread response has been to assign teachers to teach subjects or levels of schooling for which they are not qualified – that is, to teach out-of-field (OOF) (Hobbs, 2013). This practice, evidenced by recent data such as the estimated 40% of Mathematics teachers teaching OOF (Australian Institute for Teaching and School Leadership (AITSL), 2021), carries significant implications, including diminished student academic performance (Van Overschelde, 2022), increased teacher attrition (Sharplin, 2014), and compromised teacher confidence and sense of professional identity (Du Plessis et al., 2015). While existing upskilling initiatives, such as graduate certificates and micro-credentials, may offer potential avenues for addressing this issue, their efficacy is challenged by deeply ingrained cultural norms that often fail to adequately recognise and value teacher specialisation (Hobbs et al., 2022), necessitating a critical re-evaluation of current strategies.

It is critical for effective workforce planning to address the prevalence of OOF teaching through targeted professional education (PE) (professional learning and re-specialisation) and related strategies (AITSL, 2021), highlighted with specific reference to mathematics by Goos and Marchant (2025). Recognising the urgent need to understand the factors influencing OOF teachers' engagement with PE, a research project was designed seeking to map the ecosystem that perpetuates OOF teaching (Ross & Hobbs, 2024). The project, supported through an Australian Research Council grant, is entitled "Shifting the Culture of Out-of-Field Professional Education for Teachers" (www.scope-t.org). This investigation explores key elements, including school priorities for OOF teacher learning, the impact of policy on PE culture, the

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roles of stakeholders in its provision and discourse, and the systemic incentives that promote PE aimed at developing in-field teaching capabilities. The researchers operate under the premise that clearly defined pathways to “in-field” status will increase the perceived value and uptake of PE, thereby reducing OOF teaching (Hobbs, 2020). Ultimately this research aims to model an education ecology that fosters a cultural shift, valuing the re-specialisation of OOF teachers through PE and addressing the complex challenges inherent in systemic reform.

The broader project seeks to identify the systemic factors that hinder the recognition and value of OOF teacher re-specialisation through PE across the four subject domains of English, geography, mathematics, and science. This report presents a scholarly inquiry in progress, offering an emerging map of the current educational ecosystem that sustains OOF teaching in one of the four domains, that is mathematics. This paper specifically contributes to knowledge by examining how OOF mathematics teachers are positioned—or overlooked—within the current policy and organisational strategy documents that are supposed to support them. Additionally, it explores methodologies related to professional capital used during policy analysis and highlights the current state of workplace-based support for OOF mathematics teachers seeking to access PE.

Literature Review

Considering the Out-of-Field Phenomenon

The prevalence of OOF teaching in Australia has been obscured by a historical lack of systematic data, particularly concerning teacher subject specialisations (Weldon, 2016). This data gap has contributed to a tendency within government to normalise OOF teaching as a pragmatic solution to persistent teacher shortages rather than addressing the root causes. While upskilling programs like graduate certificates and micro-credentials exist, their effectiveness is limited by prevailing cultural norms within schools and education systems that often undervalue teacher specialisation (Hobbs et al., 2022). This cultural resistance is deeply embedded within the mechanisms that recognise and monitor specialisation, including data collection practices, teacher registration policies, and school improvement frameworks.

Research consistently emphasises the heterogeneity of OOF teachers (Hobbs, 2020), highlighting the variability in their support needs, willingness to remain in OOF roles, and inclination to pursue additional qualifications. Teachers are assigned to OOF positions for diverse reasons, ranging from temporary placements to expected career trajectories, and encompass both novice and experienced teachers (Donitsa-Schmidt et al., 2021; Nixon et al., 2017; Hobbs, 2013, 2020). To capture this complexity, Hobbs et al. (2020) proposed a multidimensional definition of OOF teaching, considering factors such as qualifications and specialisations, subject allocation stability, developing subject identity, and pathways to ‘in-field’ through formally recognised re-specialisation programs. Moreover, school context and support culture significantly influence teacher commitment, self-concept, and confidence.

This heterogeneity necessitates nuanced approaches to motivate OOF teachers to engage in PE. Policymakers and PE providers must recognise that reported incidences of OOF teaching (e.g., AITSL, 2021; Weldon, 2016; Thomson et al., 2017; Shah et al., 2022) do not directly translate to the proportion of teachers willing to pursue additional qualifications. Not all OOF teachers are intrinsically motivated to gain expertise in their assigned subjects (Donitsa-Schmidt et al., 2021; Lünne et al., 2021), highlighting the need for targeted interventions that address the diverse needs and motivations of this teacher population.

Mapping an Ecosystem

The diversity of OOF teacher needs has led us to employ Bronfenbrenner’s (1994) ecological systems theory to provide a comprehensive framework for analysing the complex

dynamics surrounding OOF teaching and PE. Originally conceptualised to understand child development within a nested system of influences, Bronfenbrenner's theory has proven increasingly valuable in educational and social contexts for examining intricate interrelationships between systems (Callingham et al., 2017). Bronfenbrenner's model posits that development occurs within a series of interconnected systems: the microsystem (immediate environment), mesosystem (interactions or connections between microsystems), exosystem (external settings that indirectly affect the individual), macrosystem (social and cultural values), and chronosystem (changes over time). The theory poses a series of nested environments. In our study the teacher is positioned at the centre and is shaped by their immediate context as well as the proximal and distal contexts of the broader ecological settings. Applying this lens to OOF teaching enables a shift from focusing solely on individual teacher experiences (e.g., Hobbs, 2013; Donitsa-Schmidt et al., 2021) or student outcomes (e.g., Van Overschelde, 2022) to a broader examination of the cultural and systemic factors that perpetuate this phenomenon. As Darling-Hammond (2010) argues, teacher quality is influenced by a complex interplay of factors beyond individual teacher attributes, including policy contexts and organisational structures. Bronfenbrenner's theory allows for a nuanced exploration of the PE ecosystem, encompassing initial teacher education (ITE), research communities, PE providers (universities, government, private organisations), schools, professional associations, and government agencies.

This research adopts an ecological perspective, aiming to move beyond descriptive analyses of OOF teaching to a more holistic understanding of the systemic factors that drive its prevalence. Bronfenbrenner's (1994) ecological systems theory was selected for the study, to position policies and other documents within the relevant system they influence, allowing for the development of a map of the ecosystem that underpins and perpetuates OOFness, while also providing avenues to teachers becoming in-field, that is, via in-the-workplace training, through PE, or through requalification via Initial Teacher Education. The study has developed preliminary maps based on ecological theory to portray each of the four curriculum areas being considered.

This paper reports on the initial work undertaken by the mathematics team to develop a map of the mathematical terrain, drawn from academic and grey literature, including state/territory and federal policies, industrial background and organisational strategy documents. Due to the nature of these documents, their influence centres their effects on the mesosystem and exosystem, that is, documents such as policy documents and strategy documents provide guidance about how to fix a specific issue (Parliamentary Education Office, 2023), thus they influence connections between environments (mesosystems) or external settings that indirectly affect individuals (exosystems). The documents considered for this study represented perspectives on OOF teaching in mathematics relating to workplace support (e.g., AITSL, 2021; DuPlessis, 2020; Weldon, 2015), initial teacher education (ITE) and retraining (national policies and university courses; see e.g., Australian Government Department of Education, 2022; Barker et al., 2024; Hobbs, 2020), and PE available to teachers (national and state subject associations; see e.g., Centre for Social Impact, 2022; Goos & Marchant, 2025). This paper reflects on the process used and themes uncovered from the mathematics team's analysis to answer the question, *how do policies and publications of professional organisations influence workplace representations of and support for teachers currently teaching mathematics out-of-field?*

Methodology

Context and Method

This paper describes the process used as a part of the larger study, *Shifting the Culture of Out-of-field Professional Education for Teachers (SCOPE-T)*, to identify literature and policy documents focused on research and commentary on PE for OOF teaching (Hobbs et al., 2025). The purpose of the review was to ensure a representation of the current state of PE was mapped before considering how it could be adapted to better support multiple pathways to in-field.

The process for policy review and analysis involved multiple researchers collecting a range of documents from professional associations, teacher registration bodies, government departments and agencies, unions, and other learned societies with a specific group of researchers analysing documents relevant to their area/s of expertise, e.g., the authors of this paper as mathematics teacher educators and researchers were allocated all documents related to mathematics education. Researchers undertook a process of document analysis to consider aspects of the documents that directly mentioned or were related to OOF teachers or teaching, including direct references to OOF or indirect references (e.g., new or returning teachers to a subject or learning area). Following this analysis, the researchers from the mathematics group met to discuss their findings and determine emerging themes. During the discussion, the researchers were able to simultaneously check the reliability of their analysis and draw on their professional capital to discern meaning and implications from the findings.

Data Analysis and Professional Capital

Hargreaves and Fullan (2012) conceptualised professional capital as the importance and effectiveness of professional work. They proposed three linked aspects of professional capital: human (an individual's attributes and competencies), social (the quality of professional interactions), and decisional capital (the ability to make informed judgments). In an educational context, professional capital encompasses the qualities and values of individuals within educational settings (Gillies, 2015). Hargreaves and Fullan (2012) describe professional capital as a product of the interplay between human, social and decisional capital. Beyond individual talent, professional capital emphasises the collective sharing and networking of expertise, offering a framework for analysing decision-making at both individual and group levels. Oates et al. (2021) explored the use of professional capital to describe the influences on mathematics teacher educators' decisions in course design, and their study supports the use of professional knowledge as an integral part of the discourse analysis (p. 351)

As mathematics teacher educators and researchers, the researchers drew upon a discourse analysis approach using their human, social and decisional capital to consider themes emerging from the document analysis. The three researchers leading the mathematics analysis and development of the map independently considered the documents and compared themes and the appropriate placing of these with the ecosystem. The emerging themes pertained to opportunities and challenges for teachers to access pathways to becoming in-field primarily emanating from the mesosystem and exosystem due to the nature of the documents being analysed. A contextual understanding was also required in interpreting the focus and purpose of the documents. While the thematic analysis may not have been as rigorous as commonly employed approaches (Oates et al., 2021), the analysis was drawn from the interplay of the document analysis with the professional capital of the researchers, whose expertise in the field reinforced credible contextual understanding. Further, the researchers were able to draw on the professional capital of a wider community of expert mathematics educators and researchers when testing the emerging themes in round table presentations at conferences (e.g., Oates et al., 2024; Ross et al., 2024).

Findings and Discussion

The findings and discussion report on key themes emerging from the analysis of research literature, policies and strategy documents relating to mesosystem and exosystem influences on OOF teachers in the workplace.

Mesosystem: Myths From the Workplace

When testing/analysing themes identified in the data/literature with mathematics teachers and mathematics teacher educators, the research team found emerging themes that identified a prevalent pair of contradictory myths that stem from school leadership practices (the mesosystem); contradictory to each other, but also, in some ways, to the literature that underpins them. The first myth shared by teachers and teacher educators (Ross et al., 2024) is a belief from school leadership that *anyone can teach mathematics*. It is brought about by the consideration that every person engages in some mathematics every day. Therefore, anyone should be able to teach mathematics. This notion devalues the specialised knowledge and pedagogical content knowledge required for effective mathematics instruction, suggesting that mere content familiarity is sufficient. Research consistently demonstrates that effective mathematics teaching requires a profound understanding of mathematical concepts (Hill et al., 2005). Further, mathematical pedagogical content knowledge is not just knowing the correct mathematics or how to teach it, but a deep understanding of the ways that students come to mathematics. This can be, for example, knowledge of the misconceptions and issues students typically face and expertise in how to assist students in working through the wrong answers to work towards correct ideas. The myth that anyone can teach mathematics not only disregards the complexity of mathematics education but also contributes to the normalisation of OOF teaching, hindering the development of targeted interventions and reinforcing the systemic undervaluation of mathematics teacher specialisation (Weldon, 2016).

The contradiction comes from an ongoing belief that *not everyone can learn mathematics*. This myth seems to be used in relation to students, suggesting that while we expect every teacher to be an expert in mathematics because they engage in the subject matter in incidental situations in their daily lives, we do not expect all students to be able to learn mathematics. The question is about the underpinning philosophy of these contradictory myths. Is the omnipresence of these myths a result of our belief that students are on a learning journey and may not reach the desired destination at the exact time planned for, or is it foreseeable that one myth is feeding the other? That is, if we ask OOF teachers to teach mathematics, do we believe that we have set the students up for success in mathematics? These are areas for further exploration.

Exosystem: Barriers to Targeted Workplace Interventions

A recurring theme identified in the research studies analysed for this study is that targeted interventions appear to be devalued in practice (Hobbs et al., 2022). As Barker and colleagues (2024) examined in a recent report, *Analysis of Out-of-field Secondary Mathematics Teacher Upskilling Initiatives in Australia*, there is a diverse landscape of programs designed to address OOF mathematics teaching, this includes several universities across Australia that are offering courses to support OOF mathematics teachers. The report reveals a diverse landscape with varying degrees of alignment to initial teacher education standards, highlighting that some programs focus on bridging specific content gaps while others aim to develop broader pedagogical skills necessary for effective mathematics teaching. However, even with incentives (such as no or minimal fees) and despite growing numbers of OOF mathematics teachers (AITSL, 2021), these courses are not attracting large numbers of teachers wishing to re-train or re-qualify to teach mathematics.

Addressing OOF teaching necessitates more than simply providing upskilling opportunities; it requires a nuanced understanding of the systemic and individual factors that influence teacher participation in professional education. While targeted professional development can equip OOF teachers with essential tools and support, it is not a singular solution to the broader teacher workforce crisis (Weldon, 2016). As Hobbs and Porsch (2021) highlight, even when relevant professional development is accessible, OOF teachers may choose not to engage. This reluctance can stem from various factors, including a preference to allocate professional time and resources to their in-field specialisations, a lack of intrinsic motivation to improve their OOF teaching, or the intermittent nature of their OOF assignments, which may diminish perceived long-term career relevance. Furthermore, the apprehension of being permanently reassigned to the OOF subject can act as a significant deterrent, discouraging teachers from pursuing further training in that area (Hobbs & Porsch, 2021). This reluctance is further exacerbated by the practices of hierarchical PE structures within schools that limit teacher choice and tie participation to annual development plans that may favour the teacher's in-field area. Alternatively, PE targets are frequently satisfied by mandatory PE that is generic and non-subject-specific (e.g., annual fire safety training) rather than focusing on adequately developing subject matter knowledge for teaching. This is not unique to mathematics; however, with the volume of teachers reportedly teaching mathematics OOF (AITSL, 2021), it is a concern to ensure students have appropriately qualified and/or supported teachers.

Teachers and teacher educators shared their perspectives that while on-the-job learning can offer immediate workplace support, it risks reinforcing knowledge gaps and may be insufficient for building capacity, particularly when OOF teachers lack access to high-quality mathematics teaching models within their schools (Ross et al., 2024). It can also reinforce a teacher's lack of knowledge. Goos et al. (2023) emphasise the crucial role of PE in equipping teachers to effectively manage OOF teaching. Teacher engagement in PE is often enhanced when they recognise learning as an ongoing process that leads to increased self-efficacy (Bugwak, 2021). This recognition can be challenging when the systems and structures of schools do not support ongoing development. For example, OOF teachers can often lack the necessary time to engage with the incidental PE available through engaging with in-field colleagues during school meetings. A teacher who is OOF for mathematics may only have one or two classes per week. During school subject meeting structures, they will be asked to attend in the area most aligned to their work, thereby missing the PE that emanates from discussions about pedagogy, assessment or moderation. Further consideration of the supporting structures within schools is needed to consider how these can be flexibly placed to ensure OOF teacher support.

Reflection on Themes in Relation to Professional Capital

The three aspects of professional capital (Hargreaves & Fullan, 2012) provide a frame for considering the intricacies and complexities of PE to support OOF teachers. Consideration during the document analysis was given to the importance of developing OOF teachers' individual skills and knowledge, including the current suite of courses available to teachers to address content gaps and pedagogy contributing to development of human capital. Additionally, consideration for the relational aspects of teacher PE in formal settings and on the job considers the development of social capital, while systemic factors from the exosystem explicating structures and policies influencing OOF teachers' ability to make informed decisions about their professional learning contribute to their decisional capital. In Hargreaves and Fullan's (2012) conceptualisation, all three aspects were essential for the development of professional capital, and this analysis has emphasised this necessity for the development of PE that truly supports OOF mathematics teachers.

Conclusion

The complex motivations relating to OOF teachers and PE outlined above underscore the need for a comprehensive approach that acknowledges the heterogeneous experiences of OOF teachers and that addresses the underlying systemic issues that contribute to the phenomenon. Recognising the variability in current offerings for PE, Goos and Marchant (2025) have proposed the technical specifications for a nationally consistent framework for PE to support OOF teachers to work toward qualification. The program, delivered in partnership with universities, would consist of a Graduate Diploma with options for staged entry or early exit to allow for flexibility to meet the needs of the teachers requiring it. The proposal goes beyond the qualification component to recommend supports such as subsidised tuition fees, leadership activities and teaching relief time to support teachers undertaking this study beyond the development of content and pedagogical knowledge. This is a start.

The multifaceted challenges associated with OOF teaching, as evidenced by analyses of policy and strategic documents, reveal that upskilling alone is insufficient. The complex motivations influencing OOF teachers' engagement with PE, alongside systemic barriers point toward further research to map the complex ecosystem that both supports the ongoing need for PE and to consider the breadth of possible avenues to supporting teachers to become in-field should they wish to do so.

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Affordances of a Novel Mathematical Manipulative: Teacher Perspectives on the Keyboard

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This study explores two Year 2 teachers' perceptions of the affordances of a novel mathematical manipulative, the Keyboard, based on their observations of its use in the classroom. Analysis of the semi-structured interviews indicated that the Keyboard's linear structure, design features, and hands-on nature promoted flexible number strategies, relational reasoning, and student engagement. It was viewed as a valuable complement to other tools, with potential applications extending beyond its initial design. These findings contribute to research on the role of specific manipulatives in primary mathematics education and highlight implications for instructional practice.

Mathematical manipulatives can be defined as “concrete objects that can be used to help students understand and solve mathematics problems” (Bouck & Park, 2018, p. 66). Hattie et al. (2017) argued that using physical manipulatives alongside visual, symbolic, verbal, and contextual representations can facilitate deep learning by enabling students to “make concepts concrete and visible, look for patterns, make connections, and form generalizations” (p. 170). Indeed, there is evidence to suggest that incorporating mathematical manipulatives into the teaching and learning of primary mathematics can positively impact student achievement (Byrne et al., 2023), although the magnitude of such effects is contingent on the specific ways in which manipulatives are implemented, including:

- *when* (length of instructional time, with shorter and medium durations yielding greater effects than extended periods),
- *how* (level of instructional guidance, with high guidance proving more effective),
- *who* (students' developmental stage, with greater effects observed for those at the concrete operational stage),
- *what* (mathematical topic, with stronger effects for fractions compared to arithmetic or algebra), and
- *which* (type of manipulative, with neutral designs—e.g., plain wooden blocks—being more effective than those resembling real-world objects, such as plastic money or pizzas) (Carbonneau et al., 2013).

In terms of research into teacher perspectives on mathematical manipulatives, teachers commonly believe that manipulatives improve student understanding, provide multisensory learning opportunities, and make abstract ideas more concrete (Marshall & Swan, 2008; Quigley, 2021). They are also widely regarded as a means to enhance engagement and enjoyment in mathematics (Tan, 2020; Quigley, 2021). However, manipulatives alone are not necessarily sufficient to improve students' mathematical understanding, as their effectiveness depends on how they are used (Marshall & Swan, 2008). Additionally, studies have identified several constraints on their use, including limited resource availability, the time required for setup and organisation, and teachers' confidence and pedagogical knowledge in integrating them effectively (Tan, 2020; Marshall & Swan, 2008).

We note that in previous studies into teacher beliefs about mathematical manipulatives, it is rare that specific manipulatives are named. Consequently, the reader is left wondering which (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 397–404). Canberra: MERGA.

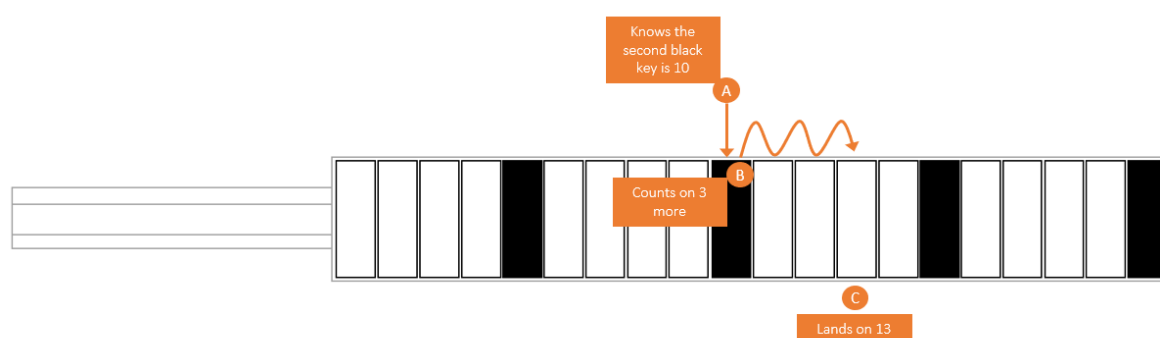
manipulatives, in which grade level, and for which content, did the teachers have in mind as they expressed their views. To address this gap, we conducted a small-scale teaching experiment in two Year 2 classrooms, introducing a novel manipulative (the Keyboard) and gathering observational insights from classroom teachers. The research examined: *What are the perceived affordances and constraints of a novel mathematical manipulative (the Keyboard) as identified by Year 2 classroom teachers?*

Design and Intended Affordances of the Keyboard

The Keyboard is a newly developed manipulative designed to support students in engaging with number relationships through visual-spatial reasoning. It consists of a handle and a linear board with 20 evenly spaced keys, each representing a number from 1 to 20, but without numerical labels. To aid navigation, every fifth key is black, providing subitising cues that may help students structure their thinking around landmark numbers (e.g., 5, 10, 15, 20). The absence of written numerals ensures that locating numbers requires students to use relational strategies rather than passive number recognition. The design further supports iterative structuring, as multiple Keyboards can be linked together by sliding the handle into the end of another Keyboard thereby increasing the number of keys. For example, joining four keyboards creates a set of 80 keys and maintains a scaled subitising framework where students can see four sets of 20, four black keys per Keyboard, and four white keys between each black key. This layered repetition has the potential to encourage flexible numerical reasoning and deepen students' understanding of number magnitude and structure (Hopkins et al., 2023).

Figure 1

A Student Locates 13 on the Keyboard



Preliminary findings from Hopkins et al. (2025) provide the first empirical evidence that the Keyboard may support students in transitioning from counting-based strategies to retrieval-based strategies when solving single-digit addition problems. That study involved a one-on-one intervention with a Year 4 student who predominantly relied on counting strategies, using microgenetic methods to track changes in strategy use across 20 sessions. Structured activities and games using the Keyboard were introduced across seven consecutive sessions, with minimal explicit instruction to allow for intuitive exploration. Analysis of trial-by-trial strategy data revealed a shift toward retrieval-based strategies, including both derived-fact strategies (e.g., knowing that $8 + 5$ is the same as $8 + 2 + 3$, and that $8 + 2 = 10$, with 3 more giving 13) and direct retrieval (e.g., just knowing that $8 + 5 = 13$). Notably, once the student no longer had access to the Keyboard, they did not revert to counting strategies, suggesting that the Keyboard may support the internalisation of more efficient mental computation strategies.

Beyond its potential to support retrieval-based strategies (Hopkins et al., 2025), the Keyboard may also foster broader aspects of number sense and relational reasoning. Because it emphasises positional relationships over explicit quantity-based grouping, students must

engage in spatially driven problem-solving when determining numerical values. For instance, to locate 13 (see Figure 1), students might first find the second black key (10) and count on three—or simply move forward three by subitising—or they might locate the third black key (15) and count back two. These actions can strengthen their understanding of numbers in relation to each other. This positional thinking could also support reasoning about numerical proximity, where students can directly compare magnitudes by placing small cubes beneath the Keyboard to visualise which number is closer to a reference point (e.g., "Which is closer to 16? 14 or 19?"). Additionally, by linking multiple Keyboards together, students may notice recurring positional patterns, such as how 7, 27, 47, and 67 always appear two keys after the first black key on each Keyboard. These features align with Siegler and Ramani's (2009) findings that linear number board games significantly enhance young children's understanding of numerical magnitude. Their research suggests that structured spatial representations, such as linear board games, help children develop a clearer sense of numerical order and relative magnitude. Similarly, the Keyboard's linear format may provide a useful spatial framework for reinforcing numerical relationships, as students must actively locate, compare, and navigate numbers through spatial reasoning rather than relying on symbolic recognition alone.

Compared to other mathematical tools, the Keyboard shares similarities with number lines and structured counting frameworks but offers unique advantages. Like bead strings, it provides a linear representation of number relationships, but unlike bead strings, it emphasises positional relationships rather than counting in groups. Similarly, while tens frames support subitising, they structure numbers in base-ten arrangements, whereas the Keyboard encourages iterative subitising across a larger scale, especially when multiple Keyboards are linked. Moreover, unlike bead frames and tens frames, the Keyboard allows multiple numbers to be represented simultaneously, enabling students to reason about relative distance and numerical difference. Perhaps most significantly, its lack of numerical labels sets it apart from number lines and number charts, as students must construct numerical meaning through spatial reasoning and relational thinking, rather than simply identifying pre-written values.

While Hopkins et al. (2025) provide promising early evidence of the Keyboard's instructional potential through a one-on-one intervention study, the current paper represents the first classroom-based investigation of the Keyboard's use, focusing specifically on teachers' perceptions of its affordances and constraints.

Method

Study Design and Participants

This study is best characterised as a teaching experiment (Cobb, 2000), in which the researchers introduced a novel mathematical manipulative (the Keyboard) into two Year 2 classrooms through a structured, two-week (eight-lesson) teaching sequence. The aim was to investigate how classroom teachers perceived the affordances and constraints of the Keyboard, based on their observations of the lessons. Giovanni (Class A) had eight years of teaching experience, exclusively in Years 2 and 3, and was more familiar with using manipulatives for developing early number sense, addition, and subtraction. Michael (Class B) had taught for seven years, primarily in Years 3 and 6, and was teaching Year 2 for the first time, with less experience using manipulatives designed for early number learning.

Semi-structured interviews were conducted at two time points and transcribed for analysis. Giovanni's interviews captured his evolving perspectives across all eight lessons observing the Keyboard in use, while Michael's interviews contrasted his observations of lessons involving a bead frame in the first four sessions with those involving the Keyboard in the subsequent four sessions, allowing for a direct comparison of the perceived affordances of each manipulative. Data were analysed using thematic analysis, with themes generated inductively from participant

responses. The analysis prioritised capturing teachers' perspectives in their own terms, ensuring that key insights emerged naturally from the data.

Throughout the lessons, both teachers were positioned in an observational role rather than leading instruction. This allowed them to focus on noticing how students interacted with the Keyboard and how instruction was structured to support learning. Stepping out of an active teaching role can help teachers attend more closely to student thinking and pedagogical decisions that might otherwise be overshadowed by classroom management demands (Clarke et al., 2013). By observing rather than directing instruction, teachers were able to form reflections based on classroom observations and interactions, which informed their insights during interviews. Finally, we note that roles were deliberately delineated in this study: the lesson sequence was collaboratively planned by the first and fourth authors, but all teaching was led solely by the first author; the Keyboard was designed by the second author; and the interviews with the observing teachers were conducted by the third author. This research was approved by the Monash University Human Research Ethics Committee (Project ID: 36277). All participants provided informed consent prior to their involvement in the study.

Lesson Structure

The broader teaching experiment consisted of eight 45-minute mathematics lessons, with the same lesson content delivered to both classes. Each lesson followed a three-part sequence:

1. Familiarisation with the manipulative (5–10 minutes): Students engaged in whole-class activities designed to help them develop familiarity with the structure of the manipulative. For example, they might be asked, "Show me 17 on the Keyboard. How did you find it? Is there another way you know it's 17?" These tasks encouraged students to develop a sense of its spatial and numerical organisation, recognising key patterns and relationships that underpin its design.
2. Mathematical game (20–30 minutes): Students played a structured game where the manipulative was used as either a game object or a thinking tool. Class A used the Keyboard in all lessons, while Class B used a bead frame for the first four lessons before transitioning to the Keyboard for the final four lessons. An example game was a modified version of Nearest to the Gnarly Number (Russo, 2017), where students aimed to get as close as possible to a target number using the available cards.
3. Mathematical investigation (10–15 minutes): The game was extended into a problem-solving task requiring deeper reasoning. For instance, students might be asked, "If the Gnarly Number was 19, and my opponent's total was 16, what might my total have been if I had won? Show me on the Keyboard (bead frame) and explain your reasoning". These investigations encouraged students to engage in strategic thinking and numerical comparisons while also deepening their understanding of the manipulative's structure.

Findings

Our thematic analysis of the interview transcripts resulted in the identification of four interrelated themes that described specific affordances of the Keyboard. These are unpacked below, with quotations from the two teacher-participants used to illustrate each theme. While the interview prompts were open to both affordances and constraints, the two teachers did not explicitly raise any limitations of the Keyboard in their reflections.

Linear Representation of Number

The linear representation of number was perceived as a particular strength of the Keyboard, especially in contrast with the bead frame. Michael noted the similarities with a number line, as students could track increasing magnitude along one dimension.

One problem with the bead frame is – well... One strength the keyboard has, it's just a number line... You don't have to worry about it going down to the next line and then losing count.

Michael also noted that, when using the Keyboard, students were less inclined to adopt inefficient counting-based strategies, and more inclined to work from known benchmark numbers and 'trusting the count', again compared with the bead frame:

I think there was a bit less of kids counting. When we used the bead frames... there is a bit of kids still counting the beads on the row. Even if it was ten, they would go one, two, three... They weren't trusting the count. Whereas because this was broken down more, it was easier for them to use. I played with the same student with the keyboard and she was able to go five... six, seven, eight...

This finding is consistent with research suggesting that linear number tracks are superior for developing early number sense, in particular, an appreciation of number magnitude, because they better map onto the desired mental representation (Siegler & Ramani, 2009). However, the Keyboard is distinct from a number line in that it did not contain number labels. Giovanni noted that this design feature served to increase student cognitive engagement in the associated activities, as students had to bring to mind the relevant number relationships:

I would be using a number line... But the numbers would be visible for the kids to see to help them understand where on that line that they are. With this, they had to use their memory, what they know, their knowledge and really visualise what position everything is, based on the pattern.

Integrated Instruction of Important Number Concepts

Beyond its linear design, teachers emphasised that the Keyboard facilitated effective instruction by allowing multiple mathematical concepts to be introduced in an integrated manner. As Giovanni stated:

I thought they were great, because in one lesson, just using the keyboards, he (first author) was able to cover a number of different strategies, not just one focus. Just by introducing the keyboard, he was able to show how to add on, how to take away, having two more and two less, seeing patterns.

Through continued exposure to the Keyboard, Giovanni noted that students' "concept of less than and more than has naturally just evolved", noting that the Keyboard supported students in reasoning about numerical proximity and comparing magnitudes:

So they would be able to say, not just by looking at it, but go, "Oh. she's closer because the gnarly number is here, or she's two away and I am one away, which means I am closer."

Additionally, teachers noted that the Keyboard was not necessarily a replacement for other manipulatives but could be used in conjunction with them to enhance understanding. Michael observed that different manipulatives provide unique affordances, and the Keyboard could complement other tools, particularly in supporting addition and subtraction strategies:

Being able to do jumping strategy for addition and subtraction, that sort of thing... it's not either/or - it's in conjunction with a tool like the bead frame. (Michael)

Although not the focus of the lessons demonstrated during the project, Michael also noted that the keyboard has the potential to support student exploration of multiplicative ideas:

It also is fantastic for doing... multiplicative thinking or splitting things into groups of five. Dividing, doing that sort of thing. I think it has a lot of utility.

Given that initial evidence suggests the Keyboard may assist some students in shifting from counting-based to retrieval-based strategies when solving single-digit addition problems (Hopkins et al., 2025)—a focus distinct from the current classroom-based study—its potential for supporting learning across a range of number concepts is particularly noteworthy.

Flexibility and Thinking Relationally

The design features of the Keyboard, in particular the fact that every fifth key is black, provided opportunities for students to think flexibly and relationally about number. Indeed,

students' ability to notice and internalise the Keyboard's structure in a relatively short period of time was surprising to Giovanni:

So, for them to be able to see the pattern of every black key in increments of five, I thought was going to be a challenge and for them to understand "Okay, what's two more than a black key?", I thought the wording for that example would have been tricky for them but I think they were able to process black keys of five. They can count by fives. One more after five must be six or eleven or so on... They were developing strategies that they didn't even realise they could do... Because, they were making a connection to the piano keys.

Students were able to identify a given number as being embedded in a rich set of relationships through their internalisation of the structure of the Keyboard. This enabled them to rapidly identify numbers despite there being no number labels. As Giovanni noted:

I feel like they could clearly see the numbers between one and twenty. It wasn't just black-and-white now, they were actually seeing clear numbers in spots where there were no numbers. So, if they're on like, "Okay, you've picked up ten and seven, that's seventeen. Where would that be?" They would, without really thinking too much, go straight to the further end of the keyboard and place the counter past the black, and I am like, "That's great."

On occasion, students went beyond the physical keyboard to estimate the position of numbers beyond 20:

Some kids looked at their keyboard... and they said "If we were going to add these two numbers, it would be" and they pointed at the keyboard but past the twenty mark on the keyboard saying "It would be roughly here". (Giovanni)

Building on from this, both teachers noted that an additional strength of the Keyboard as a tool was that numbers beyond 20 could be explored by connecting multiple Keyboards:

I love that it can be kept as just twenty and you can make them bigger as they need to. (Michael)

I loved it because, again start off real basic, introduce it on the first day and then just expand on their knowledge on the second day. As soon as you connected it, they were mind-blown... And then you go in further and add a third, and then their heads exploded. (Giovanni)

Sustained Student Engagement

Finally, the Keyboard was viewed as sustaining student engagement through allowing for 'hands-on learning', but in a manner that minimised student distraction. Giovanni indicated that the students appreciated being able to hold the mathematics in their hands:

It was very engaging the fact that it was very tactile. The kids could hold it and play with it and explore with it in different ways. It kept them engaged the whole time.

In addition to promoting general engagement, the Keyboard appeared to be particularly beneficial for certain students who struggled with other manipulatives. Michael noted that some students found the Keyboard significantly easier to use than the bead frame, which in turn improved their confidence:

I think seeing one student in particular who is typically low... seeing how much easier he found using the keyboard... He found it so, so much easier compared to the bead frame. Just seeing him play the games, he was so much more confident in using them.

This suggests that for some students, the structure of the Keyboard provided an entry point into mathematical thinking that other tools may not have afforded as effectively. Interestingly, Michael also noted that there may be design advantages associated with the keyboard compared with other manipulatives that have more moving parts, such as bead frames, as its simplicity and static nature supported students to focus on the mathematics:

They work a little bit better because there's less moving parts and there is less to fidget with and I think with the bead frames, there is a bit of doing a back and forth, shaking them, tipping them. It's not the end of the world, but just one less moving part, there is one less thing for kids to fidget with. It's just a good thing for attention, I think. Just having to be something that's static, but you can

make it dynamic with counters, is a good thing because for kids who can't focus on too many things at once, it's good.

Collectively, these findings imply that the Keyboard may be particularly effective in sustaining student focus while facilitating structured, hands-on mathematical exploration.

Finally, beyond discussing its affordances, it is worth emphasising that both Giovanni and Michael were enthusiastic about the possibility of utilising the Keyboard to support future mathematics instruction in their own classrooms going forward, if given the opportunity:

If we have those in the classroom, I would definitely use them. (Giovanni)

I really love the idea of them. I would like to spend more time using them. I would be really interested to see where else it could be used. (Michael)

Discussion and Conclusion

These findings contribute to existing research on mathematical manipulatives in several ways. First, they reinforce the view that teachers perceive manipulatives as having distinct, tool-specific affordances, reinforcing the need for research that investigates teacher perceptions of particular tools rather than manipulatives in general. The results demonstrate that not all manipulatives are interchangeable; rather, their design features shape how students interact with mathematical concepts. Second, the Keyboard's linear structure aligns with prior research indicating that structured linear spatial representations enhance number sense (Siegler & Ramani, 2009). Siegler and Ramani's work specifically found that linear number tracks are more effective than circular ones for helping young children develop a clearer understanding of numerical magnitude. The Keyboard provides a similarly structured, linear representation of number, reinforcing magnitude relationships in a way that reduces reliance on inefficient counting strategies. Additionally, its lack of numerical labels required students to engage more deeply with number relationships rather than relying on rote recognition of pre-written numerals. Teachers observed that students internalised the Keyboard's repeating structure, recognising numbers based on their position relative to black keys and other reference points, rather than through simple recall. This suggests that both its structured, linear format and the absence of numerical labels contributed to students' ability to reason flexibly about number relationships. Third, several additional hypothesised benefits of the Keyboard, particularly its support for relational reasoning and flexible number strategies, were evident in teachers' reflections. This aligns with prior research on how manipulatives can enhance conceptual understanding of key number concepts, such as reasoning about numerical relationships (Marshall & Swan, 2008; Quigley, 2021). Additionally, the study provides further evidence that manipulatives with mathematically purposeful yet simple designs may be preferable, as suggested by Carbonneau et al. (2013). Teachers noted that the Keyboard's structured but uncluttered design helped students focus on number relationships rather than being distracted by moving parts. Finally, teachers perceived the Keyboard as enhancing engagement and enjoyment in mathematics learning (Tan, 2020; Quigley, 2021), particularly through the structured learning experiences that accompanied its use. The way it was introduced—through whole-class familiarisation, mathematical games, and problem-solving investigations—appeared to sustain student interest while fostering deeper thinking about numerical relationships. Findings also reinforce the idea that a manipulative's effectiveness depends not only on its design but also on how it is embedded within instruction (Marshall & Swan, 2008). The Keyboard's lack of numerical labels further contributed to this engagement, as students were required to construct meaning through active reasoning rather than passive identification of printed numbers. This reasoning-based interaction with the Keyboard may have also supported students' enjoyment, as teachers observed students sustaining interest and curiosity while identifying and reasoning about numbers in relation to the Keyboard's structure.

While this teaching experiment provided valuable insights into how teachers perceive the Keyboard's affordances, several limitations must be acknowledged. First, notably, neither teacher explicitly identified constraints associated with the Keyboard. While this may reflect their genuine enthusiasm, it may also reflect the structure of the lessons or the framing of the interviews. Future research could explore potential limitations more systematically. Second, the study was small in scale, involving only two teachers from a single school. Although their perspectives offer depth, broader generalisability requires further research across different school contexts and with a larger sample of educators. Third, our study focused on teacher perspectives rather than student learning experiences, meaning further research is needed to examine how students engage with the Keyboard and how it influences their mathematical reasoning over time.

Looking ahead, teachers also identified potential for the Keyboard to support multiplicative thinking, which extends beyond its original design focus on number sense and additive reasoning. This raises interesting possibilities for future research into how the Keyboard might support students' understanding of a broader range of mathematical concepts. Taken together, these findings highlight how the Keyboard, when embedded within a carefully structured lesson sequence, can support teachers to perceive new opportunities for fostering students' understanding of number relationships and mathematical reasoning.

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Conceptualising Primary School Mathematics Leadership as Poly-Motivated Activity

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This paper conceptualises mathematics leadership using cultural-historical activity theory (CHAT). Qualitative data were generated with three primary school mathematics leaders and analysed focusing on motive objects of activity. Analysis showed that the leaders directed their leadership at multiple motive objects, establishing mathematics leadership as poly-motivated activity. This conceptualisation reveals the interplay between building relationships, developing teachers' practice and knowledge, and managing demands of the leadership role. The findings contribute to mathematics leadership theory and offer implications for practitioners and researchers.

Mathematics leadership in schools is crucial in improving and sustaining effective mathematics teaching and learning (Driscoll, 2022; Sexton, 2023). It is, however, a complex form of practice (Sexton, 2019). Reasons for its complexity include navigating the positionality of the middle leading role (Sexton, 2023); responding to role clarity challenges (Copping, 2022); and negotiating demands concerning access to and the use of physical, temporal, financial, and human resources (Copping, 2022; Driscoll, 2022; Sexton, 2019). As a relatively new inquiry area, researchers have called for further conceptualisations of practice to make greater sense of the complex functioning of this form of middle leading practice in schools (Sexton, 2019; Vale et al., 2023). In this paper, I use cultural-historical activity theory (CHAT), drawing on the concept of *motive object*, to interpret data about mathematics leadership. By analysing the motive objects of activity, I aim to see how a CHAT perspective offers a conceptualisation of primary school mathematics leadership as *poly-motivated activity*.

Literature and Theoretical Background

This section begins by outlining middle leadership as a form of school practice to provide the context for understanding mathematics leadership. I then describe specific actions that characterise primary school mathematics leadership before introducing a CHAT perspective of human practice. The section concludes with the research question that is answered in this paper.

Middle Leading

Middle leadership is understood as a form of practice that is enacted in the space *between* executive school leadership (principal and deputy principal) and the teaching that happens in classrooms (DeNobile, 2018; Grootenboer 2018). Middle leading is associated with formal roles of responsibility (Grootenboer, 2018), and in primary schools, this is often linked to curriculum areas such as mathematics (Copping, 2022; Driscoll, 2022; Sexton, 2023). Through their practice, middle leaders are generally responsible for influencing school organisational development, realised through improved student learning outcomes as well as teachers' professional learning (Grootenboer, 2018). Middle leaders work in and about classrooms, positioning them as closely connected to teaching and learning in schools.

Contemporary middle leadership theory has moved away from reports of leaders' traits and characteristics to a focus on the actions enacted by middle leaders (Grootenboer, 2018). A way of understanding the nature and function of middle leading actions, they can be classified using categories of *leadership* and *management* work (DeNobile, 2018). With middle leading being highly contextualised, that leadership and management work undertaken by middle leaders is

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enabled and constrained by conditions, structures, and resources available within the school. Such provisions are usually provided by principals because of their executive positioning and authority within the school leadership system (Grootenboer, 2018).

Mathematics Leadership Practice

Mathematics leadership is interpreted as a form of middle leading practice in schools that influences mathematics teaching by facilitating teachers' professional learning (Copping, 2022; Driscoll, 2022; Sexton, 2023). Mathematics leaders' actions have been categorised as *leading* and *managing* activity, with leading work focused on influencing teachers' dispositions, practices, and knowledge for mathematics teaching, and managing work focused on coordinating resources and organising conditions for professional learning (Sexton, 2019).

