Research in Mathematics Education -
Constraints on Construction?

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INTRODUCTION

The conference theme selected by the organisers is timely and ambitious: challenges in mathematics education - constraints on construction. There are many possible interpretations of this tantalisingly ambiguous topic. I have chosen to reflect on the ways in which research affects this issue. Is research relevant as we grapple with challenges in mathematics education? Does research shape, reflect or determine constraints on construction?

Educational research, it seems, is now a huge, multi-purpose enterprise. Relevant conferences attract large numbers of participants who compete for a place on the program to present their work. Library shelves are well stocked with volumes which describe the fruits of research activities. There is keen competition for research funds. Do such activities indicate that researchers are handling the challenges in mathematics education effectively? Is this the time to be complacent? 'The great lesson of history, wrote G.K. Chesterton, is that mankind never profits from the lessons of history' (Clifford, 1973, p. 34). What, then, can be learnt from the past?

LESSONS FROM HISTORY

Exhibit 1

> All research is based on the assumption that careful, systematic enquiry into a publicly acknowledged, valid problem will be seriously considered by policy makers and others with influence in educational matters. That assumption has little to support it. Educational research by and large is ignored or taken up within political expediencies. It rarely informs or extends public debate. (Adelman & Young, 1985, p. 52)

Question: *Much is currently written about the advantages and disadvantages of introducing a national curriculum. Do beliefs and value positions or the carefully documented findings of well conceived and executed research inform this debate? What determines the constraints on construction of the arguments* *posed?*

Exhibit 2

> It is now widely recognized that the careful, objective, step-by-step model of the research process is actually a fraud and that, within natural science as well as within social science, the standard way in which research methods are taught and real science is often written up for publication perpetuates what is in fact a myth of objectivity.... The reality is very different. There are now several autobiographical accounts by scientists themselves and academic studies by sociologists of science that show that natural science research is frequently not carefully planned in advance and conducted according to set procedures, but often centres around compromises, short-cuts, hunches, and serendipitous occurrences. (Walford, 1991, p. 1)

Question: *Some twenty years ago Clifford (1973) argued: 'to the old American anti-intellectualism has been added a new nihilism: loss of faith in science' (p. 35). Whose interests are best served by such devaluing of the scientific methods and its output?*

Exhibit 3

> Consider the following hypothesis. The difficulty is not that the teacher wilfully refuses to listen to the researcher, but that however carefully he (sic) listens the researcher has little of interest to tell him. The fault lies not with the practitioner, nor with the communication system, but with our current conceptions of educational research itself. (Becher, 1980, p. 65)

Question: *Is this comment still relevant today? Research methods are not static but change with time. How have the ethnographic and action research approaches now frequently encountered in mathematics education research affected the traditional* *constraints on construction of problems and methods?*

Exhibit 4

> The need for research into educational issues would seem to be unlimited. Although knowledge in this field continues to increase, the demand for further research in order to understand and thus to improve the quality of education remains. (Husen & Posttlethwaite, 1985, p. 4304)

Question: *How informative is commissioned research? Are its findings necessarily constrained by the parameters imposed by the funding agency who contracted the work?*

Exhibit 5

> By its nature, educational research is a quest for alternatives, for change, for new ways to solve old problems.... (But) there are strict rules to be followed by researchers who would suggest change and alternatives. (Wittrock, 1973, p. 5)

Question: *Is it enough to look for new ways to solve old problems? Is a move towards different research paradigms a way to overcome the constraints of traditional constructions?*

Exhibit 6

> One reason for the limited attention sometimes given to research-based alternatives in education is indigenous to the methods of reporting scientific research. The limitations, constraints, and caveats that must accompany conscientiously prepared reports of educational research sometimes dampen popular enthusiasm for research-based alternatives. (Wittrock, 1973, p. 4)

Question: *Much has been written in recent weeks about the role played by the media in moulding public perceptions of political leaders. Is media reporting of research findings accurate, simplistic, constructive or destructive? What do parents concerned about their children's education learn through the* *media? Are their views broadened or constrained by this mode of reporting?*

These brief, and randomly selected, excerpts can be interpreted in various ways. Clearly, educational research is not valued universally. Past practices have at times been found wanting. The constraints imposed by disciplined inquiry and reporting all too often lead to dull reading, readily discarded for more sensational reports. Yet MERGA, and other organisations like it, would not have been formed or continued to thrive if its members did not value research or its products. What can research in mathematics education deliver?

