CHARTING A FRACTION ENVIRONMENT FOR CLASSROOM LEARNING

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The fraction program described in this paper is characterised as an environment with resources of knowledge and skill that are available and appropriate for classroom use. The main goal of the program is that learners might be enabled to use a fraction environment confidently and in appropriate ways. The program presents activities for classroom groups to develop, select and manage fraction knowledge in purposeful tasks. It aims to encourage a learning partnership among 10- to 13-year-olds and their teachers. The intention is that knowledge and skill in fractions are developed interactively and in authentic contexts.

The project is informed by research into cognition, especially as it is manifested in social and workplace activity. Three stages completed so far have involved (1) an investigation into perceived classroom learning needs for the selected age group and their teachers, (2) the charting of a fraction environment in terms of relationships between concepts and procedures, and (3) the design and publication of a book presenting tasks involving fractions and suggestions as to how the activities might be managed in classroom groups. This paper indicates how theoretical and empirical information gathered have been integrated to provide a framework and action plans for classroom implementation.

The project reported in this paper addresses concerns about students' understanding of mathematical concepts and ways of thinking, and their ability to use mathematics as a tool in solving problems. Traditional classroom mathematics has typically been presented (and practised) as a collection of facts and rules to be memorised, and exercises to be done. Alternatively, some more recent approaches have put students into the position where they are expected to solve problems without necessarily being given informed and supportive access to the necessary mathematical tools required.

Both of these approaches present mathematics and its uses in narrowly-defined terms. Both are limiting to students' mathematical opportunities and potential. Under-developed abilities in formulating and solving problems can deprive students of much of the richness of mathematical experience. Similarly, lack of knowledge in mathematics limits students' access to the range of resources available for use in problem situations and reduces their chances to construct sophisticated mathematical meanings for ideas that might be employed.

THE PROJECT: CHARTING OF A FRACTION ENVIRONMENT FOR CLASSROOM LEARNING

In this study, a physical environment provides a metaphor for a classroom program in a mathematical domain (Greeno, 1991). An environment is a centre for human activity, involving purposeful and productive use of resources that are balanced by responsibility for their maintenance and development. In a mathematical environment learners need access to meanings and uses that enable them to participate fully in these activities: students need to develop a comprehensive range of appropriate knowledge and skill.

The task of charting an environment involves defining parameters for the area under scrutiny. Key resource bases and constraints and their relationship to each other need to be identified, along with ways to encourage flexible and productive use of the environment. In mathematics, due attention must be paid to relevant concepts and the procedures commonly employed in solving related problems. Human use of the environment involves experiences that give texture to concepts through deployment in a variety of situations. Thus effective skills in using mathematics provide a key means to enabling further growth in the understanding and use of mathematics. In the program designed, fractions have been selected as an area of mathematics for classroom implementation of these practical considerations.

A FRAMEWORK FOR THE PROJECT'S ACTIVITIES

Several interrelated strands of thinking and research activity in cognition have informed the classroom approaches being tested. A study of the relevant literature suggests that a comprehensive program for classroom learning needs to address instructional aspects of knowledge development, cognition and learning, and social interaction. These aspects have provided a base plan for the charting activity.

Knowledge Development: Cognitive research shows that understanding and competence in complex, thinking skills involves a critical and close relationship between reasoning processes and knowledge that is specific to the area (Glaser, 1984; Glaser & Chi, 1989). For example, skill in working through a problem or complex task involves activity that is underpinned by a well-organised base of relevant knowledge. Students' knowledge in fractions is therefore seen as central to fraction use in formulating questions, communicating ideas, reasoning about problems, justifying solutions, and so on. In regard to mathematics instruction, the suggestion is that the development of such fraction skills should go hand-in-hand with the systematic development of knowledge in fractions (Greeno, 1991).

Cognition & Learning: The study draws on the view that learning is situated in contexts (Rogoff & Lave, 1984; Brown, Collins & Duguid, 1989; Collins, Brown & Newman, 1989; Greeno, 1991). The suggestion is that social and physical contexts provide a 'scaffolding' for situating learners' representations of knowledge. This represents a move away from the information processing view of reasoning as mental manipulation of *symbols* of objects and events. Instead, learners' perceptions and reasoning processes are considered in relation to the learning situation, the objects in the situation and to others who are involved in the activity: mathematical meaning is developed in the context of its use.

