STRATEGY TRAINING AND MATHEMATICS LEARNING DISABILITIES

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This paper reports two investigations in which mathematics underachievers were taught to use cognitive and metacognitive strategies to facilitate information processing in two areas of mathematics learning: to read symbolic statements and to categorize their mathematics knowledge. One group of students learnt to use the relevant cognitive strategy while a matched group learnt, as well, associated metacognitive strategies (to evaluate the effectiveness of strategy use and to decide when they might use the strategy in the future). Both groups out-achieved a control group immediately after teaching. As well, the group taught both cognitive and metacognitive strategies were more likely to transfer the cognitive strategy to other areas of mathematics. The results are discussed in terms of models of human cognition and performance.

When solving a mathematics task, successful students engage spontaneously in various strategic activities (Cardelle-Elawar, 1992); they (1) sample the data defining the task in order to inform themselves of its nature, (2) relate the tasks to types that they have learnt previously, (3) plan a solution strategy and select from long term memory procedures that they apply to components of the data and (4) evaluate their solution and re-run some of these strategies if necessary. These four types of activities have been referred to as orientation, organization, execution and verification respectively (Cardelle-Elawar, 1992). They assume that students use their existing knowledge to make decisions about the nature of the task, about the data that may be relevant in classifying it and about the nature of the solution, whether it is reasonable, etc. Each area of activity involves two types of strategy; cognitive and metacognitive. The cognitive strategies involve manipulating ideas in various ways. The metacognitive strategies involve the planning, management and monitoring of cognitive strategy use (Haller, Child & Walberg, 1988). While the distinction between cognition and metacognition is frequently unclear , it permits a distinction between aspects of thinking during mathematics learning. Analyses of the mathematics performance of mathematics disabled students suggest difficulties in the spontaneous use of both types of strategies (Torgesen, 1980). Teaching them to use self-instruction strategies has led to improvement in solving arithmetic word problems (Montague & Bos 1986; Fleischner, Nusum & Marzola, 1987). Fewer studies have examined their use in algorithmic learning.

While they can frequently learn the steps comprising an algorithm, (that is, execution processes (Ackerman, Anhalt & Dykman, 1986)), these students have difficulty using this information on subsequent occasions. Arithmetic disabled students are less likely than their able peers to make spontaneously classificatory statements such "Oh, it's one of those" or "Is it like ...?" to make such statements. They are less likely to use their existing knowledge to organize the ideas that they are learning or to modify this knowledge in various ways. Effective algorithmic learning involves meaningful reading of symbolic statements, organizing the ideas learnt into semantic categories and using effective retrieval strategies. Error pattern analysis suggests difficulties comprehending and classifying tasks and selecting appropriate procedures. These students differ from their able peers in the features of tasks that they select to categorize them; they tend to use individual features while able students categorize using more general features and properties. They also have difficulty discriminating between relevant and irrelevant data. These difficulties may be associated with the use of search processes for locating and retrieving information in long term memory (Swanson & Rhine, 1985). These strategies are rarely taught directly. Most pupils learn them incidentally by trying out various "thinking actions".

One of the difficulties with studying strategy use is the extent to which it can be monitored. The technique of instructing pupils to "think aloud", that is to verbalize as they manipulate data, can interfere with learning. An alternative technique involves observing the effect of strategy teaching on subsequent performance. Teaching cognitive and metacognitive strategies as a way of ameliorating learning disabilities has been examined, particularly for word problem solution. The success of this teaching has been variable. The teaching of these strategies generally proceeds in the following sequence (Pearson & Dole, 1987); (1) the strategy to be learnt is demonstrated or explained, (the cognitive modelling phase), (2) teacher and pupils work together to apply the strategy (the guided practise phase), (3) pupils take more control of strategy use and make decisions about when and why they might use it in the future, (the overt self-guidance phase), (4) pupils independently use the strategy (the or faded self guidance phase) and (5) students apply the strategy in a range of contexts (the application phase). This phase is frequently omitted (Pearson and Dole , 1987). There are at least two possible reasons why some earlier strategy training has not been effective with learning disabled students; (1) it did not encourage students to evaluate the effectiveness of the teaching for themselves at an early learning stage (that is, develop metacognitive knowledge) and (2) it did not taken account of preferred ways of learning of individual students.

The present experiments compared learning under this teaching programme with one in which pupils evaluated the strategies that they are using and had the opportunity to accept or to reject use of them. Phase 1 was preceded by two others; one in which the need to learn new strategies as a way of overcoming existing difficulties was recognized by pupils and one in which each student selected the preferred reading strategy. The strategy was introduced 'something that some people do to make the task easier'. Pupils were invited to try it and then evaluate its value for themselves. In addition to the conventional activities at each phase (Pearson & Dole, 1987), they asked questions such as "Does it seem to work for me ? Why ? Does it help me when I do maths ? What can I do that I couldn't do earlier ? When will I use it in the future ?"

