Some Teachers' Beliefs and Perceptions of Problem Solving

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This paper reports the results of a preliminary investigation into primary school teachers' beliefs about the role of problem solving in learning mathematics. A survey was used to gather teachers' favourite problems and comments about traditional versus constructivist perspectives on using problems in classrooms. At least half of those surveyed chose problems which they believed promoted learning but most teachers adopted a position that saw problem solving as an end rather than as a means in the learning process.

The problem-solving focus to learning has been advocated for some time and yet many teachers do not seem to have responded to this call for reform. This may be because teachers are not committed to this idea. They may not believe that a problem solving approach is the best way to teach school mathematics. There is a lack of information about what teachers at large believe and understand about problem solving and how they might then try to teach it. In an effort to explore teachers' beliefs about problem solving, this study examines teachers' favourite problems and their beliefs about the role of problem solving in learning mathematics. The data reported in this paper are part of a larger study which focuses on primary school teachers' beliefs about mathematical problem solving and how these beliefs influence decision making in their classrooms. It also aims to explore what factors might promote or hinder the implementation of problem solving approaches.

Different perspectives of mathematics, learning mathematics and teaching mathematics are examined. It is suggested that the perspective adopted by teachers represents their belief system about the nature of mathematics and influences the decisions which are made when teaching mathematics. In this study a survey was used to investigate teachers' favourite problems and to obtain comments about the role of problem solving in learning mathematics. The responses give an indication of teachers' beliefs about characteristics of worthwhile mathematical tasks as well as beliefs about whether a problem solving approach to learning mathematics is acceptable.

Perspectives about Mathematics and Mathematics Learning

Two distinct views or conceptions about what mathematics is and how children learn mathematics can be described. These views represent the end points of a continuum of varied conceptions which may be held by teachers. At one end, mathematics is perceived to be a fixed body of facts and procedures which are memorised and rehearsed so that students can reproduce them when required. This view has been referred to as the instrumentalist view (Ernest, 1989) and may lead to a traditional approach to teaching. At the other end, mathematics is viewed as a "science of patterns" where students are actively engaged in pattern-seeking by exploring this dynamic, evolving discipline (Schoenfeld, 1992). This view has been referred to as the problem-solving view (Ernest, 1989) and is consistent with a constructivist approach. Each of these perspectives has implications for views about how children learn mathematics and also about how mathematics should be taught (Burton, 1993).

Traditional Perspective

The traditional perspective about mathematics leads to a teaching approach which emphasises skill acquisition, speed, accuracy and answers rather than processes. The teacher is an authority and manager of learners who does not need to know a great deal about how children learn mathematics (Ernest, 1989; Battista, 1994). It is sufficient to accept that students passively receive information and if they are able to reproduce procedures then this is evidence that it has been learned. The teacher instructs, often from

a prescribed text in which all of the necessary subskills are sequenced with sets of exercises to practise. There is little need for reflection or even awareness that there may be viable alternatives to this method of teaching (Ernest, 1989).

Some teachers who use this traditional approach incorporate problem solving into mathematics lessons but they tend to view problem solving as something which is done at the end of a topic when the appropriate skills and procedures have been mastered (Burton, 1993). Wright (1992) found that many primary school teachers saw problem solving as an 'end' in that mathematics is learned so that students can solve real-life problems. For this approach, problem solving is seen as separate from or supplementary to the traditional curriculum. Authors of many textbooks reinforce this view by placing problems at the end of chapters.

Constructivist Perspective

The constructivist perspective about mathematics has led to an approach to teaching which emphasises active engagement with mathematical tasks, with children constructing their own meanings as they are confronted with learning experiences which build on and challenge existing knowledge. The teacher's role is that of a facilitator and requires considerable reflection as the teacher must observe student responses, challenge student thinking and encourage risk taking within a supportive classroom environment. Teachers need to know about how children learn mathematics (Battista, 1994). They need to constantly review student understanding and to "guide children's constructive activities until they eventually 'find' viable techniques" (Steffe & Cobb, 1988, pvii-viii).

