CHANGING CHILDREN'S APPROACHES TO MATHEMATICAL PROBLEM SOLVING

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A year-long teaching experiment explored the possibility of changing fourth grader's approaches to mathematical problem solving. A metacognitive question-and-answer technique was used to negotiate meaning, explore problem representation, discuss possible solution strategies, and reflect on the problem-solving enterprise. Analysis of the transcript data, classroom observations and childrens' work samples revealed that while those with most to gain and nothing to lose demonstrated the greatest shift in approach, each child's approach was successfully challenged to some extent.

It would appear that there are two relatively distinct views of mathematical problem solving. One view, which might be characterised as the information processing view, describes problem solving as the process of moving from a given state (where you are) to a goal state (where you want to be) without the benefit of a pre-conceived solution path (Mayer, 1985). The other, which might be characterised as a constructivist view, regards problem solving as a particular form of learning.

It should be clear that, for the constructivist, substantive mathematical learning is a problem solving process ... In this context, substantive learning refers to cognitive restructuring as opposed to accretion or tuning (Cobb, Wood & Yackel, 1991, p.158).

These two views have variously impacted the research on mathematical problem solving as has the notion of metacognition, the term introduced by Flavell to describe one's awareness of one's own cognitive processes and products (Flavell, 1976, p.232). If an information processing perspective is adopted, metacognition tends to be seen in terms of the control and regulation of cognition (for example, Garofalo & Lester, 1985; Schoenfeld, 1985). However, if a constructivist perspective is adopted, metacognition can be viewed as cognition informed by problem-solving-relevant and/or learning-relevant knowledge and experience. That is, as an interaction between cognitive goals, cognitive actions, metacognitive experiences and metacognitive knowledge (Flavell, 1981). Viewed in this way, problem solving can be seen as a form of learning influenced by one's goals, knowledge, beliefs, actions and experiences in much the same way as any other form of learning. This implies that what an individual has learnt to value, attend to, monitor and accept in a particular setting will shape the individual's approach to learning and/or problem solving in that or a related setting.

Although children's approaches to mathematical problem solving in out-of-school settings has been the subject of considerable research in recent years (for example, Carraher, Carraher and Schliemann, 1985), what children might be observed and/or inferred to be doing while they are engaged in problem-solving episodes in school mathematics classrooms over a long period of time has only just begun to emerge as a focus of research (for example, Cobb, Wood & Yackel, 1991). Long-term teaching experiments which recognise the complexities of the classroom environment and acknowledge the critical role of the teacher in generating, modifying and endorsing the type of social interactions and norms which shape childrens learning appear to offer the best means of describing and explaining childrens approaches to mathematical problem solving in school settings.

Four distinct approaches to school mathematical problem solving were confirmed by a year-long teaching experiment involving a grade four class and their teacher (Siemon, 1992). The approaches varied according to the extent to which the children appeared to be valuing, attending to, and monitoring their procedural and/or conceptual knowledge. For example a Low Conceptual-High Procedural approach (referred to as the Player's approach) was characterised by relatively few instances of cognitive monitoring with respect to the identification and evaluation of appropriate cognitive goals (conceptual knowledge), but by relatively high levels of cognitive monitoring in relation to the implementation of a range of cognitive actions (procedural knowledge). Where there were instances of cognitive monitoring, these were more likely to be directed at cognitive actions than at cognitive goals.

