MENTAL COMPUTATION STRATEGIES FOR ADDITION AND SUBTRACTION ALGORITHMS

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The changing nature of modern technological society and the role of mathematics in its functioning is resulting in calls for increased emphasis on mental computation and computational estimation and reduced emphasis on pen-and-paper algorithms. There is strong evidence that children begin primary school with many original and creative strategies for the operations, strategies that support mental algorithms and computational estimation. However, the procedures in traditional pen-and-paper algorithms appear to inhibit these spontaneous strategies and the early treatment of pen-and-paper algorithms may be wrongly placed in modern mathematics syllabi. This paper identifies and describes the different spontaneously derived cognitive strategies for one-, two- and three-digit mental computation in addition and subtraction used by 130 children within a longitudinal study to examine changes in knowledge and use of strategies over Years 2 and 3 of traditional pen-and-paper algorithm instruction.

There have been changes to the scope and sequence of mathematics in recent Australian syllabus documents tc delete and defer some pen-and-paper algorithms and give greater focus to calculators, mental computation and estimation. Experience with materials before introducing pen-and-paper recording has been stressed. Computations with large numbers have been left for the calculator. However, unlike some initiatives overseas (e.g. Shuard, 1991), mastery of pen-and-paper algorithm procedures has tended to remain the chief goal of operations while mental computation, estimation and calculator usage have followed the pen-and-paper algorithm instruction.

The procedures in the traditional pen-and-paper algorithms for addition and subtraction appear to contradict those for mental algorithms. When adding the traditional algorithm requires: (1) the two numbers to be split into ones and tens; (2) the ones and tens to be added separately, with the ones added first and any tens formed added in with the tens; and
(3) the two sums combined for the answer. On the other hand, efficient mental algorithms tend to do all or some of the following: (1) hold one of the numbers unsplit; (2) add the tens before the ones; and (3) add the numbers cumulatively, modifying the answer as the calculation proceeds, e.g. 25+37=55+7=62. Mental algorithms appear to differ from traditional algorithms in terms of what they do to the numbers and in the order in which they add tens and ones.

There is strong evidence that children begin primary school with many original and creative strategies for the operations (e.g. Carpenter & Moser, 1984; Ginsburg, 1977). In particular, children have been able to invent cognitive strategies for the addition and subtraction algorithms and these strategies have been closer to the mental than the pen-and-paper procedures (e.g. Bebout, 1990; Cobb & Merkel, 1989). Evidence has existed for many years that strategies improve performance (Flavell, Beach & Chinsky, 1966) and that invented strategies enhance learning (Flavell, Friederichs & Hoyt, 1970). There is also evidence that such cognitive strategies can be taught (e.g. Lindquist, 1987).

Therefore, there is cogent argument that instruction in pen-and-paper algorithms may be wrongly placed in Australian mathematics syllabi, in that it may diminish the use of invented strategies and therefore act against the needs of children in a technological society (Sowder, 1990). There are strong doubts as to the need for formal pen-and-paper algorithms at all and support for restricting recording procedures to informal child-chosen techniques (Shuard, 1991).

This paper reports on an ARC funded two-year longitudinal study across years 2 and 3 of children's spontaneously-derived cognitive strategies for one-, two- and three-digit mental addition and subtraction and the effect of instruction from the traditional pen-and-paper algorithms on these strategies. It describes a range of strateges identified and the changes in strategy use across the two years.

METHOD

Subjects

The subjects of the study were approximately 65 girls and 65 boys from 3 state primary schools and 3 private primary schools, representing a variety of abilities and socio-economic backgrounds.

Instruments

The instrument used was a 'mixed cases' interview (Ginsburg, Kossan, Schwartz & Swanson, 1983). The children were presented with a sequence of one-, two- and three-digit addition and subtraction tasks to solve mentally. The sequence moved: (1) from addition to subtraction; (2) from one-digit to two-digit to three-digit numbers; (3) from non-renaming to renaming situations; (4) from a real world problem and pictorial presentation form to symbolic presentation form (horizontal to vertical); (5) from take-away to missing addend to comparison concepts of subtraction; and (6) from strategy-friendly to non-strategy examples (e.g. 26+49 to 26+47). The sequence of addition and subtraction examples was designed so that when children began to fail at mental addition, the appropriate level at which to begin the subtraction examples was evident.

Procedure

The children were interviewed six times between the beginning of year 2 and year 4. They were withdrawn for their classroom and interviewed in a vacant room within the school. The interviews lasted approximately 20 minutes and were all videotaped.

