MULTIMODAL FUNCTIONING IN NOVEL MATHEMATICAL PROBLEM SOLVING

KEVIN F COLLIS, JANE M WATSON and K JENNIFER CAMPBELL
University of Tasmania

Problem-solving in school mathematics has traditionally been considered as belonging only to the mode of thinking concerned with making logical connections between data and the mathematical model and then teasing out the relationship between the variable in the model and the concrete symbolic mode. Little, if any, attention has been given to the place of the intuitive processes in the context of mathematical problem-solving. The paper will present some of the results obtained in the early stages of the study.

The data presented in this study were collected as part of a larger study which considered in more detail the strategies used in solving different types of mathematical problems (Collis, Watson and Campbell, 1991). These strategies are considered here as is the relationship of strategy selection to performance on tests commonly used to gauge mental imagery and creativity. The selection of strategy used was judged in relation to recent work on multimodal cognitive functioning.

Biggs and Collis (1991) point out that much of our thinking in the area of problem-solving is multi-modal. In particular, students at the early high school level, when presented with a novel mathematical problem, have available three modes of intellectual functioning which they can bring to bear to seek a solution, sensori motor, ikonic and concrete symbolic. The first of these is usually not of great significance in school-based problem solving and will not be discussed here. The second is highly developed by the time students reach the age level of interest here and has reached this high level largely without school help. The third is very dependent on school-based teaching, and the subsequent learning, of the concrete symbolic systems of reading, writing and arithmetic - in this paper the last is the major concern.

The move to the concrete symbolic mode marks a significant shift from a direct imaging of reality to a written, higher order symbolization of reality. These second order symbols form a system with direct referents in the experienced world. The symbols also have a logic and an order between themselves which are represented directly in reality and give us one of our most powerful tools for acting on the environment. These symbol systems include written language, mathematics, maps, musical notations and so on. Collis and Romberg (1991), analysed a sample of the kinds of open-ended problem solving items used in a variety of recent assessment projects and found that all of the projects that they analysed in their study made use of open questions. These consisted of items in which the students had to construct their own responses. Although the items varied on many dimensions, they all set out to test the higher order aims of mathematical problem solving and to reveal the student's reasoning as he or she moves towards a solution. The basic format for this type of item came down to one in which the context was set by a series of propositional statements followed by questions to which the student was expected to construct a response. The student needed to take the given propositions and decide on a course of action which might be schematized as in Figure 1.

