Examining pre-service teachers' mathematical writing

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Abstract

In this study, samples of the mathematical writing of pre-service teachers were categorised using a system developed by van Domolen. Further information related to the teachers' understanding of a major concept involved in the writing was obtained by the use of a Link-sheet activity. The information gained from the two activities was compared. The Link-sheet examples suggest that these teachers possessed poorly developed concepts relating to decimal fractions, whereas the writing examples demonstrated the limited written mathematical genre used by these teachers.

Introduction

Understanding in mathematics learning involves knowing the concepts and principles related to the procedures being used and making meaningful connections between prior knowledge and the knowledge units being learned (Baroody & Ginsburg, 1986). Swing and Peterson (1988) described mathematical knowledge as 'characterised by logical connections among knowledge units' (p.54) and stated that 'making these connections is an integral part of learning and understanding mathematical knowledge' (p.54). When student learning experiences are structured in such a way that connections and associations are made among different representations of the ideas, they become meaningful and useful as they are associated with a connected network of ideas.

The use of writing activities as a method for exploring student understanding of mathematical concepts and procedures has been well documented in the recent research literature (Ellerton, 1988; Frank, 1990; Swinson & Shield, 1994). Johnson (1983) found that students who are able to write clearly about mathematics concepts probably understand them. Writing about mathematics provides opportunities for students to reflect on their understanding and enables them to negotiate meaning in mathematics (Hoffman & Powell, 1989). Furthermore Abel and Abel (1988) indicated that writing activities involve a way of thinking that requires students to organise their thoughts. While student writing tasks have been used to identify misconceptions (Keith, 1988; Miller, 1990) and to ascertain attitudes (Borasi & Rose, 1989), there appears to have been little attempt to describe the qualities of students' mathematical writing in a systematic way so that it may be used to describe their understanding of the concept or procedure being used.

Describing student writing

In a paper discussing presentations in school mathematics textbooks, van Dormolen (1985) developed a system for categorising 'textual' material. The following categories are from van Dormolens' work.

kernels: generalisations, rules and definitions which have to be learned.

aspects of mathematics: ways in which mathematical ideas and procedures may be presented - theoretical, algorithmic, logical, methodological, communicative.

levels of language: exemplary - demonstrative, related to a specific example;

relative - generalised, not related to a specific example. Withing each

level, the language may be procedural or descriptive.

These categories were further developed (Shield, 1995; Shield & Swinson, 1994) by examining the embellishments used in mathematics textbooks to support the *kernels*.

verbal words of explanation used to add meaning

graphical any type of mathematical graph

pictorial drawing or picture

diagrammatic line figure showing features of a concept

worked example demonstration of a procedure or algorithm

exercise practice of a procedure for students to attempt

data set set of data, table, related to the concept

Moreover each of these embellishments can be further described as being:

everyday set in a real life context

theoretical symbolic mathematics not related to a context

Using these categories student writing can be described and classified in a consistent manner. Moreover by categorising the written material in this way some measure of the level of understanding may be gauged.

Aims

The aims of this pilot study were:

1 to examine pre-service teachers' mathematical writing using an adaptation of van Dormolens' categorisation for mathematical textual material;

2 use a learning activity, the Link-sheet, to gain an understanding of the pre-service teachers' knowledge of the concept of a decimal; and

3 compare the pre-service teachers' expository writing about decimals with their knowledge of the concept of a decimal.

The study

The conceptual framework for this study was based on the notion that understanding in mathematics involves knowing the concepts and procedures used and being able to link these with various representations and prior knowledge. Writing which displays these characteristics would demonstrate understanding. Two groups of pre-service teachers were selected to participate in the study. One group consisted of 21 undergraduate students in their third year of a four year BEd the other group was composed of 16 post-graduate students undertaking a one year graduate diploma of education. During their first meeting in a mathematics curriculum unit all students were asked to undertake two mathematical writing tasks. The reason for doing these activities was not fully explained to the students though they were told that the tasks were important.

For the first writing task students were asked to imagine that a relative, who lived in the country, had a child entering year 8 and that the child did not understand how to perform addition of two decimal numbers. Students were asked to write a letter to the child, in sufficient detail to enable the decimal problem to be overcome. Using the system based on van Dormolens' work described above, the students' writing was examined and categorised.

The second writing task was given to provide further information concerning the conceptualisation of decimal numbers possessed by the students. The activity undertaken was the completion of a Link Sheet for the concept of a decimal number. The link-sheet had previously been developed as a conceptual learning

aid in mathematics and has also been used to evaluate the student's development of the concept being discussed (see Swinson & Shield, 1994).

Figure 1 Link sheet

Concept.....

Mathematics example	Everyday example
Diagram/picture/graph	My explanation

The Link-sheet (see fig.1) consists of an A4 page divided into four rectangular sections, one each for a Mathematical example, Everyday example, Diagram/picture, and My explanation. Earlier research (Shield & Harris, 1994 ; Swinson & Shield, 1994) suggested that use of this activity encouraged students to connect several representations of a mathematical idea or concept and to communicate their understanding of its meaning to others.

