Preservice teachers' construction of decimal numbers

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RUNNING HEAD: Construction of decimal numbers

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Since 1990 nine cohorts of preservice teachers in Australia and the USA have been given the task of ordering five decimals and asked to explain their reasoning. Data were collected via written explanations or interviews. The percentages of students in all cohorts with incorrect ordering of the decimals were disturbing but were similar to those from other studies with preservice and practising primary teachers. Written explanations for incorrect response were sorted into 11 categories. Patterns of responses and similarities with the results of the previous studies among upper primary/lower secondary students were found. In particular, evidence was found for usage of three rules found in previous research. The strategies used by students who were successful at the task were also examined for insights into students' constructions of decimals.

School Students' and Teachers' Knowledge of Decimals

Results of the fourth NAEP assessment of mathematics in the USA revealed that middle school and secondary school students had difficulty on items involving ordering of decimals (Lindquist, 1989). Owens and Super (1992) claim that research on learning of decimal fractions indicates that there is a problem with children's conceptual knowledge about decimals. Many children appear to have instrumental understanding which leads to application of 'rules without reason' rather than relational understanding which involves interrelationships between concepts and understanding of why a rule does or does not work (Skemp, 1978). Consequently, children exhibit many misconceptions about decimals as indicated in the NAEP study.

Research involving the ordering of decimal numbers with upper primary and lower secondary school students in France (Sackur-Grisvard & Léonard, 1985), Canada, (Vance, 1986), and Israel (Nesher & Peled, 1986) investigated the existence and frequency of use of the following three implicit rules derived by Sackur-Grisvard & Léonard (1985, p.161) for such a task.

Rule 1. The smaller number is the one whose decimal portion is the smaller whole number. [Resnick, Nesher, Léonard, Magone, Omanson, & Peled (1989) labelled this the *whole number rule* because it relies on a knowledge of whole numbers]

Rule 2. The smaller number is the one that has more digits in its decimal portion. [Resnick, et al. (1989) labelled this the *fraction rule* because it relies on a knowledge of fractions]

Rule 3. The smaller decimal has a zero immediately after the decimal point; otherwise, apply Rule 1. [Resnick, et al. (1989) labelled this the *zero rule*]

In the French study, 89% of the mistakes made on this task correspond to the use of one or other of the three rules. Rule 1 was used most frequently, Rule 2 was used less frequently, and Rule 3 appeared later than either Rule 1 or 2 and gradually tended to replace Rule 1 (Sackur-Grisvard & Léonard, 1985). In the Canadian study 48% of sixth grade students and 15% of seventh grade students used Rule 1 while 8% of sixth grade students and 19% of seventh grade students used Rule 2. With children in Israel, the researchers were able to account for both errors and correct responses of sixth, seventh, and ninth grade students in terms of their consistent use of these rules.

Investigations by Lesh and Schultz (1983) and Post, Behr, Lesh, and Wachsmuth (1985) found that many of the misconceptions about rational number concepts (including decimals) held by children could also be found among teachers. Post, Harel, Behr, and Lesh (1991) developed knowledge profiles of 218 elementary teachers' understanding in the area of rational numbers and concluded that many elementary teachers did not know sufficient mathematics, and that only a minority of those who could do the mathematics, could explain their solutions in a pedagogically acceptable manner. This led the author of this paper to examine undergraduate pre-service teachers' knowledge in the same area.

Undergraduates' Knowledge of Decimals

Lester (1984) found that 50% of preservice elementary teachers [N>600] were unable to reach the 75% criterion on a test of arithmetic competency and that most often this failure was traced to the questions which involved fractions and/or decimals.

On a test of decimal concept knowledge, Thipkong and Davis (1991) classified in the low category 45% of the 65 preservice elementary teachers investigated. They stressed the importance of identifying likely weaknesses in preservice teachers' content knowledge so that steps can be taken to prevent or correct misconceptions before they adversely influence students' learning.

Grossman (1983) reported that less than 30% of the 7,100 entering freshmen students at the City University of New York selected the correct answer on a question which asked them to select the "smallest" of a given set of five decimals. The most frequent incorrect answer was the "longest" decimal in the set and this was selected more frequently than the correct answer (Grossman, 1983, p. 32). Furthermore, this item was the most difficult on the test.

This paper reports on preservice teachers' attempts at sequencing decimals and the thinking behind their responses. This information is vital for teacher educators who wish to break into what manifests itself as a vicious circle for many students.

The Study

In 1990, beginning preservice primary teachers undertaking a subject - Mathematics for Primary Teachers (PD1210) - at a regional university in Queensland were asked the following question:

Place in order from smallest to largest:

0.606   0.0666   0.6   0.66   0.060

Nine of the 29 students in the class were able to complete the task successfully, while 10 different response patterns were obtained from the remaining 20 students.

As a result the author decided to investigate: (a) the occurrence of these and other responses among preservice primary and middle school teachers, and (b) the reasoning behind the different responses, both correct and incorrect, given by these students.

