EXAM PERFORMANCE AND THE GRAPHICS CALCULATOR IN CALCULUS

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At Swinburne University of Technology we have found that, after the introduction of a graphics calculator in introductory calculus subjects, female students for the first time significantly outperformed males on calculus tests. Students solution strategies were examined as a possible reason for the superior performance of females, but seemed to point to the opposite result. Comparison of scores of exam questions with graphical and purely algebraic content revealed that females received their better marks due to their better performance on purely algebraic questions.

There have been a number of recent studies in which a graphical calculator or graphical package was used to support the teaching of calculus (Heid, 1988; Ruthven, 1990; Ryan, 1992; Tall, 1989; Teles, 1990 (cited by Dunham, 1992); Vazquez, 1991 (cited by Dunham, 1992)). Some of these studies show that there are several ways that students appear to benefit from this more visual approach to calculus. For example, Heid (1988) found that by focussing on concepts instead of procedures for the main part of the course and by studying concepts with the help of computer generated graphs, students' conceptual understanding was improved while their performance on a regular technique based calculus test was not hampered. Ruthven (1990) found that students who have access to graphics calculators in a traditional calculus course showed a significantly better capacity to translate graphs into an algebraic form than students who did not have access to such calculators in their course. He explained his results by stating:

> Regular use of a graphic calculator is likely to rehearse specific relationships between particular symbolic and graphic forms, as it is through such relationships that the calculator itself is operated, albeit in the reverse direction to that tested. Moreover, reliable access to graphics calculators is more likely to encourage both students and teachers to make more use of graphic approaches in solving problems and developing new mathematical ideas, not only strengthening these specific relationships, but rehearsing more general relationships between graphic and symbolic form. (p. 447)

Ruthven also found that girls and boys appeared to benefit differentially from the graphics calculator: the girls in his study profited more than the boys. He explained the positive influence on women by observing that the calculator provided feedback to women which might have the effect of reducing their generally higher level of anxiety and that the greater exposure to graphical images might have increased competence and confidence of female students. Similarly in a technology enhanced pre-calculus course Dunham (1991) found that, although males showed a superior performance than females on visual items of the pretest, both groups of students gained significantly over a ten week period in their competence on visual items. Furthermore, there were no longer competence differences between males and females on visual items of the posttest. Dunham (1991) also studied the impact of the graphics calculator on calculator neutral test items, that is, those that could be solved either algebraically or graphically. She did this by interviewing eight students after each test in the subject and by asking them what method they used to solve the problems. The majority of solution methods she found were either purely graphical or purely algebraic. Only 23% of the students' solutions contained a mixture of graphical and algebraic methods; students did say that in the exam they preferred the graphical approach because of the speed with which you could find an answer, but also expressed 'algebraic-guilt' about using the calculator.

With regard to visualisation skills, Shoaf-Grubbs (1992) found that female students' visualisation skills improved with a graphics calculator. However, these tasks were not specifically mathematically related. Recently, Boers & Jones (1992) found that the introduction of the graphics calculator into the first year of a Mathematics/Computer Science degree was associated with differential performance levels of males and females. With the introduction of the graphics calculator in 1991, females in this group scored significantly higher than males ($F(1,68)=2.14$, $p=0.02$), whereas in the two consecutive years before its introduction the males of this major scored slightly higher than females, although not significantly (see figure 1). In 1991 the females scored on average 10% higher than the males. While in 1992, the difference was of a similar proportion, 9%, but due to the smaller sample size did not reach statistical significance ($F(1,34)=0.70$, $p=0.41$).

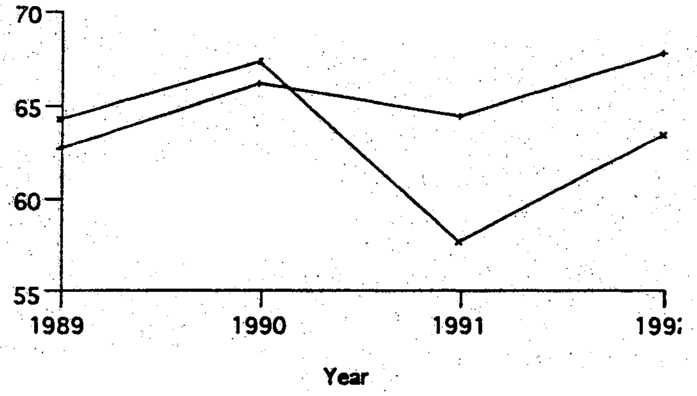

Figure 1: Comparison of mean Male and Female Math/Comp Science Scores. (+ Females, x Males)

This paper reports some recent investigations into the source of this apparent differential influence of the graphics calculator on the performance of males and females in which answers to the following research questions were sought:

1. Is there a difference in the kind of solution strategies, as defined by Boers & Jones (1993), used by males and females to answer questions with explicit graphical content?
2. Is the performance difference between males and females related to the question content?

