Using computational environments as tools for working mathematically.

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Introduction

The belief that mathematics is a social activity, in which members of a community of mathematicians engage in systematic mathematical experimentation and reflection is held by many mathematics educators (Schoenfeld 1992, p335) and underpins A National Statement on Mathematics for Australian Schools National Statement (Australian Education Council 1990). Part of teaching mathematics from this viewpoint involves modelling being a mathematician and encouraging students to practice being mathematicians. How does current practice in classrooms reflect this belief? In this paper the author reports on reflections and actions on his classroom practice aimed at creating a classroom environment where students practice as mathematicians. The use of technology, in creating potentially rich mathematical environments for such experimentation and reflection is also examined.

Modelling mathematical practice in the classroom

The teaching learning situation

The teaching learning model and software under discussion in this paper are being developed for three Year 12 Quantitative Methods classes at Casuarina Senior College in Darwin. Quantitative Methods focuses on Statistics, Growth and Decay and Operations Research. The course is administered through the South Australian Education Department and is assessed through a public examination (50%), Project (15%) and school assessment (35%).

Each student owns a graphics calculator and the classroom is equipped with one Macintosh™ computer and a grey-scale Data Show™. Students have access to a computer laboratory outside class-time. Class sizes are 15, 18 and 24.

Theoretical considerations

The constructivists' theory of learning asserts that all mental activity is constructive (Noddings 1990). The all implies that mathematics does not exist out there in objects. As all learning is an act of internal construction, there is no particular set of external actions and materials that can be labelled as constructivist teaching. Rather, the challenge is to create potentially efficient learning environments where the student is inclined to engage in meaningful, productive learning.

Although external representations such as concrete materials, texts and computer screens do not have a lump of mathematics embedded in them, they can, however, act as agents for the learner (Mason 1992). These external images can potentially help extend the thought of the learner, and act as recording and storage devices.

An important implication for teaching and learning is that shared meaning is negotiated rather than pointed at in the sense that it exists in external objects. It may be that what the expert teacher sees mathematically in an external representation is not necessarily what the novice student sees. For example Multiple Attribute Blocks (MAB) may represent units to the teacher and building blocks to the student (Cobb 1992, p4).

Developing a teaching learning model

Traditional mathematics teaching, where the teacher demonstrates and explains an example of a particular mathematical algorithm, then sets carefully crafted exercises for the student to do is well documented (for example Schoenfeld 1992,p338). This traditional mathematics teaching model is taken as shared meaning in many of our textbooks and course outlines. Students often expect to be taught in this traditional manner, especially after 11 years of the similar fare. In order to move away from this deeply ingrained teaching learning model, I set about designing a model which reflects the view that mathematics is a social practice where mathematicians work on interesting problems.

These principles are captured diagrammatically by Confrey (1991) in figure 1.

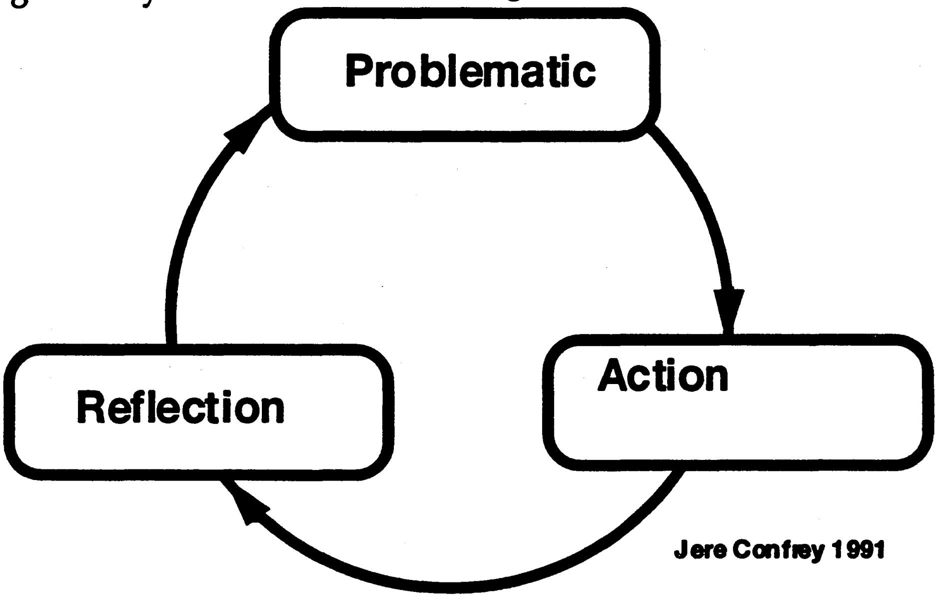


Figure 1

Confrey's teaching learning model is centred around problematics.

