CHILDREN'S UNDERSTANDING OF THE INDEPENDENCE OF RANDOM GENERATORS

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THE EXPERIMENTAL SITUATION

A simple way to investigate children's understanding of the independence of random generators is to examine successive random trials of a random generator whose probability function does not change from trial to trial. Such trials might be done by successive tosses of a coin or a die, or by successive drawings from an urn, with replacement after each draw.

Eight subjects, four male four female, from each of Years 4, 6, 8, 10 in South Australian schools were interviewed for about half an hour each. Part of these interviews is reported here. Each subject (S) was asked by the interviewer (I) to predict the results of nine successive random draws from a transparent container holding a total of 3 balls: 2 green (G) and 1 blue (B). For variety the questions asked varied from draw to draw, but all were variations on the following themes:

® Which colour is most [sic] likely to come out?
® If I asked you which colour you will draw out which would be the best [sic] colour to guess?
® Which colour do you expect to come out this time?
® Which colour do you think you will take out this time? Why?

There was no evidence that children saw the variation in questions as leading to change in meaning in the way reported by Jones (1974), but this is a possible weakness in the protocol. In practice, nine draws started to become a little boring for some SS, so this questioning procedure was not slavishly adhered to before every draw. Boredom was more likely to arise if there were a long run of Gs, and least likely to arise if there were a long run of Bs. Regardless of the amount of questioning, all SS wrote down a prediction before every draw.

The result of each draw was written down next to each prediction and S was usually asked to make some comment on what had happened using questions like:

® Were you surprised?
® Does that mean you were wrong?
® Why did this happen?
® Can you make it come out blue?
® Did you expect this to happen?
® Will you be right every time?

Once again, questions were not asked on every occasion. For example, if S had predicted G and obtained G three times in a row, there was little point in asking for further comment if the event occurred a fourth time. Since outcomes could not be pre-determined the protocol had to be flexible in order to capitalise on what actually happened and on changes in mood of S.

RESULTS

The view taken here is that each of the nine draws represented independent random actions. With this model it would be anticipated that mature SS would consistently predict G regardless of any of the preceding outcomes. Such SS would be thinking in a Pascalian way and could be presumed to have understood the independence of random actions.

However, it is clear from other research that SS who do not think in a Pascalian way adopt a very wide range of responses and it is not clearly understood what situations are likely to induce which responses. Much previous research, e.g. Fischbein (1975), Goodnow (1955), Green (1982), Turner (1979), has regarded patterns of outcome as the most likely stimuli, but it is also possible, as suggested by Brainerd (1981) that the pattern of SS's responses or the combined pattern of responses and outcomes may influence future predictions. Given the exploratory nature of the research reported, it was not considered profitable to use stochastic matrices in the way that Brainerd did.

The results of all nine predictions/outcomes for each S have been tabulated in full and are summarised in the tables below. The small sample size means that statistically significant results are unlikely to be obtained. However, the results may indicate which questions might yield significant results with a larger sample.

Initial Predictions

All SS were aware of the structure of the random generator: the urn contained 2G and 1B. All SS, except two in Year 4, predicted that the first ball drawn would be G. TD (Year 4, M, 9:3) expressed several times a strong liking for B, so it likely that affective matters were influencing his choice. The other S who predicted B was MW (Year 4, F, 8:7), whose comments suggest that she was not aware that the composition of the urn was relevant to the situation. She made comments like "Because it looks like it's the green's turn today", "It looks like it's green before blue" and "... coz it came out green then it looks like it's blue now".

Turner (1979) found that an appreciation that the preponderance of Gs means that in some sense G is more likely to be drawn than B did not appear until a child was about 9 years old. This research provides some support for Turner's finding.

Pascalian Responses

Five SS, one in Year 4, one in Year 6, two in Year 8 and 1 in Year 10, consistently predicted G for all 9 draws. All of these had at least two of their predictions refuted during the experiment. The responses given are instructive.

ML (Year 4, M, 8:10)	2G 1B Same as the other questions, cos there's 2 Gs ...
I	Can you make it come out G?
ML	No you can't.
I	Why not?
ML	Because it's hard to choose cos I know where it is at first, but when you shake it up it's hard to find them.

Although ML had five of his predictions refuted he stuck to his predictions, using an explanation remarkably good for his age. He saw that the composition of the box was constant, but that the shaking up meant that an element of randomness was inevitable. The other SS produced similar responses.

These SS who used a Pascalian approach all saw that drawing a B is possible but believed that it did not affect the properties of the random generator. These are determined by the contents of the urn and not the outcome of any particular trial. They have understood the idea of independence of random trials, at least in this situation. The early age at which appropriate arguments are used is surprising, and warrants further investigation.