Primary school mathematics leaders are pivotal agents of change, influencing teaching practices through leading and managing actions (Sexton, 2019; 2023). Research shows mathematics leaders' work primarily focuses on site-based professional learning to enhance teachers' mathematical content and pedagogical content knowledge (Cheeseman & Clarke, 2005; Copping, 2022; Driscoll & Cheeseman, 2022; Sexton, 2023). This leadership incorporates leading assessment data analysis, developing teachers' planning practices, and providing in-classroom support through team teaching (Cheeseman & Clarke, 2005; Driscoll, 2022; Sexton, 2019; Vale et al., 2021). Mathematics leaders facilitate teacher learning through collaborative practices, including staff meetings, formal and informal coaching conversations, and classroom visits (Cheeseman & Clarke, 2005; Sexton, 2023). Additionally, mathematics leaders work on developing shared understandings of mathematics teaching through professional learning they lead in their schools (Roche et al., 2022; Sexton, 2023).

Mathematics leaders' management work is understood as organising and managing administrative tasks, coordinating resources, and documenting work for accountability reasons (Cheeseman & Clarke, 2005; Sexton, 2023). This managerial dimension continues to be significant, with mathematics leaders responsible for creating and distributing documents such as timetables for school-based professional learning (Sexton, 2019), managing mathematics resources through purchasing and auditing routines (Cheeseman & Clarke, 2006; Sexton, 2019), and organising assessment data collection and storage (Copping, 2022).

Recently, middle leading research reports the role of relational trust and how middle leading is inherently relational and dialogic (Grootenboer, 2018). This is documented in mathematics leadership research, showcasing that leaders deliberately work on building relational trust (Sexton & Lamb, 2023). Mathematics leaders have emphasised the purpose of relationships, positioning them as vital when leading professional learning (Copping, 2022; Driscoll & Cheeseman, 2022). These relationships mediate engagement in dialogue about practice problems and facilitate shifts in practice that reciprocally nurture further trust (Eden, 2018).

CHAT Perspective of Human Practice

CHAT is considered a practice-based theory, suitable for researching educational leadership (Nuttall et al., 2024). Within CHAT, practice is understood as collective activity realised through joint labour processes where practical and psychological development is drawn forward simultaneously (Engeström, 2015; Nuttall et al., 2024). CHAT recognises activity as always *object-oriented* meaning that central to understanding human practice, attention must be paid to the motive objects of activity (Kaptelinin, 2005; Leont'ev, 1978). The CHAT concept of motive object is interpreted as the undertakings, tasks, or goals that form the focus of practice (Nuttall et al., 2015). Motive objects are pursued by people and provide reasons for activity and are the entity at which activity is directed, acting as driving forces of activity (Kaptelinin, 2005).

Leont'ev (1978) introduced the idea of poly-motivated activity to understand the complex and contradictory nature of human practice. Complexity surrounds because, when engaged in practice, humans often direct their activity at more than one motive object. This is due to the desires, wants, and aspirations that humans attach to motive objects in response to the ever-shifting nature of activity (Miettinen, 2005). Therefore, humans can respond to multiple motive objects that are shaped by shifting desires and goals (Leont'ev, 1978; Miettinen, 2005).

Considering the literature and theoretical background and the calls for a conceptualisation of mathematics leadership, the research question for this paper is: Using a CHAT perspective, how does focusing on the motive objects of activity provide a coherent conceptualisation of mathematics leadership practice enacted in primary school sites?

Research Design

The findings in this paper are drawn from my doctoral study, which used CHAT to examine how mathematics leadership sustained teaching reforms following participation in a large-scale, system-wide school mathematics intervention (Sexton, 2023). As a practice-based theory, CHAT is a suitable theoretical frame and offers methodological tools for studying mathematics leadership as a form of middle leading practice (Sexton, 2019; 2023; Sexton & Lamb, 2023).

Theoretical Framework

CHAT supports inquiry into the relationship between humans, their environments, and their practices, focusing on how activity shapes and is shaped by the context in which the activity is enacted (Engeström, 2015). Rooted in Marxist and Vygotskian thought and further developed by Leont'ev, CHAT emphasises the importance of motive objects, acting as the motivation for activity (Nuttall et al., 2024). Motivation, however, is not understood as an individual psychological construct but rather as an enactment of culturally meaningful, collective activity where motive objects serve as the primary carrier of motivational force (Engeström, 2015; Leont'ev, 1978). For this reason, motive objects are understood as teleological, offering a way to analyse what is done in activity and why it is undertaken (Kaptelinin, 2005).

Context and Participants of the Study

The *Contemporary Teaching and Learning of Mathematics* (CTLTM) project engaged 82 Catholic primary schools within the Melbourne Archdiocese from 2008 to 2012. Schools participated in a two-year professional learning program designed and facilitated by Australian Catholic University (ACU) and Catholic education personnel. School participation required all staff involvement in off-site professional learning and the appointment of at least one staff member to undertake the middle leadership role of *School Mathematics Leader*. Throughout CTLTM, those mathematics leaders supplemented teachers' professional development by facilitating school-based professional learning and undertaking management tasks that mediated the implementation of pedagogical ideas and advice highlighted in the CTLTM project.

Three mathematics leaders, Penny, Cindy, and Rachel (pseudonyms), who participated in CTLTM in 2011 and 2012, were participants in my study. The leaders were from three different primary schools and had sustained their leadership roles following the end of CTLTM. The participants were selected for their ongoing enactment of mathematics leadership over time and their willingness to participate in longitudinal data generation designed to generate insights into their leadership activity during and after participation in the CTLTM project.

Data Generation and Analysis

Data were generated from November 2014 to February 2018 using the methods of semi-structured interviews, observations, and document retrieval. Data generation focused on collecting information about mathematics leadership before, during, and after participation in

the CTLM project. Each participant was individually interviewed about their leadership before (~75 minutes) and during CLTM (~85 minutes). Interviews were also used before (~15 minutes) and after (~60 minutes) observations of post-CTLM project leadership practice (~70 minutes per observation) where relevant documentation was collected (e.g., meeting agendas). I visited each participant's school at least five times and observed enactment of their leadership (~65 minutes per observation) at least eight times each time during the data collection period. A total of 48 interviews were conducted, along with 21 observations of the leaders' practice. With document retrieval, 24 artefacts were offered as ones deemed important by the participants. All data were imported into NVivo™ for management and analysis.

A coding scheme including CHAT (e.g., motive object) and literature (e.g., positionality) concepts was used to drive a deductive thematic analysis (DTA) approach (Fereday & Muir-Cochrane, 2006). The concepts in the coding scheme were used as nodes within NVivo™, allowing the systematic tagging of data. The analytical process involved reading (e.g., looking for phrases like "I am working on..."), coding, and examining data in collaboration with doctoral supervisors. Guided by the CHAT principle that activity is always object-oriented (Leont'ev, 1978; Kaptelinin, 2005), particular attention was given to how participants described the nature and purpose of their leadership actions—what they were working on, trying to achieve, or directing their efforts toward. These utterances, actions, and recordings in artefacts were treated as indicative of motive objects. Within the data tagged under the 'motive object' node, inductive analysis revealed a set of recurring orientations to the participants' leadership. These orientations informed the development of themes that frame the findings in this paper.

Findings and Discussion

Given MERGA conference paper limitations, the findings are reported and discussed simultaneously in relation to the background and theoretical literature to support the claim presented in this paper. By focusing on the concept of motive object as an analytical tool (Kaptelinin, 2005), the data analysis process found that the mathematics leaders collectively pursued three motive objects of activity. I interpret these as *relational*, *developmental*, and *managerial* motive objects realised through specific leadership actions and undertakings (Nuttall et al., 2015). These are reported in this order of their prevalence within the dataset.

Relational Motive Object

The relational motive object captures mathematics leadership undertakings that build relational trust with colleagues. The mathematics leaders intentionally directed their leadership towards building relationships, confirming previous research on the centrality of collegial relationships in mathematics leadership (Copping, 2022; Driscoll & Cheeseman, 2022; Sexton & Lamb, 2023). Through their talk about and observations of their leadership, the mathematics leaders enacted actions that were deeply relational and social, shaped by a commitment to fostering relational trust for and about mathematics teaching.

This was evidenced by Cindy, who undertook her school's mathematics and general curriculum leadership roles. She shared the importance of relational trust building, especially with colleagues who may have experienced mathematics anxiety:

Relationship plays a big part in leadership, even more so with maths leadership. There is something about mathematics, trust and relationships that allows me to know how my teachers really feel about maths and their teaching of it, especially the teachers with maths anxiety. They need to know they can share their feelings with me.

The leaders actively created collaborative dialogical spaces characterised by empathy and respect (Eden, 2018). Their work was deeply tied to relational trust, as highlighted by Sexton and Lamb (2023), where providing safe spaces for teachers to discuss successes and practice problems was privileged. Rachel exemplified this through the following comment:

The relational part is really important with being a maths leader and working with teachers, and I do think it's different in maths. You're working with people with different understandings of maths, and they need to feel like it's okay to share that. Relationship plays more of a role.

The mathematics leaders enacted an approach that privileged relationship building amongst themselves and their colleagues, with a work focus on remediating teachers' relationships with mathematics. Penny revealed this through the following comment:

I'm trying to build that relationship with them (teachers) so that it allows them to see that they can be great maths teachers. I guess you can say that I am building relationships through maths. Relationships with them, and relationships with maths. Better people through maths; all of us better together through maths.

This finding reinforces that mathematics leadership is as much about relationships as it is about leading mathematics pedagogy and knowledge development (Copping, 2022; Driscoll & Cheeseman, 2022; Sexton & Lamb, 2023). The finding also reinforces that mathematics leadership is grounded in relational trust, which serves as a key motivational force of mathematics leadership activity.

Developmental Motive Object

The developmental motive object embodies actions that improve teachers' pedagogical approaches for effective mathematics teaching. Enactment of this motive object was specifically observed during teachers' planning meetings that the leaders facilitated, again revealing that mathematics leaders leverage teachers' planning meetings as professional learning opportunities (Driscoll, 2022; Sexton, 2023; Vale et al., 2021). Within those meetings, the leaders influenced how teachers planned for mathematics teaching by attending to curriculum expectations, selecting tasks, identifying differentiation prompts, and effectively using questioning, physical materials, and assessment strategies. Penny exemplified the enactment of this developmental motive object when she shared:

I work a lot on differentiation with the teachers like how to individualise and differentiate the activities and tasks, using what I know about enabling and extending prompts. Differentiation is a big issue for the teachers, so I tend to focus on that quite a bit. We talk about how to differentiate by posing different questions or using a particular concrete material with a group of kids.

Rachel highlighted activity directed at developing teachers' use of mathematical tasks, emphasising that simply selecting tasks was insufficient:

I spend a bit of time helping the teachers become aware of good tasks to use when teaching. I think they need to have a set of tasks and activities that they know are good in helping the kids learn the maths. But there's more to it than just choosing tasks, so I also help them know how to use the task or the activity in the maths lesson. I mean, you can have a great task, but if you can't use it properly, it doesn't stay a great task.

The mathematics leaders modelled strategies for using tasks as they led dialogue about approaches to make mathematics content more accessible to students. Their leadership extended to supporting teachers' use of representations for mathematical concepts, using learning trajectories to identify tasks and differentiation prompts, addressing student misconceptions, and aligning tasks and intended learning with curriculum expectations.

Enactment of leadership that pursued the developmental motive object focused on enhancing teachers' pedagogical content knowledge (PCK) and documenting shared understandings of PCK by creating *collective commitments*. The commitments acted as visions for practice (Roche et al., 2022), ensuring that teachers had a shared approach to mathematics teaching. This leadership action was evidenced by Penny through this comment:

The teachers need to understand what a good teacher of mathematics looks like, the key components of a maths lesson, and how to differentiate tasks for the kids. I make that part of our shared thinking in facilitated planning (meetings). I focus on that vision for teaching maths with the teachers.

The leaders' actions toward the developmental motive object were mediated by knowledge of teachers' professional learning strengths and needs. This knowledge was developed through ongoing work of pursuing the relational motive object and was influenced by their position as middle leaders (Grootenboer, 2018; Sexton, 2023). Their work primarily focused on developing teachers' PCK through the facilitation of planning meetings as professional learning opportunities, paying attention to lesson design through task selection and use in classrooms.

Managerial Motive Object

The managerial motive object was realised as the mathematics leaders worked on purchasing, managing, and organising a range of resources, confirming previous research that management is a required element of mathematics leadership (Cheeseman & Clarke, 2005; Copping, 2022; Driscoll, 2022; Sexton, 2019). The managing of resources included physical resources such as concrete materials for classroom use and teacher reference texts used in planning meetings; financial resources, which included maintaining yearly budgets allocated to mathematics; temporal resources that required time management of the allocation assigned to their leadership role; and spatial resources which include the management of spaces where teachers can engage in collective professional learning. This claim about the managerial motive object was supported by Rachel when she commented on her enactment of managing actions:

Look, management is part of the role. It's not a leading vs. managing thing. You need to do both. There's a bit to manage, like the materials for the teachers and students to use, along with the books that we use to help with planning. I also have a budget that I get each year, and I have to use it by the end of Term 3. I also need to manage my time as in how much release time I have in the week to lead professional learning, checking in with teachers and visiting their classrooms. I have to also check when the planning rooms are free so I can lead the facilitated planning (meetings).

One finding of interest concerned how the mathematics leaders "managed up" by enacting advocacy for mathematics. This was specifically important after participation in CTLM. The mathematics leaders reported that during CTLM participation, mathematics was a key school improvement area. However, after CTLM, the leaders reported how they managed by informing principals of the need to maintain an emphasis on mathematics. The leaders' commitment to mathematics drove them to actively manage the promotion of its significance, ensuring that mathematics remained on school improvement agendas. Cindy shared insights about this:

I say to the principal, "I understand there's other things that have had priority, but when is mathematics going to get a guernsey again?" I see it as part of my role to fly the 'maths flag' and keep maths on the agenda.

Penny also shared how the managerial motive object was worked on by advocating for mathematics when faced with competing curriculum areas of improvement agendas:

The focus has been the RE (religious education) curriculum renewal. I know part of my maths leader role is to keep managing maths, looking for ways for maths to have the status it had in CTLM and what it still needs. I keep cheering on maths.

Acting as a mathematics advocate was a key leadership action of the managerial motive object, but it did mediate tension for the mathematics leaders. Rachel exemplified this when she stated: "You have to promote maths as the maths leader and keep it as a focus area, but it's hard when you're not the principal. You get to do some things as a middle leader but not everything." This is further evidence that although they are leaders within the school site, they are not afforded the authority that executive leaders can enact due to their positionality as middle leaders within the school leadership system (Grootenboer, 2018; Sexton, 2023).

Mathematics Leadership as Poly-Motivated Activity

The identification of the relational, developmental, and managerial motive objects of activity provides evidence for the claim that mathematics leadership is poly-motivated activity (Leont'ev, 1978). The findings illustrate how mathematics leaders pursue multiple motive

objects and reflects the function of mathematics leadership as a complex and dynamic form of middle leading practice (Grootenboer, 2018; Sexton, 2019).

The poly-motivated nature of mathematics leadership is evidenced in how the three motive objects coexist and interact. For example, when the leaders worked on building relationships (relational motive object), they also created spaces to develop teachers' knowledge for mathematics teaching (developmental motive object). Similarly, when advocating for mathematics within improvement agendas (managerial motive object), they drew upon their relationships with principals (relational motive object) and the shared vision for mathematics teaching built through their professional learning leadership (development motive object).

This CHAT-informed conceptualisation advances prior understanding of mathematics leadership as leading and managing work (Sexton, 2019) by revealing that it is poly-motivated activity. Rather than serving a singular or dual function, mathematics leadership responds to multiple, interacting motive objects. This framing highlights the complexity and dynamism of the role, offering a CHAT-informed conceptual basis to guide future research and support the development of mathematics leadership as a form of middle leading practice in schools.

Conclusion and Implications

This paper conceptualised mathematics leadership as poly-motivated activity, drawing on the CHAT concept of motive object. The findings reveal that mathematics leaders direct their activity towards three motive objects that coexist and interact, reflecting the multifaceted nature of primary school mathematics leadership. The focus on motive objects provided a coherent conceptualisation of mathematics leadership as relational, developmental, and managerial activity. This conceptualisation offers a more nuanced understanding of mathematics leadership than perspectives focusing solely on leaders' traits or capturing mathematics leaders' activity as only leadership and management work.

This paper offers several important contributions. First, it provides a conceptual and practical framing for understanding the diverse undertakings enacted by mathematics leaders, demonstrating how they are motivated by different yet interrelated motive objects. Second, it highlights the importance of considering the individual and collective dimensions of mathematics leadership. Finally, it emphasises the dynamic nature of mathematics leadership, showing how leaders direct their activity to different yet related motive objects.

For mathematics leaders and principals, this CHAT-informed conceptualisation offers a lens for reflecting on practice. By recognising the poly-motivated nature of their work, leaders could develop greater awareness of how attention is allocated across motive objects. This awareness can support more intentional leadership that address the responsibilities of mathematics leadership. These findings highlight the importance of creating conditions that enable mathematics leaders to pursue all three motive objects effectively. For principals, this might include providing dedicated time for leaders to build relationships, leading teachers' professional learning, and undertaking management tasks, as well as providing the spatial and financial resources that facilitate the motive objects of mathematics leadership activity.

I acknowledge that focusing only on motive objects could be perceived as reductive. Research into the mediational means (e.g., cultural tools, rules, and division of labour) that shed light on their function, specifically how they enable and constrain leadership activity, will provide a richer conceptualisation of mathematics leadership. Further research might also identify how mathematics leaders address tensions when determining which motive objects to privilege at given times when enacting their poly-motivated leadership activity.

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A Study of Grade 10 Students' Conceptions of Proof in a Singapore Secondary School

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This paper examines Grade 10 students' conceptions of proof in a secondary school in Singapore. Using a purposive survey of 9 mathematical items, proofs by 8 students for two items, one on Number and Algebra and another on Geometry and Trigonometry, were coded using Harel's proof schemes. The findings show that the students' proofs for the two questions displayed a range of proof schemes, except the authoritative scheme. With the exception, of the perceptual scheme, the predominance of other schemes was item specific. It was also apparent from the proofs written by the students that they struggled to articulate their arguments using correct mathematical language.

Mathematical proof and the aspects of proof form the very foundations of mathematics. The arguments, justifications and reasoning typically used in a proof are ubiquitous in all of mathematics. The formulae or theorems we apply to solve problems are proven true via deductive means. Even though most of these proofs that we speak of are conceptually out of reach for the average student at secondary level, the aspects mentioned are crucial in forming solid mathematical foundational knowledge. It is surprising then that the concept and process of proving have not been ascribed sufficient importance in the secondary mathematics curricula worldwide until recently (Ball et al., 2003; Healy & Hoyles, 1998). There has always been a belief that proof should not be covered in an in-depth manner in the secondary mathematics curriculum as it is beyond students' access and teachers' expertise (Philipp, 2007; Ko, 2010; Stylianides, 2013). The applications of mathematics were deemed as more critical than the proofs of mathematical theorems since only professional mathematicians actively pursue the construction of rigorous proofs.

Students' conceptual knowledge (or conceptions) related to learning proofs encompasses their understanding of the fundamental principles and logical reasoning involved in constructing valid mathematical arguments (Basturk, 2010). It involves grasping abstract concepts, recognizing the logical structure of proofs, and comprehending the significance of proof techniques such as direct proof, proof by contradiction, and proof by induction. Such conceptual knowledge also includes understanding the role of definitions, axioms, and theorems in establishing mathematical truth, as well as recognizing the connections between different mathematical concepts and how they can be applied in the context of proof construction (Basturk, 2010).

In the Singapore secondary school mathematics curriculum (SMC), there is an acceptance that proof and its aspects of it (for example, argumentation, justification, and reasoning) are essential, as evidenced in the mathematics curricular documents (Ministry of Education, 2013a, 2013b, 2018). However, despite the statements of intent stated in the curriculum documents, there is little evidence of the enactment of proof and proving throughout the mathematics curriculum. Most proofs seem to appear mainly in the topic of Geometry in both Elementary Mathematics (EM) and Additional Mathematics (AM). All students in secondary school offer EM as a school subject, while only students in the same grades who are capable of doing more mathematics offer AM.

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The Study

The study reported in this paper is part of a larger study on proof and proving in Singapore (Thanabalasingam, 2024). As part of an investigation related to “What shapes students’ learning of mathematical proof in a secondary school” students’ conceptions, experiences, approaches, perceptions and challenges were investigated. However, in this paper we will limit the scope to students’ conceptions related to learning proof.

Conceptual Framework

The study adopted a purposive framework, shown in Table 1, for classifying the types of justification students adopt in their proof schemes. Harel's (2007) proof scheme comprises three main categories, namely i) the *External conviction proof scheme*, ii) the *Empirical proof scheme*, and iii) the *Analytical proof scheme*. In the external conviction proof scheme category, individuals prove via their existing conceptions of proof purely through an external source such as a textbook or teacher. In this process, individuals do not try to develop new proof; instead, claims are made based on rote learning. This scheme is further elaborated into three categories, which are:

- *Authoritative* - proof based on student, teacher, book or any other external authority.
- *Ritual* - proof is made based on opinion and form not on the content.
- *Non-Referential Symbolic* - individual tries to produce mathematical arguments through symbolic representations and statements without knowing the meanings of symbols.

Table 1

Harel's Proof Scheme (Harel, 2007)

External conviction	Empirical	Analytical
○ Authoritative	○ Inductive	○ Transformational
○ Ritual	○ Perceptual	○ Axiomatic
○ Non-referential symbolic		

In the empirical proof scheme category individuals make or develop claims based on their intuitions through examples. This scheme is further elaborated into two categories, which are:

- *Inductive* - individuals try to produce arguments with examples and trials. Generalisation is made with one or several examples.
- *Perceptual* - individuals try to produce a proof by making deductions based on perceptual representations.

The analytical proof scheme category consists of arguments based on logical deductions. Proofs in this class are based on definitions with formal structures. Formal concepts are developed through theoretical definitions, not in line with existing ideas. Verification is achieved using axioms, and the difference between an axiom and a theorem is grasped. This scheme is further elaborated into two categories, which are:

- *Transformational* - mental guesses are made regarding the accuracy of the proposition and deductions are made based on these guesses. It features generalisation, operational thinking, and logical deduction.
- *Axiomatic* - proof is made by using axioms and theorems.

Method

The Conceptions Survey

The conceptions survey comprised nine questions involving Algebra and Geometry. The questions were modified from past national level EM and AM examination questions. These

modified questions were more general, thus providing students with opportunities to solve them in ways they deem fit. This would allow us to decipher their proof schemes from their solutions. In this paper, we will focus on two of the questions, question 2 (Number and Algebra) and question 7 (Geometry and Trigonometry).

Subjects

A purposeful sample of eight Grade 10 students (four males and four females) from a secondary school in Singapore participated in the study. The students were representative of mathematics ability range in their respective classes. They were taking both EM and AM as school subjects. The students were keen to participate in the study and assess their readiness for their upcoming end-of-grade 10 national examination. Participation of the students in the study was governed by ethical requirements of the Nanyang Technological University (National Institute of Education) in Singapore.

Data Collection and Analysis

Students were given one week to complete their take-home survey. The survey had to be completed in one-sitting within a duration of 150 minutes. This arrangement was based on mutual trust and a belief in students' integrity. The rationale for allowing a take-home survey was due to a tight curriculum schedule at school, as they were preparing for their end-of-grade 10 national examination. The researcher collected the completed surveys from the students a week after they were given out.

The solutions provided by the students were coded using Harel's (2007) proof schemes shown in Table 1. The two coders were both experienced mathematics teachers with deep mathematical and pedagogical knowledge for teaching of mathematics at the secondary school level. The simple percentage agreement (McHugh, 2012) between the 2 coders was near perfect (greater than 85%) for all solutions provided by the students. Figures 2 and 3 show sample solutions of students for questions 2 and 7 respectively, and the proof schemes adopted by the students in their solutions together with the proof schemes identified by the coders.

Figure 1 illustrates that Student 1-1 employed ritualistic, non-referential symbolic, inductive, and transformational proof schemes in her solution. The ritualistic nature of her approach was evident in the formation of an equation rather than demonstrating that the difference between the products of opposite corners of the square is always 8, as required. Her assumption that $x = 1$, followed by the formation of an equation summing four arbitrary numbers within a 2×2 grid to 260, reflects a non-referential symbolic scheme. Additionally, the lack of distinction between x as a constant (1 in this case) and as a variable further exemplifies this scheme. The inductive proof scheme was apparent when she attempted to identify a pattern by computing the difference between vertically adjacent numbers (e.g., $9 - 1 = 8$), although this was not applicable in the given context. Her final reasoning suggested an attempt at proof by contradiction, as indicated by the conclusion that x resulted in a non-integer, which contradicts the problem constraints. The transformational scheme was evident in her indirect but logically structured approach.

Student 7-2, in contrast, demonstrated perceptual, transformational, and axiomatic proof schemes. The perceptual scheme was reflected in his introduction of the variable x without explicit definition, relying on an implicit assumption that x must be a positive integer. Similar to Student 1-1, he successfully transformed the problem and proved it by contradiction. His axiomatic proof scheme was observed in the logical deductions made in part (a) of the question.

Figure 1

The Coded Solutions of Students 1-1 and 7-2 for Question 2

Question 2. The diagram shows part of a number grid. <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr> <tr><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr> <tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td></tr> <tr><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td></tr> <tr><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>....</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40									A square outlining four numbers, as shown, can be placed anywhere on the grid. Above are 2 examples of such a case. (i) Show that the difference between the products of the numbers in the opposite corners of the square is always 8. (ii) Show that the sum of the four numbers in the square cannot be 260.
1	2	3	4	5	6	7	8																																										
9	10	11	12	13	14	15	16																																										
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33	34	35	36	37	38	39	40																																										
....																																																	
Student 1-1 	1. Authoritative 2. Ritual 3. Non-Referential Symbolic 4. Inductive 5. Perceptual 6. Transformational 7. Axiomatic																																																
Student 7-2 	1. Authoritative 2. Ritual 3. Non-Referential Symbolic 4. Inductive 5. Perceptual 6. Transformational 7. Axiomatic																																																

Figure 2 illustrates that Student 2-1 employed ritualistic, non-referential symbolic, and perceptual proof schemes in his solution. The ritualistic aspect of his approach was evident in the incorrect application of the Pythagorean Theorem when the problem required the use of its Converse. The non-referential symbolic scheme emerged in his statement, “ $\angle ABC = \text{Right angled triangle}$ ” erroneously equating an angle to a geometric figure. Additionally, his perceptual proof scheme was reflected in his assertion that “ ACB should form a right-angled triangle.” This statement suggests an internal assumption that ACB represents a triangle rather than an angle, as he did not explicitly define it. Furthermore, rather than proving the claim as required, he treated it as a given.

In contrast, Student 6-2 demonstrated perceptual and axiomatic proof schemes. His perceptual reasoning was evident in the sketching of a distorted yet conceptually intended circle passing through points A , B , and C . Despite minor errors in his argument, the overall approach

was axiomatic, as demonstrated by his application of the theorem stating that if the product of the gradients of two lines equals -1, then the lines must be perpendicular.

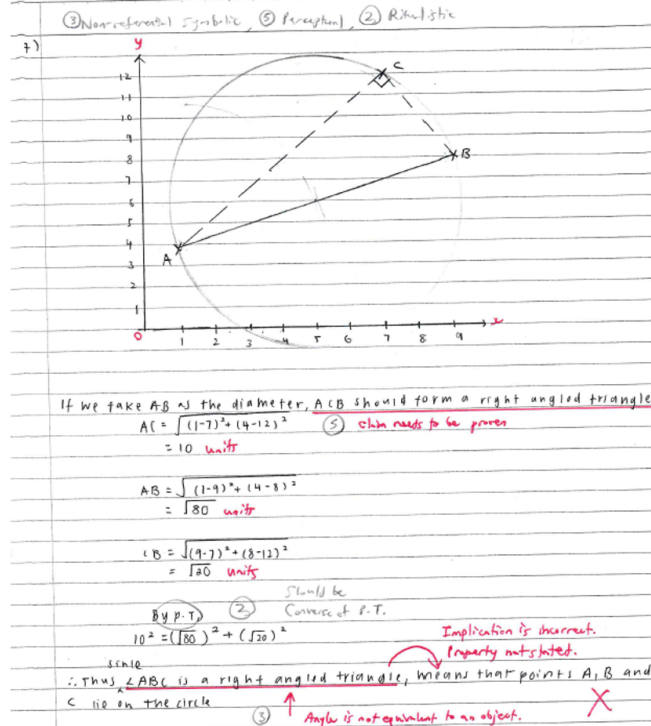
Figure 2

The Coded Solutions of Students 2-1 and 6-2 for Question 7

Question

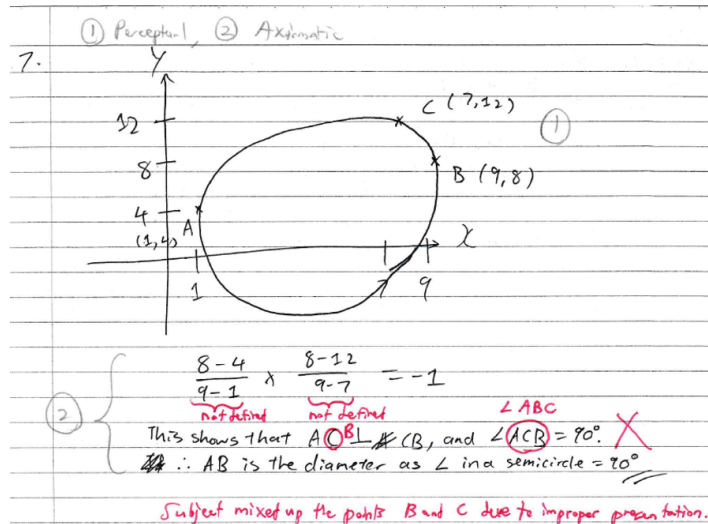
7. Three points are given by $A(1,4)$, $B(9,8)$ and $C(7,12)$. Explain why A , B and C lie on a circle with exactly two of these points forming the diameter.

Student 2-1



1. Authoritative
2. Ritual
3. Non-Referential Symbolic
4. Inductive
5. Perceptual
6. Transformational
7. Axiomatic

Student 6-2



1. Authoritative
2. Ritual
3. Non-Referential Symbolic
4. Inductive
5. Perceptual
6. Transformational
7. Axiomatic

Findings and Discussion

Question 2 (EM Number and Algebra strand) involves concepts in Number Theory. Students were required to recognise the number pattern in the number grid given and prove in general that the difference between the products of the numbers in the opposite corners of the square is always 8 (for part (i)) and that the sum of the four numbers in the square cannot be 260 (for part (ii)). The question required students to generalise and find an association between the four terms in each 2×2 grid and form the relevant equations to solve the problem. Table 2 summarises the proof schemes the eight students displayed in their solutions.

Table 2

Summary of Students' Proof Schemes for Question 2

Main	Sub	1-1	2-1	3-1	4-1	5-2	6-2	7-2	8-2
External Conviction	1. Authoritative								
	2. Ritual	√							
	3. Non-Referential Symbolic	√				√			
Empirical	4. Inductive	√		√	√				√
	5. Perceptual				√	√		√	√
Analytical	6. Transformational	√	√	√	√		√	√	
	7. Axiomatic		√	√			√	√	

Legend: √ indicates presence of respective proof scheme

It is apparent from Table 2 that the proof schemes which occurred most regularly were the Inductive (4 out of 8 times), Perceptual (4 out of 8 times), Transformational (5 out of 8 times) and Axiomatic (4 out of 8 times) proof schemes. Evidence of the inductive scheme was seen in the solutions of students trying to verify that a statement is true using particular values instead of proving the general case. Students 1-1, 3-1, 4-1 and 8-2 used arbitrary starting values and tried to explain that the statement was true in general by verifying only a few instances.

Evidence of the perceptual proof scheme was seen in the solutions of students who either misunderstood the problem or perceived the nature of the variables used in their generalisation. Student 3-1 misunderstood that the number grid is limited to the 40 numbers shown when it was supposed to be unrestricted. Students 4-1 and 7-2 did not define the variables used in their general terms. Student 5-2 took the vertical difference in each column instead of considering the difference in the product of the diagonal entries. Evidence of the transformational proof scheme was seen in all the solutions of the students except students 5-2 and 8-2. Most of them managed to transform the problem into one that required proof by contradiction and correctly proved the result. The axiomatic proof scheme was evident in the solutions of students who solved the problem using generalised terms and strung together a logically well-connected argument.

Question 7 (AM Geometry and Trigonometry) required students to explain that points A , B and C lie on a circle with exactly two of these points forming the diameter. They were expected to show that the triangle ABC is a right-angled triangle in a semi-circle with diameter AC . They could do this by either showing that line AB is perpendicular to line BC or stating the converse of the Pythagoras Theorem. Table 3 summarises the proof schemes the eight students displayed in their solutions.

Table 3
Summary of Students' Proof Schemes for Question 7

Main	Sub	1-1	2-1	3-1	4-1	5-2	6-2	7-2	8-2
External Conviction	1. Authoritative								
	2. Ritual	√	√	√	√				
	3. Non-Referential Symbolic		√		√				
Empirical	4. Inductive								
	5. Perceptual		√		√	√	√	√	√
Analytical	6. Transformational								
	7. Axiomatic			√			√	√	

Legend: √ indicates presence of respective proof scheme

It is apparent from Table 3 that the proof schemes which occurred most regularly were the Ritual (4 out of 8 times) and Perceptual (6 out of 8 times) proof schemes. Evidence of the ritual proof scheme was observed in the solutions of students who did not have a clear idea as to what was necessary to prove the problem or assumed the result that needed to be proved. Students 1-1 and 3-1 found all the three gradients, to observe which paired product will give an output of -1, when it was only necessary to prove that AB is perpendicular to BC . This was intuitively clear from the respective positions of the three points on the Cartesian plane. In student 4-1's initial attempt, he incorrectly assumed that points A , B and C lay on the circle and used that assumption to find the length of the circle's diameter, and hence, its equation.

Evidence of the perceptual proof scheme was seen in the solutions of students who were not rigorous enough in their proof or had misconceptions about the nature of what a proof should be. Students 2-1 and 8-2 showed that triangle ABC is right-angled by showing that the converse of the Pythagoras theorem holds without explicitly stating which angle is right-angled. Students 4-1, 6-2 and 7-2 used diagrams, according to their own perceptions, to explain their proof. While a diagram could have been used as an aid to their understanding, it should not have been referenced in the proofs.

It is apparent from the proofs written by the eight students for questions 2 and 7 that none of the students adopted the authoritative proof scheme in their solutions. For question 7, none of the solutions the students wrote displayed the inductive or transformational proof schemes. However, for question 2, the inductive and transformational proof schemes were present in the solutions of four or more students. It appears that the nature of the questions does lend students to varied proof schemes. For both the questions, several student solutions displayed the ritual, non-referential, perceptual and axiomatic proof schemes.

It was also apparent from the proofs written by the students that they struggled with articulating their arguments using correct mathematical language. They misused symbols, connectors, or semantics, leading to incorrect or incoherent explanations. At times when they relied on inductive reasoning, they attempted to verify a statement by using specific values or examples instead of establishing a general case. They made assumptions or drew conclusions without properly defining variables, proving statements, or considering all possible cases. For question 2, where six out of the eight students demonstrated the transformational proof scheme, they successfully transformed a problem into one that required a different approach or proof technique. Students did rely on memorized steps or formulas without fully understanding when and how to use them when proving and this was apparent when they adopted the ritual proof scheme.

As the students who participated in the study were at the end of their ten years of schooling in a secondary school in Singapore, it is apparent from their proof schemes that they lacked the knowledge and skills for the construction of robust proofs. This has implications for secondary school mathematics teachers in the school where the study was carried out.

Concluding Remarks

The findings of this study have implications for teachers enacting the school mathematics curriculum in secondary schools, specifically in Singapore. There appears to be a need for teachers to focus on the construction of rigorous proofs by students. They may do this via explicit demonstration of proof types and construction of mathematical arguments using accurate mathematical language. Proving, like other mathematical processes such as evaluating and solving must be placed at the core of mathematics instruction as proofs are the foundational building blocks of mathematics.

Acknowledgement

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An Exploration into Secondary Mathematics Teachers' Instruction of Mathematical Proof in Singapore: Understanding Their Conceptions, Beliefs, and Challenges Related to Teaching Proofs

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This study examines factors shaping teachers' instructions of mathematical proof in Singapore. Using a qualitative case study, two teachers' conceptions, beliefs, and challenges were analysed. Findings reveal reliance on empirical and axiomatic proof schemes, shaping instruction. Teachers faced challenges such as limited curriculum emphasis and lack of training. Instructional gaps include insufficient focus on proof tasks and inadequate pedagogical resources. Recommendations include enhancing teacher training, curriculum integration, and assessment reforms. Addressing these issues ensures proof is integral to mathematical reasoning in classrooms.

Mathematical proof represents a hallmark of rigorous reasoning in the discipline of mathematics. In school contexts, proof serves not only to validate mathematical truth but also as a vehicle for cultivating students' logical thinking, justification, and communication of mathematical ideas. Despite its pedagogical significance, the teaching of proof continues to present challenges in secondary school classrooms around the world (Aaron & Herbst, 2019; Ball et al., 2003; Healy & Hoyles, 1998; Mariotti et al., 2018; Sears, 2019).

How proof is taught depends largely on teachers' instructional decisions, which are shaped by their conceptions and beliefs. Teachers play a pivotal role in mediating students' engagement with proof, yet they often encounter challenges that affect how proof is introduced and emphasised in lessons (Aaron & Herbst, 2019; Ayalon & Hershkowitz, 2018; Mukuka et al., 2023). Although the Singapore secondary mathematics curriculum formally recognises the importance of proof – evident in the syllabi for Elementary Mathematics (EM) and Additional Mathematics (AM) (Ministry of Education, 2013a, 2013b, 2018) – proof remains underemphasised in enacted instruction. Furthermore, the absence of proof-related items in national assessments discourages teachers from investing instructional time in developing students' proving abilities.

The study reported in this paper is part of a larger study on proof and proving in Singapore (Thanabalasingam, 2024). As part of an investigation related to "What shapes teachers' instruction of proof in secondary school", teachers' conceptions, beliefs, and challenges were investigated there. However, in this paper we limit our attention to a proper subset of findings found therein. This paper examines how teachers' varied experiences with proof – some having had strong undergraduate exposure, others with limited formal training – affect their beliefs about teaching. It also highlights constraints such as limited curriculum materials and the lack of emphasis in assessments, both of which lead to an over-reliance on textbook demonstration and reduced opportunities for student-centred proving activities. By unpacking these factors, this study contributes to our understanding of the enacted curriculum for proof instruction and points to areas for improvement in teacher preparation and curriculum development. The

findings aim to inform ongoing efforts to strengthen the teaching of proof in secondary schools and ensure that students acquire reasoning skills that go beyond procedural competence.

Conceptual Framework

This study adopts a dual-framework approach by integrating Harel's (2007) Proof Scheme Framework and Cabassut et al.'s (2012) Belief System Framework, offering a holistic lens to examine both the cognitive and affective dimensions of teachers' proof instruction.

Teachers' Conceptions of Proof: Harel's (2007) Proof Scheme Framework

Harel (2007) prioritised proof function as a credible argument. He obtained from this idea ways to "classify individuals' argumentative behaviours" through the type of reasoning they tend to use and referred to proof schemes which were classified into three categories:

- External Conviction – Reliance on authority (e.g., "My teacher said this is true"), procedures, or symbols without understanding. This is further subcategorised into 'Authoritative', 'Ritual' and 'Non-referential symbolic'.
- Empirical – Validation through examples or visual patterns, including inductive or perceptual reasoning. This comprises subcategories of 'Inductive' and 'Perceptual'.
- Analytical – Deductive reasoning using formal logic, subdivided into transformational (structured-based) and axiomatic (based on definitions and theorems). This consists of 'Transformational' and 'Axiomatic' as subcategories.

Analysing teachers' responses to proof tasks through this framework helps identify their underlying proof schemes and the implications for their instructional practices.

Teachers' Beliefs About Teaching Proofs: Cabassut et al.'s (2012) Belief System Framework

Cabassut et al. (2012) studied teachers' beliefs about proof and devised categories to describe these distinct beliefs. They emphasised how beliefs shape teaching decisions, outlining:

- Beliefs about the nature and role of proof in mathematics.
- Beliefs about proof's place in the school curriculum.
- Beliefs about the teacher's own mathematical ability.
- Beliefs about instructional strategies for proof.

These beliefs influence how teachers prioritise, deliver, and assess proof instruction in classrooms.

An Integrated Framework for Studying Teachers' Instruction of Proof

Although Harel's (2007) Proof Scheme Framework and Cabassut et al.'s (2012) Belief System Framework originate from different strands of mathematics education research – one focusing on cognitive conceptions of proof and the other on affective and pedagogical beliefs – there is a strong theoretical alignment between them. Both frameworks recognise that how proof is taught depends not only on the teacher's conceptual understanding of proof but also on their beliefs about its role in mathematics education. The integration of these frameworks is not arbitrary; it is based on the premise that teachers' conceptions of proof influence their beliefs about proof instruction, and these beliefs, in turn, shape their enacted teaching practices. For example, a teacher who predominantly operates within an Empirical Proof Scheme (Harel, 2007) may hold a belief that proof should be taught through example-based exploration rather than formal deduction (Cabassut et al., 2012). By linking these two perspectives, the integrated framework allows for a holistic understanding of proof instruction – one accounts for both the cognitive processes involved in doing proof and the affective, pedagogical, and epistemological

considerations involved in teaching proof. Thus, it provides a structured way to analyse the enacted curriculum of proof instruction in Singapore's secondary schools and helps identify areas for improving teacher education and curriculum design.

Methodology

Research Design

This study employs a qualitative case study approach to examine the factors shaping teachers' instruction of mathematical proof in Singapore secondary schools. A multiple-case design was adopted, involving two in-service secondary school mathematics teachers. While the small sample size limits broad generalisability, case studies allow for rich, context-specific insights (Creswell, 2012). The selection of one junior teacher (10 years of experience) and one senior teacher (39 years of experience) ensures a comparative perspective, capturing a range of instructional practices. Data was collected through two survey questionnaires and semi-structured interviews, with responses analysed using both qualitative and quantitative techniques (e.g., counting the frequencies of various proof schemes). Triangulation of data sources (questionnaire responses, task performance, and interviews) was employed to enhance validity and mitigate self-report bias.

Participants

Two secondary school mathematics teachers were purposefully selected based on the following criteria: (1) Mathematics training background – Both teachers held a degree in pure mathematics and a postgraduate diploma in education; (2) Years of teaching experience – One teacher (Teacher 1) had 10 years of teaching experience, while the other (Teacher 2) had 39 years, providing perspectives from both junior and senior educators; (3) Teaching responsibilities – Both teachers were teaching Elementary Mathematics (EM) and Additional Mathematics (AM) at the upper secondary level. These selection criteria ensured that the study included teachers with a strong mathematics background and extensive classroom experience in proof instruction.

