LETTING CHILDREN DESIGN THE CURRICULUM

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INTRODUCTION

Since 1989 the Basic Skills Testing Program (BSTP) in New South Wales has gathered data on the performances of some 220 000 year 6 and 120 000 year 3 children on tests of language and mathematics. In mathematics, each of the three strands of Number, Space, and Measurement are defined separately in the reports prepared for teachers and for parents.

This paper describes an exploration of children's responses to problems in the tests in search of any underlying general cognitive performance in mathematics. Some conclusions about order of difficulty within strands and links between strands are discussed. The data used is the responses of about 160 000 students to over 120 problems from the 1989, 1990 and 1991 year 6 tests.

OVERVIEW OF BSTP

'Basic Skills' in Numeracy was defined as that content outlined in the course of study for mathematics (NSW Department of Education, 1989). This syllabus emphasizes problem-solving in real life contexts; the numeracy tests were required to reflect this emphasis. Each year 6 test had separate stimulus material (sale catalogue, magazine, scrapbook) which provided the context for the problems posed in the test.

With such a large number of children to be assessed, the decision was made to use optical mark-sense readers, which allowed children to respond to problems by marking their booklets with an ordinary HB pencil. Within the constraints of machine-readable responses, many questions were developed in as open-ended a form as possible.

Parents received a report on their child. Teachers received a report on each child and a report on the performance of their school group.

The development of the BSTP, including the nature and purpose of the reports provided, is described in the report of the 1989 program (Masters, Lokan, Doig, Khoo, Lindsey, Robinson and Zammit, 1990).

TOWARDS A "CHILDREN'S CURRICULUM"

What an analysis of system-wide assessment can give is a picture of the children's preferred sequence of the curriculum. For example, when considering fractions, many curricula recommend that unit fractions be taught in the order $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and so on. This appears to be a reasonable thing to do. On the other hand, research indicates that children's 'difficulty order' is $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{3}$ (Masters, 1987).

In much the same way, Stern proposed a 'child's order' to the teaching of basic number facts in place of the traditional order, which followed common numerical order (i.e., first the 2 times tables, then the 3 times, then the 4 times, etc.) (Stern and Stern, 1971).

In the BSTP context, the children's preferred sequence may not be just a single discrete list of topics but also include interactions between topics described in different strands.

In order to understand what it is that the children are telling us, it is necessary to build a picture from the information they have provided. An analysis of the responses from each strand (Number, Space and Measurement) using an Item Response Theory (IRT) approach makes it possible to build such a picture. (For a full explanation of these techniques see Wright and Stone, 1979. Their use in the BSTP is explained in Masters et al, 1990).

When problems are placed along a continuum defined by the difficulty that children had in answering a problem correctly, it becomes possible to understand what we are being told. Figure 1 shows these continua for each of the three strands for BSTP Numeracy in year 6.