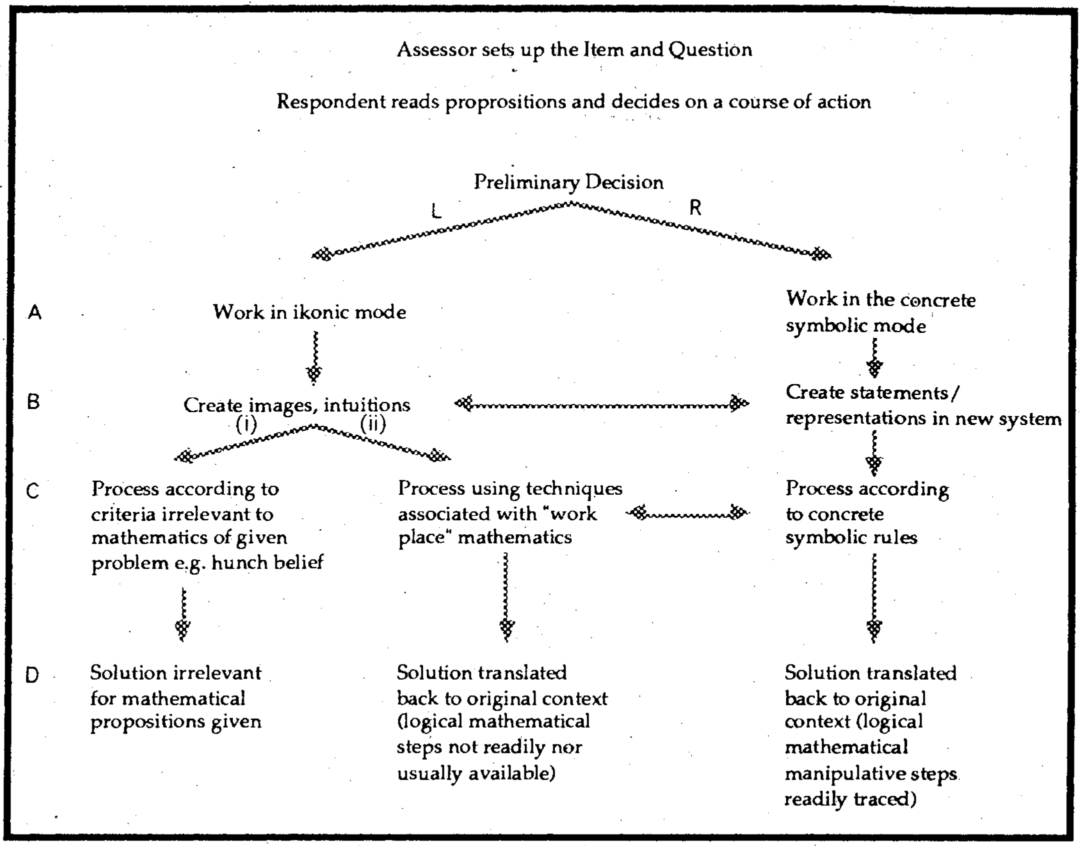

Figure 1: The Problem Solving Path (adapted from Collis & Romberg, 1991)

An initial research question in relation to the model in Figure 1 concerns the relationship between (a) the strategy choices made while problem solving and (b) scores obtained on tests traditionally used to assess visualization and intuitive creativity skills hypothesized to be important factors in ikonic functioning. The latter were obtained by using the Betts Questionnaire on Mental Imagery (QMI) (Sheehan, 1967), the Gordon Test of Visual Imagery Control (Gordon, 1949) and the well-known 'Uses of a Brick' Creativity test (see Guilford, 1967).

THE STUDY

The Items

Problems were selected which could be solved by techniques associated with the different courses of action a student might decide to take (see Figure 1). Two categories of problems were identified according to expected solution strategy, those which would suggest the use of concrete-symbolic processes (CS) and those which would suggest ikonic processing (IK). On this basis, two sets of problems were prepared for use in interviews (see Collis et al, 1991) with both categories of problem present in each set.

The following problem is an example of a CS item:

Chickens and Goats Problem. *A farmer is counting the chickens and goats in his yard. He counts a total of 50 heads and 140 feet. How many chickens and how many goats does the farmer have?*

The most likely methods of solution involve symbols and diagrams; that is, methods that are primarily school-taught, based on concrete symbolism and traditionally associated with mathematical problem solving in the school context. An example of a second type of problem, where a different set of solution methods would be expected, is the following:

Cube Painting Problem_ *A cube that is 3 cm by 3 cm by 3 cm was dipped in a bucket of red paint so that all of the outside was covered with paint. After the paint dried, the cube was cut into 27 smaller cubes, each measuring 1 cm on each edge. Some of the smaller cubes had paint on 3 faces, some on 2 faces, some on only 1 face, and some had no paint on them at all. Without drawing the cube, explain how you would find out how many of each kind of smaller cube there are.*

This problem would appear to involve a significant visual or imaging, intuitive component, even if the individual attempts to draw a diagram or picture during the course of seeking a solution.

Sample and Test Administration

Two Advanced Mathematics classes in a Tasmanian High School were selected as the sample. There was a total of thirty-eight students altogether, nineteen in year 9 (mean age: 14 years 10 months; 11 boys, 8 girls) and nineteen in year 10 (mean age: 15 years 10 months; 12 boys, 7 girls). At the first testing session the three group tests noted earlier were administered, two to assess aspects of each student's ability to image situations and one to obtain a measure of each student's creativity.

