MULTIMODAL FUNCTIONING DURING MATHEMATICAL PROBLEM SOLVING

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This study investigated the interface between ikonic and concrete symbolic functioning by studying two different aspects of visual processing associated with mathematical problem solving. The first of these involved the use of mathematically based visual images and diagrams, while the second was concerned with incidental non-mathematical visual imagery associated with the problem story. Four groups of grade 10 students were studied, representing all combinations of high and low concrete symbolic ability and high and low ikonic ability. The results suggested that success at problem solving was related to concrete symbolic, but not to ikonic ability, and that the use of mathematically based visual images and diagrams was similarly related to concrete symbolic rather than ikonic ability. Incidental visual imagery associated with the problem story was, however, linked to individual differences in ikonic processing and was not related to concrete symbolic functioning.

There is increasing recognition that intuitive and imagery based processes play an important role in all levels of mathematical problem solving from those of the child in the early stages of mathematical development (Moses, 1982; Maher & Alston, 1989; Wheatley, 1991); through the non-conventional workplace mathematics of the adult with little formal mathematical training (Resnick, 1987; Scribner, 1984); to the novel creations of highly gifted mathematicians and scientists (Hadamard, 1954; Shepard, 1988). There is, moreover, a more general recognition of a relationship between imagery ability, and problem solving and creativity in areas outside of mathematics (Shaw & DeMers, 1986; Armbruster, 1989; Kaufmann & Helstrup, 1985). These findings suggest that the development of mathematically competent and creative students involves not only the traditional focus on computational and logical problem solving abilities, but also the development of associated ikonic skills at all stages in the developmental process.

Biggs & Collis (1991) provide a theoretical model of how ikonic processes might develop in interaction with concrete symbolic ones. Following Piaget and others, they describe the development of different modes of thought, and argue that the ikonic mode of functioning begins in early childhood, after the sensorimotor mode is well established. The ikonic mode, however, is not simply associated with early childhood thinking, to be replaced during school years by the concrete symbolic mode. Rather it continues to develop in power and complexity and to interact with other modes as they develop. Collis & Biggs (1991) argue that much of our more complex thinking and problem solving is multimodal and use of the ikonic mode in conjunction with the conventional logic of the concrete symbolic and formal modes, provides richness and flexibility.

While there has been considerable research focusing on the development of the concrete symbolic mode and a firm understanding achieved (Biggs & Collis, 1982; Campbell, Watson & Collis, in press), analysis of ikonic mode development and its interaction with the concrete symbolic is still exploratory and conclusions remain tentative (Collis & Romberg, 1991; Collis, Watson & Campbell, 1992; Watson, Campbell & Collis, in press). The present study was designed to investigate the interaction between ikonic and concrete symbolic modes during problem solving. This paper looks at one aspect of this, namely visual processing.

METHOD

Subjects

A group of one hundred Grade 10 students from two public high schools in upper socio-economic class suburbs of Hobart, Tasmania were screened using the ACER Mathematics Profile Series: Operations Test to provide a measure of concrete symbolic mathematical ability, and the visual imagery scale of the Betts Questionnaire on Mental Imagery (QMI; Sheehan, 1967) to assess one factor hypothesised to be important in ikonic mode functioning. Fifty students with the highest and lowest scores on the two tests were selected and four groups of 12 or 13 students each were formed, providing all four combinations of high and low concrete symbolic ability and high and low vividness of visual imagery.

Procedure

The above fifty students were interviewed and their responses tape recorded while they solved four problems. Two of these problems were novel while two were school-based; and two of the problems required the use of diagrams or visual images for their solution while the other two could be solved using non-visual concrete symbolic processes. Responses on the following three problems are analysed here to explore the way in which students produced and used visual images and diagrams:

Hungry Man: Three tired and hungry men went to sleep with a bag of apples. One man awoke, ate 1/3 of the apples, and then went back to sleep. Later a second man awoke, and ate 1/3 of the remaining apples, and then went back to sleep. Finally, the third man awoke and ate 1/3 of the remaining apples. When he had finished there were 8 apples left in the bag. How many apples were there originally?

Painted Cube: A cube that is 3cm by 3cm by 3cm was dipped in a bucket of red paint so that all of the outside was covered with paint. After the paint had dried, the cube was cut into 27 smaller cubes, each measuring 1cm on each edge. Some of the smaller cubes had paint on 3 faces, some on 2 faces, some on only 1 face, and some had no paint on them at all. Find out how many of each kind of smaller cube there were.

Drink-Driving: The blood alcohol readings of two drivers were recorded the morning after an accident. The readings were:

Mark	Wayne
6 hours after accident: **5 units**	**5 hours** after accident: **7.5 units**
8 hours after accident: **2 units**	**9 hours** after accident: **5.5 units**

Assuming a linear relationship answer the following:
(i) Who had the highest reading at the time of the accident?
(ii) When were their readings the same?

Two different aspects of visual functioning were studied:
(i) Mathematical diagrams and visual images associated with these;
(ii) Non-mathematical visual images related to the problem story.

RESULTS AND DISCUSSION

The data were analysed with reference to the following three questions, each of which will be analysed in turn.

