THE GRAPHICS CALCULATOR IN TERTIARY MATHEMATICS

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INTRODUCTION

Technological advances are so rapid that hand-held calculators which perform symbolic algebra and graphically display functions will soon be affordable for most students. Already relatively cheap graphics calculators are available. Their use on a large scale by teachers and students at the secondary and tertiary level might therefore not be far off. Of importance before the wide-scale introduction of such technology into the classroom are several questions of which two are "How do students perceive of the calculator as a tool in mathematics?" and "How do students use the graphics calculator?"

In 1991 the TI-81 graphics calculator was introduced at Swinburne University of Technology to all first-year Applied Science students in their calculus courses. At the same time as the calculator was introduced, research was initiated to study the impact of the graphics calculator on the teaching and learning process. Several sources of information were tapped. These were: student surveys, a student 'brainstorm' session about the calculator, and the study of final examination scripts. In this article we will discuss some of the results of this research. Before presenting the research methodologies, results and discussion of the results, the calculator itself and the calculus subject in which it was used will be described.

SUBJECT

The course was a tertiary first year calculus course based on the book "Calculus with Analytic Geometry" by Hunt (1988) . The course content was similar to that of previous years, but wherever the graphics calculator could be used, adaptations in the form of lecture notes were made to the presentation of the content (Barling, 1991). For example, the calculator was used in the topics of graphing functions, equations and inequalities, and limits. In addition, particular emphasis was placed on the capacity of the calculator to act as a checking device for analytically derived knowledge. For example, derivatives of functions could be graphed and graphically compared with analytically determined derivatives.

THE CALCULATOR

The calculator used was the TI-81 graphics/scientific calculator. Graphs are produced by entering a function into the alphanumeric key board and by using the various scaling functions. A 'zooming' facility enables the user to move rapidly from the macroscopic to the microscopic view of the function and vice versa. The calculator can simultaneously display the graphs of up to four functions. Facilities exist to enable the user to read off the co-ordinates of any point on a graph with a high degree of accuracy. This is extremely useful when using the calculator to solve equations. The graphing capacity of the calculator is limited in certain ways. For example, for the function $\frac{\sin x}{x}$ with a point discontinuity at zero, the calculator will not show a gap around the point $x = 0$ unless the range of x is chosen such that $x = 0$ falls exactly on a pixel of the screen. Students not aware of such discontinuities would most likely not recognize the discontinuity from the calculator alone. Another limitation is that only functions can be graphed; relations such as circles and ellipses have to be entered in two parts, one for the bottom half and one for the top half.

There are other peculiarities that are beyond the scope of this article to discuss, but which force the students to use mathematical judgement before they can use the calculator as a mathematical tool, particularly when operating at the calculus level.

STUDENT SURVEYS

Method. In April and September of 1991 a survey was given to all calculus students attending the tutoring sessions during one particular week in each month. Except for two questions, the surveys were identical so developments in students' attitudes and opinions could be studied. The two questions in April that were deleted in September asked the students whether they had bought the calculator or not and if so how long ago. Because 98% of the students had bought the calculator at the time of the first survey, the two questions were in September replaced with items of more interest at that time.

The first four questions of the survey dealt with the students' major, sex, and first language. The remaining twenty-six items were statements concerning attitudes towards mathematics, learning mathematics with the graphics calculator, and the graphics calculator itself. Students responded to the statements by indicating their attitudes or opinions on a five point scale ranging from strongly agree to strongly disagree, with not sure in the middle. The complete questionnaires and results can be found in Boers and Jones (1992b).

Results and discussion. Several issues related to the graphics calculator were addressed in the survey, such as whether students were anxious about mathematics and whether the graphics calculator made them more confident, whether learning mathematics at the same time as learning the technology was difficult to do, whether the calculator was difficult to learn how to use, did they think the calculator was worth the price, and how the students used the calculator in the class and during problem solving: were they following on their own calculator what the teacher did in front of the class, did they use the calculator to explore problems and to check problems solved by algebra, and had making a graph of the function become an integral part of their problem solving routine. A summary of students' responses during April and September is given in Table 1.

From Table 1 we can read that the effects of the calculator were mostly positive for the students. And if there were developments from April to September they were generally in a positive direction with respect to the influence of the graphics calculator on students' attitudes and behaviour. Some statements caused significantly different reactions between males and females. The gender effects in those survey items are summarized in Table 2.

