DEVELOPMENT OF A MODEL TO ENHANCE MANAGERIAL STRATEGIES IN PROBLEM SOLVING

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The aim of this study was to investigate children's perseverance when solving difficult or unfamiliar number problems. It was concerned with those students who are referred to as 'perseverers' because they reached a stage in their problem solution where they recognised that they had not reached a satisfactory answer and decided to take some action - start again, modify their strategies or change to different strategies rather than giving up immediately.

The sample consisted of ten boys and ten girls in grade 6 and ten boys and ten girls in grade 10. The tasks consisted of number problems of varying difficulty. Data gathering took the form of clinical unstructured interviews with individual students in which they were asked to verbalise concurrently with solving a set of number problems.

Task analysis maps were used to provide overviews of the interview protocols. From observation of the maps of students who were ultimately successful, it became apparent that these children were more inclined than others to be flexible in their use of strategies. A model was developed which described the sequence of strategies used most consistently by successful students. This model formed the basis for a small-scale training programme to investigate whether the strategies could be taught. A descriptive analysis suggested that most of these children were able to be trained to use the model independently.

This paper addresses the difficulty experienced by many students with problem solving and the ease with which many give up an attempted solution when the answer is not immediately apparent. More specifically, it explores the question of managing problem solving strategies, an aspect to which comparatively little research has been directed. It identifies managerial strategies used by successful problem solvers and gives classroom teachers some direction for helping their students to enhance their own strategies. The problems used in the research are structured number problems which require some productive thinking, as opposed to either structured 'word problems' requiring just the recognition and use of appropriate rules, or open-ended problems. Before describing the research (Taplin, 1992), consideration will be given to other literature which has addressed this issue and explored why it continues to be a 'persistent and recurring problem in the practice of mathematics teaching'.

SYNTHESIS OF RESEARCH EVIDENCE

Although there has been considerable emphasis on the importance of problem solving in the mathematics curriculum (Stanic and Kilpatrick, 1989; NCTM 1980, 1989) we are in danger of not developing it to its maximum potential because not enough is known about how people best acquire problem solving skills or how they can best be taught them. There is still much to learn about how problem solvers can best develop and use appropriate processes and strategies. In particular there is a growing need to understand more about the nature of the problem solving process and specifically factors which interfere with its development. This has been approached from two main directions. First there are writers such as Polya (1957) who offer practical procedures and strategies to increase success in the problem solving process. Also, there are writers such as Newell and Simon (1972) who have engaged in scientific research investigations of the effectiveness of these procedures and strategies, trying to tease out underlying mechanics and psychological variables from which they can make generalisations. There are also some writers whose work appears in publications of classroom practice as well as in research journals. Their writing often intersects with each of the two groups described formerly, yet is separate in some respects from both. For example, the work of Schoenfeld (1985a, 1985b) which will be described in this section examines some of the research findings and suggests reasons why they cannot always be applied successfully in learning contexts. The points of view of some writers representing each of these groups will be described here.

Difficulties with Problem Solving

Although a great deal has been written and debated about problem solving, it is still clear that there are difficulties associated with teaching people how to succeed at it (Bransford, Hasselbring, Barron, Kulewicz, Littlefield and Goin, 1989; Kowplowitz, 1982; Cockcroft, 1982). For example, Bransford et. al. (1989) document evidence of students employing sophisticated task-avoidance strategies in order to prevent facing difficult or unfamiliar problems. This section will address some of the difficulties which have been described by both those who are teachers of problem solving procedures and strategies and those who are researchers. It will also consider responses to the research which have been made by writers who can be described as fitting into both categories.