RESEARCH IN MATHEMATICS EDUCATION

In the natural sciences it has typically been assumed that a mature discipline will have only one dominant research paradigm at any one time. Yet in the social sciences and in education 'it is far more likely that the coexistence of competing schools of thought is a natural and quite mature state' (Shulman, 1986, p. 5).

Kilpatrick (1992) has identified three different approaches used in contemporary research in mathematics education: empirical-analytic, ethnographic, and action research. The first - the empirical-analytical approach - embodies the traditional aims of science: to explain, predict, or control. The second - the ethnographic approach - views research as a vehicle for the interpretive understanding of a culture and 'tries ... to understand the meanings that the learning and teaching of mathematics have for those who are engaged in the activity (Kilpatrick, 1992, p. 4). The third, action research, has been described as

> a firing-line or on-the-job type of problem solving or research used by teachers, supervisors, and administrators to improve the quality of their decisions and actions; it seeks more dependable and appropriate means of promoting and evaluating pupil

> growth in line with specific and general objectives and attempts to improve educational practices without reference to whether findings would be applicable beyond the group studied. (Good, 1973, p. 494)

The approaches identified vary in the way they select, value, describe and investigate problems, in determining what is admissible evidence, and in interpretation of the data collected. Each has its preferred methodologies, instruments, assumptions, and values. And each approach imposes different constraints on the construction of issues deemed suitable for investigation and on the implications drawn from the findings obtained.

In the next section I want to illustrate how constraints are imposed on construction by the way issues are conceptualised. Changing conceptions of learning and the 'optimum' assessment practices they have spawned are used to do this. (A selection of views of learning is shown in Table 3 in the Appendix.)

BELIEFS ABOUT LEARNING

Beliefs about mathematics learning have changed over time. For many years the study of mathematics was justified in terms of its power to 'cultivate and discipline pupil's mental powers so that they would learn to reason correctly' (Willoughby, 1967, p. 13). Put simply, the mind was regarded as a muscle that needed to be exercised for it to grow stronger. The best practice, it was argued, was through studying the most difficult subjects, i.e., ancient languages and mathematics. Assessment instruments which focus on the recall of memorised material and sets of arguments are consistent with this view of learning. Such tests are still in use today.

The work of early psychologists such as William James and Edward Thorndike offered a powerful challenge to the mental-discipline view of teaching and learning. The latter relied on experimental work with animals for his theories about learning and teaching. Generalising from this work, he prescribed that certain rules should be followed when it came to teaching arithmetic:

> When the problem of teaching arithmetic is regarded, as it should in the light of present psychology, as a problem in the development of a hierarchy of intellectual habits, it becomes in large measure a problem of the choice of bonds to be formed and the best means of forming each in that order. (Thorndike, 1922, p. 70)

In this framework, responsibility for student learning rested largely with the teacher and his/her effectiveness in determining the choice and ordering of bonds to be formed. The student was assigned a passive role. The sequencing and pace of explanations were largely determined by the teacher's framework and preferences.

Preoccupation with the 'drill and practice' mode of instruction, with having students attempt lists of graded examples, and with assessment of rote learning, followed inevitably from this theoretical perspective. Contemporary descriptions of mathematics lessons bear witness to the long term impact of Thorndike's theories. The comment below is representative of those given when students were asked - not all that long ago - to describe a typical mathematics lesson:

> The teacher comes in, tells us what page to turn to, explains what we have to do and goes through a few questions with us and lets us continue by ourselves. (Grade 7 student, 1991)

Turning the focus onto children's thinking is perhaps one of Piaget's most important legacies. He hypothesised that learning was based on intellectual development and occurred when cognitive structures necessary for assimilating new information were available to the child.

