Using Data Maps to Analyse Students' Strategy Use in Problem Solving: A Visual Technique

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Qualitative data analysis has become increasingly important in mathematics education research. However the conceptualisation of the data and the fulfillment of the interactive assumption of qualitative analysis are a concern. An alternative to a linguistic approach to analysis is a visual approach. The visual analysis of data utilises wholistic data displays, referred to as data maps, and employs visual reasoning. The data maps, which are produced using drawing software, can enhance conceptualisation and facilitate the interactive process of analysis.

In recent years the use of qualitative data analysis in mathematics education research has expanded and provided useful insights into the teaching and learning of mathematics. However the use of text as the "unit" in analysis is recognised as labour intensive (Bogdan & Biklen, 1982), time-consuming and difficult (Pitman & Maxwell, 1992). An alternative to the use of a linguistic representations and sequential reasoning in data analysis is the use of visual representations and visual reasoning. Such a perspective does not seek to degrade the *status quo* but rather to recognise the limitations of tools and techniques and attempt to overcome them: "New perspectives provide new order to the things before us" (Webb & Glesne, 1992, p. 795). Therefore, the purpose of this paper is to explore whether there is any benefit in representing and analysing qualitative data visually. In order to investigate this question a pilot analysis of data is reported from a study of students' use of diagrams in problem solving. This paper specifically addresses 1) the rationale for the visual analysis of data, 2) the development of a visual tool, and 3) the technique of visual analysis.

The Rationale for the Visual Analysis of Data

Both qualitative and quantitative data analysis consist of three phases: data reduction, data display, and drawing inferences. However it is too simplistic to consider the differences between the outcomes of the analyses as a difference between the use of words and the use of numbers. The fundamental difference between these forms of analyses is in the *process* of data analysis and the *state of development* of the data analysis techniques (Miles & Huberman, 1994). Quantitative data analysis is essentially a sequential process which utilises statistical techniques that are reported as, for example, means, correlation tables and levels of significance. However the validity of the data analysis is dependent on an understanding of the assumptions of the techniques and their appropriate use. In contrast to the well-developed process of quantitative analysis, qualitative analysis is still in a "pioneering" state. Although guidance is provided for qualitative analysis, there appears to be no specific guidance on the visual analysis of data (e.g., LeCompte, Millroy & Preissle, 1992; Miles & Huberman, 1994). Quantitative analysis however utilises both well known and lesser known visual display techniques (Tufte, 1983; 1990), for example graphs and Chernoff faces respectively (Chernoff, 1973). Unlike quantitative analysis there are no shared conventions for the analysis or reporting of qualitative data, but there is agreement that the analysis should be an interactive and recursive process (Miles & Huberman, 1994). The lack of convention however is not to be interpreted as a need for convention. Qualitative studies are by their very nature diverse and accordingly require an analysis that suits the study. Thus, qualitative analysis is based on a criterion reference system rather than a norm reference system. Implicit in qualitative analysis is the obligation to provide criteria as a backdrop for an audit trail, which provides sufficient detail about the data and the analysis to enable the reader to evaluate the trustworthiness of the conclusions (Miles & Huberman, 1994).

In qualitative analysis the use of software, such as NUD.IST (1994), facilitates the exploration of patterns and relationships by positing "what if" scenarios. The utility of such software supports Pea's (1985) notion that technology is a powerful tool that extends thinking "beyond amplification" by reorganising mental functioning and thus engaging cognitive processes in qualitatively different ways. Special purpose software packages for qualitative data analyses differ, but text is normally used in the display, for example in a concept map (Richards & Richards, 1994). There appears to be no software in common use in data display that employs visual representations as a means of exploring "what if" scenarios (Miles & Weitzman, 1994), although inexpensive and easily managed general purpose drawing packages do exist (e.g., SmartDraw Pro, 1995). As cognition is affected by the use of visual representations (Barwise & Etchemendy, 1991) and visual reasoning is implicated in problem solving (National Council of Teachers of Mathematics, 1989) the visual analysis of data appears to have potential.