Results

The first task given to the pre-service teachers was the letter writing activity in which the subjects were required to explain the procedure for adding two decimal numbers. It was found that 68% of the scripts contained kernels (generalised statements of the procedure) written in relative procedural language. No definite pattern existed for the placement of the kernel in the script. About one third positioned the kernel before the worked example, another third had it after the example, while the remainder positioned it within the demonstration of the algorithm.

Methodological statements such as 'keep the decimal points under each other' were common, being used in 54% of the letters. Forty three percent of the writers realised the benefit of linking the new material to previously understood examples. Whole number addition exercises were demonstrated or described. In a few cases, just a reference to whole numbers was made. Making links to whole numbers may have been common but linking to the concept of a decimal number was not. Only sixteen percent either described decimal numbers or place value concepts as a link to previously known mathematics.

Demonstration examples were considered to be important with all but three letters containing at least one such example. However, only four of the teachers included practice examples for the reader to attempt. Four letters contained references to concrete materials, three included some form of table and two used diagrams. Three letters made use of the everyday example of money.

The link-sheets were not analysed using the categorisation discussed earlier. Rather they were examined more generally to gain an overview of the subjects' knowledge of the concept of a decimal number. Consequently, the findings in this section are reported in general terms. The *Mathematical example* section mainly consisted of an example of a number containing a decimal point, 73% being in this form with less than half of these containing any explanation or other embellishment. The remaining 27% provided a simple algorithm without any accompanying text. The *Everyday example* section was found to be similar in form to those in the previous section with 59% of the examples given relating to decimal currency and containing some brief explanation. A further 14% had some

form of simple example, sometimes mathematically incorrect, showing where decimal numbers are used in society.

The final two sections of the link-sheets required the teachers to be more innovative and as such there was a greater variety in the answers give. These have been broadly grouped as follows. For the *Diagram/picture/graph*, 42% used a diagram and of these about three quarters included some form of explanation. The number line was a popular form of diagram, being used by 35% with about one third including some form of written embellishment. The free writing section *My explanation* had 76% using reference to the fact that a decimal is a form of fraction. In this final section statements were short and simple.

Discussion

This examination and categorisation of pre-service teachers' mathematical writing provides an insight into their view of teaching mathematics. The pre-service teachers had not participated in any mathematics curriculum classes, consequently the writing undertaken in this research project should provide a 'picture' of their ideas, developed from their personal experiences of 14+ years of study, about what constitutes a mathematical explanation. Furthermore, having the teachers link several representations of the concept being explained provided an indication of their knowledge of the concept.

That 66% of the letters were written using relative procedural language and another 20% in exemplary procedural language indicates there is a strong belief that mathematics is best learned by stating the method or algorithm to be used. Moreover the significant use of the relative level of language suggests that these teachers believe explanations of a general nature, not linked to specific examples, are sufficient for effective student learning. Kernels which are general expressions of procedures that have to be learned were contained in 64% of the letters, adding further weight to this suggestion.

Only four of the 37 letters contained any embellishments such as diagrams or reference to some concrete material aid, adding further to the conviction that these teachers saw general procedural explanations as the method to use when teaching mathematics. Though there is some difference between the BEd and the post graduate groups when procedural language types are combined, this difference is minor.

If the aspects of mathematics category is considered in conjunction with the other two categories (kernels and levels of language), a clearer picture of the teachers' writing emerges. Fifty percent of the writing examples contained language of an algorithmic nature, that is explicit methods stating how to do an operation or procedure. A further 30% used methodological language which again is a 'how to do' instruction. Though all but three included demonstrations of the required algorithm, none described step by step how to do the algorithm by referring specifically to the numbers being operated upon. The 'how to do' descriptions and the demonstration of the algorithm appeared to be two discrete steps.

The link-sheet exercise suggests that these teachers possess poorly developed concepts related to decimals. In part support of this claim, it was found that there were a number of instances of incorrect information being used. For example, time as displayed on digital clocks was referred to and discussed as 'decimal time'. Also, when drawing a picture of the concept of a decimal, a simple dot was placed on the page. Other examples involved number lines without any numbers on them or with intervals marked and one or two fractional parts given using decimal notation.

The majority of examples used were simplistic in nature, with the 24% in an algorithmic form containing one or two numbers and an operation sign without

any elaboration . The 73% non algorithmic examples consisted mainly of an example of a decimal number, though in these examples about half did contain some form of explanation. It was encouraging to find in 'my explanation' that 76% were able to describe decimals as a form of fraction. However statements such as 'a whole number is made up of decimal places' and 'fractions seen as a number over 100' and 'a very tiny number, smaller than a standard number' indicate incomplete understanding of the concept.

Conclusion

The research discussed in this paper suggests that pre-service teachers may have a poorly developed understanding of the concepts that they will be teaching. More importantly it appears that these teachers believe that procedural language is the most suited for teaching. This is the genre used in many textbooks and so it is not surprising that after many years of using these books a belief structure is developed which assumes this to be the way to teach mathematics. If these teachers are to teach modern mathematics curricula in the manner suggested in the curriculum documents then some form of belief modification is an essential part of their pre-service mathematics curriculum program.

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