Beliefs and values about oneself as a problem solver, one's role as a student of school mathematics, and about mathematics were recognised as important factors in childrens' approaches to problem solving. Additional characteristics of each approach were derived from the literature and confirmed by the year-long teaching experiment (for example, differential approaches to learning (Marton & Saljo, 1976) and the tendency to premature closure

CONCEPTUAL KNOWLEDGE:
metacognitive knowledge and cognitive goals

Diver: ***High Conceptual-Low Procedural*** cognitive goals attended to more than cognitive actions comprehension strategies: checks, monitors, plans, predicts, links and reflects more on knowledge than on actions access to a variety of strategies, not always well used identifies goals tends to synthesise and analyse data some tendency to conceptually-driven premature closure questions directed more at goals than actions some undirected actions uses labels, understands structure deep approach to learning extended locus of control for actions	***Solver:*** ***High Conceptual-High Procedural*** attends to cognitive goals and actions comprehension and regulation strategies: checks, predicts, monitors, plans, links, introspects and reflects on knowledge and actions uses a variety of strategies knowingly identifies goals and appropriate actions synthesises and analyses data strong tendency to persist until reasonable solution obtained questions directed at goals and actions directed manipulation uses numbers, labels, structure deep-achieving approach to learning internal locus of control
Survivor ***Low Conceptual-Low Procedural*** unlikely to attend to either cognitive goals or cognitive actions tendency to remember and replicate, but experiences difficulty strong tendency to premature closure tends not to check, monitor, reflect or predict on actions or goals experiences difficulty identifying goals experiences difficulty identifying appropriate actions more likely to synthesise than analyse little or no questioning surface approach to learning external locus of control for knowledge and actions	***Player:*** ***Low Conceptual-High Procedural*** cognitive actions attended to more than cognitive goals remembers and replicates, often quite effectively tendency to procedurally-driven premature closure regulation strategies: checks, monitors, predicts and reflects, more on actions than on goals tends to assume goals tends to try a range of actions more likely to synthesise than analyse questions directed more at actions than at goals surface-achieving approach to learning external locus of control for knowledge

PROCEDURAL KNOWLEDGE:
metacognitive experiences and cognitive actions

Figure 1. Children's approaches to mathematical problem solving (Siemon, 1992).

(Biggs and Collis, 1982)). The Player's approach and the remaining approaches, that is, the Solver's approach (High Conceptual-High Procedural), the Diver's approach (High Conceptual-Low Procedural), and the Survivor's approach (Low Conceptual-Low Procedural) are described in Figure 1. The suggestion of orthogonality is deliberate as the approaches are not meant to imply discrete, mutually exclusive entities, but tendencies towards some particular behaviours rather than others in relation to a specific task at a given point in time. Organised in this way, the model provides a framework which allows children's approaches to a range of problem-solving tasks to be identified and monitored over time.

CHALLENGING CHILDREN'S APPROACHES

Given that different children appear to be valuing, attending to and monitoring different aspects of the mathematics classroom environment, it is reasonable to ask what can be done to challenge and change those approaches which do not value the negotiation of shared meanings and/or the use of more powerful generalisable strategies, that is, the Player's, Diver's and Survivor's approaches. In the context of the year-long teaching experiment, this question took the form, to what extent could children's approaches to mathematical problem solving be changed by a program designed to enhance metacognition, and it is this question which will be addressed here.

As indicated above, Flavell's (1981) model of cognitive monitoring was used as a basis for understanding what metacognition might mean in this context. The model consists of four interactive components, cognitive goals, cognitive actions, metacognitive knowledge and metacognitive experiences. The interaction provides for a powerful and dynamic link between cognition and metacognition and accommodates a distinction between conceptual and procedural knowledge. The model also acknowledges the critical role of prior knowledge, beliefs and values (conscious or unconscious, cognitive or metacognitive) in the generation of cognitive goals and in the implementation of cognitive actions. While metacognitive experiences allow for the conscious recognition of problem-solving-relevant or learning-relevant properties of a particular experience, this view of metacognition does not presume the degree of consciousness implied by more recognised views of metacognition in the mathematics education literature, for example, "knowledge of cognition" and the "regulation of cognition" (Garofalo & Lester, 1985, p. 164).