Visualisation and the use of diagrams are recommended in mathematics education (National Council of Teachers of Mathematics, 1989) and are credited as the source of insight for many eminent people, such as Einstein (Boden, 1990). A diagram is an external manifestation of visualisation (Presmeg, 1986) and is defined as an abstract visual representation that exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented wholistically (Winn, 1987). Diagram use in problem solving has particular advantages related to working memory (van Essen & Hamaker, 1990), the conceptualisation of the problem (Yancey, 1981), and the informational content of the representation (Larkin & Simon, 1987). A further advantage of diagrams is that they provide a visual alternative to words (Mayer & Gallini, 1990). The utility of visual-spatial methods is recognised both by expert mathematicians (Eisenberg & Dreyfus, 1986) and in cross-cultural studies (Shigematsu & Sowder, 1994). Changing the mode of a representation, for example from a linguistic to a visual form, may also be advantageous as a means of knowledge generation (Karmiloff-Smith, 1990). However whereas linguistic representations utilise sequential reasoning, visual representations utilise visual reasoning (Barwise & Etchemendy, 1991). The key difference between sequential and visual reasoning is the respective fragmentary and wholistic use of the representations.

The Development of a Visual Tool: The Data Map

Communication systems are not developed arbitrarily but are dictated by the tools available and the mode of communication. A diagram utilises graphic elements, including points, lines, shapes, shading, direction and location. Symbols that are visually associated with actions can also be incorporated to facilitate communication, for example, a hexagon can be used to signal a "stop" in a procedure. In a diagram, all elements of the display are available simultaneously and thus interpretation requires visual reasoning. In decoding diagrams, attention can oscillate between the whole diagram and sections of the diagram. Diagrams that serve to organise and display the data, and orient the decoder are henceforth referred to as data maps. These maps are not to be considered as absolute depictions of data but are relative to the question they seek to explore. Hence a variety of maps could be produced from the same data.

A Pilot Analysis of Qualitative Data Using the Data Map: A Visual Technique

Background to the study

The subjects of the study were 12 Year 5 students who were participants in a teaching experiment within the context of a case study. All subjects were interviewed individually twice and in each interview were presented with five novel problems. The problems in the two interviews were isomorphic. The first interview was conducted before the commencement of an instructional program of 12 half hour lessons spread over a four week period which was designed to facilitate the use of the strategy *draw a diagram* in novel problem solving. The second interview was conducted at the conclusion of the instructional program. The interview sessions were video-taped and subsequently transcribed. The interviews were of approximately thirty minutes duration and consisted of two phases. In Phase A of the interview the subjects attempted each of the tasks independently. During Phase B the subjects were asked to explain their solution strategies. The interviewer also probed for meaning and if warranted, provided minimal support in order to determine whether scaffolding (Vygotsky, 1978) facilitated the solution of tasks which were unable to be solved independently. The interviewer was known to the subjects through prior classroom involvement.

The evaluation of a teaching experiment requires the detection of changes that have presumably occurred as a result of instruction. Therefore, the change in the selection and success of problem solving strategies used by the subjects is of particular interest in the analysis. Although teaching experiments may employ pre- and post-tests as end product measures, a comparison of the strategies used in the interviews was considered as a more appropriate measure of change when evaluating diagram use (Diezmann, 1995).

The data maps

Two data maps are presented to exemplify the visual analysis of data from the study. The graphic code, which was developed to represent the components of the subjects' behaviour during the interviews, is shown in Figure 1. The first map displays the strategies used by a single subject across tasks and time, see Figure 2. The tasks and interviews are identified numerically at the top of the map. For example, 1/2 means Task 1 in the second interview, whereas 2/1 means Task 2 in the first interview. Although time sequence is represented by the vertical chain of strategy use, the duration of time has not been included as a component because the subjects' solution times were affected by the interviewer probing responses. The line lengths between shapes, which represent behaviours, were determined by the display and accordingly are not correlated with time taken. This particular subject was selected because of the gross differences in her strategy use between the first and second interviews. The second map displays the performance of four students, from the group of twelve subjects, who used a restricted number of strategies on a pair of isomorphic tasks, see Figure 3. These two maps are intended to be illustrative rather than exhaustive of the possible data maps which can support various mathematical tactics for generating meaning, including the discernment of patterns and relationships, counting and noting relationships between variables (Miles & Huberman, 1994). The interpretation of the data maps necessitates the dual consideration of visual and linguistic cues. The analysis that follows is by necessity brief, but provides an example of the potential use of data maps in visual analysis.