Table 1: Student opinions about mathematics, the TI-81, and learning mathematics with the TI-81 expressed in two surveys during April and September. The September responses are recorded in brackets. (SA-Strongly Agree, A-Agree, NS-Not Sure, D-Disagree, SD-Strongly Disagree) Items indicated with * had a gender effect in April, September or both months.

Issue	% of students giving response				
	SA	A	NS	D	SD
* Calculator is difficult to use	3 (3)	24 (14)	12 (13)	47 (56)	13 (13)
Prefer scientific to graphics calculator for doing numerical calculations	20 (13)	22 (19)	10 (7)	34 (43)	14 (17)
Simultaneous learning of mathematics and how to use the calculator confuses me	1 (1)	16 (7)	11 (9)	59 (67)	13 (14)
Graphical and algebraic analysis helps me to understand mathematical ideas	13 (14)	59 (63)	17 (13)	10 (9)	2 (2)
Time spent in class learning how to use the graphics calculator is worth it	10 (9)	50 (52)	20 (18)	16 (15)	5 (6)
* Due to the TI-81 I find myself exploring a mathematical problem rather than just trying to get that answer	5 (8)	39 (39)	26 (26)	23 (23)	7 (4)
The TI-81 is not worth the price we had to pay for it	18 (19)	40 (36)	23 (28)	16 (16)	3 (2)
In class I follow on my own calculator what the teacher is doing when he is using the TI-81	18 (11)	55 (56)	9 (10)	15 (17)	3 (4)
* I am generally anxious about mathematics	8 (7)	39 (29)	26 (29)	22 (30)	4 (4)
Since using the TI-81 I am more confident in doing mathematics	4 (7)	33 (40)	32 (25)	27 (25)	4 (4)
* Because of the TI-81 I am less afraid of mathematics	2 (6)	22 (28)	29 (29)	38 (30)	8 (7)
My TI-81 is useful for checking solutions to problems I solved using algebra	19 (20)	55 (58)	13 (11)	11 (9)	2 (2)
Making a graph of the problem situation on my TI-81 is becoming an important part of my problem solving	(10)	(47)	(22)	(17)	(5)

Table 2: Gender effects for survey items. Numbers in brackets indicate the responses for September

Issue		% of students giving response				
		SA	A	NS	D	SD
Calculator is difficult to use	male	2	19	9	52	18
April $p = 0.0003$, Sep. $p = 0.06$		(3)	(9)	(13)	(58)	(17)
	female	5	33	18	37	6
		(4)	(20)	(14)	(54)	(7)
Due to the TI-81 I find myself exploring a mathematical problem	male	7	43	26	17	7
rather than just trying to get that answer		(11)	(43)	(24)	(19)	(3)
April $p = 0.02$, Sep. $p = 0.06$	female	3	30	27	34	6
		(2)	(33)	(29)	(30)	(5)
I am generally anxious about mathematics	male	9	34	29	23	5
April $p = 0.3$, Sep. $p = 0.003$		(3)	(34)	(21)	(35)	(6)
	female	6	46	22	22	3
		(11)	(26)	(36)	(26)	(0)
Because of the TI-81 I am less afraid of mathematics	male	4	23	28	37	8
April $p = 0.2$, Sep. $p = 0.07$		(7)	(33)	(27)	(25)	(8)
	female	0	21	30	40	8
		(4)	(20)	(39)	(39)	(6)

'Brainstorm' Session

Method. One of the main reasons for introducing the graphics calculator was that it would support the learning of mathematics. Feedback from the students on this issue was sought through a 'brainstorming' session in which a number of students (n=18), selected at random from the calculus classes, participated in a structured group discussion. The purpose of the discussion was to identify and rank in order of importance both positive and negative aspects of learning mathematics with the graphics calculator. Input from the researchers was held to a minimum; they only guided the discussion and provided the starting questions. The two questions discussed were: What are the positive aspects about learning mathematics with the TI-81? and What are the negative aspects of learning mathematics with the TI-81? For a more detailed description of the procedure see Boers and Jones (1992b) or Jones (1992).