From this perspective, problem solving is viewed both as central to the way children learn and as a means by which they learn as they are confronted with activities which are problematic (Wright, 1992). This approach provides the opportunity for children to reflect on current procedures and develop more sophisticated meanings and strategies. Problems are used as the focus of lessons with mathematical concepts and skills being developed as required.

The two perspectives described above represent opposite ends of a spectrum of views about mathematics and how children learn mathematics but there are many possible views in between. Ernest (1989) described a middle perspective as the Platonist view of mathematics. This view sees mathematics as a "static but unified body of certain knowledge"(p250) where content is consistent, connected and objective. The teacher's role is that of explainer as the textbook approach is modified, adapted and supplemented with problems and activities.

Problems and Problem Solving

Based on the perspective adopted, teachers form beliefs which directly influence their classroom practices including lesson planning and choice of tasks. When preparing to teach mathematics, a significant decision for the teacher is to choose appropriate problems. This can be challenging as questions which are problems for some students in the classroom may not be problems for others. In choosing a worthwhile mathematical task, Lappan (1993) suggests that the teacher must judge

> ...how well the task represents the embedded concepts and procedures, how likely the students are to bump into the mathematics in the course of investigating the problem, how well the task represents what is entailed in doing mathematics, and what skill development the task will or can support (p524).

She warns that problems should not be chosen just because they are fun or use manipulatives but rather that they allow students to engage in significant mathematics. Good tasks capture student curiosity and include opportunities for mathematical thinking as well as developing concepts and skills.

Teachers may have different beliefs about what constitutes a mathematical problem. Defining the terms 'problem solving' and 'problem' has caused considerable concern for mathematics educators and teachers. Teachers give many different definitions

(Funkhouser, 1993) which must lead to differences in classroom practices including variety in the kinds of mathematical tasks which are presented to students. When asked to define problem solving, Funkhouser found that only one-third of teachers were able to give a definition which included specific problem solving terms. The remainder gave very vague definitions. Hembree's (1992) examination of a large number of studies revealed definitions that range from traditional word problems to non standard problems.

Earlier definitions tended to consider questions which were asked in words as problems whereas more recent definitions require the presence of a blockage as a necessary prerequisite for a question to be considered a problem. These are clearly conflicting definitions. Whereas on one hand a problem can be a term for routine exercises, on the other hand it can refer to a question which is difficult or creates a perplexity for the solver. According to House, Wallace and Johnson (1983)

> ...a mathematical problem is a situation that involves a goal to be achieved, has obstacles to reaching that goal, and requires deliberation, since no known algorithm is available to solve it...Problem solving is the process of attacking such a problem...(p10).

In this study one aspect of the investigation is to try to discover what practising teachers believe to be a good problem and another is to ascertain their beliefs about the role of problem solving in learning mathematics.

The Survey

A teacher belief survey was designed to gather background information, favourite mathematical problems and comments about two scenarios or vignettes. The background information included gender, number of years teaching experience, grade currently being taught, role in the school, number of days inservice attended in the last two years as well as the number of these which were devoted to mathematics.

Each teacher was asked to record their two favourite problems and to briefly describe why these were chosen. This question was designed to be an unobtrusive measure of what teachers really think about problem solving. It was anticipated that the kinds of problems chosen would give some indication of each teacher's classification of what constitutes a problem and what characteristics of these chosen problems made them particularly useful.

The scenarios were structured so that two perspectives on the role of problem solving in learning mathematics were presented (Appendix 1). Each included a teacher who believed that problem solving could be used as the focus of learning mathematics and a teacher who believed that children need practice with basic skills with some problem solving when time permits. These two perspectives reflected the problem solving as a 'means' versus problem solving as an 'end' approaches to learning mathematics. Respondents were required to comment on each scenario giving reasons why they preferred a particular approach to teaching. It was anticipated that these responses would reveal espoused beliefs about mathematical problem solving and its role in the teaching and learning of mathematics.