The Interviews

Sixteen students were selected for interviewing individually on their problem solving strategies. For each interview, the interviewer sat next to the student. Problems, or parts of problems, were presented to the student one at a time on cards. Once the problem was presented students could start solving the problem whenever they were ready; each had a pen and paper, ruler and calculator available if required. The order of presentation of the problems was counterbalanced. In addition, to check on the effect of images associated with each part of a question on the method of solution, some problems were serialised. Students were asked to tell the interviewer everything that came into their minds while reading the problem (or a part of the problem) and while solving it. During the course of the interview, the interviewer prompted the student, if necessary and where appropriate, by asking if they 'saw' any aspects of the 'story', if they were anticipating what the problem was about or what information they thought they still needed to solve the problem. The interviews were tape-recorded.

PLAN OF ANALYSIS

Categorisation of the Interview Data

A variation of Haylock's Think-Board (Haylock, 1984) was used to explore the students' responses in relation to the mode (CS or IK) which each individual statement apparently represented. The Think-Board was adapted so that it could be used to record the problem solver's actual 'moves' during the process (see Figure 2). The broken line divides the board into two regions, the upper region represents concrete symbolic (CS) responses while the lower half represents ikonic (IK) responses. Each of these regions is divided into three smaller regions by the diagonals, these smaller regions represent subjects of the CS and IK regions of which they form a part.

