SOME PLUSES AND MINUSES OF RADICAL CONSTRUCTIVISM IN MATHEMATICS EDUCATION

NERIDA ELLERTON AND MA (KEN) CLEMENTS
Faculty of Education
Deakin University

RADICAL CONSTRUCTIVISM IN MATHEMATICS EDUCATION: AN OVERVIEW

The veteran, widely respected American mathematics educator, Robert Davis, recently stated that anyone "who observes mathematics education has to be impressed by the quite sudden eruption of 'constructivism' as a central concern of so many researchers" (Davis, 1990, p. 114). As we have observed elsewhere (Ellerton and Clements, 1991, pp. 53-56), there can be no doubt that since the mid-1980s many mathematics educators in different parts of the world have identified themselves with the so-called "constructivist" movement and with radical constructivism in particular.

Each of the keynote addresses at the 1987 annual International Conference of the Group for the Psychology of Mathematics Education, for example, was dedicated to the theme, and since then three major edited collections have been prepared in which the theoretical bases and implications of radical constructivism for mathematics education have been explored. In 1990 a collection of 12 papers was published by the National Council of Teachers of Mathematics in the United States (Davis, Maher, and Noddings, 1990), and in 1991 a book edited by Ernst von Glasersfeld and containing chapters written by leading American and European writers, appeared, carrying the title *Radical constructivism in mathematics education.* Another collection of papers with strong constructivist leanings (entitled *Epistemological foundations of mathematical experience*, edited by Les Steffe, and with an international writing team) is, at the time of writing, in press.

Ackermann (in press) has pointed out that not only are there as many definitions of constructivism as there are minds to construct them, but also that attempts by teachers to develop learning environments in accord with constructivist principles have not always met with success. As Duckworth (1987, p. 31) put it, in discussing the difficulties of applying Piagetian theory in the classroom, "either we [the teachers] are too early and they can't learn it, or we're too late and they know it already." The dilemma of constructivist teaching is that while educators do not wish to impose "right" answers, they still feel compelled to teach the subject matter.

The science educator, Strike (1987), made a useful comparison of the effectiveness of the terms "democracy" and "constructivism," however loosely defined these might be, to motivate apparently disparate groups of people to coordinated and productive activity. After noting that agreement has not been reached on the meaning of constructivism in education, Strike (1987) went on to say that just as the banner of "democracy" can unite people who in reality do not agree on many social and political issues, so, too, in education can the term "constructivism" serve as a unifying theme. He wrote:

> Perhaps it [i.e. the word "democracy"] at least serves to remind people that they share a common political position, or that they share common antagonists.

> "Constructivists" may function in a similar way. Whatever it means, if we are constructivists, at least we know that it is important to be interested in children's misconceptions, to describe how they think about science, that we have Piaget as part of our heritage, and that behaviorists are the bad guys. Loyalty and group identification is made of such stuff. Clarity and intellectual progress are not. (p. 481)

Duit (in press), in commenting on Strike's analogy, pointed to the danger of constructivism being only superficially adopted in education. This paper is concerned to identify some of the strengths and weaknesses of the present-day constructivist movement in mathematics education.

RADICAL CONSTRUCTIVISM

The fundamental tenet of the *radical* constructivist position is that mathematics is not an "out there" pre-existing body of knowledge waiting to be discovered, but rather is something which is personally constructed by individuals in an active way, inwardly and idiosyncratically, as they seek to give meaning to socially accepted notions of what can be regarded as "taken-to-be shared mathematical knowledge." As von Glasersfeld (1990), probably the best known advocate of the radical constructivist position in mathematics education, has stated:

> . . . knowledge is the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication. (p. 37)

According to von Glasersfeld (1990, p. 37), all good teachers know that guidance which they give to students "necessarily remains tentative and cannot ever approach absolute determination," because, from the constructivist point of view, there is always more than one solution to a problem, and problem solvers must approach problem situations from different perspectives.

The cornerstone of radical constructivist theory is Piaget's emphasis on action (that is to say, all behaviour that changes the knower-known relationship) as the basis of all knowledge. An individual gets to know the real world *only* through action (Sinclair, 1990; von Glasersfeld, in press). Hermine Sinclair, Piaget's successor in Geneva, described von Glasersfeld, who is regarded as the high priest of radical constructivism, as perhaps "an even more radical constructivist than Piaget" (Sinclair, 1987, p. 29).

