Title

The use of a spreadsheet as an algebraic environment..

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Abstract.

Computers have provided many opportunities for new approaches to mathematics in the classroom. This project focusses on the use of a spreadsheet as an algebraic environment in which algebra can be introduced to students at year 7 level. Materials for this have been written and used with a group of mathematically weak year 7 students who had no previous experience of spreadsheets. An assessment task given the students following the use of the spreadsheets required students to explain some of their solutions. The results of this give some ideas of students learning. The project is in early stages and further suggestions for monitoring students algebraic development in a classroom situation would be appreciated.

Paper

Context.

For many years there have been concerns about students understanding of algebra. Different approaches have been and still are being tried in an attempt to enhance student understanding. There is a need for algebra to be introduced to students in a way which they perceive as relevant . The concepts and context need to be presented in a manner enabling the students to attach meaning to the symbolic language of algebra.

Algebra seems to often provoke anxiety in students. This does not however seem to apply in the same way to computer based symbols (Sutherland 1993). Rojano and Sutherland (1992), worked on a project using a spreadsheet to enable 14 and 15 year old students, who had been unsucessful at achieving in mathematics, to learn algebra. The students worked one period a week for four months and showed that they could also transfer their algebraic learning to pen and paper tasks. In the computer environment the symbolic expression related to the students own constructions of generality. Sutherland (1993) noted that in solving worded problems students were both negotiating general rules for specific cases and testing their general rules with specifics.

Learning is affected by culture and environment. The culture of the school and the classroom can impact on student's learning. Classrooms have changed and new tools are available in the classroom environment which, in the case of the computer, itself provides new learning environments within the classroom. Papert (1987) in reference to Logo and the computer claims "It sets the cultural context for personal learning." (p23) The spreadsheet on the computer sets a cultural context for learning algebra. It is a rich environment encouraging the use of variables and functions and open for development in many areas of algebra and the study of functions. There is also no doubt that the computer is very much part of our society and the student's environment and, as such, is seen as relevant.

Much of the computer research has concerned the use of Logo and programming and the use of Computer Aided Instruction. There are many advantages claimed for children learning to program. Rowe (1993) has summarised a list of claims based on research papers including rigorous thinking, greater facility with heuristics for problem solving and enhanced metacognitive awareness. The use of a spreadsheet involves programming but in a different environment. It is also necessary for algebra to be used in order to communicate with the computer in a spreadsheet environment. Like Logo the instructions necessary to begin are few but the capacity of the spreadsheet microworld/environment is great and encourages the development of algebraic thinking.

After working with older students who had experienced difficulties, Rojano and Sutherland (1992) began using the spreadsheet to introduce algebra to two small groups of pre-algebra students in England and Mexico. They reported that the children with which they were working had no difficulty accepting and using the spreadsheet package. In the beginning the students did not think spontaneously in terms of general formula but they learnt to do this with support from both the spreadsheet environment and the teacher. Once they had learnt to use columns and copy cell formula they did not have difficulty accepting that the column A5, A6, A7 etc could all effectively represent the variable number of lollies or width of river (Sutherland & Rojano 1992). In the computer context the students seem to be able to make the transfer to a symbolic language to represent mathematical relationships more easily. The spreadsheet is also a particularly useful package in that it is generally available and used widely, is useful in other areas and allows graphing and statistics as well as enabling algebraic ideas to be developed.

As with many computer based research projects the groups of children studied by Rojano and Sutherland were small and the environment was not their normal classroom as part of their normal mathematics program. Their ideas are exciting but there is a question of effectiveness in a normal school environment with the normal classroom teachers operating the program.

In 1993 materials based on those of Sutherland and Rojano were used with two year 7 classes. The students were not closely monitored but the teachers observed the students responses and discussed their feelings about the effectiveness. Advantages seen included students working at their own pace, the classroom became student based and student focussed, problem solving skills were used and developed and even two students who began with frustration because of their lack of contact previously with computers very quickly became comfortable and participated in all aspects of the class (Horne & Jeppesen 1993). Other aspects raised in discussion included the need to encourage students to take control of their own learning and to share and discuss their understandings and the realisation of the teachers that they did not need to know much about spreadsheets to use them in the classroom. As a result it was decided to incorporate the approach into the year 7 program and revise the materials for use in 1994.