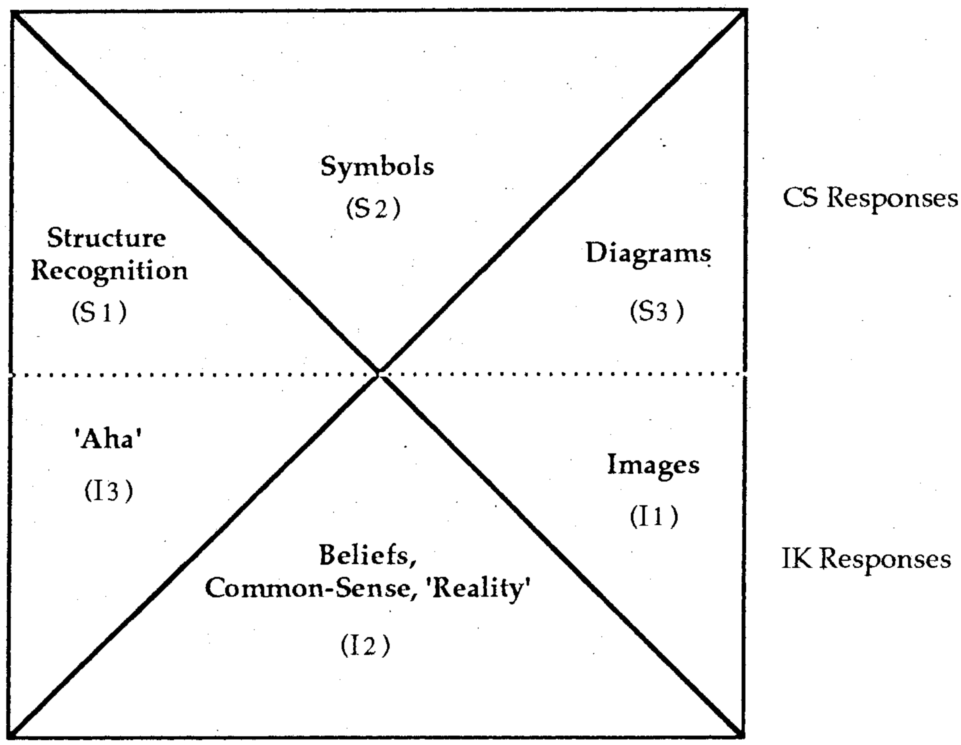

Figure 2: The Adapted Think-Board

Starting at the top left, the smaller regions are defined as follows: *Structure Recognition:* recognition of a CS structure in a problem as similar to a previous example; *Symbols:* use of a CS procedure; *Diagrams:* the use of a CS type diagram, e.g. a graph; *Images:* reporting visual images related to the problem or its context, e.g. "I can see the pieces of the cube with one side painted."; *Reality, Beliefs etc:* use of real world experience which appears to have some 'practical' relationship to the problem, e.g. "Everyone knows that you can't run a car without petrol!"; and *'Aha' Experience:* a sudden, apparently unbidden, 'insight' into the structure of the problem, usually visual.

Interviews were transcribed and each student's responses to each question were mapped onto a Think-Board. The number of responses falling in the upper (CS) half of the board and the number falling in the lower (IK) half of the board for each question were determined and used as the basis for the statistical analyses. Where a response was seen to be a combination of more than one section of the Think Board it was decided that it should contribute to the score for each of those sections. Examples of responses and their classification for the two problems noted earlier are shown in Figure 3. They illustrate two very different approaches in terms of ikonic and concrete symbolic functioning.

Example 1. Cube Problem with resulting scores IK 7, CS 1
I am imagining a whole cube that isn't cut up. (I_1) I can see the cube being cut up. (I_1) Now I can see the cube in slices coming away from each other and I can see how many have 2, and how many have 3... (I_1) It is like a Rubix cube. (I_2) [From this image the student worked out: 3 sides (I_1), no sides (I_1) and 2 sides (I_1).] Then I added the numbers together and found that there were 6 left so there are 6 with one side painted. (S_2)

Example 2. Chickens and Goats problem with resulting scores IK 2, CS 5
[On reading the question] I thought of the farmer bending down, counting the legs on the livestock after counting the heads. (I_1) [Pause] A chicken has two feet. (I_2) .. I'll call the chickens X and the goats Y. (S_1 and S_2) [Set up simultaneous equations (S_2) and solved using the substitution method. (S_2)] There are 20 goats and 30 chickens. (S_2)

Figure 3: Examples of responses and their classification.

RESULTS AND DISCUSSION

The results of the data gathered may be looked at most easily in relation to four key questions.

Question 1: Did students use the same primary strategy (CS or IK) regardless of the type of problem set?

Cochran *Q*. tests revealed that when the data were considered using all ten problems, the primary strategies used by the students were not consistent across the two different types of problems (CS & IK) ($Q = 38.86, p<.001$). This general finding also held for each set of 5 problems considered separately ($Q = 26.52, p<.001$): $Q = 27.30, p<.001$). Further analysis to check whether primarily IK strategies were applied in response to CS problems produced non-significant results. ($Q = 3.00, p<.10$). Inspection revealed that the strategies applied in these cases were primarily CS. Likewise it was found that students did not consistently apply CS strategies to IK problems ($Q = 0.00, p<.99$); rather IK strategies were primarily used to solve IK problems. These results seem to indicate that over and above any individual differences in the use of ikonic or concrete symbolic strategies, the nature of the problem itself is highly significant in determining which type of strategy will be used.

Question 2 (which supplemented Question 1): Did the strategies used depend upon the obvious characteristics of the item?

The dependent *t*-test performed on the pooled CS and pooled IK responses of the whole sample for all items supported the view that *a priori* CS problems elicit more CS responses than *a priori* IK problems ($t(15) = 5.98, p<.001$). It was clear that significantly more CS responses than IK responses were given to CS problem types ($t(15) = 2.40, p<.05$), and significantly more IK responses than CS responses were given to IK problems ($t(15) = -3.42, p<.02$). These results support the conclusion that it is the apparent nature of the problem itself that determines the strategy to be used.

Table 1: Problem Category and Type of Response: Total Number of Responses of 16 Subjects adjusted for Set Type

	CS-type Problems	IK-type Problems
CS Responses	89.2	26.0
IK Responses	56.4	55.6

Question 3: What was the typical path followed by students in the process of solving each type of problem?