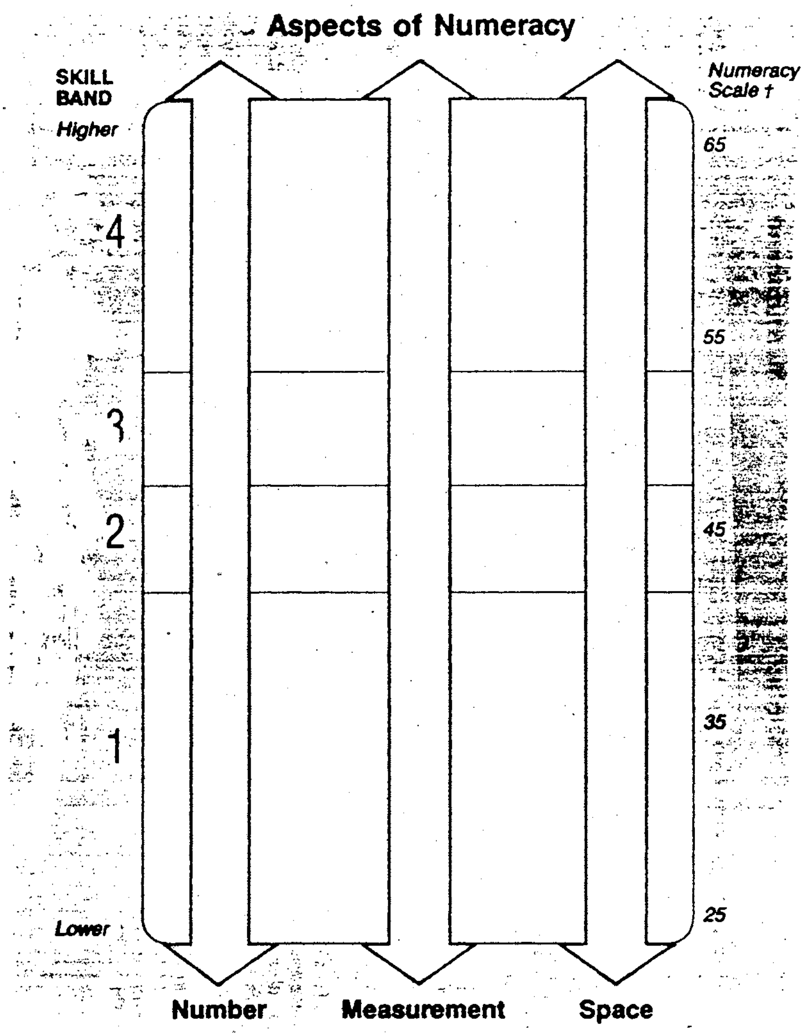


Figure 1: Difficulty continua for the three strands for Numeracy, Year 6.

It is possible now to analyze those problems which cluster at particular points along these continua. This analysis involves, in the first instance, describing the problems in terms of their curriculum content. This procedure leads to the creation of 'skill bands' or 'skill levels' (Masters et al, 1990). In this way "the ... scale developed to measure performance ... reflect[s] the student's level of maturity in that domain [curriculum content]" (Romberg, 1989: page 15). Figure 2 shows the skill band descriptions for year 6 1991.

AT BAND AND THE BANDS LISTED BELOW IT, STUDENTS IN THESE BANDS ARE GENERALLY ABLE TO:

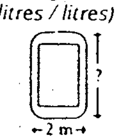
Number	Measurement	Space
• solve a problem using division when the reverse process is suggested by word clues • recognise square numbers *(e.g. 4, 9, 16, 25)* • solve problems involving several steps, including 'rounding off' • estimate a percentage *(nearly 50%)* from a pie graph	• solve problems involving a mixture of measurement units *(grams / kilograms* or *millilitres / litres)* • work out length from a scale drawing on which width is shown  • mark on a scale the increase in water level when objects *(four marbles, 25 mL each)* are dropped into a container	• interpret information shown on a line graph *(the first 100 kilometres took about ...?)* • work out how many identical 3D objects will fit into a given space *(6 cm tennis balls in a 12 cm x 12 cm x 12 cm box)* • work out the number of objects represented by part of a pie graph • choose a shape which can be used to form a pattern without gaps *(to tile a floor)*
• use several steps to locate information *(postage costs)* in a table • work out a fraction of a quantity *(in a recipe)* • estimate how many items *(at $1.35 each)* can be bought with a given amount of money *($5)*	• choose the best estimate for an area *(car windscreen in square metres)* • recognise the temperature of boiling water • convert 12-hour time to 24-hour time • estimate the masses of everyday objects *(Which object weighs about one kilogram?)*	• identify the top view of a model given a drawing which shows a different viewpoint • recognise the simple shapes which combine to make a given shape • use compass directions and landmarks to find a position on a map
• choose the calculation to give the cost of items sold in sets *(packs of 4 tins cost $1.60; 8 tins cost 2 x $1.60* NOT *$1.60 ÷ 2* or *8 x $1.60* or *$1.60 ÷ 8)* • estimate the cost of **one** item when the price for a set is given *(6 pens for $1.85)* • work out whether to use +, −, x, ÷ to solve a problem	• solve a problem involving the conversion of length units *(centimetres to metres)* • work out and compare the areas of shapes drawn on a grid • locate a date on a calendar *(where the days are shown by initial letters)*	• match a box to the cut-out from which it is made • identify the compass direction of one place from another
• add or subtract four-digit numbers where trading is needed *(2053 + 1674; 6125 − 4038)* • use word clues like **altogether** to work out that multiplication is needed to solve a problem • work out that subtraction is needed when diagrams and word clues like **difference** are given	• work out a time which is in the next hour [on a digital clock] *(6 minutes later than 12:57 p.m.)* • estimate the height *(in metres)* of an object visible in the classroom *(doorway)*	• show the reflection of a pattern on a grid • follow instructions to find a position on a simple grid *(gameboard)* • use coordinates to find a place on a simple map *(Colour in the building at position E2.)*

Figure 2: Skill band descriptions (Year 6, 1991).

