CONSTRUCTIVISM, LEARNING AND COGNITIVE REPRESENTATION; THE CASE OF FRACTION IDEAS[1]

GEORGE BOOKER
Faculty of Education, Griffith University

The assumption that mathematics is learnt by the individual construction of ideas, processes and understanding rather than through the transmission of pre-formed knowledge from teacher to student is now a commonly held belief among mathematics educators. An essential feature of this view is that existing conceptions, whether gained from everyday experiences or previous learning, guide the understanding and interpretation of any new information or situation that is met. As a result, there is often a resistance to adopt new forms of knowledge or to give up or adapt previously successful thinking, and the intuitive conceptions of children may appear very different to accepted mathematical practice. While much of the early support for constructivism has come from observations of situations where new knowledge has arisen from concrete situations, constructivism also needs to account for the more complex mathematics which has been formed by the processes of abstraction and generalisation of earlier ideas. The conventions that have emerged cannot simply be replaced by the idiosyncratic building of a host of individual learners; they need to be acquired in the same social context from which the mathematical concepts are to be drawn. This paper reports research which set out to establish a constructivist approach to the learning of initial fraction ideas, focussing on the social setting and activities which could lead to the negotiation and reconciliation of mathematics formed from historically and culturally determined generalisations.

CONSTRUCTIVISM AND MATHEMATICS EDUCATION

By viewing teachers and learners as active meaning-makers, continually giving contextually based meanings to each others' words and actions as they interact, constructivism challenges the assumption that meanings reside in words, actions and objects independently of an interpreter (Cobb, 1988). It suggests that the sharing or exchanging of mathematical thoughts and ideas is dynamic, reflecting a continually changing fit between the meaning-making of active interpreters of language and action. It also highlights the fact that old ways of thinking are not usually given up without resistance and emphasises that their replacement by or extension to new ways of thinking is guided by already existing conceptions (Duit, 1992). In contrast, traditional approaches to mathematics teaching and learning assume that students' everyday and prior conceptions have to be replaced by more mathematical ones, and there often appears to be

[1] The data reported here is drawn from a BT(Hons) study conducted by Barbara Messinbird during 1991.

no assumption of any prior knowledge or experiences at all. Seemingly, any existing ideas should simply be erased in the process of inculcating 'proper' procedures.

Much of the support for the belief in constructivism has come from observations of the learning of elementary mathematics (Steffe, von Glasersfeld, Richards & Cobb, 1983; Kamii, 1990; Cobb, Yackel & Wood, 1992). Further evidence has been drawn from the common difficulties that occur as complex procedures are built with inappropriate or inadequate generalisations of the concepts and processes that gave initial success (Confrey, 1990; Graeber, Tirosh & Wilson, 1990). It seems a short step from seeing how ill-formed constructions are made to assuming that even 'correct' mathematical forms can also be constructed individually. But seeing how appropriate constructions can be formed is not the same as knowing that they necessarily occur in this way. There is a compelling need to show and explain instances of children constructing for themselves these advanced ideas from among the various possibilities that could arise.

In particular, this requires viable models of cognitive development in specific areas of mathematics so that a learner's conceptions can be inferred and activities that might lead to a restructuring of conceptual understanding towards a particular mathematical goal can be selected or formulated. An awareness of ways in which learners might discuss, negotiate and resolve their possible constructions is essential for this, so that much current thinking in mathematics education is directed at the social conditions of learning and the specific activities in which the learning is situated. At the same time, constructivism reminds us that mathematical concepts are human inventions rather than objective truth; that there are different possible conceptual frameworks for explaining and acting on phenomena in a consistent manner. There is a need to account for the actual possibilities that arose in the development of mathematics as well as to be aware of the various possibilities that children might feel constrained to develop. In this way, the building of mathematical cognition can be viewed as a process of social (re-)construction with the conventions which rule mathematics acquired in the same social context from which the mathematical concepts are drawn. Consequently, the notion of situated, social knowing can replace the concept of knowledge as the development of routines and strategies which are only viable for coping with reduced complexities, are non-adaptive, and which have a damaging influence on self-concept and self-confidence (Bauersfeld, 1991).