Difficulties Associated with Practical Procedures and Strategies

Polya is probably the best known of the former of these groups, the practical teachers of problem solving. In his well-known discussion of problem solving strategies, How to Solve It, Polya (1957) stresses the importance of having a plan when tackling a problem. He suggests that one of the difficulties with problem solving is that even if a plan is instituted, students often fail to check its implementation during the problem solving process. He also believes that failure to check the 'reasonableness' of the answer is another common problem. 'The student is glad to get an answer, throws down his pencil and is not shocked by the most unlikely results' (p.95). Polya also offers some insight into why so many people find it hard to persevere in problem solving: 'it is easy to keep on going when we think the solution is just around the corner, but it is hard to persevere when we do not see any way out of the problem' (p.93). Not only is this often the case, but as Kowplowitz (1982) says, many students fail in the problem solving process because they do not have a sense of knowing when they have reached a correct solution and often stop when they think they have, although their solution may be inappropriate.

A further hindrance to successful problem solving is an unwillingness or inability to understand the problem and translate it into the appropriate mathematical terms (Polya, 1957; Cockcroft, 1982). In fact Polya describes this as the most common problem. It was pointed out earlier that he believes that many students either rush into an attempted solution without any overall plan, or 'wait clumsily for an idea to come and cannot do anything that would accelerate its coming' (p.95). Bransford et. al. (1989) and Sowder (1989) also suggest that many students rush into trying to solve a problem before they are really aware of what the problem requires them to do or what operations are involved. Lester (1985) takes this point further, saying that children very often rush in and do what they think the cues in a problem are telling them to do, rather than taking time to reflect on what is actually required. He uses the example of the following problem.

> 'Tom and Sue visited a farm and noticed there were chickens and pigs. Tom said, "There are 18 animals." Sue said, "Yes, and they have 52 legs in all." How many of each kind of animal were there?' (Lester, 1985, p.41).

Many of the children who attempt this problem interpret the phrase 'in all' as a cue to add, which they do without giving any further consideration to other cues.

Another possible block to successful problem solving which is often commented upon by this group of writers, and which may well be attributed to the traditional preoccupation with the outcome rather than the process, is an unwillingness by many students to regard themselves as the 'owners' of the problems they are attempting. Rather than accepting responsibility for the problems themselves, there is a tendency for students to think it is common to accept procedures at face value and not try to understand why they work.

Research and Discussion by Other Writers

Amongst the researchers, one of the lines which has been followed in the attempt to contribute to an understanding of the problem solving process is the comparison of the problem solving strategies of experts and novices in the hope that the procedures, or heuristics, used by the experts can be taught to the novices. Newell and Simon (1972), Greeno (1976, 1980), Simon and Simon (1978), Larkin, McDermott, Simon and Simon (1980a, 1980b) Chi, Feltovich and Glaser (1981), Chi, Glaser and Rees (1982), and Schoenfeld (1985a) are some who have researched in this area. However, there is little evidence to suggest that teaching the use of heuristics has effectively enhanced problem solving performance in the classroom (Schoenfeld, 1985a).

This 'novice/expert' research on heuristics has been explored extensively by many writers in an attempt to clarify understanding of why the application of heuristics does not immediately make problem solving performance more effective. One possible explanation that has been advanced is that because novice problem solvers have their own innate systems of 'raw' heuristics (Silver, 1985), the use of these might actually inhibit the development of expert heuristics (Owen and Sweller, 1985). It may also be that the 'expert-novice' studies focus on the end-point, the level of heuristics at which the experts have arrived, rather than accounting for the developmental stages which must be traversed in the transition from 'novice' to 'expert' (Silver, 1985). If this is the case, then teaching the experts' heuristics may be akin to the traditional model of teaching mathematical rules - the problem solver is expected to use the strategies derived by the experts with no insight into how they were derived. A further likelihood is that too much emphasis may have been placed on the actual heuristics and not enough on how to manage them. Schoenfeld (1985a) emphasises this point when he suggests that the knowledge of heuristic strategies alone does not lead to problem solving success, without an understanding of which ones should be used for particular situations. He indicates that the problem solver must be able to develop a structure for knowing when it is appropriate to use a particular heuristic and how to recover from making a wrong choice.

