ASSESSMENT IN MATHEMATICS - SOME ALTERNATIVES

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> As we need standards for curricula, so we need standards for assessment. We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics. By confusing ends and means, by making testing more important than learning, present practice holds today's students hostage to yesterday's mistakes.
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> (Everybody Counts,1989 p. 70)

Improvement in the teaching and learning of mathematics at the school level has been the goal of mathematics educators for many decades. In response to a great deal of research, mathematics education is currently in an exciting state of review and renewal and teachers are participating in a major restructuring of the goals and practices of mathematics education. With the recent work of the Australian Mathematics Curriculum and Teaching Program (Clarke, D. and Lovitt, C., 1989) and the release of the new Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) in the United States, together with the release of the National Mathematics Curriculum in England (Department of Education and Science, 1990) and the National Statement on Mathematics in Australia (Australian Education Council, 1990) there has developed a growing agreement amongst educators about desired directions for change, and the sorts of mathematical experiences likely to prepare students for the challenges of the twenty-first century.

The types of mathematical skills which are being advocated in these important documents are significantly different from the narrow objectives which typified the mathematical goals and attitudes of yesteryear. One such change is the increased emphasis upon mathematical process, as distinct from content, which has resulted in the developing importance of problem solving and its applications in school mathematics. Our vision of an adequate mathematical education has been changed rapidly and permanently by the advent of calculators and computers for tools in both learning and doing mathematics. Similarly it is advocated that school mathematics should become more broadly defined to include the development of co-operative group processes and the fostering of associated communication and learning skills. Underpinning many of these changes is an increased use of "active" participation by pupils in the learning process. Associated with this model of learning are a variety of classroom teaching ideas which may involve pupils in manipulative materials and an increased emphasis on the central role that language plays in the learning of mathematics. This includes the advocacy of the use of extended and relatively more realistic problem situations in the classroom and the use of co-operative learning groups.

Unfortunately, however, with each step forward in our understanding of how to teach mathematics and how children learn mathematics, timed pencil and paper tests have remained the primary means of assessing the pupil's understanding of mathematics. The identification of the subject with a single form of assessment has shaped pupil attitudes, teaching practices and even the content of the subject. The results of such assessment have been used solely to grade and rank students, to regulate entry into courses, to award

credentials and to determine selection for employment and further study. This has tended to have a distorting effect upon the aims and goals of mathematics education. It has conferred an inordinate importance upon those aspects of mathematics which can be tested in an unambiguous and straightforward way through student's performance on routine skills and algorithms. As a result, there has been a great deal of resistance to broadening the goals of school mathematics and to exploring assessment strategies which better reflect these goals.

As an example of this resistance, the recently released National Statement on Mathematics for Australian Schools (Australian Education Council, 1990) comments that:

> In the past, mathematics has been assessed mostly by pencil-and-paper timed tests. This form of testing has remained dominant in mathematics, partly because it was perceived to be objective and hence more fair than other forms of assessment.

Australian secondary mathematics teachers are particularly resistant to alternative forms of assessment and have always relied on the age-old traditional method of timed pencil and paper tests. This is what they themselves experienced when they were at school and they consider that these practices are fair and equable. Clarke (1987, p. 9) comments

> schools continue to succeed in the teaching of routine computation and to fail in the teaching of such skills as problem solving....the maintenance of current assessment procedures serves only to maintain the illusion that significant learning is taking place.

Clarke (ibid., p. 8) continues:

> it is through our assessment that we communicate most clearly to students which activities and learning activities we value

so that it is essential that our assessment be comprehensive and give recognition to all valued learning experiences.

In a recent presentation to the Mathematics Association of Victoria, Mousley (1991) relates a story as follows:

> One of my off-campus students, a secondary teacher from New South Wales, recently claimed that assessment is the tail that wags the dog in her school: it seems that this is also true in Victorian schools too. As I observe a growing number of teachers and students starting to view mathematics as a field of enquiry rather than a pre-existing set of knowledge and skills, I glimpse the power of the tail.

