METACOGNITIVE AWARENESS AND MATHEMATICAL PROBLEM SOLVING IN THE SENIOR SCHOOL

GLORIA STILLMAN
School of Education
James Cook University of North Queensland

A major aim of the study reported here was an investigation of the relationships between mathematical and cognitive processing and metacognitive activities during problem solving by Year 11 Mathematics I students. During the first year, one problem was used for video-taping sessions whilst another was used in an examination situation followed by free response interviews where students reviewed their examination scripts retrospectively. In the second year, the role of the problems was reversed and a structured interview was used. The interviews probed the students' metacognitive knowledge, strategies, decision making, beliefs and affects. Results indicated that the students possessed quite a store of metacognitive knowledge which had the potential to influence their problem solving activities. Orientation activities were crucial with many students failing to inhibit impulsive responses to initial reading of the problem. Students were more concerned with the mechanics of solution execution and the tyranny of time than with planning, monitoring and verification strategies. The study supported the notion that students acquire and develop their store of metacognitive knowledge through metacognitive experiences and social interaction. There were indications that classroom practice and assessment techniques emphasized the use of automatic routinized application of formulae and procedures at the expense of experiences where students needed to reflect on, monitor and evaluate their progress.

The importance of conscious self regulation or executive control in the problem solving process has been noted by Ashman and Conway (1989), Campione and Brown (1978), Flavell (1979), Lester, Garofalo and Kroll (1989) and Schoenfeld (1987) amongst others. The term "metacognition" has been used to refer to an individual's awareness and conscious control of these monitoring and regulating activites.

Although many authors stress the importance of metacognitive activities in problem solving, students appear to be deficient in monitoring and self regulatory skills. Lester and Garofalo (1982) found that elementary school students do not routinely analyze problem information, monitor progress or evaluate results. Schoenfeld (1983) reported the same result with American college students. Crawford (1986) attributed a similar finding in her study with upper primary school students to socialization processes in the classroom. Schoenfeld (1988) argues that, largely as a result of what goes on in classrooms, students develop a view of mathematics that is detrimental to their problem solving activities. According to Flavell's (1979) classification, metacognitive knowledge consists of both knowledge and beliefs such as this which affect the course and outcome of cognitive activities.

In order to study the role played by metacognition in mathematical problem solving, it is necessary to identify a framework or model which incorporates both the cognitive and metacognitive aspects of the problem solving process. The prototypical model on which most recent problem solving research has been based is Polya's (1957) four phase description of the problem solving process. The model does not, however, deal explicitly with the metacognitive aspects of problem solving which remain implicit. Garofalo and Lester (1985) have proposed what they call a "cognitive-metacognitive framework" which incorporates Polya's ideas but also "specifies key points where metacognitive decisions are likely to influence cognitive actions" (p. 171). The framework consists of four categories of activities that are involved in a mathematical task, namely, orientation, organization, execution, and verification. Each category is explicitly defined and illustrative subsets of the possible cognitive and metacognitive activities associated with each category are identified.

PURPOSE OF THE STUDY

The study reported in this paper was designed to investigate the relationships between mathematical and cognitive processing and metacognitive activites during problem solving by senior secondary students. The specific research questions addressed were as follows:

1. What metacognitive knowledge and activities do senior secondary school students exhibit when attempting to solve mathematical word problems?
2. How does this metacognitive knowledge and activities interact with the students' mathematical and cognitive processing?

METHODS

As the goal of the study was to examine how students think, it was decided that case study would be the most suitable method for such an investigation. A variety of data collection techniques was used as this allowed the researcher to triangulate results and interpretations and resulted in a more detailed portrayal of the problem solving process.

The study was carried out over a two year period at a Catholic Girls' School. In the first year 121 Year 11 Mathematics I students attempted a problem under formal examination conditions. Seventeen of these students were randomly selected to participate in audio-recorded open response interviews where they reviewed their scripts. A further 26 students participated in self-selected pairs in video-taped cooperative problem solving sessions using a second problem.

In the second year the problems were reversed with 120 students sitting for the second problem under examination conditions whilst 22 students were involved in the video-taped sessions using the first problem. Twenty-three randomly selected students participated in audio-recorded structured interviews which replaced the open interviews.

