Towards a Theoretical Framework for Research in Beliefs and Values in Mathematics Education

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Following several reports by the author and a colleague, this paper attempts to synthesise some aspects of research pertaining to beliefs and attitudes in mathematics education. Possible constructs that contribute towards the development of values in mathematics education are suggested and a brief account of valuable methodologies for the study of mathematics values is given. An application of a modified repertory grid technique is described and reported as a case study of two graduate students' beliefs and values. Models for research in different aspects of values in mathematics education are recognised and implications for the classroom suggested.

A burgeoning area for research in mathematics education is the values and beliefs which are held by the participants in the mathematics education setting. This includes teachers, students, administration and the community. This renewed interest has come about through a realisation that despite the new approaches to mathematics education that have resulted from such major initiatives as the Agenda for Action (NCTM, 1980), the Cockcroft Report (Cockcroft, 1982), the Australian Mathematics Education Project and concurrent or subsequent developments in many state education systems in Australia, there still seems, according to reports from teachers, to be no tangible improvement, except in isolated pockets, either in the mathematical achievement of students in school or in their enjoyment of the subject. This lack of marked improvement has forced educators to look more carefully at some of the affective factors which contribute to learning. This is not a new phenomenon as research in attitudes towards mathematics has a fairly long history compared with research in mathematics education in general (Aiken, 1970). The fact that cognitive research has not paid a great deal of attention to the studies in affect may be because the latter appears not to be grounded in cognition and is therefore not deemed to be as reliable as cognitive studies.

This renewed interest in attitudes and other affective factors has resulted from a realisation that both cognition and affect are factors in mathematics learning. If a student does not consider mathematics to be very important to them, they will not feel inclined to put much effort into it. If a student feels he or she does not have the necessary ability to achieve good results in mathematics or that the subject is difficult, that student will not feel confident enough to try to achieve good results. The beliefs students hold about mathematics as a discipline and about themselves as mathematicians have a profound effect on their mathematical achievement, and could have a detrimental effect on other aspects of learning and life because of the feelings of inadequacy and anxiety that are engendered.

The more recent studies in affect in mathematics education have incorporated not only attitude, but also beliefs and values (Schoenfeld, 1989). Some conceptualisation of the affective schema related to values in mathematics education may be helpful in providing a structure for research in the area. McLeod (1992) developed a theoretical framework for the affective domain in mathematics education in terms of the relationship between beliefs, attitudes and emotions. An extension of this framework to incorporate values is needed. Such a framework needs to be developed on three levels. There is a conceptual level which provides a reference point for research in all the concepts which contribute to values. There is also a practical task-oriented level which is designed to provide guidance in methodology for research. Finally, there is an output level which gives an indication as to how the results of research may be incorporated into the relevant educational setting.

Conceptual Level: Concepts and Constructs

The conceptual level depends on one's beliefs as to the meaning of values and beliefs and hence is, in one sense, very personal. Several explanations of what a value is, need to be considered, as there appears to be some confusion as to what constitutes a person's values and how they differ from beliefs. Sometimes the two ideas are used interchangeably and at other times their meanings are assumed and not explained at all.

Raths, Harmin and Simon (1978) reserve the word "value" for individual beliefs that one has arrived at as a result of the 'valuing process'. The valuing process is defined as deliberately choosing, prizing and cherishing beliefs and behaviours, and actually incorporating them consistently into one's every day life.

The writer sees the essence of the difference between values and beliefs as the consistency with which an individual lives according to the values held. An individual might believe certain principles or facts and not put them into practice, whereas a person who has accepted that belief as a value, integrated it into his or her value system, will act consistently according to that belief. Someone might be hoping to buy a new car. In order to make a wise choice, that person might investigate the types of cars available, their economy, their performance and their appearance. On the basis of these investigations, the person may come to believe that the best car to buy for his or her particular purpose is a particular model. When it comes to actually buying the car, however, the person chooses one mainly because of the colour available or the lumbar seat or other characteristic not necessarily relevant to cost effectiveness. This person has held a certain belief but has acted in a way not wholly in keeping with that belief. That belief, therefore, is not grounded on a value related to the performance of the car, but one which is grounded in aesthetics or convenience.

