

Investigating The Use Of Language And Mathematics In A Workplace Context

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This study investigates how the linguistic devices of metaphor and metonym produce and regulate mathematical knowledge in the performance of work related tasks. Data are drawn from the front desk operation of three metropolitan district motels in various parts of Australia, and analysed making use of 'critical incident' methodology. Findings indicate that within the workplace, language use and mathematical knowledge are deeply entwined

Background and aims of study

Edward Thorndike, in his General Introduction to the *Psychology of Arithmetic* (1922) made the following assertions:

The aims of elementary education, when fully defined, will be found to be the production of changes in human nature represented by an almost countless list of connections or bonds whereby the pupil thinks or feels or acts in certain ways in response to the situations the school has organized and is influenced to think and feel and act similarly to similar situations when life outside of school confronts him [sic] with them. (p xi)

In some ways, perhaps, this statement represents a high water mark in terms of its confidence in the prerogatives of 'the school', the power of this institution to realise its mission to influence totally the development of the learner, and the rightness of the school in so doing. Since this time, of course, each of these aspects of Thorndike's notion of elementary education has been debated vigorously. Whilst arguments have ranged over domains including psychology, sociology, anthropology, philosophy, ethics, cultural studies, they have usually not accorded a high degree of attention to learning experiences outside the school, such as in the home, the market place, the workplace, and so on. It would thus seem appropriate that in renewing consideration of Thorndike's general assertions, attention be focused on pertinent data obtained within workplace sites. Accordingly, in this study concern centred on the extent to which "already made" knowledge is merely transferred to a given task, as distinct from being produced in response to site specific needs. The approach taken was to examine certain linguistic processes involved in the production and regulation of mathematical meanings within specific workplace contexts. In so doing, the intention was to investigate how these processes were found to shape and so construct the mathematical ideas formed in these sites. If established, these points will enhance claims that out-of-school learning experiences need to be more systematically studied than is presently the case.

Theoretical background

One way to think about the way mathematics is used in a workplace context is to make use of the literature relating to the phenomenon of knowledge transfer. Researches into what transfer is, how it works, and what conditions promote its function currently range over behavioural, cognitive and socio-cultural domains. Given the limited scope of this paper, however, it will be possible to investigate only one salient aspect of this literature base, namely, the nature of epistemological assumptions underlying current research in this area of investigation. It is to this purpose that the following paragraphs are addressed.

A key concern within mathematics education research over the last two decades has surely been the close reconsideration and revision of ideas relating to the epistemological status of mathematical knowledge itself (Ernest, 1991). At one time essentialism (the doctrine that mathematical knowledge is factual in character and thus independent of the knowing subject) occupied a near universal position of dominance within the education field, yet by this time a considerable body of both empirical and theoretical material has now been adduced for a number of alternative epistemological stances; these might generally be grouped under the broad heading of social/individual constructionism. In these emerging alternatives to essentialism, mathematical knowledge is thought to be a construction involving social organisations (Tymoczko, 1986) and/or individual people (von Glasersfeld, 1995), rather than mainly or totally an objective fact of nature. Whilst the nature of these alternatives cannot be discussed here in greater detail, of interest to my immediate purpose, would be to highlight the kinds of research which have led to the rise of constructionist alternatives and relate these to the issue of usage of mathematical knowledge under circumstances of transfer. Four major lines of development are discernible, and these will be briefly considered in turn.

Firstly, mathematics curriculum theory has for some time drawn attention to 'school mathematics' and 'real mathematics' as consisting of different kinds of mathematical knowledge. For example, Newton (1983) showed that school age children are frequently able to perform elementary numerical operation when they are embedded in naturalistic contexts, but are unable to perform structurally identical tasks within the formal context of the every day school setting. Lave (1988) found similar differences in her study of how adults used mathematical procedures in shopping contexts. Likewise, Walkerdine (1988), in her study of pre-school aged children, found that the discourse between mother and child relating to the preparation of food and mealtimes organisation presented a range of mathematical concepts unique to these contexts and frequently at variance with those later encountered within the formal schooling context. In considering salient contexts for the use of mathematical knowledge, Dowling (1989) offered a rich theoretical map for considering the components of the semantic field framing the operationalisation of numerical knowledge within school contexts. Schoenfeld (1991) also considered what he termed "the unfortunate separation" between formal and informal mathematics. Both, he argued, are required for sense to be made of either. Whilst in each of the studies referred to above, authors conceptualised their data within substantially different theoretical domains, a common set of questions can be seen to arise. These may be stated as: Can mathematical knowledge can be said to occupy an a-contextual space? Does the process of realising meaning require a defining context as a preconditional requirement? To what extent does this context act to shape the mathematical knowledge itself? The general tendency, in the research literature at any rate, has been to respond to the first of these in the negative and the second in the positive, and this has fuelled the shift from belief in essentialism to social constructionism. It is to be noted that the nature of mathematical knowledge transfer/usage on this view is more concerned with the adoption of salient contexts for the operation of knowledge, and is less interested in how knowledge is actually deployed from a cognitive view.