The problem solving program used in the year-long teaching experiment explored a variety of problem types and strategies and provided an accessible model of cognitive monitoring in terms of the ASK-THINK-DO problem-solving cycle (see Barry, Booker, Parry and Siemon, 1985). Key components of the program were,

(i) the mathematics content of the particular problems considered was selected by the teacher to support her curriculum objectives,
(ii) the problems were varied according to the amount of information provided, the degree of ambiguity about what procedures or strategies might be required, the transparency of the relationships between data, and the number of steps involved,
(iii) the problem-solving process was specifically talked about in terms of the ASK-THINK-DO cycle and modelled by the teacher whenever a strategy was reviewed or introduced,
(iv) pupils were encouraged to reflect on their problem solving both individually and collectively, and
(v) key questions, strategies and observations about problem structure and process were discussed and recorded on a large ASK-THINK-DO problem solving chart which was on constant display in the classroom.

Problems were worked on individually, in small groups or as a class. Twelve children were selected on the basis of their mathematical performance (high/low attainment) and interviewed every three to four weeks. Six of the twelve children, three girls and three boys, were interviewed on a more frequent basis. Interviews generally consisted of a reflective review of the problem considered in class followed by an attempt at a similar or related problem. A response mapping technique which reflected Flavell's (1981) model of cognitive monitoring was developed and used to analyse the individual interview data (see Siemon, 1992). Patterns in the response map data were used to identify childrens approaches to mathematical problem solving.

Kieron - The Solver

At the beginning of the teaching experiment Kieron's response maps indicated that while he appeared to be monitoring his cognitive goals and actions quite effectively, he seemed to be unaware of what he was doing. For example, in response to the problem, *If Mr Applebee arrived at the bus-stop at 7:15, how long will he have to wait to catch the next bus at 8:33?*, Kieron replied, "From 7:15 to 8:15 is one hour, from 8:15 to 8:33 is .. eighteen minutes, one hour and eighteen minutes". Asked how long from 8:33 to 9:05, Kieron said, "8:30 to 9:02, thirty-two minutes". Although Kieron's approach remained fairly consistent over the course of the teaching experiment, the program appears to have had an impact nonetheless in that it seems to be associated with him becoming more aware of what he was doing and why. For example, in the fourth month of the teaching experiment, the class considered the problem, *Greg was given $175 by his uncle to buy a bike for his birthday. How much money did Greg have left over?* (an illustration provided the information that a 5-speed racer was $158 and a BMX was $137), Kieron recognised this problem as similar to a problem considered two months earlier, "It's like the ice-cream one", and suggested that an "If ..., then ..." sentence was required. Reporting what he had done in the class lesson, Kieron indicated that he subtracted to find the change for both options and wrote, "If Greg bought the BMX, he'd have $38 change. If he bought the racer, he'd get $17". In this instance, Kieron systematically partitions the problem to consider all options in turn (formulates appropriate cognitive goals), uses a computation strategy (employs a relevant cognitive action) and reports his answer in a conditional manner (applies a problem-solving-relevant item of metacognitive task knowledge). Asked to reflect on his grade four maths experience to date, Kieron acknowledges that "when I first came, I wasn't sure, now I'm really enjoying it, ... I like the way Mrs M argues with us, ... I've learnt to think for myself".

Possibly as a result of the fact that Kieron generally met with success he seemed to become complacent about attending to those aspects of the program which focussed on the analysis of problems, particularly the questioning designed to connect cognitive goals to cognitive actions. By the end of the teaching experiment, it was apparent that Kieron was much more inclined to monitor his goals and actions in situations where the relationships between data were fairly transparent (familiar problems) than he was in situations where the relationships between data were fairly ambiguous or for which a representation was required. The effect of the program in this instance seems not to have changed Kieron's approach but to have clarified and qualified certain aspects of it.