Instruments

Three instruments were used for data collection: (1) Conceptions Questionnaire – A set of nine modified Ordinary-Level (O-Level) proof-related questions from the EM and AM syllabi. The O-Level is a national examination which secondary school students sit for at the end of their secondary education in Singapore. Teachers' responses were analysed using Harel's (2007) Proof Scheme Framework to categorise their proof schemes. Inter-rater reliability was conducted to ensure that the classification process was rigorously reviewed to ensure consistency and to avoid bias. (2) Beliefs Questionnaire – a 40-item Likert-scale survey adapted from Cabassut et al. (2012) Belief System Framework (Thanabalasingam, 2024). Questions were categorised into: (i) Teachers' beliefs about the role of proof in school mathematics; (ii) Teachers' beliefs about themselves as mathematical thinkers; and (iii) Teachers' pedagogical beliefs about proof instruction. While self-reported data may be influenced by social desirability bias, the study mitigated this through data triangulation, cross-referencing questionnaire responses with task performance and interview insights. (3) Semi-structured Interviews – Follow-up interviews were conducted to probe teachers' reasoning in responding to the questionnaires and to identify additional challenges in proof instruction. The interviews were audio-recorded and transcribed for qualitative analysis, which we elaborate in the section on Data Analysis. Interview protocols were developed based on prior literature and reviewed to minimise interviewer bias.

Procedures

Teachers were briefed on the study's objectives and provided consent to participate. They completed the Conception Questionnaire as a take-home task within one sitting (150 minutes). While external influences could not be entirely controlled, interview cross-checking was used to verify the authenticity of responses. One week later, they completed the Beliefs Questionnaire. Immediately after the Beliefs Questionnaire, semi-structured interviews were conducted with each teacher (about 30 minutes per session).

Data Analysis

Teachers' conceptions of proof were analysed using Harel's (2007) Proof Scheme Framework, categorising responses into External Conviction, Empirical, and Analytical Proof Schemes. Inter-rater reliability checks were in place. Teachers' beliefs about proofs were analysed using Cabassut et al.'s (2012) Belief System Framework, examining their epistemological, didactical, and pedagogical perspectives. Interview data was transcribed and coded to identify recurring themes and challenges in proof instruction. Coding consistency was ensured through independent interviews and iterative refinement. This methodological approach acknowledges its limitations while strengthening its credibility through triangulation, cross-checking procedures, and transparency in data collection. The study provides a nuanced, in-depth examination of how teachers' conceptions and beliefs shape their enacted curriculum for proof instruction in secondary school mathematics classrooms.

Findings and Discussion

We now present the findings from the study, organised according to five themes as categorised in Thanabalasingam's (2024) study: teachers' conceptions, experiences, approaches, perceptions, and challenges related to teaching proofs. Each theme is supported by evidence from the Conceptions Questionnaire, the Beliefs Questionnaire, and semi-structured interviews.

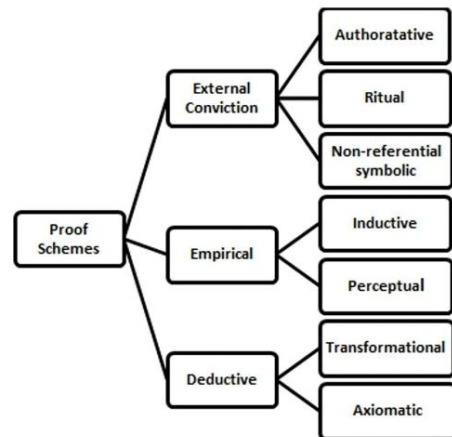
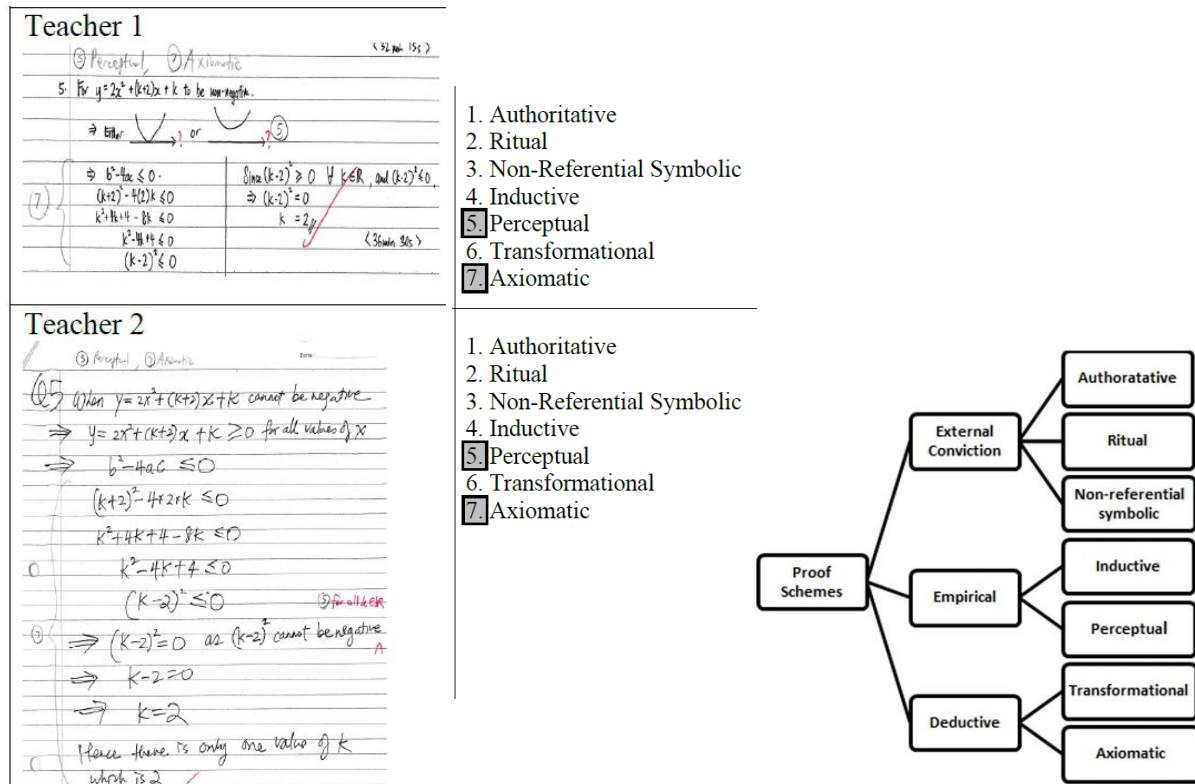
Teachers' Conceptions of Proof

Taking note of the number of times, each proof scheme surfaced for each question, the findings indicate that teachers exhibit varying proof conceptions, aligning with Harel's (2007) Proof Scheme Framework. While one teacher predominantly relied on Empirical Proof Schemes, frequently validating mathematical statements through pattern recognition rather than formal deduction, the other demonstrated a more Analytical Proof Scheme, emphasising logical transformations and axiomatic reasoning. This is exemplified in their attempts of Question 5 (Algebra Strand) which required them to prove that there exists only one value of the constant k for which $y = 2x^2 + (k + 2)x + k$ cannot be negative and to state its value (see Figure 1).

Both teachers applied axiomatic reasoning in structuring their arguments using standard algebraic theorems. However, aspects of perceptual reasoning emerged – Teacher 1, for instance, sketched quadratic graphs without explicitly defining the horizontal axis, assuming it was understood to represent x . Similarly, Teacher 2 omitted crucial definitions of a , b , and c in the discriminant expression, perceiving it as a norm to represent the discriminant in this 'standard' form. Also, she perceived that $(k - 2)^2$ cannot be negative always *without* indicating clearly that this was quantified over all real numbers – it may be negative if complex numbers are allowed. Thus, Perceptual and Axiomatic proof schemes were the most observed schemes among the teachers.

Figure 1

Responses of Teachers and Respective Proof Schemes (Question 5) and Harel's Proof Schemes



Teachers' Experiences in Teaching Proof

Both teachers had limited exposure to proof instruction during their teacher training. The junior teacher, Teacher 1 (10 years of experience), noted that proof was once emphasised more in Singapore's curriculum but has diminished over time. The senior teacher, Teacher 2 (39 years of experience), admitted that proving was largely absent from their pre-service education, leaving them to learn proof-teaching techniques through trial-and-error. Additionally, their teaching experiences related to teaching proof differed in the following ways: Teacher 1 had a more positive experience with proof instruction. He actively used diagrams and multiple examples when teaching proofs, emphasising a visual and structured approach. Teacher 2 had a mixed or neutral experience with proof instruction. She did not regularly encourage students to construct or present proofs and felt less confident in incorporating proof-based activities in class. Furthermore, findings from the teachers' beliefs system questionnaire and insights from the subsequent semi-structure interviews indicated that while both teachers regularly presented proofs, neither strongly emphasised student-centred proof construction (e.g., working in groups or presenting proofs). Notably, Teacher 2 said "... but I'm not trained to teach proofs in the in-service courses... Uh, yes. I mean lack of training courses in teaching proofs. Because there are many courses on heuristic problem-solving, but not on proof". The lack of formalised training in proof instruction suggests a gap in teacher preparation programs. Teacher 1 said "...back to the question as teaching proofs uh, I find myself needing to bring in a lot of my prior experience". So, if teachers are not systematically exposed to proof pedagogy, their instruction may be shaped more by personal experiences than by research-informed strategies.

Teachers' Approaches to Teaching Proof

The study found significant differences in how the two teachers approached proof instruction. Teacher 1 adopted a student-centred approach, encouraging peer discussions and allowing students to play a role in proof development. He emphasised logical reasoning and engagement in understanding proofs rather than passive notetaking. Teacher 2 took a teacher-centred approach, relying more on textbook examples and direct instruction with limited student involvement in proof construction. Survey results further indicated that while Teacher 1 actively encouraged student participation, he did not integrate proof construction regularly into classroom activities or homework assignments. Teacher 2, on the other hand, rarely engaged students in proof discussions and placed a strong emphasis on direct explanation. To illustrate this point, we note that in teaching congruence proofs in geometry, Teacher 1 encouraged students to verify congruence properties with physical cut-outs before moving to formal proof, fostering student engagement. In contrast, Teacher 2 followed a structured approach using textbook examples and guided practice, with minimal student involvement in constructing proofs. These findings suggest that instructional approaches to proof are strongly shaped by teaching experience. Younger teachers may be more open to student-centred discovery approaches, whereas experienced teachers may adhere to traditional didactic instruction. This highlights the need for professional development programs to encourage diverse and effective proof-teaching strategies.

Teachers' Perceptions to Teaching Proof

Teachers' perceptions of proof instruction varied. While both teachers strongly agreed that proofs are essential in mathematics, their views on student involvement differed. Teacher 1 strongly valued student engagement, encouraged classroom group discussions, and believed students should actively participate in constructing proofs. Teacher 2, though generally positive about proof, was less convinced that all students should learn to read and write proofs. She had a neutral stance on whether students should play a significant role in constructing proofs. Both teachers agreed that it is the teacher's responsibility to present proofs, rather than relying on students to construct them independently. Teacher 1 supported active student contributions, while Teacher 2 remained neutral on this point.

Teachers' Challenges to Teaching Proof

Teachers face several challenges when teaching mathematical proofs, as surfaced during the semi-structured interviews. These challenges are influenced by various factors. Curriculum materials play a role in preparation, but teachers find them lacking in explicitly demonstrating the proof process. One teacher noted that the curriculum does not sufficiently emphasise proof instruction. Training is another factor—while undergraduate studies help develop mathematical maturity, there is a gap in in-service training on how to teach proofs effectively. Personal experience also shapes teaching ability; Teacher 1 credited his struggles as a student for his understanding, more than formal training. Determining the correct approach to proofs poses difficulties, as evidenced in the conception's questionnaire, with one teacher struggling under time pressure, while another found problems easier due to prior exposure. Teaching proofs effectively is a challenge, as students often struggle with clearly expressing their reasoning and lack motivation to learn proof techniques, especially if they already find mathematics difficult. Finally, both teachers suggested curriculum and training improvements. Teacher 1 recommended access to exam markers' reports to address student misconceptions and better guidance for teachers in supporting students' proof development. Teacher 2 emphasised increasing the inclusion of proofs in the curriculum, more in-service courses on proof teaching, and greater emphasis on proof-related questions in exams. Overall, the key challenges involve

curriculum adequacy, training gaps, personal experiences, proof-solving difficulties, teaching struggles, and areas for improvement in curriculum and teacher preparation.

Summary of Findings and Identified Gaps in the Enacted Curriculum

Triangulation of data identified several factors shaping teachers' instruction of proof. Teachers' proof schemes, mathematical knowledge, reasoning patterns, and misconceptions influence their conceptions of proof and instructional approaches. Their experiences are shaped using visual aids and examples. Instructional style varied between student-centred and teacher-centred methods, differing in proof development, student involvement, and assessment strategies. Teachers' beliefs about the purpose of proof in mathematics guided their teaching priorities. Challenges such as curriculum constraints, limited training, time pressures, and assessment demands also significantly affected instruction. Understanding these factors can inform improvements in professional development, curriculum design, and assessment policies.

Key gaps in the enacted curriculum hinder effective proof teaching in secondary mathematics. From this study, some of these include: (1) Guidance on Proof Development – Teachers need structured frameworks to guide students through the proof process. (2) Teacher Training – There is a lack of specialised training in proof pedagogy; in-service programs should address this. (3) Recognising Students' Thinking – Teachers need better strategies to identify and develop students' reasoning in proof. (4) Curriculum Improvements – Teachers suggested greater access to markers' reports, more proof problems, and support for developing mathematical maturity. By addressing these gaps, curriculum developers can better support teachers in delivering effective instruction and fostering a deeper understanding of proof among students.

Implications and Recommendations

The findings from this study may have some implications for the teaching of mathematical proof in secondary school mathematics in Singapore, granted the limitation of small sample size. In this section, we present key recommendations aimed at addressing the challenges identified in teachers' conceptions, experiences, approaches, perceptions, and challenges related to proof instruction.

Strengthening Teacher Training in Proof Pedagogy

Given that teachers had limited formal training in proof instruction, pre-service and in-service professional development programs should include explicit training on teaching proof. Workshops can focus on different proof schemes, reasoning patterns, and instructional strategies to enhance teachers' confidence and competence in proof instruction. Training programs should incorporate case studies and lesson demonstrations to illustrate proof-teaching strategies.

Addressing Curriculum Gaps in Proof Instruction

The Ministry of Education and curriculum developers should consider reviewing current textbooks and instructional materials to ensure they provide adequate support for teaching proof. Additional teaching guides can be developed to offer step-by-step strategies for guiding students in proof construction. Efforts should be made to align teacher training programs with curriculum reforms, ensuring that new proof-related content is well-supported by pedagogical training.

Encouraging Professional Learning Communities

Schools should establish professional learning communities (PLCs) where teachers can collaborate, share best practices, and reflect on their proof instruction experiences. Cross-school

collaborations and lesson study groups can be formed to allow teachers to observe proof-teaching strategies in a supportive environment. Online forums and teacher networks should be strengthened to provide a platform for ongoing discussions and resource-sharing among mathematics educators.

Conclusion

This study set out to investigate what shapes teachers' instruction of mathematical proof in secondary school mathematics classrooms. By examining teachers' conceptions, experiences, approaches, perception, and challenges, we have uncovered key influences on proof instruction and identified gaps in the enacted curriculum. The findings underscore the importance of strengthening teacher training, improving curriculum resources, refining assessment practices, and fostering professional collaboration to support proof instruction.

Inevitably, this study contributes to the broader discourse on mathematics education and proof pedagogy, providing valuable insights for educators, curriculum developers, and policymakers. Future research can further explore how these proposed changes impact classroom practices and student learning outcomes, ensuring that proof remains an integral and meaningful component of mathematics education.

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Top Number or Numerator? Teachers' Choices of the Mathematics Register in Mathematics Teaching Talk

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Recent research has called for more attention in the understanding and developing of mathematics teaching talk, focusing on the specific use of language as a resource to help students learn mathematical concepts. Based on a task-based interview with eleven teachers, this paper attempts to utilise the Mathematics Register Knowledge Quartet to examine what these teachers know and may do when teaching fractions. Consequently, the analysis of the teachers' responses, in relation to the four dimensions of the quartet, provided insights on how teachers might attend to or use (or not) the mathematics register in the mathematics teaching talk of fractions and related concepts.

Language as a Resource for Teaching and Learning Mathematics

Though language as a research focus in mathematics education is a relatively young sub-field, a wide range of work has been done in this area for at least the last half of the century. This included research such as those focusing on language's nature and role in mathematics teaching and learning and those focusing on the sociopolitical dimensions of language in mathematics education (Barwell, 2021). Research perspectives on the role of language in mathematics education have also evolved from being deficit-oriented initially (e.g., Cummins, 1979, as cited in Schütte, 2018) to more resource-oriented (e.g., Adler, 2000; Planas, 2018).

The Mathematics Register (*vis-à-vis* Mathematical Language)

Particularly, the mathematics register (Halliday, 1975), often synonymously referred to as the formal or school mathematical language, has been largely researched upon, in terms of its role as an important resource in the developing and acquiring mathematical ideas or concepts (e.g., Sigley & Wilkinson, 2015). Being a complex notion, a register is often misunderstood simply as a collection of highly technical vocabulary terms in the language used to describe objects in an academic discipline (Halliday, 1975). In the same manner, the mathematics register is not just a collection of mathematics-related words or terms. Instead, the mathematics register further determines how these words or terms are used or structured within the natural language to form unique phrases or clauses that can precisely represent and communicate both explicit and implicit mathematical meanings or relationships (Wilkinson, 2018).

To sum up, the mathematics register serves the function of thinking about (and communicating in spoken or written forms) mathematical ideas and meanings (Pimm, 1987; Vygotsky, 1934/1986). Consequently, the mathematics register is a rich and useful resource for mathematics teaching (and learning), particularly in mathematics teaching talk (as defined by Planas et al., 2023a), which has the potential to “open up or close opportunities for participating in the mathematical discourse for school learners” (p. 522). However, this is only possible if teachers have actually developed an understanding of “the forms and the meanings and ways of seeing enshrined in the mathematics register” (Pimm, 1987, p. 207) or have internalised the mathematics register as a tool for thinking for themselves (Wilkinson, 2018), such that they could appropriately mediate its use in learning processes. Unfortunately, this is not necessarily true for all teachers. For instance, in a study (Lane et al., 2019) on pre-service teachers' mathematics register proficiency, it was found that these teachers generally do not have sufficient knowledge and understanding of the mathematics register. Similar findings were observed in the case study of six early career teachers' understandings of the register (Turner

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 429–436). Canberra: MERGA.

et al., 2019), which resulted in variations in terms of their practices in using the mathematics register as a resource for teaching. While both studies presented what could be seen as deficit-oriented narratives of pre-service or beginning teachers' mathematics register proficiency, they also highlighted the importance of thinking about how the mathematics register can be better used as a resource in both mathematics and mathematics *teacher* education.

The Mathematics Register Knowledge Quartet (MRKQ)

In the same study by Lane and colleagues (2019), a framework adapted from the Knowledge Quartet framework (Rowland et al., 2005) was conceptualized with the intent of analysing pre-service teachers' ability in using the mathematics register to facilitate a peer-teaching segment of a teachers' education course. In particular, they (Lane et al., 2019) argued that, while there has been much research done in terms of pre-service teachers' mathematical knowledge for teaching (e.g., Ball et al., 2008), little attention has been given to mathematics language as being part of knowledge for teaching, though it is deemed as an essential component of mathematics teacher education (Cramer, 2004, as cited in Lane et al., 2019). Hence, they attempted to provide a glimpse into pre-service teachers' knowledge of the mathematics register by "redefining" how teachers' knowledge of the mathematics register may look like in relation to the four dimensions of the Knowledge Quartet (Rowland et al., 2005). A full illustration of the MRKQ with examples can be found in their study (see Table 1 in Lane et al., 2019, p. 793). Briefly, the four dimensions of the MRKQ are defined as follows:

- *Foundation* dimension focuses on teachers' knowledge and understanding of the register and their awareness of differences between everyday language and the register.
- *Transformation* dimension focuses on teachers' *knowledge-in-action*⁵ through their planning and actual teaching, in terms of how they plan for mathematical language and use representations and analogies to elicit mathematical meaning for students.
- *Connection* dimension focuses on teachers' consistency in the use of the register within and between lessons, and across different mathematics topics, coupled with an awareness of students' difficulties with the register.
- *Contingency* dimension focuses on teachers' *knowledge-in-interaction*⁵ observed through their abilities to interpret students' register in line with the mathematics register and facilitate an adherence to the mathematics register during classroom interactions.

Notably, both Knowledge Quartets by Rowland et al. (2005) and Lane et al. (2019) were primarily developed for the purpose of understanding pre-service teachers' knowledge for teaching and their teaching practices. However, I see value in how the constructs within the four dimensions can similarly be used to better understand the existing state of experienced teachers' knowledge of the mathematics register, as well as, how and why their mathematics teaching talk are shaped in a certain manner. As argued by Rowland et al. (2005), the Knowledge Quartet "provides a means of reflecting on teaching and teacher knowledge, with a view to developing both" (p. 257) – a framework to analyse and discuss teachers' mathematical content knowledge. Correspondingly, the MRKQ (Lane et al., 2019) can be used as a frame to think about mathematics teaching practices with a focus on teachers' knowledge of the mathematics register. Thus, in this paper, I hope to inform the existing state of how teachers are attending to language as a resource in their classroom – an aspect which appears to be lacking in recent research – using the MRKQ. Specifically, I looked at the following research question: How are teachers' knowledge and potential usage (*knowledge-in-action* and *knowledge-in-interaction*) of the mathematics register featured through their responses to a task on fractions?

⁵ These terms *knowledge-in-action* and *knowledge-in-interaction* were defined in the original Knowledge Quartet (Rowland et al., 2005).

Method and Data

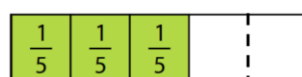
The data for this paper was part of my PhD research study, which investigated the same phenomenon in greater depth and scope. To address the research question, an overall qualitative approach was taken to collect and analyse data. The data was collected through semi-structured interviews with nine experienced mathematics teachers based in Canada in the first quarter of 2022. All the teachers had taught or were still teaching mathematics in English-medium classrooms, ranging from elementary to tertiary levels. As the data was collected during the early post-COVID19 pandemic period, I was limited in terms of my research approach, due to restrictions on face-to-face interactions with my participants. Hence, I designed a task-based interview protocol where the teachers were asked to respond to a series of eight tasks. These tasks were created and presented in the form of a fictional dialogue between a pair of student characters to illustrate classroom-based situations which might lead to language-related dilemmas and challenges when using the mathematics register relating to a range of concepts across different content foci (numbers, fractions, geometry and graphs). After reading each task, the teachers were asked to reflect upon what they noticed about the language used by the students and how they would respond if they were the teacher in that situation. Further details on the specific tasks and the methodology can be found in my doctoral thesis (Tiong, 2024).

Considering the size of this paper, the remaining discussion is based on interview data for the task which focused on the concept of fractions – an important yet challenging mathematical concept for many children and even adults (e.g., Kieran, 1992; Siegler & Lortie-Forgues, 2017). In a review conducted by Siegler and Lortie-Forgues (2017), they attributed the difficulties of the fraction concept to *inherent* factors such as its confusion with whole number arithmetic, and culturally contingent factors such as teachers' knowledge of fractions and the teaching of fractions. For example, students tend not to understand or view fractions as numbers – an important aspect in the concept of fraction which is often neglected. Instead, they may consider a fraction as being a mathematical object which is made up of two separate whole numbers. This misconception is often compounded with the colloquial use of the *top number* and the *bottom number* when referring to the *numerator* and the *denominator* of a fraction respectively. The following task simulates a fictional dialogue between two elementary-level students (S1 and S2) who are working with fraction strips to show the fraction $\frac{3}{5}$ [see Figure 1], as follows:

- S1: Because the bottom number is five, we need to use the green piece (which denotes the fraction $\frac{1}{5}$).
- S2: And we need three of them to get the top number three.

Figure 1

Diagram of Fraction Strips Given in the Task



Here, S1 and S2 seemed to understand what each other meant by the *top number* and the *bottom number* when discussing about how they should represent the fraction $\frac{3}{5}$, using the fraction strips. In other words, they appeared to have a common student mathematics register, which is more colloquial, in relation to fractions. While there did not seem to be any apparent misconception due to the use of the student mathematics register (which differs from the formal mathematics register) in the task, their understanding of the concept of fractions, due to the use of colloquial terms, was up for interpretation. In particular, the task aimed to prompt the participants to think about whether (and how) they would mediate the use of language in this instance and if such language use was appropriate in the mathematics teaching talk of fractions.

Teachers' Choices of Mathematics Teaching Talk on Fractions

In analysing the teachers' responses, I used the four dimensions of the MRKQ as the lens to identify quotes which were indicative of the teachers' knowledge and use of the mathematics register in the mathematics teaching talk of fractions, based on how they would respond to these tasks – their articulated *knowledge-in-action* and *knowledge-in-interaction*. Notably, most quotes identified are not unique to only one dimension, but often relevant to other dimensions of the MKRQ (Rowland et al, 2005). The following is an example of a quote, which showed a participant's understanding of the mathematics register surrounding the concept of fractions within the *Foundation* dimension, as well as her ability to interpret students' register and inferred their understanding of fractions within the *Contingency* dimension.

[...] there is no reference to that relationship, part-whole. So it is like that they are separate but procedurally. But they will see that the bottom, it should be divided to that one in the top, it should be divide. [...] so it feels like they didn't build a meaning for this part-whole relationship here [...] they don't develop this relationship, they see them as separate numbers.

The subsequent discussion is substantiated with teachers' (identified with participant numbers T1 – T9, in order of appearance) responses where relevant.

Foundation Dimension

All participants were cognizant of the mathematics register, such as *numerator*, *denominator*, *parts* and *whole*, surrounding the concept of fractions. They also showed understanding of what *numerator* and *denominator* referred to or meant in relation to the relative positions in the fraction representation, analogous to the top and bottom notions that the students ascribed to in the task. The part-whole relationship in fractions was also referred to when they elaborated on the meaning of the two words. For example, T1 defined a fraction as “pieces of a whole, where the whole is the denominator and the number of pieces you have is the numerator”. T2 stated that the denominator represents “how many pieces the whole [is] divided into”. Their explanations mostly focused on the definitions of *numerator* and *denominator* in the case of proper fractions, which might not be applicable to other types of fractions such as improper fractions or compound fractions. Yet, differences were observed in how they described the concept of fraction. For instance, only T3 and T4, explicitly mentioned the idea of fractions as numbers. Specifically, T3 articulated that she would want her students to “think of one-fifth as the number, not like the five as a number and the one is a number”. This concept has implications on how students may understand what the numerator and the denominator mean in relation to any given fraction due to its symbolic representation.

In terms of the awareness of the differences between the everyday and the mathematics registers, several teachers hinted at how there might be a disconnect in terms of how fractions are used within the everyday and the mathematics register. For example, T5 mentioned how “there's a gap between how we talk about fractions in real life, versus the terminology”, when asked if she would be concerned with students using terms such as *top* and *bottom numbers*, instead of *numerator* and *denominator*. She further highlighted how terms specific to the concept of fractions, are rarely mentioned or heard outside of the mathematics classroom. T5's viewpoint was similarly shared by T4, who added that perhaps even *top number* and *bottom number* did not seem to be common terms that would be heard in the everyday context. However, to T4, these terms seemed more concept-related, and she wondered “where this language (*top number* and *bottom number*) comes from”. She highly doubted that it would be “out of their natural home experience” and elaborated that parents would more likely be using names of specific fractions (e.g., a half or a third) if they were to teach or talk to their children

about fractions. One might more likely hear a phrase such as “cut that apple in half”, than a discussion on the *top* and *bottom numbers* of a fraction, in the everyday context.

Transformation Dimension

As most teachers focused on the use of *top* and *bottom numbers* as being indicative that the students likely understood the fractional representation, there was not much in-depth discussion of the approaches to how they would specifically plan for the teaching of fractions. Primarily, their articulated *knowledge-in-action* related to how they would introduce the mathematics register, specifically *numerator* and *denominator*, in relation to the task. Interestingly, though the concept of fractions is often taught with representations and analogies, most teachers did not go into that discussion. Instead, the interviews revolved around whether and how they would teach words in the register. In retrospect, I wondered if it was due to the design of the task, or due to how the connection between the learning of language and the learning of mathematics concepts might have gone unnoticed by the participants. Nonetheless, there was one teacher, T2, who elaborated on how she would use other representations to help students develop deeper understanding of the part-whole relationship in the concept of fractions. She felt that the brief dialogue between S1 and S2 was insufficient to inform her about their understanding of fractions. Other than asking students to explain what they meant, with reference to “what’s the part and what’s the whole”, she mentioned the importance of using different (and atypical) representations to reinforce their understanding of the part-whole relationship. She further described how she would “switch up the wholes and switch up the parts”, by using hexagons to represent the whole and parts of the whole, instead of the typical pie or number strip.

Connection Dimension

With the interview data to the task alone, it was insufficient to ascertain the teachers’ use of the mathematics register across topics and lessons. However, some observations relating to their consistency in the mathematics register relating to the concept of fractions could still be made. Notably, the teachers seemed to be less consistent in terms of their use of the mathematics register while relating to the concepts of fractions. Many teachers mentioned (implicitly and explicitly) that they would not mind the student mathematics register presented in the task and even code-switch between the student mathematics register and mathematics register. For example, T6 would “call it the *bottom number*” as she introduced the concept of the *denominator*. Subsequently, while she would model the term *denominator* in her teaching, she would constantly remind students that it “means the *bottom number*”, until they become “comfortable with the math”. In contrast, only T3, who explicitly mentioned the idea that fractions should be seen as numbers, shared that she was not comfortable with students using *top* and *bottom numbers*, in the context of fractions. To her, the student register, in this case, would likely be inconsistent with their prior or future understanding of rational numbers.

In relation to why the students might have chosen to use *top* and *bottom numbers*, rather than *numerator* and *denominator*, all the teachers attributed it to at least one of the following three difficulties – the unfamiliarity with the terms *numerator* and *denominator*, the perceived lack of usefulness of *numerator* and *denominator*, and the challenge with describing *numerator* and *denominator* other than as numbers. Firstly, the unfamiliarity with the terms were pointed out by both T4 and T7. Notably, the terms *numerator* and *denominator* are rarely used in the everyday context and very specific to the concept of fractions in the mathematical context. As such, students would mostly likely think of them as analogous to unfamiliar “foreign language”. Secondly, some teachers (e.g., T2 and T5) voiced their opinion regarding the perceived lack of usefulness of these terms, although they valued the precision of the mathematics register. To them, learning the terms might not necessarily be helpful in developing students’ understanding. Possibly because the part-whole relationship is not obvious in the terms

numerator and *denominator*, T5 even commented how she would not “necessarily think that the terminology itself is super-useful”. Lastly, T3 shared her struggled attempt to avoid referring to *numerator* and *denominator* as two separate numbers. She commented how “the language here is tricky” and questioned how teachers could name and refer to *numerator* and *denominator* without using the word *number*. Specifically, she asked, “one-fifth is a number, so what should we call that bottom five?” Consequently, it made me reflect about how many teachers (including myself) might likewise refer to the one and five in the fractional representation for one-fifth as numbers, for lack of a better term.

Contingency Dimension

Intentionally, the design of the task focused the teachers’ attention to the student mathematics register (*top* and *bottom numbers*) and how they interpreted it with the formal mathematics register (*numerator* and *denominator*). In responding to what they would do as a teacher in this task, the extent to which they would adhere to the mathematics register during their interactions with students varied, pending different considerations. Generally, all the teachers were able to interpret how the students were referring to the *numerator* and the *denominator* when they used *top number* and *bottom number* respectively. Two teachers further shared their inference of the two students’ understanding of fractions, based on their more colloquial mathematics register. For instance, T7 felt that by using “*bottom number* is five, *top number* is three” in the description of the fraction $\frac{3}{5}$ indicated a rather procedural understanding of fractions. The students seemed to be thinking how “the only relationship between the numbers is where they stay, top or bottom”. This lack of “reference to that part-whole relationship” further reinforce the possibility that they were seeing the *numerator* and the *denominator* as separate numbers. On the same note, T4 commented, “it is clear here from this discourse that fraction is not seen as a number, it's seen as something separate”.

In terms of how the teachers would facilitate an adherence (or not) to the mathematics register in mathematics teaching talk on fractions, many shared that they would model the use of the mathematics register. However, most of them would not necessarily correct the students’ usage of *top* and *bottom numbers* in this instance. They would instead consider students’ readiness and whether they might still be at the early stages of learning about fraction concepts. Some teachers were even doubtful about the connection between the students’ usage of the mathematics register and their understanding of the concepts as they considered these terms as specialised terms which only exist in the mathematics register. For example, T7 was not confident that students who “say denominator” would demonstrate a different (or higher) level of understanding as compared with those “who would describe it as a *bottom number*” by explaining how these terms “are so mathematical, and it’s not used in daily life that much”.

Consequently, most teachers would likely push for a greater adherence to the mathematics register after the students developed a more conceptual understanding of fractions. They generally agreed that mathematics register could be acquired gradually when students are ready. For instance, T3 commented how “the class would eventually pick up on that” while T8 suggested that the mathematics register “would be something to develop and grow” as students become familiar with fraction concepts but “not at the elementary level” necessarily. Similarly, T6 would “step in and actually emphasise the correct word because they have the conceptual understanding” and encourage them to “practise using it”, since “they’re both there and they both can practise at the same time”. T6’s actions would probably resonate with T9 who strongly believed that to “build our knowledge base, we have to know the language as well” and explained, “if they don’t use these terms, how are they going to get comfortable with them?”

Discussion and Conclusion

Collectively, all the experienced teachers demonstrated an understanding of the content-specific mathematics register related to the concept of fractions within the *Foundation* dimension of the MKRQ (Lane et al., 2019). However, this knowledge might not be translated into actions as they would still plan for the teaching of fractions without necessarily involving the mathematics register (*Transformation* dimension). Moreover, there appeared to be a consensus in how they would mediate the use of the content-specific mathematics register in this case. The teachers would generally model the use of the specialised terms, *numerator* and *denominator* while code-switching with the use of *top number* and *bottom number* (*Connection* dimension). They would also not expect or insist that their students use these terms, which are seen as specialised within the mathematics register (*Contingency* dimension).

While I am cognisant that mathematics teaching talk can occur without the use of the mathematics register⁶, I wonder if the lack of emphasis on the use of the register in mathematics teaching and learning processes might reduce the opportunities for students to engage in richer discussions and deeper reasoning around fractions. As proposed by Kieren (1992), a potential gap exists between students' "conceptual models of fractional numbers" (p. 326) and their experiences working with the symbols of fractions. On the same note, I wonder if a gap may also exist between students' conceptual understanding and their language or verbal experiences with fractions. Notably, studies (e.g., Siegler & Lortie-Forgues, 2017) have suggested how the mathematics register in some East Asian languages, such as Korean and Mandarin, might better develop students' understanding of fractions as the meanings of *numerator* and *denominator* are explicated more clearly through the way fractions are named in these mathematics registers.

Hence, in addressing the possible misconception where fractions are often not considered numbers by students, it may be necessary for teachers to bring in the etymology of the terms *numerator* and *denominator* in their mathematics teaching talk. Specifically, the word *numerator* has origins related to ideas of enumerating or counting, while the word *denominator* is linked to the act of naming. In other words, the *numerator* enumerates the number of *denominations* that has been named in the fraction. Moreover, this understanding can be further extended to that of rational numbers where integers are fractions that enumerate in denominations of ones while fractions such as $\frac{3}{5}$ enumerate in denominations of fifths. An emphasis on the *counting* of the respective denominations named in different fractions may in turn help students perceive fractions as numbers (as single entities) instead of viewing the "numerator and denominator as two independent parts" (González-Forte et al., 2018, p. 465). On the same note, Planas and colleagues (2023a) also argued how there might be "meaning encoded in the mathematics register" (p. 524) that requires teachers to decode for students during mathematics teaching talk to allow students "who have little to no experience with making specialised word use function in order to discuss and reason mathematically" (p. 523).

Fundamentally, such an approach may also help to strengthen the multiplicative relationship between the numerator and the denominator which students tend to struggle with – as seen from their erroneous thinking in the comparison of fractions where they either compare numerators and denominators separately or compare the gap between them. By speaking of and seeing a fraction as enumerating a certain denomination (equivalently, m times of the $\frac{1}{n}$ unit in the fraction $\frac{m}{n}$), it might lower the likelihood of students reasoning additively with the difference between the numerator and the denominator. Consequently, this approach may address the

⁶ In their recent discussion piece, Planas and Pimm (2023b) argue for the potential of developing the notion of a *mathematics communication register*, which encompasses both linguistic (e.g., the mathematics register) and non-linguistic (e.g., gestures) aspects of mathematical communication.

perceived lack of usefulness or relevance of the terms *numerator* and *denominator* by decreasing the seemingly arbitrary nature of these specialised terms in the eyes of both teachers and students. Hence, a next step of my research will be to expand the dimensions of the MRKQ to include an additional focus on the integration of knowledge relating to the etymology of specialised terms in the mathematics register within topic-specific mathematics teaching talk.

Acknowledgements

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How to Teach the Meaning of Limit Intuitively in Calculus: A Didactical Design Research

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Many students struggle to grasp the concept of limits, leading to persistent learning obstacles. This study employed a didactical design research (DDR) approach involving 56 mathematics education students at Universitas Khairun, divided into two groups. Data from tests, interviews, and teaching interventions revealed three key obstacles: epistemological (e.g., confusion between indeterminate and undefined forms like $0/0$), ontogenic (e.g., difficulties with factoring and graphing), and didactical (e.g., misaligned instruction). A didactical design based on the Theory of Didactical Situations (TDS) was developed to address these challenges.

Introduction

Introducing the concept of limits to students involves a variety of pedagogical approaches, each presenting distinct challenges. Visual and intuitive examples drawn from disciplines such as art, physics, and engineering have been shown to improve student engagement and conceptual understanding (Raviv, 2022). Several studies advocate for teaching limits as a notion of proximity, integrating both static and dynamic perspectives (Shigeno, 2020; Nagle et al., 2017). However, students frequently struggle to synthesise different forms of limit concepts—such as sequences, limits at a point, and limits at infinity—when these are taught in isolation (Fernández-Plaza & Simpson, 2016).

Research on the limit concept in calculus reveals substantial differences between high school and university approaches. At the high school level, limits are often taught as a process, with students relying on dynamic conceptions and procedural understanding (Nagle, 2013). In contrast, university instruction tends to emphasise formal definitions and structural understanding, with relatively consistent praxeology across different countries (Viirman et al., 2022). This shift often poses difficulties for students, who may struggle with abstract mathematical language and the coordination of multiple conceptual processes (Bansilal & Mkhwanazi, 2021). Common misconceptions include formal epsilon-delta definitions (Nagle, 2013). Researchers argue that university curricula frequently overlook students' prior learning experiences, resulting in discontinuities in their mathematical understanding (Bloch et al., 2000). Addressing these misconceptions is crucial for improving calculus instruction at both levels (Winarso & Toheri, 2017).

Students' understanding of limits in calculus reveals several recurring themes. Many struggle with formal definitions, often relying instead on intuitive or dynamic interpretations (Nagle et al., 2017; Tall & Vinner, 1981). Visual and allegorical approaches have been shown to support intuitive understanding (Shaldehi et al., 2022; Raviv, 2022), while algorithmic contexts may serve as stepping stones to deeper conceptual knowledge (Pettersson & Scheja, 2008). Language plays a crucial role, with terms like "tends to" potentially reinforcing misconceptions (Jaffar & Dindyal, 2011). The others are students' mathematical beliefs, particularly their sources of conviction, which can influence their limit comprehension (Szydlik, 2000). Common misconceptions include viewing limits as fixed boundaries or unattainable targets (Szydlik, 2000). To address these challenges, researchers suggest the use (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 437–444). Canberra: MERGA.

of mental imagery and real-world examples to build intuition before introducing formal definitions (Shaldehi et al., 2022; Raviv, 2022). This study investigates students' misconceptions and learning obstacles in developing an intuitive understanding of limits.

Learning obstacles arise when students understand concepts superficially and are unable to relate them to other mathematical concepts. These obstacles are generally categorised into three types: didactical, ontogenic, and epistemological (Brousseau, 2002; Suyadi, 2019). Ontogenic obstacles relate to students' mental maturity and readiness to acquire new knowledge. Didactical obstacles arise from the instructional sequence—such as the order in which material is presented by the lecturer or tasks are completed by students. In contrast, epistemological obstacles stem from the nature of the learning tasks and students' limited ability to extend their thinking beyond procedural knowledge.

Some studies propose strategies and approaches to tackle the misconceptions and learning obstacles. Liang (2015) used conceptual conflict strategies with graphing to introduce limits. Others suggest reformulating the definition as a local approximation (Bokhari & Yushau, 2006) or focusing on covariational reasoning (Nagle et al., 2017). Research indicates that students often rely on dynamic conceptions and struggle with formal definitions (Nagle, 2013). Common metaphors used by students include collapsing dimensions and physical limitations (Oehrtman, 2009). While dynamic interpretations can be intuitive, care must be taken to align them with formal limit concepts (Nagle et al., 2017). Combining mental imagery with puppet allegories can make understanding limits easier and more enjoyable (Shaldehi et al., 2022).

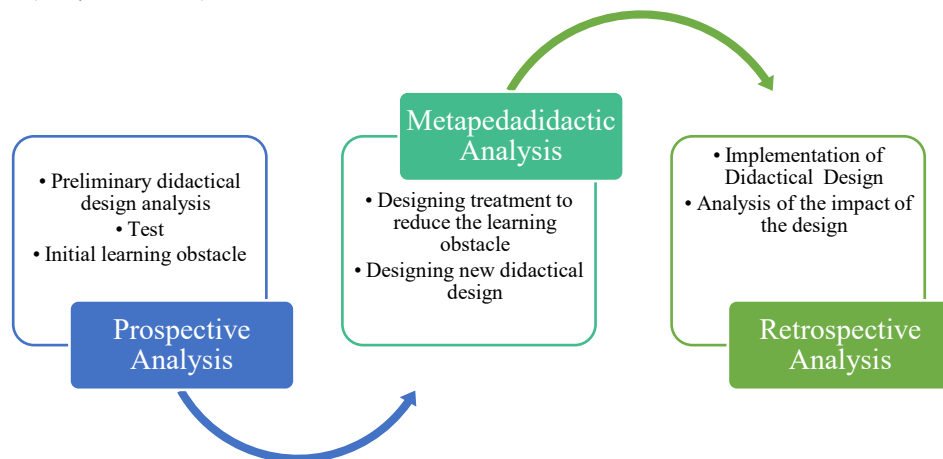
Despite the variety of instructional strategies discussed earlier, learning obstacles in students' intuitive understanding of limit concepts persist. This suggests that current teaching designs still require further refinement to better support students' thinking processes. Thus, it is necessary to develop a new didactical design to overcome problems. The fact that students face learning obstacles suggests that the didactical design that was previously used has flaws (Astriani et al., 2022). The didactical design refers to the stages of Theory Didactical Situation (TDS) by Brousseau (2002) which consist of 4 situations: Action, formulation, validation, and institutionalisation. It's important to identify the student's learning obstacles when working on introducing limits in calculus at university in order for instruction to modify challenges so that they either do not occur again or are mitigated through the didactical design in the teaching phase. These approaches aim to make limit concepts more accessible and intuitive for students beginning their study of limits. The approaches in this study also aim to bridge the gap between intuitive understanding and formal mathematical definitions of limits.

