Profiles of Some Non-Routine Problem Solving Episodes

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This paper reports the preliminary results of work undertaken for a thesis project on control-related aspects of mathematical problem solving. The study uses a non-routine mathematical problem as the focus for the development of problem solving profiles which summarise subjects' knowledge, beliefs and attitudes towards mathematics and problem solving, and uses a protocol parsing technique developed by Schoenfeld to illustrate control behaviour observed during a thirty minute think aloud problem solving session. Discussion focuses on the protocols of three sets of subjects and their achievement on the non-routine problem. The reliability of the parsing technique, and the appropriateness of a think aloud methodology are also considered.

BACKGROUND

Problem solving in a general context has been investigated in the cognitive psychology, behavioural psychology and artificial intelligence domains. Research has concentrated on activity like reading information, recognising and noting patterns, performing calculations, and constructing visual representations of aspects of a problem. Attempts at describing subjects' knowledge and beliefs about cognition (KB), or their regulation and control of cognitive actions (RC), have generally come from developmental psychology and mathematical problem solving perspectives (see Flavell (1979); Schoenfeld (1985); and the review by Garofalo & Lester (1985)).

Studies that have attempted to quantify aspects of KB and RC in terms of observable behaviours have resulted in useful, although sometimes disparate observations and discussions regarding the role of these components in mathematical problem solving. Much of the research to date has involved the use of routine problems that require little control behaviour for solution. Other studies have investigated the use of various cognitive structures including schema, domain specific and procedural knowledge but have paid little attention to control behaviours (see res[illegible]orted in Owen & Sweller (1989)). Schoenfeld (1985) is well known for his attempts to document control behaviour evident in mathematical problem solving episodes. He devised a protocol parsing technique and a method for graphically representing the various changes in behaviour that result from control decisions made during a problem solving episode. The results of such protocol parsings have already provided insights into the differences in control behaviour exhibited by novice and expert problem solvers.

The present study required the solution of a non-routine problem and the focus was on the global control and knowledge / belief resources that a problem solver brought to bear when he / she *did not* immediately see a solution path. The few examples offered by Schoenfeld provide enough evidence to indicate that consideration of problem solving ability based purely on cognitive grounds (for example, Owen and Swellers' comments regarding schemas), or only in the domain of routine problems, cannot lead to a satisfactory explanation of *all* types of problem solving behaviour. This study provided the opportunity to attempt to replicate some of Schoenfeld's results. Little research has been done on regulation and control using his parsing technique and it is unclear whether reliability in parsing can be ensured, or how applicable the technique is to a range of problem solving situations.

METHODOLOGY

The aim of this study was to provide "thick" descriptions of problem solving episodes that included all cognitive actions , as well as the control behaviours that could be observed and inferred. Results were in the form of detailed descriptions of problem solving episodes incorporating descriptions of both KB and RC. Cognitive elements like the use of schema or domain specific knowledge were also factored into the analysis where appropriate. These episodes were analysed in a framework of problem solver type (novice and expert), and achievement on the problem presented.

The problem used in the study involved investigating and generalising patterns in sums of consecutive positive integers and was based on the "Staircase Numbers" problem in Stacey and Groves (1985, pp 125-130). It could be tackled through a variety of approaches and did not require specialised knowledge for solution. Expert subjects attempted the problem on their own. Because a think aloud methodology was central to this study, novice subjects worked in pairs while they solved the problem. Schoenfeld found that this approach encouraged the subjects to voice their thoughts and so made the protocol parsing process much easier.

Conceptual knowledge and attitudinal data for each subject was collected through free and structured response surveys. Regulation and control information was extracted from verbatim transcripts of the problem solving sessions using a parsing technique devised by Schoenfeld. This method breaks up the session into six identifiable stages: read; analyse; explore; plan; implement; and verify. The process also allows local assessments (that may lead to transitions to different stages) to be identified. A graphical representation of the parsed transcript was used to summarise control behaviour and speculate on how certain decisions affected progress through the problem solving episodes. Although subjective, this parsing process highlights control activity through *changes* in observable behaviour, and also by indicating where strategic control decisions based on new information *could* have redirected and focussed actions during the problem solving session.