Kristie - The Diver

At the beginning of the teaching experiment, Kristie's response maps indicated that she was much more inclined to value, attend to, and monitor her understanding of the problem conditions and what was needed (cognitive goals) than she was to value, attend to, and monitor her cognitive actions in relation to achieving those goals. For example, in response to the problem, *Maria was given $50 to buy a doll for her birthday. If a baby cabbage-patch doll costs $35 and an ordinary cabbage-patch doll cost $49, how much change would she have left?*, Kristie said, "I'd do two sums (indicating subtraction but not proceeding) ... then, if Maria bought the baby doll she would have so and so much change, and if she bought the big doll she would have so and so". More or less as an afterthought, she adds, "If I think it over, I'll be able to work it out". Her apparent lack of interest in actually working it out and her strategy for avoiding it provide a good example of conceptually-driven closure, that is, closure prompted by the recognition that while she knows what to do she is not so confident about her ability to actually do it.

By the end of the teaching experiment, Kristie's response maps indicated that she was more inclined to value, attend to, and monitor both her cognitive goals and her cognitive actions For example, it was quite apparent that Kristie was becoming increasingly impatient with her rather inefficient and time-consuming computation strategies. In the final interview, not only did she recognise that subtraction was required, she recorded the problem using place value columns and talked her way through a decomposition recording process. Asked if there was another way she could have done it, Kristie replied, "Yes, I could have drawn, sort of like numbers for dollars (presumably referring to a tally), but that would take for ever".

Carlo - The Player

At the beginning of the teaching experiment, Carlos' response maps indicated that he was much more inclined to value, attend to, and monitor his cognitive actions than he was his understanding of the problem conditions and what was needed (cognitive goals). For example, asked what he does when he gets stuck, Carlo replies, "I try times tables .. I try lots of things (indicating algorithms)". Asked if it takes seventeen minutes to walk to school, how long would six trips take, Carlo immediately records six seventeens (vertically) and says, "six sevens are forty-two ... fifty-two minutes". Asked to explain how he arrived at his answer, Carlo replies, "Well, I said six seventeens and six sevens are forty-two, you get two down here and four up there and from the seventeen there was a ten and so four and one is five and I just put five down in their's column and I got my answer". While Carlo appears to be monitoring his actions in relation to his view of formal multiplication, it is quite apparent that he is not valuing, attending to or monitoring the meaning of what he is doing beyond the purely procedural. Carlos' knows his number facts and demonstrates a sound knowledge of place-value when asked a direct question, however, Carlo does not appear to be able to draw on this knowledge independently to validate or check his actions. Rather, he seems content to assume that his answer is correct on the basis that he has done what he believed was appropriate.in the circumstances.

By the middle of the year, Carlos' approach tended to be more typical of a Survivor's approach than a Player's approach. He appeared to be confused and threatened by program's focus on and valuing of cognitive goals (something Carlo clearly did not value or see the sense of initially). For example, in response to the Spiders and Beetles problem, *If ten bodies and sixty-eight legs, how many spiders and how many beetles?*, Carlo says, "I hate things like that ... first of all it's very confusing, and second of all I haven't done it before and I can't ... I can't remember straight off. I gotta do it many times and get it right and wrong to remember it".

However, by the end of the teaching experiment, Carlos' response maps indicated that he was more inclined to value, attend to, and monitor both his cognitive goals and cognitive actions. For example, faced with the problem, *How many days will it take Clara Caterpillar to climb a tree trunk seventeen metres high if she climbs six metres up the tree trunk during the day but slips back three metres at night ?*, Carlo provides the following response.

> **Carlo**: Id do sixes in seventeen, I think, which is two sixes, that's twelve, that's five left over, oh, no! I'm stuck, seventeen metres to the tender young leaves, so how many days did it take her ... days?, Oh, is that including days, that is, whole days and nights?
> **Interviewer**: What do you think?
> **Carlo**: I think day and night maybe ... so that's six metres up the tree and that's seventeen, so minus six from seventeen, that's six from seventeen, that's eleven, then you add three which is fifteen (records and circles 11 and 15), so that's already fifteen metres, ... then you minus six from fifteen which is nine, then you add three again which is, which is twelve (records and circles 9 and 12), then you minus three and then, no, then you minus six, get six, plus three is nine (records and circles 6 and 9) plus, minus six which is three, plus three which is six (records and circles 3 and 6), minus six that's zero, perhaps he (possibly drawing on a similar problem involving Freddie Frog) would have taken one day (counting), that's two days, three days, four days.

