Measures of mathematical knowledge students bring with them to University can contribute to better teaching

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Abstract:

The study reported here explored mathematical knowledge students brought with them to University by using both qualitative and quantitative methods. These methods provided current, richer and valid information on students mathematical abilities as compared to the traditional 'normalised' tertiary entrance score. The study data highlighted several issues that could contribute to better teaching of students having insufficient background in mathematics: students retain and recall knowledge which they perceive relevant to their current learning; a high quantitative score does not necessarily equate to high level understanding and the learning of calculus does not support the understanding of functional notation.

Introduction

More and more, Universities are accepting students with insufficient backgrounds in mathematics to enrol in study programs which involve a certain degree of knowledge and understanding in mathematics. The decision to accept the enrolment of these students is often based on their past schooling records. Some of these records can be as old as 10 years or more. Universities have endeavoured to assist these students by offering bridging courses and introductory courses in mathematics in the hope that satisfactory completion would bridge the gap. The method of bridging this gap is often a generic remediation approach. That is, individuals in this category are assumed to have the same weaknesses in mathematics. Satisfactory completion of a mathematics bridging course, therefore, would imply that the individuals have acquired abilities to cope satisfactorily in the subsequent compulsory mathematics units.

Background

One of the main concerns of tertiary educators is the insufficient information presented by tertiary entrance scores. Griffin and Nix (1991) have argued that 'the relationship between the test scores and the future performances of selected groups needs to be well established before a test is routinely used for selection purpose' (p. 15). They added that reducing 'a student's range of achievements and developed competencies to a *single number* [provides] little or no information about the student's performance... [and]... is inadequate for post-secondary selection purposes...[as such is]... unable to aid the teaching-learning process' (p.16, italics added). Another concern for educators is the lack of information on methods to assist students with insufficient background in the subject area of their choice. Weinstein and Meyer (1991) reported studies on special programs in post-secondary institutions to address the problem of students with special needs or 'academic deficit'. They concluded that these studies 'provide information about the conditions under which an individual studies best, but not the methods and cognitive processes they use to do it' (p.49).

The exploratory study reported in this paper is an attempt to provide an insight into the mathematical cognitive processes acquired by students with deficit backgrounds. Also the study examines methods to gain qualitative information on the students prior mathematical abilities. Additionally, there is a genuine concern to find ways to provide these students with appropriate assistance. This report addressed two main questions that are of concern to the authors and their client, School of Engineering: (1) How could information on the students prior mathematical abilities and conceptual understanding of the materials taught be obtained? (2) What appropriate strategies should be taken for teaching, evaluation and assessment?

The framework for addressing question (1) was based on a study, part of which has been reported in Gates (1994), of first year university mathematics students. Gates (1994) tentatively suggested that 'prior mathematical knowledge has an influence on mathematical understanding of higher order levels' (p.293). If this is so, then knowing what mathematical abilities these students have brought with them to the university course would be valuable in the planning and teaching of the materials as well as providing valid assessment on performance.

Method

The study reported in this paper evolved out of the need to determine whether a group of ten students with doubtful mathematical records should be allowed to enrol in the Engineering program. About half of the group were mature age students and their records gave very little indication of their ability level in mathematics. A course organised by the Department of Applied Computing and Mathematics was offered to the students. The 30 hour course covered introductory calculus materials to be completed within a week. This took place a week prior to commencement of the first semester. The students' enrolment was conditional upon passing this mathematics course.

How could information on the students prior mathematical abilities and conceptual understanding of the materials taught be obtained? This question was addressed by using two assessment methods: qualitative and quantitative. For this report, particular focus is on the qualitative assessment method and how this complemented the quantitative assessments to provide a richer and more meaningful information.

What appropriate strategies should be taken for teaching, evaluation and assessment? This question was concerned with the structure of the 30 hour course to allow for the gathering of information on prior mathematical knowledge, giving feedback and remediation, teaching of the units in the course, and the final assessments. There were three teaching staff involved. Two were mathematics lecturers and the third a mathematics teacher educator. The following framework was adopted for the course program:

Week prior to the course: **Pre-test** (Quantitative method) on necessary knowledge and skills. For example, calculator skills, simplifying exponents, graphing quadratics, solving simultaneous equations and trigonometric identities.