Marshall (1989) is another who believes that problem solvers not only need to be equipped with the right 'tools', but more importantly they also need to 'call upon the knowledge and skills in a nonpredetermined order to make sense of a new experience' (p.161). This will not usually happen by itself. Pupils need to be provided with the right experiences. Resnick (1989) believes that the way we structure our choice of problems can inhibit the development of these managerial skills if they are designed in such a way as to enable students to practise particular rules rather than in such a way as to encourage them to find their own.

It appears, then, that difficulties may arise, at least in part, because educators are not acknowledging that successful problem solving is in fact influenced by a combination of a number of things. Schoenfeld (1985a) suggests four factors which can interact to affect problem solving performance. These are the problem solver's mathematical knowledge, knowledge of heuristics, affective factors which influence the way the individual views problem solving and the managerial skills associated with selecting and implementing appropriate strategies. An examination of some of the research which has been done in the latter two categories will indicate the growing recognition of their importance.

Affective Factors

A set of factors mentioned above by Schoenfeld which has been discussed fairly recently are affective variables. These encompass all of those individual difference variables that have to do with affect, or feelings. They include constructs such as self-esteem, achievement motivation, anxiety, and depression, the effects of which mediate achievement in a number of ways.

To date, there has been comparatively little research exploring the role of affective factors in the problem solving process. Much of what has been done has been limited to exploring correlations between problem solving performance and attitudes, such as motivation, interest, confidence, perseverance, and risk-taking (Lester and Garafolo, 1987). Other work has been of a more speculative nature, based on the writers' observations and experiences and intended as a means of pointing out the directions in which this research should go. An example of this is a series of scenarios presented by Lester and Garafolo (1987) which suggest the 'sometimes dominant influence non-cognitive factors can have on problem solving performance' (p.10). Silver and Kilpatrick (1989) believe that it is time to develop and evaluate schemes of instruction for improving affective factors. It seems likely that 'attempts to influence a range of affective variables should have a reasonably powerful effect on problem-solving ability' (McLeod, 1985, p.276).

It has already been suggested that affective factors can assist or interfere with problem solving. For example, inability to handle emotional reactions to problem solving can hinder the search for a solution: 'students have difficulty persisting in problem solving if their reaction is intense and negative, so they tend to quit and reduce the magnitude of the emotion' (McLeod, 1988, p.134). The implications of this can be linked to Polya's (1957) statement that an environment which denies the student the opportunity to experience the range of emotions associated with problem solving is failing to contribute to a vital aspect of mathematics education. One of these emotions is related to a lack of courage to tackle a potentially difficult problem rather than taking the easiest path of stopping or asking for help (Wertime, 1982; McLeod, 1988). In fact, Wertime suggests that many children become reluctant problem solvers, to the extent that 'they would rather enjoy the consolation purchased by despair than endure the fruitful stress of confronting the [problem solving] process' (p.192). This reluctance is also mentioned by Lester (1985) and in Schoenfeld's (1985b) description of the common belief that 'mathematics problems are always solved in less than 10 minutes, if they are solved at all' (p.372). It seems true that many problem solvers give up rather than face negative emotions because they expect solutions to come to them quickly and easily (Scott, 1988). Schoenfeld (1985a) also states that many 'potentially valuable approaches are abandoned before they can bear fruit' (p.98) because students are not aware of when it is worthwhile to keep on exploring an idea and when it is appropriate to abandon it because it is leading in a wrong direction.

Also linked to affective variables are the belief patterns described by Schoenfeld (1985a) discussed earlier in this paper. One of these is the belief that not only is it important to get the right answer, but that 'getting an answer in the right way is what counts' (Thompson, 1989, p.235). For example, Silver (1985) suggests that many children approach a problem locked into the belief that they must do it in a particular way, that there is always a 'rule' to be followed and that any other approach is 'wrong' (Silver, 1985). This kind of belief pattern can be very inhibiting to a child who is finding it difficult to begin a solution or continue after encountering difficulties.