Mathematics learning in these terms involves the development of abilities in finding, selecting, interpreting and using mathematical ideas in activities and situations: classroom mathematics is seen as belonging to the world, not removed from it. Fraction learning therefore needs to be embedded in a wide range of problems, tasks and situations that are actually found in familiar activities. In this way, concepts developed are shaped and textured by a network of meanings that are grounded in their use. Furthermore, learners need to have experience in interpreting everyday problems, tasks and situations from a *mathematical* point of view. This involves deciding on the concepts that are appropriate for application, and using mathematical concepts in authentic (and increasingly complex) problem solving tasks (Collins, Brown & Newman, 1989). The intention is that mathematics is presented as a dynamic, flexible tool for human use.

Social Interaction: Studies that have focused on the development of thinking skills indicate that at even apparently elementary knowledge levels mental activities involve complex thinking processes (Scardamalia & Bereiter, 1985; Resnick, 1987). From this position Resnick presents a case for embedding instruction in thinking skills in the academic disciplines of the school curriculum, from the very earliest stages onwards. Resnick furthermore advocates that classroom instruction should seek the development of thinking skills through socially-interactive means. She carried out a study that examined a wide range of teaching programs designed to encourage and transfer 'higher order' thinking, summarised as the skills needed for complex activities requiring responsibility and judgement (Resnick, 1987). Of the programs that demonstrated some transfer of higher order skills, Resnick noted a major, common theme: the programs saw the social setting and students' social interaction as central to much of the teaching and practice. Resnick's pragmatic view is that it is therefore useful to situate skill development teaching in a deliberately planned social context. The view sits well with the Vygotskian notion that the internalisation of social activity is the basis and stimulus for growth in cognitive competence.

CHARTING OF A FRACTION ENVIRONMENT: IDENTIFYING CLASSROOM NEEDS

The first stage of the project involved an investigation into perceived classroom learning needs for the selected age group and their teachers. This comprised observation of classroom experiences involving fractions, interviews with teachers, and monitoring of students' approaches to problem solving in fractions. Relevant mathematics curricula were also reviewed. Analysis of the information collected suggested that a classroom program in fractions might appropriately focus on: (1) learners' thinking about fractions in relation to fraction knowledge as used by mathematicians (to broaden limited conceptions of fractions and to confront misconceptions); (2) fundamental concepts involved in fraction thinking (for example, the central notion of 'comparison', the distinction between a whole in terms of one object and a whole as a set of objects, and an understanding of fractions as rational number); (3) authentic and commonplace uses of fractions (to give a clear message that fractions and fraction thinking are an important part of the everyday world); (4) appropriate uses of fractions (for example, to encourage a recognition of when decimal fractions and/or calculators might rather be used).

CHARTING OF A FRACTION ENVIRONMENT: IDENTIFYING FRACTION RESOURCES AVAILABLE

In the second stage of the project, a fraction network was constructed in an effort to capture the nature, functions, and means of representation of the resources available to learners for the development of fraction thinking (see figure 1). Freudenthal's (1983) identification of fraction concepts represented in phenomena provided a starting point for the development of the network. In line with the learning aspects identified in the theoretical framework, the network was constructed to reflect the interrelated use of fraction concepts, skills and applications that is seen as critical to working effectively in the environment. The network served as a groundwork map to guide the charting activity.

CHARTING OF A FRACTION ENVIRONMENT: DESIGNING CLASSROOM ACTIVITIES

Human activity is integral to a mathematical environment. Thus the charting activity needed to reflect the close yet variable connections between fraction knowledge and learners' use of it. The outcome, a classroom book entitled *Knowing your way around the Fraction Environment* is presented as an ongoing program for fraction learning. Its contents feature central concepts and functions of fractions: these range from the *fracturing* (or partitioning) notion—the best understood and most frequently taught aspect of fractions in the classrooms observed—to *fractions as rationals*, a characteristic that was not found to be evident in learning activities or student knowledge in the classroom study. Other concepts and functions addressed include estimation, equal sharing, describing with fractions, operations on fractions, equivalent fractions, ordering of fractions and fractions as decimals.