Responses were sought from practising teachers, some of whom were part-time post-graduate students in teacher education courses. A total of 39 teachers were surveyed; 23 were chosen for analysis on the basis that the respondents were responsible for teaching a primary grade class and that they had recorded at least one favourite problem. The NUD*IST software program (QSR, 1994) was used to create an index system to manage and organise the data. Even though this was a small sample of teachers, the process of organising the data into categories highlighted emerging issues which will inform the next stage of the larger study.

Survey Results

Background information indicated that the sample of teachers represented a wide range of teaching experience and included at least one teacher for every grade in the primary school years. Analyses revealed that eleven teachers had been teaching for five years or less, another eleven teachers had been teaching from six to 15 years and one had been teaching for more than 15 years. Five were teaching in the K-2 grades, eight in grades 3 or 4 and ten in grades 5 or 6. There was a large range of inservice days attended with most teachers having attended one or two mathematics inservice days in the last two years. The ratio of females to males was approximately three to one.

Favourite Problems

The favourite problems which teachers recorded were examined to identify content and problem type. Responses ranged from very general statements, for example "money problems", to highly specific questions and were classified according to the main area of mathematical content or strand. A total of 39 problems were provided by the 23 respondents and of these 15 involved number, 12 space, 9 measurement, 2 chance and data and 1 was very general. An initial classification of problem type included the categories of 'real-life investigation', 'mathematical investigation', 'closed, content focused' and 'other' but this classification failed to provide discrete categories. An improved classification involved an open versus closed division. Each of these was then further subdivided into contextualised or decontextualised.

A total of 17 problems (44%) were classified as open as these either had several answers or many ways of approaching them. Seven problems (18%) were classified as open and contextualised since they were presented to the students in a realistic context. Examples of these were :

"Can a dinosaur fit into your classroom?"

"Redesign the classroom."

Ten problems (26%) were classified as open and decontextualised as they were removed from a real context. Examples included:

"Tell me a story from the graph."

"How many different combinations of coins can you find which total 50c."

"Find your stride length, then find objects which are the same, shorter and longer."

The remaining 22 problems (56%) were classified as closed as they usually had one correct answer. Ten problems (26%) were closed and contextualised. Examples included:

"There are 32 children in 4L, 31 in 4T and 30 in 4P, how many children are in year 4?"

"There are chickens and pigs on a farm, given the number of legs and the number of heads, how many of each?"

Twelve problems (30%) were closed and decontextualised. Examples included:

"What is the chance of throwing 3 heads with three coins?"

"Counting in twos via addition."

It was anticipated that teachers' favourite problems would reveal useful information about their definitions of a problem. The examples given did not provide this as problems need to be appropriate for the class or particular students which the teacher has in mind. One teacher recorded the problem:

"Kylie had 37 apples, she ate 12 apples, how many does she have left?"

For many students this would not be a problem but in the lower grades there would be students who would not have a readily available procedure for solving this problem. It is possible that this particular teacher may consider that a problem is a question stated in words. Other responses which did not reveal teacher's definitions of a problem were very vague, for example:

"Measurement - time".

From this information it is not possible to determine exactly what kinds of questions might be posed by this teacher.

It is worth noting that of the 16 survey responses not used in the analysis, seven of these were full-time classroom teachers who did not record a favourite problem. Two of

these teachers commented that they "use many" whereas another two stated that they "can't think of any". In a follow-up interview with a voluntary group of eight of those surveyed, a comment was made that it is difficult to think of a problem when you are not in class with the children. Another teacher said that she thought of a text-book question but she did not want to write that down because these are not the best examples of problems. Other comments suggested that problem solving is not planned but just arises out of what happens in the classroom. If this is the case, then it is not surprising that so many of those surveyed were unable to record at least one favourite question.