It seems that at least part of the challenge of problem solving instruction is to help problem solvers to break away from these belief patterns by developing an awareness of when it is appropriate to use a particular strategy, when it is worthwhile to persevere and when it is not. It is therefore necessary to consider the managerial factors which link together knowledge, heuristics and affective variables.

Management Factors

As with affective factors, the importance of understanding managerial factors has only been acknowledged recently. To date little research has been reported in this area, although some writers advocate the need for it. Lester (1985), for example, claims that there has been much emphasis in research on the 'discrete skills and procedures'(p.43) of problem solving, but little on the 'managerial aspects ... which serve as "guiding forces". He advocates the need for this to be done through qualitative as well as quantitative methods in order to obtain clear insights into what these variables are and how they can be influenced. One of these managerial aspects is the previously mentioned sense of when it is appropriate to use a particular strategy. Linked to this, and also closely linked to the emotional factors described above, is the knowledge of how to persevere when the problem solving process becomes difficult. Unless problem solvers have the desire to persevere, knowledge of heuristics and planning can become redundant because the will to use them does not exist. However, it does not appear to be enough to just help students to understand that frustration, for example, is a normal part of problem solving (McLeod, 1988), and to encourage them to spend time working on the task. They also need to know how to make '[managerial] decisions about whether to persevere along a possible solution path' (McLeod, 1988, p.138). It is desirable to encourage perseverance, but on the other hand it is possible to 'overpersevere', particularly if one becomes locked into one approach. 'Overpersevering' is an unproductive strategy to employ when it may be more appropriate when stuck to use other strategies, such as help-seeking (Nelson-Le Gall and Scott Jones, 1983). This poses not only the question of how to enhance perseverance, but as well the question of how to avoid time being wasted on 'overperseverance'. Since this appears to be an important question within the context of managerial strategies for problem solving, it is this which will become the main focus of the research described here.

THE FOCUS OF THIS STUDY

This study was directed towards exploring how the particular managerial factors associated with perseverance could contribute to problem solving success, since comparatively little is known about perseverance. It was decided to address an aspect of perseverance in solving problems by examining the ways in which persevering students chose to pursue possible solution paths - not only what strategies they used, but how they managed them. The focus was not on those students who succeeded with a problem immediately, or on those who gave up quickly. Rather, it was on those who did not arrive at a quick solution but were prepared to engage with the task for some time until they either succeeded or chose to give up. The study was not, therefore, concerned with what initially motivated this engagement with a problem. It was intended to compare those persevering students who are ultimately successful with those who give up after persevering for some time, in an attempt to investigate whether, under the specific conditions of the study, there were any different patterns in the use of strategies by those who are successful.

The initial sample consisted of ten boys and ten girls, aged 12 years, in grade 6, their final year of primary school. As well, a sample of ten boys and ten girls, aged 16, studying the top academic level of mathematics in grade 10 was interviewed in order to obtain strategy data on a group of subjects who successfully solved the problems.

The tasks consisted of ten number problems of varying difficulty, all requiring a degree of productive thinking rather than just the recognition and application of an appropriate rule. Care was taken to ensure that subjects were familiar with any necessary knowledge required for successful solution and a calculator was made available to reduce the burden of computation.

Data gathering took the form of interviews with individual subjects in which they were asked to verbalise concurrently with solving the set of problems. Where necessary, they were also asked to reflect on what they had done retrospectively to fill in gaps in the interviewer's understanding of what took place.

Subjects were invited to continue with a problem for as long as they liked and to move backwards and forwards between problems at any stage. Most interviews took place over a series of sessions, each of approximately one hour's duration. As seeking help is regarded as a legitimate strategy for approaching a problem, specific assistance was given when requested. Interviews were recorded and later transcribed for analysis.

