Negotiation of Meaning in Mathematics Classrooms: A Study of Two Year 5 Classes

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The purpose of this study was to investigate the relationship between students' classroom experiences and their construction of mathematical meanings. Data were collected from classroom observations and videotaping sessions and from subsequent video-stimulated interviews with 6 students in each of two year 5 classes. Results discussed here are those from analysis of the interviews. This analysis revealed the existence of four primary sources by which students determined the meaning or correctness of mathematical activity: the teacher, intuition, familiarity, and procedural knowledge. Second, in relation to the social level, the teacher emerged as playing the most valued role in the sense making and ratifying of procedures or answers.

INTRODUCTION

In recent years a substantial body of classroom data has emerged internationally to substantiate constructivist views of learning processes (for example, see Perkins & Simmons, 1987; Confrey, 1990). However, apart from providing an interpretative framework for the analysis of classroom learning, constructivist theories have had little actual influence on mathematics instruction. Specific mechanisms of classroom interactions by which teachers and students conjecture, criticise, explain, test and refine ideas and procedures is what is referred to by the concept of "negotiation of meaning in mathematics classrooms". Elaboration of the form these mechanisms take in practice, and hence investigation of classroom construction of meaning, is required if the relationship between student classroom experience and student constructed meanings is to be understood in a way that will inform instruction. Hence, the goal of this project was:

> To investigate the relationship between students' classroom experience and the manner in which they construct mathematical meaning.

Within this overall goal were two intertwined research foci:

(i) the individual level at which students make sense of and utilise mathematics concepts and operations, and

(ii) the social level within which teachers' and students' individual contributions play a key role in a social making sense of and utilising mathematics concepts and skills.

The two research foci (the individual and social levels) arise naturally as a consequence of the fact that learning theories which fall within the constructivist school invoke the negotiation of both academic content and social context meanings as inevitable characteristics of classrooms, rather than as pedagogical options. Thus, the dual perspective of sources of conviction (Frid, 1992) and classroom consensus processes (Clarke, 1986) was selected as a guiding interpretative framework for the study. Sources of conviction refer to how one determines facts, legitimacy, logicality, consistency and accordance with accepted mathematical principles and standards. Consensus processes are typified by group compromise, refinement, accommodation or agreement. They are taken to be those interactions whereby statements, conjectures and arguments arising in classroom discourse are compared and assessed (i.e. development of social context meanings).

METHOD AND DATA SOURCES

Since this study was concerned with the context of learning and related descriptions of teachers' and students' mathematical interpretations, qualitative research methods predominantly were employed. This approach is in line with current educational research practices as they shift away from quantitative, quasi-scientific experiments so that researchers can more explicitly document and analyse the experiences of teachers and learners in the broad encompassing social and academic complexities of classrooms. In particular, this project adopted an inductive reasoning approach (Glaser and Strauss, 1967; Powney and Watts, 1987) for analysis of data collected by videotapes of classroom lessons and video-stimulated recall interview techniques (Keith, 1988; Meade & McMeniman, 1992). The emergent key patterns were therefore grounded in primary data.

The two year five classes chosen for the study were in two different government primary schools in the metropolitan area of a large city in Western Australia. The classes included students of low and middle socio-economic levels, as well as some students of minority backgrounds (aboriginal and Asian). Nine visits were made to each class with six lessons videotaped for each class between the months of March to August 1993. Following each lesson, two students were interviewed using a video-stimulated recall technique. These students were selected by the teacher so that the entire sample included a balance of males and females as well as students representative of a range of achievement levels in mathematics. Other student characteristics guiding selection included high versus low verbalisation and participation in class activities. The overriding principle guiding sample selection was that a rich diversity of cases be provided. A process of matrix sampling of students was used to ensure maximum diversity of sample cases, with a guarantee that each student was interviewed twice. Thus, a total of 24 interviews were conducted with students, each interview of a length of 30 to 45 minutes.