Recording a favourite problem does not reveal how the problem was posed in the classroom or the way that it was explored or discussed with the students. The problem has been removed from the context of the classroom. A better question would have been to ask how the problem was used in the classroom or to describe a recent problem solving lesson. In an effort to explore what the characteristics of a favourite problem might be, teachers were asked to record why these particular problems were chosen.

Responses to the request to describe why these particular problems were chosen were categorised into four groups - affective responses, problem characteristics, teaching factors and learning factors. The affective responses included statements about enthusing and motivating students and students enjoying the challenge. Desirable problem characteristics were the open-ended nature of the question, the real-world context of a question, that problems could be solved in the context of a game and that some questions could be adapted for use with many classes. Teaching factors included statements about children working in groups which promoted use of mathematical language, lots of activity with hands-on experiences, incorporation of several content strands in one task and possible links with other KLAs. Learning factors were estimation, remediation, visualisation, practise in basic skills, introduction of concepts, challenge and extension. Half of the teachers surveyed mentioned at least one learning factor which indicates that they are aware of the role that problem solving plays in the learning of mathematics. It is unclear whether the other teachers believe that problem solving does promote learning as they focused on the alternative categories.

Teachers' Beliefs about Problem Solving

Responses to the two vignettes were also categorised. The vignettes described possible scenarios with two teachers who had quite different approaches to teaching mathematics. Surveyed teachers were required to comment on the statements by making recommendations with explanations for their decisions. Statements were classified into four groups - class, mathematics, learning and other. The 'class' category was subdivided into ability, behaviour, background and other. 'Mathematics' was subdivided into skills, real-life and other. 'Learning' was subdivided into problem-solving views, non problem-solving views and mixed views for statements which indicated that the teacher was recommending a middle position or combination of the two previous views.

Many comments related to the class or individual students. Several related to the ability of students to cope with a problem solving approach as they may need considerable practice on basic skills. Comments were made about children with learning difficulties, ESL backgrounds and the increased need to cope with language when solving problems. There was concern that children need to be cooperative and that some children were uncooperative, antisocial and immature. One teacher suggested that previous experience with problem solving was necessary before a child would be comfortable with a problem solving approach. Several comments referred to a consideration that children learn in different ways and hence respond to different approaches, this was summarised in the comment:

"All children learn in different ways and I feel we should adopt a variety of approaches to cater for these."

One teacher noted that using a problem solving approach gives students the opportunity to construct their own knowledge.

Comments about mathematics focused on skills and real-life experiences. Most teachers believed that skills need to be developed before problem solving and seemed to support the problem solving as an 'end' approach. This was reflected in the comment:

"Basic skills have to have time to be taught - has to be a priority! Teach these first, then apply problem solving to them."

Seven teachers wrote statements which supported problem solving as a 'means'. This was reflected in the following:

"Because the children are learning skills through problem solving they have a need to use basic skills - a real need, rather than simply the completion of sums in a text."

Ten teachers wrote comments relating to the relevance and importance of children developing problem solving skills and that problem solving provided an opportunity to learn mathematics in a real-life context. Other comments about mathematics included opportunities for discovery learning, investigation from open-ended questions and integration with all areas of the curriculum.

Statements about learning were organised according to their support of a problem-solving approach, a non problem-solving approach or a middle position. Almost every teacher surveyed made a statement which supported the use of problems in the mathematics classroom. It was clear that teachers were aware that problems provide students with the opportunity to use valuable skills and that this was more interesting for them than practising algorithms from a textbook. Most teachers also recommended that skills must not be ignored and that extra practice may be needed. It became clear that isolating comments into these three categories did not really give an indication of each teacher's beliefs as many teachers provided comments which could have been placed in all three categories. There were only four cases where comments were placed in only one category and these teachers each wrote very little. The comments which teachers wrote in responding to these scenarios really needed to be read as a whole.