The effectiveness of strategy teaching was examined for two aspects of using one's existing knowledge in learning algorithms ; reading meaningfully symbolic statements and categorizing knowledge of mathematics procedures.

EXPERIMENT 1 : READING SYMBOLIC MATHEMATICS STATEMENTS

Meaningful reading involves constructing an impression of the intention coded in a written statement. Readers do this by making use of what they already know. Consider the mathematical statement $2x + 3 = 19$. The coded idea be represented mentally in different ways (Gardner, 1985), Some readers may prefer to represent ideas learnt in terms of visual imagery , some may demonstrate a verbal preference, tapping into one's linguistic knowledge and verbal coding systems, some may tap into their propositional logical knowledge and others may attempt to represent the ideas kinaesthetically. Once the statement has been represented, it is available for execution processes. In other words, students can use a range of cognitive strategies to read meaningfully a symbolic statement (they can visualize or verbalize it or represent it as a series of actions), particularly if it involves small numbers, that is, numbers that make comparatively low demand on available attentional resources for their representation (Munro, 1991). In this experiment the reading strategy taught to each student was linked with that student's preferred way of learning The learning preference was identified by providing students with a range of representational formats and having them select their preferred mode. The teaching was intended to increase students' awareness of how they could use these preferences in the context of mathematics. The present discussion did not examine issues associated with preferred ways of knowing, such as the stability of preference for a particular mode of representation.

In this experiment mathematics- disabled pupils selected their preferred representational format for mathematics and learnt to use the associated cognitive encoding strategy with mathematics tasks that they had found difficult. A second group, in addition, engaged in concurrent metacognitive activity. This design permitted an investigation of the comparative influences of cognitive and metacognitive knowledge on algorithmic learning. As well, differences in retention and transfer under the two conditions were monitored.

Method

Subjects. The subjects were 48 mathematics disabled grade 8 and 19 pupils who met accepted mathematics disability criteria (Pickering, Szaday & Duerdoth, 1988).

Experimental design. The pupils were arranged into groups of three, matched on mathematics task ability, general learning ability and grade level. The mathematics tasks to which the pupils applied the reading strategy were simple linear equations typical of middle secondary mathematics courses. All pupils had, within two weeks prior to the strategy teaching programme, completed this area of study and had achieved less than 10 % accuracy in equation solution. Two members of each group were allocated to two strategy learning groups and the third to a control group. Prior to the allocation, all pupils completed an intensive arithmetic computations unit on which they achieved at least 95 % competency on computations involving at least two operations. They were permitted to use a calculator. The pupils in the strategy learning groups learnt the strategies under one of two conditions; a conventional strategy learning condition (strategy learning; Pearson & Dole, 1987) and the modified condition that included a greater emphasis on self-evaluation of strategy effectiveness (strategy learning + evaluation). The training programmes were administered individually for each student over five or six thirty-minute sessions in the pupil's school. The pupils in the control group continued to solve typical linear equations for the five sessions.

Each pupil's task performance was measured before the beginning of the strategy learning programme, at the end of each session, at the conclusion of the programme and four months after the conclusion of the programme. A priori comparisons of means were made using linear contrast techniques (Howell, 1992). This provided the basis for comparing the maintenance and retention of the strategies. As well, the ability of students to describe the strategies that they had learnt was monitored at the beginning of sessions 2 to 6.

Procedure : The two teaching sequences described above were implemented. Students selected their preferred reading strategy by being cued to read a linear equation under each of the verbalize, visualize interpret as a actions conditions. The instructions for each training condition were as follows;

(1) for the verbalize condition the task was introduced by the experimenter as follows "One way in which I do these problems is to tell myself what they say. I listen to myself as I say it. I say this ($2x + 3 = 19$) as "two times a certain number add three is equal to nineteen. I try to say it the way I talk".

(2) for the visualize condition the task was introduced by the experimenter as follows "One way I do this is to make a picture in my mind of what the equation says. For $2x + 3 = 19$ the picture is two bags of bolts and three more bolts is equal to 19 bolts. How much is in each bag ? This is how I start off".

(3) for the action condition the task was introduced by the experimenter as follows "One way I do this is to think of the actions that it says. For $2x + 3 = 19$ I think of the actions. First I begin with a certain number, then I multiply it by two and add three. I end up with 19. What is the number ?