Results and discussion. The five most important benefits of studying mathematics with the calculator from a list of fifteen this group of students came up with were, in order of importance:

1. the ease of sketching and gaining information for graphs;
2. being able to quickly check the correctness of derivatives, integrals, and answers;
3. the understanding and interpretation of graphs and derivatives is made easier;

4. screen display helps in calculating and checking difficult formulae;
5. increases confidence and enthusiasm.

From three of the five most important positive aspects of studying mathematics with the TI-81 (points 1, 2, and 4) it becomes clear that the students value the calculator as a tool for helping them 'do' mathematics. The students also valued the graphics calculator as an aid to understanding and learning (point 3) but this would appear to be of less importance to them than its technical capabilities. Yet, from the initiators' point of view, the potential of the graphics calculator as an aid to learning was the key reason for its introduction.

The fifth positive feature identified by the students was attitudinal in that they claimed that learning mathematics with a graphics calculator increases confidence and enthusiasm. This point was also addressed in the survey mentioned earlier with around one third (36%) in April and almost one half of the students (46%) in September reporting that with the graphics calculator they were now more confident in doing mathematics and 24% in April and 33% in September saying that, because of the graphics calculator they were less afraid of doing mathematics (see Table 1).

With respect to the problems of studying mathematics with the TI-81 students made five points of which the two most important were:

1. possibility of calculator dependency - removes the need to know why
2. tendency to rely on the calculator

These points related to fear that using the calculator could lead to deskilling. It is of interest that this was less of concern with the teaching staff, possibly because they tended to view the calculator primarily as a teaching and learning aid rather than a mathematical tool.

USE OF CALCULATOR UNDER EXAMINATION CONDITIONS

After requesting students' opinions about the calculator the question emerged: how do students actually use the graphics calculator while solving mathematics problems? An attempt at answering this question was undertaken by studying the examination scripts of a representative sample of 37 students chosen from 304 students who took the 1991 calculus examination and who scored 15 points or more on the exam. A quota sampling technique was used to select the students for this analysis so students from all levels of achievement were represented in the sample. For a detailed description of the selection procedure see Boers and Jones (1992a). In this paper the selected students are indicated with a number (1 through 37) assigned to them during the study.

Results and discussion. Three main results were found from studying the examination scripts of calculus students. For a complete report on this study see Boers and Jones (1992a).

Firstly, the full potential of the graphics calculator was not utilised by the students during the exam. On questions that did not specifically ask for a graphical response, such as, on a question asking for the limit of a function at a certain point, little evidence was found that the graphics calculator had been used to arrive at a solution. The calculator's potential as a graphical checking device for algebraically derived information was not realised either. None of the errors made in derivatives, limits and integrals were corrected.

On a question where a graphical response was required *and* where a purely graphical solution would have sufficed, less than half the students (43%) solved the problem by calculator alone. For this question, which required students to sketch a graph of the function $x^{1.4}e^{-x}$ and find the stationary points, most students (49%) began finding the stationary points algebraically, with about a third changing to a graphical solution when their algebra broke down. However, irrespective of how the students attempted to find the stationary points, students did use the graphics calculator to obtain their sketch graph. Apparently, while students were taught how to find stationary points using a graphics calculator, many preferred an algebraic strategy. Under examination conditions, for many, the calculator was seen as a last resort.

Under-utilisation of the calculator was therefore a much greater problem than over-utilisation. Banning the graphics calculator on the grounds of over-use seems therefore unfounded.

Secondly, the presence of the graphics calculator could make solving the mathematical problems on an exam harder instead of easier. The presence of an independent source of information could possibly bring students into conflict, which they needed to resolve. On a question that asked students to give the points where the rational function $\frac{x^2 + 2x - 3}{2x^2 + 3x - 5}$ was not defined (part (a)), the limit of the function at one of those points with a bounded limit (part (b)), and the graph of the function (part (c)), the majority of students (73%) ignored the information gained from the first and second part of the problem, or were unable to reconcile the apparent differences between their algebra and the graphical information from the calculator. The graphics calculator does not show a gap in the graph at a point where a function has a bounded limit, unless that point falls exactly on the centre of a pixel of the screen. The students who saw the conflict (27%) between algebraic and graphical information, either ignored the graphical information from the calculator or ignored the information gained from answering parts (a) and (b) algebraically (see for example the work of students #21 and #30 on this problem in Figures 1 and 2). Only 14% of the students were able to integrate algebraic and graphical information.