> 'To understand', Piaget has said, 'is to invent', to build for oneself. Although children can be helped to acquire mathematical concepts by means of special materials and teachers' questions, it is only through their own efforts that they will truly understand. (Resnick & Ford, 1981, pp. 190-191)

His focus on individually oriented testing is consistent with this perspective.

Clearly, research-driven changes in views about human learning have had an impact on constructions and constraints, including those relevant for assessment practices and activities.

USING RESEARCH TO CHALLENGE POPULAR CONSTRUCTIONS

It has been argued by Kilpatrick (1984) that

> Each generation of mathematics educators ends up wrestling with many of the same problems the preceding generations thought they had "solved", and I think that is likely to be a permanent condition of our field, not simply a product of our limited history and our lack of agreed-on criteria for what problems are "solvable". We don't *solve* problems of mathematics education, we *inter* them. (p. 45)

In this section I want to use data from a recent research study[2] about student performance on the Victorian Certificate of Education to challenge a 'traditional assertion', i.e., that, on average, males do better than females on large scale, externally set, mathematics examinations.

THE VICTORIAN CERTIFICATE OF EDUCATION

The Victorian Certificate of Education (VCE) was introduced in all Victorian schools in 1990. The two year course, which covers grades 11 and 12, serves as a common end-of-school credential to replace five earlier paths of completing year 12. (HSC Groups 1 and 2 in academic high schools, T12 in technical schools, TOP in TAFE and STC schools' year 12 and Tertiary Entrance Certificate). The common certification symbolised real structural changes in the delivery of post compulsory education in Victoria. The assessment format of the new certificate has actively supported

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[2]The bulk of this work was done jointly with Glenn Rowley while I was still a member of the Graduate School of Education at Monash University. Dr Max Stephens (Board of Studies, Victoria) also had some input into the project which was supported by the Australian Research Council. Our thanks is expressed to Ms Chris Brew for her considerable assistance with the analysis of the data.

the introduction of a substantially new curriculum.

The assessment structure of the VCE has attracted much attention - both praise and criticism. There are two quite different components to the examination: Work Requirements and graded tasks. All Work Requirements in a particular subject must be completed and rated as satisfactory or unsatisfactory by school staff whose judgements are guided by centrally set guidelines. The VCE is awarded on the basis of satisfactorily completed Work Requirements: set in 1990 at 16 semester-length units including the Work Requirements for at least three pairs of Year 12 level units. Entrance to tertiary institutions and courses is based on performance on Common Assessment Tasks (CATs) which are part of the study designs at grade 12.

VCE MATHEMATICS[3]

Originally, there were four blocks of units in the *Mathematics study design* (Victorian Curriculum and Assessment Board [VCAB], 1990). These were Space and Number, Change and Approximation, and Reasoning and Data. Each comprised four units. The fourth, the Extensions block, offered an opportunity for specialised study and extended the breadth or depth of work covered in one of the other blocks. The Extensions block contained 12 units, four of which related to one of the other three blocks. Other details about the structure of the study and combinations of units students can take can be found in VCAB (1990).

There were three work requirements for each unit of the study and four CATs for each sequence of units 3 and 4. The latter were described as follows:

> **CAT1. Investigative project**: a project based on a theme set annually by VCAB. This task is undertaken as part of the Project work requirement for Unit 3.
>
> **CAT2. Challenging problem**: a problem selected from a number of problems set annually by VCAB. This task is

[3]The VCE has been modified since its initial introduction. The information provided represents the conditions that applied in 1992, the year in which the study described here was set.