The protocol for the video-stimulated interviews asked students to respond to episodes in a 15 to 20 minute segment of the video. The episodes included teacher actions and utterances, other students' actions and utterances and the interviewee's actions and utterances. Choice of the 15 to 20 minute classroom segment was determined by the researchers so as to maximise the number of episodes which could be discussed in the interview. In relation to the video segment, students were asked to respond to a selection of questions related to what they had been thinking, what they did or did not understand, how they knew when they understood something, what they thought other students or the teacher meant by particular statements or actions, and if they agreed with these statements or actions. (For more details on the interview protocol, see Frid & Malone (1994)).

RESULTS AND RELATED DISCUSSION

Students' sources of conviction

In relation to the individual level of making sense of and utilising mathematics concepts and operations (sources of conviction), interview data revealed the existence of four primary sources by which students determined the meaning or correctness of mathematical activity: the teacher, intuition, familiarity and procedural knowledge. At the social level within which teachers' and students' contributions play a role in making sense of and utilising mathematics concepts and operations (consensus processes), one primary feature emerged from the data: the teacher. The following presentation is an outline of the results of data analysis, with supporting interview excerpts used to exemplify rather than substantially justify results. More complete documentation can be found in Frid & Malone (1994).

Teacher as a source of authority

Students saw the teacher as the primary authority on the meanings of mathematical concepts and the correctness of operations or answers. Within this context they saw the teacher as the individual with access to the "truth" or "correctness" of particular pieces of mathematics. This point is particularly clear in the following interview extracts:

(Anna)
Interviewer: Would you agree with what the teacher was saying?
A: Yes.
I: Why?
A: Because teachers are normally right.
I: Oh, I see. What makes you think they're normally right?
A: Um. Because they had to go through a school, teaching school, and they learn all about decimal points.

(Rachel)
I: Okay, how do you know that's all right?
R: Oh, because I have faith in my teacher and I believe him.
I: . . . When do things look right?
R: Things look right when I just look at them and they, and also another thing that told me it was right was that Mr Y walked around, looked at my answer and said yeah good answer. So when the teacher does that then I know for sure that it's almost right or pretty close to right. Or maybe even is right.

(Lisa)
L: Ah yeah, I will listen before I try it out myself and find out if I understand it or not. But if I don't she, she will come and if she sees that I've got um it wrong then she'll come and tell us if we, explain it to us and we'll find out if we understand it or not.

The students take what the teacher says as "right" as a reason to be convinced of the correctness of answers or procedures. Such aspects of the interview responses do not in themselves imply students do not make any use of their own sense of what seems correct. However, as evidenced in Rachel's and Lisa's words, when students were probed about how they knew they understood something, even when they referred to their own sense of something making sense, they relied on the teacher as an initial or an ultimate source of the coherency. In particular, Lisa's last comment about the teacher explaining "it to us and we'll find out if we understand it or not" is particularly revealing of her sources of conviction. It seems to indicate that Lisa does not see herself as a source of the determination of her understanding. Rather, she finds out if she understands or not by having the teacher tell or explain "what's right".

Intuition

Students included their own sense of having a "feeling", being able to "sort of know", or feeling that something "just makes sense" as means by which to determine appropriateness of a procedure or answer. Some students also brought into play how something looked or sounded. For example:

(Nathan)
N: Um. Well I have a feeling and when the teacher puts it up on the board then I sort of like, she has half the question and I sort of know it's right already. (pause) Before she starts. But when she's writing it on the blackboard and I've already got my answer down and she's just gone through it half way already, ah, put the numbers down already, I sort of like know it's right.
I: Do you have any idea how you know?
N: Um. I can sense it really.

(Alison)
I: Did you agree with him?
A: Kind of.
I: Can you say more?
A: He said that because um it just didn't sound right. It just didn't sound right.

(Walter)
I: How do you know when they're wrong?
W: (pause) I don't really know. You can sort of just see that they're wrong.

It was generally the case that students were not able to articulate reasons why something intuitively made sense. This apparent lack of metacognitive awareness cannot however be automatically attributed to an inability to think metacognitively. It might be that such aspects of working with mathematics have never been communicated to or stressed or practised with these students, in which case it would be unreasonable to expect them to somehow have naturally developed related skills and awareness.