EXPLORING THE DATA

The method used to explore the 1989,1990 and 1991 year 6 data was as follows:

* all problems relating to a strand were composed into a single list so that number, space and measurement problems could be examined separately;

* the problems on these lists were arranged in difficulty order as determined from the students' responses, using an IRT model;

* problems listed were then grouped to form four levels or 'skill bands' (Masters et al, 1990);

* each problem was examined to decide the type of mathematical skills required to complete it;

* the fundamental conceptual elements being used by children to solve a problem were inferred;

* the conceptual content of each strand for each level was compared within and between strands.

To illustrate the method, its application to the Number strand is described in detail below. The first level/skill band described is of those problems that the children found to be the easiest, and subsequent levels are in order of increasing difficulty.

Band One

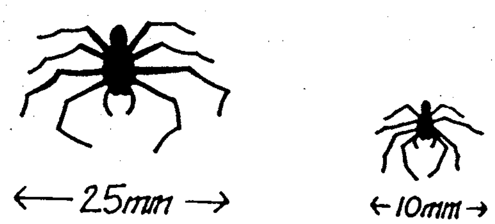
The problems at this level begin to introduce mathematical conventions as well as obvious synonyms for number operations. Key words and phrases replace the symbols of mathematics and it is the ability to translate from the real world to that of symbol and convention that marks those that succeed at this level.

38 6297 people went to the BIG X sale
In this number, what does the '2' stand for?

○ 2 ones
○ 2 tens
○ 2 hundreds
○ 2 thousands

Figure 3: Question 38, 1989

2 Here are two spiders.



What is the difference between the widths of the two spiders?

○ 250 mm ○ 15 mm
○ 35 mm ○ 10 mm

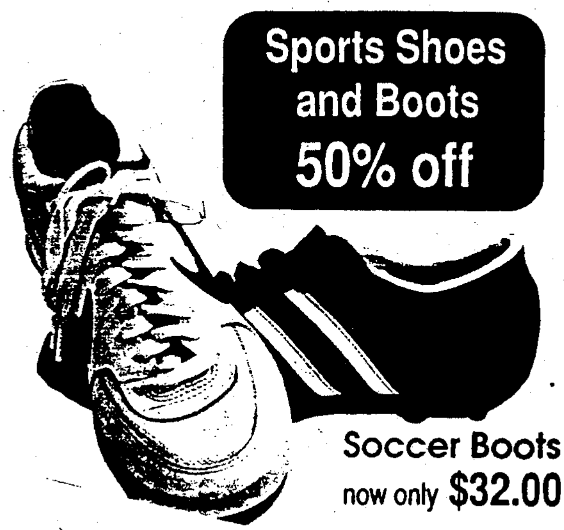
Figure 4: Question 2, 1991

Question 38, 1989: (Figure 3) - This problem is about the conventions of the decimal system of numeration. The differences between a single '2' and the '2' used in the context of a four-digit number can only be memorized like any other convention, for there are no clues to help the children here. The high success rate on this problem demonstrates that this convention is accessible to most children.

Question 2, 1991: (Figure 4) - If the word clue 'difference' is recognised as a cue to subtract, the problem is solved. Again, the high success rate demonstrates that most children do recognise this clue.

Band Two

Many problems have specific clues, such as 'altogether' indicating that some combining process like addition is required. Contextually based problems have their solution processes 'hidden' within the context of the problem, with no such obvious word clues. Although most children can perform arithmetic operations correctly, the ability to generalize the meaning of, say, addition, to all additive situations is less common. Extracting enough information in order to choose the correct operation to use is the key to success at this level. In real life this may be the most utilized facet of problem solving.



25 Mike chose some soccer boots at the BIG X sale.
His mother paid for them with a $50 note.
Colour in the notes and coins that would make up the change they got.

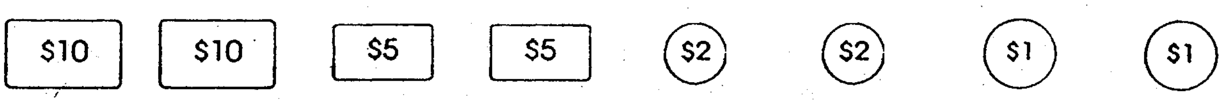


Figure 5: Question 25, 1989

Question 25, 1989: (Figure 5) - To calculate the amount of change due in a particular transaction may well be straight-forward, but to actually 'make' the change from a selection of notes and coins involves an additional degree of competency. The skills necessary to manipulate the multiple data, together with correctly performing the chosen processes in combination is typical of problems at this level.

8 Alex is doing some French Knitting. Each centimetre of knitting has five rows of stitches.

How many rows of stitches are there in 10 centimetres of knitting?