RESULTS AND DISCUSSION

Subject LI: First Year "Expert"

This profile describes the efforts of expert problem solver LI. He was a representative for Australia in an International Mathematical Olympiad and won first prize in several national Mathematics competitions while at secondary school. LI is very confident about his abilities in Mathematics. He has little difficulty with Maths and believes he could be doing more complicated work. LI produced an almost complete solution to the "Staircase Numbers" problem. In a little over sixteen minutes he:

- identified and justified the case for all odd numbers being staircase numbers;
- used a number theory parity argument to identify and justify why powers of 2 are not staircase numbers;
- developed and used a recipe for generating stairs for any staircase number; however algebraic errors meant that the algorithm did not work. (Some of his calculations were unclear , and the subject managed to get a (correct) answer that still did not agree with his own pen and paper calculations!).

A graphical representation of the parsed protocol for LI's problem solving session is given in Figure 1. It was difficult to identify overt control behaviour from this protocol. This is partially explained by the frequent 10 to 20 second periods of silence recorded during the session. These periods almost always preceded a transition from one type of behaviour to another, so it is probably the case that conscious control decisions were made, but not verbalised. Another reason relates to LI's reliance on analysis to solve the problem. From the moment he stopped reading, LI decided on an algebraic formulation of the problem that he seemed confident would result in success. It was difficult to categorise this behaviour as anything but analysis because he did not verbalise any decisions that would indicate that (say) a planning / implementation episode was in progress.

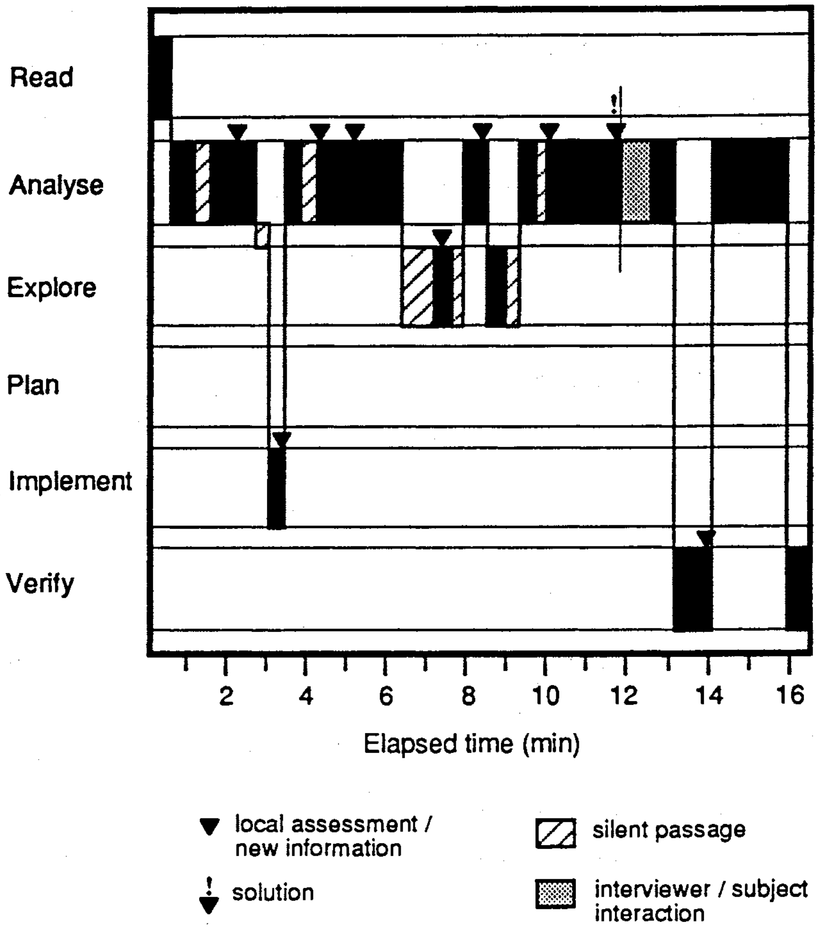

Figure 1. Graphical representation of problem solving behaviour of expert "LI".