Secondly, developments in the cultural anthropology of mathematics has also provided a challenge to essentialist thinking in mathematics. The work of D'Ambrosio (1985), Carraher, Carraher and Schlieman (1985), Gerdes (1985), Bishop (1988), Saxe (1991), Nunes, Schlieman and Carraher (1993), and others can be cited as making a major contribution to our knowledge of the mathematical ideas and processes made use of by indigenous people and persons generally outside of the hitherto confined circles of academic western dominated culture. Within these broader contexts, essentialism as a doctrine for mathematical knowledge, has appeared increasingly inadequate. Mathematical behaviours have been observed as too diverse and differentially integrated into the cultural and productive life of societies studied for such a doctrine to maintain credibility. In contrast, mathematical knowledge is seen as arising from

various interactions among the social, physical, and cultural contexts of the mathematics knowledge user. On this view mathematics knowledge use/transfer and knowledge creation are seen as part of the same process.

Thirdly, recent developments in the psychological literature have generated theories of learning which make little call on essentialist epistemologies for mathematics. For instance, whilst essentialism is not a logical requirement of the associationistic style of learning theory as represented in the works of theorists from Edward Thorndike to John Anderson, this epistemological doctrine is nevertheless both highly compatible and often implicitly assumed within the context of theory formation. In contrast, Piagetian theory posits learning to be the process of active and purposeful reorganisation of thinking and meaning in response to a problem situation, and this so-called constructivist thesis does not depend on the correspondence among accommodated schemata and objectively given facts and conditions. In Vygotsky's theory, also, no such correspondence is either required or entertained. Learning in this theory is the process of internalising inter-psychological involvements as mediated by language and the other artefacts belonging to the learner's context and culture. Arising from these influences a complex body of literature has evolved delineating a theory of 'situated cognition' according to which mental operations are not determined exclusively by factors relating solely to the stage of mental development or other individual characteristics of the person involved, but are also characteristic of the physical, social, cultural context in which they are performed (Pea, 1987; Lave, 1988; Brown, Collins & Duguid, 1989; Collins, Brown & Newman, 1989; Greeno, 1989; Rogoff, 1990, 1995). In summary, situated cognition regards the question of mathematics knowledge transfer/usage as involving two kinds of issues: the first relates to the nature of site specific rendition of knowledge and social practice (Bruner, 1990; Lave and Wenger, 1991), and the second considers the development of task competency in the light of constructs derived from cognitive theory (Pea, 1987; Greeno, 1989, 1992).

In part drawing from the above, a fourth general line challenging essentialist epistemology in the mathematics education literature can be discerned. In this approach researchers have focused on the micro details of interactions between students and their teachers as they go about tasks within normal classroom protocols (Bauersfeld, 1991; Cobb, Yackel & Wood, 1992; Voigt, 1993). Emerging from this research has been a far ranging revision of the Piagetian constructivist paradigm for learning. Cobb et al, for instance, have proposed a theory of social constructivism according to which learners, reacting to given problematic situations, engage in two knowledge building processes simultaneously. In the first they individually construct their own meanings for the mathematical concepts and processes they encounter; in the second, individual meanings are massaged ("negotiated") with reference to the multiple levels of constraint and facilitation occasioned by the many forms of social process that usually accompany learning in actual situations - for instance, interaction with peers, teachers, instructional materials, and so on. Social constructivism is thus posited on a social constructionist epistemology and mathematical knowledge is to be identified with its application within site specific contexts.