A third finding about the use of a graphics calculator under examination conditions is that it can give the examiner a deeper insight into a student's understanding. To illustrate this point we can look at the work of student #4 on the question that asked students to graph the function $x^{1.4}e^{-x}$ and indicate the two stationary points in the sketch graph. Student #4 determined the stationary points algebraically. She then most likely entered the function $x^{1.4}e^{x}$ into her calculator (which is an exponentially growing function), but still marked the algebraically found stationary point on her sketch-graph where there obviously was none (see Figure 3). If the student did not see the conflict, she must have lacked understanding of what a stationary point means graphically. Another possibility is that she saw the conflict, did not know how to resolve it, ignored the graphical information, and relied on her algebraic work instead. From her algebraic work one might have drawn the conclusion that she basically knew what she was doing. The combination of the graphics calculator and algebraic work told a different story.

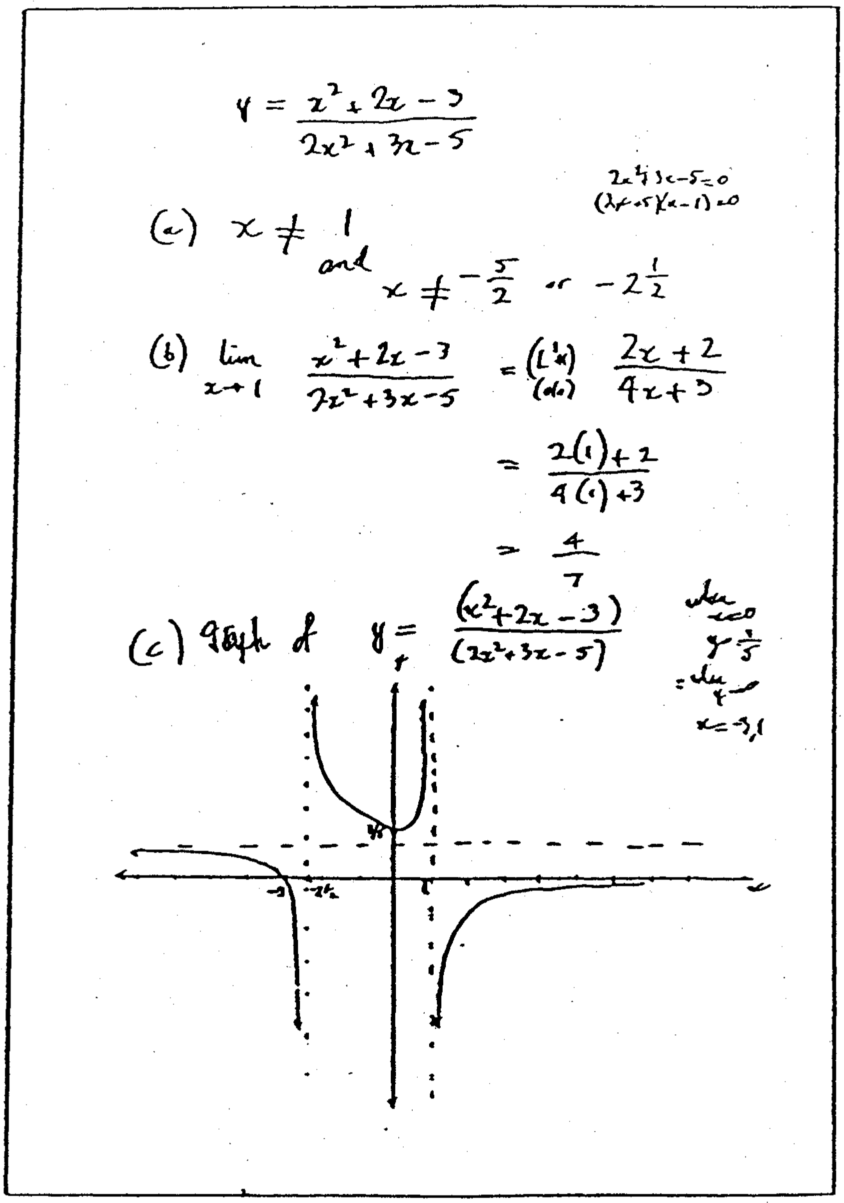

Figure 1: The work of student #21 on question 1 showing Strategy IIA, failed integration: algebraic preference.