> undertaken during a specified period of time as part of the Problem solving and modelling work requirement for Unit 4.
>
> **CAT3. Facts and skills task**: a set of 49 multiple choice questions designed to assess knowledge of mathematical concepts and skills. The task is completed under test conditions.
>
> **CAT4. Analysis task**: a set of structured questions designed to assess interpretive and analytical skills. The task is completed under test conditions. (VCAB, 1990, p. 53)

For CAT1, students were required to carry out an independent investigation and present a written report of their findings. The total time spent on the task was expected to be between 15 and 20 hours - some in class, much of it outside school hours. Many students spent considerably more time than recommended on this CAT as well as on CAT2. Initial grades on both CAT1 and CAT2 were determined within each school according to centrally set guidelines. The initial grades were subjected to a verification procedure.

The challenging problem set for CAT2 required students to undertake a problem-solving or modelling activity and to prepare a report of their work. It was expected that students demonstrate their ability to read and understand a problem, to use and interpret mathematical language, define important variables, use problem-solving or modelling strategies, find patterns, formulate hypotheses, justify, explain and interpret solutions. Recommended time to be spent on this activity was six to eight hours.

CAT3, which - as already indicated - consisted of 49 multiple-choice questions covering all content areas of a course, focused on students' facility in applying concepts and skills in standard ways.

The aim of CAT4, which like CAT3 was conducted under formal examination conditions, was to assess students' proficiency in interpretation and analysis of the mathematics contained in the compulsory content strand in each course.

There were thus several important differences between CATs 1 and 2 and CATs 3 and 4. The former were longer term projects, were attempted primarily outside school hours, had a strong language component in the presentation of the final product, and were initially teacher marked. CATs 3 and 4, on the other hand, were timed assessment tasks, done in school under examination conditions, had a much lower language component, and were marked externally.

The 1992 format of the VCE mathematics CATs allowed further exploration of the effect on student performance of assessment tasks which focus on different mathematical skills and responses.

SELECTED RESULTS

Details of the numbers of students who enrolled in the different VCE mathematics options are shown in Table 1 below.

Table 1: Sample details

Subject	Total Enrolment	% Female	% Male
Space & Number	7 181	49.5	50.5
Space & Number (Ext)	4 653	50.5	49.5
Change & Approx.	5 954	56.2	43.8
Change & Approx. (Ext)	10 385	38.6	61.4
Reasoning & Data	13 655	44.5	55.5
Reasoning & Data (Ext)	152	44.1	55.9

While outside the topic of interest for this paper, the differences in the enrolments of males and females in the different mathematics subject are worth noting.

To avoid unnecessary repetition, and to comply with space constraints, data are reported only for Space & Number, Change & Approximations, and Change & Approximations (Ext). Each of these subjects attracted large samples of students but differing proportions of females and males. Students' results on the different CATs are summarised in Table 2.

Table 2: Mean scores on VCE mathematics subjects

	CAT		1		2		3		4	
Subject	N(F)	*N(M)*	F	*M*	F	*M*	F	*M*	F	*M*
S & N	3558	*4175*	4.5	*4.0*	4.8	*4.5*	4.5	*4.7*	3.5	*4.1*
C & A	3436	*2788*	6.2	*5.5*	6.3	*5.9*	5.1	*5.1*	5.9	*5.7*
C & A (Ext)	4116	*6668*	6.6	*5.9*	6.5	*6.3*	5.5	*5.6*	5.4	*5.7*

The substantial overlap in the scores obtained by females and males on each of the CATs is strikingly apparent. Undoubtedly, within group differences are considerably greater than between group differences. Nevertheless some consistent between group differences are noteworthy. For each of the three subjects, females did somewhat better than males on CATs 1 and 2, while males did as well or slightly better than females on Cats 3 and 4, with CAT 4 for the C & A subject being the only exception to this. Collectively, these data provide a very clear example of the way in which the format of assessment tasks can influence our construction of gender differences in mathematics performance.