For some teachers, overall beliefs became clear when the responses to both scenarios were read together. By selecting individual ideas or statements it seemed that the teacher supported a problem solving approach and yet accompanying comments revealed reservations. One case is particularly noteworthy. In response to the first scenario this teacher wrote:

> "Ms Smith would probably be the teacher most favoured by myself...because the children are learning skills through problem solving."

For the second scenario she wrote:

> "Yes I would probably agree that you can teach skills through a problem solving approach in some schools and with some students. I honestly would not adopt such an approach to teach maths.
> (1) The problem solving arena is very overwhelming.
> (2) Usually involves too much language.
> (3) I am yet to find suitable problems for all sub-strands."

This teacher was prepared to be very honest. Even though she was aware of current thinking involving the role of problem solving in learning mathematics she was not prepared to teach that way. She confessed to feeling overwhelmed and she raised two important issues which caused her concern. It is possible that other teachers may have held similar beliefs but these were not revealed on the surveys. Interviews may provide a better opportunity to reveal espoused beliefs.

Another issue relates to what teachers believe to be a problem solving approach to teaching mathematics. Many respondents agreed with Bill in the second scenario and yet their comments suggested that their views of a problem solving approach were different to what was intended. One teacher responded as follows:

> "Agree with Bill - what does John do??? Short groups of problems or tasks can cover all mathematics areas - break them up into small manageable components."

This response seems to suggest sets of textbook word questions rather than using a problem as the focus of a lesson from which skills and understanding are developed. Another teacher cautioned that there may be different definitions of a problem solving approach. She stated:

> "It depends what's meant by a problem solving approach. My maths uses plenty of problem solving with open-ended problems. This is the way to go I believe."

Given that the term was meant to suggest that problem solving was used as a 'means' rather than an 'end' in teaching mathematics, responses from a majority of teachers seemed to support the problem solving as an 'end' approach. Eight teachers made direct statements indicating that they did not support the view that all mathematics could be taught through problem solving. Many teachers supported a middle position with their comments suggesting that you need to have children practising skills as well as doing problems in mathematics classrooms. This indicated that they interpreted the scenarios to mean that skills alone was one position, and problem solving alone was the other position.

Implications

The survey has revealed that most teachers support the use of problem solving as an end in teaching mathematics but it has also identified further issues for exploration. Teachers do have different beliefs about the role of problem solving in learning mathematics but in order to decipher exactly what these are requires clarification of what each teacher understands by the terms 'problem' and 'a problem solving approach to teaching'. It may be difficult to identify teachers' beliefs using only a survey although Thompson (1992) cautions that verbal responses in interviews can be equally unreliable because teachers may readily give a commitment to ideas about teaching which they know to be currently desirable.

The data about favourite problems highlighted the need to have teachers describe how the problem was used in the classroom. Teachers make decisions about appropriate tasks based on their beliefs, the class and individual students as well as the skills and concepts they are trying to develop. These were not revealed in the surveys and it would have been more useful if teachers had been asked to describe a recent problem solving lesson. Another strategy would be to present teachers with a set of problems and to have them rank these according to their usefulness with a particular grade as well as their value for promoting understanding of mathematical concepts.

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Appendix 1

Scenario 1

A close friend tells you about two Year 5 teachers at her child's school. Ms Jones is concerned that too many children finish primary school without a solid background in the basics of mathematics. She ensures that plenty of basic skills in the 4 operations are practised in her class with the occasional use of problem solving. Ms Smith, on the other hand, does not practise these skills as much but prefers her children to develop their skills by drawing on them as they need them when solving problems. Your friend wants to know which teacher she should request to teach her child. What advice would you give her and why?

Scenario 2

Two teachers are having a conversation in the staffroom. John is clearly frustrated by the number of things he has to fit into the curriculum and barely finds time to cover the essential basic skills in mathematics, let alone investigate problems. Bill responds by saying that you can teach all mathematics by using a problem solving approach. They ask you for your opinion - what do you think?