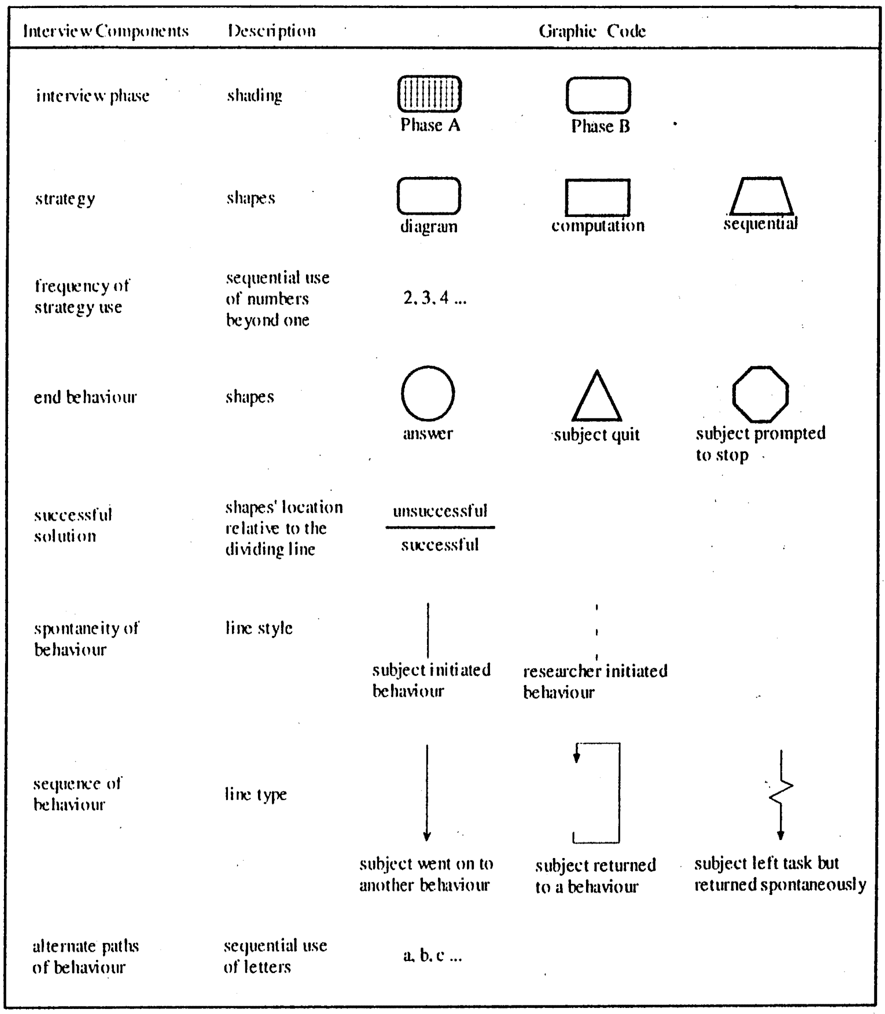

Figure 1. The graphic code used in the data maps

An example of the data analysis

Figure 2 was designed to display the strategies used by a single subject during the two interviews on the five pairs of isomorphic tasks. Performance can be compared between the phases of the interviews and between the two interviews. The analysis focuses on three areas. Firstly, what patterns and relationships can be identified from the data map? Secondly, what questions are suggested by the analysis of the data map? Thirdly, what were the critical events during the interviews?