11
15
9
12
6
9
3
6
0

Carlos' unique solution to this problem was not previously modelled or discussed in class although a similar problem had been considered in class five months earlier. An analysis of the transcript reveals many instances of conceptual and procedural monitoring. It is evident that Carlo is not only keeping track of his strategy but testing it out against the conditions of the problem as he goes, that is, he is monitoring his cognitive actions in relation to his cognitive goals. That he now appears to recognise and value such strategies as drawing a diagram, recording data, and working backwards represents a significant shift in Carlos' beliefs about the nature and purpose of school mathematics and his role as a student of mathematics.

David - The Survivor

At the beginning of the teaching experiment, David's response maps indicated that he was not particularly inclined to value, attend to, and monitor his understanding of the problem conditions and question (cognitive goals) or his cognitive actions in relation to achieving those goals. For example, in response to the problem, *If Mr Applebee arrived at the bus-stop at 7:15, how long will he have to wait to catch the next bus at 8:33?*, David replied, "from seven to eight is one hour and thirty minutes more ... ninety, and three there, ninety-three minutes". Asked if was necessary to take the fifteen minutes (7:15) into account, David said, "no, not really". In responding to the problem, *How much could you earn on a paper-round if you delivered eighty-two papers per day from Monday to Saturday at five cents per paper?*, David says, "Five dollars two cents". Asked to explain his answer, David says, "I said six eights are forty-eight and I said six twos are twelve, and that gave me fifty-two, so I said five dollars and two cents". By the end of the teaching experiment, David's response maps indicated that he was more inclined to value, attend to, and monitor both his cognitive goals and his cognitive actions. He was particularly inclined to do this in relation to problems which prompted from some form of diagramatic representation. For example, David experienced little or no difficulty with the Spiders and Beetles problem or the Fence problem, *A farmer wants to make a rectangular fence using twelve lengths of timber. Each length is nine metres long and costs $7. If fence posts cost $5 each, how much will it cost the farmer to build the fence?* David used the diagrams to clarify his cognitive goals and evaluate his cognitive actions. Asked, "What made you think of doing it like that?", David responded by saying, "I thought sketches would help me a lot more than trying to do it in my head and writing sums. It's a lot easier doing that I think" (that is, using the drawings).David had nothing to lose by attending to the program and modelling his behaviour accordingly. His overall shift in approach is best summarised in his own words at the end of the last interview, "If you think you can always aork out the answer to a sum".

CONCLUSION

While the nature and extent of the shifts in approach were different for different children, it would appear that the program did have an impact on childrens' approaches to problem solving. Given that aspects of the program were directly targeted at challenging children's knowledge and beliefs about school mathematics, such an outcome is consistent with what might be expected from a constructivist perspective of teaching and learning, that is, that learners value, attend to, monitor, and store different aspects of their shared experience depending on their prior knowledge, goals, beliefs and values. If the program had no effect on childrens' approaches to problem solving, it would have to be argued that something else occurred which did prompt the change in approach as one would hardly expect such well-established responses to school mathematics to be changed as a result of maturation or history alone. The value of the approaches for classroom teachers is that they help identify the different, goals, beliefs and values which prompt different children to make different sense of the same experience and explain why, in response to the sense they make of that experience, different children draw on different knowledge, skills and strategies in order to deal with that experience. Having some insights into the different goals, beliefs and values operating in the classroom provides a basis for challenging and changing those which actively operate against the negotiation of shared mathematical meanings and the use of more powerful generalisable strategies.

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