- ○ 2 rows of stitches
- ○ 5 rows of stitches
- ○ 15 rows of stitches
- ○ 50 rows of stitches

Figure 6: Question 8, 1989

Question 8, 1989: (Figure 6) - A thorough understanding of the concept of multiplication is necessary to see that it is the correct interpretation of this problem. Either the use of a 'one-to-five' ratio can be used (the function view of multiplication) or a 'ten lots of five' approach (an additive view). In either case a good grasp of the concept is required. The most common error at this level is to confuse multiplication with division, an indication of an uncertain grasp of the concepts involved.

Band Three

Children who have developed the necessary abilities and skills to master the problems at this level are able to extract pertinent information from competing, irrelevant information. Being able to ignore irrelevant data is a sophisticated and valuable skill especially necessary to solve those real life problems involving tables. At one level the table may simply be a list of prices, at another, more complex level, the table may be a bus or train time-table which involves relating at least two pieces of information. To simultaneously vary two aspects of tabular information to gain a solution is an even more difficult exercise in relationship building.

Delivery charges

	Up to 50 km	50 to 100 km	Over 100 km
Under 10 kg	free	$2.00	$8.00
Over 10 kg	$5.00	$10.00	$15.00

42 Sofia lives 83 km from the store. How much will it cost her to have a 3 kg Moulding Set delivered?

- ○ Free
- ○ $5
- ○ $2
- ○ $10
- ○ $8
- ○ $15

Figure 7: Question 42, 1989

Question 42, 1989: (Figure 7) - To enable a solution to be found for this problem, it is necessary to first determine which distance category is to be used. Next, it must be understood that the (3kg) parcel belongs to the 'under 10kg' class of parcels. The difficulty of combining and relating such pieces of information is evidenced by the proportion of students unable to deal successfully with this problem.

11 The best estimate for the cost of **one** sticker is

○ about 30 cents. ○ about $1.00.

○ about 60 cents. ○ about $15.00.

Figure 8: Question 11, 1990

Question 11, 1990: (Figure 8) - The relevant information here is the price of a set and the number in each set. The correct process must then be chosen and a rounding performed either before (a better strategy) or after the division.

Band Four

Children who have fully developed understandings of concepts, together with the appropriate related skills are able to solve quite complex problems. The ability to round prices to whole dollars is very important in the daily life of everyone; whether the need is to select the correct notes and coins to pay for goods, or whether it is simply to check the accuracy of the bill itself, working with approximate costs derived from such rounding is without doubt an essential skill. Further, in real life there is often no language clue to which arithmetic processes one should use to solve a problem. Solving these problems would be impossible without a deep understanding of the fundamental concepts of mathematics.

33 Sam and her mother went to the café. *Their bill is on page 13 of the scrapbook.*

What was the total cost, to the nearest dollar?