Day 1: **Pre-conceptual assessment** (Qualitative method). Immediate feedback to students on Pre-conceptual. Teaching and remediation.

Day 2 to Day 5: Teaching and remediation.

Day 6: **Post-conceptual assessment** (Qualitative method)

Post-test: Final examination (Quantitative method).

The pre and post conceptual assessments were designed to provide qualitative information on the students' mathematical ability. The SOLO taxonomy technique developed by Biggs and Collis (1982) was used for assessing the quality of the students' responses to these assessments. The items for the assessments were selected from studies that have used the SOLO technique in their evaluation. Hence, there were empirical data available for comparing and classifying the responses into SOLO categories. Also, the items closely reflected the types of mathematical concepts and skills involved in the course. The items were selected from the studies by Coady and Pegg (1993a,b,c,d, 1994) and Collis and Watson (1991), see appendix.

Results and Discussion

The pre-test given to the students prior to undertaking the course was assessed in two ways: the traditional method of quantifying the scores and by the criterion-method. For the criterion-method, each item of the pre-test was examined to identify areas of individual weaknesses and strengths. Written comments were given on each item on the correct approach, resource materials available and where to find them. These comments were the focus of the feedback discussion on the group's overall performance. It was found that the group, particularly the mature age students, had great difficulty with rounding decimals involving scientific notation and significant figures. Graphical skills were lacking. Knowledge and skill to factorise and solve quadratic equations were also shown to be lacking.

The pre-conceptual assessment was the very first task presented to the students in the program. Feedback was given through discussion of solutions immediately after the 30 minutes allowed for the task. Students were encouraged to write comments for each item indicating the degree of difficulty they encountered and provide possible reasons for these difficulties. These comments showed the functional notation items to be the most difficult, due mainly to their lack of knowledge. The written responses collected showed responses to functional notation items to be similar to those of Concrete Symbolic when compared with the empirical data on these items by Coady and Pegg (1994, p.181-184). The written responses were also used for mapping conceptual profiles for the students.

The post-conceptual assessment included five items from the pre-conceptual assessment. The inclusion of these items was necessary for two reasons: (1) to examine the affect of 'immediate feedback' on learning and retention and (2) to explore the influence of learning Calculus on understanding functional notation. The remaining four items were included in order to confirm or support the level of understanding (SOLO level) and functioning mode (Concrete symbolic or Formal mode) of students in specific contents. For example, items 1 & 2 are to confirm the functioning mode in relation to functions and variable substitution and items 3 & 4 are to give support in relation to variable substitution. The responses were mapped in the same manner as the pre-conceptual assessment. From these mapping profiles, predictions of students' performance on the final examination were made.

Comparisons of pre and post assessments indicate that 'immediate feedback' had little affect on learning and retention of functional notation items (see Appendix Part B). These results also suggested that the learning of calculus appeared to have had insignificant affect on the understanding of function notation. It is suggested that perhaps the students have perceived no connection between the learning of calculus and understanding of function notation.

The final examination was marked independently of the above assessments by another assessor. Eighty percent of the students' performances on the exam were as predicted based on the conceptual assessments. However two subjects, S4 an S10, showed marked discrepancies between their predicted and final results. S4 was predicted to achieve 80% or better but his exam result was 46%. S10 was predicted a result between 50% and 69% but his exam mark was 82%.