The definition used by the writer distinguishes between values and beliefs, with beliefs of a particular kind being accepted as subsets of values. Values are the beliefs we hold which determine how we live. Beliefs are opinions we hold but do not necessarily live by. The definition and descriptions given by Hill (1991) extend the writer's simplistic definition and indicates how values in education incorporate at least two domains.

The three elements (of valuing) are:

1 A value can be described by a statement expressing a person's belief in an idea, having to do particularly with judgements of worth or obligation. Thus it has a rational or 'cognitive' element, and the belief statement involved is often referred to as a 'value judgement';

2 The intensity with which individuals believe certain ideas, especially those affecting the priorities they attach to certain kinds of experience, such as the moral, aesthetic and physical, indicates that there is an emotional or 'affective' element to valuing, and this is often (but not always) what people have in mind when they speak of 'attitudes';

3 Because one's most deeply held beliefs dispose one to act in certain ways, there is a volitional element,

> which leads us to refer to such beliefs as 'dispositions' or 'commitments'. We tend to make choices which are consistent with our value systems, that is, to live by the beliefs and values to which we attach highest priority. (Hill, 1991, p. 4)

Accepting these definitions leads to a consideration of the variety of affective elements which contribute to a learner's or teacher's values. Naming them in random order provides an extensive list. This list would include attitudes, beliefs about mathematics, beliefs about mathematics teaching, beliefs about self, beliefs about the social context in which mathematics functions, beliefs of society, emotions, motivation, disposition, attribution, self-efficacy, confidence, anxiety, cognitive autonomy, intuition and personality factors. Researchers have studied many of these elements (Brahier and Speer, 1995; Nisbet, 1991; Taylor, 1992, 1995).

Task Level: Research Methodology

At the task level of any conceptualisation, previous studies by the writer and colleague (Southwell & Khamis, 1987; Southwell, 1988, Southwell & Khamis, 1992, Southwell & Khamis, 1994a, Southwell & Khamis, 1994b) have used surveys, interviews, repertory grid, journal keeping and stimulated recall methodologies. Others (Schoenfeld, 1989) have used extensive surveys and several other qualitative and quantitative methods. There is an acceptance of the possibility of using cognitive research methods for research in the affective domain (McLeod, 1992, p. 589). In addition, there is the possibility of using cognitive studies to extrapolate results relating to affect.

Surveys have been used for a considerable length of time. Mostly, they have consisted of a series of statements to which subjects are asked to respond on a point scale. In some studies these responses can be totalled to give an overall indication of the subject's attitude. Others are used in a more qualitative sense with the researchers interpreting the responses in terms of the concept embodied in the item. Basically such surveys deal with the affective domain and cannot validly be applied to cognitive aspects of mathematics education.

Interviews can incorporate both cognitive and affective aspects of mathematics education. Various types of interviews have been used, ranging from subjects being asked to do mathematics and say aloud what they are doing to very structured clinical interviews.

Observation may seem a simple technique but it provides a very rich source of data for both cognitive and affective studies. The observation may be of a short term nature or of an extended period, even over several years.

Stimulated recall is a particular method of observation which also incorporates the remembered thoughts and feelings of the subject. The way in which the writer has used this technique is in research on problem solving (Southwell, 1990). The subject was recorded on video whilst solving problems using two cameras. The video was replayed immediately and subjects asked to recall their thoughts and feelings as the video proceeded. The cues to affective issues revealed by the video as well as by the responses of the subjects provide rich data for studies in both the cognitive and the affective domains.

Journal keeping has the advantage of allowing the learners to reflect on their mathematical experiences and to express their understanding of that experience and its implications. Journals can be used in a number of different ways (Southwell, 1993; Waywood, 1993). An examination of a learner's journal will reveal that person's grasp of the actual mathematical content as well as the emotions and attitudes which are present and therefore is a rich source of data for both cognitive and affective studies.