This paper addresses two research questions: 1) What types of initial learning obstacles do students encounter when developing an intuitive understanding of the limit concept? and 2) How can a didactical design be developed to support students in overcoming these obstacles and enhancing their intuitive understanding of limits? The first question identifies the specific learning challenges, while the second focuses on the application of the Theory of Didactical Situations (TDS) to guide the design and its intended outcomes.

Method

Research Design

This study employs a Didactical Design Research (DDR) approach, grounded in interpretive and critical paradigms (Suryadi, 2019). The interpretive paradigm guides the analysis of how learning obstacles emerge, while the critical paradigm informs the design and evaluation of instructional interventions. Based on these paradigms, the study follows three DDR stages, outlined in Figure 1.

Figure 1*DDR Stages (Suryadi, 2019)*

Participants

This research was conducted at Universitas Khairun from May to December 2024. Participants consisted of two groups: (1) 23 third-semester mathematics education students who had completed a differential calculus course and were given test questions to identify initial obstacles; and (2) 33 first-semester students currently enrolled in the same course.

Data Collection and Analysis

There were 3 instruments used for data collection in this research. Two experts in mathematics education, DDR, and the mathematics analysis field validated the instruments.

Test for Limit at a Point

The test instrument comprised two sub-tasks focusing on finding the limit and comparing two functions. Student answers were analysed to reveal learning obstacles in their understanding of the meaning of limits

Table 1*The Question of Test Limit at a Point*

Indicators	The question
Determine limit of a function	Find the limit of $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$
Compare two functions and determine whether both are similar function	Given the function $f(x) = \frac{x^2 - 1}{x - 1}$ and $g(x) = x + 1$ Answer the following questions and give reasons! Are the functions $f(x)$ and $g(x)$ similar? Draw the graph (via application) Find $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ dan $\lim_{x \rightarrow 1} (x + 1)$ Give a conclusion for the two functions Explain the meaning of the limit for $f(x)$.

Interview

In-depth interview guidelines were used to verify students' answers (Turner, 2010). Participants were selected based on the diversity of their responses to the limit questions. Three students were chosen for in-depth interviews to explore their thought processes, understand the reasons behind their differing answers, and gain insight into their conceptual understanding.

A Didactical Design of Limit at a Point

This instrument was developed based on several key sources: 1) the results of the initial analysis of students' learning obstacles when answering the questions; 2) a comparison of calculus textbooks commonly used in Indonesia, including those by Varberg et al. (2013), Martono (1999), and Stewart (1999); 3) relevant articles on the limit at a point or intuitive understanding of limits; and 4) validation of the didactical design through a focus group discussion (FGD) with Calculus experts.

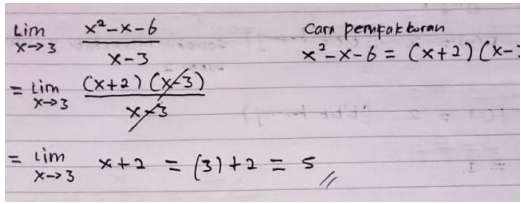
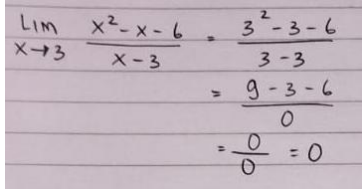
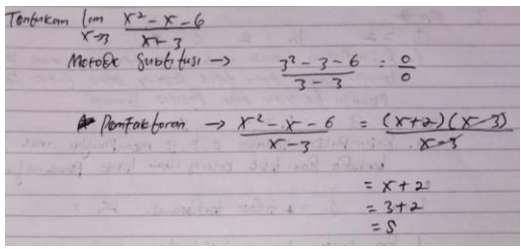
Result

The Initial Learning Obstacles

The obstacles identified at this stage offer insight into students' intuitive understanding of the limit concept. Table 2 presents responses to the first limit-at-a-point test question, while Table 3 summarises responses to the second. Based on the data presented in Tables 2 and 3, the initial learning obstacles are summarised below.

Table 2

The Answer of the Test Limit at a Point Number 1

The answer	Interview conclusion
M1 answer: 	M1 was able to correctly find the limit value, but did not fully understand that the expression $x - 3$ in the numerator does not yield an absolute value of zero, but rather approaches zero as x approaches 3. The same misunderstanding occurred with the expression $x - 3$ in the denominator, leading M1 to assume that both could simply be crossed out. M1 also mentioned that 0 divided by 0 is undefined.
M7 answer: 	Based on the answer, it was determined that M7 used direct substitution in solving the problem. M7 forgot how to factor so he solved it by substitution. M7 misunderstood that 0 divided by 0 results in 0.
M12 answer: 	Based on the answer, information was obtained that M12 did direct substitution first, but because the value was 0, they used factoring. M12 did not understand that 0/0 was indeterminate. M12 only crossed out directly if the denominator and numerator were the same, and did not understand the concept of division. M12 also could not differentiate how to do ordinary algebra with solving function limits.

Epistemological obstacle: (1) Do not know the reason for crossing out or eliminating the same terms or factors between the numerator and denominator; (2) Cannot distinguish between indefinite and definite forms; For example, by stating that $0/0$ is 0, or $0/0$ is undefined; (3) Cannot distinguish between limit and algebraic forms due to the loss of limit notation in the process of finding the limit value; (4) Cannot distinguish between undetermined and undefined; (5) Limitations in understanding the similarities in 2 different functions $f(x)$ and $g(x)$; (6) Limitations in understanding the differences between $f(x)$ and $g(x)$ are how to solve them using factoring and substitution; and (7) Cannot explain the meaning of limit intuitively.

Ontogenic obstacle: (1) Cannot undertake factoring; (2) Errors in solving factoring by not writing limit notation or can be said to be unable to distinguish between ordinary algebraic forms and limit forms; and (3) Unable to draw a function graph.

Table 3

The Answer of Test Limit at a Point Number 2

The answers

Interview conclusion

M1 answer:

2. b. $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1} x+1 = 1+1 = 2$

• $\lim_{x \rightarrow 1} (x+1) = 1+1 = 2$

c. kesimpulan terhadap Fungsi $f(x)$ dan $g(x)$

- Fungsi $f(x)$ harus dicari dengan metode pemfaktoran karena $f(1)$ tidak ada atau jika di grafik membentuk titik bintul kosong
- Fungsi $g(x)$ dapat langsung dicari nilainya dengan metode substitusi karena $g(1)$ ada

Jadi, $f(x)$ dan $g(x)$ berbeda.

d. makna limit pada $f(x)$

Limit pada $f(x)$ adalah suatu nilai yang semakin mendekati nilai fungsi $f(x)$, pada $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$ ketika nilai x mendekati 1 membentuk titik kosong pada grafik, nilai limit fungsi $f(x)$ hanya semakin mendekati angka 2.

M1 demonstrated understanding that the way to find the limit value was by substituting the value of x approaching c but because it produces a divisor of 0 which means it is undefined, then M1 used the factoring method. M1 understood that $f(x)$ and $g(x)$ are different from the solution process. $f(x)$ is sought by factoring because if the value of $f(1)$ is substituted there is no value, while $g(x)$ can be directly substituted for its value.

Translation part c) and d):

c) conclusion on $f(x)$ and $g(x)$: $f(x)$ must be found using the factoring method because $f(1)$ does not exist or if it is on the graph it forms an empty dot. $G(x)$ can be directly found using the substitution method because $g(1)$ exists. So $f(x)$ and $g(1)$ are different.

d) meaning of limit on $f(x)$: limit on $f(x)$ is a value that is getting closer to the value of the function $f(x)$ on $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$ when it approaches 1 it will form an empty dot on the graph so that the limit value of the function $f(x)$ only approaches 1.

M12 answer:

2. $f(x)$ dan $g(x) = \frac{x^2-1}{x-1}$ dan $g(x) = x+1$

Misal: $x = 0$, maka $f(x) = \frac{0^2-1}{0-1} = 1$
 $g(x) = 1$

$x = 1$, $f(x) = \text{tak tentu } (\frac{0}{0})$
 $g(x) = 2$

$x = 2$, $f(x) = \frac{2^2-1}{2-1} = \frac{3}{1} = 3$
 $g(x) = 2+1 = 3$

a. sama, apabila dimasukkan nilai 0 dan 2,

x	0	1	2
$f(x) = \frac{x^2-1}{x-1}$	1	$\frac{0}{0}$	3
$g(x) = x+1$	1	2	3

b. $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \frac{1^2-1}{1-1} = \text{tak tentu}$

$\lim_{x \rightarrow 1} (x+1) = (1+1) = 2$

c. pada $f(x)$ apabila persamaannya dimasukkan nilai 1) maka didapat nilai tak tentu, selanjutnya fungsi $f(x)$ dan $g(x)$ sama.

M12 was able to draw the conclusion that $f(x)$ and $g(x)$ are more or less the same but at $x=1$ they are different, although there are still errors because M12 cannot clearly provide a final conclusion that $f(x)$ and $g(x)$ are different only when $x=1$ and the rest are the same.

Translation part c)

In $f(x)$ if the value is entered into the equation, an indeterminate value is obtained, otherwise $f(x)$ and $g(x)$ are the same.

Didactical obstacle: The sequence of materials developed by the lecturer does not match the students' thinking process. For example, in the first meeting, the lecturer introduced the limit

intuitively by presenting a function that when x is substituted produces $0/0$, then the lecturer did not explain what $0/0$ is. Therefore, from the initial learning obstacle the following didactical design recommendations were implemented to overcome these obstacles: (1) Explain the rationale for cancelling identical terms or factors between the numerator and denominator; (2) Provide examples that require determining limit values for functions yielding defined, undefined, and undetermined.

The Didactical Design of Limit at a Point

A review of published literature has found no prior studies on teaching the concept of function limits using the Theory of Didactical Situations (TDS) that explicitly incorporates all four phases: action, formulation, validation, and institutionalisation. This design also consists of 3 situations that are deliberately designed to improve students' understanding of limits (see Table 4).

Table 4

Teaching Activities Based on TDS

Teaching activities based on TDS	Pedagogical didactic anticipation																
Action situation Situation 1: Students are presented with a function, $f(x) = \frac{x^2-x-6}{x-3}$ Students are asked to identify possible values of $f(3)$	If the students answer it as follows: $f(3)= 0/0 = 0$ $f(3)= 0/0 = \text{none}$ $f(3)= 0/0 = \text{undefined}$ $f(3)= 0/0= \text{indeterminate}$ The example of anticipation: Introducing $0/0$ is an indeterminate form. Suppose $0/0 =x$ $0=x \cdot 0$ x can be replaced by other numbers, namely 1, 2, 3. So $0/0$ is indeterminate																
Formulation situation Because the value of f for $x = 3$ cannot be determined, we need to identify the values of f for x around 3. Therefore, students are asked to draw a graph of $f(x) = \frac{x^2-x-6}{x-3}$ for $x \neq 3$ Then students are asked to determine the value of $f(x)$ for the values of x in the table below <table><tr><td>x</td><td>2.8</td><td>2.9</td><td>2.99</td><td>....</td><td>3</td><td>3.001</td><td>3.01</td></tr><tr><td>$F(x)$</td><td></td><td></td><td></td><td>...</td><td>?</td><td></td><td></td></tr></table>	x	2.8	2.9	2.99		3	3.001	3.01	$F(x)$				...	?			
x	2.8	2.9	2.99		3	3.001	3.01										
$F(x)$				...	?												
Based on the graph and table above, students are asked to draw conclusions about the value of f around $x = 3$. Situation 2: To make the method of determining the limit value more efficient, students are asked to solve the problem below $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x - 2}$	The value of $f(x)$ approaches 5 for x approaching 3 If the students answer it as follows: Students work using a table of values The example of anticipation: Using tables is inefficient. Students perform factoring and substitution $\lim_{x \rightarrow 2} \frac{x^2-2x-8}{x-2} = \lim_{x \rightarrow 2} \frac{(x\cancel{-2})(x+4)}{x\cancel{-2}} = \lim_{x \rightarrow 2} (x + 4)$ $= 2+ 4 = 6.$ The example of anticipation:																

Situation 3: To distinguish the process of simplifying functions in the limit from functions in ordinary algebra. Then the following problem is presented Given the functions $f(x) = \frac{x^2-1}{x-1}$ dan $g(x) = x + 1$ Answer the following questions and give reasons! Are $f(x)$ and $g(x)$ the same two functions? Are $\lim_{x \rightarrow 1} f(x)$ and $\lim_{x \rightarrow 1} g(x)$ the same value?	The lecturer asked why in step 2, the factor can be crossed out. If the students answer it as follows: $f(x)=g(x)$ $f(x) \neq g(x)$ The example of anticipation: The lecturer asked, is $f(1) = g(1)$? Then present the graphs of $f(x)$ and $g(x)$
Validation & institutionalisation situation To strengthen the understanding of determining limit values, students are presented with limit problems with various forms of function values and contexts. Determine the following limit values: 1. $\lim_{x \rightarrow 1} \frac{2x^2-x-1}{x-1}$ 2. $\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x-3}$ 3. $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$ 4. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2x \cos(x-\pi)}{x-\frac{\pi}{4}}$	

Discussion and Conclusion

Based on the analysis of initial learning obstacles, students were found to rely heavily on memorised procedures rather than developing an intuitive understanding of limits. This was evident in their tendency to cancel factors without grasping the mathematical justification for doing so. Furthermore, students were unable to explain whether two given functions were equivalent. Some simply stated that the functions were different because the methods used to find their values were different, such as using substitution versus factoring. Likewise, with the intuitive meaning of the limit, students said that the limit on $f(x)$ is a value that is getting closer to the value of the function $f(x)$ on $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$ when it approaches 1 it will form an empty dot on the graph so that the limit value of the function $f(x)$ only approaches 1. Students can find the limit value but are still confused when explaining the meaning of limit. These findings align with Duru (2011) and Nagle (2017) who observed that although students may apply procedures correctly, they often fail to understand the underlying concept or apply it to related problems.

Based on the findings regarding initial learning obstacles, improvements are needed in structuring the learning process. To address this, a didactical design was developed, comprising four phases: action, formulation, validation and institutionalisation. The action and formulation phases include three instructional situations. Situation 1 aimed to determine the value of a function in the form $0/0$. Situation 2 described the process of determining the limit value. Situation 3 helped distinguish between simplifying functions in the context of limits and in ordinary algebra. While validation and institutionalisation phases aim to reinforce students' understanding of limit values across various functional forms and contexts. This design, rooted in the DDR framework, was intentionally constructed to support students' conceptual thinking and reduce learning obstacles (Hendriyanto et al., 2024; Sukarma, 2024). Further research is recommended to evaluate the effectiveness of this didactical design.

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Primary Teachers' Adaptive Expertise for Teaching Interdisciplinary Mathematics and Science: Findings from a Video Questionnaire

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Adaptive expertise concerns the ability to respond effectively to novel and unexpected situations. This expertise is important for improving students' cognitive engagement, understanding and participation in meaningful learning. This paper presents findings on primary teachers' adaptive expertise in interdisciplinary mathematics and science using a video-stimulated questionnaire with multiple choice items and verbal explanations. Data were collected prior to the teachers' planning and teaching interdisciplinary lessons. Findings showed discrepancies between responses to the items and verbal explanations, indicating gaps between their espoused and enacted adaptive expertise.

Introduction

Adaptive expertise (AE) involves both innovation and efficiency (Bransford et al., 2005). Adaptive teachers apply deep levels of content and pedagogical knowledge in order to respond flexibly to unexpected, diverse and complex classroom situations (Anthony et al., 2015; Hatano & Oura, 2003; Timperley & Twyford, 2022; Yoon et al., 2019). Previous studies of adaptive expertise have used qualitative methods involving interviews, observations and videotaping of teachers and their teaching of science or mathematics lessons (e.g. Alonzo & Kim, 2016; Estapa & Amador, 2023). Others have used video analysis tasks to pose open-ended questions to analyse teachers' pedagogical content knowledge or their capacity to 'act in the moment' in research, teacher education or professional learning contexts (Borko, et al. 2015; Chan, 2021; Copur-Gencturk & Rodrigues, 2021).

In this paper we report findings from a video-stimulated questionnaire used within a design-based longitudinal study on the development of primary teachers' adaptive expertise when teaching interdisciplinary mathematics and science. We used a video-stimulated questionnaire to gather data about how teachers might respond to classroom situations to identify teachers' AE at the beginning of the longitudinal project. The research questions are:

- To what extent do primary teachers select and explain adaptive expertise practices in complex teaching scenarios in STEM lessons?
- How is their amount of teaching experience associated with their anticipated use of adaptive expertise?

Theoretical Framework

Adaptive expertise as a construct has gained significant traction in education as it foregrounds teachers' responsiveness to the diversity of students in their classrooms and their needs as part of inclusive education (Anthony et al., 2015; Parsons, 2012; Soslau, 2012). In regard to science pedagogy, Crawford et al. (2005) align adaptive expertise as a disposition with "excellence in science teaching" (p. 3). Such teachers undertake causal and data-driven forward reasoning through self-regulation and cognitive flexibility to address student learning needs. Yoon et al. (2015, 2019) argue that "high quality teaching" (p. 903) in science manifests as adaptive expertise as the combination of flexibility, deep-level understanding and deliberate practice (see Table 1). Suh et al. (2023) propose that such adaptive expertise is necessary for teachers to effectively induct students into the epistemic practices of science, which supports the development of student disciplinary-specific agency. In regard to mathematics education, Baldinger & Munson (2020) suggest that adaptive expertise is aligned with "ambitious teaching practices" (p. 1), with such expertise emerging from a process of co-construction among teacher colleagues who share professional experiences to improve practice. The results of teacher adaptive expertise, Sherman (2020) argues, are increased opportunities for students to express their mathematical thinking in meaningful ways as elicited by responsive teachers.

This study uses Yoon et al.'s (2019) STEM-based framework for adaptive expertise (see Table 1) to examine teachers' use of adaptive expertise in interdisciplinary mathematics and science lessons. Each of the components (flexibility, deep level of understanding and deliberate practice) is defined in Table 1. Firstly, opportunist planning and flexible and critical application of content and pedagogical knowledge aligns with other research concerning acting-in-the-moment in mathematics lessons (e.g., Sherman, 2020). Secondly, to be adaptive experts, teachers need a deep level of understanding of the discipline and, in our context of interdisciplinary mathematics and science teaching and learning, knowledge of the connections between these two disciplines, to act efficiently and innovatively. With this knowledge, teachers can respond to the learning needs of all students and focus on developing student argumentation and understanding (Anthony et al., 2015; Bransford, 1999). Finally, deliberate practice involves making concerted efforts to reflect on their teaching using data on students' cognitive engagement and feedback from observers or co-teachers, to purposefully plan to improve their practice (Anthony et al., 2015; Timperley & Twyford, 2002).

Table 1

Adaptive Expertise Components (Yoon et al., 2019, p. 897-898)

Component
<i>Flexibility</i> : the ability to opportunistically plan, change enactments faster than non-experts, and flexibly and critically apply their knowledge to new situations while constantly learning.
<i>Deep-level understanding</i> : addresses the need to not only have acquired content and pedagogical knowledge, but to have a deep understanding of it in order to use such knowledge effectively.
<i>Deliberate practice</i> : addresses the need for teachers to receive feedback about and reflect upon their teaching either directly from their observations, from student outcomes, or from outside perspectives, with the intent to shift their practice based on feedback and reflection.

Methods

Following Copur-Gencturk & Rodrigues' (2021) recommendation to use video excerpts of authentic lessons, we selected video episodes of two primary teachers when they co-taught an interdisciplinary sequence of three lessons, "Keeping Your Finger on the Pulse" (Hughes et al., 2022) to a Year 5-6 class in the pilot study for the larger project. These teachers did not have a

relation with the participants in the study. The broad aims of the lesson sequence included developing students' understanding of mathematics and science concepts, explaining their mathematical and scientific reasoning, and conducting investigations about heart rate. Specifically, the aims for students were to develop their understanding of the circulatory system, the use of the pulse to measure heart rate, exploring average heart rate, and the reasons for changes in heart rate along with developing knowledge and skills for proportional reasoning including rates, percentage and percentage increase.

Questionnaire Design

We designed and used a questionnaire composed of six video episodes to measure teachers' adaptive expertise including their anticipated actions regarding their deep level of understanding, flexibility and deliberate practice (Yoon et al. 2019). Vale et al. (2024) described the process and analysis used to select video episodes from the pilot study and design and validate the multiple-choice (MC) options and scoring for the video-stimulated questionnaire. Within the interdisciplinary context, the selected video excerpts focussed on the learning and teaching of mathematics (Items 1 and 3), science (Items 4 and 5) and connections between mathematics and science (Items 2 and 6). The agreed options were scored on a scale from 1-4, where 1 = low AE and 4 = high AE. In the validation and reliability process, we included open-ended questions so that teachers could explain their reasons for selecting the particular MC option. Three of the video-questionnaire items targeted teachers' deep level of understanding: one concerned mathematics (Item 1), another science (Item 5) and the third, both mathematics and science (Item 2). Two items concerned teachers' flexibility (Items 3 and 4) and one item, deliberate practice (Item 6). Each item included a description of the episode, the video excerpt (up to one minute in length) which participants clicked on to view, and the MC options for answering the question posed. The video stopped following a student comment, response or action and the participants were asked: "If you were the teacher, what would you do next? Choose one response from the list below." The teachers completed the questionnaire online and explained their choice orally rather than in writing.

Data Collection and Analysis

As part of the larger study, 15 participants from 5 metropolitan primary schools in Victoria attended a whole day professional learning session to prepare and plan for teaching one of two interdisciplinary lesson sequences. Prior to attending the session, the teachers were provided with draft lesson plans of the two lesson sequences. In the first session of the day, participants completed the video-questionnaire individually online.

Given the very small sample size, descriptive statistics were used to analyse the data collected. As discussed in Vale et al. (2024), the scores for the MC options were agreed upon prior to data collection. Following data collection, each verbal explanation was analysed by four researchers to reach agreement on the score for their explanation (EXP) from 1 (low AE) to 4 (high AE). Examples of teachers' explanations and their scores are provided in the findings. Means and medians were calculated for MC and EXP scores for each item and overall mean scores for each AE component and participant. Boxplots were used to illustrate findings.

Participants

Of the 15 teacher participants, five teachers across the five participating schools were identified as early career teachers as they had taught at Year 5 or 6 for 3 years or less. The other ten teachers had taught these year levels for more than 3 years, described as experienced teachers. Two of the experienced teachers had taught for more than 10 years. Fourteen teachers had experience of teaching Year 5 or 6 mathematics, but only five had experience of teaching Year 5 or 6 science. Two teachers had experience of teaching interdisciplinary STEM lessons.

Therefore, teaching interdisciplinary mathematics and science was not familiar to most of these teachers. All teachers had experience of co-planning either maths and/or science lessons, but only five teachers had experience of co-teaching mathematics, science or interdisciplinary lessons. Pseudonyms are used when providing examples of explanations of individual teachers.

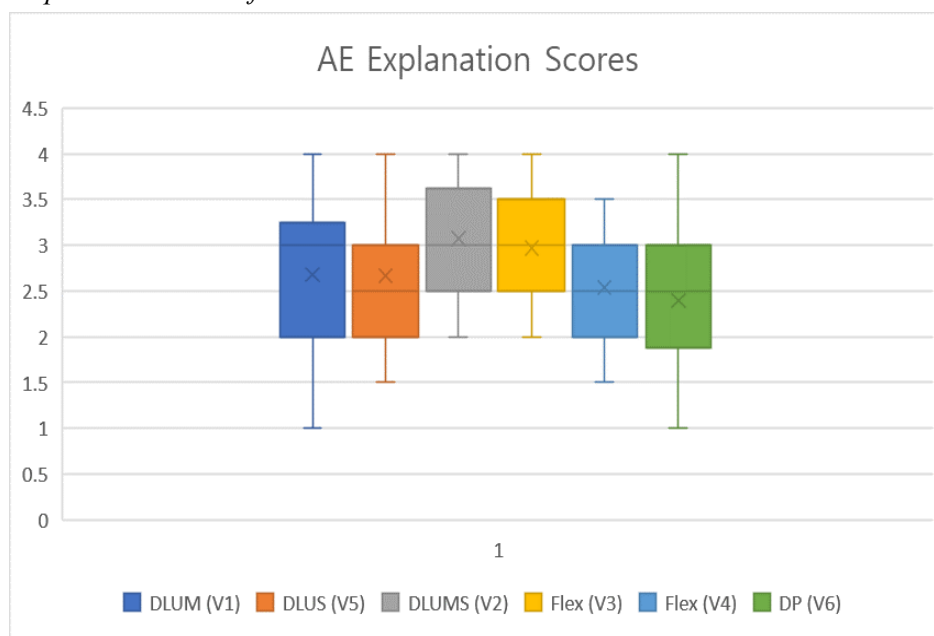
Findings

Mean scores were calculated for the MC scores and EXP scores for each of the six items and overall, along with the range, median and quartiles. The overall AE mean scores showed that the mean MC score ($m = 2.92$) was higher than the mean EXP score ($m = 2.60$). However, whilst the individual teacher's mean MC scores ranged from low ($m = 1.89$) to high ($m = 3.67$), the range of the mean EXP scores was much narrower ($2.17 \leq m \leq 3.25$). Whilst four teachers seemed to be familiar with some AE teaching practices when making their MC selections ($m \geq 3.0$), a different five teachers explained their selection to show awareness and understanding of AE practices ($m \geq 3.0$) and how they would implement them. These teachers were from three of the five schools. Seven teachers recorded higher mean EXP scores than their mean MC scores, providing a clearer indication of their familiarity with an AE practice. A comparison between the five early career teachers and the ten experienced teachers revealed a larger range of mean EXP scores for experienced teachers ($2.2 \leq m \leq 3.25$) than for early career teachers ($2.4 \leq m \leq 3.0$).

In Figure 1, the range and quartiles for EXP scores for each item are illustrated. The highest EXP scores were recorded for the item about Deep level understanding of mathematics and science (Item 2) along with the first Flexibility item (Item 3), with about half the teachers scoring 3 or more. The broadest range of EXP scores and lowest mean scores were for Deep level understanding of maths (Item 1) and Deliberate Practice (Item 6). Findings for the three AE practices are presented below.

Figure 1

Boxplot for Explanation Scores for each Video Item



DLUM: Deep Level Understanding-Mathematics, DLUS: Deep Level Understanding-Science, DLUMS: Deep Level Understanding-Mathematics & Science, Flex: Flexibility, DP: Deliberate Practice, x: mean

Deep Level of Understanding - Mathematics, Science and Mathematics and Science (Items 1, 2 and 5)

For 7 out of 15 teachers their EXP score exceeded their MC score for the three Deep level of understanding items. The lowest mean MC score was for Item 1 about mathematics knowledge ($m=2.57$), whilst the highest mean MC score was for Item 2 about mathematics and science knowledge ($m=3.13$); the mean MC score for Item 5 about science knowledge was in between ($m=2.67$). The highest mean EXP score ($m=3.07$) was also for Item 2, whilst the mean EXP score for Item 5 about science ($m=2.67$) was similar to findings for Item 1 about mathematics ($m=2.68$). Early career teachers selected lower AE responses for Item 1 than experienced teachers, whereas all teachers recorded higher EXP scores than MC scores for Item 2 about science knowledge.

The first video excerpt (Item 1) explored teachers' deep-level understanding of mathematics. It took place in the first lesson when the class began exploring the human circulatory system and the meaning, measurement, and calculation of heart rate (bpm). The mathematics aim for the lesson concerned the conceptual understanding and meaning of rate and equivalent rates as applied to heart rate and measuring heart rate. The student in this video used the method of counting their heartbeat for 10 seconds and multiplying by six rather than what the teacher had planned - counting for 15 seconds and multiplying by 4. For this item, five teachers selected one of the two higher MC options: ask another student to repeat this in their own words (option A) or ask a student why this would work (option C) and their EXP showed the same spread ($3 \leq \text{EXP} \leq 4$). Altogether seven teachers' EXP scores were rated higher than 2.5 (see Figure 1).

Two experienced teachers and one early career teacher scored the highest rating for their explanation. Afelo (pseudonym), an experienced Year 5 & 6 teacher, demonstrated both content knowledge (measuring heart rate) and pedagogical knowledge (engaging students in mathematical thinking and justification) in their explanation: "...I would get another student to repeat the procedure, um, using their own words as it might provide like a new way for students to understand the same concept". They valued different explanations: "...if it's explained in a different way, um, it probably also would show other students in the class getting an understanding of the procedure that the student suggested...". They would also seek to elicit other methods of calculating heart rate: "... to see if anybody knows another way as well as the way that was described that might help as well, even if they were to do it for 30 seconds and then multiplied by two ...".

More than half the teachers selected one of the low scoring MC options (B—restating the procedure, or D—ask for another way to calculate bpm). Most of these teachers improved their score with their explanation. For example, an early career teacher explained that they would "... asked for another way to calculate beats per minute to show that there is more than one way, um, that they can get to this answer" (Rafel, MC score=2, EXP score=3). Other teachers were more focused on students using the planned method to calculate heart rate (bpm). For example, "So I would probably see if another student had a different way trying to work towards what had been planned, which was to count by 15 and multiply by four" (Arkey). That is, they would look for another student to suggest the planned ratio to use and if no one did, then they would explain the method to use.

Item 2 was selected from the video of the second lesson in the sequence. The goals for this lesson included drawing on students' experiences of pulse rate and heart function and collecting pulse rate data to make predictions about their pulse rate for a range of physical activities. In the video excerpt, the teacher observes a student's written pulse rate measurements whilst they are predicting pulse rate for the next physical activity.

One teacher chose the most adaptive MC option (C) and explained it well to demonstrate AE for content and pedagogical knowledge for both mathematics and science (EXP score=4). They compared the other two options before choosing C as they explained this action would be best to elicit the student's understanding:

... A, B and C kind of say the same thing, but they give different amounts of scaffolding. What did you find out to me is too open and too vague. 'What do you think about your prediction for heart rate for star jumps' is better because it gives them a little lead in as to what you're looking for. But it also allows the students to really apply their understanding and allows the teacher to gauge their understanding as well (Krong).

Another teacher chose low scoring MC option B (2) but their EXP score (3) was higher. Similar to Krong above, they compared the implications of different options. They considered option A was "too open" and that their response might focus on the physical activity rather than the scientific concepts and procedure for conducting trials. They suggested a more closed rewording "...what did you think about your prediction for the heart rates for star jumps?" (Afelo). They explained "...it would be a bad idea for the students to change their prediction on star jumps as um it's a bit of a learning curve and we want to teach students that it's OK to be off in your predictions because that's how we kind of get to learning" (Afelo). Aligned with the teacher above, they also thought that option C would give "a student that needs a little bit more support" (Afelo).

Five teachers recorded a lower EXP score than MC score. For example, one teacher chose option C (MC score=4), but their explanation (EXP score=3) focussed on getting the student to identify their error rather than to elicit their understanding: "...I want to elicit from the student, why they think their prediction isn't going to be accurate" (Zella). They indicated undertaking routine expertise in science teaching practices to "...talk about scientists making predictions and hypotheses that are often wrong..." They also spoke about the scientific "... understanding of the change or the adaptation that the body is making" (Zella).

Flexibility (Items 3 and 4)

Video excerpts for Items 3 and 4 explored teachers' anticipated flexibility. Interestingly, teachers responded differently to these two items. The highest mean MC score and EXP score for flexibility occurred for Item 3 from Lesson 1. The pulse rates of each student had been collected and displayed for the class with the range from 60 to 120 bpm. During a discussion on calculating the mean, one student claimed the average would be 166.1 bpm. This item concerned the teacher's capacity to act in the moment, to opportunistically change the plan for that part of the lesson.

The video excerpt used in Item 4 occurred at the beginning of Lesson 2 while the teacher was conducting a whole-class review of Lesson 1. A student asked about measuring pulse rate at the neck or wrist. This question provided a scenario where the teacher could plan opportunistically. However, most of the teachers' explanations for their MC option did not indicate flexibility as they did not indicate that they would take up the opportunity to plan in the moment and critically apply their knowledge. Of all six items, Item 4 recorded the second lowest mean scores for both MC and EXPs.

Seven teachers selected option B (MC score=2: Ask students "How can you find this out?") and nine teachers scored 2 or 2.5 for their EXP score. Four teachers received the same score for MC and EXP. Two teachers selected option B because it was the most open-ended response. One teacher argued "So, I might ask a student how he could find this out. Then hopefully they will look at repeating the experiment using the pulse rate in the neck" (Leina). Another teacher (EXP score=3.5), showed that their flexibility was more adaptive:

Ask students how they could investigate this question themselves. Have students discuss different ways to measure this. Students ultimately compare the measurement of wrist to neck. Direct students to make a prediction prior to conducting their tests (Trent).

Deliberate Practice (Item 6)

One video excerpt explored teachers' AE with regard to DP (Item 6). It occurred in the third lesson when students were investigating activities to increase heart rate by 50%. The video excerpt is a conversation between the co-teachers during the lesson as they reflected on the students' approaches and deciding on next steps for the lesson.

The explanations showed that the teachers reflected on the observations discussed by the co-teachers in the video excerpt and decided on an action to take, however, Item 6 obtained the lowest mean MC score ($m=2.21$) and lowest mean EXP score ($m=2.39$) of all the items. The lowest MC score and EXP scores for Item 6 were for responses that did not indicate deliberate practice or did not involve engaging with the co-teacher in carrying out their chosen action. One experienced teacher without co-teaching experience, chose the most adaptive MC option (D): "to chat with the groups of students" (Dona), to collect data about students' process and findings. However, their explanation did not match this option. Instead, they described a routine practice: "I would probably stop and reiterate to the class that they need to... measure their resting heart rate..." (Dona, MC score=4, EXP score=1). Conversely, another experienced teacher, with experience of co-teaching, chose one of the least adaptive MC options: Let's stop the class and find out from the students how they are measuring heart rate (B, MC score=2) but provided one of the more adaptive explanations (EXP score=3). They focussed on students' social engagement and their engagement in mathematical and scientific thinking: "I think it is important for children to listen to their peers and discuss ways that they are measuring their heart rate..." (Rachel). Both responses show that the teachers' anticipated individual action would follow discussion with the co-teacher, but neither referred to what their co-teacher might say or do whilst they enact these routines.

Discussion and Conclusion

The inclusion of the explanation question with each video item provided a clarification of teachers' intended action with respect to AE. For some items, participants scored higher for the MC option and for other items higher for their explanation. Overall, for all six items the range of MC scores was broader than for their EXP scores, indicating that some teachers' explanations did not match their selected higher scoring AE MC option, whilst others explained something more adaptive than the MC option selected. This finding may indicate that some teachers identify the more AE option, but their explanation suggests that they were less familiar with how to enact it. Conversely, other teachers were better at explaining what they would do than selecting an option. This finding also suggests that some MC options may be improved with rewording.

As reported by Yoon et al. (2015), while some of the experienced teachers scored higher than early career teachers in this study, we should not assume that more experience leads to more AE in all complex situations. Also, similar to Yoon et al. (2015), we found differences in teachers' level of AE for the different AE components. Differences in AE for the three deep level of understanding items and the two Flexibility items showed that their likelihood of using AE depended on the disciplinary context, mathematics or science, or the degree of unexpectedness and how to act in the moment. There was only one video item for deliberate practice, and it occurred when the co-teachers consulted with each other. The five teachers with co-teaching experience recorded high levels of explanations for deliberate practice.

The video excerpts used in this questionnaire included students' learning actions and not the response or action taken by the teacher. This enabled the participating teachers to select and

explain intended actions rather than critique another teacher's actions. This provided an indicator of their level of AE rather than their assessment of another teacher. In the next step of our study, teachers will review videos of their teaching of two STEM lesson sequences. We will encourage the teachers to review their videos with regard to adaptive expertise. Findings from this questionnaire study will be compared to their developing adaptive expertise gathered through observation and videos of lessons and teacher interviews and reflections on their lesson videos of their interdisciplinary teaching.

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Gesturing Partition and Fraction Understanding: A Case of Primary Students Solving Partition Tasks

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This paper aims to understand how primary students use gestures to solve partition tasks and to what extent these gestures may promote fraction understanding. Analysis of six selected students' answers to partitioning tasks, classroom observation, and post-lesson interviews indicates that co-thought representational gestures play a critical role. The gestures help the students represent partitions that may not be readily available mentally. Gesturing partitions might allow students to extend their understanding of fractions.

Learning fractions has been challenging for students (Wijaya, 2017), while the topic is considered a gatekeeper's doorman in mathematics (Booth & Newton, 2012). The challenge appears to be rooted in multiple factors, such as the different meanings of fractions, the complex processes required to understand fractions, or instructional delivery. Partition is argued to be one of the underlying processes or concepts for fraction understanding (Lamon, 2020). In addition to partitioning, other underpinning concepts for fraction understanding are disembedding, iterating, splitting, and unit coordination (Steffe & Olive, 2010). Students who have interiorised the concepts are argued to be able to treat any fractions as numbers (Hackenberg, 2007). Many types of interventions have been implemented to address fraction learning (Roesslein & Coddington, 2019), including the use of gestures (Beilstein, 2019). The term gesture is consistently used throughout this paper to refer to hand gestures. For example, Zurina and Williams (2011) found that gesturing for oneself helps students visualise fraction concepts. Despite the positive impact of gestures, literature indicates that only a few studies have focused on gestures in fraction content (Martinez-Lincoln et al., 2018). Moreover, considering the importance of the underpinning concepts, such as partition for fraction knowledge, very little is known about how the use of gestures, either by teachers or students, could help comprehend the concept, which might lead to fraction understanding. Therefore, this paper aims to address this gap by investigating students' use of gestures in solving partition tasks and to what extent gestures on partition might promote students' fraction understanding.

Partition, Gesture, and Fraction Understanding

In general, partition could be perceived as dividing an object or collection of objects into some disjoint parts. Regarding fractions, partition is to divide fractional objects or representations, for example, a fraction bar into equal parts or sizes. The equal parts of the object denote the existence of fractions (Lamon, 2020). The term partition is used in this paper, which means equipartitioning. For instance, Martin and Hunt (2022) provided students with partition activities, such as sharing one whole chocolate among an increasing number of children, as a foundational step prior to learning other fraction concepts, including iteration and unit coordination. The activities improved students' performance on the fraction test. It appears that teaching interventions involving partitions are frequently associated with students solving fair-sharing problems. In addition to the fair-sharing context, hands-on activities such as paper folding are promising for constructing the idea of partition (Empson & Turner, 2006).

Gesture is generally perceived as body movements. It represents actions (Novack & Goldin-Meadow, 2017) and frequently involves hands with speech (McNeill, 1992) or without speech (Logan et al., 2014). Speech-accompanied gesture is spontaneous hand movements that co-occur with speech (McNeill, 1992). Meanwhile, co-thought gesture is hand movements that (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 453–460). Canberra: MERGA.

come along with students' thinking, for example, when processing task information (Logan et al., 2014). Three frameworks have been prominent in explaining gesture production and its function: gestures as representational action (Novack & Goldin-Meadow, 2017), gesture as simulated action (Hostetter & Alibali, 2008), and gesture for conceptualisation (Kita et al., 2017). As a representation action (GRA), gesture represents ideas, objects, or actions. As a simulated action (GSA), gesture emerges from spatial representations and mental images. Lastly, for conceptualisation (GfC), gesture could activate, manipulate, package, and explore spatio-motoric information for speaking and thinking.

In their review of journal articles on gesture research targeting teachers/students from preschool to grade 12 in mathematics classrooms, Martinez-Lincoln et al. (2018) suggest that the use of gestures either by the teachers or students in mathematics teaching and learning improves students' mathematical achievement. Of the inclusion studies, only one study focused on fractions (Zurina & Williams, 2011), investigating how primary students conceptualised fractions through gesturing the number of equal-shared bread pieces for oneself. After the review, there appear to be a few gesture-fraction studies. For example, Beilstein (2019) found that a student used gestures to make partitions and iterations on number lines, which could be a resource to develop students' number-line skills. The study then implemented a gesture-based fraction intervention where the teacher demonstrated a hand gesture to partition the number lines (partitioning gesture). Some other studies also embedded gestures in fraction interventions, for example, using chunking gestures to represent the magnitude of unit fractions on number lines (Barbieri et al., 2020). Given that partitioning is foundational to fractions (Lamon, 2020), the studies indicate that the use of gestures by the teachers could help students visualise the concept/process, or by the students to represent the concept/process, which might be unavailable mentally.

To sum up, partition is a concept that underpins fraction understanding (Lamon, 2022). Thus, facilitating learning that helps students interiorise partition is critical. Gestures help students improve mathematics performance (Martinez-Lincoln et al., 2018), including fractions. However, in general, the gesture-fraction relationship is under-researched (Beilstein, 2019). Specifically, concerning Beilstein's study, little is known about how gestures could help students represent/process partition and promote fraction understanding. Therefore, this paper focuses on these questions: (1) How do students use gestures to solve partitioning tasks? and (2) To what extent does the use of gestures on partition promote students' fraction understanding? The answers to the questions could likely lead to our understanding of how gesturing partition could promote fraction understanding and how gestures could be a pivotal part of teaching intervention that helps students learn and understand fractions.

Methods

Research Context and Participants

This paper is part of a larger intervention study that aims to help students develop the underpinning fraction concepts (Steffe & Olive, 2010), including partition through a classroom intervention that had spatial reasoning as its core foundation. The intervention comprised eleven lessons, two of which were devoted to partition and are reported in this paper. Thirty-one grade 4 students participated in the intervention delivered by their classroom teacher as part of a regular classroom schedule. Six students (2 males, 4 females) were purposively selected based on the result of a fraction scheme pre-test (Norton & Wilkins, 2009), in which they demonstrated a mix of abilities. To solve the fraction scheme test, students need to have developed the underpinning fractions concepts: partition, disembedding, iteration, splitting, and unit coordination. The selected students were continuously observed during the intervention

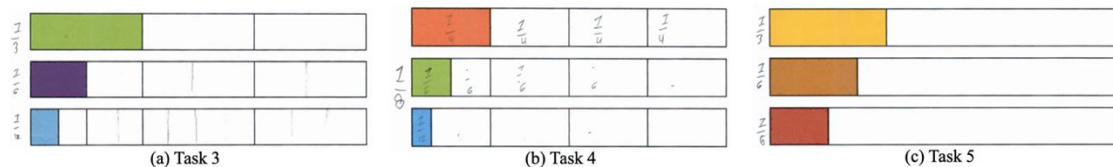
and interviewed after the pre-test, lessons, and post-test. All names reported in this paper are pseudonyms.