> *A problematic is defined only in relation to the solver and only becomes a problematic to the extent to which and in the manner in which it feels problematic to the solver... A problematic is a call to action*
>
> *(Confrey 1991 p11)*

This is different from a traditional view of problem solving where the teacher defines the problem in relation to the set content under study and then sets the problem for the student to do. Problematical situations arise when the learner attempts to actively resolve a situation where there is no ready solution available. After engaging in the problematic the learner uses internal constructions to work or act on the problem. External representations developed or appropriated by the learner have meaning only in terms of these internal constructions or schema.

During the learning process the student reflects on the work in progress in order to solve the problematic. This can lead to the construction of new and more powerful ways of thinking mathematically.

Working mathematically

In developing my own teaching learning model I have drawn on Confrey's (1991) model and the Working Mathematically Strand from *Mathematics- a curriculum profile for Australian schools* (Australian Education Council 1994)

Learning sequences begin with the learner accepting a challenge or engaging in a problem. The learner then works mathematically on the problem, using problem solving heuristics, reflection and communication. The teacher also demonstrates working mathematically on problems in order to model the process to the student. The purpose of these teacher actions is to build a community of practice (Goos 1994, p305) where students can test out their emerging ideas in small groups, with the whole class and with the teacher. One role of the teacher is to encourage dialogue and to pose focus questions that prompt the student to re-think and justify their conjectures. The teacher also offers appropriate formal mathematical language in order that students may rightfully communicate with the wider mathematical community.

Towards the end of the learning sequence more emphasis is given to evaluating the mathematics under study and its place in the students mathematical knowledge. Students are encouraged to publish or celebrate their findings to the class in the form of written reports and class presentations.

The role of technology

There has been considerable research in the past few years in the development of computer based learning resources (for example: Confrey 1991; Kaput 1992; Rubin 1989) that exploit the potential of the computer to extend the power of the mind. Such computer programs encourage the learner to build conceptual understanding through the computers ability to:

- dynamically represent objects, as opposed to static pen and paper representations;
- show multiple, linked external representations of mathematical concepts;
- carry out low level computational operations, freeing the learner to construe instead of just do;
- interact with the medium, allowing the learner to experiment (adapted from Kaput, 1992).

Specific software, based on Kaput's principles, has been written by the author for students of Quantitative Methods with the intention that it is used by students as a tool to develop their conceptual understanding.

Concept building software: the least-squares example

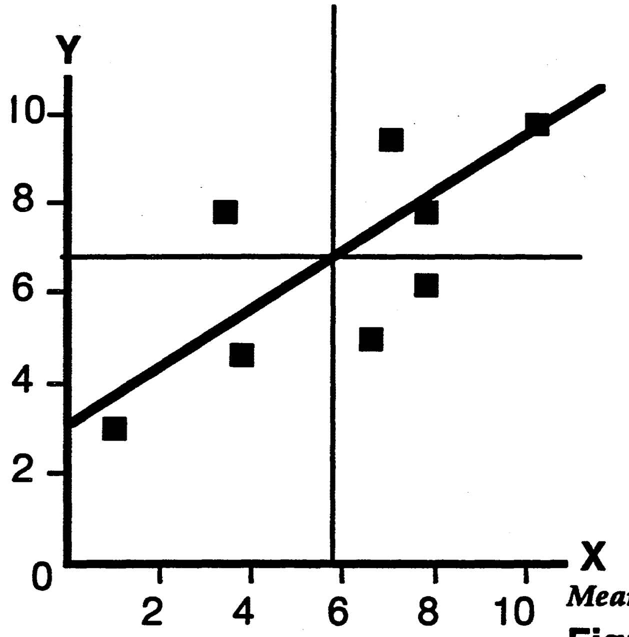
Description

The software is written in Future Basic™ for a Macintosh™ computer.

Double clicking on the software opens a window where the positive quadrant of a cartesian graph is drawn. The scale is 0 to 10 for each axis. On the graph eight observations are shown. Beside the graph is a table that shows the coordinates of the observations, the correlation coefficient, the estimated y values, residuals, slope and intercept of the least-squares regression line. Dragging an observation point fills the table and graphs a horizontal line through the mean of the y-values, a vertical mean line through the mean of the x-values and the least-squares line of best fit. Dragging a data point to a new position causes the program to recalculate, display the new values in the table and redraw the mean lines and the line of best fit. By moving a data point from or towards the line of best fit the correlation coefficient responds almost immediately, giving the user a sense of how the numerical representation of the correlation coefficient relates to the graphical representation. The corresponding row in the table is coloured red.

