DEVELOPMENT OF THE NUMBER CONCEPT \\ THROUGH NUMBER WORDS IN LANGUAGE

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Our purpose is to exploit linguistic evidence derived from theuse of number words in languages in order to formulate conjectures of how the number concept emerged in human thinking. Comparison of structures in naming numbers gives us a hint about what thoughts are at the background in the evolution toward the mathematicalconcepts. Two distinct procedures seem to activate the process; the firstconsists in developing an awareness of quantity and leads into the cardinalnumber concept, the second relies on order and introduces the ordinal numbers.Four succesive phases may be distinguished in the development of a numbersystem. Phase I is related to the recognition of a number of objects, involvingawareness of the concept without exteriorisation, without using namesfor it. Phase II comprises the introduction of number words with classifiers: thenumber concept is closely associated with objects of a certain kind.In phase III the development of a number system occurs through associationwith a standard: the human body. Eventually, in phase IV abstraction yieldsthe creation of an abstract number concept. A word for a concrete object whichis part of the vernacular is totally freed from any concrete connotation and becomes an abstract measure.

Anthropologists consider counting as a cultural universal; useless to say that mathematicians and mathematics educators are more than willing to support that opinion. Psychologists often consider numbers as archetypes (Jung). Linguistic evidence derived from the occurence, form and usage of number words in different languages allows for a comparison of structures in naming numbers and for the formulation of conjectures of how the number concept emerged in humanthinking and of what kind of thoughtstructures are at the background in devising number systems. In particular, what we know about the transition from concrete numbers to themathematical concept may be very important for the early teaching of arithmetic, as the construction of a mathematical concept in the individualmind seems to follow procedures which are closely related to the ones whichhave been in action during its historical development.

Of much importance for the development of the counting ability is the introduction of words which may be considered to be rudimentary forms of symbols for numbers, not yet endowed with all of the abstract power which our present symbols 1, 2, 3, ... have. As counting becomes very soon a symbolic process, lack of words/symbols may hamper the smooth development of the counting faculty. There is a striking similarity with another abstraction faculty of the mind: the recognition and naming of colours. Although the human eye has the ability to distinguish among 10 or 11 basic colours, many primitive languages didn't develop words for all of these, f.ex. using the same word for blue end green, and are unable to give an accurate description of the full sprectrum of the rainbow ([5] p.10, [13] p.33).

ASPECTS OF THE OCCURRENCE OF NUMBER

The intricate cultural development of an abstract concept such as number may have passed through some distinct phases, which however didn't have to be strictly successive, as parts of some of them may have evolved inparallel. It seems to us that an overall scheme for organizing the evolution of the number concept has to take into account at least four phases.

1. PHASE I:/ Recognition of a number of objects; awareness of the number concept without verbal exteriorisation, i.e. without using names for it. Groupings of objects are perceived as permanent, discrete and distinguishable items, such that the difference with another grouping is part of the conscious mind of the observer. But *number words* are not yet introduced in the language, if language has already come to exist at all ! It is likely that the counting process preceded the use of fully developed number words; study of the

content of the oldest Sumerian tablets reveals the fact that tablets containing arithmetical operations (an elementary form of bookkeeping) are one or two centuries older than the very first literary texts ([6]). The same order has been found in the development of writing in the oldest tablets of Mycen\ae an Greece, about 1500 BC ([2]).

2. PHASE II: The use of collective nouns and of classifiers. A specific term is used to signify ``a pair" or ``a ten of ". But neither the numbers 2 and 10, nor the name of the object are mentioned. Words are used to name certain collectivities of objects of a rather particular kind, often animals. The collectivity may be small (2, 3) or big (no precise number: herd, flock, crowd,...). Other examples are *a dozen, a gross*. Classifiers control the introduction of different classes: different number concepts are associated with objects of a given class. Number words exist but refer to specific objects with a common characteristic, considered as members of a well defined class, see [4]. Remnants of this phase subsist till the present day in many modern languages.
3. PHASE III: Development of a number system. The material collected by anthropologists reveals that this occurred almost universally through association with a standard, the human body. Counting of a grouping of objects is done by establishing a correspondence with the speaker him/herself, counting on fingers, toes and the whole of the body.
4. PHASE IV: Development and communication of an abstract number concept using specific terms which are integrated into the vernacular, followed by the creation of a fully developed number system.

THE EARLY BEGINNING OF COUNTING.