Method

In order to investigate (a), the same question was asked in 1991 and 1992 of similar classes of beginning preservice teachers taking the same subject - Mathematics for Primary Teachers (PD1210) - at the same university. It was also asked of three other classes of preservice teachers at the same institution. The first two were second year students (Semester 1, 1993) - one class was studying a first level methods course - Mathematics Education for the Young Child (PD2110); the other class was studying a first level methods course - Primary Mathematics Education (PD2210). The third was a class of fourth year students (Semester 1, 1992) who were studying an Advanced Language Arts and Mathematics Education course (ED4303). Comparative data were collected (1992-93) from one class of freshmen mathematics students taking Mathematics for Elementary Teachers (MAT151), and two classes of preservice middle school mathematics teachers (MAT202 and MAT309) at a university in the midwest of the United States.

In order to investigate (b), most of the students were either interviewed after completion of the task, or asked to supply a written explanation when completing the task. Transcripts from the interviews together with the written explanations were analysed to ascertain the thinking behind both the incorrect and correct responses.

Results and Discussion

The frequency of correct and incorrect responses to the question are shown in Table 1.

Table 1

Numbers of Correct and Incorrect Responses for Preservice Teachers

	AUSTRALIA						UNITED STATES OF AMERICA		
	1st Year			2nd Year		4th Year	Freshmen	Middle School	
Cohort	PD1210 1990 (n=29)	PD1210 1991 (n=64)	PD1210 1992 (n=59)	PD2110 1993 (n=16)	PD2210 1993 (n=94)	ED4303 1992 (n=64)	MAT151 1993 (n=322)	MAT202 1992 (n=38)	MAT309 1992 (n=18)
Incorrect response	19 (66)[a]	33 (52)	26 (44)	10 (62)	38(40)	25 (39)	181 (56)	7 (18)	4 (22)
Correct response	10 (34)	31 (48)	33 (56)	6 (38)	56(60)	39 (61)	141 (44)	31 (82)	14 (78)

[a] the number in parentheses is the percentage of respondents.

The percentage of students with incorrect responses in all cohorts is a matter of concern since these people are future teachers of this and related topics to primary and middle school students.

The performance of the beginning primary students (PD1210) in Australia was similar to that of the freshmen students (MAT151) in the United States, while that of the middle school preservice teachers in the USA (MAT202 and MAT309) was much better than the performance of the second year early childhood (PD2110) and primary mathematics (PD2210) education students in Australia. Students in the primary education program (PD2210) were more successful on this task than their counterparts in the early childhood program (PD2110). The high percentages of students with incorrect responses were particularly disturbing in the subject - Mathematics for Primary Teachers (PD1210) since this course, designed mainly for older students who have not completed 11th and 12th grade at high school and for students who have done poorly in mathematics in 11th and 12th grade, covers the subsets of the real numbers. However, a large number of students clearly have misconceptions about decimals which are not being identified and rectified.

Thirty-nine percent of the Australian students taking ED4303, after three years of a teacher education program, still had problems with ordering decimals. This is a situation which has been addressed in the subject ED4303 since its discovery. Since Australian students use metric measures in most situations they encounter, it is surprising that the Australian data are so similar to that from the United States which is not a metric country.

An analysis of the qualitative data obtained from interviews or written explanations was undertaken to identify students' reasoning behind their ordering of the decimals.

Interviews and Written Explanations - Correct Responses

Explanations for the correct ordering of the decimals were obtained from students in seven of the nine cohorts tested [n=297]. These explanations were then categorised by the author into 11 different strategies for performing the ordering. The two most frequently occurring strategies were based on students' understanding of place value as it relates to decimal numeration. In fact, the most frequently used strategy [88/297] involved an initial sorting of the numbers into two groups based on the digit immediately to the right of the decimal point. This was followed by an ordering within these groups according to the place value of each of the digits or, in some cases, by an examination of the common fractions which represented the decimal numbers. The second most frequently used strategy [75/297] simply arranged the numbers in order by identifying the largest and working down to the smallest or vice versa. It would appear that both of these strategies demonstrate that these students have constructed an accurate conceptual understanding of the decimal numbers.

The third most frequent strategy [37/297] seems to demonstrate a procedural rather than a conceptual knowledge of the ordering of decimals. It occurred more often in the explanations given by the American students which could indicate that it is one of the strategies taught in that country. It involved placing zeros on the right of each number to ensure the same number of digits after the decimal point, and then considering each decimal part as a whole number. A strategy similar to this saw each number multiplied by the same power of ten to convert each decimal to a whole number and then the ordering done [8/297]. Another strategy similar to this involved the multiplication of each decimal by a power of ten which did not result in a whole number in all cases, but still allowed a correct basis for comparison [11/297].

Rounding to the nearest hundredth or thousandth [22 /297] and converting to percentages [8/297] had similar frequencies to converting to fractions with common denominators [16/297] and converting to fractions with different denominators [6/297]. Reasons which could not be clearly understood [10/297] or no reason given [16/297] completed the classification of the strategies which resulted in correct ordering.

While there are a small number of dominant successful strategies which students have learned, there is evidence also of a variety of constructions of decimal ordering on the part of some of the successful students. Teachers and researchers need to be aware that students do construct their own strategies in the process of learning about decimals.