METHOD AND RESULTS

Introduction.

In 1991 the TI-81 graphics calculator was prescribed for all first-year Applied Science students taking introductory calculus. The course content remained essentially unchanged from previous years, except that graphical solutions were given greater emphasis than in the past. In addition, the calculator was used as an integral part of the teaching process to provide alternative graphical representations of processes that were previously presented in symbolic form only, for example, limits. Students were also urged to use their calculator as checking devices. The assessment was not changed from previous years, therefore comparison with previous years was possible.

Comparison of Solution Strategies

In a previous paper Boers & Jones (1993) found that students used a variety of strategies when solving graphically oriented calculus problems. Some of these strategies were mathematically more sophisticated than others. It was thought that perhaps the superior performance of Mathematics/Computer Science females was because they used different strategies than males on graphically oriented calculus problems. To test this, the solution strategies of the Mathematics/Computer Science majors, a subset of 67 students, in which the females significantly outperformed the males after the introduction of the graphics calculator, were classified according to the scheme developed by Boers and Jones (1993). Two questions on the paper were susceptible to this analysis.

The first question. The first question on which this analysis was done, was a routine graphing question involving a rational function with two discontinuities: one involving a vertical asymptote and the other being a removable discontinuity. The question was:

(a) For what values of x is the function $y = \frac{x^2+2x-3}{2x^2+3x-5}$ not defined?

(b) Find an exact value for the limit $\lim_{x \to 1} \frac{x^2+2x-3}{2x^2+3x-5}$

(c) Sketch a graph of the function y.

Due to the limitations of the graphics calculator students needed to integrate algebraic information gathered from parts (a) and (b) of the question with the graphical image produced by the calculator to form a complete picture of the function. The graphics calculator will not show removable discontinuities unless they fall exactly on the centre of a pixel of the screen as was the case in this example. In the initial study of student solutions to this problem, three main solution strategies were identified:

Strategy I (successful integration of information):

the student successfully integrated both algebraic and graphical information in reaching their solution. An indication of successful integration was the presence of a gap in the graph or a circle around the point x=1 of the graph of $f(x)$.

Strategy II (failed integration of information):

the students attempted to integrate both algebraic and graphical information in reaching their solution but failed in their attempt. When conflict arose some tended to disregard the graphical information in favour of their algebraic work (Strategy IIA, failed integration: algebraic preference), that is, adjusting the graph by showing two vertical asymptotes, while others did the opposite (Strategy IIB, failed integration: graphical preference). The latter group adjusted the answer to part (a) to only one point where the function was not defined, that is, x=-2.5 only.

Strategy III (no integration of information):

students made no real attempt to integrate algebraic and graphical information in reaching a solution. There was no apparent realisation that the solution to an earlier part of the question bore any relationship to the graph they had to produce in the end. Three subgroups could be identified: one group of students who used an algebraic method to solve the first part of the problem (Strategy IIIA, No integration: algebraic), one who used a non-algebraic method (Strategy IIIB, No integration: non-algebraic), and one for students whose solution to the first and the third part of the question seemed purely calculator generated; no evidence of algebraic work was shown (Strategy IIIC, No integration: calculator only).

A detailed description of each of these strategies with sample student responses can be found in Boers & Jones (1993). In classifying the responses of the students in this study new strategies were not found. This confirmed the validity of the initial classification scheme. The range of strategies used by males and females along with percentage usage, are shown in Table 1.

Table 1**: The number and percentages of students adopting each strategy for question 1.**

	Strategy			
	I Integration	II Failed Integration	III No Integration	Not Classified
Males N=40	6(15%)	6(15%)	23(58%)	5(13%)
Females N=27	1(4%)	6(22%)	20(74%)	0(0%)

From Table 1 we can see that in general the females used lower order strategies than males. That is, there were more males than females who were able to integrate the algebraic information with the graphical information and there were more females than males who appeared not to integrate the two pieces of information (74% versus 58%). Due to the small cell sizes appropriate statistical tests could not be performed.