Archeological findings have led to the conjecture that some form of counting had already developed in palaeolithical times (20,000-10,000 BC). Scratches on bones or stones have been found which seem systematicallyorganized in groupings, separated by gaps ([10]). The groups have 29 or 30 marks, with occurrence of subgroups of 7 or 14. It is acceptable to relate those scratches to the phases of the moon, hence that could be a primitive (lunar) calendar and implies the ability of some elementary form of counting. If there were already words for numbers at that time is a question which must remain unanswered. But it is impossible to imagine an endeavour to keep track of the passing of days and nights without at least a vague concept of number. Counting may very well have started with unnamed numbers.

Those facts bring us to the question if there is an instinctive component in counting. In [12] p.6, D.E.Smith mentions ``it is said that a shepherd may recognize that one of his sheep is missing without being able to count his flock, and even his dog could do the same. There is in such circumstances no need for counting up to large numbers". The same author refers also to the example of a deaf-and-dumb boy who acquired a knowledge of numbers from observing his fingers, even before he was taught to count, which shows us that the idea of number did not have to await the development of spoken language, and primitive man may have appreciated ``3" without having a name for numbers beyond two.

A CONJECTURE.

In the development of number two elementary but distinct factors seem to be involved, and there is no need to use words for numbers in the cultural background which supports this evolution.

1. The first factor is related to the act of ``seeing" numbers, without naming them. It seems to work only with small numbers and allows for the visual distinction between, say, a group of three and another group of five objects. The limit has been fixed experimentally around six or seven, and as such there exists already a boundary to counting. This process may be called ``the use of unnamed numbers", or alternatively ``instinctive number use" and leads into the cardinal number concept of present day mathematics.

2. The second factor relies strongly on memory and is based on remembering succession of events, f.ex.\ the succession of a number of specific days (the idea of a calendar). If the conjecture of the existence of a lunar calendar in prehistoric times is correct, we may infer that the second factor was predominant in the early development of number. The limit to the registration of succession/order seems to be much larger than the preceding one: primitive peoples remember very long lists, f.ex. of a succession of kings. An example is the list of

Mwami's of Burundi and Rwanda, starting from the 16th century and forwarded by oral tradition ([7]). Clearly, this factor is related to an idea of ``ordering" and preludes the ordinal number concept.

In some modern languages both aspects have been preserved in some form: there is still a difference in words used to name an absolute quantity and the corresponding words to designate a succession. This is the case in modern Japanese with measures of time: the names for hours, weeks, months (which require the use of a classifier). For hours, considered as a moment of the day, the classifier ``-ji" is used: sanji is 3h, but for a time-period the classifier ``-jikan" is used: sanjikan is a period of three hours (an absolute quantity). For weeks (shu) the classifier shukan is used; goshukan is a period of five weeks. But there seems to be no other classifier for a succession of weeks (neither in I.E.languages). For the succession of months in a year the classifier ``-gatsu" is used: nigatsu is the second month of the year (february). But for a time period (an absolute amount of time) the classifier ``-kagetsu" is used: nikagetsu is a period of two months.

THE USE OF NOUNS FOR A COLLECTIVITY; CLASSIFIERS

Some languages developed particular terms to express a certain number (often five, ten or twenty) of objects; using the term there is no need to express the number nor the name of the object. This is the case in Polynesia (Fidji, Florida Island), see [3]. Modern languages have conserved some relicts of this way of expressing numbers; some of these are still in use such as the terms *couple, pair, dozen*, but others have an archaic status, as a . More examples are: brace, heap, crowd, school (of fish), pack (of hounds), flock (of sheep). The existence of a dual in some, extinct as well as presently spoken, languages as a distinct form between singular and plural relies probably on the same background concept of using a special word for a set of two objects.

A possible broadening of the idea of relating specific number terms to the nature of the objects counted may have led to the use of *classifiers*: different kinds of objects are counted using different terms, mostly suffixes (see examples in [4]). It is likely that primitive man could count only by pointing to the objects counted, one by one, just as little children do. In the basic motivation for counting the object is all-important and the number may be seen as only the answer to a question raised in relation with the objects (adjectival use of numerals). There is no separation of the number word from the class of objects counted; it is not at all clear that, once primitive man had counted one collection of objects, he could immediately count another collection of the same number of quite different objects. Prelogical mentality has no abstract concepts at command; it does not distinctly separate the number from the objects numbered [9]. This process resulted eventually in linking the number concept very intimately to the counted object, using *different* words for the same number of objects of a *different* kind. This procedures is already present f.ex. in the early Sumerian system [6]. Linguistically the conceptual linking of objects (of a kind) to numbers resulted in the use of classifiers, used in many primitive languages, f.ex. in several Maya languages, but also in a modern language as Japanese. An extreme case is Tzeltal (Mexico), as reported in [1], which has 528 different classifiers.