Detailed analysis of these subjects' responses to exam items showed subject S4 to have learned fewer 'new' studied elements. These new learnings by S4 appeared to be closely related to his well learned knowledge of algorithms. For instances, algorithms on expansion of binomials, simplifying expressions were closely linked to newly learned skills, eg. the manipulation of the 'product rule' for differentiation. S4's approach to learning appears to be rote learning rules and algorithms. However, it appears that rules and algorithms involving more than two processes and used in several situations were difficult to recall. The 'chain rule' which involves more than one process and is used in differentiation and integration computations is an example of such difficulties by S4. These results seemed to suggest that S4's success in the 'conceptual assessment' was a reflection of his ability to recall well formed rote learning strategies as well as his capacity to recognise 'patterns'. For instance, a 'doubling' pattern as shown by his response to item 5, in the pre-conceptual assessment, which is a function notation involving more than substitution: *If $f(1)=5$ and $f(x+1)=2f(x)$, find the value of $f(3)$;* was **'f(1)=5, f(2)=10, f(3)=20'**. S4's response was one of the two correct responses from the group, hence he was asked to make special note of how he worked it out during the feedback discussion session in which he commented. **'Easy, when x=0, [f(0+1)], f(1)=5, which is double of f(x), ie. 2f(x); x=1 is 2f(1)=2x5=10, x=2 is 2f(2)=2x10=20'**. According to Coady and Pegg (1994), this response is *relational* (top SOLO level) in Formal mode of functioning. That is, the response 'indicated that the student was capable of using the concepts underlying function notation and could note and use the interrelationships existing within the question' (p.184). In addition, S4's responses to items 4a and 4b, (see Table 1.1), seemed to support *relational*, particularly to 4b, but perhaps in the Concrete Symbolic mode. For instance, responses to item 4a in both pre and post assessment tended to suggest a rote learning technique has been used with little understanding of function notation. Responses to item 4a are similar to those classified as Concrete Symbolic by Coady and Pegg (1994, p.181). S4's responses could be considered as procedural knowledge rather than conceptual knowledge in mathematics as described by several researchers, for example Eisenhart et al. (1193) and Hiebert (1986).

Table 1.1

Response to 4a	Response to 4a in POST-ASS	Response to 4b
$f(t)= 3y^2 + 2y$	$\frac{dx}{dy}=3x^2 + 2x$	$f(x+h)=-2(x+h)^2+3(x+h)= -2x^2-4xh-2h^2+3x+3h$

Detailed analysis of subject S10's performance of 82% on the exam showed that he recalled more of the 'new' studied elements and these were shown to have linked to each other as well as 'old' knowledge and skills. For example, the 'new' rules and algorithms for differentiation and integration were closely linked to knowledge and skills in solving equations, factorising, expansion and simplifying expressions. He was also able to recall the 'chain rule' and used it when appropriate in computations. Although his conceptual assessment revealed little understanding of function notation, he did demonstrate a sound ability in variable substitution to items 3 & 4 of the post-conceptual assessment. His high score in the exam tended to indicate his capacity to memorise rules and algorithms as well as the ability to relate new skills to old or well learned ones. His response to item 5: *If $f(1)=5$ and $f(x=1)=2f(x)$, find the value of $f(3)$;* which was: **'$f(x+1)=2f(x)$, $f(x)=f({}^{(x+1)}/_{2})$; $f(3)=f({}^{(3+1)}/_{2})=2$'**, is similar to those identified by Coady and Pegg (1994) as

Concrete Symbolic since it 'characterised by quick substitutions' (p.183).

Implication and Conclusion

The shift of focus of assessment from quantitative to qualitative or both in relation to improvements in teaching appears to be widely advocated in the pretertiary level. According to Herman (1991) 'assessment practices in schools will change substantially and productively by focusing upon improving instructions through assessing student cognitive processes, student preconceptions, and the learning of relationships and structures' (p.154). Although this statement was aimed at the pretertiary level, the processes involve are those that are much needed in tertiary as well. Herman (1991) added that the measures from such assessments 'will help to identify the causes of problems in learning and will facilitate the design of instructionally effective teaching strategies' (p.154). The educational values highlighted in these statements were those examined by this exploratory study through use of both qualitative and quantitative assessments. The analysis of responses by S4 and S10 have shown that both types of assessment provided indepth data on cognitive processes and on the ability of these students in mathematics. As such, this exploratory study has illuminated several important issues relating to assisting students who enter university with insufficient backgrounds in mathematics: (1) Their inability to retain and recall knowledge which they perceive as irrelevant to their present learning. This was demonstrated by these students inability to recall 'corrected' responses given during feedback session, to repeated items in the post-conceptual assessment. (2) A high score in final examinations does not necessarily equate to high level understanding as related to Formal mode of intellectual understanding. Rather, as demonstrated by S10's responses, the high score may be due to highly developed skills in memorisation of procedural knowledge. (3) Perhaps the most significant issue is the carrying out of both types of assessments prior to and after the program of study. As demonstrated by the analysis of S4 and S10 responses that such provided not only current and richer but also valid information about these students mathematical potentials.