During the first year of the study, an unstructured approach was used in the interviews as the researcher wanted to remain as unobtrusive as possible, allowing her subjects to respond as spontaneously as possible. As preliminary analysis of the previous years' data was completed by the beginning of the data collection stage in the second year, it was decided to use the information already accumulated to develop a structured framework to assist the interviewer maintain consistency between subjects during the second round of interviews. An interview protocol was devised using Garofalo and Lester's Framework (1985). Questions were used to elicit responses by students reviewing their solution scripts within the four categories of orientation, organization, execution and verification. Students were also asked probing questions to determine state and trait variables affecting their performance. In addition, a final series of questions were used to determine students' beliefs about mathematical problem solving in general.

RESULTS AND DISCUSSION

All students in the study who participated in the interviews or the video-taped sessions possessed a store of metacognitive knowledge. This knowledge base had the potential to affect all four categories of problem solving activities identified by Garofalo and Lester (1985) in their framework. Flavell's (1979) person, task and strategy categorization of metacognitive knowledge provided a suitable basis for examining the metacognitive knowledge possessed by the students in the study.

Person knowledge exhibited by the students included assessment of their ability as problem solvers and assessment of their ability to cope in differing problem solving situations. This personal assessment was affected by the influence of such variables as motivation, anxiety and persistence on the particular individual involved.

Task knowledge included beliefs about the nature of the expected task they had to perform and assessment of task difficulty. Their assessment of task difficulty was affected by such features as content, context, structure, surface characteristics of the problem and the problem solver's preference for working on particular types of tasks.

Knowledge of strategic behaviour to assess and understand a problem included strategies to assess the level of difficulty of the problem, to aid understanding of the problem, to organise information and to aid recall of information from Long Term Memory. Strategies to aid understanding included rereading of the problem statement, jotting down key words and numbers, mulling over the question, starting from what the student knew, relating the problem to other problems encountered in the classroom and the textbook, and looking for further information. Students were aware that organizing information into a table or some form of systematic format before attempting analysis greatly facilitated the analytic process and the development of a suitable model. Although some students were aware of the utility of graphs and diagrams for organizing information or clarifying relationships between pieces of information, these were used less frequently. Mnemonic devices were the main aids mentioned or used by students in memory recall.

In general, students lacked an awareness of strategies for generating and executing plans. In fact, almost half of those interviewed believed such planning was unnecessary. Although students often used an overall strategic plan, many were unaware they did so. The highly "event-driven" nature of mathematical problem solving precludes the implementation of a rigid plan and this could have contributed to students' beliefs that they did not use a plan or that plans were unnecessary. Similarly, if students' past experience of problem solving in mathematics had merely been the instantiation of schema-driven solutions, this reinforced the notion that planning was unnecessary.

An important area of planning where students lacked metacognitive knowledge was strategies for executing plans. For plans to become operational in a controlled and efficient manner, executive decisions had to be made about allocating resources, deciding which aspects of the plan needed to be focussed on and refined at any partciular time and the scheduling of actions. Students appeared to be totally unaware of such strategies as mental simulation which would have allowed them to review their plans for specific segments of their problem solving performance to ensure these decisions were appropriate and the best that could be made under the circumstances. Knowledge of, and experience with, strategies in this particular area of problem solving would have allayed many students' fear that they would not have sufficient time to complete their solution satisfactorily.

On the whole, students' store of metacognitive knowledge about specific strategies for monitoring progress was not well developed with many students saying they did little more than check they were still on track by checking the reasonableness of local results against a mental benchmark or "gut feeling", and/or they guarded against mechanistic errors by redoing calculations or reworking algebraic procedures with the same algorithm. As well as monitoring execution activities, students need to be able to successfully overcome feelings of stress, frustration or panic when things go wrong. To be effective problem solvers, students must be aware of the signs that show they are experiencing their typical stress-response pattern and learn how to control those responses themselves and, in so doing, better prepare themselves for improved performance in the future. Our thoughts affect our feelings and ultimately our actions. Scrambled thoughts are hardly going to lead to controlled and efficient problem solving performance. One quite useful strategy in this regard of which some students were aware, is self-talk. Controlled self-talk can be used by problem solvers to keep their attention on the task at hand, to maintain their focus by using affirmations and so control their confidence and guard against any tendency to just give up and say it is all too hard.

