Mental Computation in Australia, Japan and the United States

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Purpose of the Research

The teaching of mental computation is advocated in national statements of all three countries, but only in Japan is there consistent teaching of mental computation strategies and precedence given in the early years to mental computation over written computation (AEC, 1991; NCTM, 1989; Shibata, 1994; Reys & Barger, 1994).

Despite its perceived importance, few efforts have been made to assess performance levels on mental computation. Mental computation is not assessed on standardized achievement tests nor has it been addressed within international assessments (Shigematsu, Iwasaki & Koyama, 1994; McKnight, Travers, Crosswhite & Swafford, 1987).

The major purpose of this study was to explore performance on a variety of mental computation tasks using two presentation formats (visual and oral).

Methodology

Sample

The sample consisted of students in three countries (Australia, Japan, and the United States) at four grade or year levels. In Japan and the USA students in grades 2, 4, 6 and 8 were assessed; in Australia data were obtained from students in years 3, 5, 7 and 9. The selection of these grades/years provided data on students of approximately the same ages in the three countries.

Instruments

A Preference Survey (PS), an Attitude Survey (AS) and a Mental Computation Test (MCT) was constructed for each year level.

The Preference Survey focussed on which calculations students would prefer to do mentally and provided one perspective of mental computation. Most items in the PS were also included in the Mental Computation Test (MCT), but four very difficult 'checker' items ($^4/_7 + ^2/_5$, 14 x 83, 35 x 55 and 0.35 x 567) were included to provide a check on the validity of the PS instrument.

The Attitude Survey focussed on six aspects of students' attitudes to mental and written calculation. The MCT was designed for group administration and contained two parts: oral items (read individually by the administrator) and visual items (presented individually using an overhead projector).

Results

Preference Survey

More than 40 per cent of year 5 students said they would not attempt the item 100 x 35 mentally and between one third and one quarter of year 7 and 9 students would not calculate 945 x 1000 mentally. This suggests that many students lack conceptual understanding rather than computational skill. This lack of conceptual understanding is also apparent in the results for the item 0.1 x 45, for which fewer than half of year 7 or year 9 students would use mental computation in spite of the simple computation involved.

In order to determine whether students scoring high and low on the MCT differ in their selection of items to compute mentally, student responses were sorted by first, middle and fifth quintiles according to their total score on the MCT.

An examination of Table 1 indicates that there is a marked difference in the preferences of high and low performers on the MCT. Students who are more skilled at mental computation tend to prefer this method over others, while less skilled students tend not to opt for a mental computation approach.

Table 1. Percentages of Students in First (H), Third (M) and Fifth (L) Quintiles of Australian Third, Fifth, Seventh and Ninth Years Preferring Mental Computation for the Given Calculations

Item	Yr 3			Yr 5			Yr 7			Yr 9		
	L	M	H	L	M	H	L	M	H	L	M	H
165 + 99	24	42	61	30	67	82	55	70	88	77	93	100
945 x 1000							30	79	97	43	87	97
0.1 x 45							33	52	67	17	50	77

Table 2 shows computational preferences on one typical item according to gender. In all years more boys than girls opted for mental computation, though the results overall are only significant at years 3 and 9.

Table 2 Percentages of Students in Third, Fifth, Seventh and Ninth Years by Gender Preferring Mental Computation for the Given Calculations

Item	Year 3		Year 5		Year 7		Year 9	
	F	M	F	M	F	M	F	M
165 + 99	33	44	55	66	69	71	84	95

Attitude Survey

The categories in the Attitude Survey (AS) were Interest and Enjoyment, Perception of Competence, Perception of Value, Perception of Use and Perception of Source of Instruction. The AS was not administered to year 3.

Table 3 gives responses to selected items from the survey. Increasingly as they mature students perceive written computation to be less important than mental computation, but that more attention is paid in school mathematics instruction to written than to mental computation.

Table 3 Percentages of responses of 'Yes' 'No' and 'Not Sure' by Australian Fifth, Seventh and Ninth Year Students to some Statements Reflecting Attitudes

	Yr 5 (n=163)			Yr 7 (n=163)			Yr 9 (n=152)		
	Y	N	NS	Y	N	NS	Y	N	NS
Perception of Value									
26. It is more important to be good at written than mental computation	27	29	44	14	41	45	8	41	41
Perception of Source of Instruction									
20. I learned to do mental computation at school	56	29	15	54	25	21	39	39	22
28. I learned to do written computation at school	65	23	12	68	14	18	78	12	10

Mental Computation Test

Although the samples for this research were carefully drawn from each country participating in the study, they are not claimed to be representative of the populations of the countries involved. Therefore, comparisons between countries must be rather tentative. The discussion of the data which follows is based on trends observed in the item analysis, and on clusters of common items (same operation or number type).