The introduction of any program into a school, even though it is a small program, can be affected by constraints beyond the control of the teachers. Unfortunately the year 7 mathematics classes were timetabled against the computer science classes so what previously and in initial planning had been excellent computer access was considerably reduced. In late March/ early April 1994 one class of very weak students were introduced to algebra solely via the spreadsheet. It was intended that all classes used the approach but because of the access constraints the introduction with spreadsheets for all five classes will need to wait until 1995. The class which did use the materials is made up of the fifteen mathematically weakest students in year 7 in an all girls school. This is unfortunate as it is again a small group not normal class size. Some of the students have language difficulties as well. One child has a learning problem which makes reading and writing very difficult. Others have, for various reasons, very poor levels of arithmetical and other mathematical skills.

The materials.

The algebra which the mathematics department in the school wished to cover in year 7 was divided into two sections. The first introduced formulae (functions but without formal notation), ideas of equivalence and discovery of rules. The second section (not yet done and which all year 7 students will be using) covered the idea of inverse functions, solution of equations and applications to worded problems. The first section includes four worksheets, one of which is included here.

USING FORMULAE

As you work through, when it says RECORD write the work suggested in your book.

1. Enter a number in cell A1.

2. In cell B2 type =A1+3 then hit the return key. What happened to the cell B2?
Change the number in cell A1 (hitting the return key afterwards).What happened to B2?
Change the number in A1 again but try to predict what will happen in cell B2 before you hit the return key.

What is the formula in B2 doing to the number in A1?

Change A1 three more times using a large number, a small number and a decimal number. Each time predict what will happen to B2.

RECORD: RULE: B2 = A1+3

Number in A1	Number in B2

Write in words what the rule is doing to the number in A1.

3. Repeat 2. but this time use the rule =3*A1 in the cell B2.

RECORD: RULE: B2 = 3*A1
Record your changes to A1 and the changes in B2 in a table like the last one.

Write in words what the rule is doing to the number in A1.

4. It is your turn to write your own formula in B2. Your formula must start with = and must have A1 in it. The spreadsheet uses * for multiply and / for divide.
Don't forget to hit return when you have finished typing. What happened to the formula when you hit the return key?
Change the number in cell A1 again. Before you hit the return try to predict what will happen to B2, the cell with the formula in it.
As before change cell A1 three more times predicting each time what will happen to B2 and recording your changes in a table.

RECORD: RULE: B2 = (fill in your own rule)

Write in words what the rule is doing to the number in A1.

5. Try it again with another rule recording it in your book in the same way as before.

6. Write an explanation for a friend who is absent so that they will understand how to enter a formula and what a formula does.

The program.

The students worked in the computer room with their class teacher for five fifty minute periods and had another three periods interspersed in the classroom discussing what they had discovered and practising some of the skills with pencil and paper. This was not ideal but programs do have to fit in a schools constraints. The materials involved a lot of reading and these some of these students had language difficulties so the teacher concerned introduced the activity with the students trying the first formula together. She also explained to the students that she wished them to keep a record of their work. At various stages she called the whole group together to discuss what they had learned. This was sometimes done in the computer room but because of the access many discussions were held the following period. The spreadsheet used was Excel but any would have been satisfactory.

The assessment task

Show all working and set your work out clearly.

1. (a) If B = 4 x A - 7 find the value of B when A = 6.

   (b) If D = 20 - 2C find the value of D when C = 3

   (c) If F = 3 x E x E + 2 x E find the value of F when E = 10

   (d) If Z = 2Y + Y^2 find the value of Z when Y = 4

   (e) Explain, as if to someone who didn't know, what question (c) means and how you did it.

2. (a) Match the pairs of equivalent expressions.

A.	3 x A + 5	(1)	4 x A + 5
B.	5A + 2 - 4 A	(2)	10A - 3A
C.	A + 7 + 3 x A - 2	(3)	2 x A + 3 + A + 2
D.	3A + 4A	(4)	A + A + 4 + A + 5
E.	8 + 2 x A - 3	(5)	2 + 3 + 2A
F.	9 + 3 x A	(6)	A + 2

   (b) Write 3 equivalent formulas for B = 5A + 2

   (c) Explain how you know two expressions are equivalent.

