ENCOURAGING VISUAL IMAGERY IN CONCEPT CONSTRUCTION: OVERCOMING CONSTRAINTS

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Abstract

Some mathematics educators have pointed out that students seeing or using so-called concrete representations of concepts may not construct the expected conceptualisations. This paper presents research that specifically considered how problem solving with concrete materials could lead to the development of concepts. Students in Years 2 and 4 in primary school were engaged in spatial problem solving. A model developed by the author emphasises the interaction of concepts, imagery, heuristic thinking, and affective processes in problem solving as well as the importance of manipulation of materials and interaction with others. Students' attention to certain aspects of the problem, to other problem solvers, or to the materials was seen as particularly significant in the problem-solving process.

The Problem

During development of mathematical concepts and processes limitations can be placed on thinking if concrete materials are not used (Bishop, 1973; Bright, 1986; Dana, 1987; Fennema, 1972; van Hiele, 1986). Furthermore, using concrete materials, models, or images can aid reflective thinking (Gagatsis & Patronis, 1990; Goldenberg & Cuaco, 1992). On the other hand, concrete materials can limit conceptualisation especially when their use encourages concept images which restrict thinking (Hershkowitz, 1989; Wilson, 1986). When materials are used, links may not be made with symbols and words (Mason, 1992), and physical manipulations may not improve mental transformations (see Williford's (1972) training study).

Despite the number of training studies and studies on developmental levels of spatial thinking, some of which have made use of concrete materials, explanations of concept development and the role of concrete materials in learning have not been the focus of in-depth studies (see reviews by Clements, 1981; Clements & Battista, 1992; Eliot, 1987; Lean, 1984; Owens, 1990, 1993b).

A concept image is the visual reflection of a concept in the individual's mind and is usually based on the set of examples from which the concept was developed. Such prototypical images can be restricting. For example, students may draw prototypical examples but no other examples of a concept, or they may include as examples of the concept non-examples which have a feature of the prototype which was not relevant to the concept (for example, an internal line on an obtuse triangle for an altitude), or they may exclude special examples of a concept (for example, a square may not be regarded as a quadrilateral). Alternatively, a person can develop a prototypical image (for example, a scalene triangle having no horizontal sides) which is less visually biased because the critical attributes of the concept have been used to develop the image.

Furthermore, Cobb, Yackel, and Wood (1992) have warned that students cannot absorb concepts from merely seeing or even using "representations" of concepts in concrete materials (for example, base 10 blocks for place value). Students actively construct their own visual images and concepts, and, as Cobb (1991) has suggested, construction occurs with "metamorphic accommodation," that is to say, a change in mental schema which changes the whole schema, at which point a mental action resulting from using concrete materials changes into a cognitive modification or development. Encouraging students to make appropriate links between the physical objects, diagrammatic representations, conceptualisations and other mental representations becomes a significant challenge for the teacher.

The Study

The author has previously reported on an experimental quantitative study which provided evidence for students developing spatial thinking processes if they participated in a series of spatial problem-solving experiences in the classroom (Owens 1992, 1993a). The qualitative study referred to in the present paper complemented the quantitative study by exploring *how* children learned through spatial problem-solving experiences with concrete materials.

The qualitative study involved the same learning experiences as the quantitative study (an introductory activity, a series of ten problem-solving sessions based on activities with pentominoes, tangrams, pattern blocks, and matchstick designs, and worksheet "tests") and the same classroom organisations (some students working individually and others cooperatively). The study led to the development of a model of learning through problem solving and to an understanding of how concrete materials and visual imagery can influence each other in effective concept construction and spatio-mathematical thinking.

Methodology

The exploratory nature of the qualitative study required that a holistic approach be developed for the purpose of describing the factors involved in spatial learning. The study began by developing four categories--conceptual, heuristic, imagistic, and affective processing--to summarise the written comments made by adults on their thinking. These categories were also a way of summarising much of the problem-solving literature (Owens, 1993b). The dynamic interaction of these categories was recognised early in the study (Owens, 1990).

The four categories were used to analyse the thinking of 13 children who solved spatial problems alone, four groups of three children (half working cooperatively and half individually), and children in six classrooms undertaking the same problems in either cooperative groups or individually. As the study progressed the categories were refined.

Over 120 problem-solving sessions were coded and more than six hours of tape were coded by second coders. Frequencies and crosstabulations of occurrences of subcategories assisted in bringing to attention common patterns and relationships from which the description of learning was developed. Samples of another six classrooms were used to check the story-line for different contexts and to provide further examples of greater diversity within the subcategories.