Overview of Results

Table 4 reports a summary of statistics for the MCT from the Australian, Japanese and American samples at each of the four year levels.

Table 4 Summary of Japanese/American/Australian Performances on the MCT

	Yr 2 JA	Yr 2 US	Yr 3 AU	Yr 4 JA	Yr 4 US	Yr 5 AU	Yr 6 JA	Yr 6 US	Yr 7 AU	Yr 8 JA	Yr 8 US	Yr 9 AU
N	176	137	163	187	141	163	186	125	163	206	119	152
Max.	30	30	30	30	30	30	40	40	40	40	40	40
Range	0 - 30	1- 29	0 - 29	0 - 30	0 - 22	0 - 29	4- 40	0 - 38	2 - 40	3 - 40	2 - 40	12-40
Mean	19.45	12-15	12.27	18.09	9.17	13.55	28.39	16.34	26.55	28.85	23.2	30.63
SD	7.42	5.81	6.21	5.67	4.45	6.67	9.14	8.97	9.13	7.44	11.61	6.85

While the Japanese mean score was the highest of all three countries at each level except relative to Australia in year 8/9, the Japanese standard deviation was also the highest of all three countries at years 2/3 and 6/7, higher than the standard deviation of the USA scores at year 4/5, and higher than the standard deviation of the Australian scores at year 8/9. Thus, in spite of a general perception of homogeneity in Japanese classrooms, these data suggest that Japanese children are as diverse in their abilities in this area as their American and Australian counterparts.

At the year 2/3 level the performance of the Japanese sample far exceeded those of the USA and Australian samples in both addition and subtraction of whole numbers, which comprised 24 of the 30 questions. However this initial superiority, which is well documented by other researchers (see for example Stigler, 1990) is not maintained. Table 4 shows that the gap narrows with age and that, by year 8/9, Australian performance has passed that of the Japanese sample.

An implication may be that early instructional emphasis on formal written (or mental) algorithms is not necessarily beneficial in the longer term. On the contrary, it may represent wasted or misplaced effort if undue time and attention is given to formal computation at an early age at the expense of conceptual learning and a wider curriculum.

Effects of Visual Versus Oral Presentation

Table 5 shows that, in general for students in the USA and Australia samples, success rates were affected only marginally by mode of presentation, whereas (particularly for more complex additions and subtractions) the visual mode of presentation produced dramatically better success rates for Japanese students than for the same item given orally.

Table 5 Mean Percentage Score by Year and Country on all Items*

	Japan			USA			Aus		
Mean score	Oral	Vis.	V-O	Oral	Vis.	V-O	Oral	Vis.	V-O
Year 2/3	60	70	11	40	40	0	41	41	0
Year 4/5	55	66	11	26	35	9	45	45	0
Year 6/7	65	77	12	38	44	6	65	68	3
Year 8/9	69	76	8	58	58	0	75	78	2

* V-O refers to visual mean minus oral mean.

Items presented visually, particularly those in horizontal form, may have produced higher performance for children in the Japanese sample owing to familiarity. Japanese textbooks typically present straight computations (those devoid of context) horizontally until the middle of year 2, whereas the typical presentation in most American and Australian texts and in classroom presentations is the vertical form. It is also possible that higher performance on the visually represented items may indicate more success in applying a mental version of the standard paper/pencil algorithm, whereas higher performance on an orally represented item may indicate success in applying a more efficient mental strategy. Certainly, one could argue that the tendency to search for and use an efficient mental strategy is more likely to occur with orally presented items.

Gender Differences

Do Australian boys and girls differ in their ability to do mental computation? An examination of the means of the MCT total scores (Table 6) shows that, while the mean of the boys was higher than the mean of the girls at each year, only at the fifth and ninth year were the means significantly different beyond the 0.05 level.

