Mental Computation, Computational Estimation, and Number Fact Knowledge for Addition and Subtraction in Year Four Children

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The aim of this study was to investigate the relationship between mental computation, computational estimation, and number fact knowledge. Clinical interviews were conducted with thirty-two students who were participating in a longitudinal study of mental computation. They were interviewed for mental computation, computational estimation, and number fact strategies. Results showed that proficient mental computers were also good at computational estimation and exhibited mastery of number fact recall. In addition, the data indicated that children in year 4 are capable of inventing their own valid and efficient mental algorithms.

Background

Sowder (1988) defined mental computation as "the process of carrying out arithmetic calculations without the aid of external devices". For the purpose of this paper, Sowder's definition is refined to exclude number facts solutions as mental computation. Trafton (1978) described mental arithmetic as referring to "non standard algorithms for computing exact answers" without the use of pen and paper. Although most calculations are performed mentally to some extent, and the traditional pen and paper algorithm may be used mentally, mental arithmetic refers to the employment of a non traditional algorithm calculated mentally.

Research suggests that mental computation should play a major role in the changing curriculum (Coburn, 1989). Furthermore, there is increasing awareness of the role of mental computation as a valid computational method as well as the contribution it makes to mathematical thinking as a whole (Australian Education Council, 1991; Beberman, 1985; Cobb & Merkel, 1989; Cockcroft, 1982; Jones, 1989; Reys, 1985; Reys & Bargen, 1991; Sowder, 1990). Reys (1985) stated that the major advantage of mental computation lies in helping students develop flexibility in dealing with numbers. When students are engaged in mental computation they develop an understanding of the number system and its properties, and often engage in invention of alternative algorithms. Everyday mathematics problems are primarily solved by employing mental computation or computational estimation (Maier, 1977; Clarke & Kelly, 1989). Reys, Bestgen, Rybolt, and Wyatt (1982) reported that good computational estimators are also good mental computers; however, good mental computers need not necessarily be good estimators. Further, Sowder and Wheeler (1989), in their study on computational estimation, stated that the ability to compute mentally and estimate were related skills. Basic fact knowledge and mental computation were also identified as being related (Sowder & Wheeler, 1989; Hope & Sherrill, 1987). However, there are few studies that have considered all three skills simultaneously, especially at the primary school age.

Thus the aim of this study was to

- investigate the relationship between mental computation, computational estimation, and number fact knowledge for addition and subtraction in year 4 children,

- identify strategies students employ in mental computation, computational estimation, and number fact solutions, and
- identify the characteristics of students who employ mental strategies in relation to their accuracy in mental computation, proficiency in computational estimation, and proficiency in number fact knowledge.

Methodology

Subjects

A 5-year longitudinal project of mental computation began in 1991 with 130 year 2 students from 13 classrooms in schools serving students from mixed socio-economic backgrounds (Irons & Cooper, in progress). The year 2 teachers were invited to select a cross section of students, one-third from each of upper, middle, and lower mathematics ability. The students were interviewed twice a year to identify mental strategies for addition and subtraction.

This study reports upon 32 students from the population in the longitudinal project. After the students had been interviewed in the last term of year 4, 32 were chosen for this study on the basis of whether they employed the traditional pen and paper algorithm only (16 algorithmic mental computers - AMC) or whether they also employed mental strategies (16 mental computers - MC) in the addition component of the mental computation interviews. Students were reallocated for subtraction, as their approach to addition and subtraction were often inconsistent, that is, students who were AMC for addition were not necessarily AMC for subtraction. Each of the 32 students participated in further interviews for computational estimation and number fact knowledge.

Interviews

Each student participated in three videotaped interviews; mental computation, computational estimation, and number fact knowledge. A written number facts test was also included as part of the number fact interviews. The students were withdrawn from their classrooms and interviewed and videotaped individually in a quiet room. Piaget's revised clinical interview was used for data collection. This interview technique has been successfully used by Ginsburg (1977) to generate spontaneous verbalizations, and to elicit and describe processes used when problem solving. The mental computation and computational estimation questions (see Figures 1 and 2) were presented visually in the form of pictures accompanied by printed numbers, and then orally as the interviewer verbalized the problem. Questions included 1-, 2- and 3- digit real world addition and subtraction problems. The students were asked to calculate the answers mentally, and then explain how they arrived at the solutions. The number facts tests consisted of 8 addition and 8 subtraction facts to 20, presented in written format (for instance, 7+8, 8+5, 11-8). The students were encouraged to write the answers as quickly as possible, and after completing each set (addition facts were completed first), explain how the examples were solved.