3. Simplify each of these by writing the simplest equivalent expression.

   (a) 3A + 5A + 4

   (b) 7b + 2 - 4b + 3

   (c) 3a + 5b - a + 6b

   (d) $2c^2 + 4c - 3c + 5c^2$

4. Complete the table and write the rule for each of the following.

(a)

a	b
1	1
2	4
3	7
4	
5	
10	

Rule: b =

(b)

a	b
1	7
2	12
3	17
4	
5	
100	

Rule: b =

Report on student progress so far.

Of the eleven students for whom I have results, nine showed that they could substitute in formula, one did not show working and made errors so it is hard to locate the nature of the errors and the student who had difficulty writing showed some correct substitution. Of the nine who substituted correctly in all questions, six showed later arithmetic errors in some questions. For example 4 x 6 = 27, 3 x 10 x 10 = 90, 300 + 2 x 10 = 3020, 24 - 7 = 16. Four made errors with the order of operations. The students are allowed to use calculators.

Student explanations of the process of substituting included these given below. Notice the order of operations error for student B who in solving the question actually got the answer 3002.

A. "First I would tell them to spead the sam out on a peice of paper and say it is the same as an normel sum but we have letters filling in for the numbers."

B "the formula is F = 3*E*E + 2*E, you don't know what F represents but you do know what E represents which is 10. So you could do the sum 3*E or 10 is 30 and 10*30 is 300 + 2 is 302*10 again is 3020."

C. "If F = 3 x E x E + 2 x E and E = 10 what is the value of F. I would say that means E = 10 so when you see the letter E change that to 10 so you then work out the remaining sum

$3 \times \not{E}^{10} \times \not{E}^{10} + 2 \times \not{E}^{10}$."

D. "well if E = 10 you say what is left and it is 3 x 10 x 10 + 2 x 10. How did you know when to put the 10s in. If E = 10 then you put in the 10s where the es where. So from there it is easy. 3 x 10 x 10 + 2 x 10 = 320 so F = 320."

Equivalent formulae were not handled with the same degree of efficiency. Only one student had all correct. Five had confused two of the six matching pairs. Four had not attempted or understood the question of matching the pairs but of these four two had written correct equivalent formulae for the other questions, while two who had matched pairs correctly could not simplify later expressions. One student who did not attempt answers illustrated her misunderstanding of the matching aspect of the question when she wrote "the two on the same row are different".

Other explanations included

A. "two expressions are equivalent if they have the same answer and if they have the numbers just swapped around."

B. "they are the same but written different ways."

C. "you simplify the expressions by grouping and adding the numbers together then you group the letters together."

D. "Equiv expressions when two formulas equal the same."

One student actually substituted numbers to check which formulae were the same. Notice the wording in C is similar to that used by teachers in a traditional approach suggesting perhaps the student had other input.

For the finding the rule questions four students did not manage to write rules but all filled in at least three numbers correctly. One other student expressed the rules correctly but with the 'a' missing. Another made an error with one rule while the rest were correctly completed.

Other interesting effects were the enthusiasm the students showed for using the computers and the confidence they had in demonstrating on the computer what they were doing. The student with the writing problem was really excited and enjoyed the class showing in class she had some understanding of the tasks although on the test she had real difficulties.

The teacher concerned expressed positive response to the use of the computers but frustration with the constraints which limited access to the computers.

Conclusions

This research is in early stages but so far the results suggest continuation is warranted. The second unit has been rewritten on the basis of the last years trial. Techniques and instruments for monitoring the students algebraic development need to be refined so that the meaning students attach to the tasks can be studied in relation to their interaction with the algebraic environment of the spreadsheet. One difficulty with the data here is that it is unclear with some students whether their understanding came as a result of the use of the spreadsheet or from some other source. This indicates the need for further observation and interviews. Although asking students to write explanations provides insight into some of their thinking many students find writing difficult and it is thus unsatisfactory for this to be the only data.

This year the group was a group with no previous experience of spreadsheets but the approach is not dependent on this as the materials can be structured to allow the students to progress at a pace suitable to them and further review of the materials will take this into account.

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