However, in spite of students' general lack of facility to articulate and explicate reasons for their knowledge, they were able to associate "familiarity" with a personal sense of knowing. That is, one aspect of their learning that consistently arose in relation to metacognition was their awareness of when things were familiar, or had been "learned before". This point will be outlined next.

Familiarity

Students' recognition of mathematics concepts or operations as things they had seen or done earlier in the year or in a previous school year gave them a sense of knowledge of and competence with related ideas and skills. They would say they understood things because they had seen or heard them before, and would refer to something as "easy" or making "a lot of sense" because they had seen it before and knew what to do. That is, notions of remembering and having learned, seen, or done something before were a component of students' sources of convicion. The resultant sense for the students of "knowing what to do" appeared to give them a sense of understanding. However, it was not always clear to what extent an external observer also might describe some of the students' actions as indicative of "understanding". In particular, there were numerous instances in which it appeared that students equated the word "understanding" with instrumental knowledge as described by Skemp (1987). That is, they had "rules without reasons" (p. 153), or more specifically, they were able to perform procedures to obtain answers.

Procedural knowledge

When students believed they knew the correct procedures to yield correct answers they felt they had both understanding of and skill with the related mathematics. This aspect of their mathematics learning can be seen in the following interview excerpts:

> (Amanda)
> I: Now when you were working through these first ones, how did you know when you were right or not?
> A: Um. Well, I colour, it says to colour or shade in five pieces out of six. So I colour in one sixth, another sixth, another sixth and another sixth and another. So I've got five sixths that I coloured in. One left over equal, equal the six pieces. I have five coloured in. . . . I know something's right, like I know my times tables. And then like for a times table thing, if I know my times I know I have got it right. And if I practice them at home and I learn them, I think I've got them right so I just go through my times tables and go one times four is four, two four's is eight, and so on until I make sure I've got it right.
>
> (Yvonne)
> I: Well if it didn't make sense, how would you feel then? How do you know when it doesn't make sense?
> Y: I'd be confused and I wouldn't know like how to do it, and like I'd have to ask again.

The sense of knowing "how to do it", that is knowing a procedure, was prominent in the interviews. In fact, it was often difficult to probe further to determine if the students had conceptual understandings underlying their procedural knowledge. Not only did their explanations tend to rely on detailed descriptions of procedures, as with Amanda in the above extract, but they tended to reveal that students saw procedural knowledge as a primary goal of their mathematics learning. They sometimes referred to knowing how things "fit together", but were often not able to articulate this "fitting together" in ways an observer might say are indicative of conceptual understanding. Even after further probing by the interviewer to attempt to explicate conceptions of understanding, little more was revealed.

Classroom Consensus Processes

Examination of the transcripts for whole class examples of how students determined mathematical meaning or correctness revealed that, as with the individual level, students accepted without question that teachers' comments, guidelines, evaluations and decisions were legitimate and correct. Within this social realm, peers were not heavily utilised, except when class majority was cited as a component of particular decisions, or when the teacher was not available and a peer had to be approached for assistance to proceed through a particular set of exercises.

Teacher and student roles in consensus processes

Most of the students expressed views that peers were not important to their mathematics learning:

> (Rachel)
> I: Who do you feel you rely on?
> R: Now I have found it really better to rely on teachers and calculators. Cause they have a better and more accurate answer than my peers. . . . Because like, say Mr Y went around and he said, and he's looking at our answers to help me, make me change my mind. Sometimes he can say like say I had the wrong answer he can say have a look at it. If it doesn't look right to you then it probably isn't right. And then I look at it and I realise it doesn't look right so I try again.

> (Lisa)
> I: Do you ever ask any of your friends to help you with it?
> L: Um. No.
> I: Do they ever ask you for help?
> L: Not really. They usually just wait for the teacher to tell them.

