# Developing Collective Mathematical Thinking Within the Primary Classroom

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## Abstract

This paper reports a school-based study of a collective argumentation model of learning implemented within a year 6 math classroom. Quantitative and qualitative evidence indicated that children in collective argumentation learning environments generated a higher proportion of logical operations in their group discussions about a logico-physical task and were more adept at generalising acquired knowledge to a novel problem, than children operating in more conventional learning environments.

## Introduction

The socioculturalist viewpoint promotes the notion that cognitive change is mediated by and constituted through social interaction. Rather than ascribing cognitive development to a preordained, stage-like pattern of growth, socioculturalists such as Vygotsky (1981) emphasise the importance of the decontextualisation of language and other mediational means provided by a culture in the formation of higher mental functions. From this perspective, it is the quality of the inter-mental tools of thinking available within a culture and their sophisticated use within a culture's patterns of interaction that drives development forward. This view of development contrasts with that offered by other theorists, but what is important from a teacher's perspective is how theories are integrated to create a view of social interaction within classrooms which promotes intellectual development and avoids promoting superficial learning.

In creating such a view, it is necessary to synthesise the contributions of a number of theorists within a practical framework. The work of Max Miller (1987) provides a base for helping teachers to understand how the individual mind may be transformed by social cooperation. For Miller, the structures and processes of cooperation are a 'reality sui generis' (p. 235) and are considered by him to be a necessary factor in development. To explain such development, Miller turns to collective processes of argumentation and advocates three cooperation principles. Using the term argumentation in a broad sense to include any 'type of discourse in which the principle goal is to find collective solutions to interindividual problems of coordination' (1987, p. 231), Miller maintains that these principles have the potential to direct a group's logic of argumentation in such a way that collective thinking may be enabled.

The first principle is concerned with coordinating the contributions of participants so that a set of collectively valid statements (ie., statements that need no further questioning regarding their relevance to a given argumentation) can be agreed on. Miller refers to this coordination device as the principle of 'generalizability' and describes its function as formulating the conditions which a statement must fulfil in order to be justified. The realm of the collectively valid in the group, however, is not static and can be changed as a result of intragroup interactions. The changes that may result from these interactions are not arbitrary, but conform to the principle of 'objectivity'. This principle states that if a statement cannot be denied, it belongs to the realm of the collectively valid regardless of whether it supports or rejects a participant's point of view. The third principle is that of 'consistency'.

According to this principle, mutually exclusive points of view must not 'enter into or (once they have been discovered) must not remain in the realm of the collectively valid' (p. 234).

For Miller, these principles provide a mechanism whereby consensus building and mutual understanding can be determined by social influences. Through the processes of shared thinking brought about by this form of interaction, Miller maintains that collective learning environments can be established which have the potential to advance children's knowledge and perspective taking capabilities to higher levels of mental functioning.

In our view, Miller's cooperation principles can be developed into a structure to scaffold group discourse so that learning becomes socially assisted within the classroom by having students participate in a classroom culture of argumentation which requires the adoption and display of commitments to 'progressive discourse' which Bereiter (1994) maintains define the cultural practices of scientists. Such a structure to scaffold group discourse has been incorporated within a learning model (Brown, 1994) and is referred to as the model of collective argumentation (see Figure 1).

The model incorporates Miller's principles within a cognitive disturbance framework which is compatible with a sociocultural perspective. To translate these principles into group learning procedures, five interactive processes were identified - representing, comparing, explaining, justifying and agreeing for coordinating interpsychological activity, and a series of pedagogical and interactional supports were proposed for enabling children to engage in discourse facilitative of the appropriation of mathematical knowledge and the development of higher mental functions.