Work by Zaleska (1974), Zaleska and Askévis-Leherpeux (1976), Fischbein *et al.* (1991), among others, suggest that the investigation reported here deals with a particularly simple situation. Here the random generator is a familiar one whose structure is known to the child; here the child has to predict an outcome, rather than select a generator; here the child draws the balls from the urn and records the results; here the trials are conducted successively, not concurrently. Such a simple situation is likely to be the most efficient way of investigating SS's non-Pascalian responses.

Were the Subjects Using an Heuristic of Alternating Responses?

The answer to this question must be an unequivocal "no". All SS showed an awareness of the asymmetry of the random generator, and almost all indicated, albeit in different ways, some understanding that this asymmetry affected the outcome of each draw. Those workers, e.g. Green (1982), Cohen and Hansel (1955), who have reported the use of an alternating heuristic have been working either with random generators like coins or with two outcome probability generators whose rule is unknown to the child.

Were the Subjects Using an Heuristic of Negative Recency?

The "negative recency heuristic" is used to describe a predictive strategy in which "the probability of predicting an event [decreases] as a consequence of the event having occurred repeatedly on previous trials" (Fischbein, 1975, p. 59). This is a form of the Baconian heuristic which argues that the evidence from pervious throws changes the probabilities for the next throw. One common form of this heuristic is known as the "gambler's fallacy". For example, S is less likely to predict that the next outcome will be a B after observing just 1 G than after observing 6 Gs. The research mentioned above suggests that the circumstances in which such a heuristic might be used are not well understood.

In the work reported here there are four possible situations. A prediction might change from B to G or from G to B, and such a change may take place after a run of Bs or of Gs. A run is defined here to include the case of just one occurrence. The data provided here are of course too small for generalisations. However, they do suggest a thesis different from any reported by other workers.

Table 1 shows the frequencies of the length of run which precedes a change of prediction. It takes into account the fact that, for example, a run of 3 Gs contains within it runs of 1 and 2 Gs as well. Thus an S who has changed a prediction after a run of 3 Gs has also not changed a prediction after runs of both 1G and 2G. The five SS who consistently predicted G are excluded from Table 1 on the grounds that they are probably using a Pascalian heuristic which is not relevant to this discussion.

Table 1

Length of runs of	Number of times occurred	Change of prediction from G to B		Change of Prediction from B to G		No Change in Prediction	
		N°	%	N°	%	N°	%
1G	61	14	23	14	23	33	54
2G	40	7	18	9	22	24	60
3G	14	4	24	5	29	8	47
4G	8	2	25	2	25	4	50
5G	4	1	25	0	0	3	75
6G	3	2	67	0	0	1	33
1B	27	9	33	9	33	9	34
2B	9	6	67	2	22	1	11
3B	1	0	0	1	100	0	0

(a) Changing Prediction from G to B after an Actual Run of Gs

This table shows that the probability that S will make a change of prediction from G to B is very much the same for runs of between 1G and 5G. In other words, S is just as likely to change from G to B after a long run of Gs as after a short run. The figures for actual runs of 5G and 6G are probably too small to yield reliable results. This constancy is quite remarkable. It suggests even more strongly that the negative recency heuristic is not being used in this case.

(b) Changing Prediction from G to B after an Actual Run of Bs

The numbers in Table 1 are rather too small for reliability. However, they do suggest that a run of 2 Bs is quite likely to result in a change of prediction from G to B. This strategy is the opposite of the negative recency heuristic. I am not aware of any research workers in this field using the term "Positive Recency Heuristic". Perhaps this phenomenon has been overlooked.

(c) Changing Prediction from B to G after an Actual Run of Bs

The numbers here are too small to draw any conclusion.

(d) Changing Prediction from B to G after an Actual Run of Gs

Table 1 suggests that S's probability of changing from B to G after a run of Gs is independent of the length of the run. Either this conflicts with the negative recency heuristic or there are other factors operating. It has been shown that SS are well aware of the asymmetry of the random generator. It is also possible that the relation between the length of run and the desire to change predictions is non-linear.

The similarity of the results for changes to G and changes to B is particularly striking. It makes it clear that a negative recency effect is an inadequate description of the subjects' strategies. One reason for this has been suggested by Goodnow (1958, p. 115) who reports the results of several studies which found that

For most individuals, correct prediction of an infrequent event is not just one more correct prediction but a success worth several correct predictions of the easy-to-get more frequent alternative. It assures the subject that he is either singularly blessed by luck or else smart and on the verge of discovering the 'system'.

Goodnow touches on the issue of whether the SS are making predictions which will match the actual outcome, of trying to predict a "typical" outcome, or trying to maximise rewards under pay-off conditions which are not explicitly stated. This matter will be taken up below.

What is clear from the data analysed so far is that it is unlikely that a negative recency heuristic based on actual outcomes is sufficient to describe the pattern of SS's responses and it is possible that it is not being used at all in some situations. Other possibilities will now be examined.