Following each condition the experimenter demonstrated the strategy for an equation and the student tried out the strategy with at least one equation. Students selected their preferred way of representing the mathematical statement and engaged individually in the relevant strategy teaching programme. The learning phases were spread over five or six 30 minute sessions. The students in the control group continued to solve linear equations in their regular classroom context for the six sessions.

Results

Mean task performance (proportion of correct task solutions) prior to teaching and at the end of each session are shown in Table 1. Both teaching conditions, but not the control condition, were associated with an improvement in accurate task solution ($p < .01$). Although the strategy learning + evaluation condition achieved a higher level of performance than the strategy learning condition for the final session, this difference was not significant ($p > .01$). The strategy learning + evaluation condition differed from the strategy learning condition in that it was associated with a more rapid learning of the strategy (that is, more rapid improvement to the ceiling level) and more sustained use of it than the strategy learning condition.

Table 1 : Mean task performance at the end of each session.

Condition	Pre-teach	Sess1	Sess2	Sess3	Sess4	Sess5	Sess6	Later
strategy	0.17	0.24	0.37	0.62	0.71	0.74	0.73	0.72
strategy + evaluation)	.18	0.21	0.54	0.76	0.87	0.94	0.93	0.95
control	0.20	0.28	0.33	0.26	0.38	0.34	0.23	0.37

The more rapid learning and use of the strategies under the teaching + evaluation condition is not surprising. When a strategy is observed to be assisting the solution of problems, it is likely be learnt more quickly. The focus on the effectiveness of a particular procedure is necessary for learning disabled pupils operating as non-spontaneous learners; these students are less likely to relate the actions taken and their value without focussing directly on the connection between them.

The retention and use of the strategies on a long-term basis was examined by monitoring strategy use sixteen weeks after the conclusion of the strategy learning both on solving equations and in the content learnt at that time. Both treatment groups demonstrated superior comprehension performance over the control group ($p < .01$). In terms of strategy transfer, when given a new task, the strategy learning + evaluation group was more likely than the strategy learning group to report using the particular reading strategy for new information ($p < .05$). Encouraging pupils to monitor and evaluate the use of strategies is linked with a greater preparedness to experiment with the strategies in unfamiliar contexts and to transfer their knowledge.

Experiment 2 : Categorization strategies

The second investigation examined the effect of increasing the accessibility of mathematics knowledge. In particular it examined the effects of assisting disabled students to organize their mathematics knowledge into functional categories and to remind themselves to access this knowledge. The tendency of mathematics-disabled students to classify tasks on individual perceptual characteristics rather than on more conceptual criteria has already been noted. These students are more likely to classify tasks such as

$$\begin{array}{r} 72 \\ -\underline{48} \end{array} \quad \text{and} \quad \begin{array}{r} 76 \\ -\underline{42} \end{array} \quad \text{or} \quad 1/6 + 1/6 = \quad \text{and}$$

$1/3 + 1/2 =$ as meriting the same procedures. In this experiment students, after mastering the steps in a mathematics procedure and applying the procedure in isolation to a criterion level of acceptable performance, learnt to discriminate between tasks that had the same operational symbol and that shared superficial features and to categorize them in terms of the procedure used to solve them. The study compared the effectiveness of cognitive strategy teaching with combined cognitive and metacognitive teaching. One group of students learnt to categorize instances of the tasks (the cognitive strategy group) while a second group learnt as well to instruct themselves to classify tasks on a subsequent occasion (the metacognitive group). When given a set of mixed computational tasks, the students asked themselves : "What does the task remind me of? What is it like that I have already learnt ? What did I do in this type of problem ?"

Method

The subjects were 30 third grade and 30 sixth grade mathematics-disabled students who met accepted mathematics disability criteria (Pickering, Szaday & Duerdoth, 1988).

Experimental design. The pupils were arranged into groups of three, matched on mathematics task ability, general learning ability and grade level. The mathematics tasks to which the pupils applied the classification strategy were either the subtraction of whole numbers tasks (third grade) or the addition of fractions tasks (sixth grade) described above. The design was similar to that used in Experiment 1. Prior to allocation, all pupils completed an arithmetic computations unit to at least 95 % competency on related computations. They were permitted to use a calculator. Two members of each group were allocated to two teaching groups and the third to a control group. One teaching group received classification teaching (the strategy group) while the second group received, in addition, self-instruction teaching (the metacognitive group). A priori comparisons of means were made using linear contrast techniques (Howell, 1992).