At least four themes are evident from Figure 2. Firstly, Karen's *spontaneous and appropriate choice of the strategy, draw a diagram,* in 1/1, 3/1 and 4/2, does not mean that she can implement the strategy successfully because in each task her solutions were unsuccessful. Secondly, her *repeated use of a strategy* in 1/1, 2/1, 3/2 and 5/1 does not culminate in success, either with or without interviewer intervention, suggesting that either her choice of or implementation of the strategies was inappropriate. Thirdly, the *instructional program* seems to have had a positive effect on Karen's problem solving. After the instruction she was able to solve two problems, 1/2 and 2/2, which were isomorphs of 1/1 and 2/1 respectively, problems on which she was previously unsuccessful. However her lack of success on 4/2 needs explanation as she was successful on an isomorphic task, 4/1. A comparison of her strategy selection in 4/1 and 4/2 reveals that her initial strategy selections were different in these tasks. Thus, her success may be strategy dependent. Fourthly, in four of the ten tasks Karen "*quit*": 2/1, 2/2, 4/2 and 5/1. Although she spontaneously restarted in 2/1, 2/2 and 4/2, and was in fact successful in 2/2. Hence, "quitting" for Karen may be a period of incubation rather than necessarily the end point of an unsuccessful solution process.

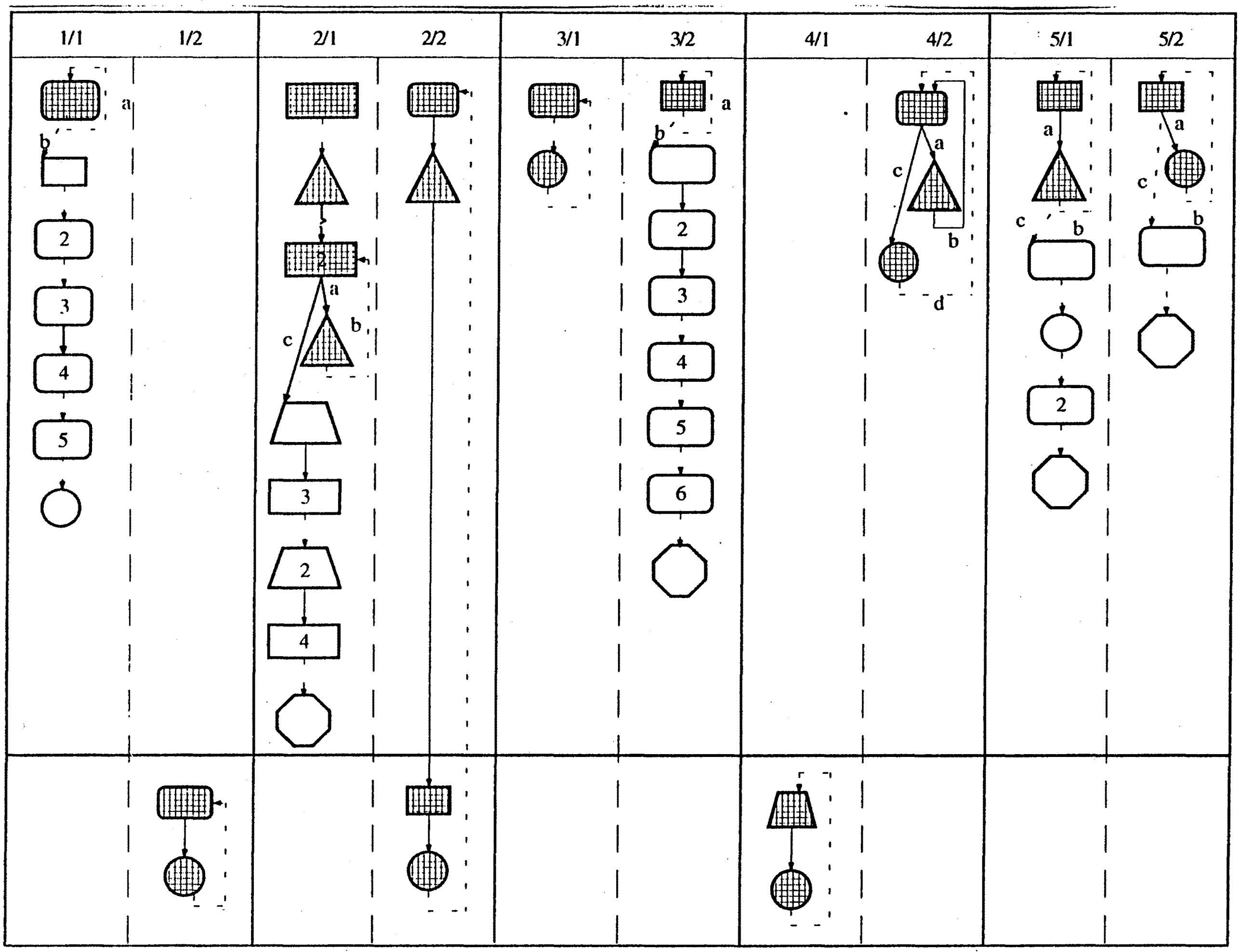

Figure 2. A comparison of a Karen's performance on five pairs of isomorphic tasks

The data map suggests further questions to explore. For example, was her unsuccessful use of diagrams after instruction in Task 4 an isolated event? No, the map reveals that she was also unsuccessful after instruction in Tasks 3 and 5. Did she ever use the strategy successfully after the instruction? Yes, the map shows in Task 1. Hence, Karen's erratic performance using diagrams needs explanation and requires a return to a more detailed data source, such as the interview video-tape or associated transcript. However there is an alternative that can be explored visually, that is Karen's performance relative to her peers on the same task, as perhaps the task was simply a difficult task for students of this age. Errors in diagram use on this task include both generating and using the diagrams (Diezmann, 1995).

The specific details of Karen's difficulties with some of these tasks is not evident from the map but what is evident is that during this interview there were some critical events which warrant further exploration. What made Karen quit? What made Karen restart after quitting? Why did the interviewer stop Karen from proceeding? The interview video-tape or transcript should provide some understanding of these events.

In order to follow up Karen's performance on a task, a second data map was prepared to show her performance relative to her peers on the first tasks in each of the interviews (See Figure 3). Figure 3 is interpreted similarly to Figure 2 and displays the performance of four students, including Karen, on the first task in both interviews.