The issue which is of a real concern for university educators is the one relating to the teaching of knowledge which is considered by educators to be relevant and perceived by students as irrelevant, as in (1). For example, an understanding of function notation is considered relevant to the learning of calculus by maths educators but the reverse was depicted by the responses in this study. According to Marton (1988), this phenomenon is rather common at tertiary level settings. He reported several studies in these settings in which students tend to favour 'acquiring huge bodies of knowledge without appropriating the conceptualisations on which those bodies of knowledge are based' (p.75).

In this exploratory study the focus was on the content of learning instead of instructional setting or other influential factors such as motivation (McCombs, 1991), however, the researchers were conscious of the influence these can have on learning. For example, the 30 hours of instruction in five days and the fact that enrolment in the Engineering study program was conditional upon passing. These factors undoubtedly would force the students to adopt strategies that facilitated their learning without appropriate conceptualisations. Hence, further investigations on the relationships among learning strategies, content of learning and instructional settings are needed to provide more information that can contribute to assessment and teaching of mathematics at the tertiary level.

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Appendix: Part A

Conceptual Mathematical Assessment (Pre-course)

1. i) $2a + 3a =$ iv) $(a + b) - b =$
   ii) $2a + 5b =$ v) $3a - b + a =$
   iii) $2a + 5b + a =$
2. Please answer the following questions:
   a) Which is larger, 2n or n+2? Explain.
   b) i. If $a+b = 43$, $a+b+2 =$ ii. If $n-246=762$, $n-247 =$
   iii. If $e+f = 8$, $e + f + g =$
3. If $(x + 1)^3 + x = 349$ when $x=6$, what value of x makes $(5x +1)^3 + 5x = 349$ true?
   [ADAPTED FROM COADY & PEGG, 1993d, 1993b]
4. a) If y is increased by t, find an expression for $3y^2 + 2y$.
   b) Given $f(x) = -2x^2 + 3x$, find $f(x + h) =$
5. If $f(1) = 5$ and $f(x + 1) = 2f(x)$, find the value of $f(3)$.
   [ADAPTED FROM COADY & PEGG, 1994]
6. Find the value of Δ in the following statement:
   $(72 \div 36) \times 9 = (72 \times 9) \div (\Delta \times 9)$
7. Given that: $2^3 = 2 \times 2 \times 2 = 8$
   $3^2 = 3 \times 3 = 9$
   $\Delta^2 = \Delta \times \Delta$
   Question a) Find the value of 4^2.
   Question b) Find the value of 5^4.

Question c) What is the value of ' Δ ' if $(\Delta + 1)^3 = 64$?

Question d) If $(c + a + 1)^3 = 512$, what pairs of whole number values can 'c' and 'a' take between 0 and 7?

[ADAPTED FROM COLLIS & WATSON, 1991]

Conceptual Mathematical Assessment (Post- course)

1. If p is a real number, discuss the following: $\frac{1}{P} \rangle P$

2. What can you say about X given the following expression:

$$\frac{\sqrt{4-x^2}}{x+1}$$

[ADAPTED FROM COADY & PEGG, 1993a]

3. If $\frac{a}{b} = 4$, find the value of $\frac{a+b}{a-b}$

4. If p = 2q and q = st, find pq in terms of t, given that $s = \frac{1}{2}$

[ADAPTED FROM COADY & PEGG, 1993c]

5. If x is increased by t, find an expression for $3x^2 + 2x$.
6. Given $g(x) = -2x^2 + 3x$, find $g(x + h) =$
7. If f(1) = 5 and f(a + 1) = 2f(a), find the value of f(3).