Despite the fact that most students appeared to believe in the importance of verification of their final solution, very few did this in any organised or systematic fashion even when they had sufficient time to do so. The reasons for this appeared to be that students' knowledge of suitable strategies was fairly limited, past experience had not reinforced the need for such verification, and verification received little attention in class and was therefore perceived by students to be of lesser importance.

How can we address these perceived deficiences in students' store of metacognitive knowledge? If the constructive learning theory is to be adopted, then we know that this knowledge can not simply be given to students. The students need to construct it for themselves; therefore, rather than attempt to teach a set of isolated strategies to students, teachers should provide their students with problem solving opportunities that allow them to reflect on and reorganize their current ways of thinking for themselves. To be able to provide students with suitable situations for this to happen we must know how students acquire and develop their store of metacognitive knowledge and how they come to use it efficiently and effectively. These questions, which arose from the investigation of students' store of metacognitive knowledge, were examined in the light of observations from the video-tapes and the interview data.

It was found that the study supported the notion that students acquire and develop their store of metacognitive knowledge through metacognitive experiences and social interaction. These findings were consistent with the views of Luria (1973), Vygotsky (1978) and Reeve and Brown (1985) who all argue that awareness of self-regulatory activity has its roots in our social interactions with others. According to Vygotsky, instruction should lead development. Learning situations should be devised so that they promote movement within a student's "zone of proximal development". The learning situation should awaken within the student a variety of internal developmental processes which are immediately possible for the student through classroom interactions with the teacher or able peers. With time, the degree of aid from the teacher decreases until students take over full responsibility for articulating their own metacognitive processes.

The degree to which students come to use their knowledge efficiently and effectively appears to bear some relationship to the quality of social interaction in the mathematics classroom. There were some indications that classroom practice and assessment techniques emphasized the use of automatic routinized application of formulae and procedures at the expense of experience in true problem solving where students needed to reflect on, monitor and evaluate their progress.

This is not advocating that automaticity of basic skills be ignored. When problem solvers perform basic skills automatically, little attention is required, freeing up memory capacity for performance of other problem solving activities. In fact, problem solvers who do not curtail ``wild goose chases" (Schoenfeld 1987) may fail to do so, not because of poorly developed managerial skills, but simply because in the particular circumstances the performance of basic algorithmic skills requires so much attention that there is no free attention to allocate to the monitoring aspects of problem solving. Thus, automaticity of basic skills affords the more able problem solver the opportunity to give attention to higher order skills. There needs to be a balance, then, between practising and testing basic skills, and giving opportunities to apply these meaningfully in a problem solving context.

The video-tapes revealed how these metacognitive behaviours interacted with the students' mathematical and cognitive processing. In addition, inferences made by the researcher in viewing the video-tapes were confirmed by comments students made in the interviews. The problem solving activities of the pairs of students in the video-tapes were consistent with a cyclic model of information processing as shown in Figure 1. Information gathering, representation, processing and validation rarely occurred in a linear progression. Instead, students would cycle back and forth through these processes as they attempted to build up their model of the problem and progress towards their goal in the light of the results they obtained and the new information that arose as their solution progressed or met a dead-end.

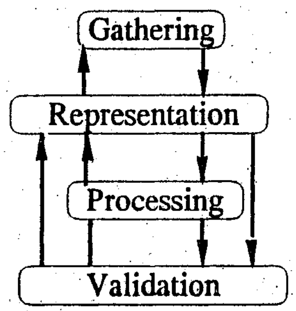

Figure 1. Information Processing Cycles during Problem Solving

Metacognitive activities were involved in all phases of the solution process. Garofalo and Lester's Framework (1985) proved valuable in identifying key points in students' solutions where metacognitive decisions were likely to influence cognitive actions by their presence or absence. Metacognitive activities during orientation appeared to be of crucial importance to the success of the solution. Many students in the study did not attempt to inhibit impulsive responses to their initial reading of the problem, jumping headlong into solution attempts without pausing for reflection. Students appeared to be more concerned with the mechanics of executing their solution and the tyranny of time than with planning, monitoring and verification strategies.