Question 1: Is success at problem solving related to individual differences in either concrete symbolic or ikonic abilities?

A two way ANOVA was conducted to test for significant differences in the total number of problems solved correctly by students with high and low concrete symbolic ability, and high and low vividness of visual imagery. There was a significant effect due to concrete symbolic ability ($F=29.16$, $d.f.=1$, $p<.0001$), but no significant effects due to vividness of visual imagery or its interaction with concrete symbolic ability. These results indicate that individual differences in ikonic processing, as measured by the QMI, have no direct effect on problem solving success, while ability with basic mathematical operations is significantly related to problem solving ability.

Question 2: How are diagrams and mathematically based visual images used in problem solving; and is their use related to individual differences in concrete symbolic or ikonic abilities?

While solving the painted cube problem, almost all students, irrespective of their vividness of visual imagery, stated that they formed images of the cube. Evidence of the use of visualisation strategies by students with low as well as high vividness of visual imagery also came from the methods by which students correctly calculated how many cubes had no painted faces. Some calculated this mathematically, while others pictured the single cube in the centre of the 3x3x3 array, a solution requiring manipulation of visual imagery. There were no significant differences between students in different categories in the use of these strategies, which indicates that an understanding of the structure of the problem was more important than vividness of visual imagery in the use of the visualisation strategy.

In addition to using visual imagery, 84% of students drew one or more diagrams to facilitate problem solution. Of the eight students who drew no diagrams, three solved part or all of the problem correctly. These students all reported a clear image of the cube and counted smaller cubes on this image. All had either high concrete symbolic skills or high visual imagery skills. In contrast, the five students who drew no diagrams and who were unable to solve any part of the problem, all belonged to the low concrete symbolic, low vividness of visual imagery category and none reported a clear picture of the cube. These results suggest that a distinction can be made between students who do not draw diagrams because they are able to generate and use appropriate visual imagery to solve the problem, and students whose imagery and understanding of the problem are so poor that they are unable to produce useful images or diagrams. The former group all had either high concrete symbolic skills or high vividness of visual imagery, while the latter group were low in both, suggesting that students in the low, low category may have fewer resources available to them when solving such problems than students in the other three categories.

Analysis of the number and type of diagrams drawn by students in different categories also indicated that the production of mathematically-based images and diagrams was dependent upon an understanding of the mathematical structure involved (a concrete symbolic process) rather than upon a more general ability to produce vivid visual images (an ikonic process).

Similar conclusions were reached when the use of graphs in solving the drink-driving problem was examined. Students with high compared to students with low concrete symbolic ability were significantly more likely to mention or use a graph while solving part of this problem (X^2=9.92, df.=1, p<.005); or actually to solve part of the problem correctly using a graph (X^2=6.64, df.=1, p<.01). There were no significant effects due to vividness of visual imagery. These results suggest that while mathematically based images, diagrams and graphs may represent an interaction between ikonic and concrete symbolic functions, the concrete symbolic component would seem to be the more important in determining whether appropriately structured images and diagrams are produced.

Question 3: Do problem solvers form non-mathematical visual images relating to the problem story; are these related to individual differences in ikonic or concrete symbolic abilities; and do they play a role in problem solving?

Students were asked whether they had any images relating to the story or its characters while solving the drink-driving and hungry man problems. Students with high compared to those with low vividness of visual imagery were significantly more likely to report such imagery on the drink-driving problem (X^2= 15.24, df.=1, p<.001); or to report clear imagery on the hungry man problem (X^2=6.35, df.=1, p<.05). There were no significant differences due to concrete symbolic ability.

The results suggest that, unlike the use of mathematically based visual images and diagrams, the production of incidental visual imagery connected to the problem story is linked to individual differences in ikonic functioning as measured by the QMI, rather than to concrete symbolic ability. Such visual imagery may play a useful role in problem solving. Comments from students in all categories suggest that visual images and stories related to the problem make maths problems more interesting. They also help in understanding the structure of the problem and in assessing the reasonableness of the answer.

CONCLUSION

The results of this study suggest that success in both novel and school-based mathematical problem solving is related to basic concrete symbolic abilities and not to individual differences in ikonic processes. In addition, while mathematically based visual images and diagrams undoubtedly have an ikonic component, their production and use are primarily dependent on a concrete symbolic understanding of the structure of the problem. This provides an example of how ikonic processes, in this case visual imagery, may be used by the concrete symbolic mode for its own logical deductive purposes. In contrast, incidental visual imagery associated with the problem story appears more closely tied to individual differences in ikonic ability and to be unrelated to concrete symbolic functioning. While the value of mathematically based visual images and diagrams in problem solving is well accepted, it is not clear whether incidental, non-mathematical visual imagery has a useful function. According to student reports, however, such visual imagery appears to motivate students, and to help them clarify the story structure and assess the reasonableness of their answers. As such it may provide another problem solving strategy for teachers to foster, much in the way described by Moses (1982), which will facilitate problem solving, especially among students experiencing difficulties. Watson et al. (in press) demonstrated that ikonic support may be particularly helpful when conceptual understanding is weak, but tends to disappear once the problem is well understood.

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