Contextual Perspectives of Secondary School Mathematics

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This study investigated the nature of conceptions of what mathematics is and the intentions of school mathematics through interviews with Years 10, 11 and 12 students and their teachers. Students and teachers held broad views of what is the content or discipline of mathematics, while their interpretations of mathematics within a wider sociocultural context reflected four additional, influential personally and socially derived interwoven factors: social status of mathematics, utility of mathematics, career aspirations, and interest or disinterest in mathematics. The existence of these factors indicate that what it means to 'understand' mathematics is related to both a context and individuals' interpretations of a context.

Introduction

School-based research has indicated that learners' experiences of mathematics, both inside and outside the classroom, influence their attitudes to and outcomes of learning mathematics (Crawford, Gordon, Nicholas & Prosser, 1993; McLeod, 1992; Nunes, 1992; Resnick, 1987). In particular, there are indications that students' conceptions of mathematics affect the quality of related cognitive activities and learning outcomes. That is, how students interpret the context of their mathematical learning and hence how they relate to mathematics endeavours inside and outside school can influence mathematics performance (Crawford, 1990; Cobb, Yackel & Wood, 1992; Lave, 1988; Steffe & Cobb, 1988).

Recent studies in Western Australia have revealed a wide variety of conceptions, even amongst high ability students, about the nature and purpose of mathematics and its study (Frid & Malone, 1993; Miller, Malone & Kandl, 1992; White & Taylor, 1994). If, as is claimed by initiators and supporters of recent curriculum changes, high school mathematics is an important foundation for later learning at post-secondary institutions, and for employment, then educators need to better understand the individuality and diversity of students who are presently studying mathematics in high school. Although new high school mathematics curricula have been developed and implemented in Western Australia and elsewhere, there has been little attempt to examine student conceptions of the mathematics they are studying, their motivations for studying mathematics, or their approaches to learning mathematics. Since the cohort of students entering upper secondary school in Western Australia has expanded and changed in the last few years, and the demand for post-secondary education has increased, it is vital that neglected factors in examination of mathematics learning be considered. This study focuses on a neglected factor – the conceptions of the recipients of mathematics curricula. Thus, the main aim of this study is:

> *To investigate the nature of high school students' and teachers' conceptions of mathematics.*

Specifically, the study focuses on the following three research questions:

1 What do students and teachers think mathematics is?
2 What do students and teachers think are the intentions of mathematics study and why mathematics is included in school programs?
3 What do teachers think students think about mathematics and how do these views compare with those of students?

The importance of addressing these questions lies in the fact that until mathematics education researchers address and come to understand what mathematics is in others' eyes we are not in a position to understand the effects of mathematics courses. By investigating students' and teachers' views, the outcomes of this study will put educators in a better position to recognise and thereby more comprehensively understand how mathematical experiences are interpreted within secondary schools. For example, until we know how students, as intended recipients of school practices, experience and interpret both the content and context of school programs, we are not in a valid position to evaluate the effects of recent trends and changes in mathematics curricula. Much research in mathematics education in relation to this issue has focussed upon designing new teaching methods or environments. The broader context of the nature of students' mathematics learning has not been examined as extensively.

Method and Data Sources

The study was interpretive in nature, and therefore, being concerned with the context of learning and related interpretations of experiences, used qualitative research methods. This approach is in line with current educational research practices as they shift away from purely quantitative, quasi-scientific experiments so that researchers can more explicitly document and analyse the experiences of teachers and learners in the broad encompassing social and academic complexities of classrooms (Moss, 1994). An inductive reasoning approach for data analysis was adopted (Glaser and Strauss, 1967; Powney and Watts, 1987).

Specifically, the study used in-depth, semi-structured interviews with students and teachers. The interview protocol asked the students and teachers to respond to a range of questions about their thoughts on: why mathematics is taught in schools, what is important about learning mathematics, what mathematics is and whether it is created or discovered, reasons why people choose particular mathematics courses, factors that facilitate mathematics learning, and specific experiences with mathematics learning or teaching. Interviews were recorded on audio tape and later transcribed.

Two high schools in the metropolitan region of a large Western Australia city were selected for the study: a large government school and an independent girls' school. A total of 40 students, 19 teachers, 2 career councillors and two administrators (one principal and one deputy principal) were interviewed. Nine students were in Year 12, sixteen were in Year 11 and fifteen were in Year 10. The sample of students was representative of high, middle and low achievement in mathematics, and at the government school was balanced in terms of gender. Approximately half the students were of multi-cultural background (generally Asian), in that they were immigrants or had parents who were immigrants to Australia. Some students were interviewed individually, while others were interviewed in groups of two or three. The reason for this difference was that the researchers found some students would speak more openly and comfortably with their peers than individually. Teachers were all

interviewed individually. Approximately half the interviews were conducted in November and December 1993, near the end of a school year, while the others were conducted in the subsequent school year, in the middle of 1994 (the Australian school year runs from February to December). The differing times enabled reliability of the data to be established in relation to end of year examinations and their potential influences on attitudes and related conceptions of mathematics experiences.