IMPLICATIONS FOR CURRICULUM DEVELOPMENT AND CLASSROOM PRACTICE

There is an obvious need to incorporate into our teaching, methods that facilitate students' problem solving performance by allowing them to make full use of their available metacognitive and cognitive strategies. If we are to adopt current theories of learning such as the constructivist perspective, we should be more concerned with how students will interpret their classroom or educational experiences than with how much of them they will absorb by maximum immersion. Students should be provided with learning situations that are going to give them experience in knowing how to use the metacognitive information they have available to them to predict future events and to plan appropriate responses in order to enhance their problem solving performance. This can be achieved by an astute selection of classroom problems and allowing follow-up time for reflection and discussion of both appropriate and inappropriate solutions including metacognitive activities that were or could have been used. It is crucial that students focus on the selection of relevant information, the construction of a feasible model and testing of that model as well as monitoring and verification strategies.

One method of raising the consciousness of students to the importance of these strategies and making them aware of the implications of their use, or lack of use, would be to view video-tapes of students in problem solving situations such as those produced in the study. Schoenfeld (1987) has found that when students view video-tapes of other students solving problems, it is easier for them to analyze and be objective about that behaviour when it is not their own but then they begin to empathize with the students and see that such an analysis can also be applied to their own problem solving behaviour. For instance, this study has shown that students should be made aware that persistence is not necessarily a virtue in problem solving. Viewing video-tapes from the study would certainly convince students of this.

Metacognition concerns decision making. Decision making involves using both strategic and temporal resources to determine the appropriate response. Both the speed and accuracy of this process of response selection is influenced and linked to the number of decisions to be made, the number of options to be selected from, the total time perceived by the student to be available for decision making and the perceived time-cost associated with incorrect decisions. As has been observed in this investigation, students' decision making about temporal resources is crucial to their success. Students need sufficient experience in problem solving during an extended time interval in order to develop appropriate mechanisms for dealing with efficient use of time

REFERENCES

Ashman, A.F. & Conway, N.F. (1989). Cognitive Strategies for Special Education London: Routledge

Campione, J.C. & Brown, A.L. (1978). Towards a theory of development: contributions from research with retarded children **Intelligence**, 2, 279-304.

Crawford, K. (1986). Simultaneous and successive processing, executive control and social experience: individual differences in educational achievement and problem solving in mathematics. Unpublished doctoral dissertation, University of New England, Armidale.

Garofalo, J. & Lester, F.K. (1985). Metacognition, cognitive monitoring, and mathematical performance. **Journal for Research in Mathematics Education, 16** (3), 163-176

Lester, F.K. & Garofalo, J. (1982). Metacognitive aspects of elementary school students' performance on arithmetic tasks. Paper read at the March meeting of the American Educational Research Association, New York.

Lester, F.K., Garofalo, J. & Kroll, D.L. (1989). The Role of Metacognition in Mathematical Problem Solving. Final Report. Bloomington, IN: Indiana University, Mathematics Education Development Centre. (Eric Document Reproduction Service No. ED 314 255).

Luria, A.R. (1973). **The Working Brain: An Introduction to Neuropsychology**. Translated by Haigh B. London: Penguin Books.

Polya, G. (1957). **How to Solve It**. Princeton, NJ:Princeton University Press.

Reeve, R.A. & Brown, A.L. (1985). Metacognition reconsidered: Implications for intervention research. **Journal of Abnormal Child Psychology, 13** (3), 343-356.

Schoenfeld, A.H. (1983). Episodes and executive decisions in mathematical problem solving. In R. Lesh & M. Landau (Eds.), **Acquisition of Mathematics Concepts and Processes** (pp. 345-396). New York: Academic Press.

Schoenfeld, A.H. (1987). **Cognitive Science and Mathematics Education.** Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A.H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. **Educational Psychologist, 23** (2), 145-166.

Vygotsky, L.S. (1978). **Mind in Society: The Development of Higher Cognitive Processes.** In M. Cole, V. John-Steiner, S. Scribner & E. Souberman (Eds. and Trans.). Cambridge, MA: Harvard University Press.