A further form of ethnographic research is the mathematical life history interview, used very effectively by Taylor, (1992, 1995). In this methodology, subjects are asked, with appropriate guidance, to report significant events and feelings related to mathematics throughout their lives and the researcher then seeks to make connections between events by analysing and discussing principles that arise from the subjects' words. This method can rely on oral recording of the subjects' responses or on their own personal written autobiography. For the latter mode, Taylor (1992) uses a double column format in which the subject writes his or her story in the left hand column and the researcher writes her analysis and discussion in the right hand column.

Repertory grid methodology arises from the work of Kelly (1955) and is a useful technique for examining the perceptions and beliefs of learners. It has been used extensively in studies on experiential learning. The writer has investigated students' perceptions of problem solving (Southwell and Khamis, 1987) and has developed a case study of two graduate students' views of mathematics, mathematics teaching and student learning.

Case Study The first subject, Catherine, is from an overseas English speaking country and has been in Australia for most of her schooling and for her teacher education program. She is currently a Year 1 teacher in a large primary school in a middle western suburb of Sydney. This school has a large population of students from over twenty non-English speaking backgrounds, of which the major language is Arabic. She has been teaching for two and a half years and has recently enrolled in a coursework MEd. She has a record of high achievement in mathematics in her undergraduate course and is highly motivated in her efforts to improve the effectiveness of her teaching of mathematics. This is evidenced by her enrolment in the MEd and also by her participation in professional development courses provided by the region or professional associations.

The second subject, Bertram, is from a non-English speaking country and has only been in Australia two years. His grasp of English is quite good. He has a fairly academic background in the formal study of mathematics and trained as a secondary teacher in his home country. He is not currently employed on a full time basis. He has provided information about the typical teaching procedures in his home country. This information indicates that the teaching is very formal and textbook oriented. Students work individually and assessment is normally in the form of paper and pencil tests.

Both subjects were asked to indicate their thoughts and feelings about several mathematical notions, namely mathematics itself as a discipline, the characteristics of good mathematics teachers, the process and experience of mathematics teaching, mathematics curricula and mathematical problem solving. They did this by following a simplified version of the repertory grid methodology in which they wrote on cards words or phrases they associated with each of the notions involved. They then were asked to take three cards at a time, decide which two were most alike and which two were most different and give their reasons. From the responses, the poles for that criterion were established and the other cards could then be located on the continuum between the two poles.

From this simple procedure, it was clear that these two subjects had very similar perceptions of what they liked about mathematics and the characteristics they value in mathematics teachers. They both enunciated the poles 'interesting' and 'not interesting' for the construct related to their perception of mathematics as a discipline. Catherine added the word 'fun' to 'interest'. It would appear that her belief is that the ability to have fun in mathematics created the interest which she valued so much. This was later confirmed in discussion concerning the polarisation process. Bertram was not so concerned with fun, leading the researcher to conclude that mathematics could be serious and interesting. Again, this was confirmed in later discussion. These attitudes are in keeping with the mathematical background of the two subjects in that Catherine's values have been largely fashioned by her experiences in teacher education and her experience in teaching young children, while Bertram's fairly formal education in mathematics and teacher education would no doubt lead to a more academic approach to mathematics and a slightly more restrained value. Further probing is needed to determine whether both subjects have the same idea of what makes the subject interesting. This question was asked of the two subjects but they were unable at the time to articulate exactly what it was that created the interest, except in the case of Catherine who simply reiterated that fun was an essential ingredient.

The two subjects also believe that teaching mathematics is interesting. In this case, the poles selected were 'interesting' and 'boring'. As with the previous construct, clarification as to what constitutes interest was limited to the idea of fun in the case of Catherine, i.e. helping the children to enjoy their mathematics, while Bertram expressed the need to enjoy his own teaching.

The beliefs of the two subjects about the characteristics of a good mathematics teacher also seem to be fairly similar. The poles selected were 'hard-working' and 'lazy' with the characteristics most closely situated to 'hard-working' on the continuum being 'knowing the subject'. The placement of this third characteristic differed only slightly for the two subjects. It would appear that they both value highly hard work and knowledge in mathematics teachers. What each meant by 'knowledge' was questioned. Bertram sees knowledge in terms of the discipline of mathematics and the content to be taught to the school students, while Catherine included strategies for teaching as well. Again these responses are in keeping with the backgrounds and situations of the two subjects. In the discussion, Catherine indicated that hard work on the part of the teacher would include having time to help students.