The total number of times that students changed modes per type of question was 35.8 for the CS-type problems and 22.6 for the IK type problems, giving means per student of 2.2 (SD=1.1) and 1.4 (SD=1.4), respectively. The dependent *t*-test on the means for each student of the number of modal changes in each type of question produced a significant result ($t(15) = 3.10, p<.02$), the direction of the differences indicating that students tended to change modes significantly more often when answering CS-type problems than when answering IK-type problems. A dependent *t*-test on the pooled CS and IK responses (see Table 1) showed significantly more IK responses to be given to CS problem types than CS responses given to IK problems ($t(15) = 3.97, p<.001$). This may suggest that students are more likely to utilize IK strategies if they find some degree of difficulty in solving CS problems and are unable to use CS strategies effectively. In the case of IK problems, however, students are unlikely to use CS strategies to assist in their problem solving pathway. In summary, closer inspection of the data summarised in Table 1 indicates that when students are working on a CS approach and find some difficulty in proceeding, they are more inclined to use an IK approach to make progress, whereas when they use an IK approach as the initial step, this either solves the problem or they discontinue their attempt.

Question 4: How did the specific individual characteristics of the students, as measured by standard assessments of imaging and creativity, relate to the nature of their strategies?

The results from the Mann-Whitney *U* tests performed on the pooled CS and IK responses for CS and IK type questions indicate that the degree of overall Vividness may be important in determining the type of strategy adopted by the student during a problem solving task. This effect was found to be approaching significance even with the small sample tested ($U = 17.5, p = 0.06$). Since Vividness is a general measure of vividness of imagery which includes such components as kinaesthetic, tactile and aural imagery, among others, the visual subscale (VisVivid) was treated as a separate variable. Further analyses using the Mann-Whitney *U* test indicated that it is this visual component of imagery that strongly influences the student's selection of the most effective strategy to be used in a problem solving task ($U = 6.5, p<.001$). It appears that students with high VisVivid scores are more likely to produce IK responses than their counterparts with low VisVivid scores. Finally, measures of both Controllability and Creativity were found not to contribute significantly to the type of strategy adopted by the student during specific problem solving tasks (U= 22.5, p>.05 and U = 27.5, p>.05, respectively). In other words, students with high Control scores in relation to visual imagery, or with high Creativity scores are not significantly different from students with low Control or low Creativity scores in the content considered in this paper.

CONCLUSION

Within the limitations set by the data and the small sample, this study highlights some important factors which must be taken into account in the teaching, learning and assessing of school mathematics especially as it relates to problem solving in novel situations.

1. There seem to be two basic approaches to problem solving at this level, one based on CS, school taught procedures, the other related to IK mode processing. The latter, in this study, means appeals to common sense, everyday life and visualisation, and the use of intuitive reasoning. It is interesting to note that students, who see what they believe to be a solution using ikonic approaches, see little point in backing up their solution by an appeal to mathematics.

2. In the course of solving a problem, students will move from mode to mode but are more likely to move from CS to IK when a school based method fails to satisfy rather than vice versa. They do however, move both ways, in the case of IK to CS procedures the move is most often made only after the problem is really solved as illustrated with the student response to the Cube Painting Problem.

3. The characteristics of the problem and its context appear to be the basic factors in determining the initial strategy, particularly when CS type problems are presented.

4. Students who show heightened overall Vividness and have the ability to obtain clear visual representations of a problem in developing ideas tend to prefer using IK strategies during problem solving tasks.

These findings, tentative as they must be because of the limitations of the study, have two important implications for mathematics teachers in particular. First, those responsible for assessing mathematical problem solving at this level, should take into account the fact that the obvious problem characteristics may determine the basic strategy of the students. Second, all teachers should be aware of the significance of the IK mode in problem solving and adjust their mathematics teaching and assessing accordingly.

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