A PHILOSOPHICAL JUSTIFICATION FOR ETHNOMATHEMATICS AND SOME IMPLICATIONS FOR EDUCATION

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This paper attempts a philosophical basis for ethnomathematics by explaining how it develops, how it is legitimised, and how it is integrated within other mathematical cultures. Ethnomathematics requires a relativistic philosophy. Questioning universal notions in mathematics can be traced back to the French philosopher Gaston Bachelard. He describes an historically relative notion of objectivity which gives rise to changing conceptions of mathematical objects and of rationality. This analysis provides a parallel which is used to explain cultural relativity, not just in mathematical practices, but also for rational thought. The history of navigation is used to illustrate such relativity. The possibility that ethnomathematics exists at the level of rational thought raises important questions in education. Questions about the practice of mathematics education, and about its socio-political function, are discussed.

INTRODUCTION

Ethnomathematics, the study of cultural relativity in mathematics, has been the subject of considerable literature in the last decade. Culturally distinct mathematical practices have been documented (e.g. Cooke, 1990, Gerdes, 1988, Zaslavsky, 1973); the politics of mathematical colonisation have been analysed (Bishop, 1990, Barton, 1991); and the structure of ethnomathematics and its place in mathematics education is the subject of a book (Bishop, 1988).

This paper attempts to establish a philosophical basis for ethnomathematics by explaining how ethnomathematics develops, how it is legitimised, and how it is integrated within other mathematical cultures. In particular we want to know under what conditions it is possible to accept the simultaneous existence of different conceptions of mathematics, particularly if they contradict one another. In addition, we must explain how one mathematics culture has come to be predominant, and, apparently, so highly developed compared with other mathematics cultures.

Ethnomathematics requires a relativistic philosophy. Writers such as Wittgenstein (1956) and Lakatos (1976) have made it possible to question universal notions in mathematics. However, the origins of such discourse can be traced back to the French philosopher Gaston Bachelard writing in the 1930's (Tiles, 1972). He describes an historically relative notion of objectivity which gives rise to changing conceptions of mathematical objects and of rationality. This analysis provides a parallel which can help explain cultural relativity, not just in mathematical practices, but also for rational thought.

The history of navigation, and the mathematics used in navigational systems, are used to illustrate such relativity. If the history of navigation can be rewritten from the perspective of mathematical relativity, and if this history can account for the way we view navigation systems of other cultures as well as the nature of contemporary navigation, then a possible alternative has been successfully argued.

The possibility that ethnomathematics exists at the level of rational thought raises important questions in education. In the final section the practice of mathematics education, particularly its assumptions of 'one mathematics', is questioned. Not only is such a curriculum questioned as to its honesty, but also its social and political effect on the people of various cultures is examined.

For the purposes of this paper, "ethnomathematics" will be assumed to have a broad meaning. As well as describing mathematical symbolisations, applications and practices which are culturally distinct, it will be assumed to include mathematical concepts, systems, modes of thinking, and meta-mathematics, e.g. what counts as a proof, beliefs about how mathematics relates to the world, and values implicit in mathematics. If the philosophical questions can be answered for this large definition, then they will also apply to any smaller definition.

PHILOSOPHICAL BASIS FOR ETHNOMATHEMATICS

Bachelard's Philosophy

Bachelard's key idea is that objectivity is an ideal rather than a reality. At any time we may think that we see clearly how things are, or that we know clearly how to make judgements, or that we know how to discover the truth, or that we undestand what makes a proof. However these ideas change over time, i.e. objectivity is illusory. With respect to mathematics, Bachelard believed that conceptions of mathematics at different times depend on changing notions of rationality, each successive change being regarded as being more objective than the last. Thus there is a progression towards a better, and then a still better, understanding of all the things which must be taken into account to get an objective view.

Thus there are many different historical standpoints from which to view mathematics, each of which are correct *at that time* and each of which explains previous views. Each such view gains its (apparent) objectivity because of its intersubjectivity (the wide agreement amongst mathematicians about the view), its rationality (the structure of causal relations within this view), and because it is seen to arise from previous views and encompass them.