LEARNING FROM A CONSTRUCTIVIST PERSPECTIVE

Constructivist approaches to learning seem to have taken on one of two forms; either conceptual change is avoided to some extent by enlarging or partially restructuring students' thinking, or some form of cognitive conflict is involved in the teaching sequence to create a situation where students exchange their initial constructions for something more akin to the mathematical viewpoint (Duit, 1992). From the perspective of constructivism as enlargement, a teacher starts with a learner's existing knowledge and, by means of a continuous chain of development helps him or her "climb up the different steps of the intended construction ... [so that] each step is an extension of accrued knowledge and this endows the learning process with continuity" (Herscovics & Bergeron, 1984 p. 195). This form of learning sees the teacher's role as a guide and raises the issue as to how much freedom the individual learner will have for her or his own constructions. But, so long as the formal end-points that the teacher conceived are seen just as that, end-points to which the learners might be oriented when they have constructed the essential mathematical meanings themselves, the danger of the teacher becoming simply a transmitter of formal knowledge can be avoided. Although, there is also the possibility that the most influential prior learning a student brings is a belief that there are exact answers, correct and efficient methods, along with the view that the major, perhaps only, task of the teacher is to hand these methods over to the learner.

The cognitive conflict form of constructivist teaching focuses on situations in which discussion, pertinent examples and examination of the limitations and advantages of particular points of view lead to the displacement of inadequate or inappropriate constructions by mathematically superior viewpoints. Situations are arranged in which contradictions in the students' own constructions can emerge in contrast to the alternative mathematical view raised by the teacher, text or some other arbiter of mathematical method. However, this is not always easily achieved, for, as Confrey (1990, p.37) says, "students cannot see their solution as erroneous until they have constructed a new problem and new solution ... this usually means revising their previous belief" and not just their method. Perhaps the notion of cognitive conflict is more appropriate when an inadequate or inappropriate construction has become ingrained, for, at an earlier stage, what is often called cognitive conflict is really just the process of argumentation and justification that is required to discuss and negotiate the mathematical meanings of a given situation. As Duit (1992) reminds us, the conflict arises first in the various possible constructions that might be formed:

> It is a central idea in constructivism that there is a dialectic relation between conceptions and perceptions. Conceptions guide perceptions and perceptions develop conceptions. Where the guidance of perceptions is concerned, one could say that humans tend to see only what their current conceptions allow them to see (p. 11).

COGNITIVE REPRESENTATION OF MATHEMATICAL IDEAS

The replacement of the notion of forming a conduit between the teacher's mathematics and the students' mental conceptions as a perspective for the learning of mathematics by a model of individual construction has had profound implications, not only for practice, but also for the relationship between mathematics learning and cognition. No longer is the object to be one of implanting an ideal structure in each student's mind, but to understand the student's conception for what it is. Cognition is necessarily concerned with individual conceptions of mathematical ideas, personally constructed from the experiences to which each individual has been subject but reflected in the particular way of viewing or interpreting these experiences based on prior knowledge and ways of looking at the world. New cognitive structures result from both the social and physical environment, and cognitive development needs to be explained by reference to principles of self-regulated change and interaction, in particular, how the artifacts and forms of social organisation, conceptual, symbolic or material products which have emerged over the course of social history, come to be interwoven with and are intrinsically related to the nature of children's intellectual constructions (Saxe, 1991 p. 4).

This is very different from the view of assimilating and accommodating new ideas to a knowledge structure by simply looking at the structure of the particular aspects to be taught to see how to best re-present them so as to replicate corresponding mental structures in learners. However, as Matthews (1992) highlights, too often "constructivism has failed to appreciate the reality or objectivity of human intellectual activity" and mathematics education also needs to be "conceived in terms of the appropriate introduction of individuals into this world of concepts, understandings, techniques and community standards". Saxe (1991) reminds us of Vygotsky's observations that scientific concepts are interconnected, comprehensive systems of understandings which have been elaborated and refined over the course of social history. They cannot simply be internalised but must undergo a complex transformation in their inward movement from artifacts external to the child's activity to the mental processes which make up an individual's cognition. Knowing is an evolving social practice, continually constituted by constructive activities of individuals, mediated by social interactions which bring about the meaningful adoption of mathematical conventions, and which allows some common perceptions to be taken-as-shared.