The analysis was based on the children's manipulation of concrete materials, their facial expressions, their spontaneous comments during problem solving, and their retrospective comments (Owens, 1990).

Results

The resulting model, represented in Figure 1, summarises the continuous interaction between the person's cognitive processing and the context of the learning experience. The term *responsiveness* is used to suggest that students respond to the task, to the materials, and to the other components of the environment by becoming actively engaged in the problem-solving activity.

The Role of Concrete Materials

The concrete materials used in the study were not used as "representatives" of visual images and spatial concepts, but as a means to encourage problem solving that would lead inherently to the development of cognitive processing and the use of different kinds of imagery. Concrete materials are not intended to develop images in any "picture" form, but rather the kinaesthetic, visual, auditory, and affective perceptions associated with the materials and with social interactions are considered as significant inputs which influence how and what individuals think. The problem solving and the deliberate manipulations of the materials encourages the making, using, changing, and storing of images, concepts, understandings, and schemas. In this way restrictive concept images can be avoided.

Concrete materials and imagery. The continuous manipulation of materials meant that students were able to see where shapes could be added or taken away and this experience encouraged their visualising of results before trying the manipulations. The making of shapes, the comparing of angles, and the finding of shapes in designs seemed to improve students' visualising. Students would flip pieces more frequently, visualise where pieces would fit more often, make more difficult pattern-block enlargements or pentomino shapes, disembed shapes and parts from more complex shapes, and use more analytical imagery. The students were required by their experiences first to perceive features of shapes such as angles and then to consider what might happen if systematic changes to shapes or configurations were made. Students were further encouraged to use both their short-term and long-term visual memories in order to achieve greater problem-solving efficiency.