In summary, the various perspectives considered above suggest that the use of mathematics can rarely be thought of as a purely technical operation in which certain "ready made" knowledge is merely accessed in order to perform a given task. Instead, knowledge is (re)formulated on each occasion a problem situation arises or a task competency is required. In general, however, the literature surveyed above does not examine what tools mathematics users need to utilise in order to produce and regulate the knowledge required within the operation of specific tasks in naturalistic settings, nor does it fully explore how these tools function. Following in the tradition of Vygotsky (Cole and Scribner, 1974; Wertsch, 1985), language can be thought of as just such a tool; it is for this reason, therefore, that the remainder of this paper will mainly focus on the use of certain linguistic devices in the performance of workplace tasks.

Examining a workplace context - methods

Data for the study were drawn from a significantly larger data base assembled for a study into the contextual nature of workplace competence (Stevenson, 1995). This larger study was conducted by the current researcher as part of a team of Griffith University based researchers and funded by the Australian National Training Authority in 1995. Motel front desk operations provided the focal point for data collection and analysis; sites were chosen from around Australia and involved motels in both metropolitan and country districts. Data consisted of the video record of transactions which took place in each site over a continuous period of 4 to 5 hours (a) among staff engaged in workplace tasks, (b) between staff and guests relating to check-in and check-out procedures, and (c) between staff and researchers relating to salient aspects of workplace operations. In the study reported here, however, the scope of this larger study was significantly reduced. More particularly, data were drawn from three Metropolitan sites only, reflecting the need felt to reduce the observed variability of the data set whilst ensuring that coverage achieved an appropriate level of depth.

In the current study, data analyses were conducted in the following way. First, transcripts of the interactions were prepared and "critical incidents" involving mathematical knowledge were identified. Criteria used for identifying these episodes were drawn from the documentation provided by the Australian Education Council, *Mathematics - a curriculum profile for Australian schools* (1994). Second, the language used in the critical incidents identified was scrutinised in order to identify uses of particular linguistic devices in the use and application of mathematical ideas and procedures in the workplace operations observed. Linguistic devices chosen for examination were selected with reference to the appropriate research literature. Third, instances of devices utilised were analysed in order to investigate how these were being used to produce and regulate mathematical knowledge within the tasks identified.

Examining the data: Use of metaphor and metonym in the production and regulation of numerical meaning within a workplace site

Linguistic theory (Jakobson & Halle, 1971, Kristeva, 1989) recognises a number of ways meaning for words and procedures can be produced and regulated. Two of these are the use of metaphor and metonym in oral and written communication, and these will be studied in this investigation. Metaphor, relies on vertical relationships between the domain of the words and symbols for which meaning is to be ascribed, and corresponding elements within the domain for which the meaning maker already possesses meaning. Possibly less familiar, however, is metonym, which relies, in contrast, on horizontal relationships established by juxtaposition among words, symbols and other possible referents eg objects (both concrete and abstract), processes etc. In the following, an illustration of findings obtained by analysis of the data will be presented, each of these tropes (as they are known in linguistics) being considered in turn.

The use of metaphorical relationships

Metaphor, as an explanatory device is quite familiar as a commonly used tool for mathematics teaching. For instance, elementary number properties are often first 'illustrated' within a concrete domain consisting of bundling sticks within which operations of grouping can be performed. Later, the bundling sticks are made to correspond to icons eg written strokes and dashes, and grouping is spoken of as 'adding'. Finally, more abstract symbols are produced in order to render the relationships first encountered in the concrete, meaning-rich domain. Notice that on this fairly classical view of mathematics instruction, the key processes revolve around efforts to build vertical relationships which preserve salient features of the meaning-rich

domain (often concrete) in the hope that these may then be transferred to the meaning-poor domain (often abstract) between familiar and unfamiliar domains. This is the representational approach to teaching employed systematically by theorists such as Dienes, for instance. Likewise, Resnick (1983) considered teaching strategies effective when, considered as representations of the mathematical knowledge they are designed to teach, they are 'transparent' to the mind of the learner - that using them, the learner will grasp the inherently metaphorical character of the knowledge encountered.

Evidence that indeed respondents made use of metaphorical devices in order to construct meaning and guide respondents in the performance of their workplace tasks was obtained. One example may suffice to illustrate use of this technique. Respondents were observed to have made structure preserving assignments between elements belonging to the price domain (numbers indicated in dollar values) and the domain of goods and services as traded by the motel. In the following episode, in which the respondent is rationalising a customer's account, the reasoning used by the respondent appears to depend critically on the establishment of a one-to-one correspondence between the number of nights the guest in question stayed in the motel and the total cost of this stay. Phone bills were counted as extra. (Note, the following conventions will be used in transcripts: RS - a respondent, G - a motel guest, R - a researcher.)