Partitioning Activities and Tasks

In general, the intervention engaged students with physical and then mental spatial activities, which aim to help students move from concrete to abstract experiences. In the physical partitioning activity, students in small groups folded paper strips equally, which introduces the idea of equal partitions. Prior to the group activity, the teacher demonstrated paper folding using the paper strips and then used hand gestures accompanied by speech to show the process of folding the paper strips (folding gestures) and the result of the paper folding (partitioning gestures). In the mental partition activity, five individual tasks were given. In Tasks 1-2, students were given four fraction bars; three were blank, and one was partially partitioned. The tasks expect students to make different partitions and reason about the relationship between the different numbers of partitions. For Tasks 3-5, given full and partial partitioned fraction bars where some parts were coloured (Figure 1a: 3/3-bar, 6/6-bar, 12/12-bar; Figure 1b: 4/4-bar, 8/8-bar, 16/16-bar; Figure 1c: 3/3-bar, 4/4-bar, 6/6-bar) and the bars are not equal in length, students were expected to identify fractions that represent the partitioned part(s). The mental partition activity was enacted through a VPC (visualise, predict, check) teaching heuristic (Patahuddin et al., 2020). At the visualise stage, the teacher helped students visualise partition by re-demonstrating the paper folding and demonstrating selected tasks using co-speech representational gestures. Afterwards, the students predicted the answers to the partition tasks individually, where they were allowed to use gestures only, no other aids such as fraction strips. At the check stage, the students checked their predicted answers using paper strips.

Figure 1

Partitioning Tasks



Data Collection and Analysis

The sources of data for this paper are six students' answers to the partitioning tasks (a total of 30 answered items), videos and transcripts of students' semi-structured interviews after the partitioning lesson (12 transcripts), and classroom observations (two lesson videos in which their activities during the lessons were captured through video recordings and the researcher's field notes). The interviews focused on how students processed the tasks and what fraction understanding they had developed during the intervention. Sample questions asked were: How did you solve the task? How did you use your hands to solve the task? Why does the part show the fraction? The steps of the analytical model of videotape data analysis (Powell et al., 2003) were referred to for the analysis of interview and lesson videos: watching the videos attentively, identifying critical events, transcribing, coding, and constructing themes/categories of gesture use and fraction understanding. For example, the students' use of gestures for solving the tasks gives rise to three categories: gesture type, gesture purpose, and gesture-task relation. The type of gestures is co-thought representational gesture (Novack & Goldin-Meadow, 2017).

Results and Discussion

The results are organised into two parts pertaining to the research questions, in which evidence on students' use of gestures in solving the tasks and the students' fraction

understanding that might be promoted by their ability to make partitions using gestures is presented.

Gestures on Partitioning

All the students gestured prior to making partitions on the fraction bars in Tasks 1-2. They placed their hands, forming a V-shape between their thumb and index fingers, and moved their hands on the bars (red curved arrow in Figure 2a). The V-shape is adjustable to the number of partitions made. The students argued that using their hands helped them make the partitions (T1 lines 1-4; T2 lines 1-2). When asked how they knew that each partition was equal, they also used gestures (for example, T2 lines 3-6). Furthermore, most students could make the inference that the more partitions on the bar, the smaller the parts.

Figure 2

Students Using Gestures for Solving the Tasks



Transcript 1 (T1) – R: Researcher, **H:** Hana

- 1 R: why was it easier to use hands [Task 1]?
- 2 H: it is just like this [placing her hands on the bar] then marking it.
- 3 R: what is the difference between just imagining the bar and marking it [directly making partitions] and using the hand first?
- 4 H: if not using hands some are bigger and the other are smaller [...]
- 5 R: how did you solve it? [Task 3]
- 6 H: it is like this [pointing the partitioned line in the 6/6-bar, Figure 1a] so it was equal
- 7 R: did you use your hand?
- 8 H: yes like this [partitioning the bar into 6 equal parts using gestures, Figure 2c]

T2 – R: Researcher, **L:** Lili

- 1 R: how did you solve it [Task 1]?
- 2 L: First I used my hands then used my hands again to find the answer in the third trial.
- 3 R: back to your answers how did you know that these [The partitioned parts of the 3/3-bar] were equal?
- 4 L: I used my hand
- 5 R: how?
- 6 L: [use gestures to show the equal size of the partitions] [...]
- 7 R: how did you know that this coloured part is 1/6 [Task 3, 6/6-bar Figure 1a]?
- 8 L: The teacher said it must be equal so how did I get 1/6 I used my hand and then drew the lines.

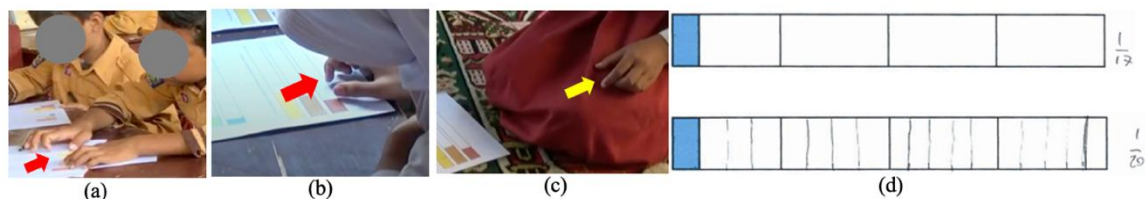
Unlike Tasks 1-2, the students processed Task 3 differently. The tasks asked them to determine fractions representing the coloured partitioned parts (Figure 1a). This task is more challenging than the previous tasks, the students need to see the relationship between the coloured part and other parts within the bar. For the 3/3-bar (Figure 1a), all the students could directly determine 1/3 without gesturing. To determine fractions in the other two bars (Figure 1a, 6/6-bar, 12/12-bar), all students used gestures. Two students were observed to gesture when solving the task (Figure 2a-b). In the post-lesson interviews, for example, for the 6/6-bar, Lili gestured and then drew partitioned lines within the bar (T2 lines 7-8, Figure 1a). Similarly, Hana gestured for determining 1/6 (T1 lines 5-8) and 1/12 for the coloured parts in the two bars. One student (Zane) directly knew the answers, did not make any drawings or partitions on the

bars, and gestured to confirm her answers. She argued in the interview that she identified connections between bars she partitioned in Task 1 and bars in Task 3. The three bars in Task 1 were partitioned into 3, 6, and 12, respectively.

Students were allowed to gesture when solving Tasks 1-3. Meanwhile, for Tasks 4-5, the students were asked to predict the answers to the tasks without gesturing. This could reveal whether the students could determine fractions without gestures. Zane was absent from the lesson. For Task 4 (Figure 1b), all five students could determine $\frac{1}{4}$ for the $\frac{4}{4}$ -bar. However, without the presence of gestures, only two students could determine correct fractions for the remaining two bars ($\frac{8}{8}$ -bar, $\frac{16}{16}$ -bar). The other three students answered $\frac{1}{17}$, $\frac{1}{10}$, and $\frac{1}{12}$ for the $\frac{16}{16}$ -bar, and some of them had $\frac{1}{6}$ for the $\frac{8}{8}$ -bar. Similarly, only one student had correct fractions for all the bars in Task 5 (Figure 1c), two students had $\frac{1}{8}$ and $\frac{1}{9}$ for the $\frac{6}{6}$ -bar, one student had $\frac{1}{6}$ for the $\frac{4}{4}$ -bar, and another student had no answers. When they were asked to check their answers during the lesson and allowed to use gestures, the student who had no answers for Task 5 was observed to gesture and determine correct fractions for the bars (Figure 3a). Another student, Lili, answered $\frac{1}{9}$ for the $\frac{6}{6}$ -bar in Task 5. After using gesture (Figure 3b), she found the bar to be six equal parts and revised her answer to $\frac{1}{6}$. Moreover, the other student, whose initial answer was $\frac{1}{17}$ for the $\frac{16}{16}$ -bar in Task 4, checked his answer using his hand and got $\frac{1}{20}$ (Figure 3d). He was able to partition the bar but made six partitions within each of the last two bigger parts. Some other students reworked their incorrect answers in the after-lesson interviews using gestures. For example, as shown in Figure 2d, Anna rechecked the answer on the $\frac{8}{8}$ -bar by gesturing to come to the revised answer, $\frac{1}{8}$ (Figure 1b). She was asked to rework the $\frac{4}{4}$ -bar where her initial answer was $\frac{1}{6}$ (T3 lines 1-6).

Figure 3

Students Checking/Reworking Tasks Using Gestures



T3 - R: Researcher, A: Anna

- 1 R: is this $\frac{1}{6}$? [pointing to the $\frac{4}{4}$ -bar, Figure 1c]
- 2 A: yes
- 3 R: could you imagine it again? [could you rework it by mentally processing the task]
- 4 A: [while reworking the bar, her hand silently forms a V-shape gesture, Figure 3c) it is $\frac{1}{6}$?
- 5 R: you can try using hands.
- 6 A: [partitioning the $\frac{4}{4}$ -bar using hand gesture] it is four

Based on the students' answers to the tasks and post-lesson interviews, there are three emerging categories regarding the students' gestures on the partition tasks: gesture type, gesture purpose, and gesture-task relation. First, the gestures used by the students, the thumb and index finger forming a V-shape, represent an action (making equal partitions) and were performed silently, called co-thought representational gestures, which might play self-oriented cognitive functions (Kita et al., 2017; Logan et al., 2014). For example, before drawing partitioned lines in Task 1-2, all the students gestured to determine equal parts (T1 lines 1-4). Another example is how Anna attempted to rework the $\frac{4}{4}$ -bar in Task 5 by self-directing her thinking through gestures (Figure 3c). Other studies also reported such a function of gestures. For example, Zurina and Williams (2011) documented how a student gestured for oneself to conceptualise fractions through the number of shared beard pieces. In addition to this study, the result of the present study provides another insight into the students' use of co-thought gestures for partition,

which is considered the foundation for fraction understanding (Lamon, 2020). Second, at the individual level, the gestures could be seen as a primary and supplementary action. When solving Task 3, the students appeared to use the gestures differently, where most of them used gestures to directly make the partitions (primary gesture), and one student gestured after solving the task to confirm her answers (complementary gesture). This aspect of the use of gesture is still questionable since the student with complementary gesture did not answer Tasks 4-5 due to her absence during the lesson. More data is required to understand how this aspect of gesture purpose is at play.

Third, at the task level, gestures are not always used by the students to solve the tasks. First, all the students gestured for making partitions in Tasks 1-2 and determining fractions within partially partitioned bars in Task 3. Second, all the students could directly determine fractions without gestures for the $\frac{3}{3}$ -bar in Task 3 and the $\frac{4}{4}$ -bar in Task 4, which are fully partitioned. Third, for partially partitioned bars in Tasks 4-5, some students used gestures while others did not. The gesture theories (Hostetter & Alibali, 2008; Kita et al., 2017) might provide explanations for the phenomenon. Hostetter and Alibali (2008) proposed three factors contributing to gestures, two of which are the strength of simulated action and the gesture threshold. For the students who did not need gestures, the tasks might not have sufficient difficulty or power to simulate their gestures. Moreover, their gesture threshold or the level of gesture activation is high due to the low cognitive load from the tasks. This is possible since the bars are fully partitioned; they could directly ‘see’ the partitions within bars. In other words, the more demanding the task, the more gestures are required. This could be relevant to fraction bars that are not fully partitioned, where they first need gestures for making partitions and then determine fractions. Research also points out this phenomenon on map tasks (Logan et al., 2014). Cognitive load as one of the determinants for the gesture threshold, appears to associate with the role of gesture for conceptualisation (Kita et al., 2017), which is to maintain spatio-motoric information. The students’ difficulty in maintaining the information from the tasks could have triggered more gestures. For the case of Tasks 4-5, where some students could determine correct fractions without gestures, two potential reasons are the individual’s difference in gesture threshold, or the students had been familiar with the tasks. Despite these possible explanations, further investigations are required to understand this phenomenon.

To conclude, the gestures help students directly/physically act on the fraction bars, rather than spatially/mentally process the partitions, which might not yet be available to them. The case of Anna (Figure 3c, T3 lines 1-6) depicts this, for example. These gesture-made partitions might allow the students to see the relationship between the partitioned parts within the bars, for example, in Tasks 1-2, where they could argue that the more partitions, the smaller the parts. This is likely to enable the students to determine fractions representing the partitioned parts. This finding could extend our understanding of how primary students use co-thought representational gestures in solving tasks involving partitioning, which is one of the concepts that underpins fraction understanding. Additionally, it contributes to gesture-fraction literature, which seems to mostly cover how teachers gestured (e.g., Barbieri et al., 2020; Beilstein, 2019) or students/participants used co-speech gestures to communicate fraction ideas (Edwards, 2009).

Gesture, Partition, and Fraction Understanding

Literature suggests that apprehension of partition, including the relationship between the partitioned parts within fraction representations, is the underpinning process for fraction understanding (Lamon, 2020). In the post-partition lesson interviews, in addition to how the students solved the tasks, they were also asked about their understanding of the process. The coding of the data resulted in five categories of fraction understanding. Due to the page limit, selected interview transcripts supporting the categories are presented. First, all the students were

aware that the equally partitioned parts represent fractions and could explain why. Second, only one student (Lili) could not argue that each partitioned part represents the same fraction and needs to fold back to the folded fraction strips to have that understanding. Third, after folding back, the student and the other four students could identify that the total number of partitioned parts determines the fraction (T4 lines 1-4). Lili had not developed the next two categories of understanding. Fourth, the other five students could observe that the smaller the partitioned part, the smaller the fraction (T4 lines 5-10). Lastly, the students were able to identify the whole (n/n) as the result of summing up all the parts (T4 lines 11-14).

T4 - R: Researcher, A: Anna, L: Lili, Rn: Rian

- 1 R: why $1/6$?
- 2 L: because it is partitioned into 6 parts
- 3 R: why this and this have different fractions? [pointing to the coloured part of the $6/6$ -bar and $3/3$ -bar in Task 3]
- 4 L: because they have different parts [...]
- 5 R: can you order the coloured parts from the longest? [Task 3]
- 6 A: $1/3$ $1/6$ $1/12$
- 7 R: is the fraction representing the shortest part the smallest?
- 8 A: [nods her head]
- 9 R: why?
- 10 A: because the part is getting shorter [...]
- 11 R: what do you find if they are added? [prior to this, he could identify two parts of the $6/6$ -bar are $2/6$]
- 12 Rn: the whole
- 13 R: what fraction is the whole?
- 14 Rn: $6/6$

It appears that the students were reliant on gestures to make partitions when solving some of the tasks. For example, some students could not directly determine fractions in the absence of gestures in Tasks 4-5 and then reused gestures in the lesson and interviews to rework the tasks (Figure 3, T3). Further interviews show that five of the six students have a solid understanding of fractions, not only arguing, for example, why the fractions represent the coloured parts but also reasoning on fraction comparisons. This might indicate the impact of gestures on fraction understanding through gesturing partitions. The students' gestures representing the action of partitioning in the present study possibly helped them build a mental model of the role of equal partition, allowing them to understand relations between the partitioned part(s) that produce fractions. For example, the smaller the partition, the smaller the fraction (T4 lines 5-10); the presence of the hands might facilitate this understanding, enabling students to directly see or visualise the partitioned part(s) that represent the magnitude of the fractions. Presmeg (2006) asserts that gestures help students visualise.

Implication and Limitation

This paper contributes to our understanding of how students use gestures for partitioning, which is one of the underpinning fraction concepts, and to what extent the partitioning gesture could possibly promote fraction understanding. The findings further emphasise the role of gestures in fraction teaching and learning, as other studies have documented (Beilstein, 2019). Moreover, the teaching intervention in the present study shed light on how gestures could be purposively and explicitly enacted in the classroom and involve partition; the teacher demonstrated co-speech representational gesture, with other resources, such as fraction manipulatives and representations. Thus, one of the potential approaches to address the challenge of learning fractions in primary schools is for the teachers to model gestures and for

students to be prompted to use gestures alongside representations. The data in this paper is from a small number of students, indicating the limitation of the findings.

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Enhancing Mathematical Wellbeing With Pedagogy

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A student's experience of mathematics in their school years has a lasting impact on many students through to adulthood. How we can improve this experience is a question of significance, as it directly impacts practice and importantly the futures of our students. This paper reports on a study exploring pedagogical approaches to support an increase in students' Mathematical Wellbeing. Findings suggest when language is used to frame what is valued, and variety of pedagogical approaches are integrated into teaching and learning sequences, students can experience an increase in Mathematical Wellbeing.

Mathematics knowledge and skills form an integral part of the Victorian Curriculum with the study of Mathematics a requirement from Foundation to Year 10. A component of the rationale for studying mathematics in the Victorian Curriculum is given as "mathematics is central to the learning, development and prospects of all young Victorians" (Victorian Curriculum and Assessment Authority, 2024). Two of the aims for students studying mathematics are to "ensure that students:

- develop useful mathematical and numeracy skills for everyday life and work, as active and critical citizens in a technological world
- develop a positive disposition towards mathematics, recognising it as an accessible and useful discipline to study" (Victorian Curriculum and Assessment Authority, 2024).

In contrast to the aims of the Victorian Curriculum, a review of the number of students completing Coursework in Unit 3/4 *intermediate mathematics* studies as part of their Victorian Certificate of Education indicated a negative trend for over a decade (VCAA, 2025).

Students lack of engagement in mathematics (Attard, 2012), finding it too hard and time consuming are reasons from practice given by students as to why they change from the intermediate mathematics course Mathematical Methods to the elementary mathematics subject of General Mathematics.

When disengagement in mathematics occurs and students change to elementary mathematics, or cease studying mathematics in their senior secondary schooling it can have "significant future implications for individual life opportunities and success, as well as economic implications more broadly (e.g. employment opportunities; contribution of mathematics to the knowledge economy)" (Zorn et al., 2022, p. 571). The investigation undertaken here set out to explore the question: Can pedagogical approaches used by mathematics teachers impact students' Mathematical Wellbeing?

Mathematical Wellbeing

Mathematical Wellbeing (MWB) is defined as "the fulfilment of students' values within the mathematical learning process, accompanied by positive feelings and functioning" (Hill et al., 2022, p. 379). Students' mathematics values have been found to align with seven wellbeing dimensions: Accomplishment, Cognitions, Engagement, Meaning, Perseverance, Positive Emotions and Relationships (Hill et al., 2020). Hill et al. (2024) have developed and validated survey questions to measure students' experiences of these values. According to a study by Hill and Hunter (2024) students have different experiences of the seven values and there is a significant decline in MWB from Year 3 to Year 8 across all values. To date there are limited studies of MWB as students progress into their senior secondary years of school.

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 461–467). Canberra: MERGA.

Engagement

Learning outcomes are significantly affected by the level and quality of engagement experienced by students (Heleme & Clarke, 2001). Attard (2014, p. 2) identifies the elements of influence that teachers have with students' engagement as "the pedagogical relationships formed between teachers and their students, and the pedagogical repertoires that are employed by the teachers". Effective pedagogical practices, that incorporate development of reasoning opportunities, discourse and challenge for all learners have been found to support engagement in mathematics (Attard, 2012; Wood, 2002; Hunter et al., 2016). As reported by Zorn et al. (2022) engagement is complex with an element including emotional engagement and students' willingness to be active learners within the classroom.

Perseverance and Mindset

Learning is achieved when students actively embody a thinking disposition. Being reflective, to form connections between new and existing knowledge and experience, supports processing of learning. Encouraging persistence with mathematics tasks has been the focus of many research studies (Clarke et al., 2014; Roche et al., 2013). Brown (2024, p. 117) discusses the impact of "increasing expectations of learners as active listeners and explainers" as a change teachers can bring about through the development of a classroom culture where students focus on controllable elements of their learning such as participation and mathematical thinking. As Perkins and Ritchhart (2005) assert "having a disposition toward thinking enhances the likelihood that one can effectively use one's abilities in new situations". Within the PISA 2022 report it is noted that students with higher levels of perseverance perform "at a substantially higher level in mathematics than students in the lowest quarter of the [perseverance] index" (De Bortoli et al., 2024, p. 113).

The Study

The research reported here is a snapshot of a year-long professional learning program undertaken by the researcher in 2024, aimed at supporting classroom teachers to transform teacher practice and student learning in their school through the implementation of a project. The project focus was on using pedagogy to enhance Mathematical Wellbeing of students in Year 9, Year 10 Advanced Mathematics and VCE Units 1 and 2 Mathematical Methods. The project took place at an independent Foundation to Year 12 school on the Victorian Mornington Peninsula. Five classes at Year 9 with between 18 to 26 students per class, two Year 10 Mathematics advanced (Year 10) classes of sizes of 18 and 23, and two VCE Mathematical Methods Units 1 and 2 (Year 11) classes with class sizes of 12 and 14 students participated in the project. The researcher taught one class at all year levels involved in the project, with five further staff teaching a combination of the remaining classes. Each of the teachers had more than 15 years teaching experience.

Staff Professional Learning and Student Workshops

Teachers participated in a professional learning (PL) session run by the researcher at the beginning of Term 3, introducing the key principles of MWB and the objectives of the project. Subsequent shorter meetings and PL occurred during Semester 2 consisting of the team of teachers or the teacher researcher meeting one-on-one with staff to discuss implementation of the series of workshops designed for staff to run with their classes and discussion of pedagogy to support students' MWB.

A series of three workshops were developed for the teachers to run with their classes to co-construct a common language and explicitly outline behaviours for engagement, perseverance and learning habits. Each workshop had a focus on one objective of the project: *Workshop 1 Developing Mathematical Wellbeing – Engagement* (objective to establish a common language

for engagement and associate student behaviours); *Workshop 2 – Perseverance* (objective to frame behaviours of perseverance); *Workshop 3 – Developing learning habits* (objective to embed a focus on the impact of mindset on learning). An objective through all workshops was casting a spotlight on student agency in the learning process. Workshops 1 and 2 were delivered during double period lessons, lasting for 30 to 45 minutes of a 100-minute double period, in July in Term 3, on a day that suited each teacher's lesson schedule. The duration of Workshop 3 was approximately 40 minutes and was delivered during August in Term 3, each teacher selected the most suitable lesson to conduct it in based on their individual class schedules.

Pedagogical approaches used within the project included, metacognition, through making time within lessons for goal setting by the students for their learning objectives within mathematics, supporting the MWB Values of *Accomplishment*, *Engagement* and *Meaning*, with students identifying behaviours that may advance the likelihood of attainment of their goal, developing a motivation for learning and path to follow. Starter questions were used by teachers to activate learning within the first five minutes of class, increasing neural links to reduce cognitive load (Newman, 2023) to enhance the MWB Values of *Cognition*, *Engagement* and *Relationships*. Custom designed reward stickers were used to reinforce behaviours of engagement and perseverance to enhance students' MWB Value of *Positive Emotions*. Learning opportunities that supported collaboration between students were drawn upon, scaffolding behaviours to promote effective communication for collaboration during the starters, or as part of the main lesson sequence occurred. The use of collaboration as a pedagogical approach by teachers was designed to build opportunity for students to experience alignment of the classroom pedagogy with the MWB Values of *Relationships*, *Positive Emotions* and *Perseverance*. Spaced-recall activities (Cooney-Horvath, 2019; Newman, 2023) were a pedagogical tool used to support and promote transfer of learning and enhancement of the MWB Values of *Accomplishment* and *Cognition*. Incorporating mathematical games to develop learning supported teamwork, competition and the MWB Value of *Positive Emotion*. Flexibility and variety in pedagogy was drawn upon to cater for the diverse learning needs of students from Year 9 to Year 11.

The use of Vertical Non-Permanent Surfaces (VNPS) (Liljedahl, 2020) provided a mechanism for collaboration, for students to explain, reason and justify their mathematical thinking. VNPS were utilized in Year 10 and Year 11 classrooms and embedded into practice with four to six non-permanent vertical whiteboards accessible within each of the classrooms. Use of VNPS promoted connections between students through group work, focussed time on task, development of knowledge mobility and high-quality talk in mathematics (Evidence for Learning, 2023). The use of VNPS supported the MWB Values of *Engagement*, *Positive Emotions* and *Relationships*.

Data

At each stage of data collection, student participation was via in an opt-in survey accessed through the school intranet. Baseline data of students' MWB was collected in May, Week 7 of Term 2, 2024, using the MWB survey developed and validated by Hill et al. (2024). The survey questions drew on a 10-point Likert scale from 1 ('Not like me at all') to 10 ('Completely like me'). A composite score for each Ultimate Value was achieved through calculation of the means of the three or four survey items.

As close as possible to Week 7 of Term 3 and Term 4 the MWB survey was again distributed to students. Week 7 was selected as the school has a whole school focus on wellbeing during this week of each term, each year. Semester examinations were completed one week after the administration of the survey in Term 2 and 4.

Results and Discussion

The number of students who completed the anonymous survey in Term 2 was 125, in Term 3, 111 students and 121 students completed the survey in Term 4. A One-way ANOVA test, using EXCEL was used to determine if the mean differences in the combined composite survey score for each MWB Value between Terms 2, 3 and 4 were statistically significant. Table 1 presents the combined composite scores for each MWB value and p-values, with $p \leq 0.05$ shaded grey to indicate statistical significance, for all classes and year levels involved in the project. The data indicates statistically significant differences between the means of the MWB Values experienced in Term 2, 3 and 4 for the values of Accomplishment, Engagement, Meaning and Positive Emotion. Across all seven MWB Values students experienced their highest MWB in August, Term 3. Factors that may be attributed to this August increase in MWB values may include, student's participation in the workshops with the use of language to frame engagement, perseverance and learning habits, the fidelity of the project related to teacher's use of pedagogy to enhance MWB, focussed learning time with reduced interruptions to the school learning program as occurs in Terms 2 and 4 and the absence of examinations within Term 3.

Table 1

Combined composite scores

Ultimate value	May (125)	August (111)	November (121)	Min	Max	p-value
ACCOMPLISHMENT	6.22000	6.88063	6.45248	1	10	0.03409
COGNITION	6.56800	7.12913	6.73003	1	10	0.08969
ENGAGEMENT	5.50933	6.41441	5.95868	1	10	0.00740
MEANING	5.56800	6.34234	5.85675	1	10	0.03490
PERSEVERANCE	6.06400	6.48649	6.34160	1	10	0.25770
POSITIVE EMOTION	4.57600	5.45946	4.96694	1	10	0.02618
RELATIONSHIPS	7.29600	7.67267	7.55647	1	10	0.23606

To explore the data further, the data collected relating to only Year 9 is reported in Table 2. A One-way ANOVA test, using EXCEL was used to determine if the mean differences in the combined composite survey score for each MWB Value between Terms 2, 3 and 4 were statistically significant. Table 2 presents the combined composite scores for each MWB value and p-values, with $p \leq 0.05$ shaded grey to indicate statistical significance. The data indicates statistically significant differences between the means of all MWB Values experienced in Term 2, 3 and 4 except for the Value of Perseverance. Some of this variation is likely to be related to less students completing the survey in August as all surveys were opt-in. Students in Year 9 sit their first examinations in secondary school in May, their second experience of examinations occurs in November, this may a confounding variable influencing students' MWB. Fidelity to the project by all teachers was likely to be at its highest in Term 3. Table 2 shows, however, an increase in all MWB Values between May to November.

The MWB Value with the greatest change from the baseline data in May to the end of year data in November was observed in the Value of Engagement, suggesting that the development of a common language, explicitly teaching behaviours of actively engaged learners and drawing on variety in pedagogy can support the enhancement of this Value.

Table 2*Mathematical Wellbeing Values combined composite score for Year 9*

Combined composite scores						
Ultimate Value	May (80)	August (59)	November (73)	Min	Max	p-value
ACCOMPLISHMENT	5.79375	6.91102	6.19863	1	10	0.00505
COGNITION	6.10417	7.10169	6.39269	1	10	0.01626
ENGAGEMENT	4.82500	6.12429	5.42009	1	10	0.00244
MEANING	5.02917	5.97175	5.23288	1	10	0.04526
PERSEVERANCE	5.65000	6.38418	5.97260	1	10	0.10193
POSITIVE EMOTION	3.80000	4.96045	4.27397	1	10	0.02332
RELATIONSHIPS	6.73750	7.58757	7.22374	1	10	0.02431

The data collected shows that it is possible to reverse the trend of declining MWB as students progress through their secondary years of school, this is in contrast to other research on MWB (Hill & Hunter, 2024). The Value of Relationships was rated the highest by students in each survey cycle and the Value of Positive Emotions the lowest. Numerous opportunities were made available for the students to collaborate on mathematical activities in classes throughout Terms 3 and 4 during the project. Increasing students' capacity to work with others was a feature of collaborative pedagogies, this is likely to have supported the increase in the Relationships and Positive Emotions Values. Goal setting and discussion of individual goals between the teachers and their students became an embedded component of pedagogy, motivating students and providing additional alignment between student's experience of mathematics and the Value of Meaning.

Conclusion

In this study a focus on developing a repertoire of pedagogical practices and a series of workshops that highlight student agency in learning mathematics has been discussed to investigate the question, 'Can pedagogical approaches used by mathematics teachers impact students' Mathematical Wellbeing?' The primary objective was to increase the fulfillment and experience of students in learning mathematics, through pedagogical practices that may enhance MWB. It has been shown that with intentional use of practices that are valued in the learning environment including, goal setting, and variety in pedagogical practices that facilitate collaboration, reflection and connection building can correlate to the enhancement of MWB.

The findings provide further insight into the potential benefits of the emerging field of MWB and the contributions that a values-based lens of pedagogy may bring to student's experience of mathematics. Further research over a longer time frame, exploring a correlation between MWB and senior secondary enrolments is recommended.

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Pema's Reflective Practices: Narrowing the Gap Between Espoused and Enacted Beliefs

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This case study examined a Bhutanese teacher, Pema's reflective practices and their effects on her beliefs and classroom practices over a sequence of four lessons sandwiching a professional development. Three instances of reflection (before, during and after each lesson) were analysed to determine the shift in her beliefs and practices. The result revealed that reflective practices challenged her beliefs and routine practices, resulting in a shift from a traditional view to a more constructivist approach in teaching.

Bhutan is a landlocked nation situated between China and India. It covers an area of about 38,000 square kilometres with a population just below a million. As Bhutan graduated from one of the Least Developed Countries (LDC) in 2023, the Ministry of Education (MoE), Bhutan, has prioritised Science, Technology, Engineering and Mathematics (STEM) as fundamental for national development and has initiated several rounds of mathematics curriculum reforms since the 1960s. Nevertheless, mathematics remained the poorest performed subject for decades in national and international assessments. Bhutanese students scored significantly lower than the PISA reference countries such as China, Singapore, and Macau.

The new curriculum advocated the importance of conceptual learning and student-centred approaches. However, most of the Bhutanese teachers are accustomed to procedural teaching (MoE, 2014), resulting in few changes in teachers' planned practices (Dolma et al., 2017). To bridge this gap, the MoE in 2014 initiated a minimum of 80 hours mandatory Professional Development (PD) per year for teachers but the focus on attaining the required hours compromised the quality and subjugated the actual purpose (MoE, 2014). Top-down In-Service Education and Training (INSET) programs may be ineffective if they do not take teachers' beliefs and practices into account.

Reflective practice is at the heart of teacher learning and it is important for their professional growth (Loughran, 1996). This case study is part of a larger study into Bhutanese secondary mathematics teachers' reflective practices, their effect on teachers' beliefs and classroom practices, and the impact of professional learning. It aims to pursue two research questions in the context of Pema (Pseudonym) who was teaching a year 8 mathematics class:

- What are Pema's beliefs of mathematics and mathematics education?
- How does the use of reflection influence Pema's beliefs and classroom practices?

Literature Review

The beliefs held by teachers influence their classroom practices (Beswick, 2005, 2007). Philipp (2007) defines beliefs as a "psychologically held understandings, premises, or propositions about the world that are thought to be true (p. 259). The espoused beliefs are teachers' ideal beliefs about teaching and learning, but enacted beliefs are those beliefs that are exhibited in teachers' classroom practices (Berk & Cai, 2019; Ernest, 1989). Thus, teachers possessing similar mathematical knowledge and teaching experiences have varying classroom practices and learning outcomes depending on the beliefs they have and their ability to enact their ideal beliefs in practice.

Ernest (1989) subsumed beliefs of mathematics teachers as beliefs about the nature of mathematics, beliefs about mathematics teaching and beliefs about mathematics learning.

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These beliefs are based on two factors: traditional and constructivist. A traditional view is where mathematics is seen as a static discipline which is taught and learned through the transmission of mathematical knowledge from teachers to the learners. A constructivist approach views mathematics as a dynamic discipline and learning as an “active and purposeful process whereby individuals adapt their constructions in order to optimise their fit with experience” (Beswick, 2007, p. 97). The beliefs of teachers about the nature of mathematics are directly related to their teaching of mathematics and selection of teaching strategies.

Beliefs about teaching refer ‘to their notions of the principles of classroom teaching and the nature of effective teaching’ (Xie & Cai, 2020, p. 749). Teachers who view mathematics as a static body of knowledge possess a belief that the learners are passive receivers of the knowledge and focus on the mastery of instrumental and basic skills by heavily relying on the textbooks or content. Conversely, teachers who view mathematics as a dynamic body of knowledge believe that students construct their own knowledge, focus on problem posing and solving, and explore different resources to supplement student learning. Beliefs about mathematics learning refer to their beliefs about the role that students should have in their own mathematics learning and the perceived attributes of good learners (Xie & Cai, 2020). Teachers who hold the problem-solving view of mathematics are more likely to accept the learners’ methods and approaches to a mathematical problem, whereas those with the Platonist or instrumental views insist on a single correct method for solving each task.

The gap between the expressed and enacted beliefs is noticeable when teachers’ beliefs are not congruent to the beliefs underpinning the educational reforms (Ernest, 1989). The change in beliefs can be expected if educational reforms are compatible with teacher beliefs but are hindered if they contradict the teacher beliefs. Reflective practices are crucial for teachers’ learning to make sense of what they observe, to challenge their existing beliefs and to appreciate associated change in beliefs and practice (Philipp, 2007). The attempt to change the practices of teachers is unreasonable without changing beliefs (Beswick, 2005). Beliefs are not consciously articulated or brought to teachers’ awareness unless they engage in reflective practices.

John Dewey defined reflective practices as an “[a]ctive, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusion to which it tends” (1910, p. 6). He distinguished between routine and reflective action and characterised reflection as comprising five phases: suggestions, problem, hypothesis, reasoning and testing (Loughran, 1996). This rational-technicist model has a drawback of overreliance on rationalism and adherence to technical rationality. These criticisms have remained as a foundation and helped to develop other models in the field of reflective practices.

Against the backdrop of technical rationality, Schön described two forms of reflection: reflection-on-action (after an action) and reflection-in-action (during action). His experiential-intuitive model underscores the process of reflection which spirals through stages of appreciation, action, and reappreciation (Zeichner & Liston, 2014). However, this has been criticised as not recognising the social process, and insufficient attention was given to social conditions that frame and influence that practice (Zeichner & Liston, 2014).

Accordingly, Loughran (1996) explored three instances of reflection: anticipatory (pre-teaching), contemporaneous (during teaching) and retrospective (post-teaching) reflection. He asserts that anticipatory reflection is a starting point for teachers to develop ways of thinking about their approaches to pedagogy. Contemporaneous reflection is the way to learn from and about their practices in action, and the retrospective reflection encompasses learning from experiences regardless of the perceived success of the episode. These three instances may help teachers compare their thinking about practice with their actual practice thus sustaining

professional growth. Teachers who engage in reflective practices consider problems in their own teaching and think about how those problems are related to their learners' understanding of mathematics (Posthuma, 2012).

Good reflective practices in mathematics education shape a meaningful and productive professional development, create opportunities to acquire new insights and skills in teaching, and are a source of professional growth (Posthuma, 2012). Conversely, reflective practice is not free of barriers. Teacher shortage, time constraints and heavy teaching periods hindered teachers' engagement in reflective practice in Bhutan (Dolma et al., 2017). A lack of subject-area specific supervisors, reflective disposition and hesitation in giving authentic feedback are some of the barriers in implementing reflective practices. However, it has the potential to assist teachers in re-evaluating their teaching practices, challenge beliefs, align theories and practices, and augment teacher professional learning.

Methods

This case study comprised two phases. In phase one, pre- and post-teaching interviews were conducted with Pema and one lesson was observed (video recorded) to establish her existing practices. In phase two, following a half-day Professional Development (PD) program, three lessons were observed. Pema was interviewed before and after each lesson to enable the observation of her enactment of reflective practices. Student work sample and her reflective journal for every lesson was also collected.

The PD program was conducted for the teachers at Pema's school. It was not based on a specific subject but included all teachers from all teaching areas. The PD focused on helping teachers understand and differentiate between routine action and reflective thinking. It was aimed not only to help teachers to understand what reflective practice in education is, but it provided an opportunity for the participants to challenge their existing beliefs. The importance of reflective practice and the prerequisite attitudes of reflection were presented. Action reflective cycle was introduced to help teachers to enact reflection in their classroom practices. The barriers and misconceptions of reflective practice were discussed. It was a platform for the teachers to learn, reflect on their beliefs and practice anticipatory, contemporaneous and retrospective reflections through group discussion and sharing their experiences based on one of their recent lessons. This was particularly relevant to Pema since she had completed the first phase of the study.

Pema's data was analysed for evidence of beliefs and practices using the three phases of reflection and compared the lessons before and those after the PD. The audio-recorded interviews and video-recorded lessons were transcribed into written form and colour coded to pieces of data that were interesting, unique, relevant and important for the study. Codes were defined in relation to the research questions and were aimed at identifying themes (Powell et al., 2003). For instance, Pema's expressed and enacted beliefs, selection of pedagogical approaches, use of mathematical reasoning and presentation, evaluation of students' learning, the influences of reflection on mathematical meaning-making process of learners and identifying the misconception through the lens of reflective practices were coded. These codes were used to construct the storyline by interpreting and making sense of it (Powell et al., 2003).

Findings

Pema was teaching Year 8 mathematics for the 2024 academic year in one of the rural schools in Bhutan. She was 27 years old and had three years of teaching experience. She completed her B.Ed. Secondary (Math/Physics) from one of the Colleges of Education in Bhutan. There were 26 students (boys=12, girls=14) in her class. The ages of the students ranged from 14 to 19 years. The age gap is due to late admission in pre-primary and low

achievers repeating year levels. The classroom was arranged in rows facing the chalkboard with two students sharing a table.

Pema's Existing Beliefs and Practices (Before PD)

Anticipatory Reflection

Pema's pre-teaching reflection involved her thinking about linking the real-life experiences of learners with the intended mathematical knowledge, which she considers as an important component of lesson planning.

Pema: Whenever I plan a lesson, I always think about how to connect with the real-life situation. If you connect with the real-life situation, students will get clear concept of the topic. Therefore, I also make related questions for students to apply in their real-life situation.

In relation to problem solving, Pema commented that:

Pema: Some students think mathematics is very difficult and some say mathematics is all about numbers. Word problem is very difficult for them. Unlike other subjects, they can't directly listen and understand mathematics. Mathematics is all about solving the problems.

To resolve this, Pema wanted to explain the concept and demonstrate how to solve the proportion problem, as she believed that first students should learn to be able to solve the given task. Her thinking about the approaches of pedagogy supported the traditional view where the teacher is expert, and the learners are passive receivers of knowledge in the learning process.

Pema attributed students' poor performance to lack of careful reading and comprehension of the questions. She highlighted the importance of connecting mathematical knowledge to the real-life experience of learners, but the pedagogical approach she described showed a transmission and teacher-centred approach.

Contemporaneous Reflection

When teaching, Pema emphasised rules and procedures instead of initiating problem-posing and solving. In consequence, the students gave the answers that Pema sought rather than asking for clarification on the topic. Pema demonstrated how to find missing terms in a proportion using two methods- scale factor and cross-multiplication.

Pema: Let's say the ratio of boys to girls in a class is '6 is to 7'. If there are 42 girls, how many boys will be there? We are going to use the scale factor method... we must either multiply or divide, so right now we can see that one numerator from this ratio is missing (wrote $\frac{6}{7} = \frac{\quad}{42}$), so we can use the other two denominators. What number times 7 will give you 42?

Students: 7 times 6 [few students answered]

Pema: 7 times what number will give 42?

Students: 6 (chorus).

Pema: Similarly, you multiply the numerator by the same number. How many boys are there in this class?

Students: 36 [chorus answer]

Pema: This number [pointing at 6] is called scale factor.

She defined proportion as two equal ratios but used no single ratios in finding the missing term of a proportion. She taught the cross-multiplication method by showing how to multiply the 'numerator of the first fraction with the denominator of the second fraction' and 'denominator of the first fraction with a numerator of the second fraction' and do cancellation of zeros. This method resonates with the traditional transmission of teaching mathematics as Pema emphasised procedures. A disparity between the espoused and enacted beliefs was observed. For instance, Pema expressed that "students are not reading and trying to understand the questions" but in the class she gave questions verbally and demonstrated how to solve the problem. Students had no opportunity to read the questions and comprehend them. The lesson

was closed by asking students to copy and complete the incomplete tasks at home. Pema could not assess students' learning.

Retrospective Reflection

In the post-teaching interview, Pema once again underscored the importance of linking mathematics to real life situation which indicated the existence of a gap between the expressed and enacted beliefs.

Pema: To understand the concept, I gave them word problems to solve which is related to real life situation. I tried to connect my lesson with real life as it helps to make learning mathematics meaningful, interesting and motivates the learners.

However, the lesson was dominated by teacher talk and demonstration. Students' participation and interaction were minimal. Student's inability to compute basic multiplication and division problems prompted her retrospective reflection.

Pema: Some students were not able to multiply and divide the numbers. I felt embarrassed to know that the students of Grade 8 were still not confident about multiplication and division which are the main components in mathematics.

Pema reported that her reflections involve thinking about what went well and what did not do well during the teaching episode. Pema shared that the presence of the observer in the class made her self-conscious and hindered the smooth flow of the lesson. She also acknowledged that there is no culture of reflection besides the MoE's mandatory observations of at least two lessons of every teacher by the principal per year.

Pema's Enactment of Reflective Practices (After PD)

Anticipatory Reflection

After attending the half-day PD program on reflective practice, Pema noted the difference between planning a lesson and reflecting on how the lesson might unfold, decisions on the selection of the pedagogical approaches, and reasons for the actions adopted (Loughran, 1996). The progression of Pema's anticipatory reflection was visible in every subsequent lesson as her reflection was influenced by learning experiences from previous lessons and PD.

Pema: Before the PD, when we plan the lesson, we usually don't think of the students. But after attending the [PD] program, I learned that it is important for the teachers to reflect not only on us [teachers] but also based on the students' thinking and [learning].

Pema acknowledged that a mathematical problem can be approached with multiple solutions, and it is important for the students to know that they need not have to depend on what the teacher has taught or demonstrated in the class.

Pema: If students get same answer using different solution, we can always accept because math is not only about the steps and procedures, and not only about the answers. There are different strategies or methods. It is important to share and learn from other students.