That secondary mathematics teachers, in New South Wales anyway, are resistant to any change in the traditional manner of assessment in school mathematics, was borne out to the author in a recent in-service activity involving about 60 government and non-government secondary school mathematics teachers which was called to discuss possible changes in the mathematics curriculum, Years 7 - 12. These teachers were the keen ones who responded to a call from their professional association to discuss these issues, and the meeting was held on an evening in the teachers' own time. An observer would think that the recent

changes to mathematics education in England, U.S.A. and in the other states of Australia, especially Victoria had never occurred as they were very dubious of any seeking of a new direction as to what and how mathematics was being taught, let alone the method of how mathematics was to assessed. The Board of Studies had conducted a survey of mathematics teachers' desire for change in Years 9 - 12 and found that most only wanted some tinkering with content at the edges. There was certainly no groundswell of advocacy for changes in assessment procedures, and when pressured, a real opposition to assessment of group work, discussion, assignments, etc. on the grounds that it would be too subjective and students might cheat! The examination of prescribed content remained paramount in Years 9 - 12.

It seems that some acceptance for changes in assessment procedures is countenanced in Years K - 6 and even within the largely mixed-ability classes in Years 7-8 in N.S.W., but once the "real" mathematics begins in Years 9 - 12, the test dominates. Mousley (ibid, p. 150) comments in a similar vein when she says that infants and primary teachers over the past twenty years have tried to gulf the divide between real and school maths, but secondary teachers have felt too constrained by the syllabus in order to meet the traditional assessment requirements in upper years. She suggests that, in her state, changes are occurring because the assessment methods in upper secondary school have changed allowing the students to think in mathematical ways and gain a sense of both the purposes of mathematics and what real mathematicians do.

In 1987, within NSW, the assessment guidelines for courses, including mathematics, in Years 11 and 12 were issued by the old Board of Secondary School Studies and these are still in force (Board of Secondary School Studies, 1987). These included a school-based assessment for each subject which carries a weighting of 50% but is moderated by performance in the Higher School Certificate examination in that subject. Advocacy did occur on assessment by other means than written tests especially in the lower prestige mathematics courses, (e.g. Mathematics in Society and Mathematics in Practice) , but in fact this rarely occurs and the timed paper and pencil test reigns supreme. Admittedly component A and B types of assessment tasks were suggested with component B being concerned with the student's reasoning, interpretive, explanatory and communicative abilities and this led to some experimentation with untimed problem solving tasks. But it seems that for the bulk of the senior students following the more prestigious mathematics courses in N.S.W. (Units 2,3 and 4), the method of assessment remains very traditional. In the most recently developed course, Mathematics in Practice (1989), alternate assessment tasks are suggested especially for the Mathematics of Early Childhood module. However this course currently attracts only 2000 out of 60 000 Higher School Certificate candidates.

In Years 9, 10 mathematics there exists, within NSW, three levels - Advanced, Intermediate and General. The Mathematics Syllabus Committee 7 -12 is currently justifying this division of students before the Board of Studies. It seems likely that it will succeed. The School Certificate is awarded at the end of Year 10 but includes a moderated externally set and marked examination in July of Year 10 in the three distinct courses in Mathematics, in Science and English. These examination grades are then forwarded to the schools but without the names of the candidates. After further assessment, the school then awards the results to appropriate students at the end of the year - Advanced Grades 1-5, Intermediate Grades 1 -5, General Grades 1-5. Again the assessment is almost always by the timed pencil and paper method and little real attention is currently being paid in these courses to alternative forms of assessment. The courses

were first developed and approved in 1983, and reflect a most traditional approach to mathematics education.

It seems that the only way to force teachers to use different teaching styles and less traditional methods of testing is to actually change the method of assessment. A number of Examining Authorities tried this in England in the mid 1980s and I am familiar particularly with the Joint Matriculation Board in conjunction with the Shell Centre at the University of Nottingham which progressively added a real problem solving question and questions into the General Certificate of Education (equivalent to our Year 10 School Certificate). This forced teachers in secondary schools presenting candidates for exams set by those authorities to teach problem solving, and teaching packages were produced to in-service teachers to do just this.