Table 6 t-test of MCT Total Scores for Australian Males and Females

Year	Sex	N	Mean	SD	DF	t-ratio	Prob
3	F	78	11.68	5.67	161	-1.16	0.25
	M	85	12.81	6.66			
5	F	84	12.17	6.73	161	-2.78	0.01
	M	79	15.01	6.32			
7	F	80	26.31	9.67	161	-0.33	0.74
	M	83	26.78	8.64			
9	F	74	29.45	6.90	150	-2.09	0.04
	M	78	31.74	6.65			

Preference Versus Performance

In the PS students were presented with a number of computations and asked whether or not they would prefer to do them mentally. At each year level, a sub-section of these items was also included in the MCT. If we assume that those students who stated that they preferred to do the calculation mentally thought they would calculate the answer correctly, then for each of these items (nine items for each of years 3 and 9 and ten for each of years 5 and 7) it is possible to classify all students into four categories:

Those who thought they could do the calculation, and could (Y Y).

Those who thought they could do the calculation, but couldn't (Y N).

Those who thought they couldn't do the calculation, but could (N Y).

Those who thought they couldn't do the calculation, and couldn't (N N).

In order to discover to what extent ability was a factor, the results were analysed by quintiles, according to the performance of each student on the complete MCT. A typical case is given in Table 7, showing the results for year 7 for the item $1/2 + 3/4$.

Table 7 Year 7 Preference Versus Performance Percentages by Quintiles for Item $1/2 + 3/4$

	YY	YN	NY	NN
Top Quintile	79	3	18	0
2nd Quintile	78	3	19	0
3rd Quintile	70	9	18	3
4th Quintile	34	25	22	19
5th Quintile	18	39	6	36
All Year 7	56	16	17	12

The most interesting result was that concerning those students who were more competent than they gave themselves credit for, correctly performing calculations which they had not felt confident they could do. In contrast to the other three groups, these students were spread quite evenly across the quintiles.

When the responses of each quintile were separated by gender, it was found that boys more often rightly expressed confidence, and girls more often unnecessarily expressed diffidence, concerning their ability to calculate mentally.

Curricular Implications

We believe the results point to certain conclusions which have direct implications for the classroom.

First, more attention should be paid to the informal teaching of mental computation, including the development of strategies, in the earlier years (for suggestions see McIntosh, De Nardi & Swan 1994).

Second, an early concentration on formal computations for all children, whether mental or written, may not be beneficial in the longer term. Indeed it may be time wasted which could profitably have been used on material at a more appropriate conceptual level.

Third, more attention should be paid to conceptual ideas, particularly with regard to fractions and decimals, since mental computation relies heavily on an understanding of the underlying principles rather than instrumental rules.

Fourth, a wider range of computations should be included in the repertoire of mental computations, since it is clear that many children have abilities in mental computation which are not harnessed by the present restrictive curricula.

Fifth, it should be recognised that presenting material visually and orally can evoke different strategies and different performance outcomes. We strongly recommend the use of oral presentation without visual clues, since it appears likely that this encourages children to explore a wider variety of solution strategies, whereas the visual presentation may incline children to lean toward a mental form of the standard written algorithm, which is often much less appropriate.

References

Australian Education Council. 1991. *A national statement on mathematics for Australian schools*. Melbourne: Australian Education Council and the Curriculum Corporation.

McIntosh, A.J., De Nardi, E. & Swan, P. 1994. *Think mathematically*. Melbourne: Longman Australia.

McIntosh, A.J., Reys, B.J., & Reys, R.E. (1995, in press). Mental computation in Japan, the United States and Australia. *Educational Studies in Mathematics*.

McIntosh, A.J., Bana, J. & Farrell, B. (1995). *Mental computation in Western Australian schools: A study of preference, attitude and performance in years 3, 5, 7 and 9*. Perth: MASTEC, Edith Cowan University.

McKnight, C.C., Travers, K.J., Crosswhite, F.J. & Swafford, J.O. 1987. *The underachieving curriculum: assessing U. S. school mathematics from an international perspective*. Champaign, IL: Stipes Publishing Company.

National Council of Teachers of Mathematics. 1989. *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Reys, B.J. & Barger, R. 1994. Mental computation: Evaluation, curriculum, and instructional issues from the United States perspective. In R.E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States*. Reston, VA: National Council of Teachers of Mathematics.

Shibata, R. 1994. Computation in Japan from the Edo Era to today: Historical reflection on the teaching of mental computation. In R.E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States.*. Reston, VA: National Council of Teachers of Mathematics.

Shigematsu, K., Iwasaki, H. & Koyama, M. 1994. Mental computation: Evaluation, curriculum and instructional issues from the Japanese perspective. In R.E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States*. Reston, VA: National Council of Teachers of Mathematics.

Stigler, J.W., Lee, S-Y. & Stevenson, H.W. 1990. *Mathematical knowledge of Japanese, Chinese, and American elementary school children*. Reston, VA: National Council of Teachers of Mathematics.