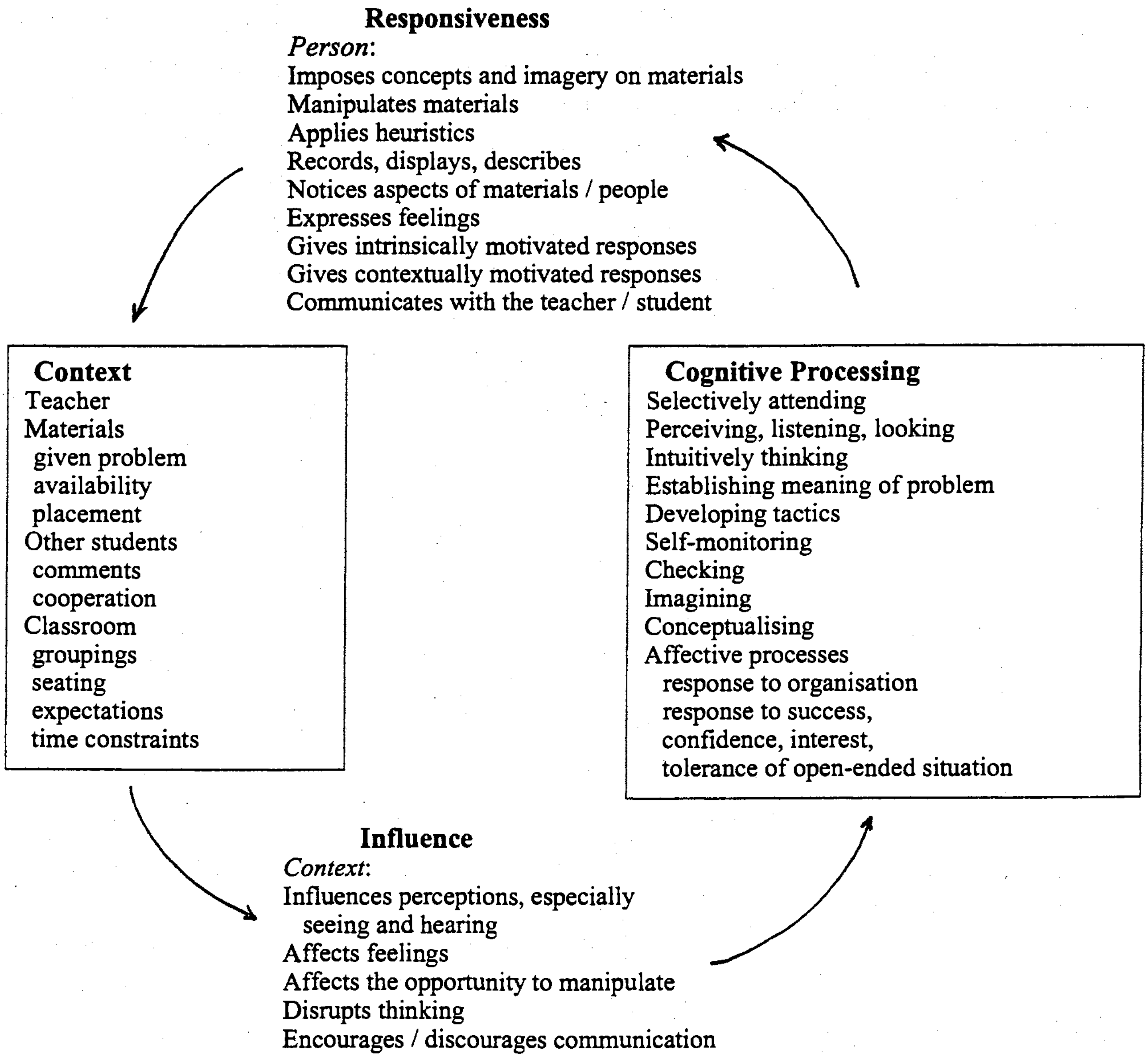

Figure 1. Aspects of learning through problem-solving.

From frequency counts of the use of the manipulatives it was found that there was a high frequency of deliberate manipulation (occurring during 57% to 86% of incidents involving manipulatives for each of the ten activities) and a reasonable frequency of incidents in which students carefully observed or physically checked the pieces. The materials were an integral part of the problem-solving exercise and a means by which students could develop problem-solving strategies. The manipulation was deliberate, not in the sense that a clear image with parts had been used but that ideas, together with the associated manipulation of images, frequently assisted the problem solving. For example, one student, Sally, commented, "In my mind, I pictured my hand moving the pieces around the shape." She had said earlier that she was using "ideas in her mind" and she did not give the imagery names as she might have done if she had evoked whole shape images. This clearly matched my observation of her efforts to obtain new pentomino shapes--she was reasonably systematic in moving the pieces around partly-made shapes searching for new pentominoes. This is an example of imagery involving *action* (Presmeg, 1986). Students knew that pieces were to be joined or moved in a particular way even though they did not know the entire procedure to make a required shape. Action imagery occurred more frequently once students began to develop and implement tactics for solving problems. This imagery was supported by concepts relating to the effects of operations or transformations on pieces.

Pattern imagery (Presmeg, 1986) was evident in several descriptions used by the students, particularly in recalling previous experiences. For example, Kathy, in Year 4, carefully counted as she made a rectangular array of eight squares with matches, and during the video playback stated, "Like the picture was in my brain but it didn't work." In fact, she had interpreted the problem as meaning that the squares had to be in a square or a rectangle. Many students used images of arrays and grids during the activities. Sally and Sam, in Year 2, both described a tessellation of triangles as one up and one down. Jodie, in Year 2, was asked why she had been able to make the triangle with pattern blocks so quickly, and she said that she had remembered that there was a similar task before. Victor, in Year 2, explained how he knew that three triangles made up the trapezium in the retention test, by referring to his making of the shape earlier. Jodie and other students called the pentomino cross "a box," relating it back to the net for an open-box given in the pretest practice item. Pattern imagery was used by Peter, in Year 4, when he was having difficulty making the hexagon outline with matches. He commented, "I know, I'll make it like the other day," and he proceeds to add one triangle next to the other as he had done with the pattern-blocks and designs-with-matches problems.

In the tangram problem, students remembered configurations such as the arrangement of the square and two triangles for making the large triangle. Students made the enlargement of the second rhombus by positioning the pieces to repeat the pattern of the enlargement of the other rhombus rather than trying other possibilities. For example,

1.01 Sam takes two rhombi and touches points symmetrically, but misses seeing the diamond and joins the sides.
1.02 He listens to the teacher talking to his friend and then concentrates on his own work and quickly puts pieces together to make a diamond. He is happy . . . .
1.03 He then describes to his friend how to do it "You put this here and this here" (touching the points of the rhombi). He goes on to describe how to make the triangle, "Up and down, up and down" to help his friend make her triangle. He is pleased with himself. . . .
1.04 He joins trapezia to make a long hexagon. The teacher asks him what is different about his hexagon and the yellow one. "It's bigger."
1.05 The teacher runs her fingers along the sides and asks about them. He says "It looks a bit like a square." . . . .he says, "It is unstraight." . . .
1.06 The teacher asks him if he can make a brown shape. He says he has made it, pointing to the blue one but she says, "No, a skinny one." So he collects the narrow brown rhombi

> and quickly follows the same pattern to make it. "I'm the best in the world," he laughs (his dimples showing).