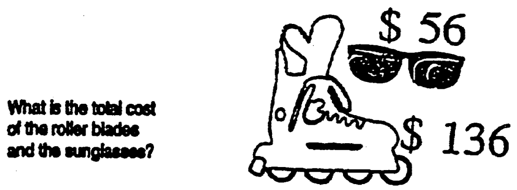

Figure 1

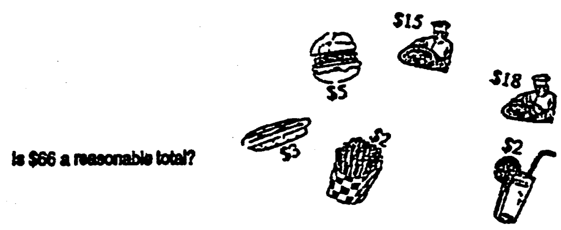

Figure 2

Analysis

After viewing the videotapes of the mental computation interviews, a coding system for strategy identification of each question was developed, based on Beishuizen (1993) and Irons and Cooper (in progress) (see Table 1).

Table 1 Strategies identified in mental computation and codes

Process	Codes	Description	Example for 49 + 42
	1	separated place value	40+40=80, 9+2=11, 51
left→right	2	separation and aggregation	40+40=80, 80+9=89, 89+2=91.
	3	aggregation	49+40=89, 89+2=91.
	4	separated place value	9+2=11, 40+40=80, 91.
right→left	5	separation and aggregation	9+2=11, 11+40=51, 51+40=91.
	6	aggregation	49+2=51, 51+40=91.
	7	compensation - undoing	50+42=92, 92-1=91.
wholistic	8	compensation - levelling	50+41=91.
	9	just know or use whole number	

As accuracy was to be considered in the analysis of mental computation, students were also allocated to levels of accuracy (level 1 - high, level 2 - medium, level 3 - low).

Secondly, after identifying the strategies employed in the computational estimation interviews (based on Reys, Bestgen, Rybolt, & Wyatt, 1980; 1982; Reys, Reys, Nohda, Ishida, Yoshikawa, & Shimizu, 1991) (see Table 2) and degree of accuracy, three levels of proficiency were identified for computational estimation.

Table 2 Strategies identified in computational estimation

Strategy
truncation
rounding
compensation during solution
compensation after solution
truncation and rounding (combination)
truncation and compensation after solution (combination)
rounding and compensation during solution (combination)
rounding and compensation after solution (combination)
wholistic
calculated
no strategy

Thirdly, to account for both speed and accuracy for number fact knowledge, scores were calculated by dividing the number correct by the time taken to complete the questions, thus a score between 0 and 1 was obtained (higher scores indicating greater mastery). Number fact strategies were also coded (based on Madell, 1985; Steinberg, 1985; Thornton, 1990) (see Table 3). These strategies could be divided into *count* (1,2,3,4, and 10), *derived facts strategies* (*DFS*) (5,6,7,8,and 12), and *immediate recall* (9).

Table 3 Number fact strategies and codes

Code	Strategy	Example
1	count back by subtrahend	9-3: 9,8,7,6.
2	count back to subtrahend	15-8: 15,14,13,12,.....;answer 7
3	count on from smaller number	4+7: 4,5,6,...11.
4	count in from larger number	4+7: 7,8,9,10,11.
5	through 10	8+5: (8+2)+3=10+3=13.
6	doubles	7+8: (7+7)+1=14+1=15.

7	use addition (for subtraction)	15-8: 8+7=15, }15–8=7.
8	use another fact	3+9: 9+2=11, }9+3=12.
9	immediate recall	
10	count on (for subtraction)	11–8: 8,9,10,11.
11	guess	
12	pattern	9+6: 1 λεσσ τηαν 6 ισ 5, }15.

In Table 4 the categories to which students were allocated for analysis are summarized.

Table 4 Categories for analysis and range of scores

Method of mental computation	1 MC
	2 AMC
Accuracy in mental computation	1 high
	2 medium
	3 low
Computational estimation	1 high
	2 medium
	3 low
Number fact knowledge	score range 0 - 1

Results

Those students who were accurate in mental computation and MC (that is, proficient mental computers) were also proficient in computational estimation, scored well in the number facts tests, and employed more advanced strategies in all three sets of interviews. In other words, proficient mental computers were also proficient in computational estimation and number fact knowledge. Wholistic strategies featured across mental computation and computational estimation interviews for these students. Further, proficient mental computers used *recall* and *DFS* for the number fact solutions, rather than *count* strategies.