[ADAPTED FROM COADY & PEGG,1994]

8. Find the value of K in the following statement:

$$\frac{72}{36} \times 9 = \frac{72 \times 9}{4k}$$

9. Given that: $2^3 = 2 \times 2 \times 2 = 8$
$3^2 = 3 \times 3 = 9$
$\Delta^4 = \Delta \times \Delta \times \Delta \times \Delta$

Question a) Find the value of 7^2.

Question b) Find the value of 3^4.

Question c) What is the value of ' Δ ' if $(\Delta + 1)^6 = 64$?

Question d) If $(x + y + 1)^3 = 512$, what pairs of whole number values can 'x' and 'y' take between 0 and 7?

[ADAPTED FROM COLLIS & WATSON, 1991]

Part B:Responses to Function Notation Items (Post-course in shade)

ID	*If y is increased by t, find an expression for:* $3Y^2+2Y$	*If x is increased by t, find an expression for:* $3X^2 + 2X$	*Given* $f(x)=-2X^2+2X$ *find* $f(x+h)=$	*Given* $g(x)=-2X^2+2X$ *find:* $g(x+h=$	*If* $f(1)=5$ *and* $f(x+1)=2f(x)$, *find the value of* $f(3)$	POST
S1	$3(y+t)^2+ 2(y+t)$	$3(x+t)^2 + 2(x+t)$	$f(x+h)=$ $-2(x+h)^2+3(x+h)$	$g(x+h)=$ $-2(x+h)^2+3(x+h)$	$f(2)=2x5$, $f(2)=10$ $f(3)=2x10=20$	SAME as
S2	$3(y+t)^2$ + $2(y+t)$	$3x^2+6xt+3t^2+2x+2t$	$-2(x+h)^2+3(x+h)$	$g(x+h)=$ $-2(x+h)^2+3(x+h)$	when $x=0$, $f(x+1)=$ $2f(1)=5/2$, when $x=2$ $f(2+1)=2(2.5+2)=9$	$f(1)=5$ $f(2)=10$ $ff(3)=20$
S3	N/A	$3(x+t)^2 + 2(x+t)$ let $y=(x+t)$, $3y^2+2y$	N/A	$g(x+h)=$ $-2(x+h)^2+3(x+h)$	$f(1)=5$, $f=5/1$, $f=5$ $f(3)=5x3=15$	$f(3)=20$
S4	$f(t)= 3y^2+ 2y$	$\frac{dy}{dx} = 3x^2 + 2x$	$-2(x+h)^2+3(x+h)=$ $-2x^2-4xh-$ $2h^2+3x+3h$	$g(x+h)=$ $-2(x+h)^2+3(x+h)$	$f(1)=5$,$f(2)=10$ $f(3)=20$, EASY double of $f(x) = 2f(x)$	N/A
S5	$3y^2t+ 2yt$	$(3x^2+t) + (2x+t)$	*difficult, no answer	$-2x^2-2h^2+3x+3h$	Difficult, no answer	N/A
S6	$(3y+t)^2$ + $(2y+t)$	$x>t$, $3x^2 + 2x$ $x(3x$ $+2);x+t(3x+3t+2)$	$-(2x-h)^2 + (3x+h)$	$g(x+h)=$ $-2(x+h)^2+3(x+h)$	15?	$f(3)=?$
S7	$3(y + t)^2$	$3(x+t)^2 + 2(x+t)$	$f(x+h)=$ $-2(x+h)^2+3(x+h)$	$g(x+h)=g'(x)=-4x+3$ $g'(x+h)=-4x + 3 + h$	$f(3)=15$,since $f(1)=5$ $f=5/1$, $f=5$	15
S8	$t(3y^2+ 2y)$	$t(3x^2 + 2x)$	$f(x+h)=-$ $2x^2+3x+h$	N/A	$f(3)=?$	?
S9	$(3y^2+ 2y)\ t$	$t(3x^2 + 2x)$	$-2(x+h)^2+3(x+h)$	$-2(x+h)^2+3(x+h)$	$f(3)=15$	?
S10	N/A	$(3x^2+t) + (2x+t)$	N/A	$(-2x^2+h) + (3x+h)$	$f(x)=f(x+1)/2$,$f(3)=2$	same as