Sam's observations of the materials, his manipulations, and his interactions and descriptions strengthen the imagery that supported his concepts, his problem solving and his positive feelings.

Developments in Conceptualisation

Sam also noted when a shape had some features like another shape, although he was unable to explain the similarities well (para. 1.5). Instead he used basic-level concepts of a *square* and *triangle* to develop his recognition and conceptualisation of other shapes. His language, on this occasion, suggested he had evoked holistic images which he *dynamically* changed into other shapes.

Observing materials also assisted conceptualisation. Tess, in Year 4, made a right angle from two tangram pieces. As she observed what she was doing with the pieces she realised she could then use this right angle as an angle of the square (Owens, 1993b). In this incident, the metamorphic change occurred when Tess realised that a right angle could be filled by two angles.

An Interpretation of Results

Rather than just imitating the teacher, students used basic concepts to solve the problems themselves. For example, they were able to make new shapes because they had the basic concept that *different configurations and drawings can physically represent a particular shape,* and they used the basic conceptual units of *square* and *triangle* from which to develop new shapes. Furthermore, as all the students participated in the same activities; discussions involving concepts were relevant to all of them. For example, the students were able to discuss the area of the large triangle as the two-dimensional space taken up by the shapes, and to estimate that it was equal to four units (the small triangles) because they had all been actively involved in trying to make the shape. In a similar way, they were able to discover other apsects of the shapes; for example, they were able to disembed angles from a shape and see that the angles of one shape could be compared with angles or the join of angles of the other shapes.

> When we notice a new aspect of an object, we do not suddenly see a property, part, or element of it which we had previously failed to register . . . but rather become aware that a new *kind* of description might be made of the object as a whole. We perceive that it can be seen as another sort of object altogether. (Mulhall, 1990, p. 130)

Images are frequently associated with the recall of the perception and manipulation of objects, generally in some order (a protocol of action). Each of the schemata associated with a term are related through the imagery (Dörfler, 1991; Johnson, 1987; Lakoff, 1987). Images may be nothing more than static images and, if this is the case, then image schemata may be restricted but imagery can be used in reasoning by linking schemata. For instance, dynamically modifying an image of a rectangle to form an image of another rectangle or to form a five-sided figure allows the rectangles to fit into different schemata for *rectangle* or into schemata for *polygon.*

It is within the context of a spatio-mathematical task, the initial shapes, and the evolving configurations that concepts become meaningful. Intention, expectation, and attention are central to responsiveness and the establishment of meaning (Johnson, 1987; Owens & Clements, in press). So, for example, Tess expected to be able to make a square, afterall the teacher had suggested they could do this. Tess's attention was caught by the right angle she had made from two pieces and she knew that this was the clue to her meeting her intention of making a square.

Students develop meaning through categorising but categories are not necessarily delineated by a predetermined set of conditions for belonging but rather the categories are indicated by members that are prototypical of inclusion (Lakoff, 1987). The categories and meaning are evolving; they are dependent on human experience of involvement and perception, and they are imagined as a result of these experiences. They are structured by metaphor, that is to say by links being made with existing identities in a student's schemata. The metaphoric links are made through dynamic imagery, action imagery and other forms of imagery together with operational concepts (for example, an angle can be made by joining two smaller angles).

> Conceptual systems are grounded in two ways--in basic-level and image-schematic understanding--and are extended imaginatively by category formation and by metaphorical and metonymic projections. . . . *Understanding is an event* . . . by which we have a *shared, relatively intelligible world.* (Johnson, 1987, p. 209)

Often the person who evokes an image does not necessarily appreciate its richness. Using external representation to communicate its meaning to another person further develops images and concepts, often as a result of the other person's responsiveness or use of language to interpret what they are noticing. It is by the use of language in an activity that meaning is structured.

It is through actively engaging in problem-solving situations that students use perceptions, ideas, and images to reflect on the properties of shapes and existing abstractions. The materials were important for the students to be able to make and create new shapes and, in so doing, they began to think mathematically and to construct meaning.

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