For radical constructivists working in the field of mathematics education, the crucial issue is *not* whether mathematics teachers should allow students to construct their mathematical knowledge, "for the simple reason that to learn is to actively construct" (Cobb, 1990a). "Rather," Cobb (1990a) says, "the issue concerns the social and physical characteristics of settings in which students can productively construct mathematical knowledge." (For further commentary on constructivism, and on radical constructivism in particular, see for example, Cobb, 1986; Confrey, 1987; Dorfler, 1987; Kilpatrick, 1987; Labinowicz, 1985; von Glasersfeld, 1983).

During the past ten years the impact of radical constructivist thinking on mathematics education researchers and, increasingly, on teachers of mathematics in schools, has been considerable. Notwithstanding certain tensions between theory and practice (Cobb, 1988), there has been a wave of research aimed at identifying the roles of teachers of mathematics who wish to adopt radical constructivist approaches (see, for example, Wood and Yackel, 1990). It is appropriate, therefore, that we pause for a moment not only to reflect on whether the claims of radical constructivism with respect to mathematics education are justifiable, but also to assess which aspects of the application of radical constructivist thinking in school mathematics have been beneficial and which have not. We shall deal with the pluses first.

RADICAL CONSTRUCTIVISM IN MATHEMATICS EDUCATION: SOME PLUSES

1. "Ownership" of Mathematics by the Learner

Probably the simplest way of summarising the radical contructivist position in mathematics education is in terms of the notion of "ownership." Radical constructivists believe that, ultimately, mathematical knowledge is not something that is acquired by listening to teachers or reading textbooks, but is something that learners themselves construct through actively seeking out, and making, mental connections. When someone actively links aspects of his or her physical and social environments with certain numerical, spatial, and logical concepts a feeling of "ownership" is often generated. In such cases a learner is likely to make comments such as "I *know* this because *I* worked it out *myself.*"

Radical constructivists believe that this notion of "ownership" is powerfully relevant to school mathematics. They maintain that in the past *too much emphasis* has been placed on a linguistic communication pattern by which the teacher explained to students what they had to do, and how they could do it. Very few mathematics educators would deny that traditionally, school mathematics has been regarded by students as a fixed body of knowledge, owned by teachers, textbook and worksheet writers, external examiners, and by great, mysterious figures of the past (like Pythagoras and Euclid). During the 1980s, however, constructivist mathematics educators around the world (for example, Confrey 1987; Dorfler, 1987, 1989; Kamii, 1985; Labinowicz, 1985; Steffe, 1987, 1990) called for teachers to establish teaching/learning environments in which students, as a matter of course, created mathematics themselves and therefore came to feel that they "owned" the mathematics that they were learning.

2. Quality Social Interaction as Basis for Quality Mathematics Learning

Pateman and Johnson (1990) have claimed that it has been "constructivist" teachers of mathematics who have led the recent important movement towards establishing mathematics learning environments that nurture interest and understanding through co-operation and high quality social interaction. Such environments are likely to foster a kind of socio-cognitive conflict and challenge that stimulate learning. Pateman and Johnson maintained, as did Steffe (1990), that the belief that children construct their own mathematics out of their own actions and their reflections on those actions (in social settings) provides a new framework for those responsible for devising mathematics curriculum and school mathematics programs. According to Pateman and Johnson (1990), three aspects of curriculum need to be considered - content, methodology and assessment:

> content (which can hardly be rigidly prescribed in advance by the constructivist teacher, methodology (which probably needs to be idiosyncratic to children and

context), and assessment (particularly difficult for those so used to competitive ratings). The constructivist teacher will need to be somewhat of an opportunist, and also an able elementary mathematician willing to continue to learn both about mathematics and children in the attempt to develop them as autonomous creators of their own mathematics. (p. 351)

We believe that this call for richer, more expressive interaction patterns in mathematics classrooms is one of the most important outcomes of the radical constructivist movement.

Steffe (1990) has elaborated ten principles for mathematics curriculum design that are in keeping with the main radical constructivist thrusts, and Cobb (1990b, p. 208) has identified the following social norms for worthwhile whole-class discussion in mathematics classrooms:

1. Explaining how an instructional activity that a small group has completed was interpreted and solved;

2. Listening and trying to make sense of explanations given by others;

3. Indicating agreement, disagreement, or failure to understand the interpretations and solutions of others;

4. Attempting to justify a solution and questioning alternatives in situations where a conflict between interpretations or solutions has become apparent.

Most experienced teachers would like to think that these norms already apply in whole-class discussions that occur in their own classrooms, but mathematics classroom discourse analyses indicate that this is rarely the case.