Pema recognised and appreciated participatory learning as she agrees that students learn when they do pair or group activities. She planned to let students show their solutions on the board and initiate discussion as a whole class to promote communication in mathematics and assess students' learning to ensure that every child learns and attains the lesson objectives.

Pema commented that the pre-lesson reflection helped her to reduce the risk of confusion when unexpected problems arise in the classroom.

Pema: Planning lessons without reflection will confuse you while explaining the concept and ideas get mixed, but when we reflect while planning, then lesson activities can be conducted as per the plan.

Identifying misconceptions and difficulties through anticipatory reflection helped her to be more confident and supported her to handle the situation when contemporaneous reflection is prompted in the class.

Contemporaneous Reflection

Pema started letting students to do pair work in the second lesson and group work in the third lesson. Due to the minimum participation of students in the group work, Pema conducted an activity in pairs in the fourth lesson. The shift of pedagogical approaches from traditional chalk-and-talk to more of a participatory learning was observed. The reflective practices assisted Pema in asking probing and guiding questions to promote students' participation in the process of learning. As a process, she realised that language appeared very important for students to learn mathematics. She explained the question in English, but the students responded in Dzongkha (Bhutan's official language). She also noticed that students were struggling with basic multiplication and division facts. However, much of her instruction remained focus on instrumental understanding. For instance, Pema taught multiplication and division algorithm instead of using other methods like factorisation method to promote relational understanding.

In another instance, Pema was challenged by a student who solved the problem using a different formula. She called two students to show the class their solution using two different approaches and explain their reasoning and initiated a whole class discussion. It was learnt that the students who solved differently were repeaters who were taught by a different mathematics teacher in the previous year.

The contemporaneous reflection of Pema was enhanced and noticeable as she learned from the experiences of previous lessons. Gradually, Pema was giving more time for students to discuss and initiated whole class discussion. Students were allowed to share and justify their answers, which assisted learners to construct their own mathematics knowledge.

Retrospective Reflection

Pema's retrospective reflection was prompted by the success of her lessons and her satisfaction derived from engaging the students in the process of learning.

Pema: I related the topic to a real-life situation. Everyone was curious and were listening very attentively. The activity was given in pair... some were explaining to their friends... I called one of the students to solve on board and explain. I felt that students enjoyed and learnt the lesson when explained by their friend and that also in Dzongkha (local) language.

The success of students working in pairs made Pema consider participatory learning. She recognised the importance of language and thinks that it is an important tool to communicate and create mathematical situations in solving a problem.

Pema delved into how her beliefs were challenged after few students solved a problem using a different approach from what she taught.

Pema: We were taught by our teachers adapting stringent methods. I never thought of other methods. I prefer to teach the way I was taught before. But one of the girls solved using different method. Her previous teacher taught the method, and she finds more comfortable with it... I think sometimes the teacher can also learn from the students.

Pema believes that the teacher can be a co-learner and facilitate whole class discussion. She recounted how her students approached the task differently and enjoyed sharing it with their friends. She agreed that students are learning genuinely and confidently when they can reason and solve problems in their own ways. Hence, it indicated a shift of the teacher's beliefs about learning mathematics from a traditional to a constructivist view.

Pema viewed reflection as more than thinking and recognised it in practice as it is a source of learning and a means to appreciate students in constructing knowledge. Reflection helped

her find her students using ineffective strategies (e.g. using fingers) to retrieve multiplication facts and prompted her to think about how to teach basic multiplication facts for quick retrieval.

Pema: Reflecting before, during and after teaching helped me to explore different methods, and let students work in groups and pairs which I usually don't do. Before this study, I was always thinking about what I know and never thought about how students learn and share the ideas.

Pema feels that all the teachers should do reflection and make it a school culture to make teaching and learning meaningful for all the students and teachers. She feels accomplished as she could use different methods of teaching and could promote student participation through reflective practices.

Discussion and Conclusion

Pema held more of a traditional view before the PD. Her beliefs were shifted towards a more constructivist approach after the PD as she started to recognise the need for looking at the student learning and challenged her routine practices through reflective practices. Pre-lesson, during and post-lesson reflection helped Pema recognise the gap between her beliefs and existing practices. The finding supports the notion that the enactment of beliefs is limited if espoused beliefs are represented in a disconnected verbal way and not integrated with the pedagogical knowledge (Ernest, 1989). Pema's teaching was associated with the traditional model of teaching besides her attempt to connect mathematical knowledge to the real-life experience of the learners and initiation of participatory learning. Pema became more aware of her own beliefs and practices through reflection. This led towards greater integration of Pema's beliefs and practices (Ernest, 1989), and indicated a shift of her beliefs about mathematics and mathematics education towards a problem-solving view as she adopted participatory and child-centred approaches in teaching. Learning about one's pedagogy through reflection can lead to a cycle of learning by doing and be a source of greater ideas and suggestions (Loughran, 1996).

The three instances of reflection, viz., anticipatory, contemporaneous and retrospective, guided Pema to understand how reflection influences the practices (Loughran, 1996). She reported that she never reflected before and during teaching until she took part in this study. Moreover, she admitted that her reflection after the teaching involved just thinking about what went well or what didn't go well. The findings support those of Loughran (1996) where he differentiated between planning a lesson and reflecting on how to teach, and thinking that a lesson was either good or bad and a teacher reflecting on that experience to learn. Reflective practices helped Pema shift her focus from teaching to student learning, leading her to notice learning difficulties (for example, multiplication and division problems) and recognise the importance of language in learning mathematics. The ability to notice problems can lead to new suggestions and testing of hypotheses strengthening the meaning making process in learning mathematics.

The findings about beliefs about teaching mathematics concurred with those of Dolma et al. (2017) where Bhutanese primary mathematics teachers perceived the role of the teacher to be an authoritative source of mathematical knowledge and students acquired knowledge by attentive listening. Pema believed that participatory learning is not appropriate as students should first learn from teachers to learn in pairs or groups. However, after participating in this study and implementing some reflective practices, Pema began to believe that teachers can equally learn from students and be co-learners as there are multiple approaches to solve mathematics problems. This supports the findings of Posthuma (2012) who claimed that reflection is a potential tool to shape a meaningful and productive professional development by creating opportunities to acquire new insights and skills in teaching. The shift of Pema's role from an explainer to a facilitator, emphasising the students constructing their own knowledge, aligns with the intention of Bhutan's new mathematics curriculum which supports participatory learning.

The gap between beliefs and practices is created when teachers' beliefs are not congruent with the beliefs underpinning the educational reforms (Ernest, 1989). Teachers' beliefs can be aligned to the beliefs supporting the curriculum reforms and PD programs by providing opportunities for teachers to develop knowledge that has a positive influence on their beliefs through reflection leading to the cycle of learning by doing. Inculcating a reflective culture in Bhutanese schools can be a teacher's source of sustaining professional development that may improve mathematics standards as teachers come to appreciate the underpinning principles of curriculum reforms. However, reflection and how it influences the thinking and practices of teachers depends on context like content knowledge, experience, time, actions, feelings and self-confidence (Loughran, 1996). It is acknowledged that findings of this study are based on Pema's context over a short period of time and have not yet determined the sustainability of any changes exhibited over time. Further studies over longer periods are needed to determine how reflective practices can influence the teachers' beliefs and bridge the gap between beliefs and practices to realise the underpinning beliefs of mathematics curriculum reforms in Bhutanese schools.

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Promoting Financial Numeracy Through Mathematics Education

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This study explores how financial numeracy can be effectively promoted within mathematics education for third and fourth graders. Using a Design-Based Research approach, it identifies challenges in students' financial decision-making and implications for the design of educational interventions. The findings underscore the need for thoughtfully designed reflective tasks to enhance students' dispositions and critical orientation towards purposefully applying mathematics in financial contexts.

Introduction and Problem Statement

Financial education has gained significant attention in recent years, as financial literacy is viewed as essential in an increasingly complex society. Unlike previous generations, children today are exposed to a changing financial landscape, including greater exposure to cashless transactions, such as tap-and-go payments and in-app purchases. Simultaneously, shrinking social benefits, aging populations, and more complex financial services mean that future generations will likely face greater financial responsibilities at an earlier age (OECD, 2020). Despite this, the definition of financial literacy and its integration into education remain subjects of debate. Narrower views focus mostly on financial knowledge acquisition and retention, while broader definitions also include required attitudes/motivations and skills/behaviour (Amagir et al., 2018). Furthermore, while a strong correlation between numeracy and financial literacy has been recognised (OECD, 2020), the role of numeracy is frequently reduced to performing basic arithmetic procedures, neglecting the broader numeracy perspective adopted in mathematics education. This study aims to address this gap by exploring how financial numeracy can be fostered within mathematics education. By acknowledging that financial numeracy extends beyond knowledge to include behavioural and attitudinal dimensions, this research seeks to identify challenges in students' financial decision-making and behaviour and thus determine the likely conditions necessary for a successful educational intervention.

Theoretical Background: Financial Literacy in Mathematics Education

Research identifies two effective approaches for embedding financial literacy within mathematics education: experiential learning and a focus on numeracy.

Experiential Learning

Experiential learning has been identified as particularly effective in meta-studies examining the most successful design elements for financial education (see e.g., Amagir et al., 2018). Practical experiences with money can not only enhance financial and economic understanding (Claar, 1996) but also improve mathematical skills (Grassmann, 2008) and engagement with mathematics (Attard, 2018). A practical example of experiential learning in primary schools is the simulated classroom economy described by Batty et al. (2020), where students manage their own money in a classroom setting. The study found significant improvements in financial knowledge, attitudes, and behaviours. However, the exclusive use of quantitative measures leaves unanswered questions about why these improvements occurred. Additionally, students participating in the program showed a significant increase—about 10 percentage points—in meeting state mathematics standards compared to those not in the program. While this improvement demonstrates the effectiveness of the experiential learning approach, the broad nature of the measures used (e.g., standardised test scores) does not provide insight into specific

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gains related to applying mathematics in financial contexts or how mathematics education can be purposefully integrated with financial literacy.

Numeracy as a Bridge Between Mathematics and Real-World Contexts

Numeracy is often narrowly defined, particularly in terms of its significance for financial literacy, where it is frequently reduced to basic calculations. Lusardi (2008), for instance, differentiates financial numeracy, which involves basic knowledge, from financial literacy, which encompasses advanced understanding. In contrast, mathematics education takes a much broader view, recognising numeracy as a critical connection between mathematics and real-world contexts. Steen (1998), for example, emphasises that numeracy is distinct yet complementary to mathematics: while mathematics focuses on abstraction, numeracy grounds learning in real data and diverse situations. In the last years, the study of numeracy practices—how people actually use mathematics in their daily lives—has gained increasing attention. Surveys like the Programme for the International Assessment of Adult Competencies collect data on how frequently adults use mathematics in everyday life, such as calculating prices or budgets. Besides analysing the frequency of numeracy practices, there are also studies investigating how mathematics is used, such as the one by Northcote and Marshall (2016), which highlights how mathematics in practice often relies less on exact written calculations and more on estimation and mental strategies. Numeracy is increasingly understood as embedded in cultural, societal, and power dynamics; a perspective referred to as Numeracy as Social Practice (Yasukawa et al., 2018). Social and contextual influences significantly shape how mathematics is applied in everyday contexts, adding another layer of complexity to its study and teaching. Teaching numeracy in schools poses challenges because classroom mathematics practices often fail to align with real-world practices. In terms of the context of money/finance, there have been several studies addressing the gap between school and everyday mathematics, for instance, Nunes et al. (1993) observed that children working as street vendors, could effectively solve money-related problems using their informal strategies outside of school but struggled when these tasks were reframed within school procedures. Similarly, Brenner (1998) highlights that children's understanding of money in their daily lives often diverges from how financial problems are presented in classrooms.

Bridging this gap requires educational strategies that foster numeracy. The Critical Numeracy Framework developed by Goos et al. (2014) for classroom-use, provides a comprehensive approach to this by conceptualising numeracy through five inter-connected dimensions: Mathematical Knowledge (concepts, problem-solving, estimation); Contexts (real-world application); Dispositions (confidence and adaptability); Tools (physical, digital, and representational aids); and Critical Orientation (using mathematics to make decisions, build arguments, and evaluate positions). In recent years, there have also been studies specifically focused on the teaching and learning of financial numeracy in financial contexts. Research by Sawatzki and Sullivan (2017) and Blue et al. (2018) emphasises the importance of a critical orientation in financial decision-making. This involves navigating choices while fostering key capabilities such as accepting challenges, comparing options, and integrating mathematical and social reasoning. Furthermore, when considering context, it is recommended to avoid overly simplistic "border contexts" that merely disguise mathematical problems in superficial financial settings. Instead, emphasis should be placed on tasks that engage seriously with financial contexts (Burkhardt et al., 2024). As there are only limited insights into how financial educational interventions for younger students (aged 8 – 10) should be designed, especially in terms of fostering financial numeracy, the overall study addresses the following research question:

- How can financial numeracy be promoted in mathematics lessons in third/fourth grade?

To answer this question, the aims of this paper were to:

- Examine challenges to financial numeracy practices in students' financial behaviour and decision-making (for hypothetical financial decisions and experiential learning).
- Infer success conditions for experiential learning approaches and implications for financial numeracy task design.

Methods

To answer the research questions, a Design-Based Research (DBR) methodology was employed to iteratively develop and refine an educational intervention aimed at promoting financial literacy through mathematics education. DBR was chosen as it can provide researchers with detailed insights into students' thinking processes, enabling the identification of learning progressions and challenges necessary for refining the intervention (Prediger et al., 2012). The DBR process in this research involved two iterative cycles:

- Cycle 1 (2023): Implemented with 16 students (9 third-graders, 7 fourth-graders), identifying barriers to financial and mathematical reasoning.
- Cycle 2 (2024): Refined intervention implemented with 14 students (9 third-graders, 5 fourth-graders), focusing on improved task design and structured reflection.

Participating students were selected from two rural schools in Baden-Württemberg, Germany, chosen due to previous partnerships in financial literacy initiatives. The students can best be described as predominantly middle-class, though they came from diverse cultural backgrounds and, based on their mathematics grades, exhibited varying levels of mathematical achievement. Unlike countries such as Australia, where financial literacy is explicitly embedded in the curriculum, Germany has not yet taken this step. However, money is included in the measurement strand of the mathematics curriculum. Moreover, their financial experiences are likely to differ as well. For instance, in Sweden, cashless transactions are the norm, whereas in Germany, over half of all payments in 2023 were still made in cash (Ego et al., 2024). Participation in the lessons was voluntary and part of an after-school programme. Pre-and Post-Interviews and Classroom Observations were used to provide a holistic view of changes in financial numeracy and influencing factors. Data was analysed using qualitative content analysis (Mayring, 2010).

- Semi-Structured Pre-Intervention Interviews: The interviews were used to explore students' baseline attitudes, knowledge, financial decision-making, and numeracy practices through self-reports and hypothetical scenarios. The shopping scenario required students to decide on, and justify, the best course of action when funds were insufficient, involving cost estimation, contextual interpretation, and product exchanges based on price comparisons and proportional reasoning.
- Classroom Observations: The researcher observed students' decision-making and reasoning during the simulated classroom-economy. In this simulation, students earned and managed money (income and expenses) to reach a savings goal. Furthermore, they could spend part of their money at a classroom store. Participation in spending at the classroom store was not guided by specific mathematical procedures or shopping scripts. Each group of students had six opportunities to participate in shopping activities; however, as the activity was voluntary, not every child engaged in each session. Data was collected through recordings of the lessons and field notes. Since pre-interviews revealed that most children had only handled cash payments, the classroom store was designed around cash transactions while also exploring behaviours that could present challenges in cashless interactions.
- Semi-Structured Post-Intervention Interviews: As per the initial interviews, the same questions were asked, and similar scenarios were used to evaluate the effectiveness of

the intervention by analysing changes in attitudes, knowledge, and financial decision-making and numeracy practices.

This paper examines students' decision-making in shopping contexts during Cycle 1, focusing on their responses during the pre-intervention interviews to the shopping scenario and their behaviours whilst shopping at the classroom store. Instead of quantitatively counting strategies or problems encountered, this analysis qualitatively describes typical approaches and problematic behaviours used by multiple students. These observations were complemented by insights from the interviews regarding students' self-reported financial experiences and numeracy practices.

Findings

Financial Experiences and Socialisation

Students' financial experiences and socialisation varied significantly, influenced by parental involvement and societal norms. Some parents actively involved their children in budgeting and financial decision-making, while others excluded them, often citing beliefs that money-related topics were private or inappropriate for children. Parental values, such as thriftiness, shaped students' financial attitudes. Thus, the degree of autonomy in money management varied widely among students. Student interview responses indicated that some lacked access to their own funds and relied entirely on their parents for financial decisions, with parents frequently vetoing purchases they deemed unnecessary. Others managed their money more independently, with parents offering guidance rather than control. These students could more often reflect on a purchase they regretted and why, for example differentiating between price and value and weighing price against personal value or utility considerations.

Attitudes Toward Mathematics and Numeracy Practices

Even though students were explicitly asked about the use of mathematics in their everyday lives, they frequently mentioned school-related applications. For example, instead of recounting instances where they needed to estimate ingredients for baking or prices, students perceived mathematics as necessary primarily for completing homework and preparing for tests. Furthermore, the need for mathematics was more often associated with their future aspirations, such as achieving dream jobs, rather than immediate practical applications. Regarding numeracy practices with money, students reported applying mathematics in certain financial contexts, such as counting the money in their piggy banks. In fact, money was the most mentioned link between mathematics and daily life and the primary context in which students reported using mathematics outside of school. However, while many students regularly accompanied their parents when shopping, this did not consistently lead to numerate behaviours, such as comparing prices or calculating total costs. Instead, their engagement was often limited to asking their parents whether specific products could be purchased.

Challenges for Financial Numeracy

The challenges for Financial Numeracy are listed in Table 1. Observed challenges were first coded inductively and then categorised within the dimensions of the Critical Numeracy Framework (Goos et al., 2014). For example, students who consistently relied on the same calculation strategy, ignoring contextual efficiency, were assigned the inductive code "rigid/inflexible strategies." This code was subsequently mapped to the deductive category "disposition" in the Critical Numeracy Framework, as "rigid/inflexible strategies" would be the opposite of a flexible disposition. Since the dimensions of the framework are interconnected, assigning each observed challenge to a single category was not always straightforward.

However, this categorisation was chosen to highlight how financially numerate behaviour relies on more than just basic calculation skills and to illustrate the aspects involved.

Table 1

Observed Challenges for Financial Numeracy

Numeracy dimension	Specific challenge	Observed behaviours	Excerpts from this study
Dispositions	Rigid, inflexible problem-solving approaches	Overreliance on one approach without considering efficiency, such as exact calculations using place value partitioning or the written algorithm.	To check how much two items for 1,99 € each cost together, child calculates $1\text{ €} + 1\text{ €} = 2\text{ €}$, $90\text{ ct} + 90\text{ ct} = 180\text{ cts}$, $9\text{ ct} + 9\text{ ct} = 18\text{ ct}$ and afterwards adds these amounts. (E.L. and P.I.)
	Compensation strategies to minimise/avoid mathematical effort	Substituted mental effort with physical effort by going through several iterations of shopping script Taking all desired items without checking sufficiency of money beforehand	Child buys item for 5,99 €, received 4,01 € change, then buys another item for 3,99 €, and receives two cents change. (E. L.) Child buys items totalling at 6,98 € and only notices at checkout that he can't pay for everything. (E.L.)
	Financial values impeding numeracy practices (Value of Money)	Change was not checked for small amounts, as it was perceived as insignificant.	Child buys items totalling 8,91 €, pays with 9 €, and does not check if the three cents change are correct, stating 'It is only a few cents, it is not worth much'. (E.L.)
Financial context	Simplistic interpretation of solutions in context	Applied rounding rules without considering contextual implications, leading to incorrect assessments of sufficiency.	Child estimates total costs by rounding individual prices according to the rounding rules to the nearest euro, interpreting 5 € as sufficient for items costing 1,29 €, 2,39 €, 1,49 €, and 1,19 €. (P.I.)
Critical Orientation	Simplistic Validation of mathematical solutions	Validated decisions based on ease or simplicity rather than feasibility or contextual appropriateness.	Relating to the last example: The estimation is deemed appropriate as it was easy to calculate and meant that money sufficed. (P.I.)
	Only social, no mathematical reasoning.	Relied solely on social reasoning instead of mathematical reasoning/problem-solving.	Instead of checking receipts or substituting a product for a cheaper one, child suggest asking for more money from parents/teacher. (P.I. and E.L.)

Note. For each challenge it is noted in the excerpt whether this observation was made during the hypothetical scenario in the pre-intervention interview (P.I) or during the experiential learning environment (E.L.).

Discussion

Influence of Prior Socialisation and the Role of Financial Experiences

Personal financial experiences can serve as valuable learning tools for children, allowing them to reflect on past decisions, identify mistakes, and differentiate between the price and value of products. However, financial socialisation and prior experiences of students varied greatly, and this diversity must be carefully considered when designing educational interventions or learning experiences. Experiential learning through simulations, such as a classroom store, can create a more level playing field for students to engage in financial

decision-making and to develop foundational skills in a supportive, low-risk environment. Nevertheless, simply providing opportunities for shopping experiences may not be enough to foster numerate behaviours. This was evident from students' self-reports, where, despite recognising the importance of mathematics in managing money, students still engaged infrequently in financial numeracy practices, such as checking change or comparing prices. This finding aligns with the work of Grassmann (2008) who observed that children in Germany have limited opportunities for independent financial decision-making. Even in supportive contexts such as the shopping store, these behaviours do not automatically emerge—particularly when children only accompany their parents rather than shopping independently. Some students in the simulation resorted to strategies that bypassed mathematical reasoning. This tendency was enabled by the realistic environment, which, while engaging, also highlighted further challenges in effectively promoting numeracy through experiential learning. For instance, some avoided mental calculations by making multiple store trips until their money ran out, while others handed all their money to the “cashier” without verifying if it was sufficient. These behaviours echo Brenner's (1998) findings, where children substituted mental effort with physical actions in real shopping contexts. However, unlike Brenner's preschool participants, who likely lacked the necessary mathematical knowledge, the students in this study avoided/did not see any value in applying their existing mathematical knowledge. Such avoidance is particularly concerning in the context of increasing cashless transactions. Without the disposition/inclination to estimate costs or track expenses, children may overspend or accrue debt without realising it, as they cannot rely on immediate cues like running out of physical cash. This highlights a critical limitation of experiential learning: while it encourages engagement, it may not inherently promote robust numeracy practices or a productive disposition towards mathematics.

Challenges in Bridging School Mathematics and Real-World Applications

When students were encouraged to apply mathematics in shopping contexts, several challenges emerged. Certain dispositions, such as inflexible problem-solving approaches, impeded effective financial decision-making in both hypothetical scenarios and experiential learning settings. For example, students often relied on the same strategy for all problems, such as using written algorithms or place value partitioning, without considering number relationships or the context to find more efficient strategies – for example, using an algorithm when calculating $1,99\text{ €} + 1,99\text{ €}$ where an efficient strategy would be $2\text{ €} + 2\text{ €} - 2\text{ ct}$. Financial dispositions also played a significant role in decision-making. For instance, some students dismissed small amounts of money as insignificant and did not check their change, reflecting how their perception of monetary value influenced whether numeracy practices were enacted. Social reasoning often replaced mathematical reasoning. Students frequently relied on parental support or negotiated for more money instead of verifying their calculations. Sawatzki and Sullivan (2017) emphasise that integrating social and mathematical reasoning is essential for fostering balanced financial decision-making, as decisions based on beliefs rather than calculations leave individuals vulnerable. A further challenge was students' tendency to view mathematics as a set of fixed procedures rather than as a reflective and adaptive tool for problem-solving. For example, students applied rounding rules mechanically without interpreting or validating the mathematical solutions in context. Such a procedural focus does not align with the skills needed in contemporary everyday life, including the changing financial landscape, where mental calculation, strategic estimation, and reflective reasoning are far more important (Northcote & Marshall, 2016).

Implications for Design and Conclusion

The findings suggest that while experiential learning engages students, having the experience on its own might not be sufficient in fostering financial numeracy. Thus, in this intervention, experiences were accompanied by reflective tasks, addressing common challenges observed before and during the intervention. These should foster a productive disposition toward mathematics by emphasising its practical value in financial contexts and empower students to approach financial decisions with greater confidence and agency, not only relying on social reasoning. Furthermore, the findings suggest that students should adopt a more critical orientation to how mathematics is applied purposefully in context. Some exemplary tasks can be seen in Figure 1. Unlike lower-order tasks that focus on following predefined procedures (e.g., calculating sum of products using written algorithm), tasks in the intervention were designed to encourage students to share their reasoning and engage in discussions to find and evaluate different approaches or judge the reasonableness of mathematical solutions in context.

Figure 1

Examples of Designed Tasks



Pre- and post-interviews were compared to identify chances and limitations of this design-approach. Key markers of improvements and remaining challenges include:

- Enhanced flexibility: Many students began to demonstrate a more flexible use of mathematics. For example, when calculating prices, they recognised when estimates would suffice instead of always resorting to exact calculations.
- Improved interpretation and validation: More students tried to interpret and validate mathematical solutions in context, though understanding compensation in estimations (e.g., how repeated rounding up affects accuracy) remained challenging.
- Experiential learning and numeracy practices: Students appreciated the hands-on experiences, and some linked them to improved self-efficacy in managing money. Some students reported an increase of personal numeracy practices, such as checking receipts, but for many, engagement was limited by the lack of opportunities provided by parents.

This study underscores findings from the literature that financial numeracy is not automatically improved by merely exposing students to financial experiences. Meaningful integration requires well-designed tasks combining experiential learning with structured reflection to develop flexibility and a critical orientation about mathematics in context. Financial numeracy, in a broader sense, (see Savard & Cavalcante, 2021), involves understanding financial concepts through mathematical investigations and examining their connections to the economic system and other systems (e.g., politics, ethics), encouraging consideration of the social and ethical implications of financial decisions (Blue et al., 2018). Future research will include more pre-intervention interview questions to explore students' understanding of prices and price formation, addressing misconceptions, such as seeing prices as objective measures. Tasks will require critical analysis of price formation, including factors like production costs and profit margins. Post-interviews will assess whether students have developed a better understanding of the subjective nature of pricing and its social implications, laying the groundwork for a critical evaluation of market practices, such as dynamic pricing.

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Planning for Mathematical Problem-Posing: Exploring Practice Through the Lens of Practice Architectures

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Mathematical problem-posing (MPP) offers an alternative to teacher-directed approaches by encouraging students to create and solve their own problems. While MPP is supported by the Australian Curriculum and empirical research has increased over the last decade, implementation in classrooms is still limited. Using The Theory of Practice Architectures (TPA), this paper reports on data from the initial phase of teacher planning for MPP, part of a larger doctoral study exploring teacher practices during MPP and solving. This paper offers insights into the practice of planning for MPP and how the TPA can be used to capture the nuances of how practice is enacted in specific settings.

Problems presented in mathematics classrooms frequently require students to apply particular algorithms or techniques, focusing on obtaining correct answers rather than exploring mathematical concepts (Schoenfeld, 2017). Mathematical problem-posing (MPP) offers an alternative to this more commonly used teacher-directed approach. Although posing problems is recognised as an aim of the Australian Curriculum: Mathematics, as well as being noted in the proficiency strands and under Mathematical Processes: Mathematical Modelling (Australian Curriculum, Assessment and Reporting Authority [ACARA, n.d.]), it is less commonly used in classrooms. While MPP has gained recognition as an essential component of mathematics education research, a Google Scholar search using various terms (mathematical problem solving – ca. 5 460 000; mathematical modelling – ca. 5 050 000; mathematical thinking – ca. 4 620 000; mathematical reasoning – ca. 3 500 000 and mathematical problem-posing – ca. 439 000) illustrates it is relatively underdeveloped when compared to other topics (Rott et al., 2024). Further exploration of this also indicates increasing interest with the number of search hits for “mathematical problem posing” continuing to increase each year (Rott et al., 2024). An increase in research activity has also been noted by Cai et al. (2024) who highlight several recent journal special issues, books and conference papers focused on a wide range of problem-posing topics. However, little research has focused on teacher practices for implementing MPP in the classroom. As part of a larger doctoral study exploring teacher practices during MPP, this paper offers insight into how the practice of planning for MPP, during one ninety-minute teacher and researcher planning session, was enabled and constrained by the practice architectures within this site. It offers suggestions for teachers looking to implement MPP in their classroom.

Background Literature

Problem-posing in mathematics education dates back several decades with researchers and mathematicians recognising that in real-life, problems are generated by the solver (Kilpatrick, 1987). MPP includes the generation of new problems, as well as the reformulation of existing problems (Silver, 1994), and can be categorised into free, semi-structured, and structured problem-posing (Stoyanova & Ellerton, 1996). Free problem-posing, implemented in this study, encourages students to create mathematical problems based on a provided situation (e.g., an image prompt) without limitations (Stoyanova & Ellerton, 1996). Problem-posing literature is generally categorised into three focus areas: as a cognitive activity, learning goal, or instructional approach. In relation to this study, problem-posing as an instructional approach focuses on the methods and strategies that teachers use to engage students in generating their own mathematical problems. Zhang and Cai (2021) analysed 22 teaching cases that focused on (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 485–492). Canberra: MERGA.

the role of the teacher, the types of prompts used, and how teachers responded to students posed problems. They found that most real-life prompts were presented in words and that a similar lesson sequence was used: present the task, students' problem-posing, teacher guidance to solve, summary, and reflection. Jia and Yao (2021) also looked at problem-posing as an instructional approach, examining the inclusion of problem-posing in Chinese textbooks published by the same publisher over seventy years. They found that although still rare, problem-posing activities have begun to be purposefully included in these textbooks. Mathematics textbooks and teaching resources provide teachers with an important tool for preparing lessons, the rarity of inclusion highlights one of the challenges faced by teachers when planning for and implementing MPP. In a longitudinal study, Cai and Hwang (2021) investigated the impact of problem-posing professional development on teachers' conceptions of problem posing, their design of lessons to teach mathematics using problem posing, and the impact on students' learning. While Cai and Hwang recommend ways to integrate problem-posing in classrooms, they do not focus specifically on teacher practice. Whether teachers choose to use pre-designed problem-posing tasks, or design them themselves, innovating their practice in this way presents a challenge. Li et al. (2020) explain that this is likely due to a variety of factors including, limited resources or teaching materials, lack of familiarity with teaching problem-solving, difficulty shifting their practice, and lack of a supportive learning community. Through the Theory of Practice Architectures (TPA) (outlined below), the doctoral study from which this paper is drawn investigates teacher practices in one site for MPP. The following question frames this paper: *How do the practice architectures enable and constrain the practice of planning for problem-posing?*

Research Design

This study utilised a Critical Participatory Action Research (CPAR) methodology (Kemmis, McTaggart et al., 2014) to identify and examine teacher practices used when implementing MPP, and how those practices changed through iterative cycles of implementation. The TPA was chosen as an analytical framework as it considers the interplay between various elements of practice, including cultural-discursive, material-economic, and social-political arrangements (Grootenboer & Edwards-Groves, 2023). It provides a way to capture the nuances of how practice is enacted in specific moments and settings and identifies enablers and constraints influencing educational practices and how they can be transformed (Grootenboer & Edwards-Groves, 2023).

Theoretical Framework

The TPA has been used to examine practices and identify what enables and constrains practices in various fields (e.g., Rönnerman et al., 2023). First theorised by Kemmis and Grootenboer (2008), it is one of many practice theories that focus on “what happens: how life unfolds - and how practices unfold, in the intersubjective space in which we encounter one another and the world” (Kemmis, 2021, p.7). TPA provides a theoretical way of understanding education practices (e.g., teaching), with the discernment that practices are held in place by practice architectures - the cultural-discursive, material-economic, and social-political arrangements and conditions in the site (Kemmis & Grootenboer, 2008). These arrangements and conditions enable and constrain the language (sayings), activities (doings), and relationships (relatings) that form a practice (Edwards-Groves & Kemmis, 2016). The cultural-discursive arrangements exist in the dimension of semantic space and relate to how the distinct language and discourse is situated in and about particular practices, for example, shared technical language, or discourse around inquiry learning. The material-economic arrangements exist in the dimension of physical space-time and involve the kinds of activities or work that occurs within the practice, for example, the physical set up of classrooms. The social-political

arrangements exist in the dimension of social space, how we connect and contest in relation to power and solidarity, and the patterns of relationships with living and non-living things that occur in practices, for example, the relationship between students and teachers or students and a digital mathematics program. Kemmis, McTaggart et al. (2014) explain that it is not these individual elements in isolation that constitute the arrangements and conditions that enable and constrain practices, but how they are shaped by their dynamic interplay as they combine within a site. Whether the site is a lesson, classroom, or school community, people are engaged in particular practices within those sites. In these sites or intersubjective spaces (Grootenboer & Edwards-Groves, 2023), the practices and practice architectures “hang together” with Kemmis, Heikkinen et al. (2014) describing this “hanging together” as ‘the project of a practice’- the melding of practices and practice architectures, explaining that it encompasses the intention that motivates the practice, the actions (sayings, doings and relatings), and the end aims the practitioner hopes to achieve through the practice.

Context and Participants

This study was conducted in a P-12 independent school in South-East Queensland with three Year 6 teachers: Seth (6 years teaching experience), Ella (14 years), Kelly (17 years) (pseudonyms) and their classes. The teaching team was in their fifth year of working together. Ella had previously participated in a research project exploring instructional strategies that supported Year 5 students to problem-pose (Zorn, 2022). In casual conversation after this earlier project, Ella shared the difficulties she faced when attempting to implement the practices in her classroom, particularly the challenges related to posing problems based on a prompt. It was through these discussions that further interest was generated and this CPAR project was conceived.

Data Analysis

Fereday and Muir-Cochrane’s (2006) hybrid approach to thematic analysis was used in the broader study to analyse transcripts from planning meeting observations and audio recordings, video observations of lessons, researcher field notes, and teacher reflections collected over seven months. Using NVivo™ data management system, preliminary themes were identified for the code manual before coding the transcript corpus. During this process, themes were continually refined and adapted based on new insights. Through this process, the practice of planning was identified as a key teacher practice for implementing problem-posing and solving. The second stage of analysis used the TPA (Grootenboer & Edwards-Groves, 2023) as a pre-determined coding system. It assisted the researcher in identifying how the practice architectures enabled and constrained the practice of implementing MPP and problem-solving. The data reported on here draws on the initial, 90-minute planning session. It offers insight into how the practice of planning for MPP was enabled and constrained by the cultural-discursive, material-economic, and social-political arrangements in this site.

Results and Discussion

The teaching practice of planning here focuses on the intentional process of preparing prompts suitable for free problem-posing, anticipating student responses, and considering how ideas are presented. In this section of the paper, transcript extracts will be presented that illustrate the teachers’ sayings, doings, and relatings during the planning process. While the data reported here is not extensive, it provides a glimpse into the process of planning for MPP. An analysis and discussion follow analysing how teacher planning was enabled and constrained by the practice architectures in the site. It is important to note that, while analysing the different architectures of a practice allows for careful examination of the influences that enable or constrain the practice, in reality they are enmeshed and do not operate in isolation.

The Practices of Planning for Mathematical Problem-Posing

During the initial phase of planning, the researcher provided the teachers with a shared folder of images and website links to explore as potential prompts. After a brief look, the teachers selected a ‘lifeguard service sign’ based on its perceived familiarity to the students.

Figure 1

Lifeguard Service Sign



Excerpt 1

Seth : I think maybe the lifeguard thing. Just like they would all be familiar with that, like from living around here.

Ella : And it's summer.

Seth : And there are already identifiable numbers on there and things like that that they can see as a prompt. The only thing is that it is only a picture, is it going to hook them in as much as a video would? But then is it less ambiguous than a video? Like if it was that chicken one then that might be too broad for them whereas...

Ella : Yeah, I like it as a first one because...

Seth : ...just one point of focus.

Ella : ...it's just one like if we want to do just a short one, maybe going over two or three days, I think that's a good one. Kelly agrees, Kelly agrees [said louder], ha-ha [Kelly nodding]. OK. Yeah, cool. Let's do that one, then because they all love the beach. It's summer. Yeah, yeah.

Kelly: And they did beach safety in Year 5.

Once they had selected the prompt, the teachers brainstormed potential mathematical problems that could be generated from the prompt. While the teachers brainstormed together, Ella recorded the ideas. The extracts below illustrate two instances during the discussion that related to potential questions.

Excerpt 2

Kelly: They could look at the range of temperatures.

Seth : Yeah, across a week or a month.

Ella : I've got the prompt as well but keep looking at you.

Seth : There are people in the background too, so they might...

Ella : They could do like a...

Seth : Their question might be, how many people...

Ella : How many go to the beach?

Seth : ...go to the beach in summer? Or people...

Kelly: Well, they could do a survey.

Seth : Yeah, they could do, how many people in our class went to the beach?

Ella : Which one was it? Was it the Broadbeach beach sign?

Kelly: Could they look at wind speed?

Excerpt 3

Seth : Even the stuff like the unstable conditions, what, what's the metric for a lifeguard that makes it pleasant to unstable to beach closed? Like what are the considerations that have to go into that?

Ella : What would you call that? Because it's about, now, it's about, I could call it...

Seth : comparison?

Ella : No, I mean, like when, ah I can't think, when you go from, not a span. It's not mode, it's not median, it's not

Seth : Range?

Ella : Yes, range. So what range is considered hot, warm, cold or dangerous, or whatever. How would we write that?

Seth : What conditions, what range of conditions fits in unstable, what range of conditions fits in dangerous and beach closed? What would be a perfect day? That'd be a good question, what would be a perfect day? Or is there ever a perfect day?

Kelly: And does it depend on what you're looking for? It's very different to the surfer.

The teachers also discussed how to get the students to pose questions, suggesting and agreeing that co-constructing questions with the students may be a good strategy.

Excerpt 4

Kelly: I think you should say, this is an example and then do it collaboratively.

Seth : Maybe it's, this is an example because...

Ella : We could even co-construct.

Kelly: You could even analyse the word choice, this makes it a mathematical question or why does it make it a mathematical question?

Practice Architectures

Cultural-Discursive Arrangements

Cultural-discursive arrangements are realised in the semantic space, encompassing the ways in which language is used to communicate, express ideas, and construct meaning. Examining the language in Excerpt 1, the teachers' understanding of free problem-posing is highlighted. Seth comments that "there are already identifiable numbers on there and things like that, that they can see as a prompt", indicating that the presence of written numbers may make it easier for students to problem-pose in mathematics. Later he asks, "is it going to hook them in as much as a video would?", illustrating his awareness that the prompt is used as a stimulus to provoke curiosity and wonder and can include either a video or an image, accompanying the written text. While Ella's language, "if we want to do just a short one, maybe going over two or three days", suggests that the students may be somewhat limited by the lifeguard service sign, posing problems that could be solved and shared within a short timeframe. Limited experience with having students engage in problem-posing creates a duality, both enabling and constraining the practice of planning. While creating space for innovation by demonstrating a willingness to experiment with a new approach, the practice may also be constrained by the lack of teacher knowledge, preventing the teachers from optimising opportunities or envisioning how problem-posing will unfold in the classroom.

The use of technical language such as "survey, bar chart, mean, median, how many, and comparison" is also noted in Excerpts 2 and 3. This language, and corresponding knowledge of these concepts, enables the teachers to discuss ideas freely, contributing to the ongoing construction of the practice of planning for MPP. However, confusion around mathematical terminology is also evident. For example, while discussing a potential model that describes the different levels of safety and suitability, the teachers use the word 'range' (Excerpt 3) but express uncertainty about its accuracy, which constrains their practice.

Discourse associated with specific pedagogical approaches additionally enabled the practice of planning. The words and phrases used during the planning meeting such as, “workshopping”, “collaboratively”, “co-construct”, “analyse the word choice”, and “a mathematical question” (Excerpt 4) are all used without further explanation, suggesting a familiarity amongst the group in working in this way with their students. The specific pedagogical language enabled the practice of planning and influenced the way in which the individuals interacted and constructed meaning. Additionally, it is reflective of the cultural-discursive arrangements visible at the school, evident on the school’s website, which emphasises practices of “student-centred learning” and an “advisory approach” in relation to the student-teacher relationship.

Material-Economic Arrangements

Material-economic arrangements refer to the physical and economic conditions that exist, or are brought into the site, and they directly influence the types of activities that can occur, concurrently enabling and constraining the sayings and relatings. During this planning session, the seating arrangements (i.e., small group tables), facilitated the collaborative nature of the Year 6 teachers and task (i.e., planning practices). The easy availability of technology in this site, including personal devices and internet access, allowed the teachers to access the suggested websites and prompts saved on the school’s intranet. However, while suggested websites and prompt ideas were provided, teachers were also encouraged to look beyond these suggested ideas. However, despite this encouragement, the teachers chose to limit their search to the provided prompts and made their decision within the first three minutes of planning for the prompt. Although the provided resources enabled quick decision making, this likely constrained the practice of selecting a potentially more suitable prompt that may have been discovered if the teachers searched for prompts beyond those provided. During the planning session, the teachers brainstormed questions while Ella volunteered to record the responses using her laptop. While the use of the devices, and a shared file of resources facilitated the practice, it simultaneously constrained it. During the discussion Ella made two comments, “I’ve got the prompt as well but keep looking at you” and “Which one was it? Was it the Broadbeach beach sign?” (Excerpt 2) indicating that she found it difficult to look at the prompt while simultaneously recording and contributing to the discussion. Consequently, having the shared digital file with the prompts and planning document, and trying to view it on one personal device, constrained the practice.

Social-Political Arrangements

Social-political arrangements, highlighted by the relatings among individuals and groups, play a crucial role in influencing how practices are developed and sustained within a particular site. The familiar relationship and social dynamics between the teachers observed throughout the project, and the ease of decision making as evidenced in Excerpt 1, supported the teachers to collaborate during planning sessions. However, within this excerpt, Seth and Ella are engaged with the task of selecting a prompt, while Kelly appears to be distracted. Ella brings Kelly back into the conversation in a direct but friendly way as represented in the language used by Ella, “Kelly agrees, Kelly agrees [said louder], ha-ha [Kelly nodding]”. This re-engages Kelly into the conversation and the collaborative session continues. While in some circumstances, being distracted or having this interaction may constrain the relationships within the group and practice, here it did not appear to shift the dynamic or flow of the practice. Additionally, in the initial segment of Excerpt 2, the teachers are seen to overlap in conversation and pick up where the other left off, and in Excerpt 3 they support each other with knowledge building and recording. This is noted when Seth offers suggestions to Ella when she is trying to remember the concept of range and asks, “how would we write that?” referring to documenting the discussion, and again in Excerpt 4 when the teachers openly discuss and share ideas. As noted in researcher field notes, and evidenced in meeting transcripts, the Year 6 teachers openly

acknowledged not knowing exactly what to do at times, which perhaps indicated they felt comfortable seeking one another's opinions and advice. Here, positive group dynamics enabled collaborative practices, and these in turn helped to sustain the practice of planning. However, positive group dynamics can sometimes constrain practice, leading to 'groupthink' or creating pressure for group members to conform to maintain cohesion.