DEVELOPMENT OF A NUMBER WORD SYSTEM

The way primitive number systems have developed is basically such that new words were in general not introduced until there was a need for them and there was no attempt at, and no idea of, a continuing sequence but only a pragmatic introduction of new terms as the need arose. Two typical examples are the introduction of words for large numbers in Chinese under the influence of Buddhism, which on religious grounds cultivates a predilection for such numbers ([11], p.451) and the borrowing of terms for tenths, hundreds or thousands in many primitive languages during the 19th century due to the developing contact with western cultures. The salient features of the particular primitive systems may be summarized as follows ([5], p.9):

1. New terms are added as the need arises; one may consider this ability as based on an instinctive number use.
2. There is no obvious stopping place; once the idea of counting has emerged then the idea of going on counting is not far away. There is clearly (from our modern viewpoint) the possibility of continuing the sequence if needed.
3. There are attempts at a regular, systematized continuation.

NUMBER WORDS: FROM BODY PARTS TO ABSTRACT NOUNS

The vast majority of anthropological findings converge to the statement that the origin of number words is closely related to names for parts of the human body. A detailed study may be found in a report about the use of counting systems in Papua New Guinea ([8]). There are however exceptions: the word *sorok* for 40 in Russian refers to to a certain, standardized bundle of fur.

Primitive words used in counting were at first tied to concrete groups of objects but in thousands of years they entered the abstract stage in which the group ceases to be a factor. We think of five no longer as of a certain group of objects (five fingers) but as of a word in a succession, following four and preceding six. It is acceptable that number words have developed from abstractions, using words for a concrete object and then loosing the original meaning of the word. Awareness of this process comes from historic times; in the development of units for measures, as foot and ell (elbow) the evolution is still visible. The evolution of number words may have gone along a similar path, as is exemplified by languages which use terms such as *hand* for five, *two hands* or *half a man* for ten, *one man* for twenty. There is even some linguistic evidence that *eye* might have been the original term for two. In due time, such terms lost their meaning and we think of them as abstract measures; nowadays we can do arithmetic without looking outside the number concept.

Counting, together with the words for it, seems to have developed on the basis of a radix, the most popular being the radix ten, but the oldest seems to be radix two, and three is often conceived as a large number, at a time when the fingers of a hand were not yet recognized as a base for counting. Examples are abundant in primitive languages: in some Aztec languages number names mean *one stone - two stones - three stones*, whereas counting in Niue goes *one fruit - two fruits - three fruits* and in Java *one grain - two grains - three grains*. See [12] for an extensive discussion of the variable radix, among which some very peculiar ones, which is used in many primitive and modern languages.

In modern mathematics to dispose of symbols as a shorthand for the study of concepts is of crucial importance. In particular, symbols representing numbers are needed to achieve the construction of an abstract number concept, and the introduction of words for numbers may be seen as a first step toward symbolisation. Grammatically this step corresponds to the shift of the adjectival status of number words to the status of a noun, which according to Wilder ([13] p.41), had never been realized without the previous introduction of some kind of ideographs for the representation of numbers.

A particular problem in the development of number which requires thorough understanding of the cultural background is the existence of an upper bound to counting. A boundary to counting exists in all languages, except in mathematics, but this boundary increases with time as languages develop and become appropriate to express more intricate concepts. The expression ``boundary to counting" may be interpreted ([5],p.22) as the practical boundary determined by the smallest number which has no name in a given language or counting system or alternatively as the situation where no clear way to continue counting is available. Some boundaries are rather unexpected: according to Menninger the number eight may have been an old boundary to counting in Japanese; the same is said of Finnish where number words above 8 are borrowed from other languages.

A possibly related phenomenon is the existence of jumps in the usage of number words: in Indo-European languages jumps from 10 to 100 to 1000 to one million, etc. occur. This is different from f.ex. Japanese which has jumps of 10,000. A jump may have been an old boundary to counting which became obsolete ([11],p.450).

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