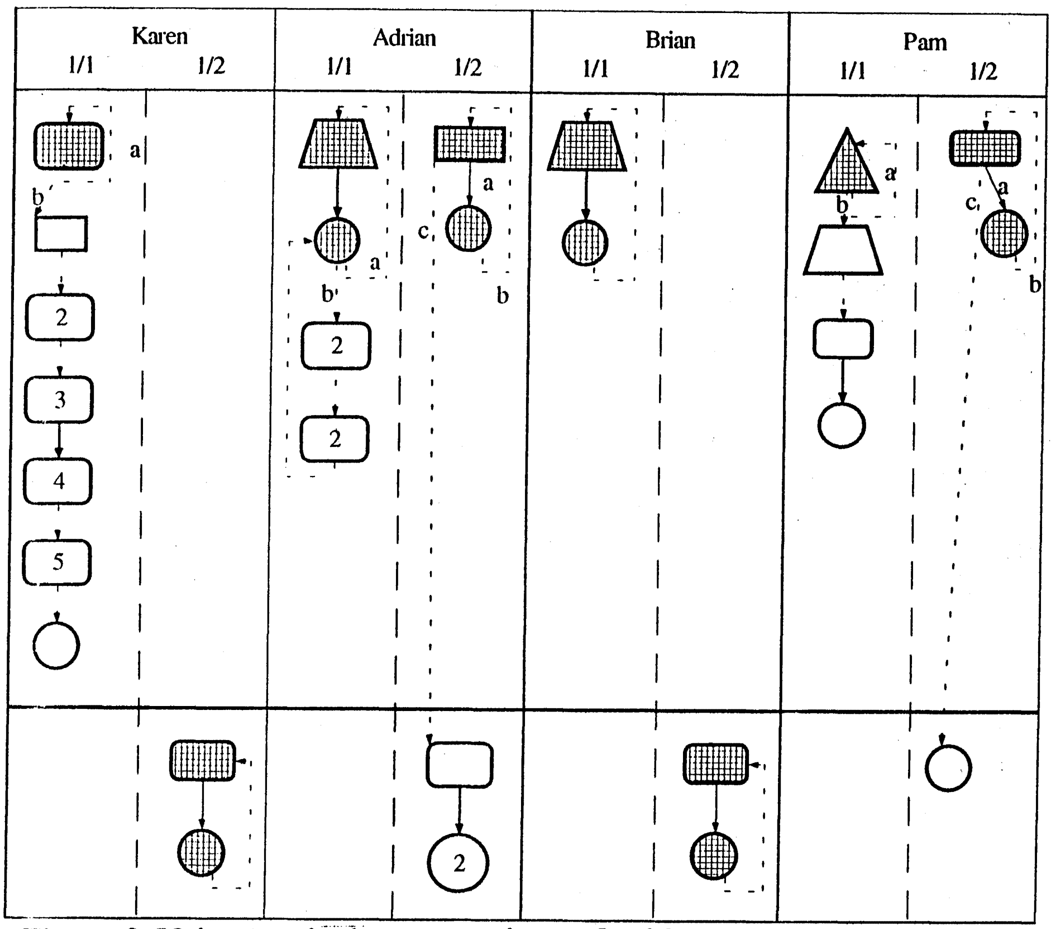

Figure 3. Using vertical cross sections of subjects' maps to compare the whole groups' performance on a pair of isomorphic tasks

A comparison of Karen's performance with three other students indicates that prior to instruction no students were able to complete the task successfully. Karen however was the only student to spontaneously use the strategy *draw a diagram* prior to instruction. All students however were able to use the strategy *draw a diagram* successfully on this task after instruction, two independently and two with interviewer intervention.

In summary, although this visual analysis was only illustrative of what can be done some understanding of the subjects' strategy use in novel problem solving was revealed. The analysis was a two way process in that specific questions were explored through the maps and the maps themselves revealed patterns and relationships. The process was interactive as the interpretation of the maps suggested follow up questions to explore. For example, the comparison between subjects' performances on Figure 3 resulted from a query in Figure 2 regarding the task difficulty. Additional questions could be investigated rapidly by generating further maps using the cut and paste facility of Smartdraw Pro (1995). For example, which strategies resulted in a successful solution? Thus, the software provides a tool to generate the data map and the data map in turn becomes the tool for analysis.

The facility of data maps to provide a wholistic account of the data is particularly useful. When a data map is considered together with the linguistic data, for example a video interview or a transcript, the map can act as an overlay "pinpointing" critical periods within the interview that can be followed up.

Conclusion

The question under discussion has been-is there any advantage in analysing data visually? In the spirit of qualitative research this question is rhetorical and can only be answered personally, although the process can be evaluated by the reader, given the criteria of the study and an adequate audit trail. This study has however served its purpose with regard to the development of a visual tool for data display and a visual analysis technique. The investigation has, to my mind at least, advanced the issue of whether visual analysis of qualitative data is plausible to-when is visual analysis useful or even preferable?

Pragmatically, the visual analysis of data is dependent on well organised and information rich data maps. Although the data maps can be created by hand and for convenience may be if coding directly onto a transcript, the software enables the rapid generation of further data maps to investigate specific questions thereby fulfilling the interactive assumption of qualitative analysis. Data maps and the technology appear closely entwined as demands on the technology or development within the technology may impact upon the data maps which in turn may affect the visual analysis.

The creation of data maps and the associated technique of visual analysis were borne out of need and are still in their infancy. However with further development and more understanding of their utility, the visual display and analysis of data appears to have enormous potential in qualitative studies.

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