The beliefs of the two subjects about what should constitute a mathematics curriculum were different. Catherine on one hand displayed a fairly sophisticated belief that connectedness is an essential element of a mathematics curriculum and that mathematical concepts, skills and principles should be related to each other. Bertram concentrated his views on topics and applications of mathematics and could not articulate any general curriculum principles. This is not surprising considering his background and his limited experience of Australian education. When pressed for further information concerning additional elements of a mathematics curriculum, Catherine gave a list of mathematical notions, including numbers, multilinks, problems, algebra, budgeting and even said 'yippee'. Bertram resonated to the ideas of budgeting, problems and algebra.

When asked to list the elements of mathematical problem solving, both subjects suggested 'number', 'quantities' and 'teacher'. From these Catherine selected the poles 'number' and 'teacher' and placed all the suggested elements on the continuum as in Figure 1. Bertram was very unsure about this construct and merely followed Catherine's lead.

Number	Quantities	Shapes	Equipment	Teacher

Figure 1 Continuum for mathematical problem solving construct

From this it would appear that the two subjects see mathematical problem solving as being related to different branches of mathematics but not having a great deal to do with the teacher. Problem solving may be regarded as an individual activity which the students do alone without a great deal of input from the teacher. This may be an appropriate response in that problems are personal, i.e. they are relative to each person. The insertion of the element 'equipment' is interesting and appropriate for Catherine, teaching as she is in a Year 1 class. It would appear that instructional materials of a concrete form are seen to be necessary, at least part of the time, for problem solving.

From these brief descriptions and conclusions concerning various constructs in mathematics education, a general impression can be obtained of the values and beliefs held by one of these subjects and, to a lesser extent, by the second subject. It would appear that they both value mathematics as an interesting subject that they like to teach because it is interesting. They value hard work and knowledge of their subject in teachers and, at least one of them believes that the mathematics curriculum should emphasise the connectedness of various aspects of mathematics and its relationship to other disciplines. Mathematical problem solving is viewed as an personal activity incorporating different branches of mathematics. The general impression gained through the polarisation process and the subsequent discussion was that both subjects value mathematics as a whole and are highly motivated to teach mathematics effectively. This motivation is apparent despite some unhelpful negative emotional memories of previous experiences and past feelings of inadequacy in relation to algebra expressed by Catherine.

The process described above, though it is only a modification of the repertory grid technique, has produced quite a considerable amount of affective information about the two subjects. The stimulus of the polarisation process can be further developed and quantified if felt desirable (Cohen & Manion, 1980).

This exploratory study has incorporated a conceptual level with the various constructs considered, a practical level in that the subjects had to relate their thoughts and feelings to their experiences in order to develop the continuua, and an output level in which the subjects were given the opportunity to reflect on their actions, experiences and thoughts. In this way, this study has exemplified the necessary elements of a possible framework suggested above.

This activity has involved a cognitive element in the constructs considered and the value judgements implied, an affective element in the degree to which feelings or emotions have been indicated, and a volitional element in the obvious motivation held by the subjects in their quest to become better teachers of mathematics. In this way, these two subjects have been involved in values.

Output Level: Classroom Implications

There appears to be a link between students' values and beliefs and achievement in mathematics. Teachers who have an understanding of their students' emotions, attitudes, beliefs and values related to mathematics and mathematics education will be in a better position to provide maximal learning conditions for those students than those who do not have that understanding. It follows, then, that strategies which assist teachers develop that understanding are important for all teachers. Several valuable strategies are available, each with particular benefits for specific situations. Those that incorporate both cognitive and affective aspects of learning mathematics will be valued above those that involve only cognition or affect. Those who reinforce positive attitudes will enable students to move through the cognitive, affective and volitional levels with ease and pleasure, thus enhancing mathematical learning.

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