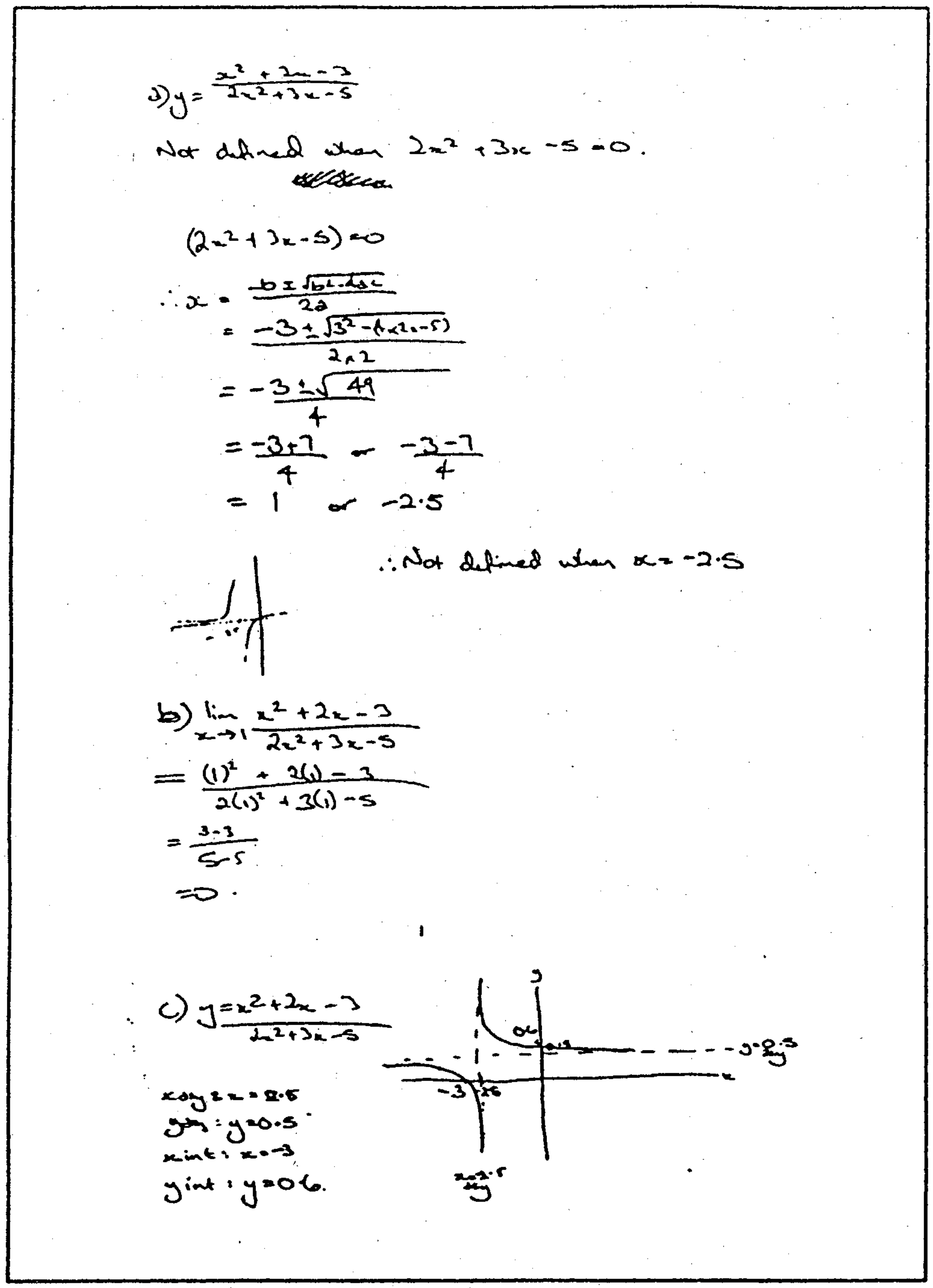

Figure 2: The work of student #30 on question 1 showing an example of Strategy IIB, failed integration: graphical preference.