A variety of mental computation strategies was identified in this study. Although the pen and paper algorithm approach was predominantly used, mental strategies exhibited diversity for each question and over all questions. Further, a far greater variety was evident for subtraction than for addition. The pen and paper algorithm performed mentally was more conspicuous in addition than in subtraction. However, this strategy, although resulting in fewer errors, produced more short term memory errors than mental strategies. In contrast to the high frequency of *separated place value* strategies (particularly *right to left*), *aggregation* was employed by few students. This is consistent with findings of Beishuizen (1993) who also reported that Dutch children found *aggregation* a difficult strategy to learn. The lower error rate for this strategy may be due to less load on working memory, as the answer is assembled as calculation progresses.

The number fact strategies identified in this study reflect those in the literature. *Immediate fact recall* was the dominant strategy, although accounting for only 50% of the solution strategies. *Immediate fact recall* and *DFS* featured more with the higher scores for the number facts tests, and *count* strategies more with the lower scores. The *count back by subtrahend* strategy was by far the most popular count strategy employed. This conflicts with research findings of Baroody (1984), Thornton (1990), and Carpenter and Moser (1984) that indicated the *count up* strategy is more popular than *count back* with younger children. The reason for this may be that classroom teaching practice in Queensland emphasises the *count back* strategy. Initial instruction for modelling subtraction encourages students to remove counters from the original set. In the mental computation interviews, most students reported using *recall* for the calculations of the problems.

The predominant computational estimation strategies were *truncation, rounding, compensation,* and *truncation and compensation after solution. Truncation* and *rounding* resulted in the greatest percentage of errors, as it appears these strategies were often indiscriminately and inappropriately applied (*rounding* more so than *truncation*). This may reflect classroom emphasis on *rounding* as the predominant estimation strategy, often practised out of context.

Discussion

Although MC were no more accurate than AMC, their facility to work with numbers appeared to be better developed as evidenced by their flexibility in the mental computation interviews, proficiency in computational estimation, and the use of more advanced number fact strategies. Proficient mental computers (MC and level 1 accuracy in mental computation) selected from a variety of more advanced mental computation, computational estimation, and number fact strategies. Thus, proficiency in mental computation, computational estimation, and number fact knowledge are related. Whether number fact knowledge is a prerequisite or whether it develops in conjunction with computational estimation and accuracy in mental computation cannot be deduced from this study. However, it could be suggested that each may be improved in the context of developing all three.

Reys et al. (1982) reported that good computational estimators are also good mental computers, however, good mental computers need not necessarily be good estimators. In contrast, evidence from this study has shown that all the accurate students who employed mental strategies (that is, proficient mental computers) were proficient computational estimators. However, not all good estimators were accurate or used mental strategies. That is, proficient mental computers were proficient estimators, but not all proficient estimators were proficient computers. Furthermore, students who employed *wholistic* strategies for computational estimation were almost exclusively those students who used mental strategies in the mental computation interviews.

Sowder and Wheeler (1989) and Hope and Sherrill (1987) also identified basic fact recall and the ability to compute mentally as related skills. When the number fact strategies were analyzed, it became evident that the MC generally used more advanced number fact strategies (*immediate fact recall* and *DFS*) than AMC. Thus, this study supports the notion that the ability to compute mentally and basic fact mastery are related skills.

Evidence suggests that students are capable of formulating their own legitimate strategies for mental computation and computational estimation. Although many students were not confident in computational estimation (they preferred to calculate or guess), many of the strategies reported are not taught in the classroom. Further, it was evident in this study that students had developed mental computation strategies without formal classroom instruction. Given the importance of mental computation and computational estimation in the context of number sense, it is imperative that methods which enhance and build upon natural skills be developed. To do this, it is essential that students verbalize strategies in classroom/group discussions regularly, and be able to choose from a variety of procedures. It is also important to recognize that, while particular strategies are meaningful to one student, teaching alternative and prescriptive strategies may cause confusion. Individual students had developed idiosyncratic strategies that were meaningful to them, and imposition of alternative strategies may not be effective.

Many students resorted to the traditional pen and paper algorithm without first considering the numbers involved, presumably because of the emphasis of the traditional pen and paper algorithm in the curriculum. Less emphasis on the algorithm and more on developing students' legitimate, spontaneous strategies may result in a better understanding of number. Curriculum changes should recognize that students construct mathematical knowledge; it cannot be transmitted. Thus, teachers should allow students to actively participate in their own learning and construct their own knowledge.

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