Interviews and Written Explanations Incorrect Responses

The responses of the students who incorrectly ordered these decimals [344 out of 704] were sorted according to which number was chosen as the largest. Seventy-four percent [256] of these students selected 0.6 as the largest decimal; 17% [57] identified 0.66 as the largest decimal; 8% [28] chose 0.606 as the largest decimal; while the remaining 0.9% [3] selected 0.0666 as the largest.

Evidence of usage of the three rules mentioned earlier in this paper was sought when the transcripts of the interviews and the written explanations were analysed. Of the 256 who chose 0.6 as largest, 198 (77%) gave an explanation for their ordering which exhibited reasoning consistent with the use of Rule 2, a variation of Rule 2, or a combination of Rules 2 and 3. The following excerpts illustrate the type of responses given by such people (the upper case letter is the person's initial):

M. "The main thing I always thought was the further you go back, like the more decimal places its got, the smaller it gets; not closer to the whole number." [Rule 2]

R. "I tried to make them into a whole number and what I've done is move the decimal place. 0.6 only moving one decimal place I assumed that would be the largest, 0.66 needed two places so ... I just continued like that and however many it needed to make it a whole number that meant it was the smallest." [variation of Rule 2]

P. "If there is a 0 in front of it and its longer, more numbers in it, then it is smaller. When you have a fraction with the big number on the bottom it's smaller, it's the same sort of thing." [Rule 2 and Rule 3]

D. "Place value is opposite in relation to counting in ones, tens, and hundreds etc... Therefore the bigger the number looks the smaller it really is in place value." [variation of Rule 2]

J. "Firstly, I changed all of the decimal fractions into common fractions. Then I looked to find the fractions with the denominator of 10,000 as this will be the smallest group of fractions." [Rule 2]

There is evidence that the use of these rules has persisted from school to university in the thinking of large numbers of students.

Of the 57 respondents who chose 0.66 as largest, 13 (23%) had the correct order for the numbers containing tenths but their ordering was reversed for the numbers with zero tenths. Some explanations from people in this group revealed strategies which should have led to a correct ordering if used consistently. For example:

Y. "Make all numbers with 4 numbers after the decimal point e.g.,

0.606	---->	0.6060	0.0666	---->	0.0666
0.606	---->	0.6060	0.6	---->	0.6000
0.66	---->	0.6600	0.06	---->	0.0600 ."

K. "0.66 is larger than 0.6 because if you change the first example to one decimal place, then it changes to 0.7 etc..."

Of the 28 people who indicated 0.606 as the largest number 26 recognised that numbers with tenths were larger than numbers without tenths but had difficulty ordering within these subsets. Some of the explanations given suggested that the incorrect response may have been an aberration because the strategy should have led to a correct ordering.

Y. "If you are able to subtract the 2nd to last largest from the largest you will get a positive number. And you keep doing it on down the line."

C. "I went by the decimals and like the 0 before the 6, that's how I picked out 0.060; 0.0666 had a higher value because of the sixes after that; 0.6 is higher because the 6 is just straight after the point; the 0.66 could have higher numbers after it as well. I don't know how I got 0.6 and 0.606 wrong seriously because going by that I should have gotten that right."

The three people who selected 0.0666 as largest appeared to have a pattern of ordering which put the largest number as the one with most digits after the decimal point.

Lesh and Schultz (1983) and Post et al. (1985) found that many children's misconceptions about rational number concepts were also evident among teachers. These data supplement their findings and indicate that the misconceptions found in teachers are most likely present when they are undergraduates. Furthermore, there is a pressing need for their identification and remediation during their preservice teacher education courses.

Implications for Instruction and Research

Confusion in students' understanding of decimals and fractions at the upper primary, lower secondary, and preservice levels requires instruction which focuses on conceptual development which links concrete or pictorial representations with the oral names and the written symbols for decimals. Modelling of decimals using concrete materials or pictorial representations and displaying them on a calculator should enable students to make equivalence connections between 7 tenths, 70 hundredths, and 700 thousandths either in fraction or decimal form, for example, $0.7 = 0.70 = 0.700$, and also prevent students from confusing the number of places after the decimal point with the size of the decimal.

By allowing students to manipulate concrete and pictorial models, they have the opportunity to construct meaningful, personal mental representations of decimals. Researchers can ask students to draw pictures which demonstrate their understanding of decimals and can thus ascertain whether their knowledge is incorrect or incomplete. Resnick (1987) and VanLehn (1986) have shown that "errorful rules" such as those discussed in this paper are often "intelligent constructions based on what is more often incomplete than incorrect knowledge" (Resnick et al., 1989, p. 26).

The evidence from this investigation signifies the importance at all levels of mathematics education of observing students' written responses and listening to their explanations of how they are thinking about a particular concept. Both teachers and researchers need to regard responses which are based on "errorful rules" (Resnick et al., 1989, p. 26), such as those discussed in this paper, as a natural outcome of students' efforts to construct meaning for concepts to which they are exposed. Indeed, they can serve as valuable diagnostic tools for detecting students' understanding of a mathematics concept.

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