CONTEMPLATING CULTURAL CONSTRUCTS

BILL BARTON
Mathematics Education Unit
University of Auckland

PHILOSOPHIES OF MATHEMATICS

A Wide View

The philosophy of mathematics has, since Plato, been concerned with the objects of mathematics and the truth of its statements. Are numbers real or are they idealised thoughts? Are complex numbers images in our mind with no reality or are they valid mathematical realities? Does mathematics show us God's glory or is it a creation of human minds? Were the Mandelbrot pictures and the patterns of Chaos waiting to be discovered or were they created by our number representations and the coordinate system? Is <1+1=2> a true statement about the world or a logical tautology? Is it true that there is always another term in an infinite series? Is Goldbach's Conjecture either true or false? Is it reasonable to divide infinitely small quantities in calculus? How do we know that the Four Colour Theorem is true?

The debates between Platonists, logicists, formalists, intuitionists and others have tried to answer these questions definitively and without unpalatable consequences like logical paradoxes or circularities. Such dêbates have produced vast fields of mathematical endeavour such as Russell's propositional calculus and the field of recursive functions. It has been studded with breathtaking inventions and insights such as Turing Machines and Gödel's Theorem. But, despite 2000 years' work from the top minds of humankind, the fundamental issues of the nature of mathematical objects and the role of logic are far from being resolved (see Körner (1960) for a general review of these debates)

In the later half of this century there have been movements away from such questions. One of the starting points of this movement was the French philosopher Bachelard, writing in the 1930's, who proposed a philosophy of science which focused on the historical development of rationality. He suggested that what counts as 'objective' changes over time: thus a subjective element is introduced into the notions of proof and truth. These notions do, however, develop in a rational way (see Tiles (1984) or Smith (1982)).

The basic theme of the historical relativity of mathematical and scientific concepts is pursued in different ways by Kuhn and Popper. The latter sees the development of science as a process of humankind approaching closer and closer to some form of Platonic reality. Popper's idea that scientific ideas are posed to be criticised and thereby improved (sometimes called 'critical fallibilism') have been taken up, more recently, by Imre Lakatos'. Lakatos' contribution was to suggest that mathematics was like science in that mathematical statements are made to be refuted. They describe things as we see them, meaning 'how we think them' rather than 'how we perceive them with our senses'. He describes mathematics as 'quasi-empirical', and his interest is with the question: How do we criticise the statements of mathematics? What would have to happen for us to say that a mathematical truth was wrong? (Lakatos (1978))

At about the same time as Bachelard, Wittgenstein was writing about the way that meaning is negotiated. He suggests that any term, for example 'prime number', is in a constant process of definition as it is used in mathematical discourse. Mathematical concepts are defined by proofs and theorems, not the other way round. That is, there are no mathematical results "out there" waiting to be discovered, rather the construction, statement and proofs of theorems are part of the development of a mathematical concept (see Wright (1980) or Bloor (1983)). Paul Ernest (1991) is arguing a position which merges the social constructivist philosophies with the critical fallibilism of Lakatos.

Such a sketchy outline as the above cannot do justice to the sweep and depth of scientific and mathematical philosophy. However, it may be sufficient to make the point that mathematical philosophy, after 2000 years of concern over the nature of mathematical objects and the role of logic, has begun to shift focus onto our changing conceptions of these objects and their relationships. This can be seen as shift from objective concerns to subjective ones, (although there are many realists and intuitionists continuing to forcefully argue their views). Another way of describing this change is to say that instead of asking the questions "What are triangles?" and "How do I know that they contain 180°?", we are asking the questions "How do we talk about triangles?" and "What are the limits to the statement 'triangles contain 180°'?".

Ethnomathematics

What has this to do with ethnomathematics? Well, in the first place, it gives a context within which mathematics educators have begun to talk about a concept of the cultural relativity of mathematics. With the evolution of the idea that mathematics may be a changing subject, it is not surprising that teachers have begun to question what mathematics they are teaching. Teachers are familiar with the social perspective of cultural difference in educational ideas and practices, so it is natural for them to introduce the idea into their subject areas.

The difficulty is that, philosophically, neither classical theories, nor, it appears, those of Bachelard, Wittgenstein, Kuhn, Popper or Lakatos, offer any help in seeing how there might be culturally different mathematics. These theories either lead to a concept of a one true (ideal) mathematics, or they lead to a concept of a developing subject forever guided along a path towards a rational ideal of mathematics. The concept that there might be different mathematics' (of equal rational standing) sustained simultaneously cannot be accomodated in these theories. At least none of the proponents of these theories have tried to defend such a position (with the exception of David Bloor who argues for a social relativity, see Bloor (1976))

The result is that writers on ethnomathematics have developed a variety of conceptions as they try to reconcile cultural mathematics with the ideal, rational conception of mathematics which is the historical legacy. Some notions of ethnomathematics include: an elementary (pre-mathematical) stage of development (Ascher), a contributor to some universal and encompassing mathematics (Gerdes), an educational construct (Bishop), a political construct (Mellin-Olsen/Gerdes), or an historical development borne from the mathematical practices of groups of people (D'Ambrosio). Geraldo Pompeu has detailed the differences in six versions of ethnomathematics according to their educational implications. (Pompeu (1992)).