Rachel and Lisa clearly see the teacher's role as including "telling" students what to do or what is right. From their perspective, classroom "negotiation" is not useful. However, although the students did not see peers as a key component in mathematics learning, they did not necessarily exclude working with peers as a legitimate or useful activity. This latter point can be seen in the following:

> (Rachel)
> I: So do you think it's important to try to help your neighbour or what?
> R: I only try to help, I only help my neighbour when I'm told to by the teacher and we're allowed to because sometimes we're not allowed to and we could get our name on the board. Like say if it was a test. Then we aren't allowed to help our neighbour.
> I: What about today? Do you think it would have been okay to help your neighbour?
> R: I'm not really sure. So. And I didn't take the risk, so I didn't. Cause I wasn't sure.

It is interesting to note that Rachel's perspective on the notion of peer consultation is highly intertwined with the larger context of the social norms of the classroom. That is, Rachel's views of both her own and other students' roles in sharing ideas and helping one another are influenced by the behaviour and action rules of the classroom. She will not "take the risk" of helping another student unless she has been informed by the teacher that it is an acceptable or appropriate action for that mathematics lesson or point in the lesson. The same awareness of and adherence to the classroom rules was evident in interviews with other students, although not all of them expressed simultaneously the devaluing of peer assistance that Rachel expressed. There was also indication that some students disliked peer interaction, rather than devaluing it. Finally, although the discussion thus far has highlighted the absence of students valuing the role of peers in their mathematics learning, there was evidence that students on occasion did help one another. They would help if while working on exercises a neighbour "got stuck on something" (Amanda) and needed to be shown an example. Students also saw peers as playing a guiding role in a whole class context in that the "bright kids . . . lead the not so bright kids along" (Yvonne). Students referred to listening to and being helped by a peer if it was someone perceived to be "good at their mathematics".

CONCLUSIONS

Within the social realm of the two year five classes studied the process of "negotiation" of meaning was such that the conjecturing, criticising, explaining, testing and refining of ideas and procedures was primarily the responsibility of the teacher. The role of students within this "negotiation" process was minimal in comparison to that of the teacher, so that students did not regard themselves or peers as a source of mathematical knowledge. Although some group consensus occurred via group majority agreement on procedures or answers, a metaphor of classroom meaning making as "negotiation" could be said to be inappropriate for these two year five classes. A more appropriate descriptor is "ratification" in that, although ratification incorporates a notion of acceptance or agreement, it primarily denotes a rite of endorsement and approval to officially validate the agreement.

That these notions of endorsement and agreement are more appropriate descriptors than negotiation indicates a need to examine the intents and expectations of both the teachers and the students within the classroom processes of these two classes. What are a teacher's intents in teaching a particular lesson, or in teaching mathematics in general? What are her beliefs and experiences in relation to mathematics learning and what are her expectations of students? What do the students expect the teacher to do or say, and what do they believe are his intents and expectations for learning? These questions and related ones must be addressed before classroom processes can be interpreted in ways which capture the classroom contexts as experienced by participants themselves. For example, a number of students in this study expected the teacher to provide clear explanations of how things work and what to do with them. They expected the teacher to prescribe procedures, and hence, the issue of negotiation was not an issue at all. That is, the students expected the teaching to be what educators call direct instruction. Direct instruction has a number of practical and effective educational features, but the issue of negotiation is not an anticipated component of such a mode of delivery. This last point highlights that the concept of negotiation requires intention to negotiate, which in turn highlights the role of classroom social contexts. The social rules and norms of a classroom, and therefore the resulting classroom practices, are necessarily key components of ways in which students construct mathematical meaning.

The notion of constructing viable knowledge and actions is an aspect of constructivist theories. However, since teachers generally want students to learn with "understanding", if viability from a student's perspective is determined by ratification or endorsement, then the nature of what students learn is problematic. It is problematic in that what teachers and students see as mathematical understanding and the intended outcomes of mathematics lessons are possibly in discord with each other or with actual classroom practices. Vital to these issues are teachers' and students' beliefs about mathematics learning and the nature of mathematics, for these beliefs will shape how they see "understanding". Thus, if construction of mathematical meaning is to be an object of research, then an integral component of examination must be individuals' beliefs about the nature of mathematics and what constitutes mathematics learning. Only then will educators be equipped to appropriately and adequately study mathematics classrooms in ways which might enlighten or transform mathematics education.

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