○ $4 ○ $6
○ $5 ○ $7

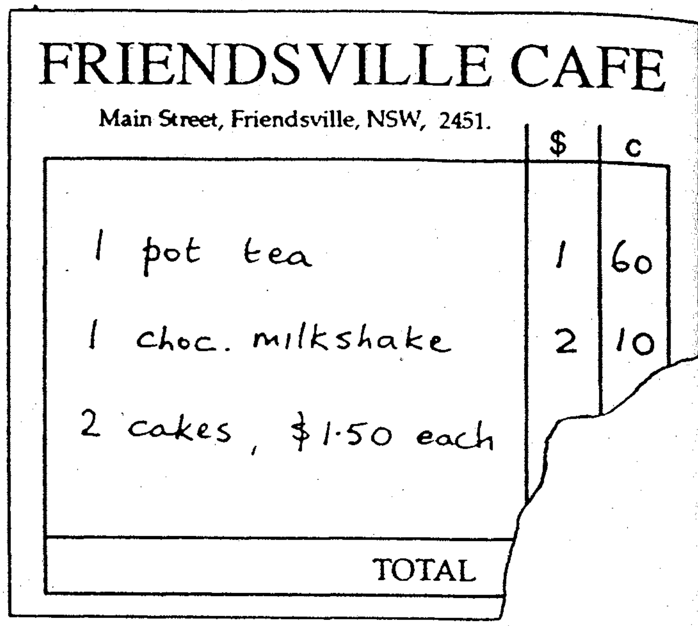


Figure 9: Question 33, 1991

17 If you become a subscriber to the magazine (see back page), how much do you save in a year?

○ 35 cents ○ $4.15

○ 75 cents ○ $4.50

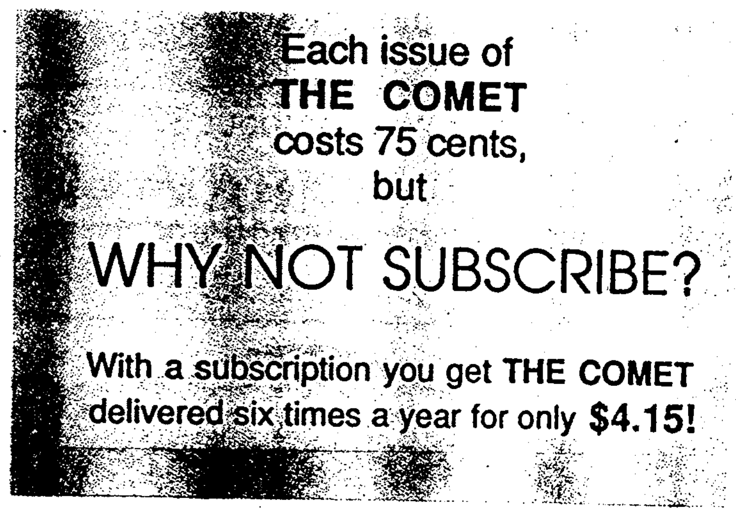


Figure 10: Question 17, 1990

Question 33, 1991: (Figure 9) and Question 17, 1990: (Figure 10) - Both these complex problems involve the use of several operations in combination. In the first, doubling, adding and rounding are required. In the second, multiplication, dollars/cents conversion then subtraction are required.

The same procedure was applied to problems in the space and measurement strands.

INTERPRETING THE RESULTS

What can we learn from all this information that the children have provided through their responses to these problems?

The most obvious information concerns the order of the assessed content, the increasing complexity of the tasks, or more interestingly, the increasing maturity and sophistication of the children's thinking. This is precisely the notion underpinning Principle 4 for new models of assessment explicated by Romberg (1989), which says: 'based on the tasks administered to a student in a domain, their complexity, and the student's responses to those tasks, the information should ... yield a score for that domain ... Note that this score is not just the number of the correct answers a student has found. ... The intent of the score is that it reflect the degree of maturity the student has achieved with respect to that domain.' (page 14). It would be a mistake though to think only in terms of discrete domains. As Romberg (1989) points out 'there are several different aspects of doing mathematics within any mathematical domain.' (page 14).

Table 1 below summarises all levels for the three strands of the Year 6 assessment. The most complex level is first, with complexity decreasing going down the table. Thus Table 1, read vertically within each strand, summarises the children's order of difficulty of concepts contained in the NSW mathematics syllabus.

NUMBER	SPACE	MEASUREMENT
Solve problems involving several steps	Picture a complicated object from a drawing in which parts are hidden.	Solve problems involving a mixture of measurements
Select information from a table.	Picture how shapes would look when reflected.	Work out sizes on scale drawings.
Know which operation to use.	Use knowledge of East-West compass directions.	Estimate measurements in everyday use.
Understand four figure numbers.	Compare the lengths of paths.	Read everyday instruments.

Table 1: Strand Content for Numeracy

In developing Table 1, typical problems that illustrate the response required have been used to facilitate the uncovering of any links between the strands.

A further simplification/generalisation of the level descriptors, focussing upon fundamental elements within the strands, makes even clearer any possible inter-relationships between strands, as can be seen in Table 2 below.

NUMBER	SPACE	MEASUREMENT
Multiple step problems.	Work with three dimensions.	Measure with several magnitudes.
Read data from two scales simultaneously.	Work with two dimensions.	Work with two different scales at the same time.
Know which operation to use.	Work with one dimension.	Estimate single measures.
Understand conventions.	Compare one dimensional attributes.	Read everyday instruments.

Table 2: Simplified Strand Content

Information about the types of cognitive functioning, and their generalizability across what are usually accepted as being different aspects of Numeracy, are clearly visible in Table 2. This information, provided by the children's responses, parallels the ideas of Collis and Biggs (1980), and provides a possible framework for a new curriculum, based upon children's 'preferences', as well as having implications for future curriculum planning and research.

Tables 1 and 2 are not complete: some possible areas for further investigation are

extend the levels downwards to include the year 3 data
try to identify other links between strands.

In curriculum planning

should the order in which topics, or parts of topics, are tackled in the present syllabus be revised?

is it possible to form a curriculum which deals, in order of complexity (from Table 2), with

simple facts and conventions,
application of basic knowledge,
problems with only two variables,
problems involving three or more variables?

In classroom activities

can the order of presentation within topics take account of children's preferences?
can topics making similar 'cognitive demands' be integrated?
can the frequency with which number, measurement, space topics are presented in isolation be reduced?
can assessment more often use tasks which require skills from several (sub)strands?

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