This paper really reports just the beginning of a preliminary study designed to evaluate the desire of secondary mathematics teachers in NSW to change their assessment techniques. Currently the author and six third year Bachelor of Education (Honours) students (including two primary and four secondary students) are conducting such an investigation in some twelve Sydney metropolitan High Schools, involving about 100 practising teachers. It is envisaged that a questionnaire will be designed and analysed and a number of follow-up interviews will be conducted in an attempt to learn what alternative assessment methods are being used or at least will be possibly used in the future. The result of this research will be reported at subsequent MERGA Conferences.

Recent research has resulted in a great deal of general criticism being levelled at conventional assessment procedures such as the written test. Most researchers agree that achievement tests are created within the framework of an obsolete concept of school mathematics. According to the old conception, mathematics is a set of symbols and rules, or algorithms to internalise. Therefore achievement tests are merely testing the extent to which pupils have learned symbols to produce answers. Assessment of a student's ability to replicate neatly a learned procedure applied to a routine task in a familiar context is therefore not sufficient. Tests, however, cannot assess any of the richer areas of mathematics which are fundamental to good practice. A test, for example, cannot assess the way that an individual carries out mathematical activity, their mathematical attitudes or their ability to evaluate the work that they do. Kamii and Lewis (1991, p.4) believe that tests tend to become ends in themselves, not a means to assess educational objectives. Knowing this, teachers often teach to the test, not to the curriculum or to the students. Ollerton (1991, p.5) asserts that tests undermine confidence in all but the most able student and therefore reinforce failure amongst the majority. In fact, he believes that a test can prevent students from demonstrating their ability to function as a mathematician. Furthermore, if students are actively encouraged to perceive that the most important aspects of mathematics are narrow skills that are immediately testable, it will inhibit their confidence to recognise the value of and therefore participate more actively in the broader aspects of mathematics.

Many suggestions for change in assessment techniques have been made in the past four years and these new methods are currently being used in the United Kingdom, USA, Victoria and in other Australian states and even in K - 8 classrooms within N.S.W., but not in Years 9 - 12 within this state. Back in 1988, Max Stephens, (Stephens, 1988, p.1) on behalf of AAMT, talked about the formative aspect of assessment as well as the summative. The formative aspect was concerned with the continuing appraisal of students' work and the summative approach is related to giving a clear record of achievement across

the full range of learning in mathematics. Summative and comparative assessment become significant at those points where schooling connects with the world of work and the requirements of tertiary studies, when there is a need to provide clearly interpretable records of student achievement.

Keeping these two aspects of assessment in mind, researchers advocate that assessment should occur along a continuum from the least to the most formal, with a range of modes being used in a blend that is appropriate for the age of the student and to the teaching-learning situation. Writing very recently Clarke (1992, p.25) asserts that the distinction between formal and informal assessment is a critical one, and that it is vital that both forms of assessment are seen as being of value in the classroom situation. He describes formal assessment as that which involves the cessation of instruction, typically for the whole class, while an "assessment" event is held. Informal assessment, on the other hand, involves the collection about students' learning coincident with instruction and without disrupting the learning process. He and his colleagues (Clarke, Clarke and Lovitt, 1990) believe very strongly that informal assessment, particularly in the form of direct teacher observations, offers the teacher a greater wealth of information than can be provided by more formal assessment using tests, which often does little more than legitimise and quantify the assessment made through extended classroom contact. However, informal assessment generally lacks structure, and the information it produces is not systematically recorded and lacks the status accorded to a test score. They recommend that, by introducing some structure into their informal procedures, teachers can maximise the information they collect and minimise the time squandered on redundant assessment.

With the recognition of the great value that lies in informal assessment procedures has come an attempt by researchers to structure these in such a way that they can be of great benefit as assessment tools. It is suggested that all of these techniques can be applied in the secondary classroom:

INFORMAL METHODS OF ASSESSMENT IN MATHEMATICS

1. **Observing students.** As Lester and Kroll (1991, p.278) note:
   observing students while they are engaged in mathematics activities can yield invaluable information not only about their skills abut also about their thinking processes, their attitudes, and their beliefs.....teachers can learn a great deal about students by circulating unobtrusively as students work in small groups and by interjecting questions to clarify their observations.