For the different areas, addition, subtraction, real world and symbolic, the children were given more difficult examples until they were unable to provide a strategy or unwilling to attempt the example. The interview procedure was designed to challenge not threaten the child.

RESULTS

Analysis

The videotapes were transcribed into protocols and the children's solution behaviours categorised. These categories of performance were then related to similar findings from the literature (e.g. Carpenter & Moser, 1984; Ginsburg, 1977) and to analyses of expert bahaviour, and competence categories involving the presence of cognitive strategies hypothesised to explain the behaviours. The analysis focused on findings; (1) across children for the various example types; (2) within children across the different example types; and (3) across the six interviews for individual children and the cohort as a whole.

Strategies

In the first interviews in year 2, most children used counting strategies even for reasonably sized two-digit examples. This caused many errors as children were unable to keep track of the numbers they were counting. However, some children had efficient strategies for keeping track of these numbers, breaking them into smaller parts, and were able to complete reasonably sized two-digit examples with accuracy. In addition, a small percentage (about 10%) exhibited sophisticated mental computation strategies and a few students were able to use these strategies to efficiently answer three-digit examples with speed and accuracy.

From the results of these initial interviews, two predispositions to mental computation and six strategy clusters were identified. The first predisposition was associated with visualisation, some children appeared to have a strong ability to visualise numbers, while others did not appear to use it. The second predisposition was associated with three primary ways the children perceived the examples: (1) through number, breaking down the numbers as the first step; (2) through strategy, activating a strategy as the first action; and (3) through process, focusing on the larger picture to obtain an overall idea of what the item required of them. The six strategies were: (1) simplified understanding, choosing to do something simpler but incorrect (e.g. adding instead of subtracting); (2) counting, simple counting procedures even with two-digit numbers, counting on and back, by ones, twos, fives, etc.; (3) basic fact strategies, near doubles, near ten and think addition, used mainly with the one-digit sections of the computations; (4) using tens, adding numbers using tens first in some way (e.g. 34+27 is 34+20=54 and 54+7=61 and 34+27 is 30+20+11=30+31=61); (5) pen-and-paper procedure, the classical pen-and-paper procedure of the ones first and then the tens (e.g. for 34+27, 4+7=10+1 and 30+20+10=60, equalling 61); and (6) estimation-recall, situations where the children appear to recall, guess or estimate the answer in one step.

However, in the later interviews in year 3, the above categorisation was unable to classify many solutions for three-digit examples and classified many interview-different solution behaviours in the same category. Furthermore, in behaviour, the students separated into three groups: (1) a section which after two years of instruction were still counting by ones and still not able to compute two digit addition and subtraction mentally; (2) a group which used the pen-and-paper algorithm mentally; and (3) a smaller group who efficiently used sophisticated mental computation strategies on three-digit examples. Therefore, to meet this situation, a more complex categorisation was constructed based around the following three perspectives.

1. Approach
   This covered how numbers were used and included the sub-categories simplistic, partitioned, place value R->L (lowest place value first), place value L->R (highest place value first), and wholistic (e.g. adding 98 in one step).
2. Process
   This covered how the operation process was achieved and included the sub-categories simplistic, separated place values, aggregation (e.g. 38+15=38+5+5+5), separated place values and aggregation (e.g. 38+25=38+20+5), compensation-undoing (e.g. 23+98=23+100-2), compensation-levelling (e.g. 23+98=21+100), and benchmark (e.g. 41+59=110).
3. Calculation
   This covered "basic-fact" style strategies and included the sub-categories counting (all, on, back, in groups, with materials, with fingers, etc.), near doubles, near tens, inverses (e.g. think addition), relating to known fact, and memory.

These categories enable complex solutions to be categorised in detail. For example, solution 244+359=544+59=600+3=603 would be categorised as place value L->R, benchmark and near tens, while solution 244+359=240+360+4-1=603 would be categorised as wholistic, compensation-undoing and relating to known fact.

CONCLUSIONS

The analysis of the two years of data collected from the six interviews is not yet complete, however some conclusions are evident. Firstly, many children lack even the most basic understanding of concepts and procedures even after years of instruction. Secondly, many children are very creative with numbers, manipulating them to suit the purpose of the task. Thirdly, children's use of strategies is complex. Many children do seem to be able to use a variety of strategies, choosing the most appropriate for the task in hand. However, they tend not to vary in predisposition or approach, using different specific strategies within a fixed framework. Fourthly, a small but significant number of children can compute effectively and efficiently before instruction. Finally, pen-and-paper instruction is appearing to have a strong influence on students' strategy choice for mental computation.

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