Results

What is mathematics?

Students initially gave many 'expected' responses to the question: What do you think mathematics is? These responses were 'expected' in that they reflected views of mathematics held by students reported in other studies (for example, see Crawford et al., 1993; Chant and Galbraith, 1993), including views that mathematics is: (1) numbers, rules and formulae, and (2) a logical process or way of thinking. Many students also spoke of the applied nature of mathematics, and how numbers, rules or formulae, or logic and a way of thinking can be applied to solve problems. An additional conception of mathematics as a connected hierarchy that studies relationships or patterns was spoken of by some students, and these were generally students achieving at average or above average levels.

The question that elicited further insight into students' conceptions of mathematics was: Where did mathematics come from and what is it for? Many students responded to this question in a way that pointed to a view of mathematics as a sort of technology, that is, a human endeavour for addressing human needs and solving human problems. These needs might be due to a desire to understand one's environment, or they might arise from wanting to accomplish a specific task such as determining the amount of wallpaper to paper a room. What is noteworthy about the students' comments is they indicate students have a fairly broad view of mathematics. Their conceptions of mathematics, when they respond beyond an initial immediate response, include much more than the stereotyped view of mathematics as numbers, formulae, logic or problem solving. That is, students appeared to have formed views of mathematics that encompassed more than the topics explicitly taught and assessed. Many see mathematics more inclusively, taking into consideration aspects of the nature and purpose of mathematics. It could be argued that engendering in students an awareness and appreciation of this broader context is a goal of many mathematics curricula (for example, National Council of Teachers of Mathematics, 1989; Australian Education Council, 1991), and therefore an expected outcome of learning. However, other research into students' beliefs about mathematics has not often indicated students hold views beyond the narrow, stereotypical views of mathematics as numbers, formulae, rules, logic and problem solving (for example, Crawford et al., 1993; Frank, 1988; Kouba & McDonald, 1991).

It is possible that students in these other studies held broader views, but the data collection methods were not appropriate for probing beyond initial expressions of views. As will be highlighted in upcoming sections of this paper, the broader contexts within which students interpret their mathematics experiences cannot be ignored if mathematics educators are to interpret research results in ways informative simultaneously to teaching practice, curriculum development and further research.

Teachers' views of what mathematics is were similar to those of students in that they included notions of problem solving, analyzing, studying patterns and relationships, and learning that

mathematical skills and reasoning can be powerful tools for solving human problems. However, they did not believe their students held the same views. They thought that most students see mathematics as boring and irrelevant, although some noted that individual students or groups of students, particularly those who are successful with their mathematics achievement, might be more in accordance with themselves as to the nature and relevance of mathematics.

What is striking is that the teachers included social factors in their comments on what they believe students think mathematics is. That is, they see students' views as related to and derived from much more that the cognitive elements of mathematics learning. Mathematics to students, as interpreted by teachers, is "just another course" that you "just have to do" as a "means to an end", even though the topics learned might not be used again. The teachers' views of students' views did not separate students from the encompassing environments in which they live. Personal factors such as enjoyment or motivation, or social factors such as school requirements or parental expectations, were not seen as isolated from more cognitively oriented factors such as learning about numbers, rules, arithmetic or algebra. It will be seen in upcoming summaries that what mathematics means to students, that is what they 'understand' as mathematics, is not formed in isolating the academic content of mathematics from personal and sociocultural contexts.

Why is mathematics studied?

Analysis of interview data revealed four major interwoven and overlapping contributors to conceptions of the intentions of mathematics studies: social status of mathematics, utility of mathematics, career aspirations, and mathematical interest or disinterest. The factors were interwoven in that each appeared to be an element in defining each other. That is, although each factor had strong identifying characteristics, they were not independent of each other. Altogether the factors create a web of beliefs that are the individuals' conceptions of mathematics and what it means to 'understand' mathematics. Each of the four factors are outlined here and detailed analysis and discussion of them can be found in Frid and White (1995).

Social Status of Mathematics

Students' conceptions of mathematics reflected a social norm that mathematics is an 'important' and essential subject to study. They saw it as a subject with much prestige in the eyes of the community, especially employers. Many had questioned the validity of this status, but had accepted that it was a social value or convention that they must acknowledge in their choice of subjects for upper secondary or post-secondary studies. They believed society at large (ie. people in general) highly values mathematics success, and sees mathematics success as something people should strive for.