Cobb (1990b, pp. 209-210) called for constructivist mathematics educators to develop a new context - a "mathematico-anthropological context" - that will assist coherent discussion on the specifics of learning and teaching mathematics. According to Cobb there is research support for moving to establish mathematics classroom environments that incorporate the following qualities:

1. Learning should be an interactive as well as a constructive activity - that is to say, there should always be ample opportunity for creative discussion, in which each learner has a genuine voice;

2. Presentation and discussion of conflicting points of view should be encouraged;

3. Reconstructions and verbalisation of mathematical ideas and solutions should be commonplace;

4. Students and teachers should learn to distance themselves from ongoing activities in order to understand alternative interpretations or solutions;

5. The need to work towards consensus in which various mathematical ideas are co-ordinated is recognised.

Many teachers of mathematics would accept all five of these points. But much still needs to be done, for all too often in school mathematics rhetoric and classroom realities do not bear much resemblance to each other (Desforges, 1989). Nonetheless radical constructivists are determined to refine and apply their ideas to mathematics classrooms however difficult and time-consuming this process might prove to be (Ackermann, in press; Steffe, 1990).

3. *Principles for Improved Mathematical Discourse*

We would wholeheartedly endorse the radical constructivist educator's desire to establish learning environments that result in students owning the mathematics they learn. There have been other positive features arising from the application of radical constructivist theories to mathematics education, though these are all related to the "ownership" issue. These pluses have been well summarised by Paul Cobb, whose writings have attempted to draw together the threads of the constructivist movement in mathematics education. Cobb and his co-workers at Purdue University sought to explore how the theoretical positions held by radical constructivists might most advantageously be interpreted in mathematics classrooms (see, for example, Cobb, Yackel, and Wood, 1992).

In the second half of the 1980s, there was a veritable flood of articles in the mathematics education literature on the nature of discourse in mathematics classrooms that built on earlier theoretical works (for example, Chomsky, 1957; Mehan & Wood, 1975; Bauersfeld, 1980). Cobb (1990a), in a paper entitled *Reconstructing elementary school mathematics*, summarised research which attempted to assess the effectiveness of the application of radical constructivist ideas to mathematics teaching and learning. He made five main points:

1. To claim that students can discover mathematics on their own is an absurdity.

2. Students do not learn mathematics by internalising it from objects, pictures, or the like. Mathematics is not a property of learning materials, structured or otherwise.

3. The pedagogical wisdom of the traditional pattern of first teaching mathematical rules and skills, and then providing opportunities to apply these in real life situations, is questionable. An alternative approach takes seriously the observation that from a historical perspective, pragmatic informal mathematical problem solving constituted the basis from which formal, codified mathematics evolved.

4. The teacher should not legitimise just any conceptual action that a student might construct to resolve a personal mathematical problem. This is because mathematics is, from an anthropological perspective, a normative conceptual activity (see Shweder, 1983), and learning mathematics can be seen as a process of acculturation into that practice. (Cobb (1990a) notes that certain other societies and social groups have developed routine arithmetical practices that differ from those taught in Western schools.)

5. Mathematical thought is a process by which we act on conceptual objects that are themselves a product of our prior conceptual actions, and from the very beginning of primary schooling, students should participate in and contribute to a communal mathematical practice that has as its focus the existence, nature of, and relationships between mathematical objects. For Cobb (1990a), understanding mathematics is constructing and acting on what he calls "taken-to-be-shared" mathematical objects.

The pluses identified in this section of the paper refer to important developments in mathematics education that can be linked, at least partly, to the radical constructivist movement. We turn now to some of the minuses.

SOME MINUSES OF RADICAL CONSTRUCTIVISM IN MATHEMATICS EDUCATION

1. The Constructivist Bandwagon

Despite the fact that much good has derived from the radical constructivist movement in mathematics education there is a possibility that the movement has questionable philosophical and linguistic foundations (Matthews, 1992; Suchting 1992), and that therefore the drawing of clear pedagogical implications at this stage is premature. Certainly, there has been considerable debate over how "constructivism" should be defined (see Cobb, 1986; Pateman and Johnson, 1990, p. 353). Furthermore, despite the detailed investigations of the group at Purdue University, led by Cobb (see Cobb, 1990b; Cobb, Yackel, and Wood, 1992; Krummheuer and Yackel, 1990; Wood, 1990), into what a constructivist mathematics classroom might look like, it is still the case that "constructivism remains relatively untried in the everyday classroom setting" (Pateman and Johnson, 1990, p. 352).