RS: (Adding machine noise over conversation.) *Yes it's \$421.45. Thank you.*

RS: *Hello reception.*

Inaudible conversation.

G: *How much did you say it was?*

RS: *Yes that's including last night's accommodation, then one week which is \$350.00 then of course your phone calls plus last night, which is already there. So that's \$421.45.*

More conversation but inaudible due to adding machine and radio.

So you still need to include those at \$71.45.

Adding machine.

In this transcript, the respondent was able to justify the bill by demonstrating how the process of tabulation preserved the logic of the guest's pattern of consumption. Of significance here is the way in which the consumption pattern was grouped into one block of "one week" followed by "phone calls plus last night". Use of the term "plus" here illustrates the strength of the implicit assignment operating between the numerical domain on one hand, and on the other hand, a domain in which items of consumption are routinely grouped. According to the metaphorical structure implicitly working here, "plus" in the former domain is assigned to the grouping operation which takes place in the latter. On one interpretation, this example illustrates the way metaphor is used to access mathematical concepts in order to better regulate workplace tasks. On another view, it illustrates just the opposite, namely, how metaphor can be used in workplace tasks to produce and regulate mathematical meanings (in this case for "plus") applicable over an extended range of highly contextualised variables. Choice between these views revolves around epistemological issues. In the first, the presumption is that mathematical knowledge exists as a separate entity and is transferred to the task, whereas in the second, it is believed that this knowledge is distributed across elements constituting the task in its particular context.

The use of metonymic relationships

Metonym is a less familiar, though frequently used way of producing and regulating meaning in communicational interactions. In this approach, the meaning to be ascribed to one term derives from a known meaning already ascribed to a second term in circumstances where both terms are juxtaposed, perhaps physically, either by accident, custom or design. In contrast to metaphor, in which is posited a structure preserving map between two domains, metonyms are based on juxtapositional, and hence horizontal relations between terms belonging to the same or similar domains. For

example, reference to 'the crown' could be taken as a reference to the monarchy and/or its institutions. In this instance, the 'crown' as an object is juxtaposed with the person of the monarch and thus with the system of monarchical government. An horizontal relation of terms 'crown-monarch-institution of government' is thus created and, by virtue of this chain, the semantic scope of each included term is greatly extended.

Analysis of the data revealed extensive use was made by the respondents of this type of meaning production. Two broad schemes for the use of metonyms were identified. The first involved constructing a nomenclature for motel guests which exploited the numbers of the rooms in which they stayed. In this case the scheme of juxtaposed relationships was the following: 'Motel guests - motel rooms in which they stayed - the numbers of the motel rooms'. Metonyms made possible by these relations were as follows. The first allowed respondents to keep a record of guests using motel room numbers as appropriate labels ("*No charge for Room 37 for a late checkout.*"); the second allowed respondents to label rooms with their corresponding room numbers ("*You're in 326 aren't you no 329?*"); and the third allowed respondents to label guests with a numerical identification ("*... what are your numbers Margaret? 23's gone 24's gone 26 has gone, what else? 28's gone, ..., 22's still there. Thanks bye.*").

Situations arose when guests were misidentified or expenses arising from office, bar or restaurant transactions were not properly ascribed to the correct accounts. In these instances, respondents were observed to permute assigned numbers in order to make the appropriate designations. As one respondent noted, in such cases

RS: You can either try the number again in case you put it in backwards, or you can just ask it to print and it will print out exactly what you put into it and you can check why you don't agree with it.

In another instance, the respondent reported a more complex engagement with the numerical aspect of the assigned nomenclature.

R: So how did you find out that was a problem?

RS: When the charges come down, you put them in and as you're putting them in. The only way you find out its a problem id if you put the room no. in and it doesn't come, there's no guest in the room. So there's two alternatives, either a) the guest is already gone, or there was no guest in the room and they've written the wrong number, or the guests get mixed up and they give the wrong number, sometimes they transpose the number or whatever. So you sort of fiddle around with it. Try the numbers the other way.