This is an historical explanation. It accounts for the development of mathematics over time, for the changing, creative nature of mathematical notions while retaining the objectivity required of the discipline. We can also see this explanation as sequential: it describes mathematical development as proceeding from one step to the next, each step being an outcome of the previous one and each step being more objective than the last.

Another important idea in Bachelard's philosophy is that of the knowing subject. In order to gain objectivity, or to claim rationality, the person must reason in accordance with logical laws, but also they must be aware of the demand for rational thought. They must realise that they are thinking rationally. Furthermore, Bachelard realised that this created an inevitable link with the culture of the subject: "The detailed features of a subjects' rational cognitive capacities cannot be disengaged from their conception of those capacities, and thus conception is shaped by and revealed in the rational discourse of their culture."(Tiles, 1972, p. 41) Bachelard does not, however, propose a relative philosophy. Although his mathematicians are bound by conceptions of rationality which are revealed in their culture, they can overcome them because they are aware of the 'objective' requirements for rational thought.

Bachelard's view of the vital place of history in the present conception of science can also be echoed for mathematics. He describes a 'recurrent history' of mathematics which is continually retold, continually evaluated and thereby contributes to our understanding of present thought.

Application to Ethnomathematics

Ethnomathematics requires a non-temporal and non-sequential explanation. Culturally different conceptions of mathematics exist at the same time and do not necessarily follow on from one another. (There is a sense of ethnomathematics which describes historical developments, tracing the origins of mathematical ideas from the cultural communities in which they arose. This should perhaps be called the cultural history of mathematics).

It is important to explain how they can co-exist in cognisance of each other. That is, it is not enough that there is a basis on which differing views may exist in far-flung corners of the earth without knowledge of each other. It is also necessary to establish a basis on which holders of these views may know about the other views and accept them as right in some rational sense.

Mathematics carries only internal universality: it is thought by its practitioners to be universal, but this does not prove that it is so. Having arrived at a conception of mathematics where universality is debunked temporally, universality in any sense is brought into doubt. It is possible, and seems likely, that different groups will arrive at different conceptions of number, of shape, of relationships, of proof, of rationality, of mathematics. Bishop's description of 'environmental activities which lead to mathematics' explains this process(..). What must be explained is the relationship between these conceptions: what happens when someone holding one conception becomes aware of another ?

The echo of Bachelard's historical vitality now becomes important. Cultural vitality performs a similar, and parallel, function. As well as a recurrent history, there is a recurrent cultural dynamic which acts in the present in a critical fashion. In this dynamic mathematicians will evaluate their own practices and conceptions in the light of other practices and conceptions, will modify, reinterpret, discard or adopt particular practices. More importantly, they will retain the knowledge of how and why this was done as part of their mathematical understanding.

There may be a consciousness of change, of what motivated particular thoughts, new ideas and so on, but not necessarily a consciousness that this is a culturally relative process. This is possible because of the ethnocentricity of any particular view. It is still the case that most mathematicians regard their subject as universal. From their point of view this is true, for, if a discontinuity arises (be it temporal or cultural) then there needs to be a cognitive shift to accommodate the clash of domains. When this is achieved the sense of universality returns. It is only when this process is reflected upon that we see the relativity of the past situation.

Culturally Different Mathematics

What, then, is the status of a culturally different mathematics. For example, is there such a thing as Maori mathematics? Again a parallel with the historical case is helpful. Particular conceptions of mathematics are temporary: they begin and end although they live on in the critical role played by historical definition of present conceptions. The end occurs when a new conception encompasses the past ones and resolves the conflict of domains which has arisen. Ethnomathematical conceptions are similar. They are temporary in that they end when a new conception arises out of two conflicting ones, but they live on in the critical role now played by that conflict on the present conception. In the case of ethnomathematics, however, there are two simultaneous conceptions held by distinct groups. Thus the possibility exists for one group to recognise and accommodate the conflict while the other does not. It is this possibility which gives rise to judgements about another group's mathematics being inferior (see below) and also explains the widespread belief in *one* mathematics.