Task analysis maps developed by Chick, Watson and Collis (1987) and modified by Collis and Watson (1991) were used to provide overviews of the interview transcripts. These recorded information about the way in which the initial data provided in the problem were used, the concepts, processes and strategies used in attempting to solve the problem and the structure of responses, both correct and incorrect, made at various stages in the solution of the problem. Four stages in the problem solving process were considered. These are shown with the symbols used for their representation:

(i) ▲ the use of cues. This includes cues from the problem which indicated what the subject was required to find. It also includes data which were given or which the subject needed to imply in order to attempt a response,

(ii) ● concepts and/or processes used by the subject to operate on selected data,

(iii) ■ interim responses which were made as the result of applying a particular process. The symbol] was added if the interim response led to a 'dead end', requiring the subject to return to the cues and start again,

(iv) the final response made before the subject either accepted the (correct or incorrect) answer or abandoned the question. Three final response symbols were used:

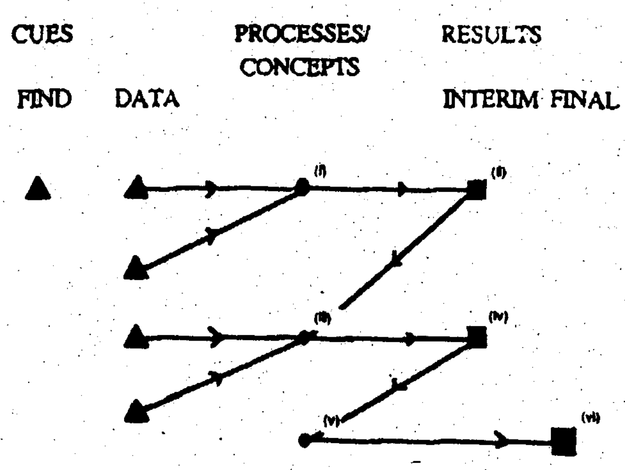
■ correct solution

□ incorrect solution with which subject was satisfied,

]] abandoned question.

Figure 1 gives an indication of how a map can be interpreted. The student selects data to which a process (i) is applied. This leads to an interim result (ii). More data are needed for the next piece of processing to occur (iii). This leads to a further interim result (iv). The student then checks this result against the initial cues (v) and is satisfied with a correct solution (vi). Lines were used to indicate single or groups of cues used for an interim response. The process or concept used to obtain this response was noted. Because the maps for the actual problems solutions were complex and lengthy, no examples have been included here, but all followed this basic format. One example of such a map has been included in Appendix 1.

Figure 1: Simple example of a task analysis map



From observation of the maps of subjects who were ultimately successful, it became apparent that these children were more inclined than others to be flexible in their use of strategies. In order to investigate this phenomenon, analysis was made of strategies which subjects employed after having reached a "dead end", that is explored a particular strategy until they could go no further with it.

A descriptive analysis using the constant comparative method (Glaser and Strauss, 1967) indicated nine problem solving strategies within two categories:

(i) repeating the same approach as in the previous strategy and:

(a) using the same set of data (cues) but bringing in additional cues in order to complete the data required for successful solution,

(b) using the same data but introducing more cues than previously, as in (a), but not having the complete set of data required for successful solution,

(c) using exactly the same data, including instances where subjects altered numbers but still used the same ideas,

(d) using a different set of data,

(ii) changing to a different approach and:

(a) using the same data but bringing in additional cues in order to complete the data required for successful solution,

(b) using the same data and introducing more cues than previously, as in (a), but not having the complete set of data required for successful solution,

(c) using exactly the same data,

(d) changing to a different set of data,

(e) using a different set of data which involved returning to an approach which had been used and abandoned previously.