The eleventh annual conference of the International Group for the Psychology of Mathematics Education, held in Montreal in 1987, had, as its theme, the theory of constructivism, and one of the plenary speakers, Jeremy Kilpatrick (1987) began his paper by comparing the constructivist movement in mathematics education in the United States with the waves of religious fundamentalism that had swept that nation over the past three-and-a-half-centuries:

> A siege mentality that seeks to spread the word to an uncomprehending fallen world; a band of true believers whose credo demands absolute faith and unquestioning commitment, whose tolerance for debate is minimal, and who view compromise as sin; an apocalyptic vision that governs all of life, answers all questions, and puts an end to doubt - these are some of the parallels that might be drawn. (p. 4)

Such comment raises the question of the extent to which research findings support constructivist theories of how mathematics is learnt. It is also pertinent to ask whether modern analytic philosophers are united in supporting the major radical constructivist contentions, and we shall turn to that issue later in this paper.

Both Kilpatrick (1987) and Matthews (1992) have called some, though not all, radical constructivists to task for claiming that their theories represent something new in mathematics education. After drawing attention to many of the laudable, insightful, and progressive aspects of constructivist theory, Matthews went on to say that nonetheless one does not need to be a constructivist to agree with most of the pedagogical claims of the constructivists. He stated:

> Socrates' teaching of Pythagoras' theorem to the slave boy is an early and enduring model of teaching for understanding, Aquinas and the medievals stressed personal engagement with and attachment to the subject matter being taught, they spoke of a 'love of learning', Montaigne's essays on education advocate dialectical rather than didactic teaching, Dewey and the progressives emphasised class discussion, debate,

the testing and scrutiny of teacher and pupil opinions, as of course has Paulo Fréire in his many publications. Recently, Richard Peters, Paul Hirst, Israel Scheffler, Jane Martin and other philosophers of education stressed the importance of pupil comprehension and understanding for education.

Unfortunately, there is a tendency for those who march under the banner of radical constructivism to accuse all others of advocating and practising transmission modes of education. As Kilpatrick (1987, p. 22) has noted, radical constructivists "ought to be more tolerant towards competing theories" because "people who claim there are many possible ways to construct knowledge ought to be more friendly and understanding toward people who have failed to construct their theory."

2. Confusion on the Communication Issue

The hardline radical constructivist position on communication. Radical constructivist mathematics educators maintain that there is an inherent and inescapable indeterminancy in linguistic communication, and that the notion of "understanding" is more concerned with fit, rather than match. According to von Glasersfeld (1990, p. 36), "once we have come to see this essential and inescapable subjectivity of linguistic meaning, we can no longer maintain the preconceived notion that words convey ideas or knowledge and that the listener who apparently 'understands' what we say must necessarily have conceptual structures identical with ours." This line of argument has been maintained, without modification, by von Glasersfeld for almost a decade (see, for example, von Glasersfeld, 1983; 1988; 1989; 1990; 1991).

On this basis, conceptual schemas - and indeed concepts in general - cannot be conveyed or transported from one to another by words, but can only be abstracted from experience. That is to say, "language is not a means of transporting conceptual structures from teacher to student, but rather a means of interacting that allows the teacher here and there to constrain and thus to guide the cognitive structure of the student" (von Glasersfeld, 1990, p. 37).

Or, as Wheatley (1991, p. 10) puts it, "knowledge is not passively received, but is actively built up by the cognizing subject," and ideas and thoughts "cannot be communicated in the sense that meaning is packaged into words and 'sent' to another who unpacks the meaning from the sentences." Thus, Wheatley states, "our attempts at communication do not result in conveying meaning but rather our expression *evoke* [Wheatley's emphasis] meanings in another, different meanings for each person."

Opposition to the hardline position on communication. von Glasersfeld (1990) summarised radical constructivist beliefs on communication in a paper that was originally presented at the Sixth International Congress on Mathematical Education, held in Budapest. The official report of the discussion that followed the paper (see Steffe, 1988, pp. 106-108) does not reflect the considerable controversy that arose at the Congress with respect to the hardline radical constructivist position on communication. Many of those who took part in the discussion (one of the present authors, Clements, was present) believed that mathematical concepts can, in fact, be conveyed from one person to another by words.

There is a sense in which it is professionally dangerous to be seen to challenge radical constructivist dogmas on knowledge, understanding, and communication, for those who hold this point of view are usually so convinced that they are right that they cannot imagine the possibility that they could be wrong; furthermore, there is a tendency among radical constructivists in the field of mathematics education to view the unconverted as conservatives who hold to a transmission view of education. Despite this possibility, in a recent publication (Ellerton & Clements, 1991) we opposed the hard-core positions on language put forward by von Glasersfeld and Wheatley and, in particular, disputed Wheatley's (1991, p. 10) claim that "ideas and thoughts cannot be communicated in the sense that meaning is packaged into words and 'sent' to another who unpacks the meaning from the sentences."