Thus Maori mathematics either did exist (and is now part of a wider conception of mathematics adopted by one or both of Maori and European mathematicians) or currently exists but is recognisable only to Maori mathematicians. The term 'Maori mathematician' does not imply that there is, in Maori culture, a body of knowledge recognisably equivalent to European mathematics and conducted by people who hold a parallel relation to their cultural community as European mathematicians do in theirs. It refers to those people in the Maori culture who would identify themselves as having some equivalent types of understanding, or those who perform functions recognisably similar to the functions of mathematicians in European cultures. It is likely that non-European cultures will have different categories of discipline which cannot be mapped onto European categories.

It is also important to draw a distinction between mathematical conceptions and mathematical practices. The interplay between practices, either historically or culturally, is easily seen and many examples can be quoted. Newly discovered practices are very quickly accommodated within existing mathematical conceptions and can usually be transported across historical or cultural boundaries without much difficulty. This is because mathematical conceptions have very broad areas of applicability, are very generalised, and can therefore explain a wide range of activities. However the interplay between conceptions is not so visible, nor is it so easy. This is because the resolution of conflicting conceptions gets played out through the mutual accommodation of many practices. Mathematical practices embody implicitly the mathematical conceptions. It takes the interplay of many practices to carry the full impact of the conceptions behind them. This explains why mathematical conceptions of minor cultures become subsumed or colonised. The mathematical conception with the wider range of applicability will accommodate different practices more readily than the minority conception will accommodate its opposing practices. What is more, in accommodating the practices of a minor culture, the dominant culture will 'rub out' the conceptions behind those practices. The imported practices will become examples of the conceptions of the dominant culture and are often therefore 'devalued' in that their original richness and potential is eliminated.

The sense of one mathematics being 'better', 'more true' than another rests on the status of the understanding. If ethnomathematical community A can see and understand the views of ethnomathematical community B without the reverse occurring, then community A will feel that their mathematics is more advanced. This will be true in the sense that they have achieved a meta-level of development whereby both ethnomathematics' can be understood. It does not affect the quality of the mathematics, it concerns only the level of understanding, and may be equalised at any time if/when community B reach a similar understanding.

However, if there is a time difference in this awareness by the two communities, the enduring feeling is that the second has been subsumed/colonised by the first. What is more, if one community is much larger than the other it is therefore more likely to quickly accommodate many smaller communities. The result is an intellectual 'black hole' into which all other practices fall and are accommodated and from within which it is impossible to see the existence of independent conceptions. This is exactly what has happened in mathematics.

THE HISTORY OF NAVIGATION

The Received History

The development of mathematical models of navigation has been one of increasing differentiation and decreasing integration. Where once the weather, directions, sea conditions and coast were an integrated system of knowledge, there is now a collection of highly developed sciences: meteorology, positional navigation, oceanography and cartography. This is reflected in the etymology of the word 'navigation' (through Latin, past participle of navigare: to drive a ship). Compare this with the current meaning which refers to positional navigation alone.

The accepted history of navigation acknowledges the many skills of ships' captains and their "men of knowledge" sailing in most of the world's oceans up to 4000 years ago. The history records the first use of maps (probably the Greeks in the first century BC although sailing directions (periplus) were common well before then); the development of instruments for observing the elevation of sun and stars (and hence determine latitude); tide charts; the development of lode-stones and compasses (although lode-stones were known up to two centuries BC the compass is not recorded as a sailing instrument before the Crusades); the search for accurate marine clocks (including a large sum of money for the first one of sufficient accuracy and the establishment of the Board of Longitiude in Britain); through to the use of radar and satellite systems. It is now possible for blue-water yachts to use a hand-held, battery powered receiver which gives them an immediate digital readout of their latitude and longitude to an accuracy of a few metres.

At the conclusion of Per Collinder's history (1954), there is a flight of imagination in which a modern ocean-going yacht takes on board an old Arab sea-captain who navigated during the Great Flood. Modern navigational aids are explained to the old man in terms he understands, i.e. by relating them to techniques and equipment which he would have used. The old man's amazement is recounted, but, he concludes "My boat is smaller than yours and does not need all these wonders." The implication is that old sailors did not need the sophistication of modern navigational aids, that their methods were sufficient unto their needs.