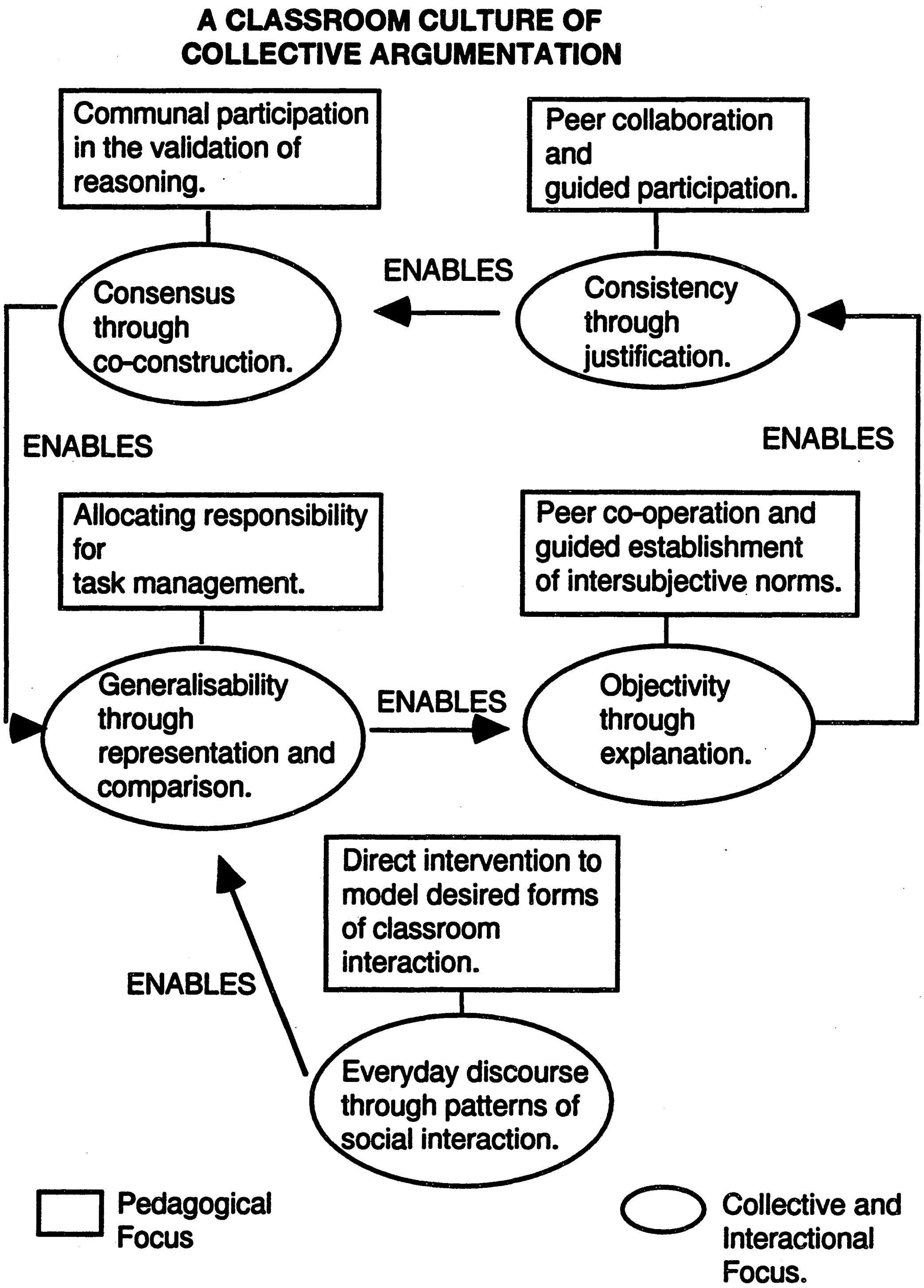


Figure 1: The model of collective argumentation embedded within a classroom culture which enables pedagogical and interactional support systems to scaffold cognitive development.

As seen in Figure 1, the processes of collective argumentation act as an interpsychological coordination device. Although the device can be considered sequential in that a group of students needs to achieve generalisability before achieving objectivity and so on, it does not imply that all groups need to be operating concurrently on the same process. On the contrary, one would expect that between group differences allow social environments at various levels of challenge to be established. That is, at any particular time groups would be operating in various sectors of the model (eg., justifying to achieve consistency, or explaining to achieve objectivity, or comparing representations to achieve generalisability). The occurrence of such diversity within a classroom enables groups to observe and participate in ('legitimate peripheral participation') or revisit aspects of task activities promoting awareness of the goals of collective argumentation. Students with less abstract representations are provided with multiple opportunities to consider the problem from the more consensual, objective and consistent perspectives offered by the more experienced members of the community. It is in this way that students' actual levels of development are linked to the social experience of the classroom, thereby allowing them to go beyond the immediate and the personal to gain control over their own learning through the continuing development of the higher mental functions of voluntary attention, logical reasoning and self-regulation.