Does the Negative Recency Operate with the Predictions?

It is possible that S may be using a negative recency heuristic based, not on the actual outcomes of the trials, but on his or her predictions. A further factor may be whether or not the prediction matched the actual outcome; this will be considered later.

Table 2 has a similar structure to Table 1, but is concerned with runs of predictions rather than runs of outcomes.

Table 2

Length of predicted runs of	Number of times occurred	Change of prediction		No Change in Prediction	
		Number	%	Number	%
1G	44	13	30	31	70
2G	31	12	39	19	61
3G	19	9	47	10	53
4G	10	4	40	6	60
5G	6	1	17	5	83
6G	5	4	80	1	20
7G	0	0	0	0	0
8G	1	1	100	0	0
1B	41	33	80	8	20
2B	9	7	88	1	12
3B	1	1	100	0	0

(a) Changing Prediction from G to B after Predicting a Run of Gs (Independent of any Confirmation or Refutation of Such Predictions)

Of the 44 changes of prediction to B after a run of predicted Gs 25 occurred after a predicted run of 1 or 2 Gs, 13 after a predicted run of 3 or 4 Gs and 6 after a predicted run of 5 or more Gs. The mean length of run was probably a little higher for the older groups.

The pattern revealed by Table 2 is quite different from that revealed by Table 1. Table 1 suggested that the probability that S would change a prediction from G to B was independent of the actual length of run of Gs (at least for runs up to 5 long). But Table 2 suggests that the more Gs a subject has predicted the greater the probability that S will change to predicting a B.

Table 3 gives a break down of Table 2 by Year level to see if older SS are more likely to predict longer runs of G before changing their prediction to B. The numbers are small but they do indicate trends.

Table 3

Length of predicted runs of	Change of prediction			Change of prediction			Change of prediction			Change of prediction		
	Year 4			Year 6			Year 8			Year 10		
		No	%		No	%		No	%		No	%
1G	10	4	40	12	4	40	9	1	11	13	4	31
2G	6	5	83	8	3	38	8	3	38	9	1	11
3G	1	1	100	5	4	80	5	1	20	8	3	38
4G				1	0	0	4	1	25	5	3	60
5G				1	0	0	3	0	0	2	1	50
6G				1	1	100	3	3	100	1	0	0
7G										1	0	0
8G										1	1	100
1B	12	10	83	11	9	82	10	8	80	8	6	75
2B	2	2	100	2	2	100	2	2	100	2	1	50
3B										1	1	100

This table suggests that the younger the child the more likely he or she is to change prediction after a short run of predicted Gs. There does not seem to be any age difference affecting the length of a predicted run of Bs. These results support the findings of Turner reported above.

It would be possible to argue that SS are attempting to give a representative example of a possible set of 9 draws. If so, we would expect them to predict 6 Gs and 3 Bs. Table 4 shows the distribution of their predictions.

Table 4

Prediction	Frequency				
	Year 4	Year 6	Year 8	Year 10	Total
9G 0B	1	1	2	1	5
8G 1B	2	2	3	3	10
7G 2B	4	2	2	1	9
6G 3B	0	2	1	1	4
5G 4B	1	0	0	2	3
4G 5B	0	1	0	0	1

It can be seen that among those who did not predict 9G 0B there is a distinct bias to predict more Gs than one might expect from a typical set of 9 draws. This suggests that SS are not trying merely to predict a typical set of outcomes, not are they seeking to maximise rewards where a B pays more than a G in the way suggested above by Goodnow. It is almost as if they want to show that they realise the bias of the random generator, but they are aware that less likely events do happen from time to time.

This tension can be seen when looking at the responses of SS who changed from predicting G to predicting B after five or more successive predictions of G In some cases the change was clearly a result of previous outcomes, not predictions. For example, AP (Year 8, F, 13:8) changed from predicting G to B immediately after having had four out of those six predictions being refuted by a B. Her reason was "Seem to be getting B all the time".

However, in two cases, it is not clear whether the outcomes or the predictions were influencing S. JM (Year 8, F, 13:7) changed after 6 predictions of G including a refutation of the 3rd prediction with the reason "Because it's been G all the time and its due for a change". Similarly, JW (Year 10, F, 15:3) changed after eight predictions of G of which the first two were refutations with the reason "I only picked B because it's been all Gs before". Since in both cases it had not been all G outcomes before the change, it appears that SS were taking their previous predictions into account.

(b) Changing Prediction from B to G after Predicting a Run of Bs (Independent of any Confirmation or Refutation of Such Predictions)

Of the 41 changes to G after a predicted run of Bs 33 followed a predicted run of just one B, 7 followed a predicted run of two Bs and 1 after a predicted run of 3 Bs. So, there was a strong tendency not to predict a large number of Bs in a row.