Within the content framework presented, the book provides a folio of exemplars for the planning and implementation of classroom developmental and assessment activities. The activities are designed for a social setting where fraction knowledge is talked about, interpreted in different ways, applied flexibly and in different contexts, and used to formulate and solve problems. Specifically, knowledge and skill development are addressed through task-based approaches that seek to engage learners' thinking and interest (eg, figures 2 & 3).

The program throughout includes notes to teachers regarding classroom organisation and learning approaches. The suggestions made are derived from the theoretical framework used. It is believed that teachers as well as students will benefit from a learning program involving fractions. In an initial testing of the program, teachers' judgements of student performance in fractions are being monitored and documented.

REFERENCES

Brown, J.S., Collins, A. & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher,* 18(1), 32-41.

Collins, A., Brown, J.S. & Newman (1989). Teaching the craft of reading, writing and mathematics. In L.B. Resnick (Ed). *Knowing and learning: Issues for a cognitive science of instruction. Hillsdale, New Jersey: Erlbaum.*

Freudenthal, H. (1983) *Didactical Phenomenology of mathematical structures.* Dordrecht: D. Reidel.

Glaser, R. (1984). Education and Thinking: The role of knowledge. *American Psychologist,* 39, 93-104.

Glaser, R. & Chi, M.T.H. (1988). Overview. In Chi, M.T.H., Glaser, R.& Farr, M.J., *The Nature of Expertise.* Hillsdale, New Jersey: Erlbaum.

Greeno, J.G. (1991) Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education,* 22:3, 170-218.
Resnick, L.B. (1987). *Education and learning to think.* Washington, DC: National Acadamy Press.
Rogoff, B. & Lave, J. (Eds.) (1984). *Everyday cognition: Its development in the social context.* Cambridge, MA & London: Harvard University Press.
Scardamalia M. & Bereiter C. (1985). Fostering the development of self-regulation in children's knowledge processing. In S.F. Chipman, J.W. Segal & R. Glaser (Eds) *Thinking and learning skills: Vol. 2. Research and open questions* (pp563-578). Hillsdale, New Jersey: Erlbaum.
Vygotsky, L.S. (197). *Mind in Society.* Cambridge, MA: Harvard University Press.