Conclusion and Implications

Examining the practice of planning through the lens of TPA reveals that the practice, which is site based, does not happen in isolation; instead, it is embedded within broader cultural, material, and social contexts. For example, the practice of planning in this instance required the teachers to work together, which is both influenced by, and held in place by, the practice architectures. While the teachers' limited knowledge of MPP potentially constrained them from optimising opportunities or envisioning how things could unfold in the classroom, the shared pedagogical knowledge (cultural-discursive) and positive group dynamics (social-political) supported the practice of planning, providing space for innovation. The unfamiliarity with MPP and their familiarity with each other, may have also contributed to the decision to quickly choose one of the provided prompts. The material-economic arrangements (provided shared file of resources) were seen to be held in place; supported and constrained simultaneously by the cultural-discursive and social-political arrangements.

By examining practices through the TPA, the researchers and practitioners gained a holistic understanding of the interconnectedness of practices and, consequently, a better understanding of the dynamics involved. As the complexities and nuances of the practice are revealed, informed interventions and changes can be made to improve practices over time. Focusing solely on the practice, without focusing on the factors or arrangements that enable or constrain it, is likely to lead to that practice being unsustainable. While only a small subset of data is explored here, this limitation does not diminish the validity and contribution to meaningful insights into using the TPA in mathematics education to examine how practices unfold within a specific site. Additionally, it provides insight for teachers looking to implement MPP in their classrooms. Shifting pedagogical practice is challenging (Fry et al., 2025), and while it may feel daunting to find the time and energy to do so, it has been illustrated here that prior experience, pedagogical and content knowledge, and healthy interpersonal relationships support the process of planning when innovating practice. More generally, it is important to take stock of available resources, both physical and human, seek the opinions of others, and be open to change. In relation to free MPP, the teachers highlight the initial steps of finding a prompt which their students will connect with, brainstorming mathematical connections, questions, and concepts, and most importantly, the importance of getting started with MPP, even when unsure of all the next steps.

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Round Tables

Key Features of the Mathematics Growth Team Program: Insights From an Academic Partnership

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In 2024, University of New South Wales, Sydney (UNSW) and New South Wales (NSW) Department of Education (DoE) collaborated to evaluate the Mathematics Growth Team (MGT) program, a multi-phase professional learning (PL) program funded by the NSW DoE. This research partnership determined the most impactful program features.

Since 2020, the MGT program has aimed to improve mathematics student outcomes through expert mathematics teachers in a school-based professional learning (PL) approach. An extensive body of research argues that effective PL should be embedded in practice, enable teachers to develop their knowledge and application of new ideas (e.g., Ball & Bass, 2003; Durksen et al., 2017; Kennedy, 2016), and be tailored to individual needs for successful implementation (Byman et al., 2021; Desimone & Garet, 2015).

The MGT program supports sustained, dynamic and agentic PL beyond expert teachers' involvement. Key features include school-based coaching roles, dedicated time for teachers and experts, and tailored support for developing mathematics teaching practice. These elements are essential for strong professional communities (Coburn & Russell, 2008; Munro & Campbell, 2022).

In this round table we will present key features of the MGT program spanning five different topics. After a brief overview of the five features of the MGT program, participants will engage in an interactive session to share insights our combined insights on practical recommendations based on the findings. We will conclude by inviting participants to contribute ideas for sharing and applying these findings within the mathematics education community.

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Conventional Practices of Mathematics Vs. Deliberate Practice

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Conventional models of mathematics practice in schools involve the rigid application of a procedure that had been previously learnt to solve similar questions (Lehtinen et al., 2017). Questions are usually drawn from sources such as textbooks and students are considered to have to follow and memorise the procedure previously demonstrated by the teacher (Nachlieli & Tabach, 2019). The general belief is that practice helps students to automate solving non-routine problems (Garelick, 2013). Teachers who follow this model of practice will likely find it difficult to create practice resources that go beyond the use of a particular algorithm to solve questions (Bridge et al., 2012), and this will limit their professional capabilities in the long term. Moreover, these practices impact students' perceptions of mathematics as they will become to believe that mathematics understanding is all about getting a correct answer after mimicking the teacher (Howard & Whitaker, 2011). If students' mathematics understanding is to go beyond automatised basic skills, then practice should focus on developing adaptive skills and relational understanding not only routine skills (Schmidt & Glaser, 2021). Such practice models, for example deliberate practice, can provide opportunities for continued improvement and attainment of deeper mathematical understanding (Han & Paine, 2010); nevertheless, there is limited research on deliberate practice in mathematics education (Lehtinen et al., 2017).

Deliberate practice seeks to develop experts in domains through building skills, competences and knowledge. In this round table, we will introduce the notion of deliberate practice, its intentions and mechanism. The participants then will be invited to discuss strategies on how deliberate practice can support and inform conventional practice models in mathematics education.

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How Do People Learn Mathematics?

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Recently we made the claim that learning is about being “stirred into practices”, or learning “how to go on in practices in the world” and this sees learning as something that is happening all the time in particular sites in time and space (Kemmis, Edwards-Groves, & Grootenboer, 2024). This means that learning is coming to practice differently. This ontological practice view of learning provides a perspective that is situated in the everyday happeningness of life (Grootenboer, 2022). Thus, learning is consequential for the places we inhabit as well as for individuals, and “learning is ontologically realised, shaped, and consequential in histories, in lives, and in sites” (Kemmis et al., 2024).

In this view, the product of mathematical learning (e.g., knowledge), is not understood as existing (e.g., in memory) separate from the practicing of mathematics – learning is about “ontological transformation”. Fundamentally, this means coming to practice differently in the sites we inhabit. Consequentially, “what is conventionally called ‘knowledge’ arises from, recalls, anticipates, and returns to its use in practices” (Kemmis et al., 2014, p. 58). This perspective would see learning as being fundamentally concerned with enabling learners to *go on* mathematically and be function members of the broad mathematics *community of practice*. This does not usurp the significance of mathematical knowledge, but rather sees it as emerging from, integral to, and realised in mathematical practice.

In this roundtable we will discuss in an open and robust manner the following questions:

- Is this conceptualisation of learning sound and/or useful?
- What might be the implications for mathematics education?
- What might be the implications for mathematics education research?

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What Matters When: Establishing a Value Proposition for Mathematics

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The pursuit of effective mathematics education is perpetually challenged by the tension between aspirational learning goals and the limitations of finite resources. This is particularly evident when a new curriculum introduces fresh demands on teachers, who must navigate these changes within existing resource constraints. Crucially, this balancing act shapes the mathematics curriculum identity of the teacher and curriculum leader. Curriculum leaders face the challenge of supporting this change, while simultaneously identifying and providing the necessary resources for curriculum implementation, thereby reinforcing and reshaping this identity. Similarly, at a system level, the support available to teachers remains finite, necessitating strategic decisions regarding resource allocation and prioritisation. This session will focus on the crucial process of establishing a strong value proposition for mathematics, investigating how perceived educational needs shape the prioritisation of mathematical content and skills across these levels. A key focus will be determining “what matters when” at each stage of learning, ensuring consistent application of value across varied educational settings.

The Shape of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2009) outlined distinct content emphases across the learning phases, each with a stated purpose: foundational learning in the early years, extending foundations in the upper primary, abstract reasoning in junior secondary, and career-focused pathways in senior secondary. However, the extent to which these purposes reflect a robust value proposition for mathematics and effectively connect learners to learning remains a critical question. Does a cohesive mathematics curriculum identity emerge from these phases, or are they simply reflective of the broader purpose of schooling at that stage of schooling?

This round table aims to collaboratively explore the challenges in establishing a cohesive mathematics curriculum identity across schools. We will critically examine whether a unifying value proposition can be developed to transcend the stated purposes of mathematics learning and effectively reconnect with the formation of mathematics curriculum identity. Participants will collaboratively examine how perceived educational needs shape the prioritisation of mathematical content and skills. Furthermore, we will consider how resource allocation and curriculum implementation shape mathematics curriculum identity, drawing upon research that highlights the impact of teacher beliefs (Handal & Herrington, 2003). Participants will be encouraged to share their experiences and perspectives, contributing to a shared understanding of how to effectively manage the complexities of establishing and maintaining a robust value proposition for mathematics education within resource limitations.

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Teacher Content Knowledge for Teaching (Algebra)

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Over the years, there has been global interest in pedagogical content knowledge (PCK) (see Depaepe et al., 2013). Considering that mathematics content knowledge (CK) can impact significantly on PCK and on the capacity to teach mathematics effectively, I believe teacher CK requires equal attention. However, there is much to be known in our context (Australia) where mathematics teachers' CK has been less studied. In their investigation into the extent to which lower secondary pre-service teachers' school mathematics level influences their mathematics dispositions and pedagogical views, Lowrie and Jorgensen (2016) made mention of a notion: "the 'silencing' of content knowledge". The authors claimed that the research in this field has been largely focused on teachers' PCK; this "PCK fever" (p. 205) has drawn attention away from the role of CK.

Research studies have examined the CK of secondary mathematics teachers' (both pre-service and practicing) in the context of solving algebraic word problems involving simultaneous equations (e.g., Hatisaru et al., 2025; Hatisaru et al., 2024). Valuable insights into the strengths and limitations of teacher knowledge for teaching algebra have been gained, through analysing teachers' solutions to purposefully selected problems and their reflections on teaching strategies. These studies propose an operationalisation for examining teacher CK that extends relevant prior work and offer a tool for future research on mathematics knowledge for teaching.

In this round table I will present the reflection forms and analysis methods used in these studies that can be adapted for use in teacher education courses, professional development workshops, and future research on teacher knowledge. The participants will reflect upon ways in which using the structured reflection forms, and associated data analysis approach, may be useful in their own research. The round table will then move into discussion about further potential lines of development—for examining CK of large samples of teachers and investigating the associations between teacher CK and student mathematics learning, how others could get involved, and possibilities for collaborative research.

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Mathematical Cognition and Learning

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There is a lot of rich mathematical cognition and learning research occurring in Australia and New Zealand. The aim of this roundtable is to bring this research together in an effort to develop community and formal infrastructure to raise the impact of this work. The roundtable will explore current mathematical cognition and learning themes in Australia and New Zealand, develop a network to identify where there might be critical masses of researchers, and consider how can we support this research and collaboration both nationally and internationally. In particular, we are interested in supporting complimentary and overlapping areas of research that can drive synergy and joint initiatives between MERGA and the Mathematical Cognition and Learning Society (MCLS).

MCLS's mission is to "advance the interdisciplinary scientific investigation of mathematical cognition and learning across countries, cultures, and communities, to disseminate the resultant knowledge, and to promote its application globally for improving the learning of mathematics and enriching its development" (MCLS, 2015). This roundtable may interest researchers working in a wide range of areas, such as embodiment, representation, visualisation, fractions, patterning, working memory, math anxiety, math learning difficulties, and assessment. This session invites participation from any researcher interested in any areas related to mathematical cognition.

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Assessment to Support Learning in Mathematics

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The potential of the effective use of formative assessment strategies to improve student outcomes has been well established (Black et al., 2004; Black & Wiliam, 1998) along with the characteristics and principles of effective assessment to support learning (Assessment Reform Group, 1999). By informing teachers and students about learning progress, assessment can be used to enable students to regulate their learning and teachers to respond to the learning needs of their students. Assessment can support student to become independent learners able to draw on their learning skills beyond the confines of formal education (Boud, 2000).

Despite acknowledgement of the importance of assessment to support learning in educational policies internationally, uptake of alternative assessment at the classroom level remains limited. In this Roundtable we will explore some of the challenges to implementation of alternative assessment strategies that are particular to the secondary mathematics classroom. We invite discussion around the 4 themes:

1. What challenges do mathematics teachers face when choosing assessment strategies?
2. My research has shown that teachers frequently refer to formal tests and topic tests when discussing the assessment that they use in their classrooms. What factors drive their focus on this assessment type?
3. The benefits of student self-regulation are well recognised but these are not often reflected in approaches to assessment. What are mathematics teachers experiences of peer and self-assessment and what are their beliefs about the potential of peer and self-assessment to improve learning?
4. Are there subject specific nuances that make some forms of assessment more or less valuable in mathematics classrooms?

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Future Directions: Feedback and Reflection Cycles With Pre-Service Teachers to Enhance the Planning of Rich Mathematics Learning Experiences

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Student success in teacher education requires providing rich learning opportunities, authentic assessments and strong relationships between experiences, assessment and feedback. Research shows that reflective practice is vital in teacher education as deep reflection is shown to improve professional practice and transform practice (AITSL, 2013). Cavanagh (2021) in their study on pre-service teacher noticing, reflection and feedback, implies that oral and written reflection after their lesson planning helps pre-service teachers review what happened providing them opportunities to develop a plan for future action, tying into the notion of ‘feedforward’ (Price et al., 2010), allowing pre-service teachers to analyse their teaching, connecting theory to practice. Our study developing feedback and reflection cycles, now in its 5th year, was conducted with 2nd year pre-service teachers. We aimed to explore how three cycles could influence lesson planning in a primary mathematics curriculum studies topic, each prompting reflection and refinement of the lesson they designed. The final feedback and reflection cycle included teaching the lesson to children on campus and more recently in schools, with feedback from classroom teachers. Models of reflection to help articulate thoughts around feedback have been developed over time and are still being refined each year (Rowtcliff & Garnett, 2023).

In this Round Table discussion, we will present an overview of the three cycles along with the development of our reflection prompts and the logistics of children being taught on campus versus in schools. Participants will be invited to share their experiences of sustainability with partnerships between schools and universities with the aim of developing future directions with the wider mathematics education community in order to enrich the experience of preservice teachers, building confidence in teaching mathematics before their formal placement.

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Mathematics Leadership During Educational Reform: Navigating Pedagogical Tensions With MERGA Support

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The growing interest in mathematics leadership in schools as a field of research within MERGA recognises the pivotal role that mathematics leaders play in leading and sustaining professional learning and practice development. Recent educational reforms across Australia have intensified a national push for explicit teaching before applications of learning across all curriculum areas, including mathematics. This shift in pedagogical emphasis presents unique challenges and opportunities for primary school mathematics leaders.

As middle leaders, mathematics leaders are positioned at the intersection of policy implementation, research translation, and classroom practice (Grootenboer, 2018). The current reforms raise questions about how leaders can maintain enactment of mathematics education research-informed approaches whilst serving accountability measures that mandate the use of explicit teaching before problem solving and reasoning. This is concerning due to the lack of empirical studies (outside of cognitive science and educational psychology contexts) about the impact of only using explicit teaching when teaching mathematics in classroom settings.

Some of our preliminary observations suggest that mathematics leaders in primary schools face significant pressure to implement government and sector mandates that emphasise explicit teaching before the teaching of and for problem solving and reasoning. These mandates are surfacing what we believe to be tensions in mathematics leadership practice.

As part of this round table, we invite MERGA members who work directly or indirectly with mathematics leaders to discuss how we might support leaders during these times of reform. We seek to identify how the MERGA community might leverage its collective expertise to mediate mathematics leaders' capacity and capability to use mathematics education research and MERGA's *Position Statement on Effective Mathematics Teaching* as leadership tools as they navigate pedagogical tensions they are facing within their leadership practice.

Our questions for this session include:

- From your experience, how are mathematics leaders navigating the tension between mandated explicit teaching and evidence-informed practice? How are we noticing this phenomenon from the perspective of mathematics leadership?
- What strategies might support mathematics leaders to access and use mathematics education research beyond politically supported evidence sources?
- In what ways can the MERGA community provide practical support for mathematics leaders as they navigate pedagogical tensions mediated by "explicit teaching" before problem solving and reasoning as mandated by governments and sectors?

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Students as Subjects of Their Mathematical Activity: Revisiting Biesta

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We follow up on our 2024 round table in which we opened a conversation on theorising mathematics education with Gert Biesta, specifically through the ideas included in his 2022 book *World-Centred Education*. Many MERGA community members strive for mathematics education in schools, where specific ways of doing, communicating, and looking at the world—rather than merely assessment outcomes—would be central to students ‘becoming mathematical’, and where criteria of judging success would foreground beauty and intricacy of one’s ideas. We want to use this round table as part of our efforts of exploring how the language and theory grounding offered by Biesta could be helpful in pursuing mathematics education research and in promoting mathematics education of the nature described above.

We thus consider some of the core questions, including *What is education for?* (Biesta, 2020, 2022) and provide a brief introduction to Biesta’s argument about three domains of purpose in education: *qualification*, *socialisation*, and *subjectification*, which are currently not all attended to overtly in how education systems are organised. Subjectification, Biesta (2020) argues, is the most important of the three domains, because of the impact education can have on a young person. It is the realisation of young people as subjects of their own lives rather than the objects of the actions and desires of others.

What does it mean to do the work of awaking/provoking in children the aspiration to be a ‘subject’ in their *mathematical activities*, rather than being an ‘object’ responding to the requests of others? What consequences may students’ subjectness (or lack of it) have for their mathematical learning? Biesta connects the notion of one’s subjectness to the notion of one’s freedom, and responsibility, to make choices, but also points out how being an object brings one a comfort and security of not having to make them. The experiences of uncomfortable freedom of making choices are well familiar in mathematics classrooms, and students’ acceptance of and comfort they find in being mere objects of what others want them to do is present in every “Can you just tell us how to do this?” request.

In the session, we will consider, and seek, illustrations from mathematics classrooms, which could help us to ‘put the finger on’ subjectification. We will invite discussion about how the notions well established in mathematics education literature attend to the same, or similar, phenomena. By opening these discussions, we hope to help participants to identify researchers with similar interests that could potentially form ongoing collaborations.

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Experiences With Publishing in High Quality Journals

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There is little doubt that writing with the intention of publishing is one of the core practices in our field. As a result, concerted attention is devoted to supporting HDR students' and early career researchers' writing, as well as to establishing peer-based writing support structures. At MERGA46 last year, we launched a round table focused on early (and not-so-early) career advice on writing and publishing as an additional means of supporting academic writing within the MERGA community; This year's round table is organised by Vince Geiger, the Editor-in-Chief of Mathematics Education Research Journal, in collaboration with the MERGA Vice Presidents for Publication (Jana Visnovska) and Development (Kristen Tripet).

In 2024, we focused on reviewing and learning to write adequate reviews as an intentional path to becoming a better writer. This year we will focus on some of the processes that are a part of the publication journey. We will attend to understanding journal quality (e.g., Nivens & Otten, 2017; Williams & Leatham, 2017), including its counterpart—which journals to be weary of, and how to tell the difference. We note that the timeline to publication as well as the style of the journal review process differ across journals, and discuss how insights on these characteristics may matter in identifying the right journal for a specific manuscript.

Drawing on the experiences of (and as) journal editors we will talk about when a submission is most likely to be successful (e.g., Simon, 2004), overview the most frequent mistakes emerging authors make, and share advice on how to avoid or work through some of these. In addition, we will touch on extending MERGA Proceedings publications for submission to stronger outlets, and on support available to authors for whom English is a non-dominant language (Geiger et al., 2022). We will invite emerging researchers who have recently succeeded in getting published in some of the high-quality journals to share their experiences and advice. Participants will be invited to share their questions and suggestions related to writing to publish with the aim of building the strength of our community.

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Expanding MERGA's Position Statement on Effective Mathematics Teaching Special Working Group: Effective Pedagog(ies) in Mathematics

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In February 2025, MERGA published the Position Statement on Effective Mathematics Teaching, affirming that effective mathematics teaching requires multiple pedagogical strategies, used in response to student needs, local contexts, and learning goals. This Round Table will engage in critical dialogue about the future research directions emerging from this position statement and the development of a position paper.

This round table invites the MERGA community, including but not exclusive to the regular Special Working Group (SWG) members, to contribute to the scholarly conversation following the release of MERGA's Position Statement. We aim to generate a robust research agenda that advances understanding of effective mathematics pedagogies and informs evidence-based practice across the educational landscape.

In lieu of the MERGA SWG monthly meeting, this interactive session will facilitate scholarly exchange through three key activities:

- **Position statement implications:** Discussion and analysis on how the Position Statement aligns with or challenges current discourse in mathematics education research and policy. Participants will identify tensions and opportunities to extend theoretical understandings of effective pedagogies beyond dichotomous perspectives.
- **Further research considerations:** Participants will collaboratively identify critical research gaps and potential collaborative initiatives that could strengthen the evidence base for effective mathematics teaching in Australasia. Discussion will focus on innovative methods for investigating the interplay between multiple pedagogical strategies and their impact on mathematical understanding across diverse contexts.
- **Position paper development:** The session will culminate in a planning and scoping discussion for the accompanying position paper. Participants will have the opportunity to self-select into a focus group aligned to research interests to effectively capture the complexity of responsive pedagogical approaches in mathematics education. We will explore opportunities for a series of journal articles to accompany and illustrate the key ideas articulated in the position statement, providing exemplars and theoretical depth.

By engaging the collective expertise of the MERGA community in this round table, we aim to strengthen the impact of the Position Statement and contribute to a more nuanced understanding of effective mathematics teaching that moves beyond dichotomous debates toward evidence-informed, contextually responsive pedagogical approaches.

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Embedding the Computational Thinking Process in Mathematics

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Computational thinking in the last decade has become a key component of mathematics curricula, and assessment frameworks across the globe. Although considered a relatively new area of research, early studies internationally have linked computational thinking to the mathematical modelling process, supporting productive dispositions, extending mathematical exploration and providing a means for deepening students understanding of mathematical structures.

In the Australian Curriculum Version 9.0 (ACV9) computational thinking, coined by Wing (2006), is recognised as both a core concept and way of thinking that draws on various skills and practices developed through the learning of specific curriculum content within the learning areas of Mathematics and Digital Technologies (Australian Curriculum Assessment and Reporting Authority [ACARA], 2022). The ACV9 also recognises computational thinking as a mathematical process that can be used to investigate, experiment, and solve problems that draw on students' proficiency in mathematics and the use of algorithms and digital tools (ACARA, 2022; Whitney-Smith, 2023). The Programme for International Student Assessment (PISA) 2022 Mathematical Literacy Assessment identified computational thinking as a key component of mathematical literacy in the 21st century (OECD, 2018). The draft framework claims "By foregrounding the importance of computational thinking as it applies to mathematics, the framework anticipates a reflection by participating countries on the role of computational thinking in mathematics curricula and pedagogy" (p. 5). Rycroft-Smith & Connolly (2019) posit that there is an interrelationship between computational thinking and mathematical thinking, with both being considered forms of abstract problem-solving approaches with commonalities and overlap. Little is known about Australian mathematics teachers' beliefs, attitudes, or practices concerning the use of computational thinking in mathematics.

This round table will provide an opportunity for rich discussion on ways to support mathematics educators, classroom teachers and resource developers to understand the importance of providing opportunities for students to develop and draw on their computational thinking skills when learning mathematics. I invite participants to share the foreseeable challenges schools and jurisdictions may face in implementing this area of the curriculum, including the need to build teacher capacity, and discuss ways in which ACARA could provide curriculum implementational support. I also encourage discussion about what areas of research may be useful to inform future resource and professional learning development.

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Short Communications

Transfer From Spatial Training to Mathematics Achievement: A Qualitative Approach to Exploring Mechanism

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Many studies have demonstrated that spatial reasoning is malleable and that improvements in spatial reasoning transfer to mathematics (Hawes et al., 2022). A recent review of the spatial training literature (Resnick & Adams, 2024) found that most studies explain their findings with respect to underlying cognitive skills, suggesting that transfer is achieved through improvements in directly applicable spatial skills (e.g., Cheung et al., 2020). Only two studies considered other pathways, such as changes in students' strategic approach (Lowrie et al., 2019; 2023). Despite increasing interest in explaining why spatial training supports mathematics, most studies did not examine mechanism (Resnick & Adams, 2024). This may be driven, in part, by the quantitative experimental methodologies employed (Young et al., 2018).

The present study utilised qualitative methods to examine Year 11 mathematics students' strategic approaches toward solving mathematics problems before and after completing a spatial training intervention. Students' work samples and verbal explanations were first used to develop response maps (Stillman & Galbraith, 1998) outlining students' solutions and coded based on the strategic approach employed. Subsequently, students' cognitive and metacognitive behaviours were coded using Garofalo and Lester's (1985) framework. Improvements were observed in students' ability to correctly encode spatial information. Furthermore, students' improved representational skills helped them to avoid unhelpful, naïve, non-spatial strategies.

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Justifying Fraction Ordering: Language in Student Reasoning

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Justifying, as a process of reasoning (Jeannotte & Kieren, 2017), is a language-based action (Bragg et al. 2016), making language a crucial aspect of reasoning. Chow and Jacobs (2015) emphasised the role of language in students' performance with fractions. They highlight fraction language as a predictor of students' future learning with mathematical registers essential for understanding fractions. Regarding mathematical language, Pimm (1988) provided insight into an everyday language as a mathematical metaphor. The extended use of everyday words can bring forth new meanings, shaping mathematical language metaphorically. In this regard, we seek to examine how students use both mathematical and everyday language when working with fractions and explore how these linguistic resources are interconnected with their reasoning processes.

We designed a written task on fraction magnitude comparisons for students aged 11-12 years. The task required students to order six fractions with different denominators from smallest to greatest and explain their response. In this presentation, we examine 18 students' written responses to the task. We initially observed the use of fraction-specific vocabulary, such as *denominator* and *numerator*. Many students also relied on everyday language expressions, such as *the top*, *the bottom number*, and *how many pieces*, instead of mathematical terminology. Common nouns related to fraction comparisons included *common denominator*, *simplification*, *smaller*, and *biggest*. Pimm and Wagner (2003) described a tendency in mathematical writing where actions and processes become nominalised – that is, turned into nouns. Borden (2011) further argued that an extensive use of verbs can indicate a focus on the act of thinking itself rather than what we think of. Building on this, we also paid attention to action verbs and phrases, given that justifying is a reasoning process. Three categories of verbs emerged. One group is fraction-related verbs, including *simplify*, *time*, and *make equivalent*. Another type is descriptive verbs illustrating their reasoning actions, such as *I counted*, *shaded out from*, *I wrote down*, and *I changed back*. Lastly, we identified meta-level verbs that reflect metacognitive aspects of reasoning, such as *which gave me this*, *drawings help me see*, *I realised*, *I had a mistake*, *I decided to*, and *because it is easier*. Based on these initial findings, we will share more students' language and open a space to explore analytical approaches for interpreting the language.

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Primary Teachers' Self-Efficacy and Outcome Expectancy in STEM Education

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Effective STEM teaching relies on teachers' self-efficacy, impacting teaching approaches (Bandura, 1977). Despite the importance of enhancing teachers' confidence, a research gap exists in understanding teachers' perceptions across Australia. Previous research established a positive correlation between teacher confidence and instructional performance (Nadelson et al., 2013; Margot & Kettler, 2019). This study examined 87 primary teachers across Australian jurisdictions, investigating their self-efficacy and teaching outcome expectancy in teaching STEM using the Teacher Efficacy and Attitudes toward STEM (T-STEM), a survey that provides insights into educators' self-efficacy/beliefs and teaching outcome expectancy (Friday Institute for Educational Innovation, 2012). The findings revealed variations in participant responses across career stage (i.e. Graduate to Lead), across all four STEM disciplines. The findings demonstrated varied confidence levels when teaching Science, Technology, Engineering and Mathematics as standalone subjects, compared to implementing STEM as a transdisciplinary approach (English, 2016; Kelley & Knowles, 2016). The study's findings also revealed several distinct pedagogical approaches employed by teachers when delivering STEM instruction (Radloff & Guzey, 2016). Analysis of teaching outcome expectancy showed variations in beliefs about the impact of effective teaching on student achievement (Siregar et al., 2019). This presentation will include a discussion of the study's findings, and their implications for the current STEM landscape across Australia.

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Spatial Visualisation Difficulties in Area of Composite Figures With Quarter-Circles

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Students face difficulties solving area of composite figures when the region which they are required to work out could not be solved by simply using the formula. Researchers such as Spiegel and Ginat (2017) attribute this difficulty to students' unfamiliarity with the diversity of composite shape configurations and low spatial visualization skills. Battista (2007) describes spatial visualisation as the ability to 'see', inspect, and reflect on spatial objects, images, relationships and transformations. As put together in a study by Harris, Logan and Lowrie (2023), students with low spatial visualization skills would therefore produce more errors, despite adequate procedural knowledge.

In our study, we aimed to gain insights into students' spatial visualisation difficulties and how these difficulties hindered them in solving an area of composite figure task involving overlapping quarter circles. As part of a preliminary study, 15 tertiary students from non-mathematics and non-science courses were recruited to complete tasks to find the area of composite figures involving semi-circles and quarter-circles. The tasks were adapted from Grade 6 school textbooks as it is the year which Singapore elementary school students are first introduced to the concept of the area of circles. In order to successfully solve the composite area task examined in the current study, participants had to recognize the relevant shapes which, when rotated and inverted, have the same shape and area. The task also required the participants to compose and decompose a composite shape as a part-part-whole structure. Students also had to recognize the shapes or objects as independent as well as combined units.

Findings of the study revealed that for one of the questions with overlapping quarter-circles in a square, some of the participants were able to recognize the common shapes and therefore had partial progress in working out the relevant areas. However, they experienced some difficulty in decomposing and composing the composite shape as a part-part whole structure and were hence, unable to progress in their solving. Instances of participants' solutions for this particular task would be shared during the presentation, which shapes our recommendations for classroom teaching so as to develop students' spatial visualization skills.

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Assessment for Learning in Mathematics: Expert Teacher Reflexivity

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Assessment for Learning (AfL) is recognised as one of the most powerful approaches for supporting student learning. It features in a range of curriculum and policy documents; however, adoption remains a challenge (Marshall & Drummond, 2006). Broad pedagogical elements have been described in the literature, but effective AfL also relies on teachers' reflexivity that stems from deep knowledge, noticing and decision-making skills which can be difficult to identify and implement (Black & Wiliam, 1998; Heitink et al., 2016). Furthermore, AfL practices are likely to look different depending on the context (Hodgen & Marshall, 2005), indicating that more discipline-specific professional learning inquiries into AfL are needed to develop teachers' capacity for implementation (Black & Wiliam, 1998).

This study is part of a larger investigation into opportunities for cross-disciplinary learning based on discipline-specific AfL practices in English and mathematics. It aimed to identify and describe mathematics AfL practices and the teacher noticing and decision making that supports effective implementation. The participants included an experienced high school mathematics teacher and his class of year 8 students in a public high school in metropolitan Queensland. Data included video footage of an AfL cycle in the classroom and recording of a video-stimulated interview with the teacher after the event. Adopting Archer's three dimensions of reflexivity (2003) and Mason's teacher noticing framework (2001), the interview protocol prompted the teacher to notice AfL-related actions and identify decisions made prior to the lesson and in-the-moment before reflecting on their impact.

The data revealed how an expert teacher develops students' capacity to provide peer feedback by illuminating quality in mathematics that goes beyond a correct answer. This is significant since providing effective peer feedback is a complex AfL skill that relies on the provider's domain knowledge and, without training, is often limited to the correctness of an answer (Hattie & Timperley, 2007). Teacher reflections unveiled critical insights into the social safety that underpins the development of novice learners' evaluative expertise in mathematics. This study highlights AfL practices in mathematics and will inform resources for cross-disciplinary investigations and teacher learning.

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An Investigation of Secondary Mathematics Teachers' Implementation of Challenging Tasks

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Although mathematics education has traditionally entailed a passive acquisition of knowledge (Bird, 2023), this approach no longer meets the needs and demands of an “increasingly volatile, uncertain, complex and ambiguous” world (OECD, 2018, p. 3). Over the last few decades, mathematics education has responded to these changes through reforms in mathematics curricula to emphasise 21st century priorities such as reasoning and non-routine problem solving (Goos et al., 2020). Research indicates that challenging tasks in mathematics are an effective medium for developing these higher order skills, while preparing students both academically and holistically for life (Sullivan et al., 2012). Despite this, there is limited research on challenging tasks in the secondary context. To date, research has focused on challenging task interventions, related teaching strategies and their impact, primarily in primary education settings (e.g., Clarke et al., 2014; Russo & Hopkins, 2017).

Therefore, this research will focus on secondary mathematics teachers' implementation of challenging tasks through an investigation of their conceptions of and orientations towards these tasks. Adopting an exploratory design using both qualitative and quantitative data sources, this study will use an online questionnaire ($n = \sim 100$, Stage 1) and semi-structured interviews ($n = \sim 15$, Stage 2) to help understand how these conceptions and orientations influence teachers' pedagogical decisions to implement or not implement challenging tasks. Classroom observations at case study high schools ($n = \sim 3$, Stage 3) will be conducted to observe challenging tasks in action. Bronfenbrenner's (1979) Bioecological Systems Theory will be used to identify and categorise enablers and barriers to the implementation of challenging tasks that emerge from the triangulated findings. The results of this study may provide insights into the successful implementation of challenging tasks in secondary mathematics classrooms and help develop recommendations on multiple system levels to support implementation.

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Illustrations of Mathematics Instructional Coaches' Routine and Adaptive Expertise for In-Service Teacher Professional Development

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Instructional coaching has become a common practice internationally as a form of high-quality professional development for teachers (Desimone & Pak, 2017). Beyond the literature on mentoring and inducting pre-service and beginning mathematics teachers, instructional coaching for in-service teachers has been shown to be a valuable vehicle for diffusing innovative teaching practices and instructional reform (Hopkins et al., 2017). Thus far, research on coaching in-service mathematics teachers has focused largely on its effects on improving teachers' practice and student achievement (Panfilio-Padden et al., 2025). *But what makes for effective coaching, and what does it look like during instruction?* Studies have aimed to distil the knowledge domains involved in coaching expertise, the nature of professional development for instructional coaches, and the wider contextual factors that may influence the dialogue that occurs between coaches and teachers (e.g., Saclarides & Kane, 2021; Sutton et al., 2011). A significant proportion of this research draws on the dialogue during the co-planning of a lesson and post-lesson reflections, which are often conducted as interviews or focus groups and typically involve observing recordings of teaching (e.g., Gillespie & Amador, 2024). However, less has been reported on the coaching dialogue that occurs in-the-moment during a lesson, which is arguably the most critical stage of the coaching cycle that has a direct impact on the learning opportunities available for both students and teachers.

In this presentation, we discuss illustrations of mathematics instructional coaches' practices through the lens of *routine and adaptive expertise* (Anthony et al., 2015) to unpack what makes for effective mathematics coaching. We focus on the nature of the coaching dialogue, including how coaches and teachers attend to and reflect on students' mathematical thinking, and how coaches initiate dialogue that contributes to effectively building trust and rapport for genuine co-construction of the lesson.

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Identifying Bishop's Mathematical Activities in the AERO Mathematical Thinking Early Childhood Learning Trajectory

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Learning trajectories are structured to show the goal or aim of the learning, the developmental pathway, and indicators of how children may continue to develop their understandings (Clements & Sarama, 2009). The Australian Education Research Organisation (AERO) Mathematical thinking early childhood (MTEC) learning trajectory outlines four subdomains in mathematical thinking. These are patterns and predictions, shapes and spatial thinking, measurement and data, and quantity and counting (AERO, 2024, p. 1). Each of the subdomains has three strands that provide the capabilities for the subdomain within indicators presented in a four-step progression (AERO, 2024, p. 2). The three strands provide a broad description of what children may do within that subdomain and the indicators provide stepped descriptions what might be observed of children as they travel along the four levels of the trajectory for that strand (AERO, 2024, p. 8). The three strands and the indicators within those strands were the focus of the analysis in this short communication.

Early childhood mathematics can be linked to Bishop's (1988, 1991) six mathematical activities – Counting, Locating, Measuring, Designing, Playing, and Explaining – even for very young preverbal children (Cooke & Jay, 2021). Descriptions from Cooke and Jay's (2021) reframed version of Bishop's (1988, 1991) mathematical activities and the examples provided by Cooke and Jay's (2021, 2022) reframed version of Bishop's (1988, 1991) mathematical activities were used as a lens through which to analyse the strands and indicators for each of the subdomains of the MTEC learning trajectory (AERO, 2024).

The analysis found that each of the reframed versions of Bishop's (1988, 1991) six mathematical activities were evident in all subdomains of the MTEC learning trajectory (AERO, 2024). Of the 12 strands over the four subdomains, all of the reframed versions of Bishop's six mathematical activities were evident in five of the strands. Of Bishop's six mathematical activities, Explaining was the only mathematical activity of the six that was evident in all strands for all subdomains. There are two limitations – although careful attention was paid to the words used and checks made to ensure rigour, it is one author's interpretation; and, Cooke and Jay (2021, 2022) deemphasise language but language was preferenced as it was the means of communicating the actions evident in both the AERO MTEC learning trajectory and Cooke and Jay's (2021, 2022) reframing of Bishop's (1988, 1991) mathematical activities.

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Students' Algebraic Thinking as Reading of Expressions to Comprehend the Reality of the Phenomenon

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Access to mathematical objects is made possible by representing the characteristics of the objects by means of symbols and expressions (Duval, 2006). The way of thinking that uses symbols and expressions to investigate mathematical objects is called algebraic thinking in mathematics education research (e.g., Kieran, 2004; Kaput, 2008). In algebraic thinking, it is important not only to represent mathematical objects in expressions and to transform them but also to read characteristics of objects in expressions. In this study, we focus on reading mathematical expressions and address the following research question: How do students observe and comprehend the characteristics of a mathematical object while reading mathematical expressions?

To clarify the research question, we planned and conducted a design experiment. The lesson we designed dealt with a problem from “Jinkoki”, a Japanese mathematics textbook from the Edo period (Yoshida, 1977). The problem was to determine the number of thieves and the number of bolts of cloth stolen from the content of the thieves' conversation about the distribution of the bolts of cloth. The problem said: "If we give everyone eight, we will be short of seven, and if we give everyone seven, we will have eight left". The Solution described in “Jinkoki” was to add 7 and 8. To consider the reason for the solution in “Jinkoki”, students represented the numerical relationship in the thieves' conversation as simultaneous equations and solved them. While changing the conditions of the problem and comparing results, they comprehended that the solution in "Jinkoki" is a method that only works when the difference in the number of cloths to be distributed is 1.

Students' thinking was analysed in this study using a framework based on Tondorf et al. (2022) and Duval (2006). As a result, we identified a link in the representation, which is different from what Duval (2006) calls Treatment and Conversion. In other words, this study identified the process of comparing the series of expressions used to solve simultaneous equations as a “module” and considering the phenomenon while reading the expressions. This thinking process is related to the expansive generalization referred to by Harel and Tall (1991).

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Teaching Numeracy With Intention

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The expectation for numeracy to be embedded across all learning areas in Australian secondary schools is well established. The increasing recognition of numeracy as a critical competency across all disciplines has placed greater expectations on teachers to understand its relevance within their subject areas and be equipped to address it effectively (Geiger, 2018; Goos et al., 2012; Thornton & Hogan, 2004). However, enactment of this mandate remains inconsistent and often ambiguous. In many school contexts, numeracy is sometimes perceived as either peripheral or difficult to integrate meaningfully (Gough, 2007; Liljedahl, 2015).

Drawing on a subset of interview data from a larger doctoral study (Ferme, 2023), this short communication explores how secondary teachers from four disciplines (English, Science, Humanities and Social Sciences, and Technologies) perceive and describe numeracy in their classroom practice. It presents a focused analysis of a key theme that emerged from the data: the distinction between intended versus unintended numeracy.

Intended numeracy refers to deliberate, planned teaching of numeracy-related concepts, often aligned with curriculum documents or assessment requirements. Unintended numeracy refers to instances where numeracy occurred naturally through the task but was not planned or framed in the instructional narrative. Although numeracy is present in many aspects of secondary teaching, its role remains inconsistent and largely dependent on individual teacher interpretations. This implicit approach, combined with time constraints and competing subject priorities, limited opportunities to engage with mathematical thinking and develop awareness of numeracy as a learning objective.

The distinction between intended and unintended numeracy offers a useful conceptual lens for understanding the gap between curriculum expectation and classroom realities: It highlights the implicit nature of authentically embedded numeracy and the uncertainty teachers face when trying to incorporate numeracy beyond basic compliance. These findings point to the need for clearer guidance and targeted professional learning to support more deliberate and effective numeracy teaching.

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I've Got the Map – Let Me Tell You Where to Go

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Navigation is an important skill that we all use on a daily basis to make our way around the physical world (Foti et al., 2023; Negen et al., 2020; Nori et al., 2023). To navigate successfully, individuals need to access two frames of reference – egocentric, where the perception of space is taken from the point of view of the navigator, and allocentric, where the perception of space is taken from a perspective that is not aligned with the position of the navigator (Negen et al., 2018; Nori et al., 2023; Ring et al., 2018).

This talk will report on a research project that addressed the question: Do successful navigators use more spatial skills than less successful navigators? Given the complexity of navigation from an allocentric perspective, it was anticipated that successful navigators would use more spatial skills than less successful navigators to complete a maze task.

A group of 29 Grade 3 (8-9 years old) students participated in the study. Students were grouped according to their success in navigating their partner through the maze. Three groups were identified – highly successful navigators (0-1 errors); moderately successful navigators (2-4 errors) and less successful navigators (5-9 errors).

The frequency of use of spatial language was the area where a significant difference was observed between highly successful navigators (Group 1) and the least successful navigators (Group 3), $\chi^2(2) = 10.11$, $p = 0.005$. The highly successful navigators used fewer spatial strategies, including fewer verbal commands, than the least successful group.

Greater fluency in the use of spatial language may assist in improving navigation and might be a consideration for classroom teachers. This would also be an interesting topic for more detailed research in the future. Successful navigation requires fluency with a range of spatial tools. Successful navigators demonstrated greater efficiency in the use of verbal instructions than less successful peers.

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Exploring Primary Teacher Education Students' Self-Perception of Readiness to Teach Mathematics

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The tertiary education of Teacher Education Students (TES) plays a vital role in preparing highly qualified educators, especially in terms of mathematics education. Current practice in how to develop TES during their studies has been very under researched (Norton & Allen, 2020), with some research addressing only the secondary perspective (Hine & Thai, 2019). The findings of this study will determine the extent to which primary-trained TES feel professionally supported and prepared in their tertiary education in relation to mathematics. This short communication will report on the findings of a doctoral study, investigating TES' self-perception and readiness to teach primary mathematics. This project investigated the self-perceptions of TES (n = 107) enrolled in a Bachelor of Primary Education, Bachelor of Education (Birth to Twelve Years) or Master of Primary Teaching at the University of Notre Dame Australia (UNDA) Sydney campus as they prepare to teach primary mathematics. Interviews were conducted to evaluate TES' readiness, focusing on three key areas: mathematical content knowledge, mathematical pedagogical knowledge, and mathematical knowledge for teaching. The findings from this research identified TES' feelings of readiness of throughout their degree, the areas TES identified they require further training in, and suggested strategies on how this training can be achieved during TES' tertiary education.