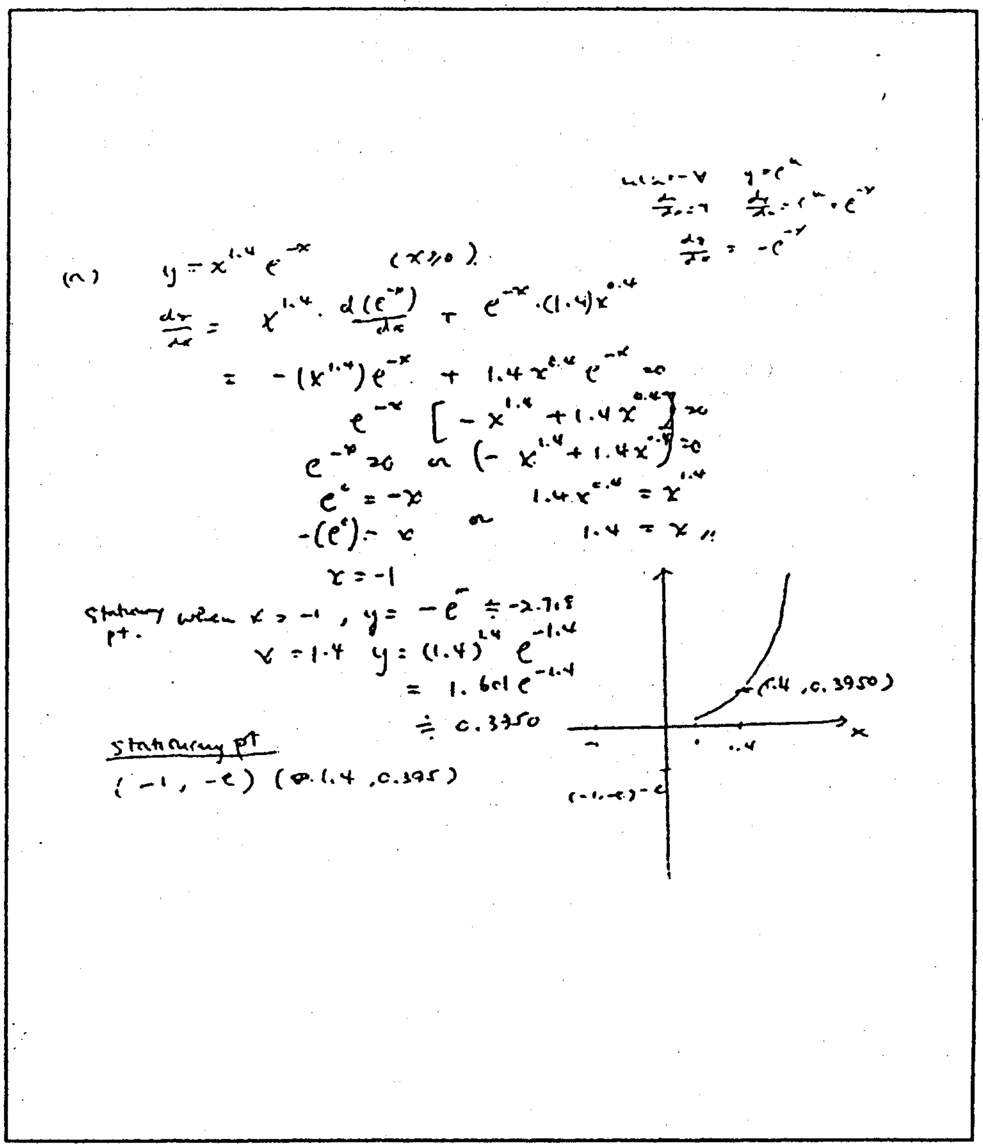

Figure 3: Work of student #4 on question 2(a) showing reasonable analytical skills but lack of integration of the analytical and graphical meaning of stationary point.

CONCLUSION

From the survey, one might draw the conclusion that the graphics calculator has had positive influences on the students' mathematical behaviour and attitudes towards mathematics. The 'brainstorm' session, although reinforcing those positive aspects found in the survey, pointed also to some of the dangers of the graphics calculators, such as, calculator dependency, which is often cited as a reason for banning its use. The evidence from studying the use of the graphics calculator under examination conditions seems to suggest that over-use and dependency were not a problem, at least during the exam. Students who did rely on the calculator as their main source of information, did not score highly in this traditional calculus subject; their strategy did not pay off. Of more interest to mathematics educators should be that the presence of the graphics calculator might make the mathematics more difficult because students might need to integrate two independent pieces of information, and that it can help examiners to gain insight into the students' levels of understanding.

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