   Observation, to be effective, should frequently be sharply focussed. Observation should be both individual and in small groups. Stenmark (1989, p. 22) suggests that observation could include student learning styles, student ideas, communication, co-operation and use of manipulatives.

2. **Annotated Records**
   This strategy focuses on recording significant events that are observed by the teacher. Teachers can draw up a checklist of the skills, behaviours or attitudes that they wish to encourage in their students and then record those significant moments that either extend or challenge their image of the particular student. The original checklist helps teachers to know what to look for and also provides a means to summarise the significant moments into a systematic record for each student.

3. **Asking Questions**
Classroom questions offers probably the best chance to monitor the development of meaningful mathematical understanding. However as Thompson and Briars (1989, p. 22) point out most questions asked by a teacher in a typical lesson require only simple rote responses which fail to give insights into students' understanding. To discover the extent to which these students are making sense of the material, questions which require more thoughtful, elaborate answers are required. Clarke and Sullivan (1990) refer to these as "good" questions and characterise them as requiring, firstly, more than simple recall of a fact or replication of a procedure, secondly, having several acceptable answers and thirdly, as allowing the students to learn by doing them. It is important to allow plenty of wait time to allow the students to give thoughtful answers. (Stenmark, 1989, p. 24)

4. **Interviews**
These interviews allow the teacher to assess mathematical understanding by probing the depths of that understanding. Is the student merely parroting back memorised responses, or has the student interacted with the ideas and built them in to his or her own conceptual structures? (Stenmark 1989, p. 23) The opportunity for positive rapport is increased while allowing the teacher to assess the level and accuracy of student understanding. These interviews could be seen by teachers to be very time consuming and quantification methods will have to further developed.

5. **Students' Work Folios**
One of the most effective ways to document progress is to collect representative examples of student work. These are well used in other curriculum areas, but until now have rarely been used for assessment purposes in mathematics. (Stenmark, 1989, p. 8) Teachers and students should be allowed to choose most of the items to include in the portfolio.

6. **Student Journals**
These enable students to write in mathematics and enable them to record what they are thinking, to summarize key topics and to internally engage in a dialogue through which they can reflect on and explore the mathematics that they have encountered. This a brand new activity in the mathematics classroom and secondary teachers will need to be convinced of its worth.

As well as these six informal methods of assessment, there are at least another three formal methods of assessment which are alternatives to the more traditional timed pencil and paper test. It is suggested that these techniques are also applicable to the secondary mathematics classroom:

ALTERNATIVE FORMAL TECHNIQUES IN ASSESSMENT

1. **Investigations in Mathematics**
These activities are very appropriate for some topics in the curriculum and provide an opportunity for the student to demonstrate their ability to use the skills that they have learned in solving a problem. The Nottingham Shell Centre investigational material is a good example of this technique. (Problems with Patterns and Number, 1984 and the Language of Functions and Graphs, 1985)

2. **Performance Testing**
This involves giving a group of students, or an individual, a mathematical task that could take from half an hour to a couple of days to complete or solve. An assessor could videotape or tape record the activity and the object of the assessment would be to look at how students are working as well as the completed tasks or products.

3. **Student Self-Assessment**
This is a brand new form of formal assessment in the secondary classroom. Students are certainly capable of self-assessment and mathematical power comes from knowing how much we know and what to do to learn more. Students can also obtain feedback from their peers by making constructive comments on one another's work. (Vale, 1987, p. 7)

By developing a range of assessment alternatives in secondary mathematics - formal and informal - a much wider picture will be built of the students' performance in mathematics. Teachers will oppose these changes vehemently because they represent a huge change in direction from past practice. Many trials of the new techniques will need to occur to convince them of the worth of change. However, over time, teachers will become persuaded that the bigger picture of their students' performance gives a more true account of the students' grasp of mathematics.

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