In the cases discussed above, respondents were seen to use numbers to form a system of labels. Use of certain numerical properties of numbers constituting labels (eg listing by numerical order, using a place value coding system which allows for error correction) was also observed. In these cases the metonymic relationships established were exploited in order to regulate better progress through workplace tasks.

A second scheme observed for the production and use of metonyms was also observed. In this scheme use was made of the juxtaposition of particular numbers, usually expressed in units of currency, the prices of particular goods and services offered by the motel, and the merchandise offered for sale itself. According to these juxtapositions, dollar amounts were taken to refer to merchandise, as in the following example in which a respondent is explaining to a researcher the method adopted to make a cash balance of transactions conducted in the day.

RS: I always take my cash out and I balance what I've got in my till. And this should be \$250. [float] If its not then I know there's a mistake somewhere. If I've got too much cash it means I've either cash for something and forgotten to post it or I've perhaps made a change error. The scary thing is if you've got money and you've forgotten to post it. because you've got no way of really knowing- you've got to actually - so much happens you've really got to sit and think - 'what did I do this morning?' You've got to look at each individual amount and get them to jog your memory and you'll remember

certain things about that person and then you've got to think OK fine - I know I'm up \$13 too much - what could \$13 be . \$13 is probably minibar - who was s'posed to pay minibar and didn't? That's the only way of doing it.

As in the previous example, metonymic structures involving numbers were not only used to construct a convenient system of nomenclature, but they also assisted in actually performing work related tasks. In the following transcript the respondent is seen to make use of the chain of juxtaposed relationships 'merchandise-price-number' in order to produce a richly worked set of highly situated task oriented procedures.

- R:** Do you do much in your head or you use that a lot?
RS: Oh, well to tell you the truth I've got a lot better. Because on check-out when people are telling you, when I say "what did you have for minibar?" "Oh I had a packet of chips, two cans of coke and a beer" - see you haven't got time to get the calculator out you've got to go, \$3.00 + \$2.80 + \$1.40 so you can do it in your head.
- R:** So you don't enter each of those items into
RS: Oh no - you just enter minibar total amount - you've got to do it that way - it would take too long if you entered - I mean some of them drink like fishes so you could end up with 15 or 20 entries for the mornings minibar - it's not worth it. So you do get better - you don't even mean to - it just happens that way. And I find actually, if its quiet like this I work best under pressure - I mean I work fine now, but when things are all happening I find that you don't have time to think, your brain just does it - like when they say that you just know
- R:** But you are thinking because you've still got to prioritise that task
RS: Oh yes - but I mean - like with maths, if I started adding up those numbers I'd be sitting here thinking, thinking, thinking. Whereas, if he said to me "I just had two beers a drambuie and a packet of chips - you just kinda know what it is"...
 OK and now we'll try the balance again.

In this episode, number operations are performed on elements within the site specific context (beer drambuie chips) rather than merely on elements within an essentialised framework for mathematical knowledge (numbers). It is concluded that the mathematical knowledge deployed by the respondent in this case was produced entirely within the confines of the domain belonging to the specific task. Moreover, this production was facilitated by the metonym afforded by the following chain of juxtaposed elements sited within the workplace: merchandise-price-number. Put another way, number relations are seen to apply both to numbers and to an augmented set of salient signifiers as designated within the particular context of the workplace. Thus, in this instance metonymic relations are seen to have produced a highly contextual conception of numeration in which tasks of addition are regulated by their association with concrete elements within the domain of the workplace task in which the respondents operated.

Conclusion

This paper has reported on a study into the use of language and mathematics in a workplace context. Two concerns have permeated the work: the extent to which workplace tasks make use of site generated rather than "already made" knowledge; the power of language to produce and regulate of mathematical meanings appropriate to the requirements of a given task. The first of these was investigated both in review of the research literature and in an examination of data obtained for this study. Evidence was provided in support of non-essentialist epistemologies for mathematics, and this finding is consistent with the literature reviewed above. The second concern investigated the linguistic devices of metaphor and metonym and showed how these were used in the sites observed both to produce and regulate mathematical knowledge as respondents performed tasks required of them within a specific workplace setting. Starting from a constructionist epistemology for mathematical knowledge, questions arising from this

study are: What limits the range of meanings produced by the linguistic devices observed? How is this limitation expressed within the task competencies observed? It is suggested that answers to these questions will enable us to better understand the nature of effective relationships among the processes of task performance, knowledge assembly, and mathematics curriculum.

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