To support our contention we invited readers to consider (see Ellerton and Clements, 1991, p. 55) the following statement, which would not cause too much surprise if it were found in a secondary mathematics textbook:

> I want you to imagine two regular polygons, whose sides are each 3 cm long. One of the polygons (Polygon A) has 10 sides, and the other (Polygon B), has 12. What is the size, in degrees, of each interior angle of Polygon A? What is the size, in degrees, of each interior angle of Polygon B? Would the interior angles be different if each side length had been 6 cm (rather than 3 cm)?

We contended that the above package of words *does* have the power to communicate unambiguously, *provided* the reader has prior knowledge of the terminology and concepts that are mentioned. Having received the "message" it is the reader's task to "unpack the meaning from the sentences." Certainly, different learners will do the unpacking in different ways, but that is not really the point being debated.

We recognise, of course, that many school children, on being asked to read the above paragraph (on polygons), would not construct the same meaning as that intended by the writers of the paragraph. Furthermore, we recognise that it is difficult (if not impossible) to get to know the meaning that another person would ascribe to that paragraph when reading it. But this does not contradict our claim that paragraphs such as the one on polygons *can and do* communicate shared meanings in a relatively unambiguous way to those who have the necessary background understandings.

Radical constructivists assert that with any communication, the person receiving the communication only makes sense of what is being communicated by actively and inwardly constructing an idiosyncratic interpretation of the input. We agree with that view, but would add that this is nothing more than expressing the meaning of the word "communication."

The many thousands of people around the world whose task it is to develop questions for external written mathematics examinations are constantly faced with the challenge of writing questions that define situations that demand the application of relatively complex mathematical operations in an unambiguous way. Rightly or wrongly, questions on such external examinations that can be interpreted in various ways, and therefore have more than one possible correct answer, are regarded as poor questions (unless, of course, open-ended questions, with many possible answers, are deliberately set - see Sullivan and Clarke, 1991). Clearly, the prevalence of written examinations in mathematics around the world over the past 150 years demonstrates that most educators believe that mathematical meaning can be communicated in unambiguous ways through words (and associated diagrams).

Like Cobb (1990a), we believe that "the hard-core Piagetian perspective downplays the role of linguistic activity in the development of abstract thought," and would support his contention that this "is a major weakness of current constructivist theorising."

Criticisms by Suchting and Matthews of the radical constructivist position on communication. We do not stand by ourselves in opposing the radical constructivist viewpoint on communication. Suchting (1992, pp. 224-226), for example, maintains that von Glasersfeld does not make sense in his utterances on the matter. After quoting a passage that appeared in a paper by von Glasersfeld in 1989 which asserted that constructivism denies "the existence of objective knowledge and the possibility of communicating it by means of language," Suchting (1992) then proceeded, in the manner of the analytic philosopher, to subject the meaning of the passage to intensive analysis:

> Firstly, what is the point of the last four words? Taken by themselves they would naturally be understood to state or imply a qualification of some sort. But there is no hint as to what this might be intended to be, and it seems impossible to see how 'objective knowledge', were it to exist, could be communicated *except* [Suchting's emphasis] by means of language of some sort . . . Secondly, consider the five words immediately preceding those just commented upon. They are also very puzzling in the context of the cited passage in which they appear. For if 'it', that is, 'objective knowledge', is asserted not to exist, and does not exist, then there can be no question of the possibility of communicating 'it' (linguistically or any other way), as distinct from the possibility of merely *seeming* [Suchting's emphasis] to communicate it. (p. 224)

Suchting (1992, p. 247) concluded that much of von Glasersfeld's doctrine of constructivism "is simply unintelligible" or, "to the extent that it is intelligible enough to provide some foothold for understanding and criticism it is simply confused."

3. *Oversimplication of the Ontological Question: What is Mathematical Knowledge?*

Another characteristic feature of radical constructivist theory is its rejection of the idea that some kind of absolute mathematical reality exists independent of the physical world (Labinowicz, 1985, p. 5). For most radical constructivists, mathematical knowledge is constructed as a result of mental activities that precipitate accommodation (that is to say, a change in an individual's mental model). From this perspective, the teacher's main task is to infer models of students' conceptual constructs, and then to generate hypotheses on how the students might best be given the opportunity to modify their mental structures in desirable ways (von Glasersfeld, 1990, pp. 33-34).