At the same time, by drawing attention to the need to account for cognition in terms of the learner's experiences and interpretations, a constructivist approach reveals the distinctive manner in which ostensibly similar tasks may be carried out in quite different ways or with differing degrees of success. The activity and context in which the learning takes place is more than just a pedagogical vehicle for discussion and meaning-making. Rather than being separable from, or ancillary to learning, the situation is integral to what is learned, co-producing knowledge through activity; learning and cognition are fundamentally situated (Brown, Collins and Duguid, 1989). Knowledge is, in part, a product of the activity, context and culture in which it is developed and used. Hence, to account for the developing cognitive representations an individual is building, it is necessary to consider socio-historical precedents for the ideas, the relativity of the context to the learner and the authenticity of the activity in terms of the experienced culture of the learner.

DEVELOPING FRACTION IDEAS

Fraction ideas are the first abstracted mathematics met by the young learner, and the difficulties experienced by child and teacher alike as the differences and similarities to the earlier whole numbers are met, grasped and reconciled are well known. A full understanding of fraction ideas would seem to require exposure to numerous rational number concepts (Kieren, 1988; Behr, Harel, Post and Lesh, 1992). An analysis by Messinbird (1991) provided a framework for the development of initial fraction ideas through the use of models, manipulative materials and games involving chance designed to give situated meaning as opposed to rule-like procedures that often dominate the learning of fractions. It was postulated that this would allow the children to:

> construct and reconstruct their knowledge, returning to earlier stages of development as reflection and observation identify areas of incompleteness. The complexity of the situation, with its established social norms, actions and interactions, cognitive conflicts and agreement ... is best experienced in a collaborative classroom. (Messinbird, 1991)

Although many children did manage to construct their own understanding of the fraction concept, including meaningful naming and manipulation of the ideas, the provision of rich experiences, both mathematical and social cannot be assumed to readily lead to mathematics governed by socio-historical conventions. Even with the use of carefully planned materials and engaging activities, children can acquire rote routines for forming answers rather than build for themselves the underlying concepts. Further, not all children will engage in the social actions required for negotiation of meaning, let alone participate in the conflicts needed to overcome cognitive obstacles resulting from inadequate earlier concepts. Indeed, children may well alter the conditions of the situation in which they are placed to avoid perceived difficulties altogether or simply to allow them to achieve their own objectives, which may be very different from those mathematical end-points the teacher had in mind.

These points are illustrated in the following excerpt from the video-taped records of the study in which Jo, Lizzie, Rebecca and Lisa played a game which used cards showing fractions more than one in pictorial form. Each player was required to read a card in two ways: in common fraction and mixed number form.

Jo:	*Picking up a card* *Um. Three and ... yeh, three and one third*
Rebecca:	*And what's the other way? You got to say the other way to say it too.*
Jo:	*Do you?*
Rebecca:	(picking up the card and pointing) *Yeh, and you got to count how many are shaded and then say the fraction.*
Jo:	*Okay, one third.* (ignoring the fraction that made up the three completely shaded rectangles)
Rebecca:	*No, like count all them that are shaded.* **Rebecca tries to impose her knowledge**
Jo:	*Oh, four thirds* (counting the completely shaded rectangles each as one third instead of as three thirds) **Rebecca and the others did not appear to notice. Where does argumentation and resolution enter into the constructive activity when there is not a knowledgeable guide present?**
Rebecca:	*Yeh, that's better.*
Rebecca:	(picking up a card for her turn, counts) *Thirteen eighths or one and ... one and five eighths* (Jo watches carefully) **Rebecca models her knowledge for Jo.**
Lisa:	(Picking up a card) *Um ... one and three fourths or um ...*

Jo: *Three fourths* **Despite watching Rebecca, her conception persists.**

Lisa: *Yeh* (seeming to follow Jo's cue and looking only at the partially shaded shape but then reconsidering) **Is it her level too, or does she just go along?**

Rebecca and Lisa together: *No*

Rebecca: *Count up...* **Direct instruction again - if teacher does not transmit, children often will!**
Jo seemed to guess instead of counting as Rebecca had suggested:

Jo: *Thirteen fourths?* (The fraction under consideration was 1 and 3 fourths)
Or had she? She appeared to have generalised from her recent experiences with decimal fractions, taking one whole as 10 equal parts, then added the three fourths to make thirteen fourths.