Procedure. The classification teaching was implemented on an individual basis as follows. Two instances of each type of task were written on a card and the student discussed differences between them. In the case of the subtraction tasks, the third grade students noted that both involved subtraction but for one type they needed to do something else before they subtracted; they had to "get it ready". They also discussed the 'getting ready' procedure. The four cards were sorted into two groups and the students suggested names for each (such as 'ready / not ready to take away'). The students sorted physically 10 tasks, written one per card into these categories. They were given a set of ten written tasks and categorized each by saying either "Ready / Not ready to take away". The self-instruction group, following classification teaching, were shown sets of randomly mixed tasks and learnt to ask themselves, for each task "What does this remind me of ? What is it like that I have already learnt ? " The learning phases took either five or six 30 minute sessions. The control group completed mixed sets of tasks and received corrective feedback. Task completion and classification ability were measured at the beginning and end of the strategy teaching program and four months after the conclusion of the program. As well, the nature of error patterns and the transfer of learning to other areas was monitored.

Results

Mean task performance (proportion of correct responses) of each group prior to teaching , at the end of teaching and four months later (later) are shown in Table 2.

Table 2 Mean task performance for each group

Performance	Strategy teaching group			Self-instruction group			Control group		
criterion	Start	End	Later	Start	End	Later	Start	End	Later
Task completion	0.18	0.43	0.62	0.14	0.95	0.94	0.08	0.13	0.17
Classifications	0.07	0.73	0.85	0.09	0.91	0.96	0.04	0.09	0.08

The three groups did not differ on either criterion prior to teaching (planned comparisons, $p > .05$). Both teaching conditions, but not the control condition, were associated with an improvement in accurate task solution, with the self-instruction condition achieving significance ($p < .01$). The retention and use of the strategies on a long-term basis was examined by monitoring the ability to solve mixed sets of tasks sixteen weeks after the conclusion of the strategy teaching. Both treatment groups demonstrated superior comprehension performance over the control group ($p < .01$) and the self-instruction group out-performed the strategy learning group ($p < .01$). In terms of strategy transfer, when given an unfamiliar type of mathematics task, the self-instruction group was more likely than the other groups to report attempting to categorize and to use what was already known. These findings support the importance of both cognitive and metacognitive strategy teaching. Teaching these students how to categorize was insufficient; it was necessary as well to teach self-instruction strategies that facilitate access to this knowledge.

Discussion

The findings of the present study support the claim that mathematics disabled students do not use spontaneously a range of cognitive and metacognitive strategies for processing mathematical data. The superior performance under the metacognitive condition indicates the need for both conditions in mathematics teaching. Why do mathematics-disabled students have difficulty learning to use these types of strategies, for example, to make use of what they already know ? Failure to use effective cognitive strategies has been attributed to inadequate metacognition (Cardelle-Elawar, 1992). Particular cognitive strategies are often used inflexibly and without selection according to the task at hand. Students may have access to strategies but don't use them spontaneously.

A possible explanation for the lack of spontaneous use of the most appropriate strategies and for the effectiveness of the type of strategy teaching described here may lie in the allocation of attention during task completion. These resources need to be invested in those processes not automatized. The allocation of attention and the executive component of metacognition are related. The importance of automatizing aspects of mathematics knowledge, so that these can be manipulated without the investment of mental attentional process for subsequent mathematics learning has been noted by several investigators (Ackerman, Anhalt & Dykman, 1986). Mathematics underachievers have difficulty meeting this demand particularly for the manipulation of "basic number facts" (Fleischman, Garrett & Shepard, 1982).

It is reasonable to expect that the issue of automaticity can be applied to strategy use during mathematics; any cognitive or metacognitive skill may demand attentional investment. The student who needs to invest these resources in processes that peers implement relatively automatically, may have proportionately less to allocate to 'building the new idea'. Increasing use of strategies is likely to lead to their attaining automaticity. Thus, pupils who continue to allocate a disproportionate amount of their attention to the manipulation of subordinate mathematical ideas may have less to allocate to the use of particular strategies and the opportunity to monitor their effectiveness. From this perspective, the effectiveness of the strategy teaching is not surprising; it teaches students to allocate their attention to essential aspects of data in a systematic way.

The effective allocation of attention can provide a basis for the restructuring of knowledge stored in long term memory. The students learning to read algebraic statements may have build 'templates' of these statements that they could apply to other instances. These templates may have permitted the students to process greater amounts of information at once. In a corresponding way, teaching these students to categorize what they already knew may have assisted them to build better-defined categories in verbal semantic memory and to access these. Rabinowitz's (1988) observations in relation to seeing strategies as isolated entities are relevant here; strategies are not used in isolation but rather are anchored in particular knowledge domains. The use of any strategy is related to the individual's ability to access related domain-specific knowledge, as well as students' perceptions of these strategies. Strategy use is higher when the strategy was being applied to conceptual knowledge that was easily accessible.

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