Mathematical knowledge, seen from this perspective, becomes highly personal and individualistic. But advocates of radical constructivism also say that the conceptions of the world acquired by humans are unlikely to be totally idiosyncratic, because contemporary constructivism argues in favour of social interactions which will effectively minimise the number of solely idiosyncratic constructions made by learners (Duit, in press). It is the teachers' role to guide learners towards the construction of "taken-to-be-shared" mathematical knowledge, that is to say, towards forms of knowledge that are regarded as mathematical, both in the domain of professional mathematicians and in the domain of everyday living where "mathematical" skills are needed to survive with dignity.

It should not be imagined, however, that this constructivist view of mathematical knowledge is supported by all contemporary philosophers. Matthews (1992), for example, maintains that the relativist conclusions of constructivists follow from an epistemological error. He draws attention to Lerman's (1989) suggestion that the core epistemological theses of constructivism are:

1. Knowledge is actively constructed by the cognising subject, not passively received from the environment.

2. Coming to know is an adaptive process that organises one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower.

After commenting that the first claim above is in the realm of psychology, and the second in the realm of epistemology, Matthews (1992) argues that despite Lerman's (1989) assertion to the contrary, (1) does not necessarily imply (2).

An alternative ontological analysis of the nature of mathematical knowledge is provided by Bigelow and Pargetter (1990), whose theory of Realism argues that mathematics can be understood realistically if it is seen to be the study of universals, of properties and relations of patterns and structures, the sorts of things that can be in several places at once. This is a kind of scientific Platonism, and it suggests, for example, that the theorem attributed to Pythagoras concerning the areas of triangles is a logical truth that would summarise properties of our world whether or not humans recognise this to be the case.

While it would appear to be impossible to prove whether an ontological theory such as the one argued by Bigelow and Pargetter is superior to the radical constructivist view of knowledge, it is important to note here that the relativism of radical constructivism is being seriously questioned by contemporary professional philosophers - a point that certainly does not emerge in current mathematics education literature. Furthermore, there are data to suggest that the radical constructivists' relativist view of knowledge is counter-intuitive. Del Campo and Clements (1988) reported data that indicated that although a high proportion of a sample that included university staff professed to hold social constructivist views, they found themselves admitting that they thought that Pythagoras' Theorem was true even before the standard Pythagorean relationships concerning right-angled triangles were ever recognised by humans.

Interestingly, von Glasersfeld (in press) admits that he has never denied that an "absolute" reality exists. He states:

> I only claim, as the sceptics do, that we have no way of knowing it. And as a constructivist, I go one step further: I claim that we can define the meaning of "to exist" only within the realm of experiential world and not ontologically. When the word "existence" is applied to the world that is supposed to be independent of our experiencing (i.e. an "ontological" world), it loses its meaning and cannot make any sense.

This kind of argument is rejected by contemporary philosophers such as Bigelow and Pargetter (1990), Matthews (1992), and Suchting (1992), who assert that there are no logical grounds for the claim that reality derives solely from the experiential world.

Indeed, Suchting (1992, p. 247), after a detailed analysis of a recent paper by von Glasersfeld (1989), concludes that with von Glasersfeld "there is a complete absence of any argument for whatever positions can be made out." Suchting goes on to assert that von Glasersfeld repeats certain words and combinations of words like *mantras*, "and while this procedure may well eventually produce in some what chanting is often designed to do, namely, produce a certain feeling of enlightenment without the tiresome business of intellectual effort, this feeling nearly always disappears with the immersion of the head in the cold water of critical interrogation."

SUMMARY

This paper, after drawing attention to the sudden rise in interest by mathematics educators in the notion of radical constructivism and its application in mathematics classrooms, and to the lack of clear definitions of the terms "constructivism" and "radical constructivism," identifies a number of strengths and weaknesses associated with the radical constructivist movement in mathematics education.

Strengths

It is our view that the most important strength of radical constructivism in mathematics education is the way it seeks to make the notion of "ownership" powerfully relevant to learners of mathematics. Radical constructivist educators seek to establish teaching and learning environments in which students, as a matter of course, create mathematics themselves, and therefore come to feel that they "own" what they learn.

A related strength is the desire to create quality learning environments in which rich social and linguistic interaction provides the basis for socio-cognitive challenge.

The paper also summarises a number of important conclusions that constructivist researchers have reached. Among these conclusions are (a) to claim that students can discover mathematics on their own is an absurdity; (b) students do not learn mathematics by internalising it from objects, pictures, or the like (that is to say, mathematics is not a property of learning materials, structured or otherwise); and (c) understanding mathematics is constructing and acting on "taken-to-be-shared" mathematical objects.