CONCLUDING COMMENTS

In this paper I have highlighted the role that can be played by research in meeting the challenges we face in mathematics education. Reference to earlier work served as a useful reminder of the strengths and weaknesses of educational research. A brief survey of research-driven, changing views of learning confirmed that constraints on constructions can change over time. The case of assessment was used to show how research can challenge findings shaped by the constraints of earlier assessment practices. The assertion that females, as a group, do not perform as well as males, as a group, on large scale examinations was the specific illustration used. Examples of other challenges to constructions generated by constraints considered appropriate at earlier times can be found on the well stocked library shelves to which I referred at the beginning of this paper.

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APPENDIX

Table 3: Views of learning: a selection

Source	Year	Reason for inclusion	Summary
Colburn	1830/ 1970	Author of 'possibly the most popular arithmetic text book ever published'. The strategies advocated were strongly influenced by Pestalozzi.	'Arithmetic, when properly taught is acknowledged by all to be very important as a discipline to the mind'. Few exercises strengthen and mature the mind as much as arithmetical calculations. Advocate of a new way of teaching arithmetic: *'instead of studying rules in the book ... the scholar makes his (sic) own rules (which) are a generalization of his own reasoning'. *All illustrations should be given by practical examples having reference to sensible objects. *Many teachers seem not to know that there is more than one way to solve a problem and stop students from using a method that differs from their own.
James	1890/ 1950	Challenged the doctrine of faculty psychology with 'experimen-tal' evidence	Intensive study of one topic or area does not necessarily improve performance in other areas.
Thorn-dike	e.g., 1924	Described as the founding father of the psychology of mathematics instruction	Learning is essentially the formation of connections or bonds between situations and responses. Teaching arithmetic is primarily a problem of the choice of bonds to be formed and the order in which this should be done. Repetition and reinforcement of appropriate responses are essential components.

Source	Year	Reason for inclusion	Summary
Brownell	1935	Challenged the drill and practice approach to teaching arithmetic	Success in quantitative thinking requires 'a fund of meanings, not a myriad of "automatic" responses'. Learning arithmetic should challenge students' intelligence, not only their memories. *Understanding* should be encouraged above all. Sequence and pace of instruction should be planned carefully; relationships (e.g., between number facts, topics) should be emphasised.
Piaget	e.g., 1952, 1956, 1960	Described as 'the foremost contributor to the field on intellectual development' this century	Children's intellectual development progresses through distinct stages: sensorimotor, preconceptual, intuitive, concrete-operational, and formal. Maturation, experience, and social transmission are considered to be important for intellectual development. The notions of equilibration, assimilation, and accommodation - integral components of his theory - have attracted much research activity. Although children can be helped to acquire mathematical concepts through teachers' questions and access to special materials, it is only through their own efforts that they will really understand. 'To understand is to invent, to build for oneself'.

Source	Year	Reason for inclusion	Summary
Wert-heimer	1959	Applied Gestaltist principles to mathematics teaching	Students should not merely be expected to copy or repeat given procedures inside the mathematics classroom. '(A)nything that might introduce a mechanical state of mind, an attitude of drill' should be avoided. Solutions to problems sometimes follows behaviour which is not clearly goal directed.
Skinner	1954	His theories led to the popularity of programmed learning	Students work through carefully sequenced (by others) materials at their own pace. Regular monitoring of progress is encouraged. When deemed necessary, remedial loops are provided.
Gagné	e.g, 1968	Well known for his work on learning hierarchies	Appropriately sequenced instruction leads to six levels of learning outcomes: specific responding chaining multiple discrimination classifying rule using problem solving. Practice is a necessary but not sufficient condition for learning.
Bruner	e.g., 1966	His work underpinned the spiral curriculum model	Knowledge can be represented in three forms: enactive, iconic, and symbolic. Generally, these represent a hierarchy of cognitive growth. Transition between the different stages needs careful planning.

Source	Year	Reason for inclusion	Summary
Simon	1979	Strongly associated with the information processing approach to cognition	Thought processes are described in terms of symbol manipulation. The processing and representation of information are of prime concern. A high degree of precision in describing cognition is aimed for. Early work was narrow in its focus (puzzles and games). With better developed tools mathematics text book tasks were also studied. It appears that humans process material both serially and in parallel.