The students' interpretations of the status attributed to mathematics success within society also indicated they saw such status as, at least in part, invalid. They felt that part of the reason mathematics is seen as an important and paramount subject is that it is generally seen as a difficult subject, and thereby a mechanism by which to filter students. That the study of 'abstract' mathematics, as opposed to 'vegie' mathematics is a filter for segregating people was something of which the students were clearly aware. Most teachers expressed views about the status of mathematics that were similar to those of the students. In particular, they generally saw the high status attributed to mathematics as unwarranted.

Utility of Mathematics

Students' views of the utility of mathematics fell within two distinct categories: (1) the use of basic

mathematics concepts and related skills in one's daily life, and (2) the potential use of more advanced mathematics concepts and skills within professional endeavours. What they often actually described as relevant mathematical knowledge in relation to needs within daily life was mathematics taught primarily in elementary school. A third category was evident in the teacher interview data: (3) the use of mathematics as a reflection of human thinking or culture. Both students and teachers frequently questioned the degree to which it would actually be necessary to use advanced mathematics within professional activities. However, although they questioned the reasons for mathematics to be a major requirement for entry into particular career paths or related programs of study, they also acknowledged the reality of these requirements. Thus, career aspirations and the relationship of mathematics study requirements to career aspirations was an integral component of their conceptions of mathematics.

Career Aspirations

Many of the answers in interviews on what mathematics is and why mathematics is taught in school were comments on career aspirations. There was both diversity and conformity in students' and teachers' conceptions of mathematics in relation to career aspirations. Conformity appeared in the form of the recognition of the pre-requisite necessity of mathematics study for certain career paths, particularly in relation to university entry. There was also a high degree of belief that many of these pre-requisites were an unwarranted constraint. Thus, students were motivated to study mathematics to enhance their prospects for a particular profession or job, to keep their career options open and maximised, or to assist their chances of gaining acceptance at a post-secondary institution. However, they were also often frustrated. Many of the students, as well as many of the teachers felt as though someone else, or the system, had imposed unnecessary or unreasonable demands upon students' choices. Students who were not aiming at university study were more explicit in their condemnation of the value of high school mathematics. Thus, the diversity or conflicts that arose amongst students were a result of the extent to which they either needed or desired to "leave their options open". Choosing whether to study mathematics or what specific courses to study at upper secondary level then became a dilemma for some students. They did not see how mathematics would be useful to them, or did not enjoy mathematics study, yet were faced with externally imposed requirements or societal values and expectations.

Mathematical Interest or Disinterest

A few students expressed an interest in mathematics related to it as a discipline of thinking and they referred to a sense of intellectual fascination and challenge it provided. These students were not necessarily the most able students. In general however, few students enjoyed mathematics for its own sake. A strong interest in mathematics was sometimes expressed in relation to career aspirations and the social importance of mathematics, with enjoyment achieved through being successful in relation to these other key components. In particular, there was evidence that interest in mathematics is linked with a sense of personal success in mathematics, findings that are not new within the realm of mathematics education research. For the students of this study, there was correlation between students who had been successful with mathematics and were intending to or who were already enrolled in university entry mathematics courses.

Conclusions

Students have 'understanding' of mathematics that is dependent upon context. That is, they define mathematics in a school sense as well as a personal sense. From a mathematics 'discipline' perspective of what mathematics is and where it comes from they generally have well-formed conceptions. However, these conceptions do not play a prominent role when students are asked to give personal meaning to mathematics in relation to their lives. Students' personal perspectives give meaning to mathematics only in relation to a number of social factors, including the social status of mathematics, the utility of mathematics, career aspirations, and interest (of lack of interest) in mathematics. That is, students incorporate into their conceptions of mathematics a wider cultural and community perspective of mathematics than that upon which mathematics educators generally focus. More specifically, students put mathematics in a whole school, career and life perspective, but mathematics education research does not presently adequately address this broad viewpoint.

Thus, it could be said that 'understanding' mathematics is neither a goal nor a necessary component of students' mathematical learning, at least not in the sense mathematics educators might define 'understanding'. Students 'understand' mathematics when they are meeting their goals as described in relation to their career aspirations and sense of the social importance of mathematics.

What also is of interest is how the four factors identified within the context of the intents of mathematics study conflict with the ideas of how mathematics educators identify problems present in mathematics education practices. For example, understanding generally is associated with a range of interconnecting, cognitive frameworks that can be utilised to explain concepts and solve problems (National Council of Teachers of Mathematics, 1989). Further self-analysis is needed by the research community into their own interpretations of what mathematics is, what it means to learn mathematics, what it means to understand mathematics, what is the nature and role of mathematics education research and how mathematics education research functions in research and other educational contexts.

Recognition is needed for how students view curriculum because this study indicates students do not separate mathematics from their personal or social contexts. They do not perceive of mathematics as one might describe mathematics as a discipline, but rather, describe mathematics in relation to a range of socially derived components. Researchers' capacities are restricted whilst they persist with disregarding the whole school context and a view of mathematics that virtually ignores students' views.

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