Some preliminary classroom observations and reflections

The traditional approach to the teaching of the least-squares regression line is to give a formal definition, represent this graphically and then give the algebraic formula. This is followed by some calculations in tabular form. (see for example Moore, 1993 pp117-125). My approach was to de-emphasise the manipulation and attempt to build understanding of the concept through



Intercept	3.15
Slope	0.629
r^2	0.766

	X	*Y*	*Y^*	***residual***
	1.20	*3.20*	*3.91*	*-0.71*
	3.60	*4.80*	*5.42*	*-0.62*
	6.20	*5.20*	*7.05*	*-1.85*
	7.60	*6.40*	*7.94*	*-1.54*
	3.60	*8.00*	*5.42*	*2.58*
	7.60	*8.00*	*7.94*	*0.06*
	7.20	*9.20*	*7.68*	*1.52*
	10.00	*10.0*	*9.45*	*0.55*
Mean	*5.9*	*6.9*	*...*	*0.00*

Figure two

dynamic interactive graphical representations coupled with students undertaking a relevant Statistical Investigation. The investigation chosen was the relationship between the length of the radius bone and the height of students studying Quantitative Methods. The challenge to the students was in the form of two focus questions:

- Can you predict a person's height given the length of their radius bone?
- If so, how good is your prediction?

This investigation requires students to first collect and then display their data using a scatter plot. The table and graph were represented both using the graphics calculator and on paper. Each student was asked to draw in an estimate of the line of best fit on their paper representation and then to draw in the line of best fit using the slope and intercept as displayed on their graphics calculator. This statistical investigation gave students experience in working mathematically, and opportunities for practicing mathematical skills in context (such as graphing, measurement, data collection, report writing, entering and retrieving data from a graphics calculator). However in the end they had had only one example of fitting a line of best fit.

The purpose of developing the least-squares software was to offer students the opportunity to gain experimental experience in fitting least squares lines of best fit. Interestingly students have experienced a great deal of difficulty in moving between representations, particularly from the screen to paper and from a graph to an algebraic formula. Moving, on paper, from an algebraic formula to a graphical via a table of values is well understood by most students. This may be because it is the traditional path adopted by the traditional mathematics lesson.

Preliminary observations reveal that representations on any screen (calculator or computer) have an additional problem of scale. It may be that students have to have a good knowledge of the shape and behaviour of the function being graphed in order to study (further) its shape and behaviour on the screen.

Using the software

Within a few minutes of dragging around the data points the user has a movie (Mason 1992) of experience in the form of many lines of best fit. What is different about this movie is that the learner is in control of it: able to stop and look, vary the speed or drag to places in order to test awareness's of patterns in the data. The user is able to create unusual situations, such as outliers and then to move the outlier towards the least-squares line, observing the corresponding changes in the value of the correlation coefficient, and the slope of the least squares regression line.

The sensitivity of the least-squares line to outliers was obvious and noted by students. Students began to predict what would happen to the line if certain movements were made. Anwen noticed that the means and the line of best fit always intersected, no matter where she dragged the points. (Is this a form of proof?) Rodney noticed that the program was unstable with a slope of zero. Is this a fault of the program or where the function is undefined?

Later in the learning sequence, when Stephanie was using the least-squares regression line in the context of a real situation she drew on her experience of the software to understand the situation and in particular the influence of an outlier on the least squares regression line. Students began to make statements such as "it is pulling the line off course" and "it is distorting the trend". As asserted by Fey (in Boers 1994, p515), the ability to draw on an experience from one context and apply it to another is a good indicator that the learner is understanding the deeper semantic meaning of the concepts under study.

Conclusion

Computer applications based on the design principles of inter-activity and multiple-linked dynamic representations have the potential to act as agents for learners of mathematics. Such applications, used in the context of mathematical experimentation, can enrich the mathematical experiences of the learner in a way that is not possible with static pen and paper representations. In order for the software to have mathematical meaning to the learner a supporting teaching learning model is being developed. The teaching learning model's main purpose is to enable a community of learners to form where members can freely conjecture, justify and celebrate their learning. Within this community the teacher works towards creating a classroom of apprentice mathematicians, and to help this process the teacher, at times, models being a mathematician.

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