Weaknesses

The tendency for mathematics educators to accept radical constructivist notions in superficial ways has meant that there has not been adequate discussion at all levels of some of the questionable philosophical and linguistic foundations. We believe that the hard-line radical constructivist position on communication, that is to say the view that mathematical concepts and principles cannot be communicated by language, is at best an exaggeration and at worse false. More seriously, any association of the pejorative term "transmission" with teachers who prefer to adopt largely verbal styles of teaching is to be deplored.

At the philosophical level, we have argued that the refusal of many radical constructivists to acknowledge the possibility that mathematical knowledge is not entirely relativist, is short-sighted. Recent philosophical critiques of the writings of radical constructivists by Matthews (1992) and Suchting (1992) have been summarised, and implications of the theory of Realism by Bigelow and Pargetter (1990) discussed.

It was argued that the two fundamental theses of radical constructivism, namely (a) knowledge is actively constructed by the cognising subject, and not passively received from the environment; and (b) coming to know is an adaptive process that organises one's experiential world, and does not discover a pre-existing world outside the mind of the knower; are such that (a) does not imply (b), despite assertions to the contrary by radical constructivists.

A Final Comment

We realise that this paper might generate more heat than light. If indeed that proves to be the case, then that would merely bear out one of the contentions in this paper that it is professionally dangerous to be seen to be challenging constructivist dogmas. Mathematics educators need to be aware of the danger of polarising discussion with terms such as "behaviourist," "transmission" and "Platonist" at one pole, and "dialogue," "constructivist" and "generative" at the other.

REFERENCES

Ackermann, E. (in press). Construction and transference of meaning through form. In L. P. Steffe (Ed.), *Epistemological foundations of mathematical experience.* New York: Springer-Verlag.

Bigelow, J., & Pargetter, R. (1990). *Science and necessity.* Cambridge: Cambridge University Press.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics,* 11(1), pp. 23-41.

Chomsky, N. (1957). *Syntactic structures.* The Hague: Mouton

Cobb, P. (1986). Making mathematics: Children's learning and the constructivist tradition. *Harvard Educational Review,* 56 (3), pp. 301-306.

Cobb, P. (1988). The tension between theories of learning and the constructivist tradition. *Educational Psychologist,* 23 (2), pp. 87-103.

Cobb, P. (1990a, February). *Reconstructing elementary school mathematics.* Paper presented at the annual meeting of the Research Council for Diagnostic and Prescriptive Mathematics, Washington DC.

Cobb, P. (1990b). Multiple perspectives. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 200-215). Hillsdale, NJ: Lawrence Erlbaum.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education,* 23 (1), pp. 2-33.

Confrey, J. (1987). 'Misconceptions' across subject matters: science, mathematics, and programming. In J. Novak (Ed.), *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics* (Vol. 1, pp. 81-106). Ithaca, New York: Cornell University.

Davis, R.B. (1990). Discovery learning and constructivism. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 93-106). Reston, VA: National Council of Teachers of Mathematics.

Davis, R.B., Maher, C.A., & Noddings, N. (Eds.) *Constructivist views on the teaching and learning of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Del Campo, G. & Clements, M.A. (1988, July). *Mathematics, reason, social construction of knowledge, and mathematics education.* Paper presented at the Eleventh Annual Conference of the Mathematics Education Research Group of Australasia, held at Deakin University, Geelong.

Desforges, C. (1989). Classroom processes and mathematical discussions: A cautionary note. In P. Ernest (Ed.) *Mathematics teaching: The state of the art* (pp. 143-150). Barcombe (East Sussex): Falmer Press.

Dorfler, W. (1987). Empirical investigations of the construction of cognitive schema from actions. In J.C. Bergeron, N. Herscovics, & C. Kieran (Eds.) *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 3-9). Montreal: International Group for the Psychology of Mathematics Education.

Dorfler, W. (1989). Protocols of actions as a cognitive tool for knowledge schemata from actions. In G. Vergnaud, J. Rogalski, & M. Artique (Eds.) *Actes de la 13e Conférence Internationale, Psychology of Mathematics Education* (Vol. 1, pp. 212-219). Paris: International Group for the Psychology of Mathematics Education.

Duckworth, E. (1987). *The having of wonderful ideas and other essays on teaching and learning.* New York: Teachers' College Press.

Duit, R. (in press). The constructivist view: A fashionable and fruitful paradigm for science education research and practice. In L.P. Steffe (Ed.) *Epistemological foundations of mathematical experience.* New York: Springer-Verlag.

Ellerton, N.F., & Clements, M.A. (1991). *Mathematics in language: Language factors in mathematics learning.* Geelong: Deakin University.