But perhaps the old man would have been amazed for other reasons? Perhaps he would wonder at the effect of these new technologies on the art of sea-faring. Perhaps he would have seen those things which had been lost from that art because particular instruments were developed? Perhaps he would have had a wish-list of aids which is quite different from the aids which have, in the event, developed?

These different views of the parable illustrate the different perspective of the relativist philosophy outlined above. The ethnocentric view of the wonders of modern navigation systems is countered by a recognition that the development may have been different, indeed may yet be different in the future.

The sea-farers of old (and of different cultures) navigated using different assumptions. However, as equipment developed to better locate oneself, the early navigation methods became judged using the criteria of accuracy of position, were found wanting and were consequently discarded. It is instructive to ask whether modern techniques (e.g. satellite systems) would be useful if judged on some of these different assumptions? For example, Micronesian sea-paths do not always cover the same ground: they depend on weather, seasons and sea conditions. Thus knowing exact position on a imaginary grid is not useful information. In other words, the development of mathematical navigation proceeded along one particular line out of many possible lines. Some of those discarded methods may well have developed into modern mathematical systems which would have been just as effective in terms of "ship-driving".

The Changing Concept of Navigation

In the history of navigation there have been changes in the idea of what it is to navigate. Three key changes are outlined.

The change mentioned above, that from an integrated body of knowledge and skills to more specialised collections of knowledge and technology (but concentrating on the positional knowledge afforded by astronomical observations), has been a gradual development. The old art of haven-finding has transformed into the technology of nautical astronomy. For early navigators the well-known winds and sailing destinations were used to name directions, stars were used as path-finders rather than position fixers, the plumb-line was used to determine depth but also had tallow affixed to show what the sea floor was like.

The criteria of a good navigation system have changed. Proximity (nearness to land), familiarity (nearness to known conditions), or reckoning (nearness to known paths) were all valid measures of navigation. They have all been replaced by location (nearness to known position). A modern ship's captain reported to me that the mathematics of positional navigation (i.e. the calculations involved) have never been very difficult. The main problem has been gathering accurate data in difficult sea conditions. But this only applies if you are interested in position fixes. If, on the other hand, you are interested in reading the sea to determine your path, or determining proximity to land, then sight is not always needed. Kyselka (1987) and Thomas (1987) report

Micronesian navigators using the feel of swells to maintain a correct path. Collinder (1954) notes the use by Spanish fishermen of the taste of mud attached to plumb lines to determine proximity to land. What would have happened if the technological development had concentrated on these aspects of navigation?

It is true that position lines have always played a role in navigation. Early sailors in most regions are recorded as using sightings on promontories to determine their position, the direction to begin voyages or the entrance to a channel. Such place-lines became augmented by bearings (where the compass rose replaced promontories) and later altitude lines (latitudes) where the sun, moon or stars became the reference points. Clearly these habits led to the development of an intersecting position-line for latitude (i.e. longitude). What limiting effect did this have on the use of position lines? Promontories (or lights) are still best for entering harbours.

This change in the meaning of navigation can be expressed as a change from other-centredness (where is the destination) to self-centredness (where am I).

A second change is that from an oral culture, where navigational skills are memorised and passed from person to person, to a written culture where knowledge is written in books, maps and tables, and is passed on through the media independently of those who wrote it.

Collinder (1954, p. 25) refers to the secrecy of navigational knowledge in early times, and attributes it to the need to have an advantage over enemies or competitors. But in an oral culture the greatest danger is not from enemies, but from poor memory or inaccurate transmission of knowledge. If everyone has access to knowledge then the chances of inaccuracy are vastly increased. If it is retained by a trusted few, and only passed on under stringent conditions of testing and confirmation, then accuracy is preserved.

The result is a close association between the knower and the knowledge. Such an association gets progressively distanced as the ability to draw and write improves. The need of a ship's master changes from "I need a navigator" to "I need navigation". In those sea-faring cultures where oral systems remain, the navigator is the centre of social organisation. Kyselka's stay in Micronesia records such a society in detail (Kyselka, 1987).