There was one instance where LI verbally indicated a decision to "try the first few examples". This is shown in Figure 1 by a brief silent transition stage followed by an implementation episode (3:05 min to 3:34 min). At the end of this episode LI used the new information gained to move back into analysis. Later in this episode he lapsed into a short period of unfocussed exploration. However he soon returned to analysis, integrating the information from this exploration into his original conceptualisation of the problem. At the 11:45 minute mark LI gave his solution and indicated that the problem solving session was concluded. He did not appear interested in pursuing the investigation any further and so the interviewer prompted him to clearly state his solution and then verify it with an example. It was during this time that LI discovered several algebraic slips. These led him into another analysis episode where he demonstrated his algorithm for calculating the stairs for the number 23. Even then his work didn't quite "add up", but it was decided that there would be little to be gained from continuing the session further as he had essentially solved the problem bar a few minor algebraic slips.

LI's success did not rely on well controlled problem solving behaviour, although he was clearly aware of his actions and worked to an (internal) plan most of the time. The reason is much more likely to be due to supreme confidence in his original conceptualisation of the problem. LI had seen many similar problems through his experience with Mathematical Olympiads. While he was unable to simply apply some routine solution procedure, he did have enough knowledge (via a sort of "super schema") to know that a "parity argument" would work. Thus the majority of time in his session was involved in "variable-driven" analysis which led to a natural evolution of the solution.

Subjects NW and JM: First Year "Novices"

LI's performance can be contrasted with that of two female first year Science students, NW and JM.

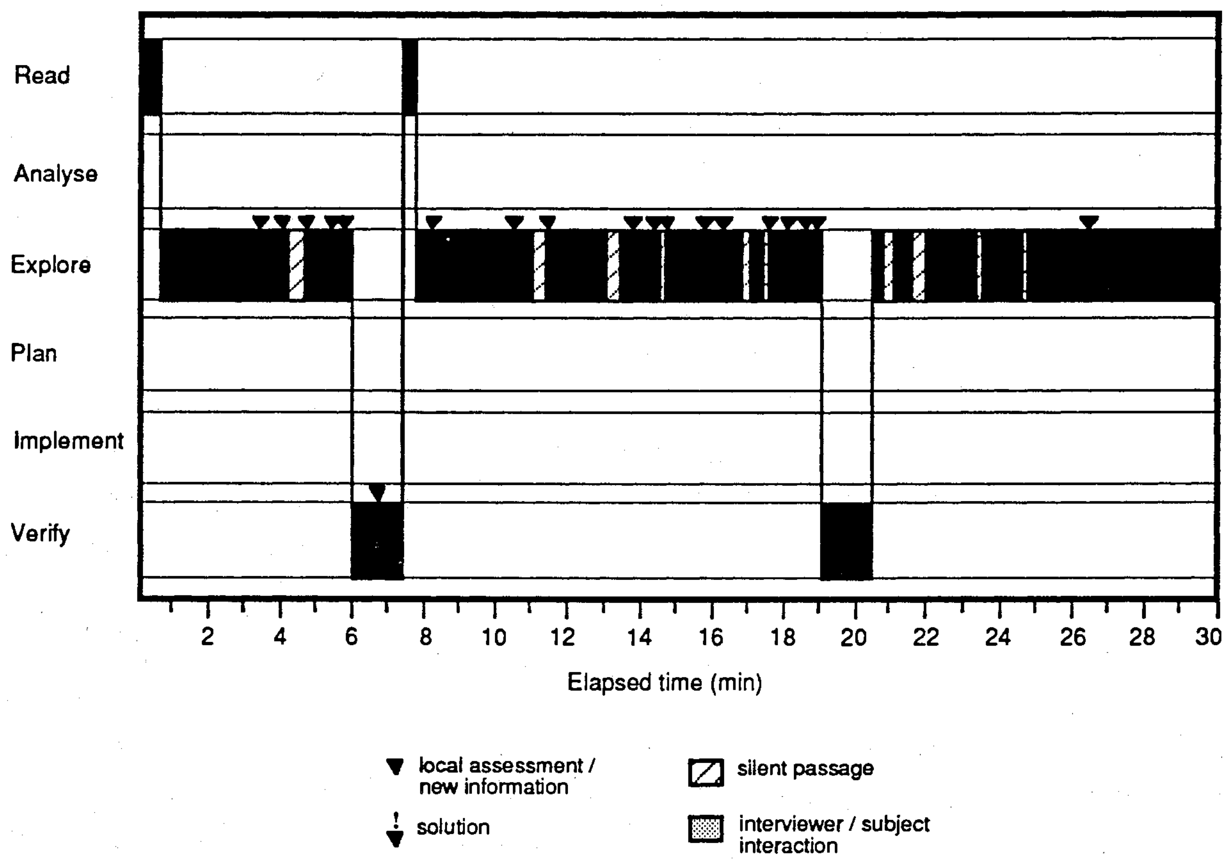

Figure 2. Graphical representation of problem solving behaviour of novices "NW" and "JM".