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Harnessing Opportunities to Explore Early Statistical Literacy in Initial Teacher Education: Teaching Statistics Using Zoo Animals and Loose Parts Media With Pre-Service Teachers

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Statistical thinking begins when young children collect, sort, classify and make meaning from tangible objects in their immediate environment (Leavy & Hourigan, 2019). Therefore, it is argued that a strong foundation for using and representing data (data handling) will lead to stronger reasoning and thinking skills in older children (Makar, 2016). Much of children's early exposure to data comes from their teachers, who may be able to teach statistics, which is often explicitly taught through a STEM project. But the teacher may also not be able to teach statistics if they have never been taught. This is compounded for early years teachers, where explicit statistical techniques are not taught in a prior to school setting. Therefore, this study sought to identify the level of understanding of initial teachers in their final year of study, focusing on their modelled pedagogical practices using real data from a zoo visit. By practising the process of working with data (Curcio, 2001), we provided pre-service teachers with opportunities to identify a problem, gather data at the zoo (animal behavioural data/ethogram), analyse data on a graph using loose parts (mobile open-ended objects) and interpret for meaning.

As STEM teacher educators in an Australian university, we explored how $n=50$ pre-service teachers (PSTs) were intentionally taught how to teach statistics through real data obtained during a series of Zoo STEM Teach workshops. By collecting data through ethograms at the local zoo (Curran et al., 2016), and then following back in the classroom with opportunities to visualise their results using loose parts (Nicholson, 1971). Our results revealed that the process of collecting data was useful in capturing animal behaviours (the ethogram) using images, text or tally counts options, but some PSTs had difficulty representing this data using counts (loose parts). By using a content analysis of the students' workbooks and observations as photos taken by the tutor, we identified $n=25$ PSTs (50%) who were able to successfully summarise and show their understanding of animal behaviour graphically, thereby showing the data-graph process as a way to model how they would teach data representation in the junior school classroom. Future directions will be presented.

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Development of an Operational and Reliable Tool to Measure Student Mathematical Resilience

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Developing student resilience in mathematics is important since every school subject requires students to recognise and understand the role of mathematics in the real world (Australian Curriculum Assessment and Reporting Authority 2023). Lee and Johnston-Wilder (2017) propose the construct of mathematical resilience as being specific to teachers and students when participating in mathematics teaching and learning. Theorisations of how to foster mathematical resilience in students varies depending on context. Lessard et al. (2009) have established that resilient students in mathematics are resourceful and persistent, knowing how to seek and obtain help. Buckley and Sullivan (2021) found that students require support to regulate their negative emotions and develop resilience when facing challenges in mathematics classrooms. These findings provide some insight about how to develop student mathematical resilience however Xenofontos and Mouroutsou (2023) found there is no single clear working definition of the multifaceted construct of student mathematical resilience, and thus further research is needed about how student mathematical resilience is developed.

Given students’ resilience in the face of mathematical challenges is crucial for educators and researchers alike, this study aims to better understand middle school students' mathematical resilience. This aim will be investigated through three research projects, each with its own targeted research question. The overarching research question guiding this study is: In what ways can new insights into student mathematical resilience inform an operational and reliable tool to measure middle year students’ mathematical resilience and can this be used to inform teacher practice? With ethics approval from the South Australian Department for Education and participating South Australian Independent schools, my first research project focuses on gathering teacher and student perceptions of the factors that support mathematical resilience through survey responses.

In this presentation I will share key findings from my literature review on student mathematical resilience, outline the methodological approach guiding this research, explain the survey design approach and seek expert feedback to ensure alignment with contemporary goals and visions in mathematics education.

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Spatial Enrichment: An Enduring Model to Build Mathematics Achievement

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Spatial reasoning is defined as the imagined movement and transformation of objects and their relations and is an integral part of mathematics (Battista et al., 2018). Research has highlighted the strong relations between spatial reasoning and mathematics. Consequently, there is a growing body of theoretical and empirical work examining the mechanisms that connect spatial skills and mathematics (Resnick & Adams, 2024). Despite a historic focus that elevates the position of mental rotation, recent literature has identified that mental rotation, perspective-taking, and fluid reasoning all contribute to student success in mathematics. However, these effects differ for girls and boys (Harris et al., 2025) and may have different affordances for different mathematics content (Harris et al., in prep). Such nuances in spatial-mathematics relations are difficult to untangle in psychological intervention studies, which have been driven by improving performance on spatial skill measures in single sessions of isolated training (such as mental rotation of blocks) and assessing transfer to mathematics.

The current study compares the effects of training three different cognitive skills: mental rotation, perspective-taking, and fluid reasoning on a range of mathematics concepts. This will allow us to examine if there are different pathways that explain the space-mathematics relation for certain individuals (e.g., girls and boys). In the current intervention study, we developed three separate digital games to improve the mental rotation, perspective-taking, or fluid reasoning skills (respectively) of Year 7 and 8 students. Three experimental groups (one for each digital game) completed five weeks of game play (3 times per week) during their mathematics lessons, compared to a business-as-usual control group. The games were designed to progressively build spatial skills with mathematical language built into game play. This talk will present the principles underlying the spatial intervention design and explore some preliminary findings from the study.

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Analogy to the Concept of Function and Their Affordances

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Analogy is often used by teachers in their instruction of the concept of function. The explanatory affordances of these analogies for student learning of the formal definition of function, however, are less known. In a planned research paper, we will present a self-generated ‘child-mum biological relationship’ analogy that has been found popular among a sample of Turkish secondary mathematics teachers to describe function and illustrate the potential of such an analogy to unpack the formal definition of function.

Hatisaru (2022) undertook an investigation into the analogies made by a group of Turkish secondary mathematics teachers ($N = 42$) in describing the concept of function, by applying the structural-mapping theory of analogy. As elaborated comprehensively in Hatisaru (2022), according to the structural-mapping theory, an analogy is the comparison of the knowledge between a source (or base) and a target domain that share relational similarities (Gentner, 1983). These two domains share a relational structure independent of the objects in which the relations are embedded. It is posited that such similarities convey that the attributes held by the base object are also held by the target object (Gentner, 1989).

An open-ended questionnaire including the following sample items was used where the teachers were prompted to define and exemplify a function: “How do you think students should define the concept of function?”, “What kind of examples do you think students should give for functions?”, and “What questions would you ask your students in order to find out if they understand the concept of function?” (Hatisaru, 2022, p. 13). A total of 61 analogies were identified in the teachers’ descriptions. There are analogies conveying the taking inputs to outputs definition of function—such as ‘machine/factory’, ‘schooling’ and ‘bus’—and analogies conveying a mapping from one set to another definition of function—such as ‘child-mum biological relationship’, ‘postman’, ‘mail-mail box’, ‘room allocation’ and ‘restaurant’. The most popular analogy in the former group is the conventional ‘machine’ analogy, while it is a novel ‘child-mum biological relationship’ analogy in the latter group (for more details, see Hatisaru, 2022).

These analogies or their derivations are often used by teachers in their instruction of function. The question then arises, in the ‘child-mum biological relationship’ analogy—or in its derivations such as the ‘restaurant’ or Sand’s (1999) ‘mail carrier’ analogies—what structural similarities do the source and target domains convey? In our planned paper, and in this presentation, we endeavour to address this question to illustrate the potential explanatory affordances of these analogies for the teaching and learning of the formal definition of function. This also includes the challenges that this analogy may bring such as cultural norms.

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The Re-Silencing of Girls in School Mathematics: Gendered Intimidation and Its Impact on Participation

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Recent media attention has highlighted a troubling rise in misogyny and sexist behaviour in schools, raising concerns about their impact on girls' experiences. Despite significant progress through initiatives such as the *Girls in STEM Toolkit* (Education Services Australia, 2024) and the *STEM Equity Monitor* (Department of Industry, Science and Resources, 2024), the rise of classroom misogyny threatens to undermine efforts to promote gender equity in mathematics education. Emerging research suggests this trend may be "re-silencing" girls in mathematics—suppressing their voices, confidence, and participation due to gendered intimidation, even in equity-focused schools (Mendick et al., 2021; Renold & Allan, 2006). While studies have explored teachers' views on misogyny in schools (e.g., Australian studies: Roberts & Westcott, 2024; Variyan & Wilkinson, 2022), scant research examines its direct impact on girls' participation in mathematics.

This study investigates how these dynamics influence Australian girls' feelings of safety and their participation in mathematics classrooms. Framed by liberal feminism (Beasley, 1999) and hegemonic masculinity (Connell & Messerschmidt, 2005), this research explores whether boys' behaviour—including threats, derogatory comments, and gendered intimidation—influences girls' willingness to engage in mathematics. Using a mixed-methods approach, the study analyses quantitative data on enrolment trends alongside qualitative insights from late primary and secondary girls' lived experiences to identify patterns and potential causal links.

By focusing on the intersection of gender dynamics and educational participation, this research aims to uncover the subtle yet pervasive ways in which girls may be re-silenced in mathematics—a subject critical to future STEM opportunities. This study seeks to address the research gap by providing empirical evidence on the re-silencing of girls in mathematics classrooms. If girls, particularly those from disadvantaged backgrounds, are being discouraged from participating in mathematics due to feelings of intimidation, such injustices must be urgently addressed. By shedding light on this issue, the research aims to inform policies and practices that safeguard and advance gender equity in STEM education.

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Gender Equity in Mathematics Education in Australia: Renewing the Dialogue

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The status of gender equity in mathematics education in Australia has re-emerged as a critical concern, particularly in light of the 2023 Trends in International Mathematics and Science Study (TIMSS) results. This cycle marked the transition to a fully digital assessment (Australian Council for Educational Research, 2024). Alarming, Australian boys significantly outperformed girls in mathematics at both Year 4 and Year 8, with Australia and France exhibiting the largest gender gaps in Year 4 mathematics achievement among the 58 participating countries (Wernert et al., 2024). Concurrently, girls' participation in senior high school mathematics is at an all-time low and has not returned to pre-COVID levels (Australian Mathematical Sciences Institute, 2024).

Despite longstanding efforts to promote gender equity in STEM education (Department of Industry, Science and Resources, 2024), the persistent decline in girls' performance and participation in mathematics warrants further investigation. Contributing factors include learning loss due to COVID-19, the impact of digital learning environments, teacher shortages, and the prevalence of out-of-field mathematics teachers. Additionally, well-documented influences such as societal stereotypes, a lack of female role models and differences in career aspirations continue to shape girls' engagement with mathematics. Classroom dynamics, teacher attitudes, and parental expectations further affect girls' confidence and willingness to pursue advanced mathematics pathways, with long-term implications for career opportunities and Australia's STEM workforce.

This study reviews findings from quantitative and qualitative research, as well as reports from the last five years, to investigate the potential impact of COVID-19-related online learning disruptions, learning loss, and current teacher shortages on gender disparities in Australian mathematics education. By synthesising recent evidence, it aims to identify key barriers and enablers influencing girls' participation in mathematics. The study highlights the ongoing challenges and opportunities for targeted intervention, emphasising the necessity for renewed dialogue on gender equity in Australian mathematics education to support girls' sustained engagement and achievement.

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The Development of a Scale to Measure Motivations in Mathematics Enrolment

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Enrolment in higher-level mathematics in senior secondary school has been declining in Australia for several decades (Office of the Chief Scientist, 2020) and continues to be at an all-time low (Australian Mathematical Sciences Institute, 2024). Although many studies have looked at factors that might be influencing this enrolment downtrend (Centre for Education Statistics and Evaluation, 2017), only a few studies have sought the perspectives of students regarding their choice of enrolment in senior secondary mathematics (Hine et al., 2023). Furthermore, there is currently no validated instrument to assess students' reasons to choose or not choose to study mathematics, or any other subject, at the senior secondary school level. In this short communication, we report on the development of a scale to examine students' motivations to enrolment in certain senior secondary mathematics courses and the pilot of this scale in Western Australia, New South Wales, and Victoria. The findings themselves have been generated using both a confirmatory factor analysis and exploratory factor analysis on a large data set ($n = 2156$).

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Mathematics Anxiety and Mathematics Metaphors: Examining Pre-Service Teachers' Perceptions of Mathematics

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Mathematics anxiety (MA) is the feeling of tension and anxiety towards mathematics that can interfere with a person's ability to perform a mathematical task. It occurs in all age groups and is common among primary pre-service teachers (PST's; Swars et al., 2006; Wilson, 2015). In this study, 72 PSTs across early childhood, primary and secondary initial teacher education courses completed the modified Abbreviated Mathematics Anxiety Scale (mAMAS; Carey et al., 2017) at the start of a compulsory first-year mathematics unit. The participants also wrote a metaphor describing their perceptions of mathematics. The use of a metaphor has been shown in previous research to be a powerful tool to uncover people's feelings towards mathematics (Adams & Logan, 2021; Brady & Winn, 2014; Latterell & Wilson, 2017; Noyes, 2006). These measures were analysed together to determine if students' levels of MA were associated with specific metaphorical themes. The metaphors were thematically analysed and compared with quantitative statistics of the mAMAS results. The use of metaphors to explore PSTs perceptions of mathematics is not a new concept, nor is the use of a measure to determine the level of MA. However, to the authors' knowledge, this is the first time that the two measures have been analysed together to examine the relation between the two. This short communication discusses this novel approach and its results.

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Improving Students' Learning Experience in Mathematics Through Engagement With steaM

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Science, technology, engineering, and mathematics (STEM) education has been proposed as a potential solution to improve student engagement in school mathematics (Murphy et al., 2019). Research shows that utilising an enriched and diverse range of STEM contexts is an approach that can excite students, help them to relate content to real contexts, cultivate their curiosity and improve engagement (Marmon, 2019). However, concerns have been raised by mathematics educators that the depth and breadth of the mathematics itself may be lost in an integrated STEM (English, 2016) or, in the case of the current project, steaM task ('a' = Arts). The capitalisation of the M in steaM reflects an emphasis on mathematics. This study addressed the research questions: (1) What are the characteristics of steaM tasks that Grade 2 students perceive as engaging? and (2) How do Grade 2 students perceive their learning of mathematics content through high quality steaM tasks?

By applying the principles of variation theory and drawing upon extant research literature surrounding mathematical tasks, engaging and high-quality steaM tasks were developed, trialled and refined prior to their application in an intervention study. Variation theory can enhance mathematics education by structuring mathematical experiences around patterns of variation (Watson, 2017). According to variation theory, carefully designed tasks and activities that highlight what changes and what remains constant can help students develop a deeper understanding of mathematical concepts. The research was carried out in an international school in Hong Kong. The study encompassed 23 second-grade students, aged 6 to 8 years, over a six-week intervention period. Data were collected via whole-class surveys, interviews of six focus students after each of eight lessons, and student work samples.

This presentation will address initial findings from the intervention component of the study. The results suggest that: (1) well-designed steaM tasks can effectively engage students in cultivating a deep understanding of mathematical concepts; (2) harnessing young children's interests from other subjects can enhance their engagement for learning mathematics. Additionally, the implications for curriculum material designers and educational practitioners will be discussed.

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Taught by Experts: Relying on the Discipline to Counter a *New Wave of Problematic Evidence*

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The singer-songwriter Paul Kelly (1992) has a song titled *Taught By Experts* in which the narrator laments the cynical nature of learned behaviour. He rarely sings the song's last stanza, in which he changes one word of each line of the chorus of Bob Dylan's *Like a Rolling Stone*. Dylan famously borrowed phrases from folk singers he admired, Kelly was sarcastically demonstrating how you could really get it wrong if you were not an expert yourself. Advances in social media have changed the narrative of what an expert is, whereby impact on the profession is best gained with a successful YouTube channel or Blog. At the same time, "evidence-based" has become a phrase commonly (and loosely) used by anyone with an opinion about education. It is increasingly challenging to counter this new wave of "expert" opinions, especially when they gain political and general-public momentum.

It is unsurprising that views about what constitutes rigorous and "best-practice" research differs across and within disciplines, even by conventional academic experts (Lowrie, 2024). The Australian Education Research Organisation (AERO, 2022) maintains that rigorous evidence occurs when research methods isolate the actual impact of an approach. Most folk in our education community would agree with that. However, AERO go on to say that when evidence does not fit within their defined confidence levels, they will draw on "expert research guidance to make an assessment about how confident we are in the effectiveness of an approach" (p. 2). Presumably, AERO decide what research is worthwhile based on their confidence in the expert. Typically, research organisations such as the National Health and Medical Research Council (NHMRC; 2000) are more explicit about the approaches that demonstrate the highest levels of evidence. For them, and many other organisations, "gold standard" research includes meta-analysis, systematic reviews of literature, and randomised control trials (RCTs). Noteworthy, the weakest form of evidence is expert opinions.

For our discipline, and profession more generally, it is becoming increasingly challenging to counter the wave of experts demanding changes to the curricula and pedagogical approaches based on expert opinions (at best). However, we do not seem to possess the research rigour to counter the arguments, given our field rarely conducts this work. In a review of publications over the last ten years published in two of the leading journals in our field, less than 5% include research designs that would meet the "gold standard". This presentation considers some of the challenges faced by the mathematics education community in a changed environment where the term evidence is used carelessly, and evidence-based practices are ill-defined. "Expert" opinions dominate the discourse, rather than studies with rigorous methodological design. This presentation examines how well prepared our discipline is to counter such challenges.

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Data-Based Differentiation: How Data Informed Teachers' Beliefs and Practices

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Differentiation encompasses a range of practices to ensure that learning is accessible and meaningful for all students. While teachers value opportunities for all students to engage in cognitively demanding tasks, they often encounter barriers in providing these opportunities equitably (Russo et al., 2021). Using assessment data narrowly can contribute to teachers' fixed view of students' abilities (Fitzgerald et al., 2021). As part of a larger study investigating teachers' data usage beliefs and practices, and their influence on mathematics teaching and learning, semi-structured interviews were conducted with eight participants. Participants were from two government schools in Victoria, Australia, including two teachers from Grades 3 and 5, a numeracy leader, and an assistant principal from each school. Employing Braun and Clarke's thematic analysis with an inductive coding approach (Braun & Clarke, 2022), the first phase involved coding interview transcripts to capture any elements from the research question: *How does data contribute to the teaching and learning of mathematics?* Subsequently, codes with shared patterns of meaning were grouped together. The emerging themes were further reviewed through several refinement cycles to identify categories within overarching themes. One of the themes that emerged from the analysis was how data informed teachers' differentiation beliefs and practices. Findings from this theme are reported here. This study defines data as any information that can be utilised to promote students' learning and well-being. Findings indicated that the type of data teachers referenced influenced their differentiation practices, such as how they organised student grouping and the tasks they employed. When participants utilised assessment data from a commercial test-based website, they organised students in ability-based groupings or in-class streaming. Teachers shared their perception that grouping students in this manner allowed for targeted reteaching of questions or topics students had answered incorrectly or needed more support. Conversely, informal qualitative observations of student agency, classroom dialogue, and affective data guided the implementation of mixed-ability groups. For task selection, when emphasis was placed on assessment data, it resulted in the selection of procedural tasks. In contrast, open-ended and problem-solving tasks were utilised based on teachers' informal assessment and affective data. This is consistent with previous findings that when teachers relied on assessments emphasising memorisation of procedures or strategies, they employed homogeneous ability groups based on a deficit view of students' capabilities (Fitzgerald et al., 2021). In this presentation, I will share insights into how teachers interpreted and used various forms of data to differentiate teaching and learning in mathematics.

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The Role of Repeating, Growing, and Spatial Patterns in Early Mathematics Learning

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Studies on mathematical patterning have found that young children from the pre-school through to early primary school can create, extend, represent and generalise pattern structure for repeating, growing and spatial patterns (e.g., Larkin et al., 2024; Lüken & Sauzet, 2020; Wijns et al., 2021). Patterning is also related to the development of spatial skills and reasoning (Mulligan et al., 2020), and numerical competencies (Junker et al., 2024; Lüken & Sauzet, 2020; Rittle-Johnson et al., (2019). Mathematical concepts and patterning have been shown to co-develop and are interrelated, but different pattern types influence the individual's understanding of pattern beyond 'alternating parts' and 'unit of repeat' structure. Given the important relationships between patterning and spatial reasoning, and with mathematical development we conducted a qualitative analysis of 369 children's responses to conceptually and spatially-different patterning tasks based on the following research question: *How do different pattern types (repetitions, spatial and growing) account for differences in mathematical thinking and spatial reasoning?* Our broader aim was to support mathematical engagement and connected mathematics learning from an early age.

A descriptive interpretive study analysed variations in response from five different interview-based tasks: Repeating, Border, Triangular, Rotational and Growing patterns across a sample of 173 Kindergarten and 196 Grade 1 Australian students, aged from 5y 9mo to 6y 11mo. Analysis of five increasing levels of response revealed strong interrelationships between all types of patterns and spatial reasoning and in particular, between repeating and rotational patterns. Our findings confirm that while the unit of repeat is fundamental to pattern structure and multiplicative thinking, the ability to visualise and construct growing patterns can develop early and can advance other mathematical skills. The analysis revealed that children's visualisation, prediction, and/or dynamic processes such as 'growing', 'moving' or 'turning' accounted for differences in response levels in unique and embodied ways.

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Investigating the Experiences of Early Years Teachers in Teaching Mental Mathematics

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Mental mathematics offers an opportunity to promote flexible calculation strategies, yet the shift from basic methods of calculating to formal algorithms reveals gaps in teachers' Pedagogical Content Knowledge (PCK), especially in supporting diverse strategies (Venkat, 2024). Addressing these gaps is key to enhancing learners' mathematical reasoning and skills. Considering this, the research aims to explore teachers' understanding of mental mathematics and its role in supporting learners' mathematical abilities. Specifically, the study investigates: What knowledge do teachers have of mental mathematics, and how can they effectively teach learners mental mathematics? What are the enablers and constraints teachers face in supporting the learning of mental mathematics?

This study is a qualitative interpretive research project, with six female teachers who are primarily responsible for teaching mathematics in the early years. To gather data, the first author collected teachers' views and experiences of teaching mental mathematics through questionnaires and interviews conducted within a Professional Learning Community.

The more recent Refined Consensus Model, developed by Carlson et al. (2019), was used to analyse the data. The model critiques the notion that PCK is a fixed body of knowledge, rather suggesting that PCK is developed and refined whilst teaching and reflecting on specific topics. The Refined Consensus Model disaggregates PCK into three distinct categories: personal PCK, collective PCK, and enacted PCK. Personal PCK refers to the knowledge of individual teachers on a specific topic, collective PCK is the knowledge about teaching a specific topic that is generally agreed on in the field, and enacted PCK is the knowledge that an individual teacher applies in the context of the classroom.

The study found that the teachers' personal PCK was restricted and unclear. While they explained that mental mathematics includes basic counting and calculations "in the head," their descriptions of their enacted PCK suggest that their learners use external devices (e.g., fingers and counters). Reflecting on their personal PCK, teachers identified challenges in developing children's mental mathematics, such as language barriers, a lack of professional development, and students' difficulties with basic mathematical concepts. The findings highlight the need for more targeted training and support in mental mathematics instruction, revealing a critical gap in the teachers' collective PCK.

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Leveraging Near-Peer Teaching: Supporting Secondary Mathematics Pre-Service Teachers' Pedagogical Development

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Professional experience during initial teacher education (ITE) plays a critical role in shaping the pedagogical practices of beginning secondary mathematics teachers (Jao et al., 2021), yet often limits them to a narrow range of teaching methods (Little & Anderson, 2016). To broaden pre-service teachers' (PSTs') experiences, this study examined the use of *near-peer* teaching—an approach where students further along in their studies provide instruction to their peers (Dekker et al., 2023). In this study, Secondary Mathematics Education PSTs (Secondary PSTs) from the same university and nearing the end of their ITE assisted university tutors in teaching a first-year foundational mathematics course (*Foundations*) for Bachelor of Primary Education PSTs. This co-curricular program, *Teaching through Whiteboarding*, aimed to enrich Secondary PSTs' pedagogical development. Initial findings from a broader 2021–2023 study are presented here. The cohort of Bachelor of Mathematics Education students (<20) were invited to engage with the program, and all who agreed also consented to be part of the research: 2021 ($n = 7$), 2022 ($n = 4$), and 2023 ($n = 4$). Semi-structured focus group interviews before and after the experience explored prior teaching experiences, program enablers and constraints, and perceived impacts on content and pedagogical content knowledge. All participants valued the experience, contrasting it with their individual experiences of high school placements which reflected traditional instruction (i.e. teacher-led, individual practice outside class, minimal group work; Dove & Dove, 2015). In contrast, the *Foundations* classes used *Thinking Classroom* approaches (Liljedahl, 2021). PSTs appreciated these opportunities but acknowledged challenges in clearly explaining concepts, often seeking support from tutors. The program also prompted them to revisit and deepen their foundational mathematics knowledge to better support Primary PSTs and address misconceptions.

Overall, *near-peer* teaching through *Teaching through Whiteboarding* shows promise in enhancing Secondary PSTs' pedagogical and content knowledge, offering insights for enriching ITE programs.

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Promoting a Mathematical Modelling Culture in a Queensland School

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National statements have flagged common ideals for mathematics education – to provide students with applicable skills for the workplace, as responsible citizens, and to enhance decision making in their personal lives (OECD, 2021). A common goal is to extend learning to include independent solving of problems in the world outside the classroom, through the development of abilities that will endure beyond formal education. Such ideals are a motivating force behind the International Mathematical Modelling Challenge (IMMC, 2025). IMMC is based on the belief that students and teachers need to experience the power of mathematics to better understand, analyse and solve real-world problems.

Performance of teams in the IMMC is rated annually. We explored how one Queensland school achieved consistently strong performance. Significantly, this spanned a time period which saw a complete change in teacher personnel – both in school mathematics leadership and in team advisors. Our research questions:

- What actions are being employed by school personnel to assure the continuance and growth of a mathematical modelling culture within the school?
- What characterises the performance of this high-performing student team – both in terms of mathematical activity and as members of a collaborative group?

Data sources for this exploratory study were the 2024 team modelling report and structured interview data provided by a leadership staff member, the team advisor, and student team members. The interview data were coded using the QCAA mathematical modelling framework and the socio-emotional dimension of collaborative problem solving (Rogat & Adams-Wiggins, 2015). We also conducted thematic analysis (Clarke & Braun, 2017) to identify any common themes not covered by the coding framework. This study highlighted the role played by team composition, team cohesion, prioritised planning, and thorough execution. It was found the school supported a mathematical modelling culture at secondary level by building on the open-ended nature of teaching developed in the junior school, by prioritising time in the mathematics curriculum for open-ended problems, by supporting out-of-school and cross-curricular opportunities, and by supporting student teams during intense mathematical modelling periods where dedicated time was needed.

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Generative AI Usage Among Mathematics Teachers: A Demographic Analysis

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Generative AI (GenAI) is reshaping mathematics education through capabilities such as automated feedback, personalised tutoring, and instructional scaffolding (Memarian & Doleck, 2023; Yoon et al., 2024). Freely accessible tools like ChatGPT present new opportunities to enhance student learning and teacher development by fostering reflective thinking, problem-posing, and pedagogical content knowledge (Biton & Segal, 2025; Gurl et al., 2025). This potential is particularly relevant in developing countries, where limited access to traditional resources may be offset by the accessibility of GenAI tools.

This study investigates how demographic characteristics—including gender, age, teaching experience, school type, and school level—shape in-service mathematics teachers' adoption, usage, and preferences for GenAI tools. It also explores teachers' reasons for use and non-use and preferred applications. Survey data were collected from 264 in-service mathematics teachers across diverse school contexts in Indonesia. Quantitative analysis using chi-square tests examined associations between demographic variables and GenAI engagement.

Findings show a near-even split in GenAI adoption, with 49.6% of teachers reporting use. Users were typically aged 31–40 with 11–20 years of experience. Non-users cited limited tech skills (39.4%), lack of GenAI knowledge (25%), and poor infrastructure (18.9%) as key barriers. GenAI was mainly used for creating learning materials (27.2%) and supporting problem-solving and assessment (21.1%). No significant links were found between demographics and reasons for use or non-use, indicating that engagement is shaped more by personal and contextual factors. Canva AI and ChatGPT were the most widely recognised and used tools across both groups.

This study also highlights a critical disconnect: despite GenAI's free availability, its meaningful integration is limited by knowledge gaps, trust issues, and a lack of evaluative skills—particularly in under-resourced contexts. The findings emphasise the need for equitable AI capacity-building for teachers in developing countries and provide a data-driven basis for inclusive professional development that supports teacher agency and realises GenAI's transformative potential in mathematics education.

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Bringing Together Mathematical Education and Cognitive Science Research to Examine How Secondary Students Interpret Modern, Big-Data Visualisations

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Big-data visualisations are increasingly prevalent in scientific research, media, and industry. Unlike traditional graphs and charts, big-data visualisations often present large, complex, and dynamic datasets using multidimensional representations, colour gradients, and interactive features. Developing students' ability to navigate and interpret these representations is critical for fostering data literacy and informed decision-making. However, how secondary students come to understand, analyse, and extract meaning from big-data visualizations remains an open research question; our research study worked to answer this question.

For our study, 35 students spread across Years 6 through 12, were asked to engage with and reason about real, modern big-data visualisations in task-based interviews. The students read six visualisations that were static, dynamic (i.e., temporal information could be manipulated), or interactive, and were either map-based or axis-based. As they read the visualisations, the students were asked to think aloud, describing their thoughts throughout the process. Follow-up questions to clarify students' approaches or utterances were posed by the interviewer, along with a summative query of the students' main takeaway from reading each visualisation.

In a separate session, students completed measures of spatial reasoning and fluid reasoning to examine how these skills might support understanding visualisations. Spatial reasoning refers to a suite of cognitive skills involving the ability to mentally manipulate 2D and 3D information (Resnick et al., 2020). Given that big-data visualizations often rely on spatial representations of abstract concepts, spatial reasoning may aid students in recognizing patterns, understanding scale, and mentally transforming visual information. Fluid reasoning refers to the capacity to reason, solve novel problems, and identify patterns or logical rules without relying on learned knowledge (Pallentin et al., 2023). Fluid reasoning may help students adapt flexibly, integrate multiple sources of information, and make inferences beyond surface-level data interpretation.

This presentation will share how a Fulbright scholarship has brought together mathematics education and cognitive science research to complete this project. In this talk, we will present on how we are analysing a rich video dataset to characterise students' approaches, developing conceptions, and strengths when making sense of big-data visualisations. We will consider several frameworks for how to analyse this data to better understand how students make sense of big-data visualizations. We will also consider the potential roles of spatial reasoning and fluid reasoning in students' depth of reading modern big-data visualisations.

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Understanding How Pre-Service Teachers Respond to Students' Mathematical Errors and Why

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When developing mathematical understanding, learners' interactions with mathematical errors can promote conceptual understanding and enhanced reasoning skills (Borasi, 1994). Studies have found that learners' level of engagement with their own mathematical errors is heavily influenced by their views of mathematical error-making (Tulis et al., 2018), and that teacher handling of student errors can positively predict student views of mathematical errors, as well as predict improvement of reasoning skills (Heinz & Reiss, 2007). Research into teacher interactions with students' mathematical errors has found that various factors influence teacher error-responses; in addition to teachers' pedagogical content knowledge and constraints imposed by their working environment, their beliefs about mathematics learning and teaching play a significant role in how they respond to student errors, with beliefs that align with constructivist approaches to teaching mathematics tending to lead to error handling behaviours that promote conceptual understanding (Bray, 2011).

How, then, do we support teachers to develop positive error-response strategies? Initial teacher education programs already aim to promote strong pedagogical content knowledge, but do they, or can they, also promote positive views and concept-focused beliefs about mathematical teaching and learning in a way that would translate to positive error-response practices? This research aims to explore the factors that influence pre-service teachers' espoused error-response practices in mathematics classrooms in an attempt to better understand the role ITE plays in promoting positive error-response practices in new-career teachers.

In this study, data was collected from ITE students who were studying degrees that included K-6 teaching across several NSW universities. In an online mixed-methods survey, participants rated the likelihood of using specific error-response strategies in mathematics classrooms, as well as providing contextual information relevant to their mathematics teaching journey. Analysis of variance techniques were used to determine whether factors such as year of degree, beliefs about mathematics, and prior mathematics learning or teaching experiences significantly influence pre-service teachers' views of mathematical errors and error-responses in the primary classroom. The results provide a foundation for further discussion of how ITE can better prepare pre-service teachers to respond to student errors in mathematics classrooms.

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Supporting Primary Students' Mathematical Learning in English: The Role of Parents with a Second Language

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Parental involvement is a key factor in children's mathematical success (Cai, 2003; Fan & Williams, 2010). However, second-language parents may face linguistic and cultural challenges when supporting their child's mathematics learning. While mathematics is seen as a universal language (Adoniou, 2014), research indicates that mathematical discourse is deeply embedded in language and culture (Moschkovich, 2010). Transitioning between home and school mathematical language can create barriers for students and their families (Barwell, 2016; Gorgorió & Planas, 2001). This presentation reports findings from a scoping review (2015-2025) addressing the research question: *How do parents with a second language support their primary-aged children in understanding mathematical concepts?* The review identifies key themes, including the challenges of differing mathematical terminology and representations across languages. Limited proficiency in the instructional language can reduce parents' confidence and access to school-provided resources. Abstract and context-dependent nature of mathematical vocabulary further hinders their ability to explain concepts effectively (Gorgorió & Planas, 2001). Despite these barriers, the review highlights strategies parents use to support their children's learning, including visual and concrete representations, such as diagrams, manipulatives, and real-world applications (Bragg et al., 2020). Digital tools, including multilingual educational apps and online tutorials, provide scaffolding in both the home language and English. Community-based initiatives, such as culturally responsive numeracy programs and peer support networks, enhance parental engagement and confidence. Schools can support second-language parents by providing accessible, multilingual resources and ensuring culturally responsive home-school communication. Professional development for teachers on engaging diverse families in mathematics can strengthen parental involvement in children's numeracy development.

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From Silos to Synergy: Exploring Inquiry in Mathematics, Science, and Geography to Unlock Interdisciplinary STEAM

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In an increasingly interconnected world, STEAM education offers a powerful framework for equipping students with the skills to navigate complexity and change. Our research investigates the pedagogical intersections and distinctions among mathematics, science, and geography, focusing on inquiry-based learning. While each discipline brings distinct epistemologies and methodologies, they are also committed to fostering critical thinking, creativity, and problem-solving. We are exploring how interdisciplinary integration can enrich mathematics teaching and learning by drawing on research highlighting the curriculum potential of Humanities subjects—such as geography—for design thinking in STEM education (Kidman et al., 2024). Examining theoretical frameworks and classroom practices, we identify strategies for enhancing collaboration across disciplines and promoting holistic, student-centred learning. Our work contributes to the discourse on integrated STEAM education, offering practical insights for educators seeking to design innovative, inquiry-rich pedagogies that unlock students' potential across diverse knowledge domains.

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Issues in Solving Tasks Involving the Area of Composite Figures

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The topic of composite figures, shapes composed of two or more simple shapes, is a crucial component in Singapore's mathematics curriculum. Success in this topic requires more than just knowing procedures but internalising their meanings and making connections with other skills and topics (Tan-Sisman & Aksu, 2016). For the mathematics teacher, the ability to identify inefficient strategies, errors and their root cause is crucial in developing student knowledge. However composite figure tasks tend to be tested using formula-centric assessments (Nasser, 2022), which may mask underlying misconceptions, limiting teachers' ability to address root causes of errors.

This preliminary study investigated how root issues in the problem-solving process for composite figure questions could be identified using questions following the Newman Error Analysis Guidelines (NEAG), a structured approach to examining the thought process of students when solving a problem (Newman, 1977). Undergraduates from non-Mathematics or Science-based degrees were interviewed and tasked to solve several composite figure problems. Data collected included written workings, final solutions, and verbal responses regarding their understanding at each NEAG stage, in the form of fieldnotes. Errors and inefficient strategies were identified and categorised corresponding to their occurrence in their respective NEAG stages.

In this presentation, we will highlight three key findings from this preliminary study. Firstly, processing-stage errors observed mirrored those documented by previous studies (Lehmann, 2023). Secondly, there was a tendency for participants to fixate on using familiar procedural methods and formulae from the start rather than analyse and apply adaptive reasoning, even if the former method was significantly tedious. Lastly, several participants had misinterpreted terms, which affected their ability to plan coherent solutions and fixate on unnecessary subgoals. The use of NEAG questioning proved useful in identifying the source of these issues in the problem-solving process that would otherwise be hidden or attributed to a different cause just from analysing workings. This preliminary study highlights the usefulness of weaving diagnostic questioning into lessons and assessments to assess specific understanding and informs the next stage of the study.

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Mathematics Learning for Adults With Down Syndrome: A Matter of Independence

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Experiencing a positive mathematics education at school is fundamental for individuals with Down syndrome as they prepare for life, post-school. Faragher and Gil Clemente (2019) express the idea that receiving a mathematics education is a right that “enables access to the cultural capital of a society” (p. 114). They propose therefore, that individuals with Down syndrome have the right “to share the culture of their community” (p. 114). Arguably, Gil Clemente and Cogolludo-Agustin (2019) propose that mathematics should, for this population, “contribute significantly to improving communication skills as well as abilities to think, reason, and learn...offering the possibility of sharing a cultural world, which is valuable for them as human beings” (p.187). Unfortunately, the mathematics experiences for many individuals with Down syndrome, is in contrast with this pursuit.

This session outlines the findings from a section of a larger study, investigating the mathematics experiences of students with Down syndrome in Australian schools. For this presentation, data drawn from semi-structured interviews, conducted with six adults with Down syndrome and their parents/caregivers will be explored to ascertain the overall school mathematics experiences of this population, and the subsequent consequences that these experiences have had, post-school, for these participants and their parents/caregivers.

Findings suggest the need for continued opportunities, post-school, for individuals with Down syndrome to engage with and learn mathematics, to allow them to function independently in their day-to-day life. Subsequently, analysis revealed the critical role that parent/caregivers play in this endeavor, often needing to ‘adopt’ the role of mathematics teacher to teach their child the mathematics they need, to navigate their world independently. Additionally, the idea that persistent parental advocacy is required for individuals with Down syndrome to experience mathematics success at school emerged as a consistent theme across cases.

These findings have implications for how students with Down syndrome can be included in mainstream mathematics classroom and for how parents/caregivers can be supported, when teaching their child mathematics in post-school settings.

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Young Children's Identification of Functional Relationships in Repeating Patterns

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The benefits of supporting young children to engage in early algebra are well established within the research literature (Blanton et al., 2015; Kieran, 2022). Patterning is a recognised pathway into functional thinking and often a focus of curriculum in the first years of formal schooling. However, studies (e.g., Rittle-Johnson et al., 2015) have found that young children can succeed in more sophisticated patterning than they are normally encouraged to do at home or school. Other research in the functional thinking field (e.g., Blanton et al., 2015) has also found that young students can seek and express recursive, covariational, and functional relationships at more sophisticated levels than previously thought. Collectively, the field shows that young children are more capable in patterning and functional thinking than many curricula require. Given that many children in their first years of schooling are introduced to algebra through repeating patterns, in this presentation, we focus on student identification of functional relationships in repeating patterns.

The current study aims to examine the capabilities of 5- 7-year-old students in their first two years of school. Data was drawn from a larger study within the Developing Mathematical Inquiry Communities project. It was collected through a written task-based assessment grounded in the repeating pattern structure of AAB. A sample of $n = 735$ students completed items relating to predicting near and far positions, completing a table of data, and making a statement about the patterns they noticed. Two conceptual frameworks from Warren et al., (2012) and Stephens et al., (2017) informed the development of a coding structure.

In this communication we will share the emerging findings of our analysis. This will include examples of how young students can make sophisticated descriptions of pattern, along with trends in how this develops over the first two years of school. Notably, there was no significant difference between the achievement of European, Māori, Asian, and Pacific Peoples in this study. This shows the potential of using flexible grouping and challenging tasks as an equitable way to implement early algebra instruction within diverse New Zealand classrooms.

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A Model of Professional Learning for Teachers of Mathematics in Remote and Rural schools

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Professional learning for teachers in remote and rural schools is both critical and challenging—critical because it enables ongoing development in mathematics pedagogy, and challenging due to limited access compared to metropolitan counterparts (Aldous et al., 2006). Since 2019, the Mathematical Association of South Australia (MASA), funded by the Department for Education, has addressed this gap through the Remote and Rural Schools Mathematics Project (RRSMP), which delivers targeted, structured, and sequenced professional learning locally to individuals, small groups, and whole staffs. This approach acknowledges the logistical constraints faced by these teachers and aims to improve student outcomes by meeting teachers where they are (Wayne & Youngs, 2003). Feedback and data highlight the value of face-to-face delivery and district-level collaboration to ensure relevance and accessibility.

While the program has been running for 5 years, there is still a considerable need for the provision of PL to include problem-solving strategies, inquiry, investigations and mathematical modelling. PL that highlights the use of concrete manipulatives to introduce and reinforce mathematical concepts, problem-solving and reasoning, are in high demand. Principals have also identified their continued need for PL due to the high number of early career teachers, and continuous staff turnover.

Through feedback it has been identified that workshops are having a positive impact on the schools. One leader commented that:

“It is really helpful to have someone hear our ideas and challenge them based on his knowledge. This kind of rigorous, evidence - based discussion is lacking in small schools because there are only a few people with the knowledge and teaching base to engage, we know each other’s perspectives. Having an outsider challenge us based on evidence is really useful”.

Other feedback highlights the need for the PL to be delivered face to face and targeted for example:

- A range of strategies will be used based on the session with a focus on concrete/pictorial/abstract methods
- Opportunity to talk and share with others. Discuss ways of learning
- showed how you can differentiate the same activity/game but ensure it’s at a level that your students can engage in
- The discussions you can have with students to deepen their understanding

The approach used and the feedback received will be shared and feedback sought on refinements and alternative models.

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The Influence of Ability Grouping on Primary School Mathematics Teachers' Beliefs and Practices and the Evidence of Nurturing Environments

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Despite evidence regarding the ineffectiveness of ability grouping practices in benefiting all students (Francis et al., 2017), grouping students by attainment level remains a persistent organisational approach in many primary school mathematics classrooms. As schools strive to improve numeracy levels and reach higher mathematics standards, it would seem counterproductive to utilise organisational approaches that work to the detriment of most students. If ability grouping practices are to remain a common organisational and structural approach in mathematics classrooms, is it possible to enhance and improve the pedagogical practices of teachers in this context, and thereby increase student learning and achievement?

This presentation will focus on a section of a comparative, multiple case study drawn from a larger doctoral research project that investigated the ways in which targeted professional learning can influence the beliefs and pedagogical practices of primary school mathematics teachers in ability-grouped classrooms. This session will draw on the experiences of two primary school teachers, one who taught the lowest ability mathematics group and the other who taught the highest ability mathematics group.

Findings of this study showed that a nurturing environment was evident in both the lowest and highest ability group where teachers exerted a higher level of control through teacher-centred practices. In both cases, the teacher worked to 'save' their students from learning that was perceived as too difficult (lowest group) and from making any careless errors (highest group).

These findings have implications for teachers and school leaders as the nurturing environments that evolved from ability grouping in this study were shown to impede the implementation of effective teaching practices in the mathematics classroom.

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