FRACTIONS

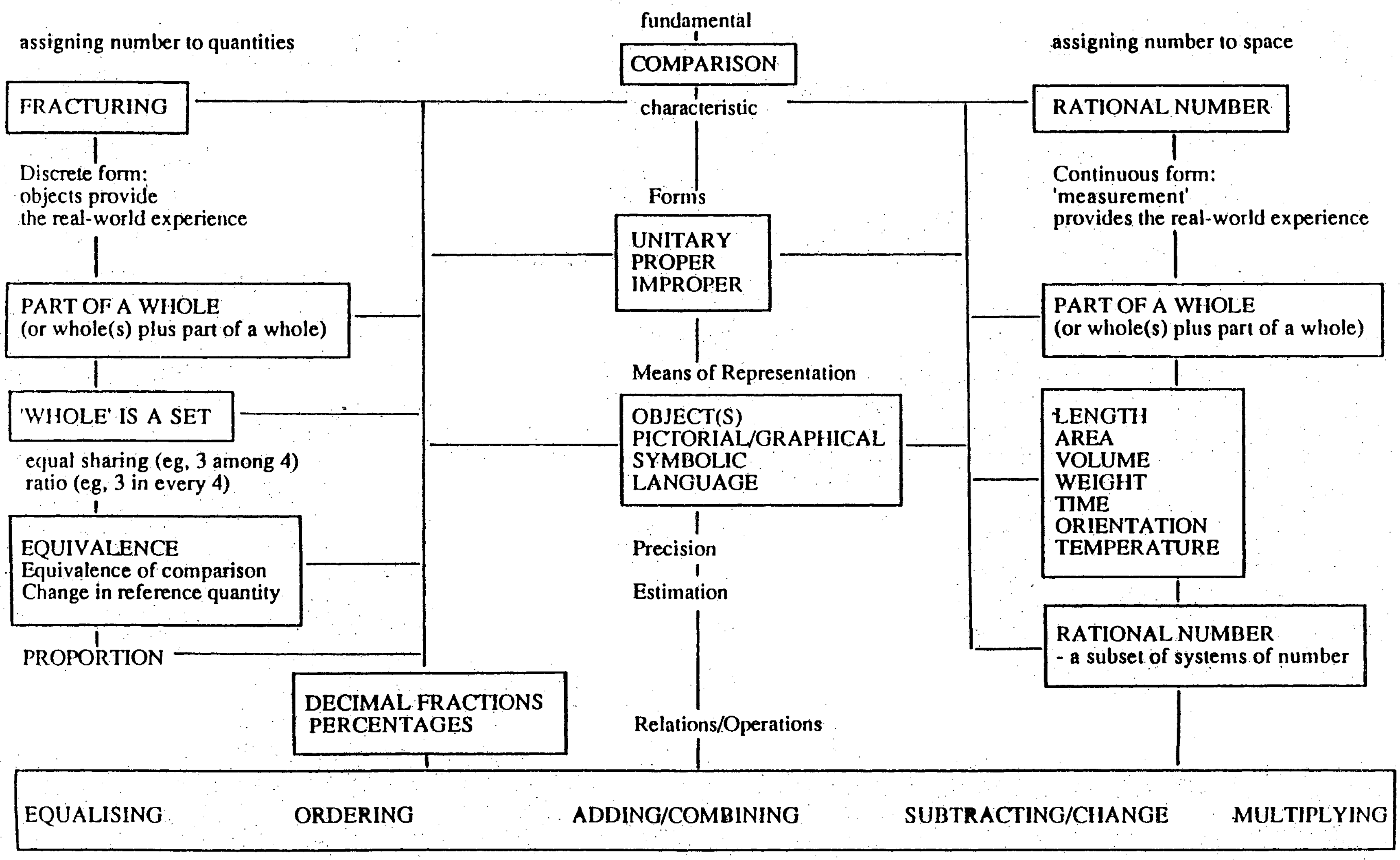


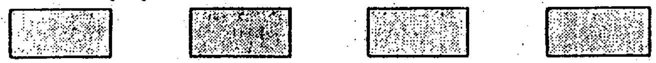
Fig 1

1. Four rooms (equal in area) need to be vacuumed.

☞ Show how you could equally share the workload.

between 3 people.

between 5 people.



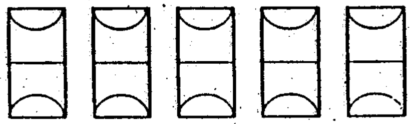
between 6 people.

✍ Write fractions to describe how the workload for each person changes as the number of helpers increases.

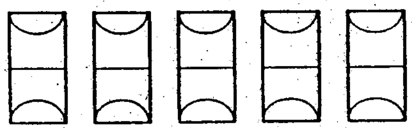
2. Five outdoor basketball courts need to be swept before the games begin.

☞ Show how you could equally share the workload:

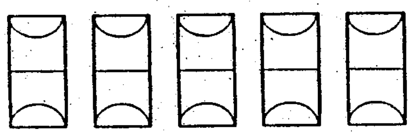
between 4 people.



between 6 people.



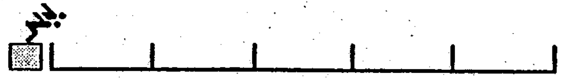
between 10 people.



✍ Write fractions to describe how the workload for each person changes as the number of helpers increases.

Fig 2

1. This length shows how far you swam on Sunday

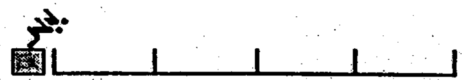


250 m

Below are lengths that show how you swam on other days of the week.

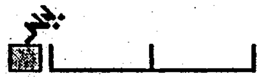
✍ Write fractions alongside each length to compare the length you swam that day with Sunday's swim.

Monday



200 m

Tuesday

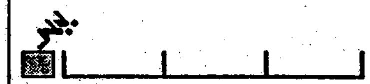


100 m

Wednesday

300 m

Thursday

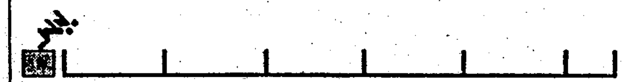


150 m

Friday

225 m

Saturday



275 m

✍ In these boxes, write each of these comparisons as *tenths*.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Fig 3