In terms of achievement, this novice team was able to identify and speculate that powers of 2 were not staircase numbers and write a recipe for finding a staircase representation for odd numbers. They had little success devising a similar algorithm for even staircase numbers. Their protocol (see Figure 2) makes an interesting contrast because it shows how the subjects were able to collect and summarise useful information relating to the problem but could not integrate this information through any form of analysis. The majority of their time was involved with pattern spotting activity and they made few attempts to move out of this mode of behaviour in order to get a "bigger picture" view of the situation. Their limited success in the problem can be attributed to the efficient summary of relevant information collected along the way. NW and JM rarely acted on the numerous local assessments made during their exploration of the problem.

Subject LR: Higher Degree "Expert"

A PhD topology student, LR, in comparison, frequently acted on assessments of the status of his work. His protocol (see Figure 3) indicates a great deal of control behaviour. While he was prone to

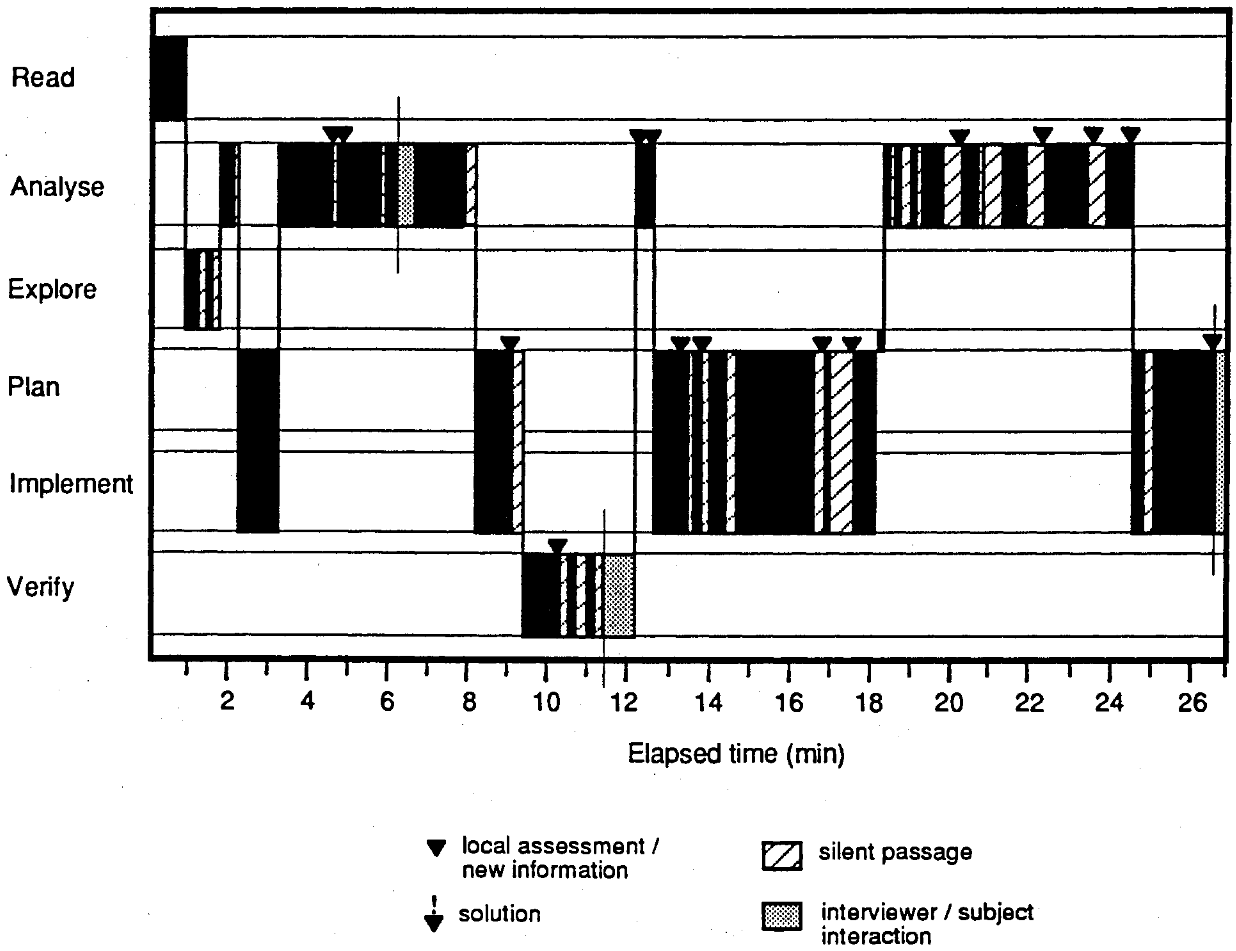

Figure 3. Graphical representation of problem solving behaviour of expert "LR".

periods of silence, unlike LI, he always indicated when he was moving into a different mode of problem solving behaviour. This made it easy to decide when he was involved in analysis rather than planning / implementation. In terms of achievement, LR was the least successful of the three subjects. He justified an algorithm for writing all odd numbers as staircase numbers and made preliminary assertions about staircase numbers with three columns needing to be divisible by three. Despite his control, LR never thought to look for a pattern in the first few examples of staircase numbers. Thus he was unable to determine that powers of two were not staircase numbers. Much of the work in his session involved trying to develop divisibility rules for staircase numbers based on the number of columns under investigation.

Some General Comments on Schoenfeld's Parsing Technique

Schoenfeld (1985) claims that it is easy to obtain reliable parsings of problem solving episodes using his technique. While his description of the process is by no means complete, and is illustrated through only a handful of examples, fundamental ambiguity remains regarding the meaning of some of the parsing categories. This is most evident when trying to determine whether behaviour is of the plan / implement type, or simply analysis. For this study, goal driven behaviour based on an initial formulation of the problem was considered, by default, to be analysis. The plan / implement categories were only used when it was clear from the transcript that the subject had made a short term plan, and was focussing all activity towards implementation. However