The second question. The second question on which comparisons between strategies were made, was:
(a) Sketch a graph of $y = x^{1.4}e^{-x}$ (x _ 0), labelling the coordinates of the two stationary points.
This question was more straight forward, requiring less integration of information than the previous question. Three strategies were identified:

Algebraic
: the student primarily relied on their algebraic skills to arrive at their solution, only using the calculator to help determine the general shape of the function and possibly check the reasonableness of their solution.

Failed algebraic to calculator
: the student attempted an algebraic solution but failed, then attempted a primarily graphically based solution.

Calculator only
: solution appeared to be purely graphically and calculator generated; no evidence of any algebraic work shown.

The strategies found for the two questions are different because the second question demanded fewer integration of information skills from the students than the first question. The second question could be solved by calculator alone, whereas the first question needed either algebraic supplementation or other reasoning to solve entirely by calculator.

In classifying the responses of the students no new strategies were found, again confirming the validity of the initial classification scheme. The range of strategies used by males and females along with percentage usage, are shown in Table 2.

Table 2: **Percentage and number of students adopting the three strategies for question 2.**

	Strategy			
	Algebraic	Algebraic/ Calculator	Calculator only	No graph
Males N=40	13(33%)	5(13%)	18(45%)	4(10%)
Females N=27	10(37%)	3(11%)	11(41%)	3(11%)

From the table it can be seen that women had a slightly higher preference for the algebraic strategy, whereas men were slightly more inclined towards the calculator strategy. This finding is similar to that of Dunham (1991). The mixed solution method (algebraic/calculator), which was very similar to the calculator only strategy in that the final solution was based on the calculator alone, was similarly preferred by males. For this question statistically significant differences between the choices of strategies of males and females were not found.

Conclusion. Comparison of the strategies used by males and females to answer questions with a graphical content does not seem to support the idea that the strategies chosen by females would make their scores on the exam higher than those of males. Based on the first question one might actually expect females' scores to be lower due to their less sophisticated responses to the question. Therefore a comparison of scores that males and females received on graphically and algebraically oriented problems might shed more light on the reasons for the difference in overall performance. This is done in the next section.

Comparison of performance on graphical and non-graphical questions.

In this section we investigate the relationship between the question content and the performance of males and females on the exam. The sums of scores of males and females on questions with a graphical content and those questions which required only algebraic work were compared. These are shown in Table 3. Problems that could be solved either graphically or algebraically, such as limits, were excluded from this analysis. In 1991, the females outscored the males on both sorts of questions, marginally on the graphical and significantly on the non-graphical questions. This does not support the contention that females outscored the males on the whole test because the graphics calculator helped their performance on questions with a graphical content. The difference was primarily due to their superior performance on non-graphical questions. When we look at a similar analysis for 1989, we see the reverse. In 1989, males and females scored equally well on algebraic questions, whereas on questions with graphical content females outscored males (see Table 3). It might therefore be the case that the performance of females did not necessarily improve, but that the males performed more poorly with the introduction of the calculator. Possible explanations could be that the males were more interested in the tool and therefore no longer as keen to study and exercise the problem solving skills necessary for a calculus exam that had as its major focus the assessment of manipulative skills.

***Table 3*: Comparison of scores of males and females on graphical and non-graphical questions in 1989 and 1991.**

	1989		1991	
	Algebraic	Graphical	Algebraic	Graphical
Males	24.2 (0.8)	10.0(0.4)	17(1)	5.3(0.3)
	N=71	N=71	N=27	N=40
Females	24 (1)	11.1(0.6)	20(1)	5.6(0.3)
	N=30	N=30	N=40	N=27
	p=.9	*p*=0.13	*p*=0.10	*p*=.42

SUMMARY AND CONCLUSION

In this article we have tried to explain a difference in performance of males and females that occurred when a graphics calculator was introduced into a first year mathematics program. We have looked at differences in strategies males and females use on questions requiring a graphical response and at differences in performances on graphical and algebraic questions. With respect to the strategies, when integration of graphical and algebraic information was required, more males than females were successful. When students had an option to solve a problem either algebraically or graphically, slightly more females preferred the algebraic strategy and, vice versa, slightly more males preferred graphical strategies. Based on the analysis of performances on graphical and purely algebraic questions we must draw the conclusion that the better performance of females was related to a better performance on algebraic questions. Considering the trend of the performance scores over the last four years, one might come to the conclusion that with traditional technique based calculus testing, males are disadvantaged by the introduction of the graphics calculator.

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