Rebecca: *No, like um ...*

Lisa: (Looking at Rebecca and asking questioningly) *Seven fourths?*

Jo: *No, I thought ... No, it would be three fourths, one and three fourths is three fourths.* **What is a fraction to Jo? Only part/whole? This belief is rife in out-of-school experiences and is situated in the introductory fraction activities with which she had success.**

Rebecca: *Yeh, but ...*

Lisa: *Yeh, but when ...*

Rebecca: *You gotta count up how many to get a four ...*

Lisa: *So there'd be four, five, six, seven, ...***Lisa has internalised the relationship of wholes to parts in general (in this case, 1 one is 4 fourths) and counts to show why, rather than counting all like Rebecca.**

Jo: *Oh, Yeh.*
Jo gives in to the pressure of the other girls' insistent answers

Rebecca: *Four and three ... seven fourths.*
Whereas Rebecca begins to reconstruct her view from Lisa's explanation and by re-thinking her own procedure.

Lisa: *Seven fourths ... Yes, that's what I said.*

Lizzie: (Picking up her card) *One and three fifths.*

Jo: *Or what else?*

Lizzie: *Um ...*

Rebecca: *Or ...* (waving her hands)

Lisa: *What's five and three?* **Lisa tries to impose her method on Lizzie in order to get to the social purpose of playing the game.**

Jo: (Answering for Lizzie) *Or eight fifths*
Beginning to respond to Lisa's procedure.

Lizzie: *Yeh.* (Putting down the card)

Jo: *No, it wouldn't.* (picking up the card)
No, because ... **This conflicts with her earlier view**
But Rebecca overrules the doubt, now using the thinking she has constructed from Lisa.

Rebecca: *Yeh, because it's five in it and three, five and three is eight.*
Jo seldom persisted when she did not understand. Even when she could solve a problem successfully, she was quite prepared to be overruled by the more dominant students. She seemed to regard her social position in the class as very important and rarely created or continued a conflict. To her, capitulation was more acceptable than confrontation, negotiation and resolution. As a result, Jo's construction of fraction knowledge remained incomplete.

Jo: (Picking up the card for her turn) *Okay* (pointing and counting) *Um, one, two, three, four. Four and one third or ...* (pointing and counting again) *one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve and one fifth ... Oh! ... or thirteen and three, thirteen ...* (looking puzzled)

Rebecca: *Thirteen thirds if they're equal ... Yeh, thirteen thirds.* (sounding impatient)

Jo: *Yeh, would it be?* (in a questioning tone to Rebecca)

Although Lisa and Rebecca constructed appropriate, sophisticated strategies for renaming improper fractions and mixed numbers, Jo and Lizzie often avoided situations where other players checked each person's results, ostensibly to speed up the playing, but actually to avoid the conflict with previous ideas. In this way, the social demands of playing were used to avoid constructing and coming to terms with the social conventions of the more advanced mathematics.

TENSIONS BETWEEN THE SOCIAL CONSTRUCTION OF MEANING AND THE SOCIAL CONVENTIONS OF THE CLASSROOM

This episode shows how, even with the use of carefully planned materials and engaging activities, children can acquire rote routines for forming answers rather than build for themselves the underlying concepts. Further, not all children will engage in the social actions required for negotiation of meaning, let alone participate in the conflicts needed to overcome cognitive obstacles resulting from inadequate earlier concepts. Indeed, children may well alter the conditions of the situation in which they are placed to avoid perceived difficulties altogether or simply to allow them to achieve their own objectives, which may be very different from those mathematical end-points the teacher had in mind.

It also raises concerns about the manner in which we might need to structure a constructivist classroom if the type of learning theoretical considerations lead us to anticipate are going to occur. Our object as mathematics educators is now to provide rich situations and experiences out of which particular mathematical concepts might be able to emerge, to guide and facilitate the construction of mathematical ways of looking at these situations, and to come to terms with the ways of thinking that each individual has formed in order to do this. In this way we may be able to make sense of each student's mathematical world-view and endeavour to assist students to build for themselves the socially-accepted concepts, processes and, sometimes, even the procedures of what is felt to constitute mathematical knowledge. In time, and with further experience, we may be able to develop situations in which "the outcomes or products, which from a psychological perspective are described as "mathematical knowledge" appear as social accomplishments of the specific culture" (Bauersfeld, 1991 p. 11).

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