this approach merely avoids the ambiguity, it does not remove it. The solution may lie in the idea that analysis can be made to appear like a string of successive planning / implementation episodes when viewed in "microscopic" detail. The ability to make very focussed plans, implement them, and then use the results in further goal-driven activity, is a fair working definition of analysis. In any case, these categorisations are fairly arbitrary as they do not affect the interpretation of results to any extent. For the protocols of this study the plan / implement and analysis categories are interchangeable. The same *degree* of control behaviour can be postulated using either categorisation.

Suitability of the Think Aloud Methodology

The debate on the validity of think aloud procedures is extensively covered in papers by Ericsson & Simon (1980) and Genest & Turk (1981). Of all the procedures for collecting cognitive data, *concurrent verbalisation* with *no interviewer intervention* is least prone to the effects of the study environment, and to the incompleteness and inconsistency of some verbal data. The subjectivity associated with think aloud procedures relates more to the researcher's interpretation of verbalisations than to the data itself. In this study, completeness was improved by having novice problem solvers work in pairs. This approach produced more verbal data than was the case for all the single-subject expert protocols.

CONCLUSION

There is little in the way of suggested methodologies for describing control behaviour during problem solving episodes. Schoenfeld's protocol parsing technique is one of the few available and deserves more investigation. There are clearly problems of interpretation in several of his original behaviour categorisations. These do not destroy the overall utility of the technique however because patterns in control behaviour can still be observed. The concurrent verbalisation device used to collect data for protocols is the most suitable of the approaches available. Results can be complemented by information collected on conceptual knowledge and subjects' knowledge and beliefs about problem solving. A more complete profile of the problem solver can then be constructed.

The protocols discussed provide little evidence that control, on its own, can necessarily promote the solution of a non-routine problem. Experience with similar classes of problems can clearly make a difference, although in this study such experience was only observed in conjunction with expert control behaviour. Also, effective control behaviour relies on knowledge of the problem solving strategies or heuristics available to the problem solver. Although he demonstrated a high degree of control of his actions, an expert in this study did not think to "try a few examples" - a strategy that was put to good use by the novice subjects.

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