To test the effectiveness of the model for promoting cognitive change within a primary classroom, a study was designed employing quantitative and qualitative measures. The first issue investigated was the effect of the model on children's use of transactive dialogue. A transactive dialogue can be defined as one in which an individual uses reasoning that operates on the reasoning of another or that significantly clarifies ideas (Kruger & Tomasello, 1986). As the model was designed to enable children to appropriate knowledge through 'progressive discourse' (see Bereiter, 1994), we expected that children participating in collective argumentation contexts would display a higher proportion of logical operations (transacts) in their group discussions about a balance-scale task than children learning within a less structured context of collaboration.

The second issue investigated was the effect of the model on the levels of cognitive functioning displayed by children when solving a novel balance problem. Since the model purports to facilitate the development within children of the higher mental functions, we expected that children who participated in collective argumentation contexts would display a more mature approach to solving a novel problem than children operating in either unstructured collaborative or teacher instruction contexts. Unstructured collaboration was similar to collective argumentation in that children worked in dyads with the teacher providing guidance, feedback and opportunities for groups to produce collaborative solutions, explanations and empirical validations. However, no discourse structure (represent, compare etc) was provided for these children. Teacher instruction was defined as children working individually to solve problems with the teacher providing feedback, direct intervention (when thought necessary) and assistance to produce representations, explanations and empirical validations.

## Method

*Subjects:* Eighteen sixth grade girls (Mean C.A.= 11.25yrs) whose responses on a pretest, designed to classify knowledge about the balance-scale task, predominantly conformed to what Ferretti and Butterfield (1989) describe as Rule III, were rated by their teachers as displaying high, average, or low-ability reasoning and assigned to same-ability dyads in each of the Collective Argumentation (C.A.) and Unstructured Collaboration (U.C.) conditions and individually to the Teacher Instruction (T.I.) condition.

*Materials and Procedure:* The study was conducted over three school weeks by the subjects' teachers in their home classrooms (approx. 20 children per room). Each condition was allocated the same number of learning sessions per week with each of the eight sessions lasting one hour. Session content focused on a variety of conflict-balance, -weight and -distance word problems (see Ferretti et al., 1989). Materials provided were the same for each condition except for the problem sheets provided to the C.A. dyads which contained added lists of the terms 'represent', 'compare', 'explain', 'justify' and 'agree'. To allow children to test their hypotheses, each classroom was provided with a balance-scale. At no time during the study was a teacher permitted to directly teach an addition or multiplication balance-scale rule (see Ferretti et al., 1989). Pupils' verbal interactions when solving the conflict-distance problem from session six were audio-taped for analysis.

A day after the last session subjects completed a post-test to measure each child's knowledge about the balance-scale task after instruction. Three days later each subject was required to solve for the novel balance word problem to establish the quality of learning. The criterial problem was administered in a separate room and audio-taped for analysis.

## Results

As seen in Table 1, most subjects displayed increased levels of competence on scale tasks from pre- to post-test as measured by Ferretti et al's. (1989) rule taxonomy, with Rule V being considered as a mature response.

Table 1: Pretest,posttest and criterial problem levels for children per condition.

| Condition/ Ability Status | Pretest Level | Posttest Level | Criterial Level |
|---|---|---|---|
| Collective Argumentation | | | |
| Low | III | V | Multistructural III |
| | III | V | Unistructural I |
| Average | III | V | Abstract V |
| | III | V | Relational V |
| High | III | V | Abstract V |
| | III | V | Relational V |
| Unstructured Collaboration | | | |
| Low | III | V | Multistructural II |
| | III | III | Unistructural I |
| Average | III | V | Multistructural III |
| | III | V | Unistructural I |
| High | III | V | Abstract V |
| | III | V | Abstract V |
| Teacher Instruction | | | |
| Low | III | III | Multistructural II |
| | III | V | Multistructural II |
| Average | III | V | Abstract V |
| | III | IV | Multistructural II |
| High | III | V | Abstract V |
| | III | IV | Unistructural I |

The tapes from session six were transcribed in a script format and the conversational turn used as the unit of analysis. Each turn was coded either as a transact or as an 'other type' of communication, with all communications not initiated by a subject or unrelated to the problem being eliminated from the analysis. The dependent measure (see Table 2) was the number of transact statements per subject expressed as a proportion of the total number of conversational turns for the dyad. For statistical analysis these proportions were transformed (Arcsine). A correlated *t* test revealed that the proportion of conversational turns identified as transacts for subjects in the C.A. condition was significantly greater than that identified for subjects in the U.C. condition, $t$ (5) = 3.8011, $p$ < .0063. Relevant data are presented in Table 2. This analysis indicates that the quality of the discourse of the children in the C.A. condition shared a greater congruence with the progressive discourse identified by Bereiter (1994) than the quality of discourse displayed by the children in the U.C. condition.