At this stage of the investigation it was considered that sufficient information about strategy use could be obtained by refining the nine strategies into four categories. This refinement is consistent with the model of qualitative analysis discussed by Glaser and Strauss (1967):

(i) using the same approach as in the previous attempt and using the same set of data (cues),
(ii) using the same approach as previously but a different set of data,
(iii) changing to a different approach from that used in the previous attempt and using the same set of data,
(iv) changing to a different approach and using a different set of data.

Maps illustrating these four strategies are included in Appendix 2. Descriptive analyses suggested that the most useful strategy used by successful "perseverers" was the "different approach/same data" one, which was often the final strategy which led to success. The "different approach/different data" strategy was also used, but to a lesser degree, by many of the successful students. The "perseverers" who eventually gave up were more inclined to use the "same approach/same data" strategy repeatedly. A closer examination was made of the way the "different approach/same data" strategy was used in relation to the others, to investigate whether successful problem solvers used it systematically and whether it was used at a crucial stage in the problem-solving process, presumably near or at the end.

Observation of the patterns of strategy use revealed that a commonly used sequence of strategies was that presented in Figure 2. Step (ii), refinement, was included because successful students commonly repeated the "same approach/same data" strategy approximately three times to refine an approach before trying something different.

Figure 2: Strategy sequence commonly used by successful problem solvers

(i) select first strategy
(ii) (if necessary) refine strategy, using 'same approach/same data strategy 1-3 times
(iii) try a different approach ('different approach/same data'or 'different approach/different data')
(iv) repeat from (i) if necessary

IMPLICATIONS FOR THE CLASSROOM

The next stage of the investigation was to explore whether the quality of children's perseverance time could be enhanced by training them to use this model. A small scale training programme was conducted with six 13-year-old grade 7 children. These children worked individually in a clinical interview situation with the experimenter on a series of four of the number problems used in the earlier part of the study. On the first problem none of the children could use the model without prompting. However, all were able to use it independently by the third or fourth problem. These patterns in the development of subjects' use of the model suggest that it is feasible to "train" children to use it. Clearly a long-term study would be required to determine the effects of such a training programme on children's ability to use the model to increase successful perseverance over a period of time.

Currently the model is being trialled in group and individual situations with groups ranging from primary to tertiary. Analysis is indicating that it is regarded by those who have used it as a useful tool. The procedure which is followed for implementing the model is indicated in Figure 3.

Figure 3: Recommended procedure for introducing problem solving model

(i) Give the student a preliminary problem to solve without guidance. Observe whether the student instinctively used the model.
(ii) Introduce the student to the model and demonstrate using the example provided.
(iii) Ask the student to repeat the first problem while you guide him/her to use the model, i.e. prompt the student to change to a different approach after a maximum of three repetitions of the previous approach
(iv) Give two more problems, monitoring the strategy pattern and reminding students, when necessary, to follow the model.
(v) Give a fourth problem and ask the student to try to follow the model, changing approach when appropriate, without any prompting from you.

The presence of the teacher is essential in the early stages of introducing the model, as intervention at the appropriate time to sugest changing to a new strategy is crucial. This can be done on a small group basis. It is also important for the teacher to be equipped with a range of different approaches to the problem solution, as in the early stages problem solvers who are willing to switch approaches are not necessarily able to think of alternatives. Interestingly, as they become more confident in using the model they also seem to become more confident and adept at finding their own alternative strategies. The teacher must be wary of telling the student how to solve the problem, but rather suggest when it might be appropriate to change approach and, if necessary, provide hints that will lead to the selection of a new approach. This gives the teacher a more positive role in offering encouragement, rather than simply telling the student to 'try again'. As problem solvers begin to use the model more automatically, teacher intervention becomes less necessary. At this stage it can be useful to have pairs of students working together to solve the problem, which encourages them to decide together when they should try something different or pursue an idea for a bit longer.

More time is needed to fully explore the long term effects of the problem solving model in the classroom setting. However there is already some evidence to suggest that it can be incorporated easily and effectively into the problem solving programme in the manner outlined above.

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