Kamii, C.K. (1985). *Young children reinvent arithmetic: Implications of Piaget's theory.* New York: Teachers College Press.

Kilpatrick, J. (1987). What constructivism might be in mathematics education. In J.C. Bergeron, N. Herscovics, & C. Kieran (Eds.) *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 3-27). Montreal: International Group for the Psychology of Mathematics Education.

Krummheuer, G., & Yackel, E. (1990). The emergence of mathematical argumentation in the small group interaction of second graders. In G. Booker, P. Cobb, & T.N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference* (Vol. 3, pp. 113-120). Oaxtepec, Mexico: International Group for the Psychology of Mathematics Education.

Labinowicz, E. (1985). *Learning from children: New beginnings for teaching numerical thinking*. Menlo Park, CA: Addison-Wesley.

Lerman, S. (1989). Constructivism, mathematics and mathematics education. *Educational Studies in Mathematics*, 20, pp. 211-223.

Matthews, M. R. (1992). *Old wine in new bottles: A problem with constructivist epistemology.* Unpublished paper, Education Department, University of Auckland.

Mehan, H, & Wood, H. (1975). *The reality of ethnomethodology.* New York: John Wiley & Sons.

Pateman, N. A., & Johnson, D. C. (1990). Curriculum and constructivism in early childhood mathematics: Sources of tension and possible resolutions. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 346-356). Hillsdale, New Jersey: Lawrence Erlbaum.

Shweder, R. A. (1983). Divergent rationalities. In D.W. Fiske & R.A. Shweder (Eds.) *Metatheory in social theory* (pp. 163-196). Chicago: University of Chicago Press.

Sinclair, H. (1987). Constructivism and the psychology of mathematics. In J.C. Bergeron, N. Herscovics, & C. Kieran (Eds.) *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 28-41). Montreal: International Group for the Psychology of Mathematics Education.

Sinclair, H. (1990). Learning: The interactive recreation of knowledge. In L. P. Steffe & T. Wood (Eds.) *Transforming children's mathematics education: International perspectives* (pp. 19-29). Hillsdale, New Jersey: Lawrence Erlbaum.

Steffe, L.P. (1987, April). *Principles of mathematical curriculum design in early childhood teacher education.* Paper presented at the Annual Meeting of the American Educational Research Association, held at Washington, D.C.

Steffe, L.P. (1988). Action Group 1: Early childhood years. In A. Hirst & K. Hirst (Eds.), *Proceedings of the Sixth International Congress on Mathematical Education* (pp. 101-115). Budapest: Janos Bolyai Mathematical Society

Steffe, L.P. (1990). Mathematics curriculum design: A constructivist's perspective. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 389-398). Hillsdale, New Jersey: Lawrence Erlbaum.

Strike, K.A. (1987). Towards a coherent constructivism. In J. Novak (Ed.) *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics* (Vol. 1, pp. 481-489). Ithaca: Cornell University.

Suchting, W.A. (1992). Constructivism deconstructed. *Science & Education, 1*, 223-254.

Sullivan, P., & Clarke, D. (1991). *Communication in the classroom: The importance of good questioning.* Geelong: Deakin University.

von Glasersfeld, E. (1983). Learning as a constructivist activity. In J.C. Bergeron & N. Herscovics (Eds.) *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 41-69). Montreal: Psychology of Mathematics Education, North American Chapter.

von Glasersfeld, E. (1988, July). *Environment and communication.* Paper presented at the Sixth International Congress on Mathematical Education, Budapest.

von Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese, 80(1),* 121-140.

von Glasersfeld, E. (1990). Environment and communication. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 30-38). Hillsdale, New Jersey: Lawrence Erlbaum.

von Glasersfeld, E. (1991). Introduction. In E. von Glasersfeld (Ed.) *Radical constructivism in mathematics education.* Dordrecht: Kluwer.

von Glasersfeld, E. (in press). A constructivist approach to teaching. In L.P. Steffe (Ed.) *Epistemological foundations of mathematical experience.* New York: Springer-Verlag.

Wheatley, G. (1991). Constructivist perspectives on science and mathematics learning. *Science Education,* 75 (1), pp. 9-21.

Wood, T. (1990). The development of mathematical discussion. In G. Booker, P. Cobb, & T. N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference* (Vol. 3, pp. 147-153). Oaxtepec, Mexico: International Group for the Psychology of Mathematics Education.

Wood, T., & Yackel, E. (1990). The development of collaborative dialogue within small group interaction. In L. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 244-252). Hillsdale, NJ: Lawrence Erlbaum.