The contrast between the influence of previous outcomes and previous predictions on future predictions, which has been noted above, is even more marked when considering SS's change from a predicted run of Bs to a prediction of G. The probabilities that such a change will take place after actual runs of 1 or 2 Bs are 0·33 and 0·22 respectively. However, the probabilities that such a change will take place after predicted runs of 1 or 2 Bs are 0·80 and 0·88 respectively. It is true that there is no difference when considering predicted and actual runs of 3 Bs with both probabilities being 1, but the sample size here is very small and the difference for 1 and 2 runs remains quite striking. It suggests that SS do not feel confident with predicting Bs but feel that they need to from time to time, perhaps to incorporate their awareness that B will happen from time to time.

This tendency may arise because a long run of Bs is not expected or because predictions of B are more likely to be refuted, and hence discouraging in the sense discussed by Goodnow above. So it is worth examining the effect of refutations on SS's behaviour.

Does the Heuristic of Negative Recency Operate with Refutations?

The effects of refutation and confirmation of a prediction on the subsequent prediction are listed in Table 5. There were no obvious differences between the year groups.

Table 5

		Next Prediction			
Refuted Prediction of	**Total**	G		B	
G	79	59	75%	20	25%
B	41	29	71%	12	29%
Confirmed Prediction of					
G	124	99	80%	25	20%
B	16	12	75%	4	25%

The figure 12 in the second row includes 4 choices by TD, who, as already mentioned, stuck very strongly to his preference for B on animistic grounds. It can be seen that a refuted prediction for G is very much less likely (25%) to result in a change of prediction for the next draw than is a refuted prediction for B (71%). It can also be seen that a confirmed prediction for G is much less likely (20%) to result in a change of prediction for the next draw than is a confirmed prediction for B (75%).

Put another way 75% of refuted predictions of G led to the next prediction also being for G, compared with 80% of confirmed predictions for G. Similarly, 29% of refuted predictions of B led to led to the next prediction also being for B, compared with 25% of confirmed predictions for B.

These percentages are very close, and there seems to be no evidence that a specific refutation has a strong influence on the next prediction. Such a result is consistent with the earlier finding summarised in Table 2 that the probability of a change of prediction is independent of the length of the actual run, at least for runs up to 5 predictions long.

Perhaps the possible change after longer runs means that a chain of refutations may have some influence on subjects' predictions. Table 6 examines how many confirmations or refutations a subject had experienced before changing his or her prediction.

Table 6

Result of Preceding Predictions	**Change Prediction from B to G**		**Change Prediction from G to B**	
	No	%	No	%
Previous Prediction Confirmed	9	23	25	64
Previous Prediction Refuted	11	28	8	21
More than one Previous Prediction Refuted	19	49	6	15
Total	39		39	

This table states that there were 39 changes of prediction from B to G. Of these 9 (23%) occurred after a confirmed prediction of B, 11 (28%) after exactly one refuted prediction of B and 19 (49%) after 2 or more refuted predictions of B. Similarly, there were 39 changes of prediction from G to B. Of these 25 (64%) occurred after a confirmed prediction of G, 8 (21%) after exactly one refuted prediction of G and 6 (15%) after 2 or more refuted predictions of G.

These differences are striking. A confirmed prediction of G predicts a change to B much more strongly than a confirmed prediction of B predicts a change to G. It is possible to argue that subjects may have changed their predictions in accordance with whether they felt that their luck was in or not, but such an argument cannot be verified by this type of data.

SUMMARY

All of these results suggest that these children were well aware of the asymmetry of the random generator. For example, when all results are combined 77·8% of predictions of G are followed by another prediction of G, while 28·1% of predictions of B are followed by another prediction of B.

As mentioned above, favourable Baconian probabilities increase with the weight of the evidence. For this experiment, a Baconian approach would mean that subject's predictions would attempt to ensure that the ratio of numbers of G and R would approach 2:1. It is clear from this investigation that while children's predictions are influenced by preceding outcomes they are influenced by preceding predictions and possibly by runs of refutations.

Some of the strategies observed here have not been observed by other workers. This may well be because the nature of the investigation is different from that employed by others. It is not a very cost-effective method because it is labour intensive, it has outcomes which cannot be controlled without resorting to some sort of deception, and because it is likely to bore children if it goes on for too long. My own experience suggests that the process of asking for a prediction followed by testing the prediction and recording and reflecting on the actual outcome can only be done from 8 to 10 times before the subject starts to lose interest for a very wide variety of reasons. So while it would be expensive to test the hypotheses suggested here, the fact that some of these differ from those of other workers suggests that such testing would be worth while. In particular, if a large sample of children were able to be tested then it would be possible to obtain statistically useful results for separate age groups.

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