A third change is in the 'nature' of ships and shipping. Collinder claims that navigation comes into being with the ship. Because kayaks, canoes or coracles can go as close inshore as they like and can be steered off, and because they are not used to make journeys of more than a day's length, Collinder infers that they do not need navigational knowledge. Larger ships require such knowledge because they are often out of sight of land and cannot always be recovered if they get too close inshore.

In fact kayaks and canoes often make ocean voyages and sail out of sight of land. Just because large ship navigation has dominated development, it does not mean that small ship navigation is valueless. This is an example of the colonisation of an ethnomathematical system as described above.

Another difference is that trade shipping requires particular harbours as destinations and exact berthing facilities. Polynesian navigators seek a whole island and any beach provides berthing.

Navigation Systems

David Turnbull (1991, p. 23) asks the question "What is a navigation system"? when considering Micronesian navigation. Some of the characteristics are: it should be symbolic (and therefore transmittable); it should be manipulable (and therefore adaptable), it should be generalised (and therefore non-localised), and it should be open (and therefore innovating). Gladwin(1970) described the system of navigation on Puluwat atoll. His (and others') descriptions were further analysed by Hutchins (1983) in a way which made it clear that these characteristics are met. To quote Hutchins "The Micronesian technique is elegant and effective. It is organised in a way that allows the navigator to solve in his head, problems that a Western navigator would not attempt without substantial technological support."(ibid, p. 223)

Another question is: on what basis can we judge or compare navigation systems? By considering historical methods and navigation systems from other cultures (e.g. Micronesia) it is possible to come up with several criteria: accuracy, consistency and explainability come to mind. However it is important to see that even these criteria are relative. For example accuracy depends on what you are trying to find. Positional accuracy is different from path accuracy or proximal accuracy.

It is possible to step outside the ethnocentricity of modern navigation both at the level of practice and on a meta-level where different systems can be judged according to their own criteria and can be seen to be valid simultaneously. This raises the question of what other mathematical systems can also be simultaneously acknowledged? Can rationality itself depend on culture?

EDUCATIONAL IMPLICATIONS

Curricula

A relativistic basis for mathematics raises several questions for the way in which mathematics is presented and taught. The 'one mathematics' curricula common in our schools must be seriously questioned because it limits the possibilities of mathematics, it does not reflect our present understanding of mathematical development, and it curtails the debate of the nature of mathematics.

Bachelard's notion of history being critically active in the present, and the parallel notion of a cultural component to our present understanding of our subject, provide important tools for the classroom. By bringing out the interaction between culturally different systems (and historically different ones) our understanding of present knowledge becomes richer.

There is a pedagogical question which also requires consideration. It is necessary for good education to acknowledge the prior learning and cognitive framework of students. If mathematical frameworks are culturally dependent, then mathematics education must start with the framework of the student and use it to reach a higher level framework which incorporates both that of the student and that of the teacher.

Social-Political Considerations

If there is a dominant mathematical culture which colonises (and devalues) alternative mathematical cultures, then good education requires that such notions are at least made explicit, and, preferably, are countered. The difficulty is that this must be done while accepting that the dominant mathematics is that which opens doors in the education system and the labour force. Furthermore, it is not just the content of the system which is relative. Modern mathematics carries a set of techno-scientific values which must also be questioned.

If the colonisation process is as described above then part of an ethnomathematical education must be to convince ethnomathematical communities that their contribution to mathematics is on equal footing. This is done by gaining a meta-awareness of the process, not by evaluating the nature of their input. It is the potential content which is of value.

The identification of this potential content is the biggest challenge to mathematics educators concerned with ethnomathematics. It will not be easy to step outside the mathematical mainstream and 'do' mathematics using other rational systems, other cultural values and other symbol systems. Perhaps the best we can do is to open up resource opportunities for people from other cultures to do this work, educate ourselves so that we recognise it when we see it, and appropriately value the work when it appears?

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