ANOTHER PERSPECTIVE

Mathematics As Process

The idea that I would like to pursue is that ethnomathematics is much more pervasive than that. I want to suggest that ethnomathematics is a constant lens through which we see the world, and, in particular, the mathematical activities of those around us. I am led to this view by taking the social, relativist philosophies of this century and applying them to the notion that mathematics is a process.

Rather than concentrating on the objects of mathematics, or the way we talk about mathematics, I'd like to ask philosophical questions which surround the action of mathematics. To use the examples above, I want to ask: "What do we think we are doing when we talk about triangles" and "Why do we investigate the angle properties of a triangle?".

I want to ask mathematicians whether the theorems of mathematics embody what they think they are doing, or whether theorems are just a record of where they have been? If the latter is the case, then what is the nature of the process of mathematics, and how do they know when they have done some worthwhile mathematics? What are the characteristics of successful mathematical action? Is it only possible to tell afterwards, or is the process itself valuable? Where do new mathematical ideas come from?

The philosophical programme I have in mind is to describe the workings of mathematicians (and metamathematicians), thereby opening up the processes of mathematics to examination so that possible limitations to the development of mathematics can be identified. Thus new ways of doing things, or new ways of thinking about things, might be explored. This is an active, creative programme, unlike those of classical mathematical philosophers which are post-hoc, descriptive programmes.

The Role of Culture

Asking philosophical questions such as this brings the actor into mathematics. It becomes impossible to separate the actor and their language, their preconceptions, their experience; mathematics is inextricably linked with the mathematician's images, concerns, metaphors, or their values, their perceptions and their view of the world. Now the sociology and anthropology of mathematics become concepts which we can understand and which we realise we know very little about. The developing literature on this aspect of mathematics is overdue and very welcome (Fang & Takayama (1975), Restivo (1992)).

The consequence of this view is that mathematics will be culturally determined to the extent to which a community of mathematicians share cultural characteristics (e.g. language, values, perceptions). It may be possible to "see" others' cultural influences, but it is never possible to escape some cultural influence or other. Mathematics as process, mathematics as action can only be relative in this sense.

It is possible to understand this view of ethnomathematics in relation to individual and social constructivist views. If we consider the processes of mathematics, and the actors in this process, these actors as operate at several levels simultaneously. They are individuals, members of an intimate group (e.g. a class or Maths Department), members of a wider community of those doing similar things, part of a cultural group and so on. These groups overlap and interact. Any one mathematical action is liable to be affected by any or all of these groups in some way - the action cannot be isolated from them. Similarly, it is difficult to separate the influence of any one group. So, how might an analysis be achieved? Under what conditions is it possible to isolate a cultural influence and call the resulting view ethnomathematical? Perhaps the answer lies in describing aspects of mathematical processes as ethnomathematical, rather trying to find ethnomathematics?

Another result of considering the processes of mathematics (and the actors involved) is that it opens up the question "What is mathematics". Instead of focussing on mathematics as it is already defined, it is possible to look at ways of behaving that have mathematical characteristics. The results of those processes then become mathematics. Taking a cultural view, this means that cultures which do not have a "mathematical" division of knowledge are not immediately dismissed from the sphere of mathematics, since some of the things members of that culture do are clearly mathematical in the sense of Bishop's six activities (Bishop (1988)).

SUMMARY

It is argued that classical philosophies of mathematics do not provide a basis from which it is possible to argue for different, simultaneous views of mathematics on cultural (or any other) grounds. A shift in the focus of philosophical attention from the objects and logic of mathematics to its processes provides a more active programme which includes a place for the actor in mathematics. Through the actor subjective influences are introduced at many levels, influences which it is not possible to escape. The influences that can, in some way, be identified as cultural, define the ethnomathematical component of the processes being considered.

It remains to illustrate these ideas using specific cultures and specific processes, and thus carry out the philosophical programme which is aimed at widening the mathematical creative process.

REFERENCES:

Bishop, A (1988) *Mathematical Enculturation*, Dordrecht: Kluwer Academic Publishers

Bloor, D (1976) *Knowledge and Social Imagery*, London: Routledge & Kegan Paul

Bloor, D (1983) *Wittgenstein. A Social Theory of Knowledge*, London: MacMillan Press

Ernest, P. (1991) *The Philosophy of Mathematics Education*, London: Falmer Press

Fang, J & Takayama, K (1975) *Sociology of Mathematics and Mathematicians. A Prolegomenon*, Hauppauge, N.Y.: Paideia Press

Körner, S (1960) *The Philosophy of Mathematics. An Introductory Essay*, London: Hutchinson University Library

Lakatos, I (1978) *Mathematics, Science and Epistemology. Philosophical Papers Vol.2.*, J. Worrall & G Currie (eds), Cambridge: Cambridge University Press

Pompeu, G (1992) Unpublished Doctoral Thesis

Restivo, S (1992) *Mathematics in Society and History: Sociological Enquiries*. Dordrecht: Kluwer Academic Publishers

Smith, R (1982) *Gaston Bachelard*, Boston: Twayne Publishers

Tiles, M (1984) *Bachelard: Science and Objectivity*, Cambridge: Cambridge University Press.

Wright, C (1980) *Wittgenstein on the Foundations of Mathematics*, London: Duckworth