Table 2: Percentage of conversational turns coded as transacts in the collective argumentation and unstructured collaborative conditions.

| Ability Status | | Collective Argumentation | Unstructured Collaboration |
|---|---|---|---|
| Low | 1 | 30.4 | 25.9 |
| | 2 | 37.7 | 25.9 |
| Average | 1 | 24.6 | 18.2 |
| | 2 | 27.5 | 20.5 |
| High | 1 | 40.7 | 17.7 |
| | 2 | 38.9 | 30.4 |
| | Mean (S.D.) | 33.3 (6.7) | 23.1 (5.1) |

The move from intermental to intramental processes is indexed by moving from an analysis of the conversational quality of the dyads to the quality of thinking displayed by each child on the think-aloud criterial problem. The think-aloud protocols were evaluated in terms of a scheme derived from Ferretti et al's. (1989) taxonomy and the SOLO levels associated with Biggs and Collis's (1982) taxonomy for discriminating the quality of the observed learning outcome (see Figure )

As Table 1 shows, two-thirds of the C.A. children displayed mature understanding (i.e., Relational V or Abstract V responses) of the task, whereas in each of the other conditions only one-third of the children displayed such understanding. These results are consistent with the sociocultural assumption that forms of mediated social interaction have a determining influence on the resulting quality of learning. All but one of the children from the C.A. condition displayed protocols which evidenced ability to recognise key structural elements of the problem, thereby allowing them to apply some degree of knowledge about balance tasks. The majority of children from the other conditions, however, displayed approaches embedded within the specific context of the problem, thereby limiting their application of balance-scale knowledge. Further examination of the protocols suggested that the voluntary employment of higher mental functions such as planning, the monitoring of subgoal achievements and the evaluation of predicted outcomes were the features which most distinguished the problem solving efforts of the C.A. children from those of their counterparts. These results support the hypothesis that children participating in collective argumentation contexts are more adept at generalising acquired knowledge to a novel problem situation than are those who participate in more conventional learning contexts.

Prestructural Unistructural Multistructural Relational Abstract

|---------|---------|---------|---------|---------|---------|

Rule I | Rule II | Rule III | Rule IV | Rule V

**Figure 2:** Criterial levels response continuum relating to novel problem analysis.

## Discussion

The present study provides evidence to support the efficacy of the model of collective argumentation for enhancing children's appropriation of mathematical knowledge and their development of higher mental functions. In addition to suggesting that the form of interpsychological functioning has a determining influence on the resulting form of intrapsychological functioning, our study highlights the importance of the following aspects of the model for collective classroom learning Firstly, the five processes of collective argumentation provide each child in the dyad with a structure to support their expression of ideas and concepts, and enable each child to actively participate in negotiating consensus and developing socially shared understandings. Secondly, the role of the teacher is crucial at every stage of the process of collective argumentation. The teacher participates by reminding the children of the five processes, by modelling possible representations, by challenging the consensus reached by dyads, by introducing more abstract terms, by paraphrasing childrern's ideas, and by guiding children to adopt the mathematical conventions of the classroom. Thirdly, it is our experience that the introduction of the collective argumentation model transforms classroom activity from private endeavour to social participation enabling children to value and enjoy collective learning of mathematical ideas and skills.

As such, the potential benefits of the model for assisting teachers and students to bridge the gaps between old and new knowledge, between everyday and scientific discourse and between school curricula and the mathematical community of practice would appear to be limited only by teachers' levels of participation in the culture of the discipline. However, the issue of how to encourage teachers and disciplinary scholars to participate in the formation of such a community has, as yet, to be satisfactorily addressed. This paper contributes by allowing teachers and disciplinary scholars to see that learning can be operationalised within the classroom as a social reality and provides some sense of the articulation between the personal and cultural practices that form the individual mind.

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